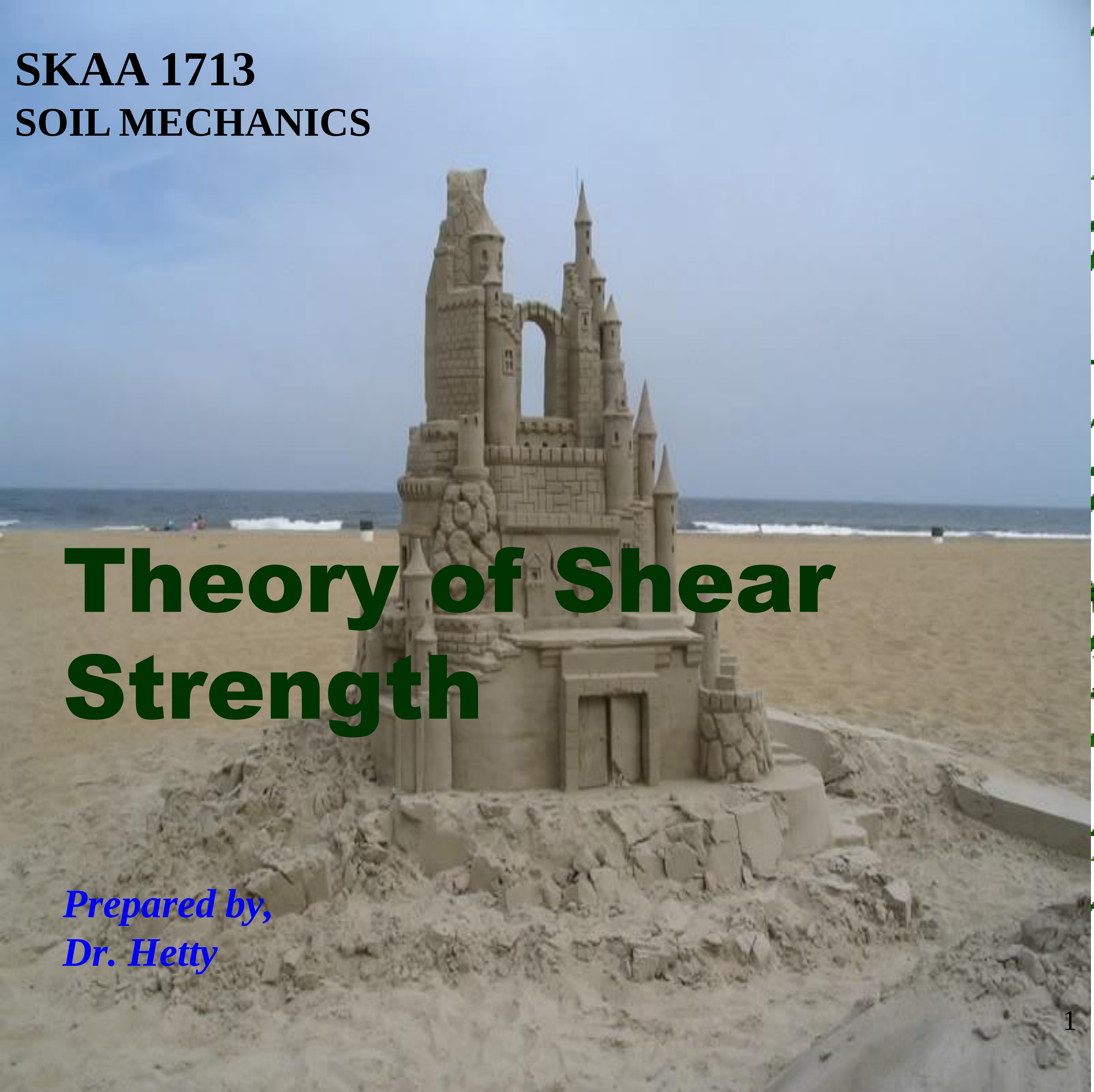
SKAA 1713
SOIL MECHANICS



Theory of Shear Strength

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SOIL STRENGTH

- DEFINITION

  Shear strength of a soil is the maximum internal resistance to applied shearing forces

  The maximum or ultimate stress the material can sustain against the force of landslide, failure, etc.

- APPLICATION

  Soil Strength can be used for calculating :

  – Bearing Capacity of Soil
  – Slope Stability
  – Lateral Pressure

Why it is important???????????

The safety of any geotechnical structure dependent on the strength of the soil. If the soil fails, a structure founded on it can collapse, endangering lives and causing economic damage

SHEAR STRENGTH OF SOIL

- PARAMETER
  - Cohesion (c)
  - Internal Friction Angle (ϕ)
- CONDITION
  - Total (c and ϕ)
  - Effective (c' and ϕ')
- GENERAL EQUATION (COULOMB)

$$\tau = c + \sigma_n . \tan\phi$$

- PRINCIPAL STRESS

σ_1 (major principal stress)

σ_3 (minor principal stress)

SOIL TYPES

- COHESIVE SOIL
  - Has cohesion (c)
  - Example : Clay, Silt
- COHESIONLESS Soil
  - Only has internal friction angle (ϕ) ; c = 0
  - Example : Sand, Gravel

SHEAR STRENGTH PARAMETER

- COHESION (C)

  Sticking together of like materials.

- INTERNAL FRICTION ANGLE (ϕ)

  The stress-dependent component which is similar to sliding friction of two or more soil particles

Factors controlling shear strength of soils

- **soil composition (basic soil material)**: mineralogy, grain size and grain size distribution, shape of particles, pore fluid type and content, ions on grain and in pore fluid.

- **state (initial)**: Define by the initial void ratio, effective normal stress and shear stress (stress history). State can be describe by terms such as: loose, dense, overconsolidated, normally consolidated, stiff, soft, contractive, dilative, etc.

- **structure:** Refers to the arrangement of particles within the soil mass; the manner the particles are packed or distributed. Features such as *layers, joints, fissures, slickensides, voids, pockets, cementation,* etc, are part of the structure. Structure of soils is described by terms such as: *undisturbed, disturbed, remolded, compacted, cemented; flocculent, honey-combed, single-grained; flocculated, deflocculated; stratified, layered, laminated; isotropic and anisotropic.*

- **Loading conditions:** Effective , i.e., drained, and undrained; and type of loading, i.e., magnitude, rate (static, dynamic), and time history (monotonic, cyclic)).

