MAJ 1013
ADVANCED SOIL MECHANICS

Theory of Shear Strength

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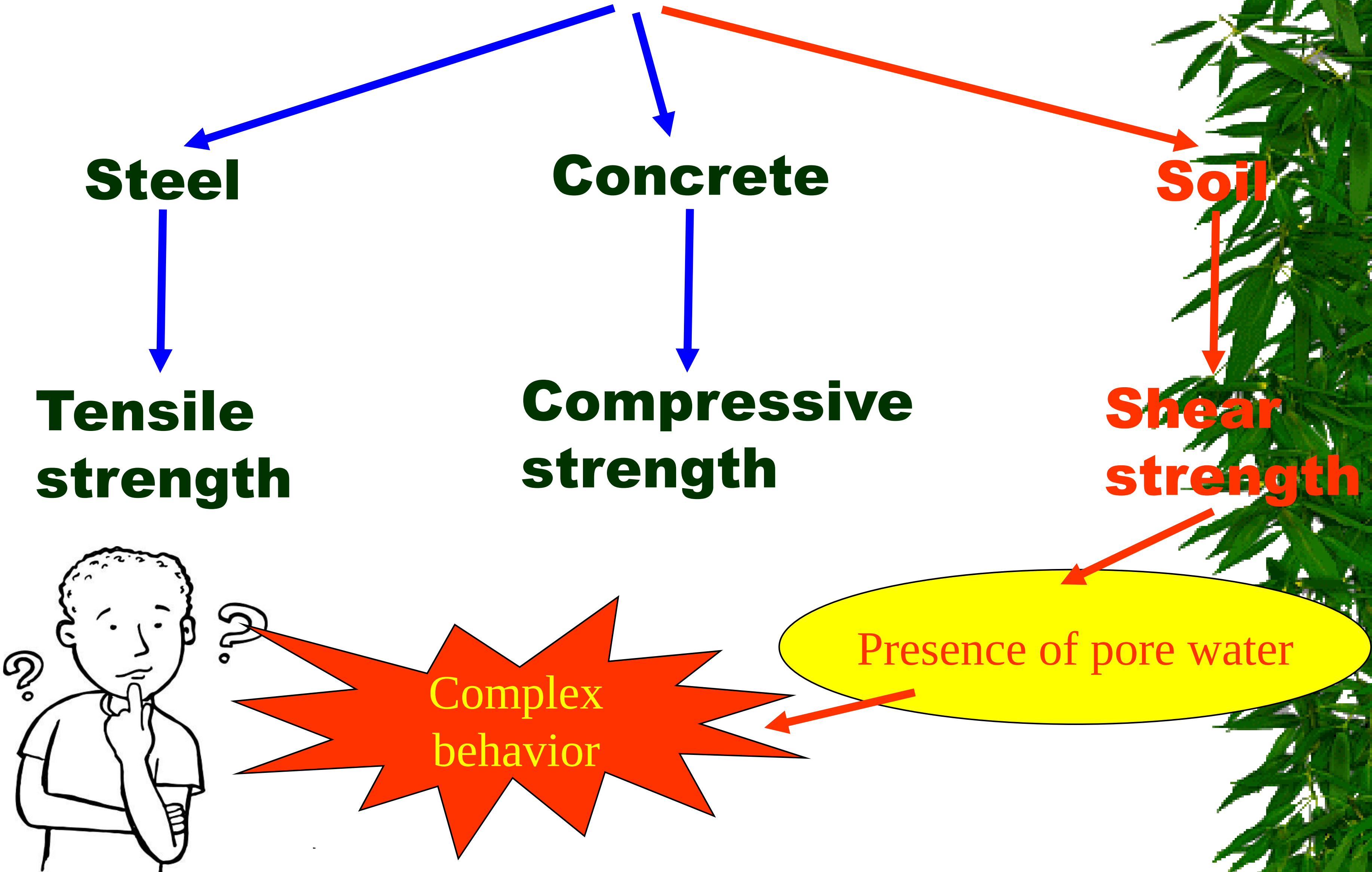
Strength of different materials



SOIL STRENGTH

- DEFINITION

  Shear strength of a soil is the maximum internal resistance to applied shearing forces

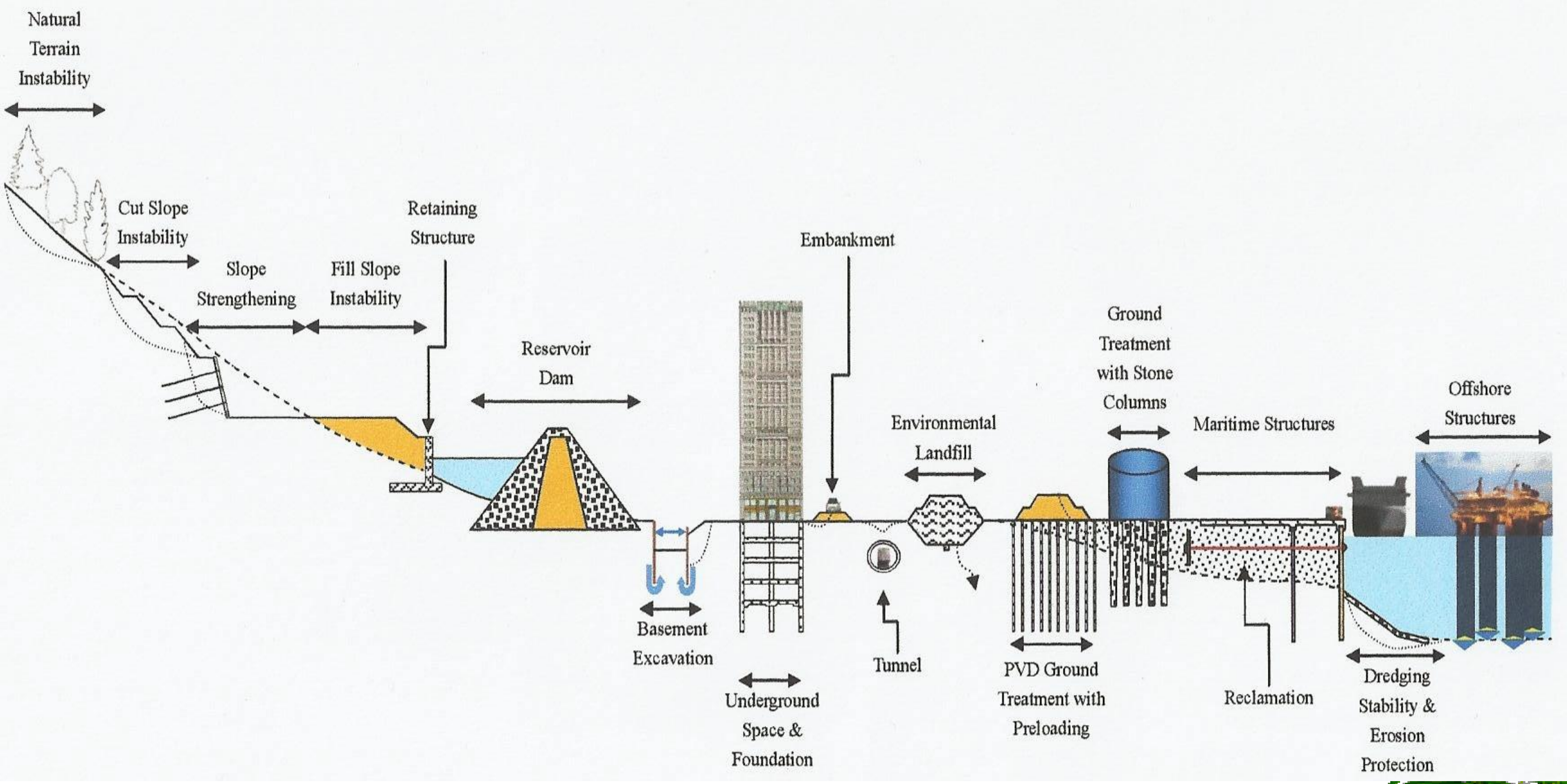
  The maximum or ultimate stress the material can sustain against the force of landslide, failure, etc.

