Analysis and Design of Precast Concrete Structures to Eurocode 2

Precast and Prestressed Floors and Composite Floors

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Precast & Prestressed Floors and Composite Floors

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• Hollow core floor units & slab fields
• Double tee units
• Half-slab (precast + in-situ topping)
• Composite floors
• Load vs. span data

Fixing Rates at 2000 m² per week?
40 × 11 feet Wide HCU onto Precast Walls at MGM Hotel, Las Vegas, 1992

Hollowcore Units

Daily production:
200 m² – 1500 m²
Double Tee Units

Twice the price but up to 4x capacity than hollow core

Prestressed Half Slab

Popular for housing and awkward shapes
Prestressed Half Slab

Jetty at Pasir Gudang, Johor

Propping required for > 5 m
Prestressed Hollow Core Floor Units

- 400 – 3600 mm wide; typically 1200 mm
- 90 – 730 mm deep; typically 150, 200, 250, 300 mm
- Self weight 1.5 to 5.0 kN/m²
- Void ratio 40 – 60 % of solid section
- Spans 6 – 20 m (economical range)
Splitting cracks due to sawing restraints as prestress in transferred
Too much plasticiser!! in 450 deep units

Actually air-entrainment agent is used by several producers as a plasticiser.

Strand Pull-in:

An important indicator of success

Should be about 1 mm, irrespective of length

Theoretical pull-in limit for zero prestress:

\[
\frac{PL}{AE}
\]

* Use a linear scale between the extremes
Bearing onto neoprene or mortar for spans more than about 15 m onto insitu or masonry

20 m long HCU

Section for Analysis for Prestressed

- Pretension and losses (about 18 – 25%)
- Service moment (bottom tension critical)
- Ultimate moment (usually $> M_{sr} \times 1.5$)
- Ultimate shear uncracked & flexurally cracked
- End bearing and transmission length
- Deflection and camber (long-term, creep)
- Live load deflection after installation
Section for Analysis for Prestressed

- \( M_{Sr} = (f_{bt} + f_{ct})Z_b \)
- \( M_{ur} = 0.87f_{pb}A_{ps}(d - d_n) \)
- \( V_{co} = \frac{l \cdot b_w}{A \cdot y} \sqrt{(f_{ctd})^2 + \alpha_1 \sigma_{cp} f_{ctd}} \)

Load vs Span Curve

Imposed load

Allowable span
Load vs Span Curve

- Bearing limit
- Handling limit
  span/depth = 50

Prestressed Hollow Core Floor Units

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Prestressed Hollow Core Floor Units

Load vs Span Curve

- Imposed load
- Service moment
- Deflection
- Handling

Possibly shear? Service moment control
Deflection control

Allowable span

Prestressed Hollow Core Floor Units

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Example 1

Calculate $M_{sr}$ for the 203 mm deep prestressed HCU shown below. The initial prestressing force may be taken as 70% of characteristics strength of the standard 7-wire helical strand. Take the final long-term losses as 24%. Geometric and data given by the manufacturer are as follows:

\[ A_c = 135 \times 10^3 \text{ mm}^2, l = 678 \times 10^6 \text{ mm}^4, y_t = 99 \text{ mm}, f_{ck} = 50 \text{ N/mm}^2, E_c = 30 \text{ kN/mm}^2, f_{ck}(t) = 35 \text{ N/mm}^2, E_{cl} = 27 \text{ kN/mm}^2, f_{p0,1k} = 1750 \text{ N/mm}^2, E_{ps} = 195 \text{ kN/mm}^2, A_{ps} = 94.2 \text{ mm}^2 \text{ per strand (}\phi = 12.5 \text{ mm}) \text{ and cover = 40 mm.}

Example 1 - Solution

Section Properties

\[ Z_b = \frac{l}{y_b} = \frac{678 \times 10^6}{104} = 6.519 \times 10^6 \text{ mm}^3 \]
\[ Z_t = \frac{l}{y_t} = \frac{678 \times 10^6}{99} = 6.848 \times 10^6 \text{ mm}^3 \]

