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The Non-Abelian Tensor Square Graph Associated to a Symmetric Group and its Perfect Code

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Abstract A set of vertices and edges forms a graph. A graph can be associated with groups using the groups' properties for its vertices and edges. The set of vertices of the graph comprises the elements of the group, while the set of edges of the graph is the properties and requirements for the graph. A non-abelian tensor square graph of a group is defined when its vertex set represents the non-tensor centre elements' set of *G*. Then, two distinguished vertices are connected by an edge if and only if the non-abelian tensor square of these two elements is not equal to the identity of the non-abelian tensor square. This study investigates the non-abelian tensor square graph for the symmetric group of order six. In addition, some properties of this group's non-abelian tensor square graph are computed, including the diameter, the dominating number, and the chromatic number. The perfect code for the non-abelian tensor square graph for a symmetric group of order six is also found in this paper.

Keywords Graph Theory, Non-Abelian Tensor Square, Chromatic Number, Diameter

1. Introduction

This section is divided into three parts. The first part is about the non-abelian tensor square, followed by properties of graphs and lastly, the perfect code. This paper is constructed in the following manners: The first section is the Introduction section which includes a literature review related to the non-abelian tensor square, properties of graphs and perfect code. The second section shares some basic concepts and definitions needed, followed by the Main Results of the research. The last section is the conclusion, followed by the list of references.

1.1. Non-abelian Tensor Square

The non-abelian tensor square of a group G, $G \otimes G$, is produced by $g \otimes h$, where $g, h \in G$, with respect to the following relations:

$$gg' \otimes h = ({}^{g}g' \otimes {}^{g}h)(g \otimes h) \quad \text{and}$$
$$g \otimes hh' = (g \otimes h)({}^{h}g \otimes {}^{h}h'),$$

for all $g, g', h, h' \in G$, with ${}^{g}g' = gg'g^{-1}$. Various studies have been conducted on the non-abelian tensor square of a group. For example, in 2008, Ramachandran *et al.* [1] determined the non-abelian tensor square of the symmetric group of order six, S_3. They constructed the Cayley table of $S_3 \otimes S_3$, where the elements are $(1) \otimes (1)$, $(23) \otimes (23)$, $(23) \otimes (12)$, $(23) \otimes (13)$, $(123) \otimes (23)$, and $(132) \otimes (23)$. The Cayley table of $S_3 \otimes S_3$ is given in Table 1. Based on Table 1, the centre of the non-abelian tensor square of S_3 is $(1) \otimes (1)$.

Table 1. The Cayley table of $S_3 \otimes S_3$

$\bullet$	$(1)\otimes(1)$	$(123)\otimes(23)$	$(132)\otimes(23)$	$(23)\otimes(23)$	$(23)\otimes(13)$	$(23)\otimes(12)$
$(1)\otimes(1)$	$(1)\otimes(1)$	$(123)\otimes(23)$	$(132)\otimes(23)$	$(23)\otimes(23)$	$(23)\otimes(13)$	$(23)\otimes(12)$
$(123)\otimes(23)$	$(123)\otimes(23)$	$(132)\otimes(23)$	$(1)\otimes(1)$	$(23)\otimes(12)$	$(23)\otimes(23)$	$(23)\otimes(13)$
$(132)\otimes(23)$	$(132)\otimes(23)$	$(1)\otimes(1)$	$(123)\otimes(23)$	$(23)\otimes(13)$	$(23)\otimes(12)$	$(23)\otimes(23)$
$(23)\otimes(23)$	$(23)\otimes(23)$	$(23)\otimes(12)$	$(23)\otimes(13)$	$(1)\otimes(1)$	$(123)\otimes(23)$	$(123)\otimes(23)$
$(23)\otimes(13)$	$(23)\otimes(13)$	$(23)\otimes(23)$	$(23)\otimes(12)$	$(123)\otimes(23)$	$(123)\otimes(23)$	$(1)\otimes(1)$
$(23)\otimes(12)$	$(23)\otimes(12)$	$(23)\otimes(13)$	$(23)\otimes(23)$	$(123)\otimes(23)$	$(1)\otimes(1)$	$(132)\otimes(23)$