- APPLICATION

  Soil Strength can be used for calculating :

  – Bearing Capacity of Soil
  – Slope Stability
  – Lateral Pressure

Why it is important??????????



SHEAR STRENGTH OF SOIL

- PARAMETER
  - Cohesion (c)
  - Internal Friction Angle (ϕ)
- CONDITION
  - Total (c and ϕ)
  - Effective (c' and ϕ')
- GENERAL EQUATION (COULOMB)

$$\tau = c + \sigma_n . \tan\phi$$

SOIL TYPES

- COHESIVE SOIL
  - Has cohesion (c)
  - Example : Clay, Silt
- COHESIONLESS Soil
  - Only has internal friction angle (ϕ) ; c = 0
  - Example : Sand, Gravel

SHEAR STRENGTH PARAMETER

- COHESION (C)

  Sticking together of like materials.

- INTERNAL FRICTION ANGLE (ϕ)

  The stress-dependent component which is similar to sliding friction of two or more soil particles

Factors controlling shear strength of soils

- **soil composition (basic soil material)**: mineralogy, grain size and grain size distribution, shape of particles, pore fluid type and content, ions on grain and in pore fluid.

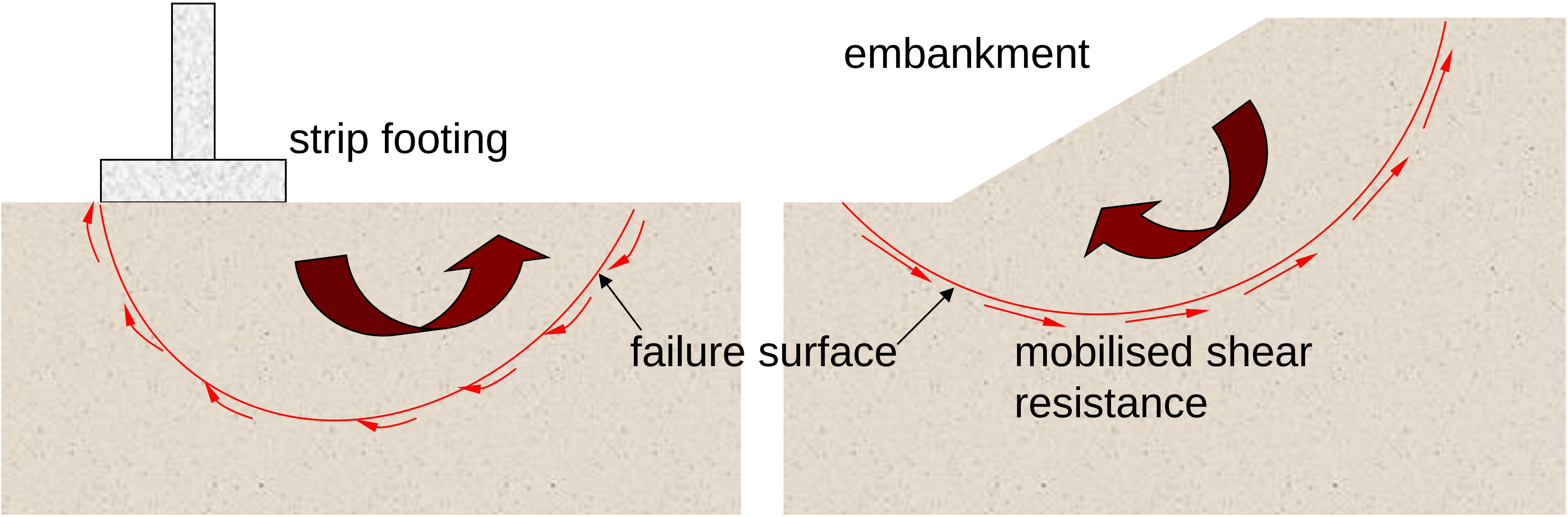
- **state (initial)**: Define by the initial void ratio, effective normal stress and shear stress (stress history). State can be describe by terms such as: loose, dense, overconsolidated, normally consolidated, stiff, soft, contractive, dilative, etc.

- **structure:** Refers to the arrangement of particles within the soil mass; the manner the particles are packed or distributed. Features such as layers, joints, fissures, slickensides, voids, pockets, cementation, etc, are part of the structure. Structure of soils is described by terms such as: undisturbed, disturbed, remolded, compacted, cemented; flocculent, honey-combed, single-grained; flocculated, deflocculated; stratified, layered, laminated; isotropic and anisotropic.

- **Loading conditions:** Effective , i.e., drained, and undrained; and type of loading, i.e., magnitude, rate (static, dynamic), and time history (monotonic, cyclic)).