Eccentricity, $e = 104 - 40 - 12.5/2 = 57.7 \text{ mm}$

Initial prestress in tendons, $f_{pi} = 0.70 \times 1750 = 1225 \text{ N/mm}^2$

Initial prestressing force, $P_i = f_{pi} \cdot A_{ps} = 1225 \times 7 \times 94.2 \times 10^{-3} = 807.8 \text{ kN}$
### Example 1 - Solution

**Check Stress at Transfer**

\[
f_{bc} = (807.8 \times 10^3 / 135 \times 10^3) + (807.8 \times 10^3 \times 57.75 / 6.519 \times 10^6)
\]

\[
= +13.14 \text{ N/mm}^2 \text{ (compression)} < 0.6f_{ck}(t) \text{ (21 N/mm}^2\text{)}
\]

\[
f_{tt} = (807.8 \times 10^3 / 135 \times 10^3) - (807.8 \times 10^3 \times 57.75 / 6.848 \times 10^6)
\]

\[
= -0.83 \text{ N/mm}^2 \text{ (tension)} > f_{ctm}(t) \text{ (3.20 N/mm}^2\text{)}
\]

**Final Prestress in Bottom and Top**

\[
f_{bc} = +13.14 \times (1 - 0.24) = +9.97 \text{ N/mm}^2 \text{ (compression)}
\]

\[
f_{tt} = -0.83 \times (1 - 0.24) = -0.63 \text{ N/mm}^2 \text{ (tension)}
\]

**Service Moment**

At the bottom fibre, \( M_{sr} \) is limited by the tensile stress limit of 0 N/mm²

\[
M_{sr} = (f_{bc})Z_b = (9.97) \times 6.519 \times 10^6 \times 10^{-6}
\]

\[
= 65.0 \text{ kNm}
\]

At the top fibre, \( M_{sr} \) is limited by the compressive stress limit of 0.6\( f_{ck} \) = 30 N/mm²

\[
M_{sr} = (f_{tt} + 0.6f_{ck})Z_t = (0.63 + 30) \times 6.848 \times 10^6 \times 10^{-6}
\]

\[
= 209.7 \text{ kNm > 65.0 kNm}
\]
Example 2 - Solution

Calculate $M_{ur}$ for the section given in Example 1. Manufacturer’s data gives the breadth of the top of the HCU as $b = 1168$ mm.

Effective depth, $d = 203 - 40 - 12.5/2 = 156.75$ mm

Stress in steel after losses $= (1 - 0.24) \times 0.70 \times 1750 = 931$ N/mm$^2$

Therefore, strain in steel after losses $= \frac{f_s}{E_s} = \frac{931}{205 \times 10^3} = 0.0045$

which is less than $\varepsilon_y$, the yield strain.

Example 2 - Solution
As a first attempt, try \( x = 75 \text{ mm} \), approximately equal to 0.5d

(a) Steel Strains

Final stress strain, \( \varepsilon_s = \) Prestress Strain + Bending Strain, \( \varepsilon'_s \)

Bottom layer at the tendon:

\[
\varepsilon_{sb} = 0.0045 + \varepsilon'_{sb}
\]

\[
= 0.0045 + \frac{(d-x)}{x} \varepsilon_{cc}
\]

\[
= 0.0045 + \frac{(156.75-75)}{75} \times 0.0035 = 0.008315 > \varepsilon_y
\]

\[
\therefore \text{Take} \quad \varepsilon_{sb} = 0.00742
\]

(b) Steel Stresses

Bottom layer at the tendon:

\[
f_{sb} = \varepsilon_{sb} \times E_s
\]

\[
= 0.00742 \times 205 \times 10^3
\]

\[
= 1522 \text{ N/mm}^2
\]