Besides that, Ghorbanzadeh *et al.* [2] computed the non-abelian tensor square for *p*-groups focusing on those of order p^4. They found that the tensor centre contains only the group's identity for $p \geq 2$. Next, in 2013, Zainal *et al.* [3] found the non-abelian tensor square of groups with order p^4, where *p* resembles an odd prime. Finally, in 2013, Jafari *et al.* [4] characterised finite *p*-groups using its non-abelian tensor square.

Most of the researches focused on the generalisation and characteristics of the non-abelian tensor square of its group. However, until today, there has been no evidence of researchers finding the algebraic structure of non-abelian tensor square.

1.2. Properties of Graph

A graph contains a set of vertices and edges. Many types of research have been conducted on graphs related to groups, including Cayley, cyclic, power, and order product prime graphs. For example, in 2013, Ma *et al.* [5] investigated the finite group's cyclic graph. The vertices of the finite group *G*'s cyclic graph are the elements in *G*. Suppose *x*, *y* are the vertices of this graph. Then, $\langle x, y \rangle$ resemble a cyclic subgroup of *G* if and only if the vertices *x* and *y* are adjacent. They found that if G_1 is isomorphic to G_2, then the cyclic graphs for both groups are also isomorphic. Moreover, in 2015, Pourghali *et al.* [6] studied the finite group's undirected power graph. This graph contains a vertex set, where two vertices are adjacent if one of the vertices is the other vertices' power. They found that the power graph of a *p*-group *G* is 2-connected if and only if *G* resembles a generalised quaternion group or cyclic.

In 2020, Bello *et al.* [7] defined a new type of graph named the order product prime graph. If $|x||y| = p^{\alpha}$ for $\alpha \in \mathbb{N}$ and *p* prime, then two vertices *x* and *y* are adjacent, and vice versa. In addition, they found that the order of product prime graph of a dihedral group is regular and complete if and only if the degree of the group is 2^{α}. Meanwhile, if the degree of the dihedral group is p^{α} for all *p,* then the order product prime graph of a dihedral group is connected.

All graphs have their own properties. These properties of graphs can give many benefits in real-life applications. Some of the graph properties are identified first before applying them to solve real-life applications**.**

In 2006, Miklavič and Potočnik [8] explored the properties of the Cayley graph. The distance-regular Cayley graphs on dihedral groups have been classified in this paper. It turned out that the graph is bipartite and has a diameter of 3. Moreover, in 2018, Khormali [9] explored the local distinguishing chromatic number. They found that the complete graph's local distinguished chromatic number of *n* vertices is indeed *n*. In 2019, Cardoso *et al.* [10] also studied the properties of the graph. In the paper, the authors focused on the coloring of edges for graphs. They only investigated the simple graph with two minimum vertices and one minimum edge. The injective edge coloring of graphs denotes the minimum number of colours required to colour three consecutive edges. By considering a graph Γ with *m* edges and *n* vertices, they also found that the injective edge coloring of Γ equals *m* if and only if Γ it is a complete graph.

Then, Pham [11] explored the (*d,s*)-edge coloured graph *G*. If a *d*-regular graph *G* follows a proper *d*-edge coloring where every edge of *G* is contained in at least (*s*-1) 2-colored 4-cycles, then the graph is called as (*d,s*)-edge colourable. They found that $G = G_1 \times G_2$ of a graph G_1 and G_2 resemble a (*d,s*)-colourable graph having certain conditions. Also, Coll *et al.* [12] explored the proper diameter of a graph. They found that the proper diameter equals 2*m*-1, where *m* is the diameter for any properly connected 2-colouring of a cycle graph. Finally, Cameron and Jafari [13] studied the independence number and connectivity of groups' power graphs. They found that the clique cover number and independence number of the power group of a group *G* that has a finite independence number is equal.

