Mohr Coulomb failure criterion with Mohr circle of stress

\[
\left[ c' \cot \phi' + \left( \frac{\sigma_1' + \sigma_3'}{2} \right) \right] \sin \phi' = \left( \frac{\sigma_1' - \sigma_3'}{2} \right)
\]

\[
(\sigma_1' - \sigma_3') = (\sigma_1' + \sigma_3') \sin \phi' + 2c' \cos \phi'
\]

\[
\sigma_1' (1 - \sin \phi') = \sigma_3' (1 + \sin \phi') + 2c' \cos \phi'
\]

\[
\sigma_1' = \sigma_3' \left( \frac{1 + \sin \phi'}{1 - \sin \phi'} \right) + 2c' \left( \frac{\cos \phi'}{1 - \sin \phi'} \right)
\]

\[
\sigma_1' = \sigma_3' \tan^2 \left( 45 + \frac{\phi'}{2} \right) + 2c' \tan \left( 45 + \frac{\phi'}{2} \right)
\]
Determination of shear strength parameters of soils ($c$, $\phi$ or $c'$, $\phi'$)

**Laboratory tests**

1. Direct shear test
2. Unconfined Compressive Strength test
3. Triaxial shear test
4. Laboratory vane shear test
5. Laboratory fall cone test

**Field tests**

1. Vane shear test
2. Torvane
3. Pocket penetrometer
4. Fall cone
5. Pressuremeter
6. Static cone penetrometer
7. Standard penetration test

Most common laboratory tests to determine the shear strength parameters are,

1. Direct shear test
2. Unconfined Compressive Strength test
3. Triaxial shear test

Other laboratory tests include,

Direct simple shear test, torsional ring shear test, plane strain triaxial test, laboratory vane shear test, laboratory fall cone test
Field conditions

Before construction

A representative soil sample

\[ \sigma_{hc} \rightarrow \sigma_{vc} \rightarrow \sigma_{hc} \]

\[ \sigma_{vc} \]

After and during construction

\[ \sigma_{hc} \rightarrow \sigma_{vc} + \Delta \sigma \rightarrow \sigma_{hc} \]

\[ \sigma_{vc} + \Delta \sigma \]
Simulating field conditions in the laboratory

Step 1
Set the specimen in the apparatus and apply the initial stress condition

Step 2
Apply the corresponding field stress conditions
DIRECT SHEAR TEST
Direct shear test

Schematic diagram of the direct shear apparatus
Direct shear test

Direct shear test is most suitable for consolidated drained tests specially on granular soils (e.g.: sand) or stiff clays

Preparation of a sand specimen

Components of the shear box

Preparation of a sand specimen
Direct shear test

Preparation of a sand specimen

Leveling the top surface of specimen

Specimen preparation completed
Direct shear test

- **Shear box**
- **Loading frame** to apply vertical load
- **Dial gauge** to measure vertical displacement
- **Proving ring** to measure shear force
- **Dial gauge** to measure horizontal displacement
Step 1: Apply a vertical load to the specimen and wait for consolidation.
Direct shear test

Test procedure

Step 1: Apply a vertical load to the specimen and wait for consolidation

Step 2: Lower box is subjected to a horizontal displacement at a constant rate
Direct shear test

Analysis of test results

\[ \sigma = \text{Normal stress} = \frac{\text{Normal force (P)}}{\text{Area of cross section of the sample}} \]

\[ \tau = \text{Shear stress} = \frac{\text{Shear resistance developed at the sliding surface (S)}}{\text{Area of cross section of the sample}} \]

Note: Cross-sectional area of the sample changes with the horizontal displacement
Direct shear tests on sands

How to determine strength parameters c and φ

Mohr – Coulomb failure envelope
Direct shear tests

Stress-strain relationship

Shear stress, $\tau$

Shear displacement

Change in height of the sample

Expansion

Compression

Dense sand/OC clay

Loose sand/NC clay

Dense sand/OC Clay

Loose sand/NC Clay
Direct shear tests on sands

Some important facts on strength parameters $c$ and $\phi$ of sand

Sand is cohesionless hence $c = 0$

Direct shear tests are drained and pore water pressures are dissipated, hence $u = 0$

