The time to the minimum or *critical point* on the sag curve is given by equation 8.24.

$$t_c = \frac{1}{K_R - K_D} \times \log_{10}\left[\left(\frac{K_D \mathrm{BOD}_u - K_R D_o + K_D D_o}{K_D \mathrm{BOD}_u}\right)\left(\frac{K_R}{K_D}\right)\right] \qquad 8.24$$

The ratio of K_R/K_D is known as the *self-purification coefficient*. The *critical oxygen deficit* is given by equation 8.25.

$$D_c = \left(\frac{K_D \mathrm{BOD}_u}{K_R}\right) 10^{-K_D t_c} \qquad 8.25$$

Knowing t_c and the stream flow velocity will locate the point where the oxygen level is the lowest.

Example 8.4

A treatment plant discharge has the following characteristics:

15 cfs
45 mg/l BOD (5 day, 20°C)
2.9 mg/l DO
24°C
$K_{D,20°C} = 0.1$ per day (when mixed with river water)

The outfall is located in a river with the following characteristics:

0.55 ft/sec velocity
4.0 feet average depth
120 cfs
4 mg/l BOD (5 day, 20°C)
8.3 mg/l DO
16°C

Determine the distance downstream where the oxygen level is minimum, and predict if the river can support fish life at that point.

step 1: Find the river conditions immediately after mixing. Use equation 8.20 three times.

$$(\mathrm{BOD})_{20°C} = \frac{(15)(45) + (120)(4)}{15 + 120} = 8.56 \text{ mg/l}$$

$$DO = \frac{(15)(2.9) + (120)(8.3)}{135} = 7.7 \text{ mg/l}$$

$$T = \frac{(15)(24) + (120)(16)}{135} = 16.89°\mathrm{C}$$

step 2: Calculate the rate constants. From equation 8.8,

$$K_{D,16.89°C} = (0.1)(1.047)^{16.89-20} = 0.0867$$

From equation 8.22,

$$K_{R,20°C} \approx 3.3(0.55/(4)^{1.33}) = 0.287$$

From equation 8.23,

$$K_{R,16.89°C} = (0.287)(1.016)^{16.89-20} = 0.275$$

step 3: Estimate BOD_u. Using equation 8.6,

$$(\mathrm{BOD}_u)_{20°C} = \frac{8.56}{1 - (10)^{-(0.0867)(5)}} = 13.56$$

From equation 8.9,

$$(\mathrm{BOD}_u)_{16.89°C} = (13.56)[(0.02)(16.89) + 0.6] = 12.72$$

step 4: Calculate D_o. From appendix B, the saturated oxygen concentration at 16.89°C is approximately 9.7 mg/l. So, $D_o = 9.7 - 7.7 = 2.0$.

step 5: Calculate t_c from equation 8.24.

$$t_c = \frac{1}{0.275 - 0.0867} \times \log\left[\frac{(0.0867)(12.72) - (0.275)(2) + (0.0867)(2)}{(0.0867)(12.72)} \times \left(\frac{0.275}{0.0867}\right)\right] = 1.70 \text{ days}$$

step 6: The distance downstream is

$$\frac{(1.70 \text{ days})(0.55 \text{ ft/sec})(86{,}400 \text{ sec/day})}{(5280 \text{ ft/mile})} = 15.3 \text{ miles}$$

step 7: The critical oxygen deficit is found from equation 8.25.

$$D_c = \frac{(0.0867)(13.56)}{0.275}(10)^{-(0.0867)(1.77)} = 3.0$$

step 8: If the temperature 15.9 miles downstream is 16°C, the saturated oxygen content is 10 mg/l. Since the critical deficit is 3, the minimum oxygen content is 7 mg/l. This is adequate for fish life.