The studies on graphs for non-abelian tensor squares can lead to many real-life applications. One of the applications is in-network. In the review paper by Sadavare and Kulkarni in 2012 [14], the graph theory is used in creating the shortest path algorithm to increase the efficiency of the system. Another application of graph theory is in computer science. Apart from that, in 2014, Singh [15] reviewed applications of graph theory in computer science and engineering. The concepts in graph theory, including graph coloring and directed graphs, can be used in the operation system. Then, in 2018, Chakraborty [16] explored the application of graph theory in social media. The relations of users in social media can be observed using the concept in graph theory. Next, graph theory also can be used in chemistry. In 2016, Prathik *et al.* [17] reviewed a paper on the application of graph theory in chemistry. The molecule structure can be studied in detail by transforming it into a graph.

Some of the properties of the graph are also related to perfect code. More literature reviews on perfect code are written in the following subsection.

1.3. Perfect Code

A subset C of $V(\Gamma)$ denotes a perfect code in the graph Γ if and only if C resembles an independent set of Γ, where every vertex in $V(G \setminus C)$ is adjacent to exactly one vertex in C. The perfect code can be found using some of its properties. The researches on perfect code have been done since the 1970s.

For example, Biggs in 1974 [18] studied non-trivial perfect codes comprising distance-transitive graphs. A perfect e-code in a graph denotes a subset C of the graph's vertices, where $V(\Gamma)$ forms a partition provided that $\partial(u,v) \le e$ for $v \in V(\Gamma)$, $u \in C$ and e is the distance. They found that if the graph is distance-transitive, then the polynomial of distance must divide the polynomial of diameter. Moreover, in 1986, Kratochvíl [19] explored the generalisation of perfect codes over graphs. They found that there are no non-trivial 1-perfect codes over complete bipartite graphs with at least three vertices.

Perfect code is also known as a perfect domination set. In 2001, Lee [20] investigated on perfect domination set of the Cayley graph for an abelian group. Let S_i and S_j be independent domination sets of a group G, where both are pairwise mutually disjoint. They found that the order of S_i and that of S_j are the same. In 2020, Chen *et al.* [21] studied the characterisation of subgroup perfect code in Cayley graphs. They found that group G has no non-trivial proper subgroup as a perfect code for a group of composite order.

1.4. The Conclusion of the Introduction

In this paper, a new graph called the non-abelian tensor square graph has been introduced. This graph has been found for the symmetric group of order six. Moreover, this study elaborates some of the graph properties, including the diameter, the dominating number and the chromatic number that have been computed. Lastly, the perfect code for this graph has been determined.

2. Preliminaries

Some definitions used throughout this study are stated in the following.

Definition 1.0 [22] Symmetric group

Let A be the finite set of n letters. The symmetric group on n letters, as labelled by S_n has $n!$ elements, which are $n! = n\cdot(n-1)\cdot(n-2)\cdots 3\cdot 2\cdot 1$. S_n consists of the group of all permutations of A.

Definition 2.0 [23] The tensor centre of G

The tensor centre of a group G, $Z^{\otimes}(G)$, is defined as follows:

$$Z^{\otimes}(G) = \{g \in G : g \otimes x = 1_{\otimes}, \forall x \in G\},$$

where $1_{\otimes}$ is the identity of the non-abelian tensor square of G.

Definition 3.0 [24] Diameter

The maximum distance of adjacent vertices of a graph Γ is the diameter of Γ. Therefore, the diameter of a graph is denoted as $diam(\Gamma)$.

Definition 4.0 [25] Chromatic number

The least number of colours used for the colouring of two distinct vertices in such a way that no adjacent vertices have the same colours is denoted as the chromatic number $\chi(\Gamma)$.

Definition 5.0 [26] Dominating number

A set S of vertices for the graph Γ is a dominating set of Γ provided that all vertices in $V(\Gamma) - S$ are adjacent to several vertices in S. A dominating number of Γ, denoted as $\gamma(\Gamma)$ resembles the cardinality of a minimum dominating set.