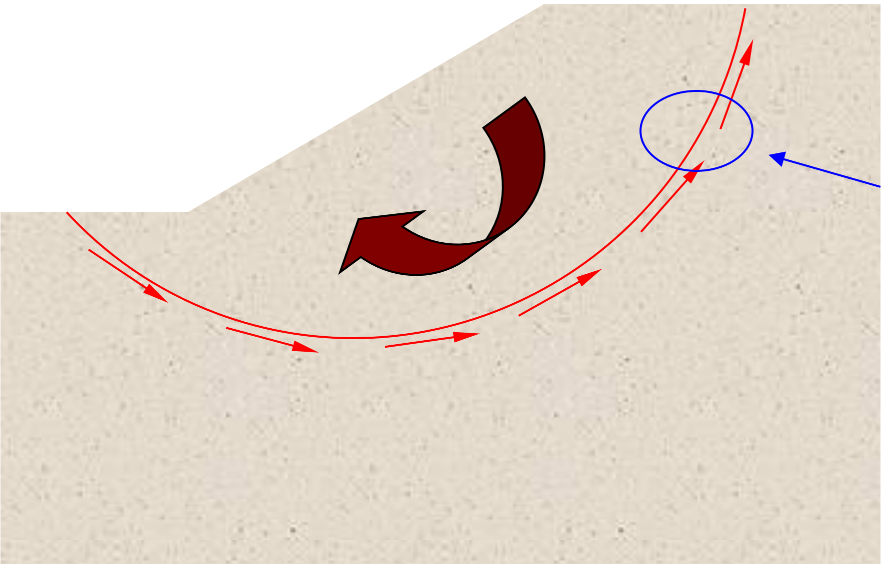
Shear failure

Soils generally fail in shear



At failure, shear stress along the failure surface reaches the shear strength.

Shear failure

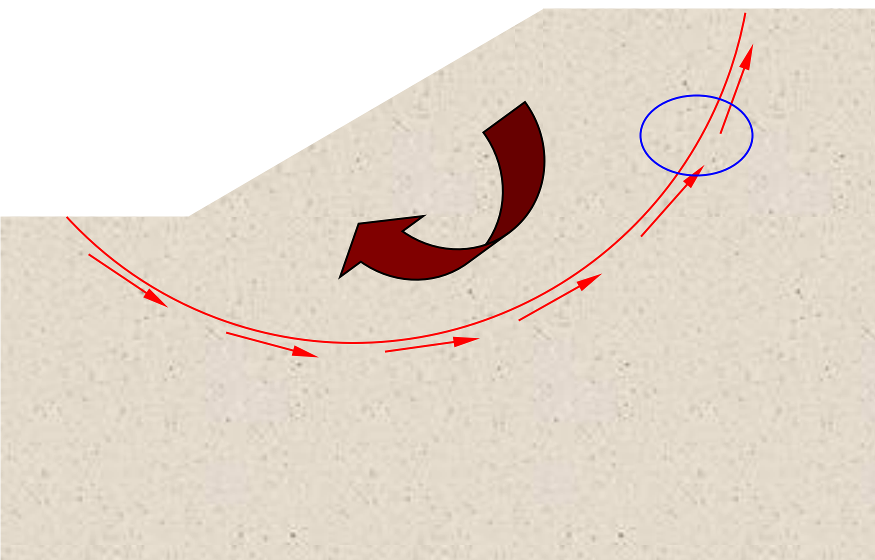


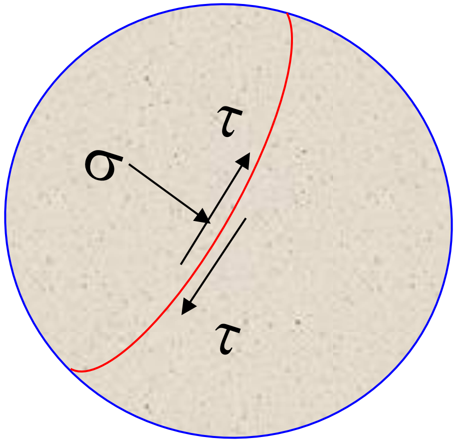
failure surface

The soil grains slide over each other along the failure surface.

No crushing of individual grains.