(c) Forces in Steel & Concrete

Steel tensile forces, \( F_{st} = \sum f_s A_{ps} = (f_{sb}) \times 7 \times 94.2 = 1003 \times 10^3 \text{ N} \)

Concrete compressive forces, \( F_{cc} = 0.567f_{ck} b \times 0.8x \)

\[
= 0.567 \times 50 \times 1168 \times 0.8 \times 75 = 1987 \times 10^3 \text{ N}
\]

Since \( F_{cc} > F_{st} \), a smaller depth of neutral axis, \( x \) must be tried.
Example 2 - Solution

Try \( x = 38 \text{ mm} > 28 \text{ mm} \) (upper flange) and then carry out the previous analysis.

Final Forces in Steel & Concrete

Steel tensile forces, \( F_{st} = \sum f_s A_{ps} = (f_{sb}) \times 7 \times 94.2 = 1003 \times 10^3 \text{ N} \)

Concrete compressive forces, \( F_{cc} = 0.567 f_{ck} b \times 0.8x \)
\[ = 0.567 \times 50 \times 1168 \times 0.8 \times 38 = 1006 \times 10^3 \text{ N} \approx F_{st} \Rightarrow \text{OK} \]

Ultimate Moment of Resistance:

\[ M_{ur} = F_{st} z = F_{st} (d - 0.4x) \]
\[ = 1003 \times 10^3 \times (156.75 - 0.4 \times 38) \times 10^{-6} \]
\[ = 142 \text{ kNm} \]

Shear Analysis

Shear failure in 400 deep units

Shear searches out the weakest web
Example 3 - Solution

Calculate the uncracked shear capacity, \( V_{co} \) for the section in Example 1.

\[
\sigma_{cp} = \frac{\gamma_{cp} \eta P_1}{A} = \frac{0.9 \times (1 - 0.24) \times 0.70 \times 1750 \times 7 \times 94.2}{135 \times 10^3} = 4.09 \text{ N/mm}^2 < 0.133 f_{uk} (6.65 \text{ N/mm}^2)
\]

\[
f_{ctd} = \frac{f_{ctk,0.05}}{\gamma_m} = \frac{2.90}{1.5} = 1.40 \text{ N/mm}^2
\]

\[
\alpha_1 = \frac{l_x}{l_{pt2}} = \frac{800 + (203/2)}{961} = 0.94 \leq 1.0
\]

\[
\begin{align*}
\mathcal{L}_{pt2} &= 1.2 \mathcal{L}_{pt} = 1.2 \left( \frac{\alpha_1 \alpha_2 \phi_{pm0}}{f_{bpt}} \right) = 1.2 \left( \frac{1.0 \times 0.25 \times 12.5 \times 1225}{4.78} \right) = 961 \text{ mm} \\
\alpha_1 &= 1.0 \\
\alpha_2 &= 0.25 \\
\sigma_{pm0} &= 0.70 \times 1750 = 1225 \text{ N/mm}^2 \\
\phi &= 12.5 \text{ mm} \\
\mathcal{L}_m &\text{(transmission length)} = 800 \text{ mm} \\
f_{bpt} &= \eta_{p1} \eta_{T1} f_{ctd}(t) = \eta_{p1} \eta_{T1} \frac{f_{ctk,0.05}(t)}{\gamma_m} = 3.2 \times 1.0 \times \frac{2.20}{1.5} = 4.78 \text{ N/mm}^2
\end{align*}
\]
Example 3 - Solution

Total breadth at centroid, \( b_w = 1168 - (6 \times 150) = 268 \text{ mm} \)

Area above the centroidal axis, \( A = 67500 \text{ mm}^2 \)

\( y' = 64.5 \text{ mm (calculate from geometry)} \)

\[
V_{co} = \frac{I \cdot b_w}{A \cdot y'} \sqrt{(f_{ctd})^2 + \alpha_1 \sigma_{cp} f_{ctd}}
\]

\[
= \frac{678 \times 10^6 \times 268 \times 10^{-3}}{67500 \times 64.5} \sqrt{1.4^2 + 0.94 \times 4.09 \times 1.40}
\]

\( = 113.1 \text{ kN} \)

Shear Analysis

Watch out for flexural-shear \((V_{cr})\) failure!