Definition 6.0 [27] Complete graph

A complete graph on n vertices K_n resembles a graph where every two distinct vertices are adjacent.

The following section shows the construction and finding of properties and perfect code for non-abelian tensor square for the symmetric group.

3. Results and Discussion

The main results throughout this research are presented in this section, starting with a newly introduced notion of the non-abelian tensor square graph of a group.

Definition 6.0 Non-abelian tensor square graph

Let G be a group and $Z^{\otimes}(G)$ denotes the tensor center of G. The non-abelian tensor square graph of a group G, denoted as Γ_G^{nts}, resembles the graph where the set of vertices, $V(\Gamma_G^{nts})$ is $G \setminus Z^{\otimes}(G)$, where any two distinct vertices, g and h, are connected if and only if $g \otimes h \ne 1_{\otimes}$ for all $g, h \in G$.

The tensor center of S_3 is stated in the lemma below.

Lemma 1.0 Let $S_3 = \{(1), (12), (23), (13), (123), (132)\}$. Then, the tensor center of S_3, $Z^{\otimes}(S_3)$ is $\{(1)\}$.

Proof Let $g = (1)$ and x denote any elements in G. Then, $(1) \otimes x = 1_{\otimes}$ for all x in G as in the following:

$$(1)\otimes(1) = (1)\otimes(1) = 1_{\otimes}, \quad (1)\otimes(23) = (1)\otimes(1) = 1_{\otimes},$$
$$(1)\otimes(12) = (1)\otimes(1) = 1_{\otimes}, \quad (1)\otimes(123) = (1)\otimes(1) = 1_{\otimes},$$
$$(1)\otimes(13) = (1)\otimes(1) = 1_{\otimes}, \quad (1)\otimes(132) = (1)\otimes(1) = 1_{\otimes}.$$

Using the relation $gg' \otimes h = ({}^{g}g' \otimes {}^{g}h)(g \otimes h)$, choose $(123)\otimes(123)$ and let $g = (123)$, $g' = (1)$, $h = (123)$. Then,

$$(123)(1)\otimes(123) = ((1)\otimes(123))((123)\otimes(123)),$$
$$(123)\otimes(123) = ((1)\otimes(123))((123)\otimes(123)).$$

Therefore, $(1)\otimes(123) = (1)\otimes(1) = 1_{\otimes}$.

The other elements are not in the tensor center for S_3. Choose $g \in G \setminus \{(1)\}$ and x be any element in G. Then, some of the relations are not equal to $1_{\otimes}$ as in the following:

$$(12)\otimes(23) \neq (1)\otimes(1),$$
$$(13)\otimes(23) \neq (1)\otimes(1),$$
$$(123)\otimes(23) \neq (1)\otimes(1),$$
$$(132)\otimes(23) \neq (1)\otimes(1),$$
$$(23)\otimes(13) \neq (1)\otimes(1).$$

Therefore, based on Definition 2.0, the tensor center of S_3 $Z^{\otimes}(S_3)$ is $\{(1)\}$.

Next, the non-abelian tensor square graph of S_3 is constructed using the definition stated in the following theorem.

Theorem 1.0 The non-abelian tensor square graph for S_3 denotes a complete graph with five vertices K_5.

Proof Based on Lemma 1.0, the tensor centre of S_3 is $\{(1)\}$. Then, $\{(1)\}$ is not included in the set of vertices for the non-abelian tensor square graph of S_3. Hence, the set of vertices for the non-abelian tensor square of S_3 is given as below:

$$V(\Gamma_{S_3}^{nts}) = \{(12), (23), (13), (123), (132)\}.$$

The edges of the graph are formed if any two vertices satisfy the condition given in Definition 6.0. Using the relation, $gg' \otimes h = ({}^{g}g' \otimes {}^{g}h)(g \otimes h)$, the calculations are shown below.