Shear failure



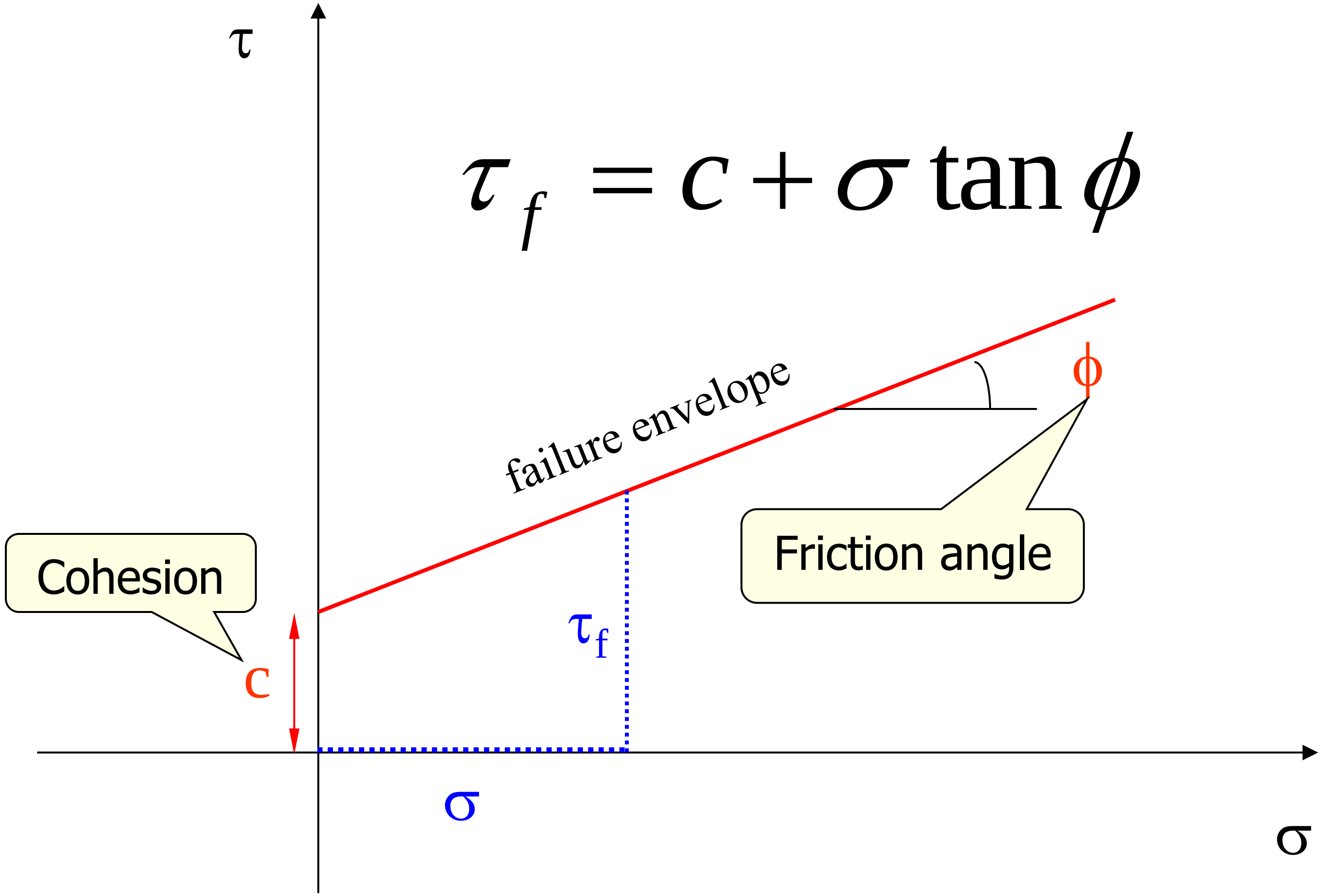


At failure, shear stress along the failure surface (τ) reaches the shear strength (τ_f).

Mohr-Coulomb Failure Criterion

(in terms of total stresses)

$$\tau_f = c + \sigma \tan \phi$$



τ_f is the maximum shear stress the soil can take without failure, under normal stress of σ.

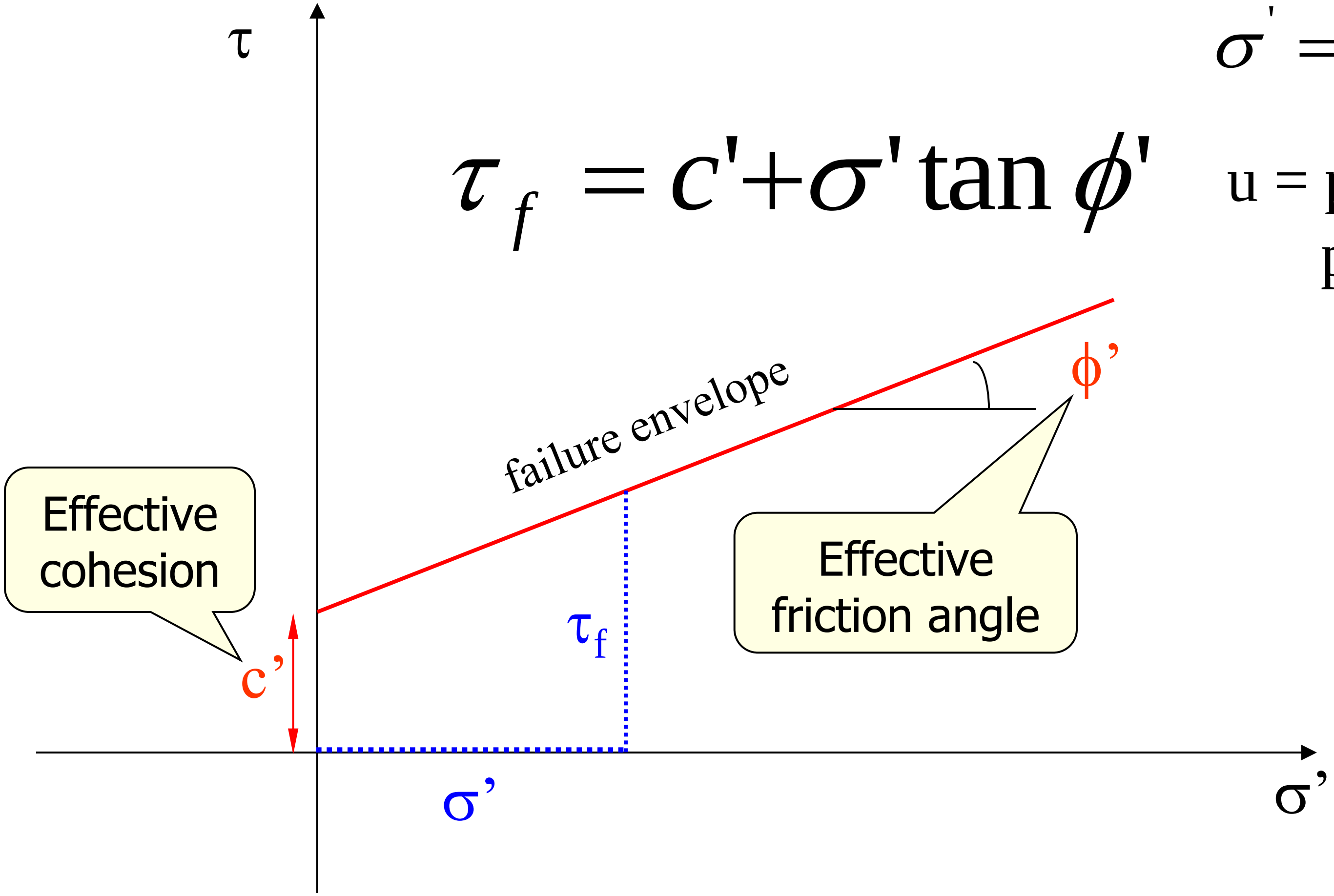
Mohr-Coulomb Failure Criterion

(in terms of effective stresses)