Strength of different materials

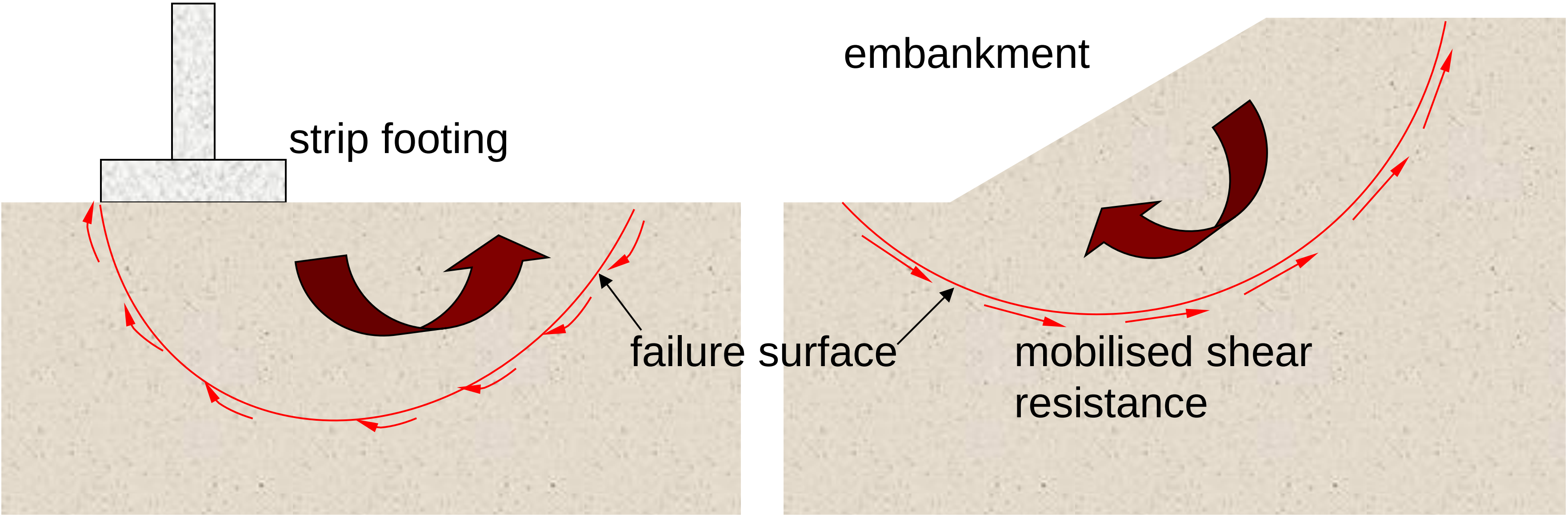
Steel → **Tensile strength**

Concrete → **Compressive strength**

Soil → **Shear strength** → Presence of pore water → Complex behavior

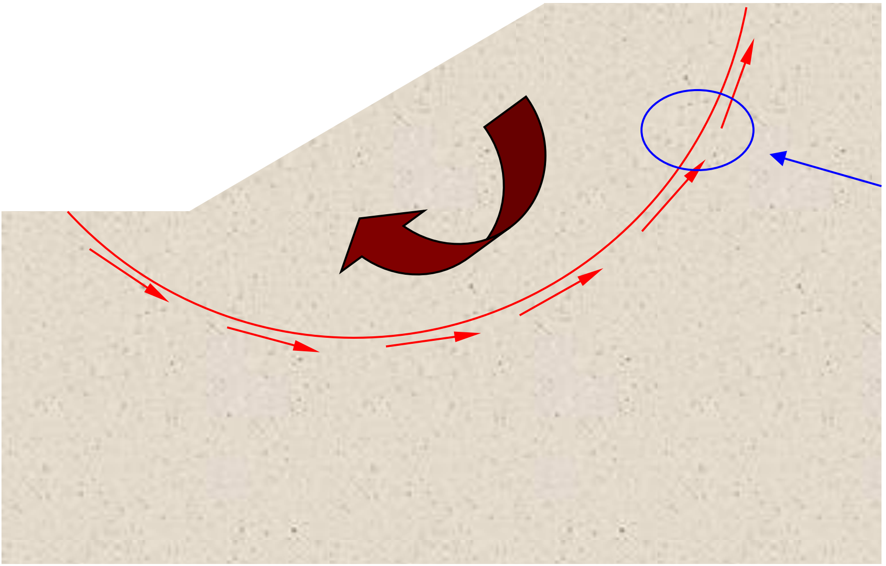
Shear failure

Soils generally fail in <u>shear</u>



At failure, shear stress along the failure surface reaches the shear strength.

Shear failure

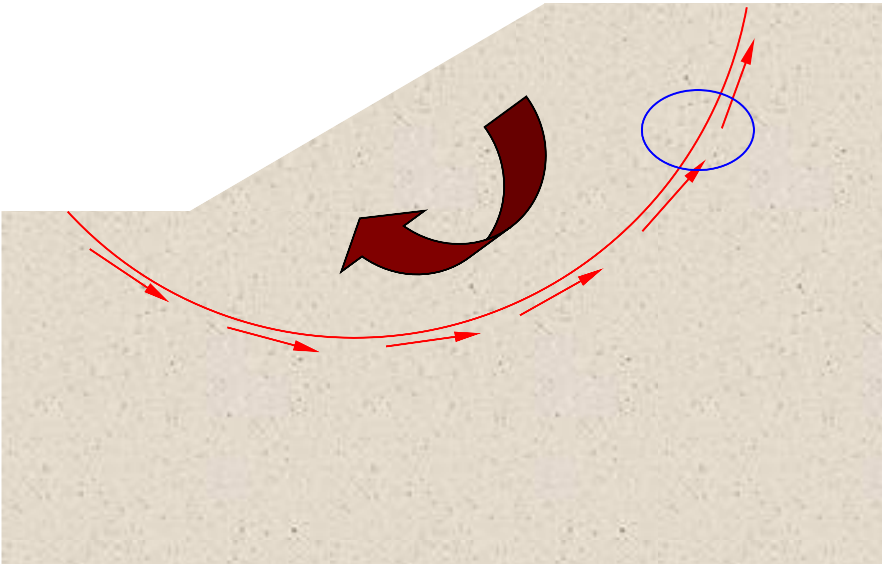


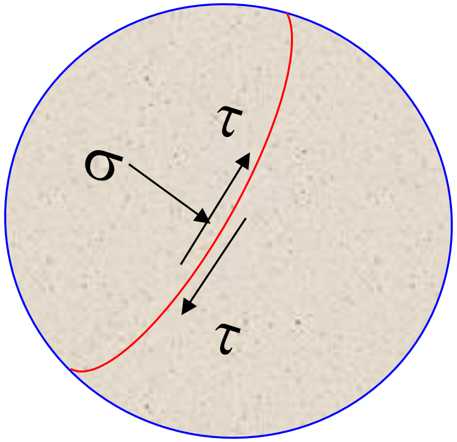
failure surface

The soil grains slide over each other along the failure surface.

No crushing of individual grains.