First, choose $(132)\otimes(23)$ and let $g = (12)$, $g' = (13)$, $h = (23)$. Then,

$$(12)(13)\otimes(23) = \big((12)(13)(12)\otimes(12)(23)(12)\big)\big((12)\otimes(23)\big),$$
$$(132)\otimes(23) = \big((23)\otimes(13)\big)\big((12)\otimes(23)\big).$$

Based on Table 1, $(132)\otimes(23)$ and $(23)\otimes(13)$ are not equal to $(1)\otimes(1)$. Hence, $(12)\otimes(23)$ it is also not equal to $(1)\otimes(1)$. It is because if $(12)\otimes(23) = (1)\otimes(1)$, then

$$(132)\otimes(23) = \big((23)\otimes(13)\big)\big((1)\otimes(1)\big),$$
$$(132)\otimes(23) = (23)\otimes(13).$$

This is a contradiction since $(132)\otimes(23) \neq (23)\otimes(13)$. Based on Definition 6.0, the vertices (12) and (23) are adjacent in $\Gamma_{S_3}^{nts}$.

Second, we choose $(23)\otimes(23)$ and let $g = (132)$, $g' = (12)$, $h = (23)$. Then,

$$(132)(12)\otimes(23) = \big((132)(12)(123)\otimes(132)(23)(122)\big)\big((132)\otimes(23)\big),$$
$$(23)\otimes(23) = \big((13)\otimes(12)\big)\big((132)\otimes(23)\big).$$

Based on Table 1, $(23)\otimes(23)$ and $(132)\otimes(23)$ are not equal to $(1)\otimes(1)$. Hence, $(13)\otimes(12)$ it is also not equal to $(1)\otimes(1)$ because if $(13)\otimes(12) = (1)\otimes(1)$, then

$$(23)\otimes(23) = \big((1)\otimes(1)\big)\big((132)\otimes(23)\big),$$
$$(23)\otimes(23) = (132)\otimes(23).$$

This is again a contradiction since

$$(23)\otimes(23) \neq (132)\otimes(23).$$

Based on Definition 6.0, the vertices (13) and (12) are adjacent in $\Gamma_{S_3}^{nts}$.

Third, choose $(123)\otimes(13)$ and let $g = (13)$, $g' = (12)$, $h = (23)$. Then,

$$(13)(12)\otimes(23) = \big((13)(12)(13)\otimes(13)(23)(13)\big)\big((13)\otimes(23)\big),$$
$$(123)\otimes(23) = \big((23)\otimes(23)\big)\big((13)\otimes(23)\big).$$

Based on Table 1, $(123)\otimes(23)$ and $(23)\otimes(23)$ are not equal to $(1)\otimes(1)$. Hence, $(13)\otimes(23)$ is also not equal to $(1)\otimes(1)$. It is because if $(13)\otimes(23) = (1)\otimes(1)$, then

$$(123)\otimes(23) = \big((23)\otimes(23)\big)\big((1)\otimes(1)\big),$$
$$(123)\otimes(23) = (23)\otimes(23).$$

This is a contradiction since $(123)\otimes(23) \neq (23)\otimes(23)$. Based on Definition 6.0, the vertices (13) and (23) are adjacent in $\Gamma_{S_3}^{nts}$.

Fourth, choose $(13)\otimes(23)$ and let $g = (123)$, $g' = (12)$, $h = (23)$. Then,

$$(123)(12)\otimes(23)=\big((123)(12)(132)\otimes(123)(23)(132)\big)\big((123)\otimes(23)\big),$$
$$(13)\otimes(23)=\big((23)\otimes(13)\big)\big((132)\otimes(23)\big).$$

Based on Table 1, the inverse of $(23)\otimes(13)$ is $(23)\otimes(12)$. Hence, the product of $(23)\otimes(13)$ and $(132)\otimes(23)$ is not equal to $(1)\otimes(1)$. Therefore, $(13)\otimes(23)$ is not equal to $(1)\otimes(1)$. Based on Definition 6.0, the vertices (13) and (23) are adjacent in $\Gamma_{S_3}^{nts}$.