$$\sigma' = \sigma - u$$

$$\tau_f = c' + \sigma' \tan \phi'$$

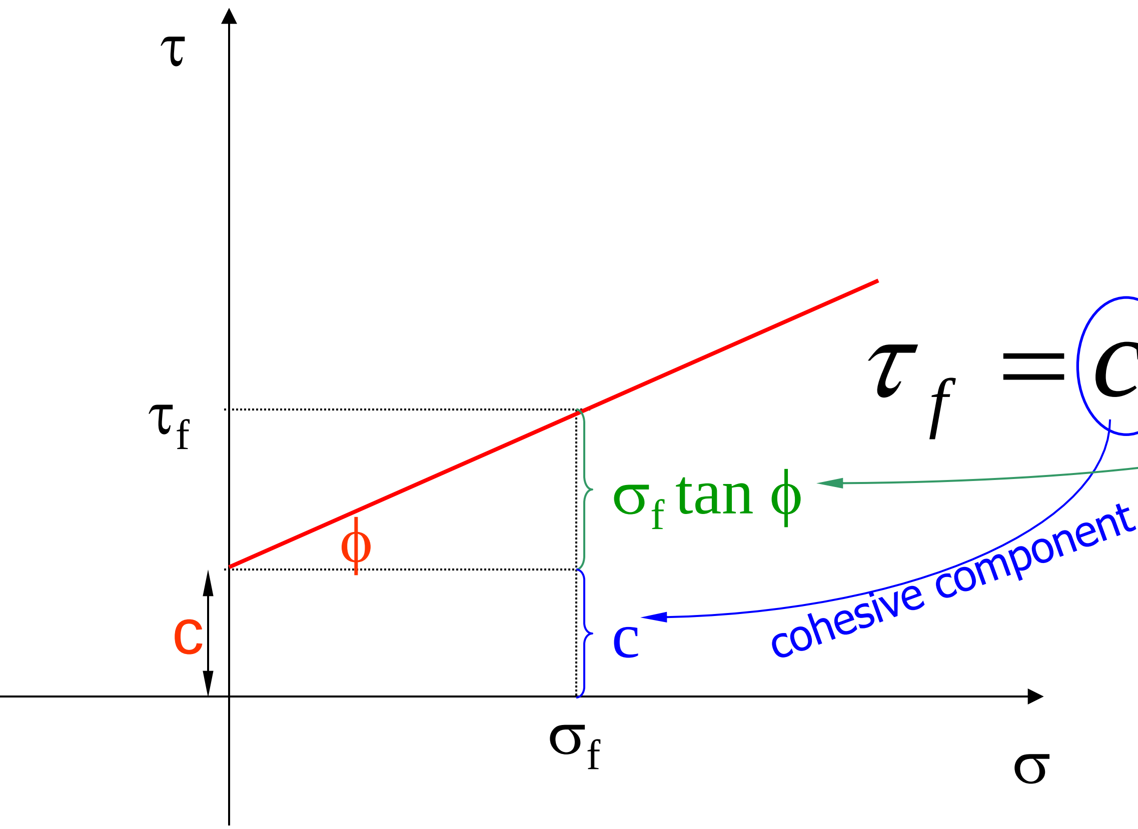
u = pore water pressure



τ_f is the maximum shear stress the soil can take without failure, under normal effective stress of σ'.

Mohr-Coulomb Failure Criterion

Shear strength consists of two components: cohesive and frictional.



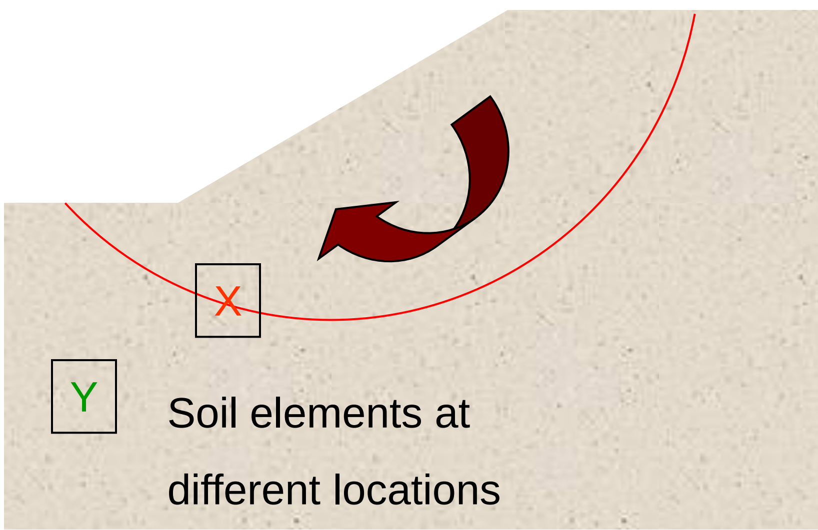
$$\tau_f = c + \sigma_f \tan \phi$$

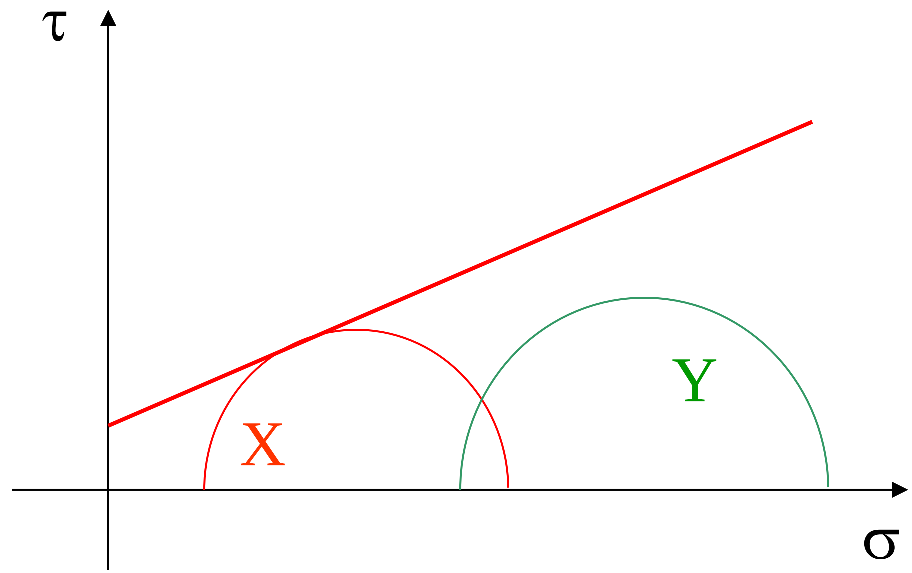
frictional component

c and ϕ are measures of shear strength.

Higher the values, higher the shear strength.