Therefore, $\phi' = \phi$ and $c' = c = 0$

A GENTLE REMINDER ...
Direct shear tests on clays

In case of clay, *horizontal displacement* should be applied at a very slow rate to allow dissipation of pore water pressure (therefore, one test would take several days to finish)

Failure envelopes for clay from drained direct shear tests

Shear stress at failure, $\tau_f$ vs Normal force, $\sigma$

- Normally consolidated clay ($c' = 0$)
- Overconsolidated clay ($c' \neq 0$)
Interface tests on direct shear apparatus

In many foundation design problems and retaining wall problems, it is required to determine the angle of internal friction between soil and the structural material (concrete, steel or wood).

\[ \tau_f = c_a + \sigma' \tan \delta \]

Where,
- \( c_a \) = adhesion,
- \( \delta \) = angle of internal friction
Advantages of direct shear apparatus

- Due to the smaller thickness of the sample, rapid drainage can be achieved
- Can be used to determine interface strength parameters
- Clay samples can be oriented along the plane of weakness or an identified failure plane

Disadvantages of direct shear apparatus

- Failure occurs along a predetermined failure plane
- Area of the sliding surface changes as the test progresses
- Non-uniform distribution of shear stress along the failure surface
Example

A direct shear test when conducted on a remolded sample of sand, gave the following observations at the time of failure; Normal force = 288 N; shear force = 173 N. The cross-sectional area of the sample = 36cm².

Determine the angle of frictional. Solved in 2 ways, namely graphically and analytically

(31 degrees)
UNCONFINED COMPRESSIVE STRENGTH (UCS) TEST
Unconfined Compression Test (UCS Test)

\[ \sigma_1 = \sigma_{VC} + \Delta\sigma \]

\[ \sigma_3 = 0 \]

Confining pressure is zero in the UCS test
Unconfined Compression Test (UC Test)

\[ \sigma_1 = \sigma_{VC} + \Delta \sigma_f \]

\[ \sigma_3 = 0 \]

Shear stress, \( \tau \) = \( \frac{q_u}{2} = c_u \)

Note: Theoretically \( q_u = c_u \), However in the actual case \( q_u < c_u \) due to premature failure of the sample
Let's continue later....
TRIAXIAL TEST
Triaxial Shear Test

Soil sample at failure

Failure plane

Piston (to apply deviatoric stress)

Perspex cell

Soil sample

O-ring

Impervious membrane

Porous stone

Water

Cell pressure

Back pressure

Pedestal

Pore pressure or volume change
Triaxial Shear Test

Specimen preparation (undisturbed sample)

Edges of the sample are carefully trimmed

Setting up the sample in the triaxial cell
Triaxial Shear Test

Specimen preparation (undisturbed sample)

Sample is covered with a rubber membrane and sealed

Cell is completely filled with water
Triaxial Shear Test

Specimen preparation (undisturbed sample)

Proving ring to measure the deviator load

Dial gauge to measure vertical displacement

In some tests
Triaxial Shear Test

Contains of 3 stages:

1. Saturation
2. Consolidation
3. Shearing

Main stages
Triaxial Shear Test

Saturation & use of back pressure

1. Reason for saturation
2. Principle of saturation
3. Maintaining saturation
4. Advantages of saturation
Typical values for parameter $B$