![Image of shear failure](image-url)
Flexural-Shear Formula, $V_{cr}$

$$V_{cr} = [0.12k(100\rho f_{ck})^{1/3} + 0.15\sigma_{cp}] b_w d$$

$$V_{cr, min} = [0.035k^{3/2}(f_{ck})^{1/2} + 0.15\sigma_{cp}] b_w d$$
Example 4 - Solution

Calculate the minimum value of $V_{cr}$ for the slab in Example 1.

$$\sigma_{cp} = \frac{Y_{cp} \eta P_i}{A} = \frac{0.9 \times (1 - 0.24) \times 0.70 \times 1750 \times 7 \times 94.2}{135 \times 10^3} = 4.09 \text{ N/mm}^2 < 0.133f_{ck} (6.65 \text{ N/mm}^2)$$

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{156.75}} = 2.12 \leq 2.0 \text{ (From Example 2, } d = 156.75 \text{ mm)}$$

From Example 3, $\sigma_{cp} = 4.09 \text{ N/mm}^2$

$$V_{cr, \text{min}} = [0.035k^{3/2} (f_{ck})^{1/2} + 0.15 \sigma_{cp}] b_w d$$

$$= [0.0035(2.0)^{3/2} (50)^{1/2} + 0.15 \times 4.09] \times 268 \times 156.75 \times 10^{-3}$$

$$= 28.7 \text{ kN}$$

Bearing Capacity

- Refer to Cl. 10.9.5
- Refers to contact bearing pressure at the bearing ledge
- **Non-isolated** – components are connected to other components with a secondary means of support
- **Isolated** – components rely entirely on their own bearing for total support

Ultimate bearing capacity, $F_{Ed} = f_{Rd} b_1 a_1$
**Bearing Definitions**

Figure 10.6: Example of bearing with definitions.

\[ a = a_1 + a_2 + a_3 + \sqrt{\Delta a_2^2 + \Delta a_3^2} \]

**Bearing Definitions**

Prestressed Hollow Core Floor Units – Bearing Capacity

**A**

Net Bearing Length, \( a_1 \) (Perpendicular to the Floor)

Table 10.2: Minimum value of \( a_1 \) in mm

<table>
<thead>
<tr>
<th>Relative bearing stress, ( \sigma_{Ed} / f_{cd} )</th>
<th>( \leq 0,15 )</th>
<th>0,15 - 0,4</th>
<th>&gt; 0,4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line supports (floors, roofs)</td>
<td>25</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>Ribbed floors and purlins</td>
<td>55</td>
<td>70</td>
<td>80</td>
</tr>
<tr>
<td>Concentrated supports (beams)</td>
<td>90</td>
<td>110</td>
<td>140</td>
</tr>
</tbody>
</table>

\( a_1 \) must **NOT** be less than the value in Table 10.2
Limiting Value of $a_2$

Table 10.3: Distance $a_2$ (mm) assumed ineffective from outer end of supporting member. Concrete padstone should be used in cases (-)

<table>
<thead>
<tr>
<th>Support material and type</th>
<th>$\frac{\sigma_{Ed}}{f_{cd}}$</th>
<th>$\leq 0.15$</th>
<th>$0.15 - 0.4$</th>
<th>$&gt; 0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>line</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>concentrated</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Reinforced concrete</td>
<td>line</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>concentrated</td>
<td>10</td>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>Plain concrete and</td>
<td>line</td>
<td>10</td>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>Rein. concrete $\leq$ C30/37</td>
<td>concentrated</td>
<td>20</td>
<td>25</td>
<td>35</td>
</tr>
<tr>
<td>Brickwork</td>
<td>line</td>
<td>10</td>
<td>15</td>
<td>(-)</td>
</tr>
<tr>
<td></td>
<td>concentrated</td>
<td>20</td>
<td>25</td>
<td>(-)</td>
</tr>
</tbody>
</table>