Mohr Circles & Failure Envelope



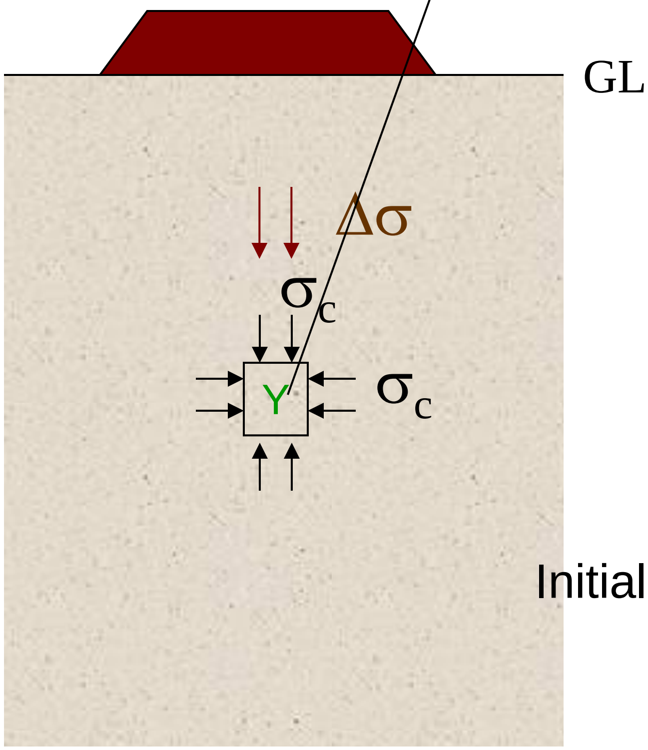


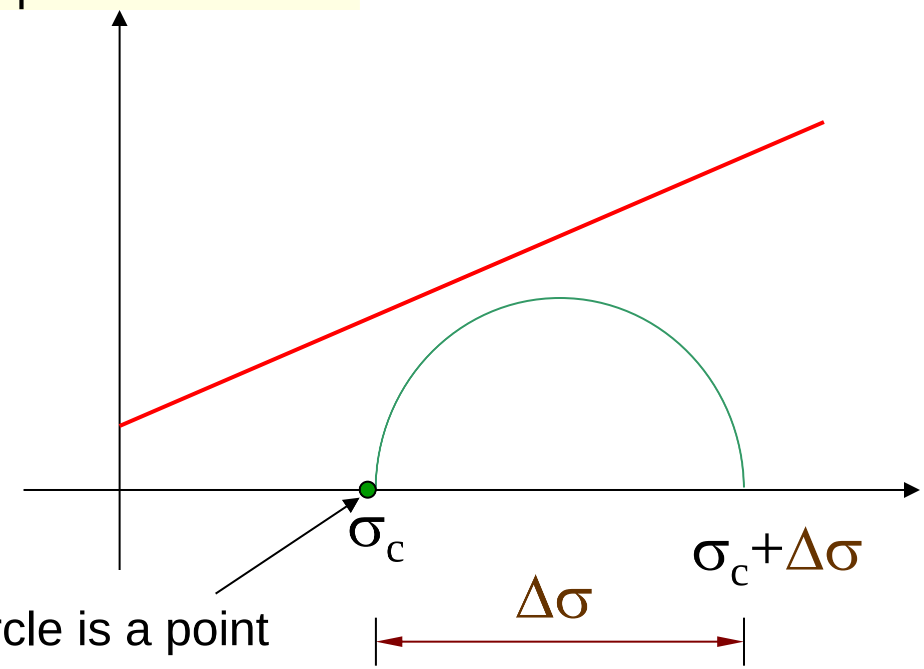
X ~ failure

Y ~ stable

Mohr Circles & Failure Envelope

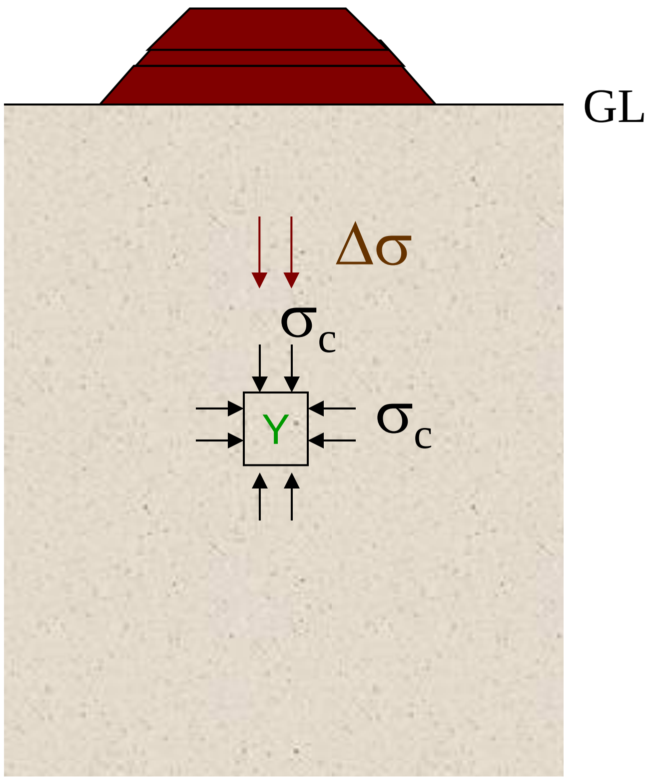
The soil element does not fail if the Mohr circle is contained within the envelope