Shear failure

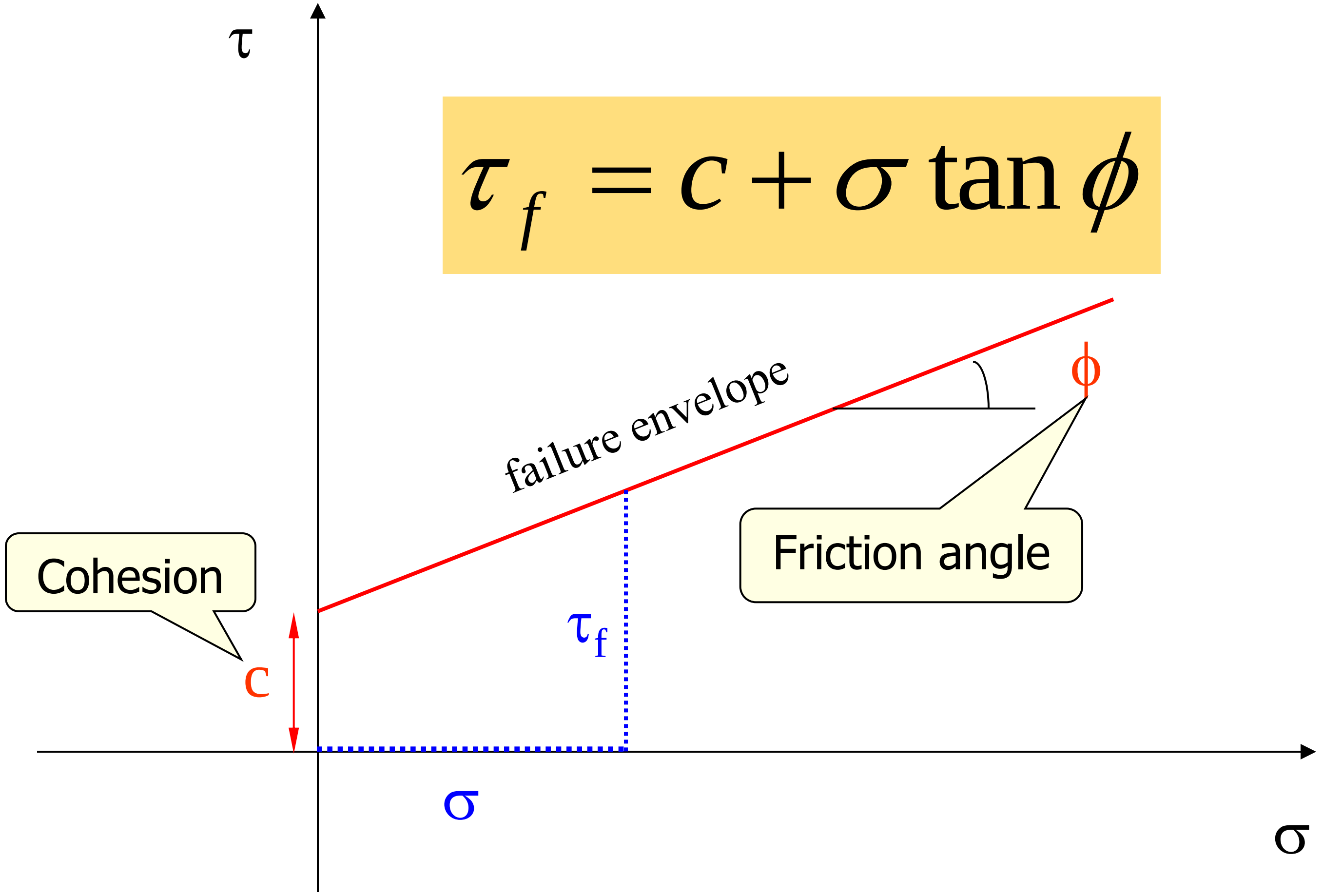




At failure, shear stress along the failure surface (τ) reaches the shear strength (τ_f).

Mohr-Coulomb Failure Criterion

(in terms of total stresses)



τ_f is the maximum shear stress the soil can take without failure, under normal stress of σ.

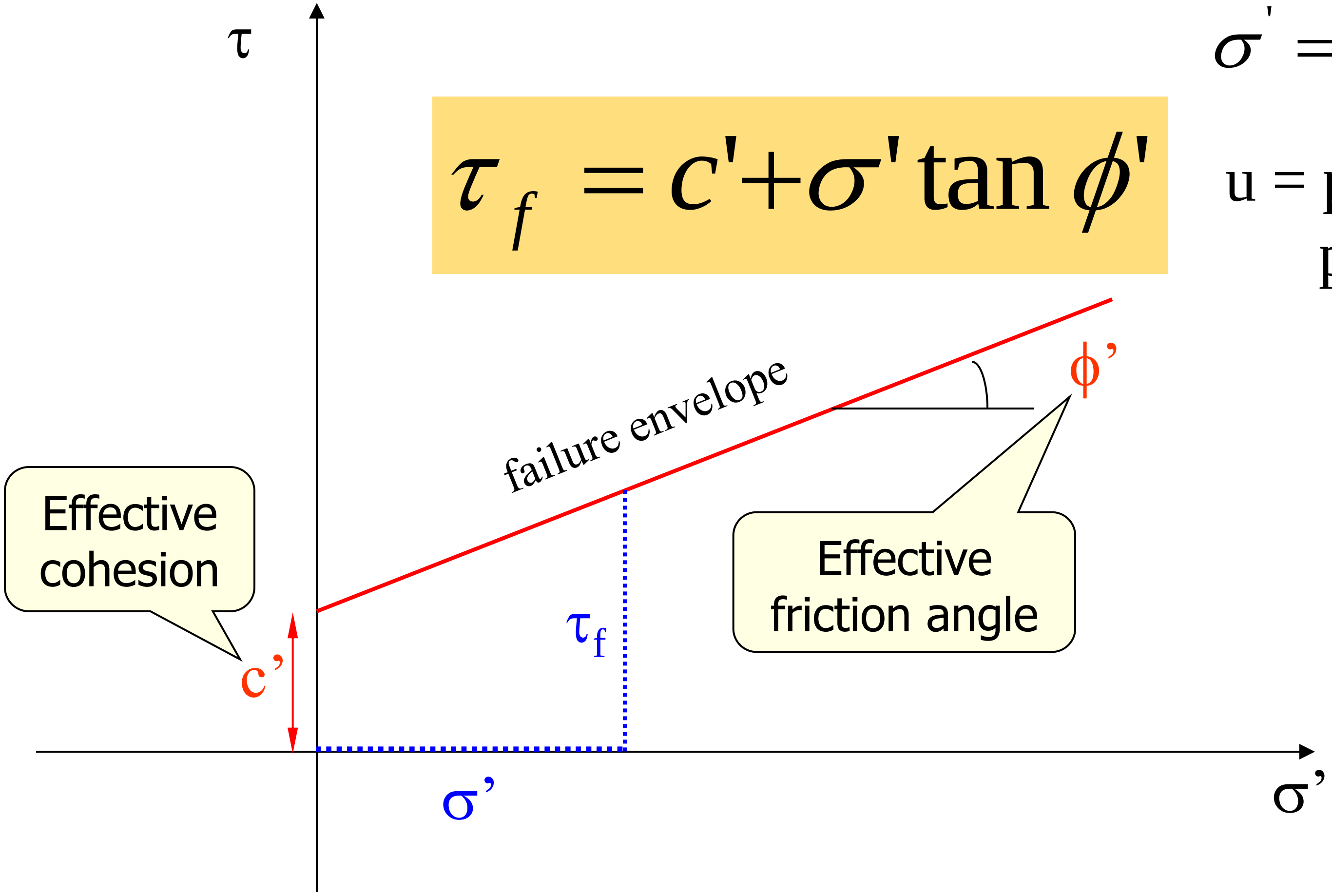
Mohr-Coulomb Failure Criterion

(in terms of effective stresses)