Typical relationship between $B$ and degree of saturation.
# Triaxial Shear Test

<table>
<thead>
<tr>
<th>Test Condition</th>
<th>Stage 1</th>
<th>Stage 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconsolidated Undrained (UU)</td>
<td>Apply confining pressure $\sigma_3$ while the drainage line from the specimen is kept closed (drainage is not permitted), then the initial pore water pressure ($u=u_0$) is not equal to zero</td>
<td>Apply an added stress $\Delta \sigma$ at axial direction. The drainage line from the specimen is still kept closed (drainage is not permitted) ($u=u_d\neq 0$). At failure state $\Delta \sigma=\Delta \sigma_f$; pore water pressure $u=u_f=u_0+u_{d(f)}$</td>
</tr>
<tr>
<td>Consolidated Undrained (CU)</td>
<td>Apply confining pressure $\sigma_3$ while the drainage line from the specimen is opened (drainage is permitted), then the initial pore water pressure ($u=u_0$) is equal to zero</td>
<td>Apply an added stress $\Delta \sigma$ at axial direction. The drainage line from the specimen is kept closed (drainage is not permitted) ($u=u_d\neq 0$). At failure state $\Delta \sigma=\Delta \sigma_f$; pore water pressure $u=u_f=u_0+u_{d(f)}=u_{d(f)}$</td>
</tr>
<tr>
<td>Consolidated Drained (CD)</td>
<td>Apply confining pressure $\sigma_3$ while the drainage line from the specimen is opened (drainage is permitted), then the initial pore water pressure ($u=u_0$) is equal to zero</td>
<td>Apply an added stress $\Delta \sigma$ at axial direction. The drainage line from the specimen is opened (drainage is permitted) so the pore water pressure ($u=u_d$) is equal to zero. At failure state $\Delta \sigma=\Delta \sigma_f$; pore water pressure $u=u_f=u_0+u_{d(f)}=0$</td>
</tr>
</tbody>
</table>
Types of Triaxial Tests

Step 1

Under all-around cell pressure $\sigma_c$

Is the drainage valve open?

- yes
- no

Consolidated sample

Unconsolidated sample

Step 2

Shearing (loading)

deviatoric stress ($\Delta\sigma = q$)

Is the drainage valve open?

- yes
- no

Drained loading

Undrained loading

$\sigma_c$

$\sigma_c$

$\sigma_c$

$\sigma_c$

$\sigma_c + q$
Types of Triaxial Tests

Step 1

Under all-around cell pressure $\sigma_c$

Is the drainage valve open?

yes

no

Consolidated sample

Unconsolidated sample

CD test

Step 2

Shearing (loading)

Is the drainage valve open?

yes

no

Drained loading

Undrained loading

UU test

CU test
Consolidated-drained test (CD Test)

Step 1: At the end of consolidation

\[ \sigma_{VC} = \sigma_hC \]

Drainage

Step 2: During axial stress increase

\[ \sigma'_{VC} = \sigma_{VC} \]
\[ \sigma'_h = \sigma_hC = \sigma'_3 \]

Drainage

Step 3: At failure

\[ \sigma'_{VF} = \sigma_{VC} + \Delta\sigma_f = \sigma'_{1f} \]
\[ \sigma'_{HF} = \sigma_hC = \sigma'_3f \]
Consolidated-drained test (CD Test)

\[
\sigma_1 = \sigma_{VC} + \Delta\sigma
\]

\[
\sigma_3 = \sigma_{hc}
\]

Deviator stress \((q \text{ or } \Delta\sigma_d) = \sigma_1 - \sigma_3\)
Consolidated- drained test (CD Test)

Volume change of sample during consolidation
Consolidated-drained test (CD Test)

Stress-strain relationship during shearing

Deviator stress, $\Delta \sigma_d$

Axial strain

Volume change of the sample

Compression

Expansion

Axial strain

Dense sand or OC clay

$\Delta \sigma_d f$

Loose sand or NC Clay

Dense sand or OC clay

$\Delta \sigma_d f$

Loose sand or NC Clay
How to determine strength parameters $c$ and $\phi$

Deviator stress, $\Delta \sigma_d$

Confining stress = $\sigma_{3c}$

Confining stress = $\sigma_{3b}$

Confining stress = $\sigma_{3a}$

Mohr–Coulomb failure envelope

\[ \sigma_1 = \sigma_3 + (\Delta \sigma_d)_f \]

\[ \sigma_3 \]

\[ (\Delta \sigma_d)_{fa} \]

\[ (\Delta \sigma_d)_{fb} \]

Axial strain

Shear stress, $\tau$
CD tests

Strength parameters $c$ and $\phi$ obtained from CD tests

Since $u = 0$ in CD tests, $\sigma = \sigma'$

Therefore, $c = c'$ and $\phi = \phi'$

c_d and $\phi_d$ are used to denote them

A GENTLE REMINDER ...
CD tests  

Failure envelopes

For sand and NC Clay, $c_d = 0$

Mohr – Coulomb failure envelope

Therefore, one CD test would be sufficient to determine $\phi_d$ of sand or NC clay
CD tests  Failure envelopes