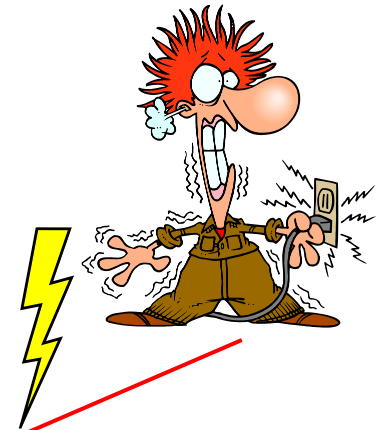


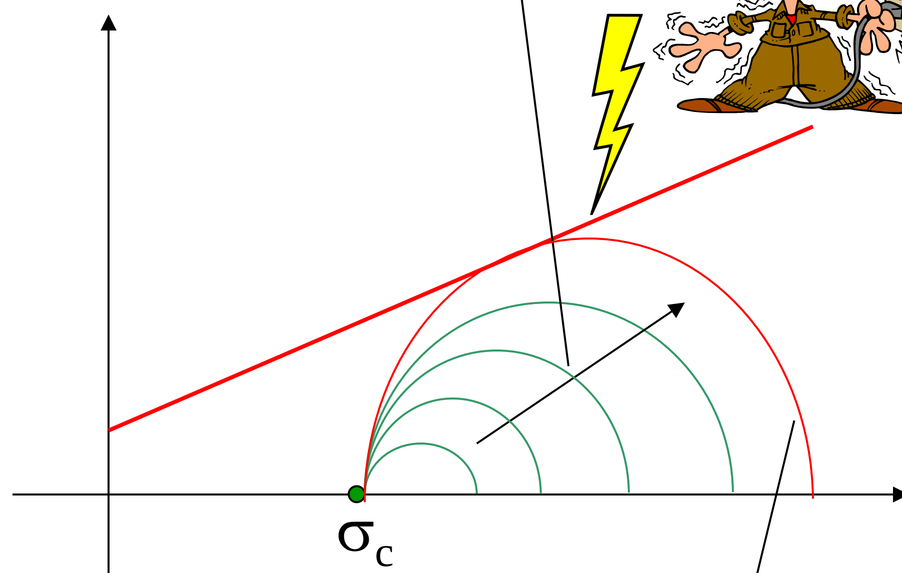
Initially, Mohr circle is a point

Mohr Circles & Failure Envelope



As loading progresses, Mohr circle becomes larger…

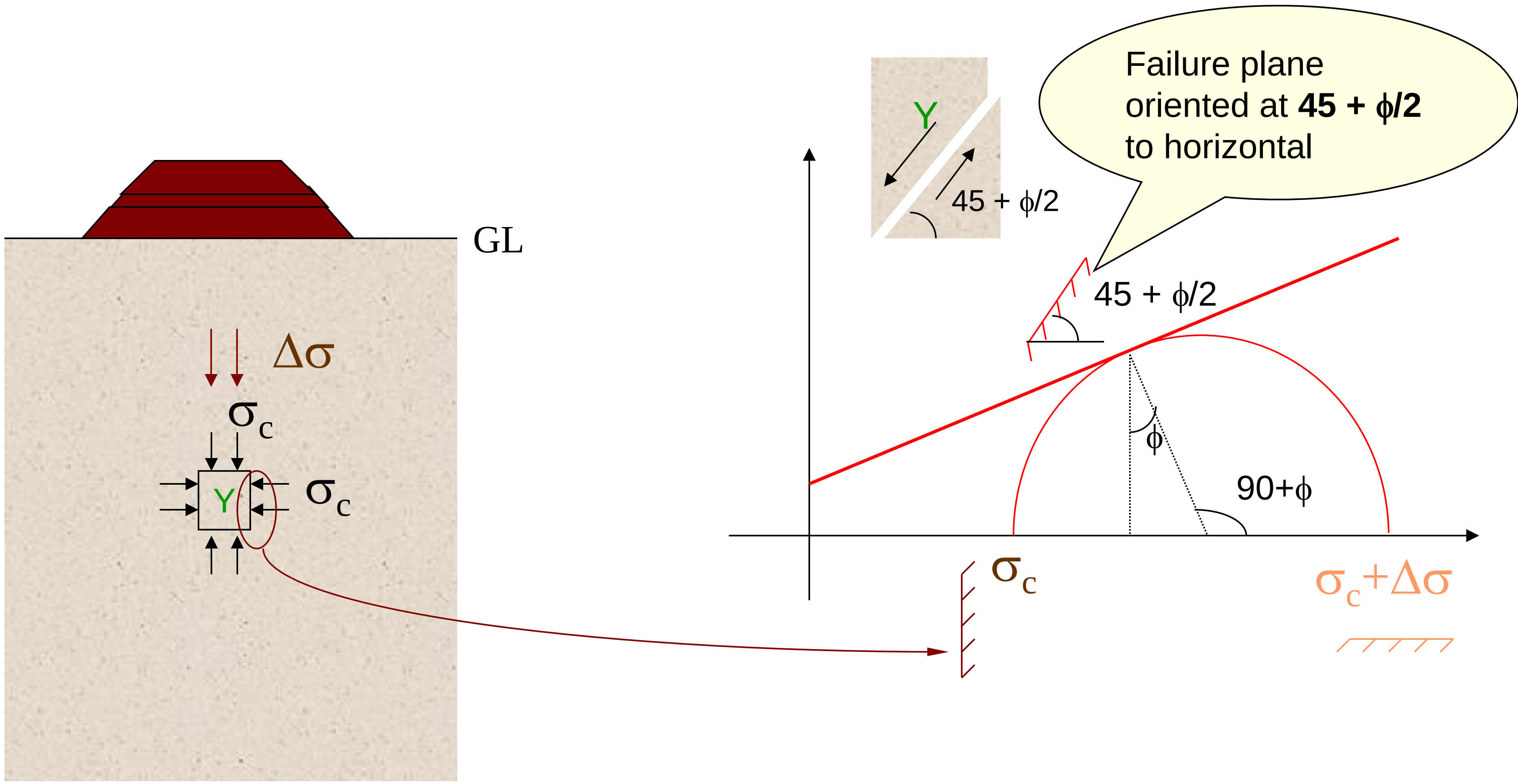




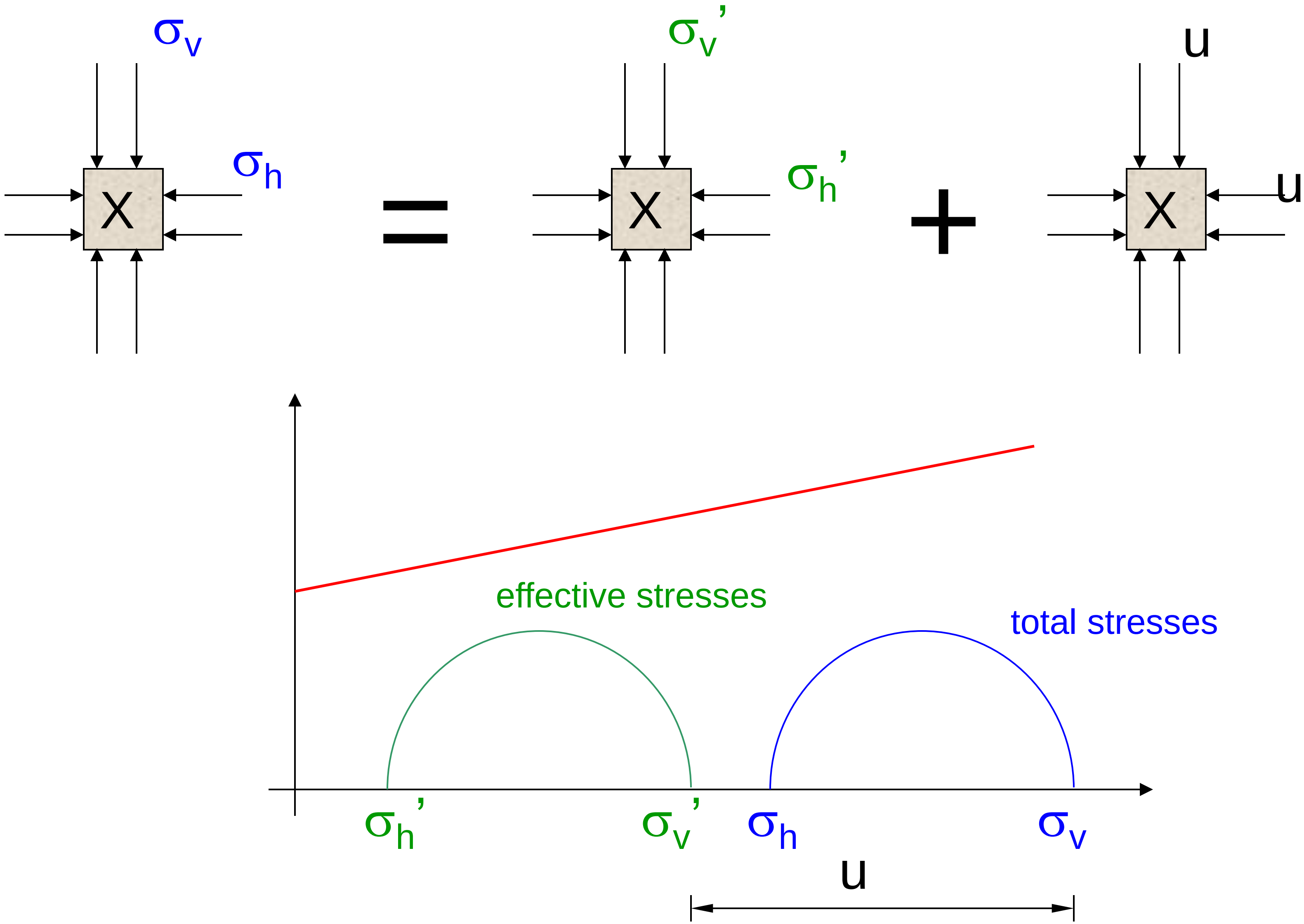
.. and finally failure occurs when Mohr circle touches the envelope