$$\sigma' = \sigma - u$$

$$\tau_f = c' + \sigma' \tan \phi'$$

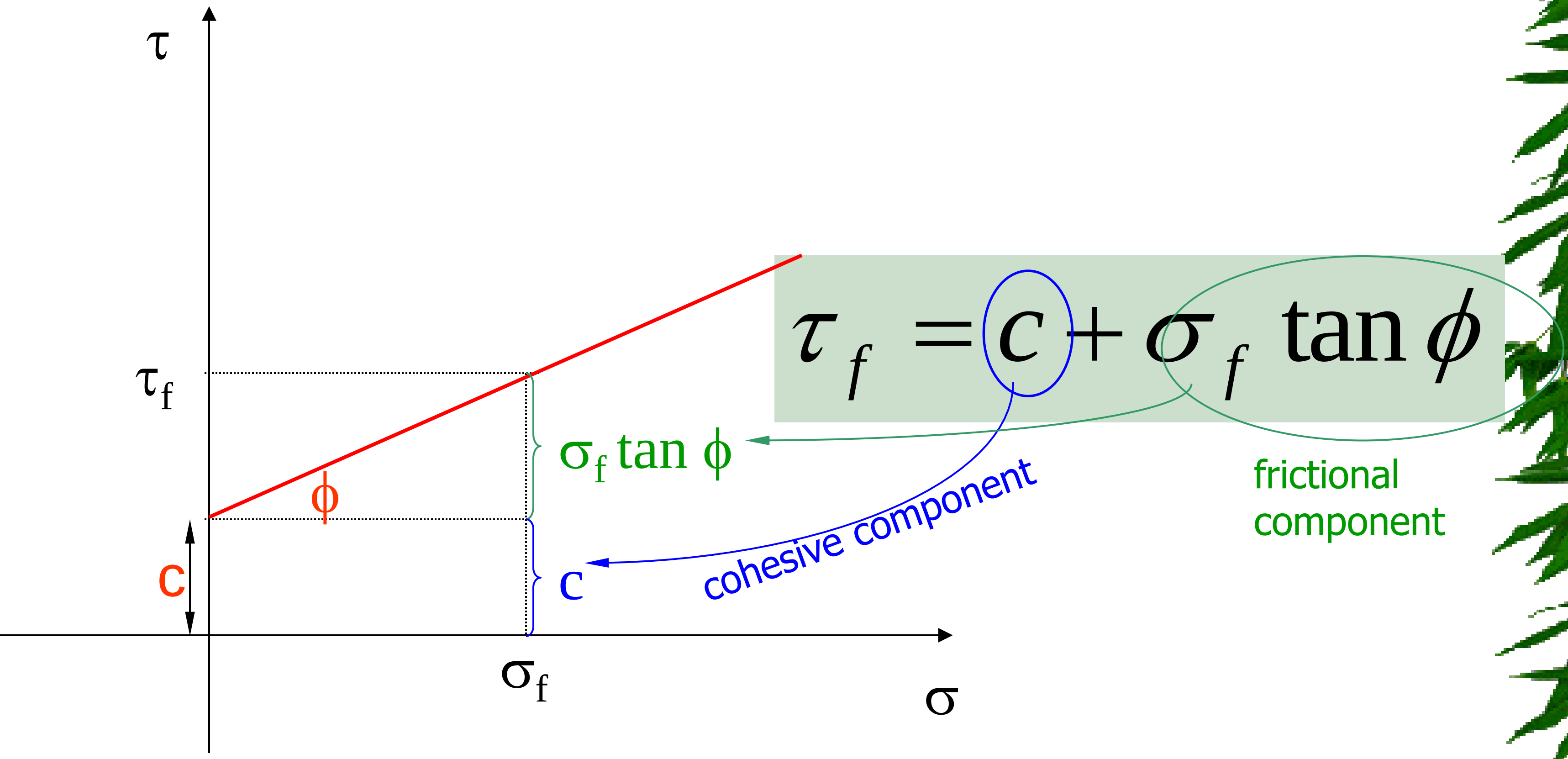
u = pore water pressure



τ_f is the maximum shear stress the soil can take without failure, under normal effective stress of σ'.

Mohr-Coulomb Failure Criterion

Shear strength consists of two components: cohesive and frictional.

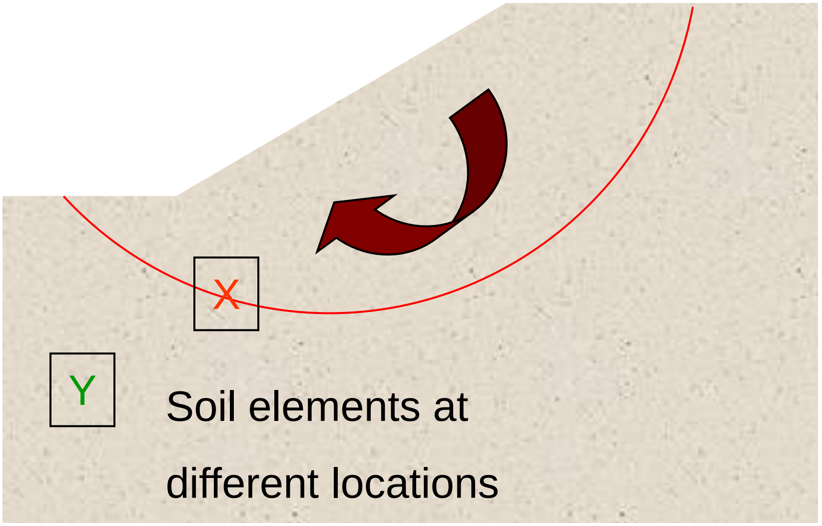


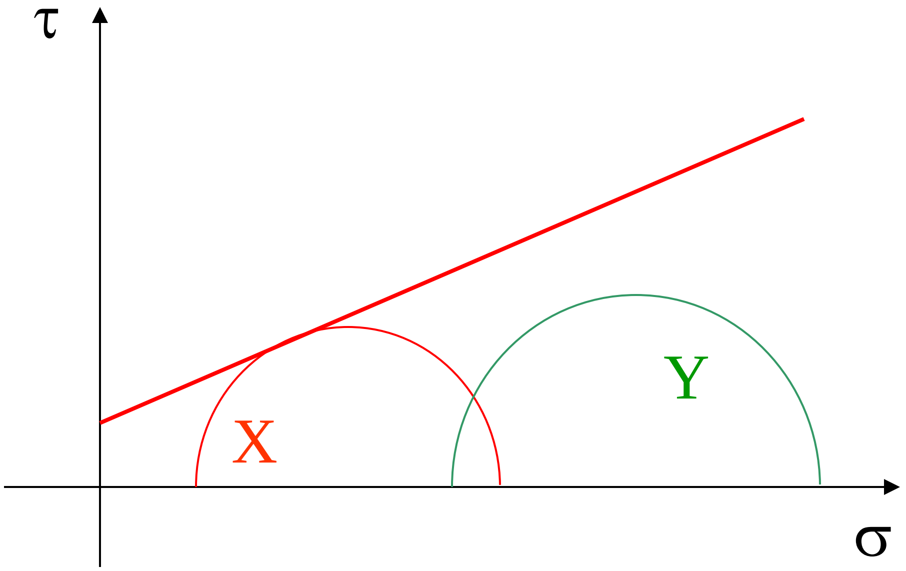
$$\tau_f = c + \sigma_f \tan \phi$$

c and ϕ are measures of shear strength.