For OC Clay, $c_d \neq 0$
Some practical applications of CD analysis for clays

1. Embankment constructed very slowly, in layers over a soft clay deposit

$\tau = \text{in situ drained shear strength}$
Some practical applications of CD analysis for clays

2. Earth dam with steady state seepage

\[ \tau = \text{drained shear strength of clay core} \]
Some practical applications of CD analysis for clays

3. Excavation or natural slope in clay

\[ \tau = \text{In situ drained shear strength} \]

Note: CD test simulates the long term condition in the field. Thus, \( c_d \) and \( \phi_d \) should be used to evaluate the long term behavior of soils.
Consolidated- Undrained test (CU Test)

**Step 1: At the end of consolidation**

\[ \sigma_{VC} = \sigma_{hC} \]

**Step 2: During axial stress increase**

\[ \sigma'_{VC} = \sigma_{VC} \]

\[ \sigma'_{hC} = \sigma_{hC} \]

\[ \sigma'_{V} = \sigma_{VC} + \Delta \sigma + \Delta u = \sigma'_{1} \]

\[ \sigma'_{h} = \sigma_{hC} + \Delta u = \sigma'_{3} \]

**Step 3: At failure**

\[ \sigma'_{VF} = \sigma_{VC} + \Delta \sigma_{1} + \Delta u_{f} = \sigma'_{1f} \]

\[ \sigma'_{hf} = \sigma_{hC} + \Delta u_{f} = \sigma'_{3f} \]
Consolidated- Undrained test (CU Test)

Volume change of sample during consolidation
Deviator stress, $\Delta\sigma_d$

Axial strain

Dense sand or OC clay

Loose sand or NC Clay

Stress-strain relationship during shearing

Consolidated- Undrained test (CU Test)

$\Delta\sigma_d$ vs. Axial strain

$\Delta\sigma_d$ vs. Axial strain

$\Delta u$ vs. Axial strain

Dense sand or OC clay

Loose sand or NC Clay
CU tests  How to determine strength parameters $c$ and $\phi$

- **Deviator stress**, $D\sigma_d$
- **Axial strain**
- **Mohr – Coulomb failure envelope in terms of total stresses**

**Confining stress** $\sigma_3a$

**Confining stress** $\sigma_3b$

**Total stresses at failure** $\sigma_1 = \sigma_3 + (\Delta\sigma_d)_f$

**Mohr – Coulomb failure envelope** $c_{cu}$, $\phi_{cu}$
CU tests How to determine strength parameters $c$ and $\phi$

$$\sigma'_1 = \sigma_3 + (\Delta\sigma_d)_f - u_f$$

$$\sigma'_3 = \sigma_3 - u_f$$

Mohr – Coulomb failure envelope in terms of effective stresses

Mohr – Coulomb failure envelope in terms of total stresses

Effective stresses at failure
CU tests

Strength parameters $c$ and $\phi$ obtained from CD tests

Shear strength parameters in terms of total stresses are $c_{cu}$ and $\phi_{cu}$

Shear strength parameters in terms of effective stresses are $c'$ and $\phi'$

$c' = c_d$ and $\phi' = \phi_d$

A GENTLE REMINDER ...
CU tests  Failure envelopes

For sand and NC Clay, \( c_{cu} \) and \( c' = 0 \)

Mohr – Coulomb failure envelope in terms of effective stresses

Mohr – Coulomb failure envelope in terms of total stresses

Therefore, one CU test would be sufficient to determine \( \phi_{cu} \) and \( \phi'(= \phi_d) \) of sand or NC clay
Some practical applications of CU analysis for clays

1. Embankment constructed rapidly over a soft clay deposit

\[ \tau = \text{in situ undrained shear strength} \]
Some practical applications of CU analysis for clays

2. Rapid drawdown behind an earth dam

\[ \tau = \text{Undrained shear strength of clay core} \]
Some practical applications of CU analysis for clays

3. Rapid construction of an embankment on a natural slope

Note: Total stress parameters from CU test ($c_{cu}$ and $\phi_{cu}$) can be used for stability problems where,

Soil have become fully consolidated and are at equilibrium with the existing stress state; Then for some reason additional stresses are applied quickly with no drainage occurring
Unconsolidated- Undrained test (UU Test)

