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On the Planar Property of an Ideal Based Weakly Zero Divisor Graph

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Abstract

Let R be a commutative ring and I be an ideal of R. Beck [1] defined the zero-divisor graph of R as the graph whose vertices are zero-divisors of R, where two vertices x and y are adjacent if and only if xy=o. Anderson and Livingston [2] modified Beck's definition by excluding zero from the set of vertices and presented fundamental results on the zero divisor graph. The notion of a zero-divisor graph is generalized in [3], where products which are zero are replaced with products which are in some ideal I of R. The planar property of this generalized graph, called the ideal-based zero-divisor graph,

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is studied in [3] and [9], and that of zero-divisor graph in [6] and [7]. As in [4], the weakly zero-divisor graph of R is the graph whose vertices are non-zero zero divisors of R and two vertices x and y are adjacent if and only if there exist r in ann(x) and s in ann(y) such that rs=0. We define an ideal-based weakly zero-divisor graph which contains the ideal-based zero-divisor graph as a subgraph and is identical to the weakly zero-divisor graph when I=0. We will determine when the ideal-based weakly zero-divisor graph is planar by introducing restraints on the size of I and girth of weakly zero-divisor graph of the factor ring R/I.

Keywords:
zero-divisor graph; commutative ring; planar graph; girth; graph theory.

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