Higher the values, higher the shear strength.

Mohr Circles & Failure Envelope



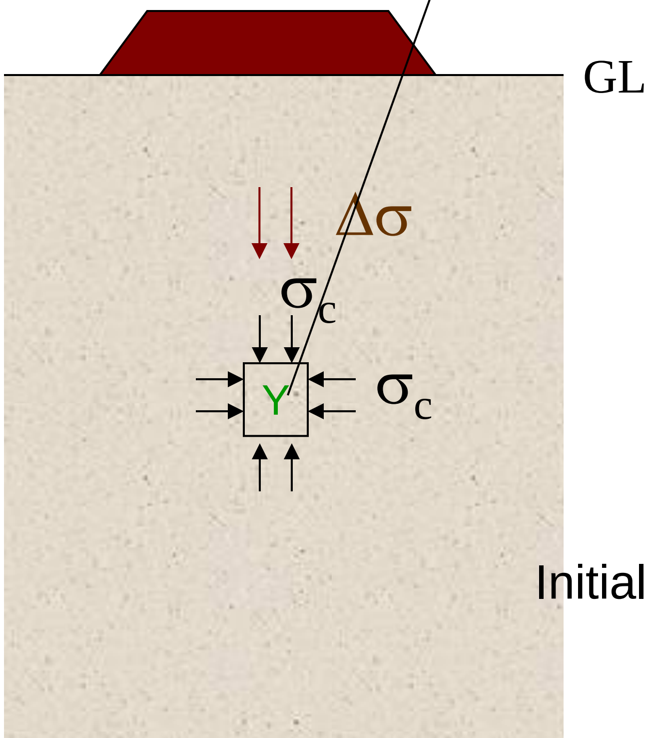


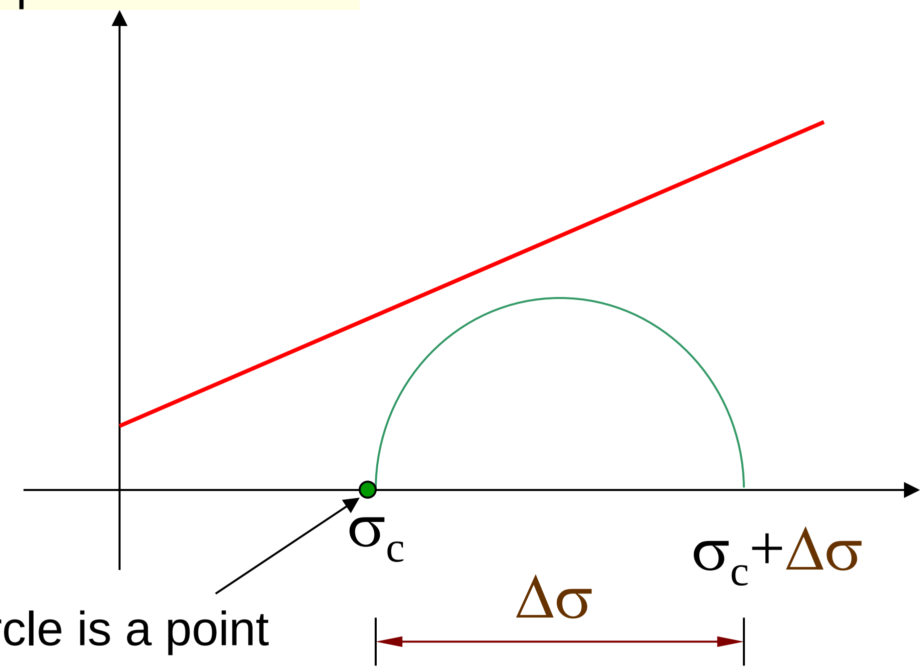
X ~ failure

Y ~ stable

Mohr Circles & Failure Envelope

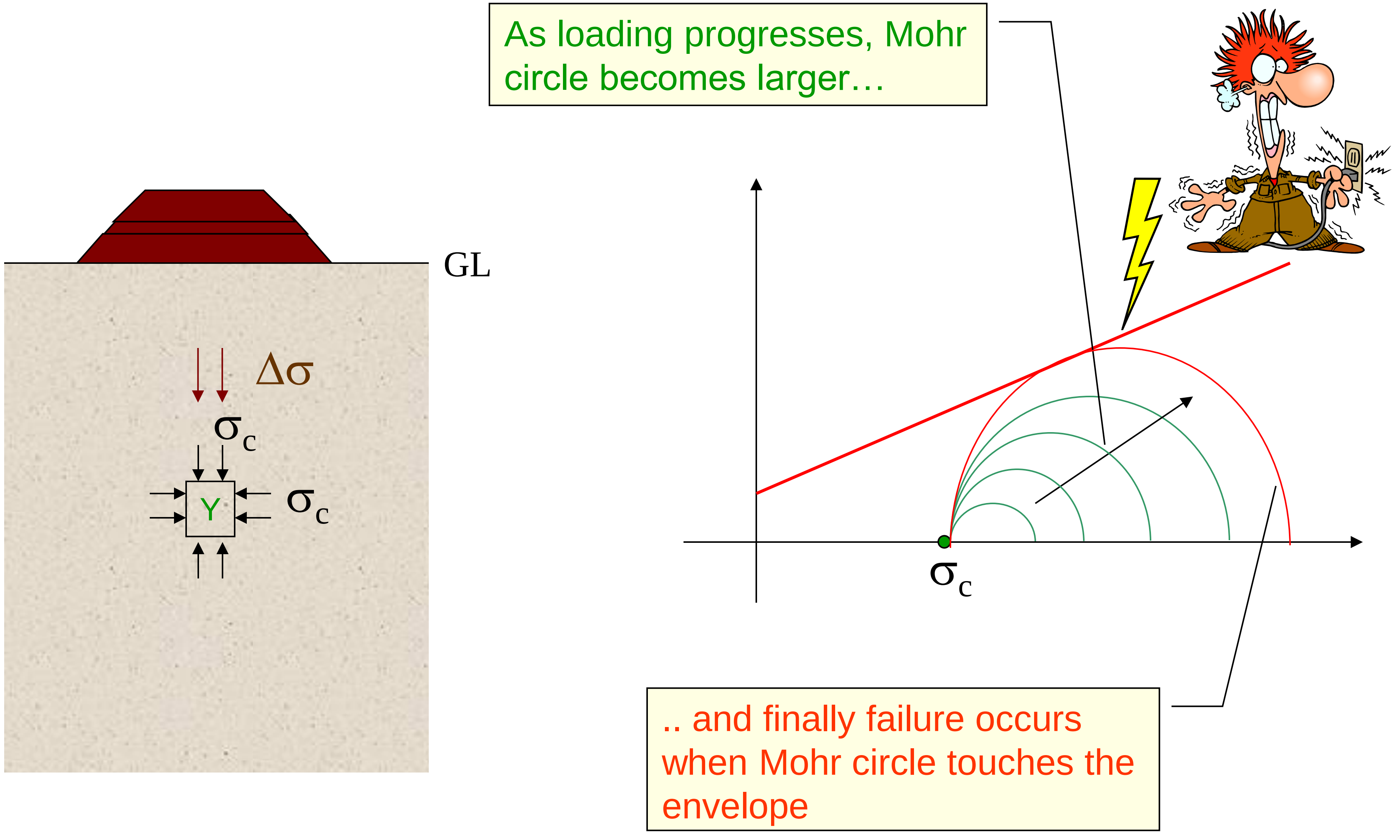
The soil element does not fail if the Mohr circle is contained within the envelope