Limiting Value of $a_3$

Table 10.4: Distance $a_3$ (mm) assumed ineffective beyond outer end of supported member

<table>
<thead>
<tr>
<th>Detailing of reinforcement</th>
<th>Support Line</th>
<th>Support Concentrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous bars over support (restrained or not)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Straight bars, horizontal loops, close to end of member</td>
<td>5</td>
<td>15, but not less than end cover</td>
</tr>
<tr>
<td>Tendons or straight bars exposed at end of member</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>Vertical loop reinforcement</td>
<td>15</td>
<td>end cover + inner radius of bending</td>
</tr>
</tbody>
</table>
Limiting Value of $\Delta a_2$ and $\Delta a_3$

<table>
<thead>
<tr>
<th>Support material</th>
<th>$\Delta a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel or precast concrete</td>
<td>$10 \leq l/1200 \leq 30$ mm</td>
</tr>
<tr>
<td>Brickwork or cast in-situ concrete</td>
<td>$15 \leq l/1200 + 5 \leq 40$ mm</td>
</tr>
</tbody>
</table>

$\Delta a_2$ is an allowance for deviations for the clear distance between the faces of the supports, $l$ = span length.

Net Bearing Width, $b_1$

(3) If measures are taken to obtain a uniform distribution of the bearing pressure, e.g. with mortar, neoprene or similar pads, the design bearing width $b_1$ may be taken as the actual width of the bearing. Otherwise, and in the absence of a more accurate analysis, $b_1$ should not be greater than to 600 mm.

10.9.5.3 Bearings for isolated members

(1) The nominal length shall be 20 mm greater than for non-isolated members.

(2) If the bearing allows movements in the support, the net bearing length shall be increased to cover possible movements.

(3) If a member is tied other than at the level of its bearing, the net bearing length $a_1$ shall be increased to cover the effect of possible rotation around the tie.
Example 5 - Solution

Calculate the bearing capacity of the HCU in Example 1. The unit has an actual 75 mm dry bearing length onto a reinforced concrete. Assumed that the unit is 6 m long and has secondary support on steel beams.

Ultimate bearing strength, \( f_{Rd} \):

\[ f_{Rd} = 0.4 f_{cd} \]

for dry connections (see 10.9.4.3 (3) for definition)

\[ f_{Rd} = f_{bed} \leq 0.85 f_{cd} \]

for all other cases

where

- \( f_{cd} \) is the the lower of the design strengths for supported and supporting member
- \( f_{bed} \) is the design strength of bedding material

\[ f_{Rd} = 0.4 \times 50 = 13.33 \text{ N/mm}^2 \]

Net bearing width, \( b_1 = 1200 \text{ mm} < b_{1,\text{max}} = 600 \text{ mm} \)

Use \( b_1 = 600 \text{ mm} \)

Net bearing length for non-isolated component, \( a_1 \):

From

- \( a_2 = 0 \text{ mm} \)
- \( \Delta a_2 = 10 \text{ mm} \)
- \( a = 75 \text{ mm} \)
- \( a_3 = 5 \text{ mm} \)
- \( \Delta a_3 = 2.4 \text{ mm} \)
Example 5 - Solution

\[ a_1 = a - a_2 - a_3 - \sqrt{\Delta a_2^2 + \Delta a_3^2} \]

\[ a_1 = 75 - 0 - 5 - \sqrt{10^2 + 2.4^2} = 60.3 \text{ mm} > 25 \text{ mm} \]

⇒ Refer to Table 10.2 for the minimum values of \( a_1 \)

\[ F_{Ed} = 13.33 \times 60.3 \times 600 \times 10^{-3} = 482.3 \text{ kN} > V_{co} (109.3 \text{ kN}) \]