Data analysis

Initial specimen condition

\[ \sigma_C = \sigma_3 \]

No drainage

Specimen condition during shearing

\[ \sigma_3 + \Delta\sigma_d \]

No drainage

Initial volume of the sample = \( A_0 \times H_0 \)

Volume of the sample during shearing = \( A \times H \)

Since the test is conducted under undrained condition,

\[ A \times H = A_0 \times H_0 \]

\[ A \times (H_0 - \Delta H) = A_0 \times H_0 \]

\[ A \times (1 - \Delta H/H_0) = A_0 \]

\[ A = \frac{A_0}{1 - \varepsilon} \]
Unconsolidated- Undrained test (UU Test)

Step 1: Immediately after sampling

\[
\sigma_c = \sigma_3
\]

Step 2: After application of hydrostatic cell pressure

\[
\Delta u_c = B \Delta \sigma_3
\]

\[
\sigma'_3 = \sigma_3 - \Delta u_c
\]

No drainage

Increase of pwp due to increase of cell pressure

Skempton’s pore water pressure parameter, B

Note: If soil is fully saturated, then \( B = 1 \) (hence, \( \Delta u_c = \Delta \sigma_3 \))
Typical values for parameter $A$

During the increase of major principal stress, pore water pressure can become negative in heavily overconsolidated clays due to dilation of specimen.
Unconsolidated- Undrained test (UU Test)

**Step 1: Immediately after sampling**

\[ \sigma_{0} = 0 \]

\[ -u_{r} \]

\[ \sigma'_{h0} = u_{r} \]

**Step 2: After application of hydrostatic cell pressure**

No drainage

\[ \sigma_{C} \]

\[ -u_{r} + \Delta u_{c} = -u_{r} + \sigma_{c} \]

\[ (S_{r} = 100\%; B = 1) \]

\[ \sigma'_{h} = u_{r} \]

**Step 3: During application of axial load**

No drainage

\[ \sigma_{C} + \Delta \sigma \]

\[ -u_{r} + \sigma_{c} \pm \Delta u \]

**Step 3: At failure**

No drainage

\[ \sigma_{C} + \Delta \sigma_{f} \]

\[ -u_{r} + \sigma_{c} \pm \Delta u_{f} \]

\[ \sigma'_{hf} = \sigma_{C} + u_{r} - \sigma_{c} \pm \Delta u_{f} = \sigma'_{3f} \]
Unconsolidated- Undrained test (UU Test)

Total, $\sigma$ = Neutral, $u$ + Effective, $\sigma'$

Step 3: At failure

No drainage

Mohr circle in terms of effective stresses do not depend on the cell pressure.

Therefore, we get only one Mohr circle in terms of effective stress for different cell pressures
Unconsolidated- Undrained test (UU Test)

Total, $\sigma$ = Neutral, $u$ + Effective, $\sigma'$

Step 3: At failure

$\sigma'_v = \sigma_C + \Delta \sigma_f + u_r - \sigma_c + \Delta u_f = \sigma'_{1f}$

$\sigma'_h = \sigma_C + u_r - \sigma_c + \Delta u_f = \sigma'_{3f}$

Mohr circles in terms of total stresses

Failure envelope, $\phi_u = 0$
Unconsolidated- Undrained test (UU Test)

Effect of degree of saturation on failure envelope

\[ \sigma_{3c} \quad \sigma_{3b} \quad \sigma_{1c} \quad \sigma_{3a} \quad \sigma_{1b} \quad \sigma_{1a} \]

\[ \tau \]

S < 100%  \quad S > 100%
Some practical applications of UU analysis for clays

1. Embankment constructed rapidly over a soft clay deposit

\[ \tau = \text{in situ undrained shear strength} \]
Some practical applications of UU analysis for clays

2. Large earth dam constructed rapidly with no change in water content of soft clay

\[ \tau = \text{Undrained shear strength of clay core} \]
Some practical applications of UU analysis for clays

3. Footing placed rapidly on clay deposit

\[ \tau = \text{In situ undrained shear strength} \]

Note: UU test simulates the short term condition in the field. Thus, \( c_u \) can be used to analyze the short term behavior of soils.
THE END