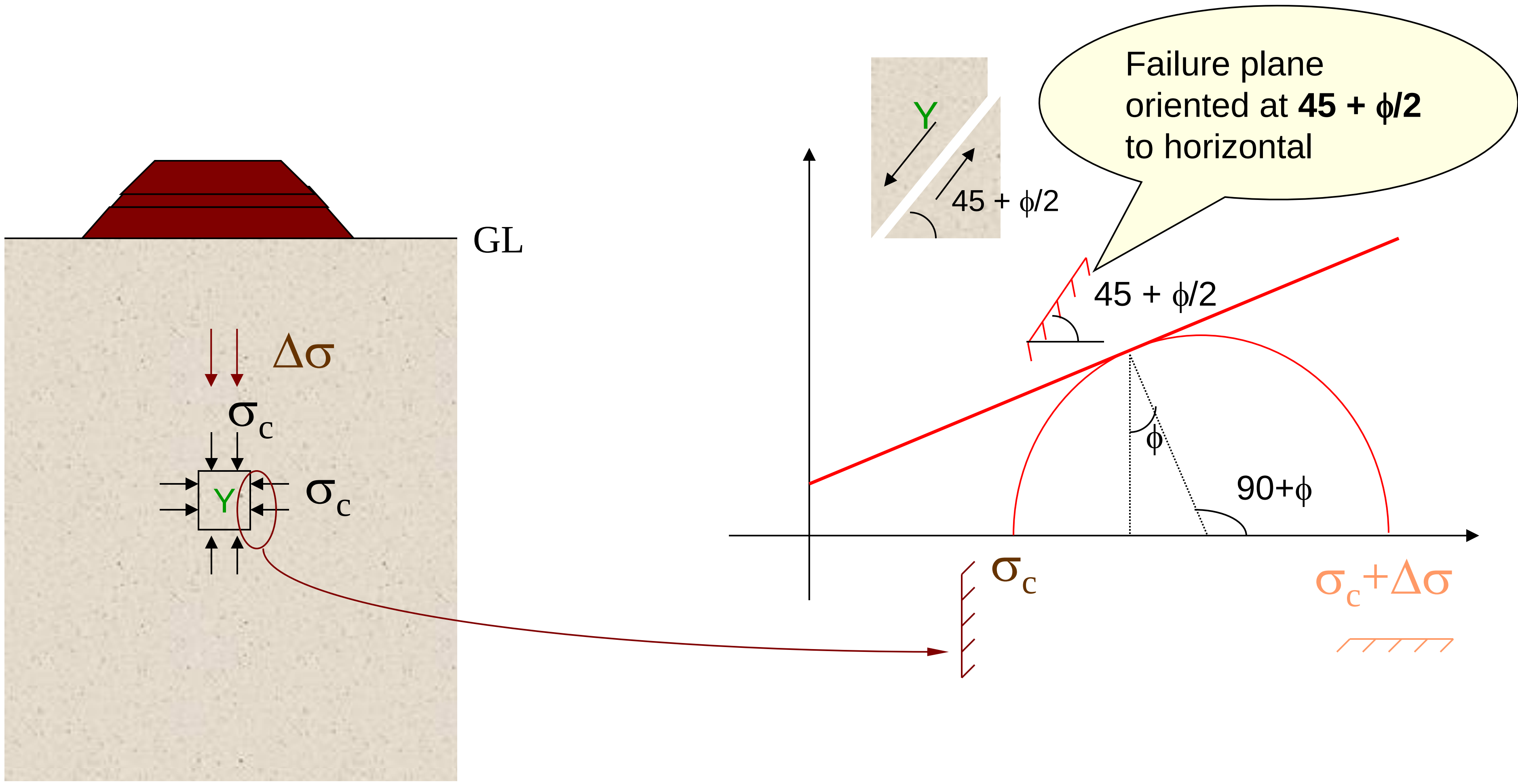


Initially, Mohr circle is a point

Mohr Circles & Failure Envelope

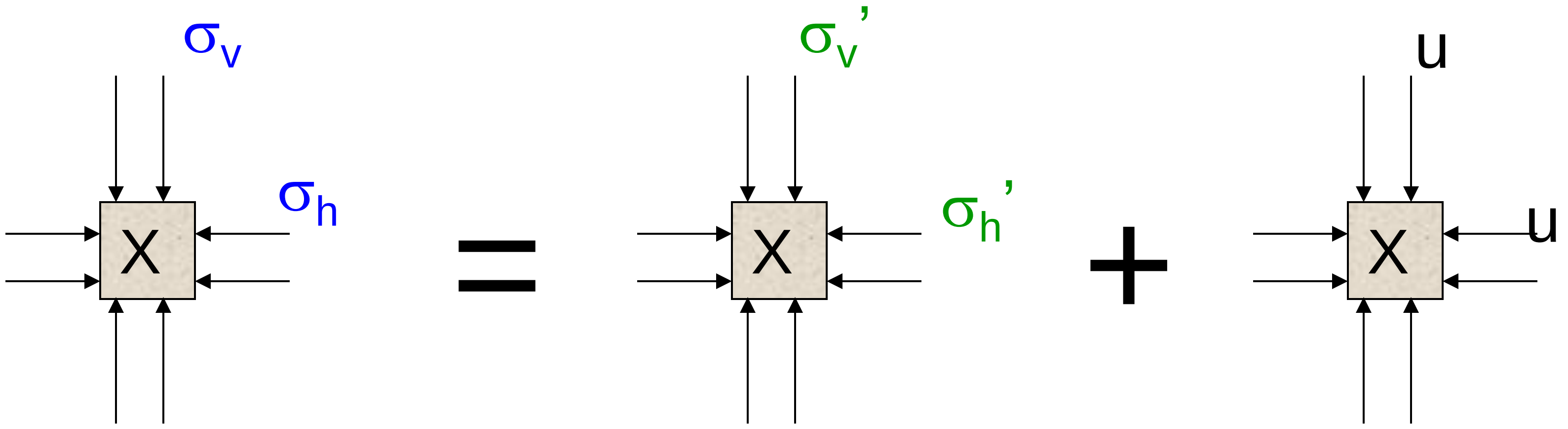


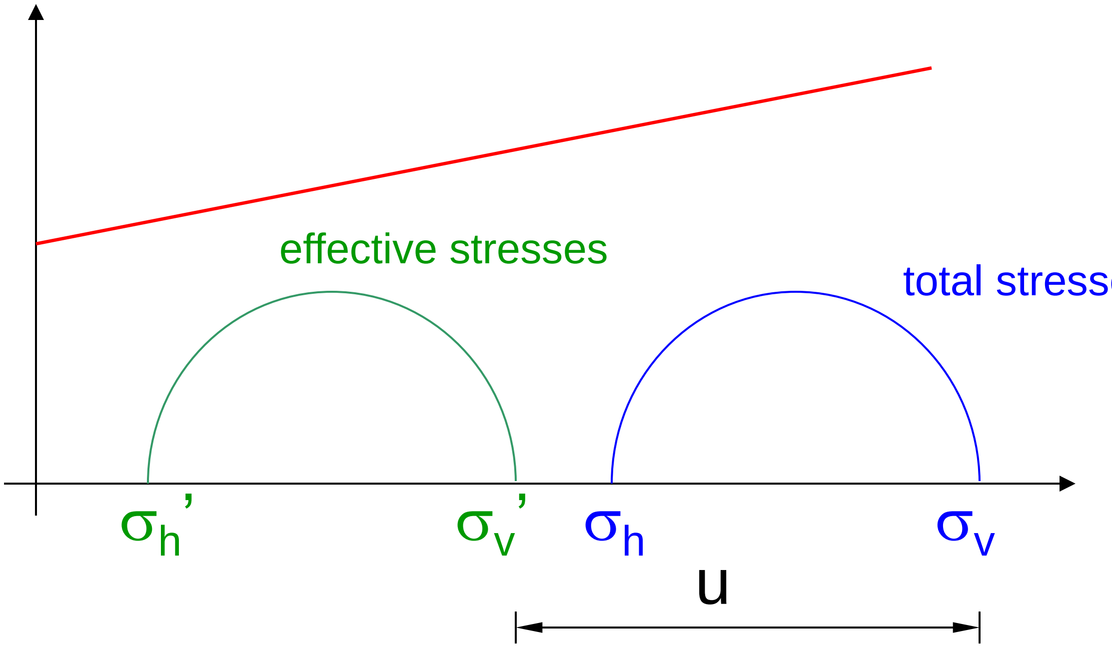
Orientation of Failure Plane



$$\theta = 45 + \phi/2$$

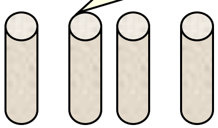
Mohr circles in terms of σ & σ'

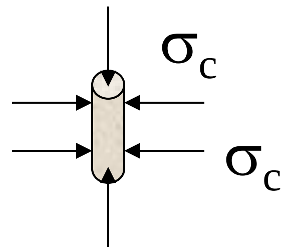




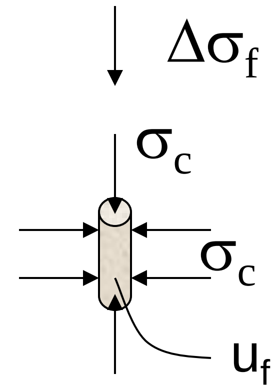
Envelopes in terms of σ & σ'