⇒ OK

Note:

\[ \sigma_{Ed} = \frac{V_{co} (From \ Example \ 4)}{1200 \times 75} = 1.25 \text{ N/mm}^2 \]

\[ \frac{\sigma_{Ed}}{f_{cd}} = \frac{1.25}{50/1.5} = 0.0375 \]

Composite Floors Required for Double Tee, but Optional for Hollowcore
Surface Laitance due to Cutting Slurry

50 mm minimum at the highest point, increasing (with slab and beam cambers) to about 80 mm
Composite Construction

- Minimum thickness of topping ≥ 40 mm
- Average depth of topping allowances for camber should be made – allowing span/300 will suffice
- Concrete grade – C25 to C30
- Minimum mesh reinforcement area = 0.13% × Concrete area

Composite Construction

- Advantages to composite construction:
  (a) Increase bending resistance and flexural stiffness
  (b) Improve vibration, thermal & acoustic performance
  (c) Provide horizontal diaphragm action
  (d) Provide horizontal stability ties across floors
  (e) Provide a continuous and monolithic floor finish
Composite Design

Depth to neutral axis \( X < h_s \)

with 75 mm topping
Composite Design - 2 Stage Approach

Stage 1
- Selfweight of slab + in-situ concrete topping + construction traffic allowance (1.5 kN/m²)
- Section properties = Precast unit

Stage 2
- Superimposed load
- Section properties = Composite section

Stress Diagram at Serviceability

- Topping
- Precast
- Prestress
- Imposed Stage 1 stresses
- Imposed Stage 2 stresses
- Total stresses
- $0.6f_{ct} \text{ limit}$
Stage 1 – Service Moment, $M_1$

Bottom fibre stress of the HCU:

$$f_{b1} = f_{bc} - \frac{M_1}{Z_{b1}} < +0.6f_{ck}\ (t)\ (\text{transfer})&\ 0\ (\text{service})$$

Top fibre stress of the HCU:

$$f_{t1} = f_{tc} - \frac{M_1}{Z_{t1}} < -f_{ctm}\ (t)\ (\text{transfer})&\ +\ 0.6f_{ck}\ (\text{service})$$

Stage 2 – Service Moment, $M_2$

Bottom fibre stress of the HCU:

$$f_{b2} = -\frac{M_2}{Z_{b2}}$$

Top fibre stress of the HCU:

$$f_{t2} = +\frac{M_2}{Z_{t2}}$$

Top fibre stress of the in-situ topping:

$$f'_{t2} = +\frac{M_2}{Z'_{t2}}$$

$$b_{eff} = b \times \frac{E_c'}{E_c} \text{ where } b \text{ is the full breadth}$$
Total Service Moment, $M_S$

Adding Stage 1 + Stage 2:

$\begin{align*}
  f_b &= f_{b1} + f_{b2} = f_{bc} - M_1/Z_{b1} - M_2/Z_{b2} > 0 \\
  f_t &= f_{t1} + f_{t2} = f_{tc} + M_1/Z_{t1} + M_2/Z_{t2} > 0.6f_{ck} \\
  f'_t &= f'_{t2} = + M_2/Z'_{t2} > 0.6f_{ck, in-situ}
\end{align*}$

Service Moment, $M_{sr}$

Bottom of the slab:

$M_{s2} = M_2 > (f_{bc})Z_{b2} - [M_1(Z_{b2}/Z_{b1})]$  

Top of the slab:

$M_{s2} = M_2 > (0.6f_{ck} - f_{tc})Z_{t2} - [M_1(Z_{t2}/Z_{t1})]$  

Example 6

Calculate the Stage 2 service bending moment that is available if the HCU in Example 1 has a 50 mm minimum thickness structural topping. The floor is simply supported over an effective span of: (a) 4.0 m; (b) 8.0 m. The precamber of the HCU may be assumed as span/300 without loss of accuracy. Use $f_{ck, in-situ} = 30$ N/mm$^2$ and $E_{c, in-situ} = 26$ kN/mm$^2$. Self weight of concrete = 25 kN/m$^3$. Selfweight of HCU = 3.24 kN/m run. What is the minimum imposed loading for each span?
Example 6 - Solution