Fifth, choose $(12)\otimes(23)$ and let $g=(132)$, $g'=(13)$, $h=(23)$. Then,

$$(132)(13)\otimes(23)=\big((132)(13)(123)\otimes(132)(23)(123)\big)\big((132)\otimes(23)\big),$$
$$(12)\otimes(23)=\big((23)\otimes(12)\big)\big((132)\otimes(23)\big).$$

Based on Table 1, the inverse of $(23)\otimes(12)$ is $(23)\otimes(13)$. Hence, the product of $(23)\otimes(12)$ and $(132)\otimes(23)$ is not equal to $(1)\otimes(1)$. Therefore, $(12)\otimes(23)$ is not equal to $(1)\otimes(1)$. Based on Definition 6.0, the vertices (12) and (23) are adjacent in $\Gamma_{S_3}^{nts}$. Using the same calculation for all vertices, it is found that $g\otimes h\neq(1)\otimes(1)$ for all $g,h\in V(\Gamma_{S_3}^{nts})$.

Therefore, all vertices in $\Gamma_{S_3}^{nts}$ are adjacent to each other and form a complete graph with five vertices $\Gamma_{S_3}^{nts}=K_5$.

Three properties, namely the diameter, the chromatic number and the dominating number, can be determined from this graph. Based on Theorem 1, the non-abelian tensor square graph for S_3 is the complete graph with five vertices. Hence, every two distinct vertices are adjacent to each other. Thus, the maximum distance between two distinct vertices is 1. Based on Definition 3.0, the diameter for the non-abelian tensor square graph of S_3 is 1, as shown in Proposition 1.0.

Proposition 1.0 The diameter for the non-abelian tensor square graph of S_3 is 1.

Based on the non-abelian tensor graph of S_3, all the vertices are adjacent to each other. Thus, the minimum colour needed to colour all the vertices so that every adjacent vertex has a different colour is 5. Therefore, based on Definition 4.0, the chromatic number for the non-abelian tensor square graph of S_3 is 5, as shown in Proposition 2.0.

Proposition 2.0 The chromatic number for the non-abelian tensor square graph of S_3 is 5.

Since the non-abelian tensor graph of S_3 is a complete graph, then the minimum number of vertices in dominating set of $\Gamma_{S_3}^{nts}$ is 1. Therefore, the dominating number of $\Gamma_{S_3}^{nts}$ is 1, as shown in Proposition 3.0.

Proposition 3.0 The dominating number for the non-abelian tensor square graph of S_3 is 1.

Proposition 4.0 Let $\Gamma_{S_3}^{nts}$ denote a non-abelian tensor square graph of S_3. Then, the perfect codes of $\Gamma_{S_3}^{nts}$ are

$$\{(12)\},\ \{(13)\},\ \{(23)\},\ \{(123)\},\ \{(132)\}.$$

Proof Since $\Gamma_{S_3}^{nts}$ is a complete graph with five vertices by Theorem 1. Hence, there are five independent sets, $C_i, i=1,2,3,4,5$ that can be obtained from $V(\Gamma_{S_3}^{nts})$ in a way that every vertex is adjacent exactly to one vertex in C_i as written in the following form:

$$C_1=\{(12)\},$$
$$C_2=\{(13)\},$$
$$C_3=\{(23)\},$$
$$C_4=\{(123)\},$$
$$C_5=\{(132)\}.$$

Therefore, the perfect codes are {(12)}, {(13)}, {(23)}, {(123)} and {(132)}.

4. Conclusions

The non-abelian tensor square graph for the symmetric group of order six, S_3 is found to be a complete graph with five vertices. Then, the properties of the graph are determined where the diameter is one, the dominating number is one, and the chromatic number is five. The perfect codes are {(12)}, {(13)}, {(23)}, {(123)}, {(132)}.

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