Identical specimens initially subjected to different isotropic stresses (σ_c) and then loaded axially to failure





Initially…

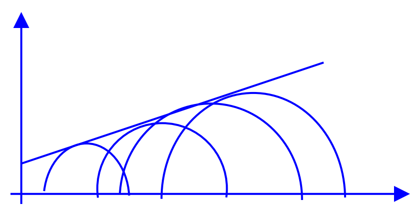


Failure

At failure,

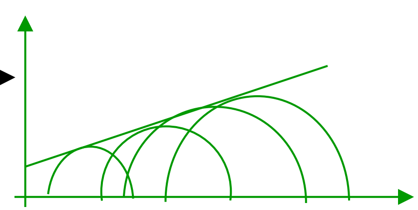
$\sigma_3 = \sigma_c$; $\sigma_1 = \sigma_c + \Delta\sigma_f$

$\sigma_3' = \sigma_3 - u_f$; $\sigma_1' = \sigma_1 - u_f$



c, ϕ

in terms of σ

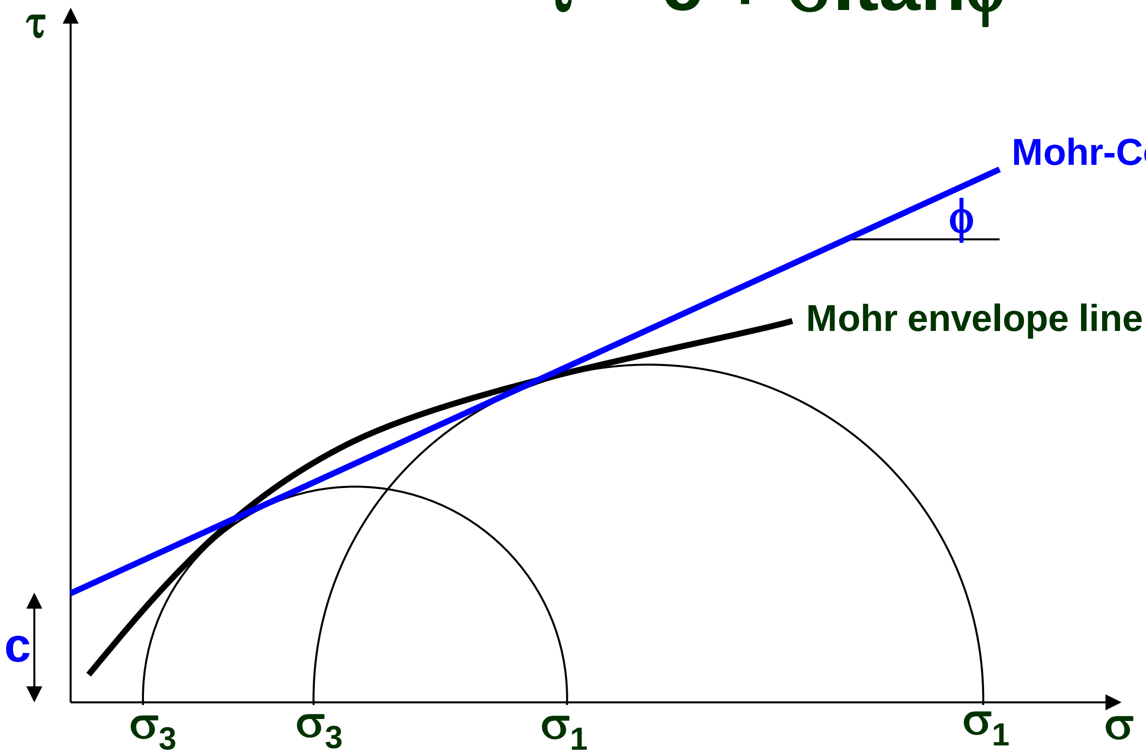


c', ϕ'

in terms of σ'