Effective breadth of topping, \( b_{\text{eff}} = 1200 \times \frac{26}{30} = 1040 \text{ mm} \)

Total depth of composite section = 203 + 50 = 253 mm

Depth to neutral axis from top of composite section, \( y_{t2} \)

\[
= \frac{(1040 \times 50 \times 25) + (135000 \times (50+99))}{135000 + (1040 \times 50)} = 114.5 \text{ mm}
\]

Second moment of area of composite section, \( I_2 = 1266 \times 10^6 \text{ mm}^4 \)

\[
Z_{b2} = \frac{1266 \times 10^6}{(253 - 114.5)} = 9.14 \times 10^6 \text{ mm}^3
\]

\[
Z_{t2} = \frac{1266 \times 10^6}{114.5} = 19.63 \times 10^6 \text{ mm}^3
\]

\[
Z_{b2}/Z_{b1} = 9.14 \times 10^6 / 6.519 \times 10^6 = 1.40
\]

\[
Z_{t2}/Z_{t1} = 19.63 \times 10^6 / 6.848 \times 10^6 = 2.86
\]
Example 6 - Solution

(a) 4.0 m

Precamber = $\frac{4000}{300} = 13$ mm

Maximum depth of topping at supports = $50 + 13 = 63$ mm

Average depth of topping = $\frac{(50+63)}{2} = 57$ mm

Self weight of topping = $0.057 \times 25 \times 1.2 = 1.71$ kN/m run for 1.2 m wide unit

Self weight of HCU = 3.24 kN/m run

Stage 1 moment, $M_1 = \frac{(3.24+1.71)\times4^2}{8} = 9.90$ kNm

---

Example 6 - Solution

**Bottom of the slab:**

$M_{s2} > f_{bc}Z_{b2} - M_1 \left(\frac{Z_{b2}}{Z_{b1}}\right)$

$M_{s2} > 9.97 \times 9.14 \times 10^{-6} - [9.90 \times 10^6 (1.40)] \times 10^{-6}$

$= 77.46$ kNm
Example 6 - Solution

**Top of the slab:**

\[ M_{s2} = M_s > (0.6f_{ck} - f_{tt})Z_{t2} - M_1 \left( \frac{Z_{t2}}{Z_{t1}} \right) \]

\[ M_{s2} > (30 - (-0.63)) \times 19.63 \times 10^6 - [9.90 \times 10^6 (2.86)] \times 10^{-6} \]

= 573.35 kNm

The allowable maximum imposed load = 8 \times 77.46/4.0^2 = **38.73 kN/m**

*Additional notes:*

Total \( M_s = M_{s1} + M_{s2} = 9.90 + 77.46 = 87.36 \text{kNm} \)

Ultimate Limit State - 2 Stage Approach
Ultimate Limit State - 2 Stage Approach

Total area of reinforcement, \( A_{ps} = A_{ps1} + A_{ps2} \)

Effective breadth, \( b_{eff} = b \times f_{cu \text{ in-situ}} / f_{cu} \)

**Stage 1**

\[ f_{pd} A_{ps1} = 0.567 f_{ck} b (0.8X) \]  
\[ \Rightarrow \text{but } d_{n1} = 0.4X \text{ or } X = d_{n1}/0.4 \]

then \( d_{n1} = f_{pd} A_{ps1} / 1.134 f_{ck} b \)

then \( M_{u1} = f_{pd} A_{ps1} (d - d_{n1}) \)  
\[ \text{------------------ (1)} \]

**Stage 2**

\[ A_{ps2} = A_{ps} - A_{ps1} \]

\[ M_{u2} = f_{pd} A_{ps2} (d + h_s - d_{n2}) \]  
\[ \text{------------------ (2)} \]

or by replacing (1) into (2):

\[ M_{u2} = f_{pd} [A_{ps} - M_{u1}/f_{pd}(d - d_{n1})] (d + h_s - d_{n2}) \]