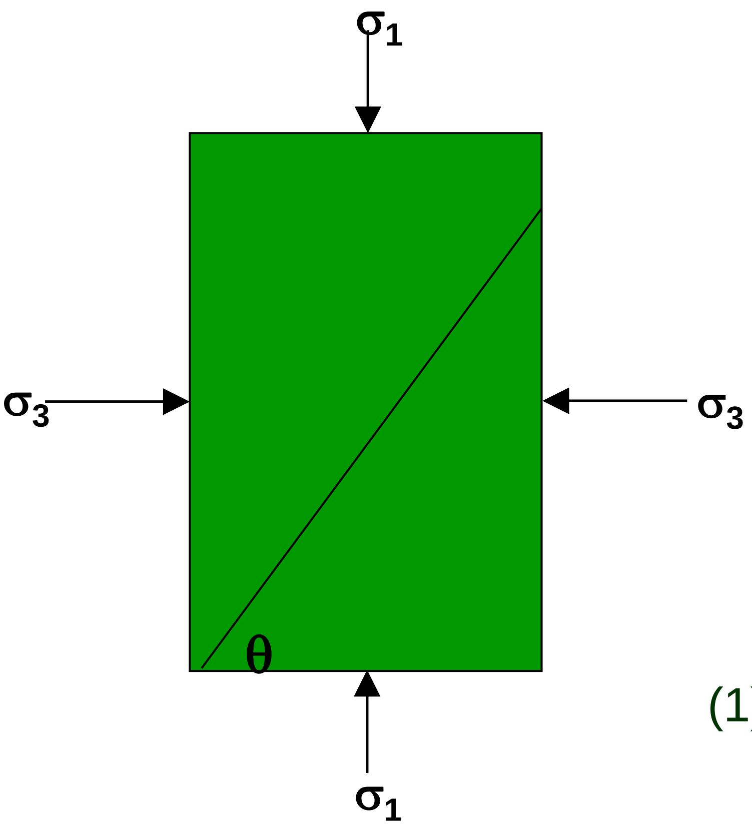
MOHR COULOMB CONCEPT

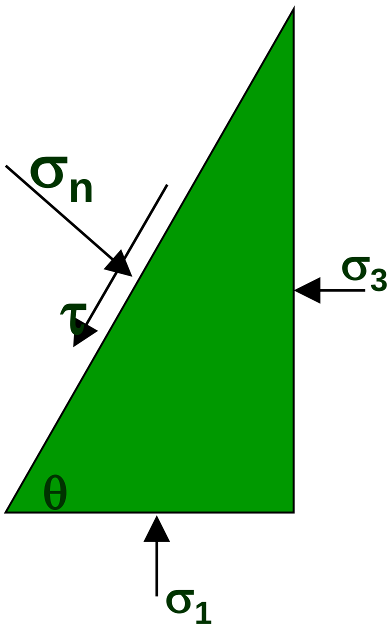
$$\tau = c + \sigma . \tan\phi$$



$$\sigma_1 = \sigma_3 + \Delta\sigma$$

MOHR COULOMB CONCEPT





(1) $$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}.Cos2\theta$$

@

$$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3)kos^2\theta$$

$\sigma_1 > \sigma_3$

(2) $$\tau = \frac{\sigma_1 - \sigma_3}{2}.Sin2\theta$$

MOHR COULOMB CONCEPT

(1) and (2) → $\tau = c + \sigma_n . \tan\phi$

$$\sigma_1 = \sigma_3 + \frac{(\sigma_3 \ . \ \tan\phi + c)}{(0.5 \ . \ Sin2\theta - Cos^2\theta \ . \ \tan\phi)}$$

The failure occurs when the value of σ_1 is minimum or the value of (0.5 . Sin2θ - Cos$^2\theta$. tanϕ) maximum

$$\theta = 45^o + \frac{\phi}{2} \longrightarrow \sigma_1 = \sigma_3 . \tan^2(45^o + \phi/2) + 2.c.\tan(45^o + \phi/2)$$

Equations.......

$$\tau_f = c + \sigma \tan\phi$$

$$\theta = 45 + \phi/2$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2}.\mathrm{Sin}2\theta$$

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}.Cos2\theta$$

$$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3)\mathrm{kos}^2\theta$$

$$\sigma_1 = \sigma_3.\tan^2\left(45^o + \phi/2\right) + 2.c.\tan\left(45^o + \phi/2\right)$$

Example 1

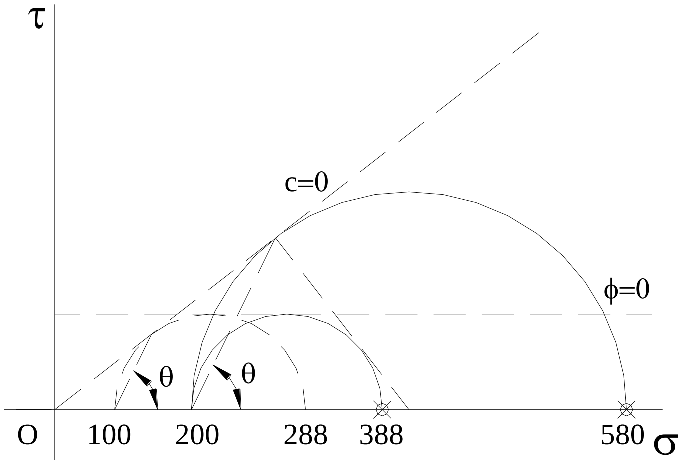
A soil failed at σ_3 = 100 kN/m2 and σ_1 = 288 kN/m^2. If the same soil is given σ_3 = 200 kN/m^2, what is the value of the new σ_1 when the failure is for:

i) Cohesive soil

ii) Cohesionless soil

Solution 1

Graphically



a.From the Mohr's circle,

$\sigma_1 = 388\ kN/m^2$

b. From the Mohr's circle,

$\sigma_1 = 580\ kN/m^2$

Solution 1

Analytically

i) Cohesive soil ($\phi = 0°$)

$$\theta = 45 + 0/2 = 45°$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2}.Sin2\theta$$

τ = (288 – 100) / 2 . Sin 2 (45) = 94 kPa

$$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3)kos^2\theta$$

σ_n = 100 + (288 – 100 $_3$)kos^{2}45 = 194 kPa

Solution 1

Analytically

for soil, where σ_3 *= 200 kPa*

$$\sigma_1 = \sigma_3 . \tan^2\left(45^o + \phi/2\right) + 2.c.\tan\left(45^o + \phi/2\right)$$

σ_1 =200 tan^2 (45 – 0/2) + 2(94)tan (45)
= ????? kPa

Solution 1

Analytically

i) Cohesionless soil (c = 0)

From graph:

τ_f = 80 kPa & σ_n=145 kPa

$$\tau_f = c + \sigma \tan \phi$$

$\phi = \tan^{-1} (80/145) = 29°$

$$\sigma_1 = \sigma_3 . \tan^2\left(45^o + \phi/2\right) + 2.c.\tan\left(45^o + \phi/2\right)$$

σ_1 =200 $\tan^2$ (45 – 29/2) + 0 = 576 kPa

Example 2

<u>Given:</u>

C= 86 kPa

ϕ= 17°

σ_3= 70 kPa

σ_1= 346 kPa

Determine angle of failure (θ), shear tress at failure (τ_f) and mormal stress at failure (σ_n)

Solution 2

$$\theta = 45 + \phi/2$$

$$\theta = 45 + 17/2 = 53.5°$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2} . Sin 2\theta$$

τ = (346 – 70) / 2 . Sin 2 (53.5) = 132 kPa

$$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3) kos^2\theta$$

$\sigma_n = 70 + (346 - 70) kos^2 53.5 = 167$ kPa