**One Step Approach**

\[ M_u = f_{pd} A_{ps} (d + h_s - d_n) \]
**Interface Shear Stress in Composite Slabs (Cl. 6.2.5)**

In the neutral axis is below the interface, \(X > h_s\)
\[ V_{Ed} = 0.567f_{ck}b_{eff}h_s \]

In the neutral axis is above the interface, \(X < h_s\),
\[ V_{Ed} = 0.567f_{ck}b_{eff}0.9X \]

The shear stress at the interface should satisfy:
\[ V_{Edi} < V_{Rdi} \]

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**Interface Shear Stress in Composite Slabs (Cl. 6.2.5)**

Design shear stress at the interface; \(v_{Edi} = \frac{BV_{Ed}}{zb_i}\)

Design shear resistance at the interface;
\[ v_{Rdi} = c \cdot f_{cd} + \mu \cdot \sigma_n + \rho \cdot f_{yd} \cdot (\mu \cdot \sin \alpha + \cos \alpha) \leq 0.5v \cdot f_{cd} \]

If \(v_{Edi} > v_{Rdi}\); provide shear reinforcement (per 1 m run) projecting from the precast unit into the structural topping. The amount of steel required:
\[ A_f = \frac{1000b\tau_h}{0.87f_{yk}} \]
Interface Shear Stress in Composite Slabs (Cl. 6.2.5)

where:
- $\beta$ is the ratio of the longitudinal force in the new concrete area and the total longitudinal force either in the compression or tension zone, both calculated for the section considered.
- $V_{Ed}$ is the transverse shear force.
- $z$ is the lever arm of composite section.
- $b_i$ is the width of the interface (see Figure 6.8).

where:
- $c$ and $\mu$ are factors which depend on the roughness of the interface (see (2)).
- $f_{std}$ is as defined in 3.1.6 (2).$^P$
- $\sigma_n$ is stress per unit area caused by the minimum external normal force across the interface that can act simultaneously with the shear force, positive for compression, such that $\sigma_n < 0.6 f_{std}$, and negative for tension. When $\sigma_n$ is tensile, $c f_{std}$ should be taken as 0.
- $\rho = A_b / A_i$

$A_b$ is the area of reinforcement crossing the interface, including ordinary shear reinforcement (if any), with adequate anchorage at both sides of the interface.
$A_i$ is the area of the joint.
$\alpha$ is defined in Figure 6.9, and should be limited by $45^\circ \leq \alpha \leq 90^\circ$.
$\nu$ is a strength reduction factor (see 6.2.2 (6)).

**Figure 6.9:** Indented construction joint

A - new concrete, B - old concrete, C - anchorage
(2) In the absence of more detailed information surfaces may be classified as very smooth, smooth, rough or indented, with the following examples:

- Very smooth: a surface cast against steel, plastic or specially prepared wooden moulds: $c = 0.025$ to $0.10$ and $\mu = 0.5$
- Smooth: a slipformed or extruded surface, or a free surface left without further treatment after vibration: $c = 0.20$ and $\mu = 0.6$
- Rough: a surface with at least 3 mm roughness at about 40 mm spacing, achieved by raking, exposing of aggregate or other methods giving an equivalent behaviour: $c = 0.40$ and $\mu = 0.7$
- Indented: a surface with indentations complying with Figure 6.9: $c = 0.50$ and $\mu = 0.9$
Double Tee Floor Units

- 2400 – 3000 mm wide
- 300 – 2000 mm deep; typically 400 – 800 mm
- self weight 2.6 to 10 kN/m²
- void ratio 70 – 80 % of solid section
- spans 8 – 30 m (economical range)

Mostly prestressed, but RC if manufacturer prefers

Bearing pads required 150 x 150 x 10 mm