Shear Failure in Soils

Orientation of Failure Plane

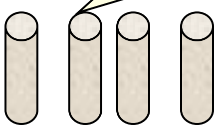


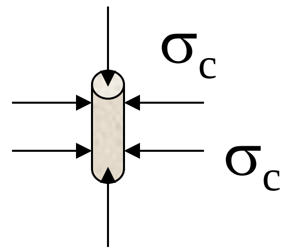
Mohr circles in terms of σ & σ'



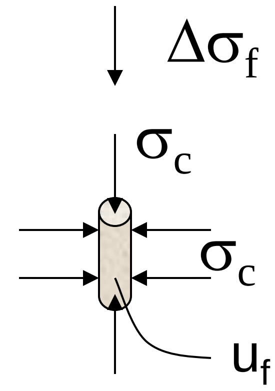
Envelopes in terms of σ & σ'

Identical specimens initially subjected to different isotropic stresses (σ_c) and then loaded axially to failure





Initially…

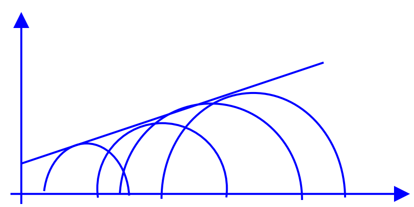


Failure

At failure,

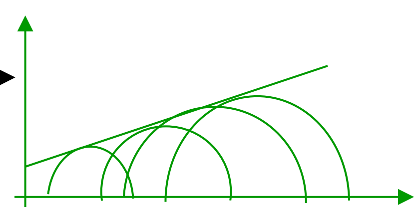
$\sigma_3 = \sigma_c$; $\sigma_1 = \sigma_c + \Delta\sigma_f$

$\sigma_3' = \sigma_3 - u_f$; $\sigma_1' = \sigma_1 - u_f$



c, ϕ

in terms of σ

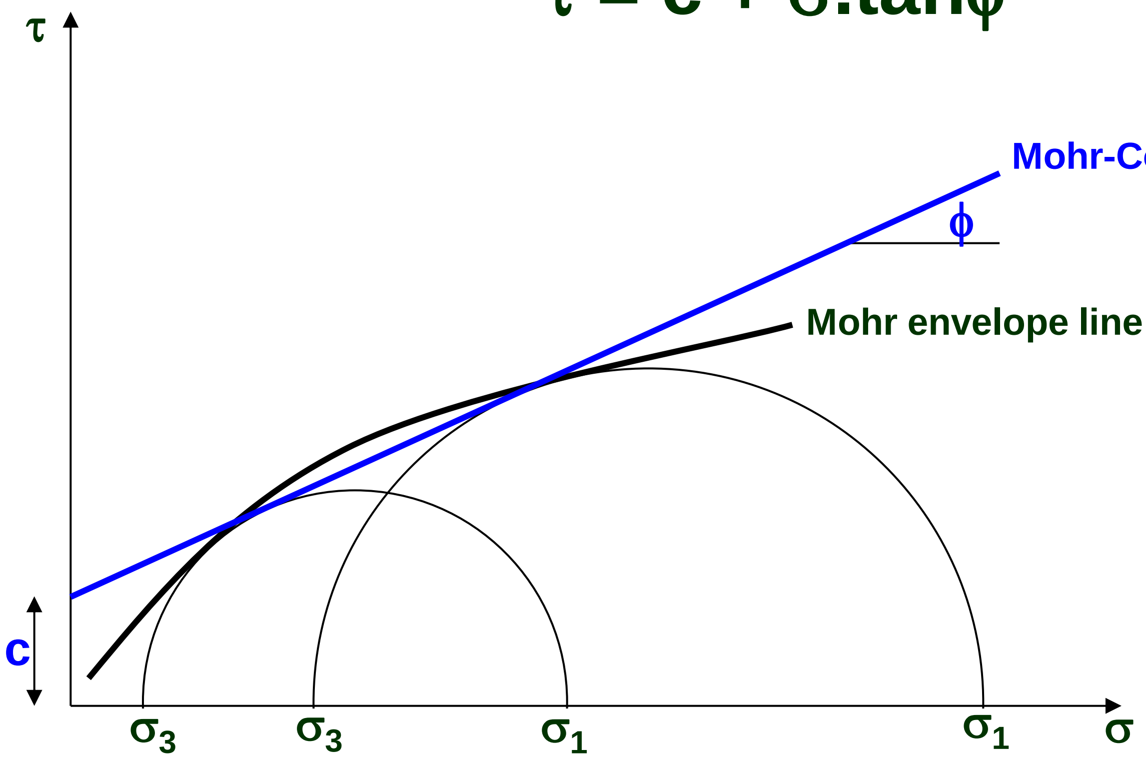


c', ϕ'

in terms of σ'

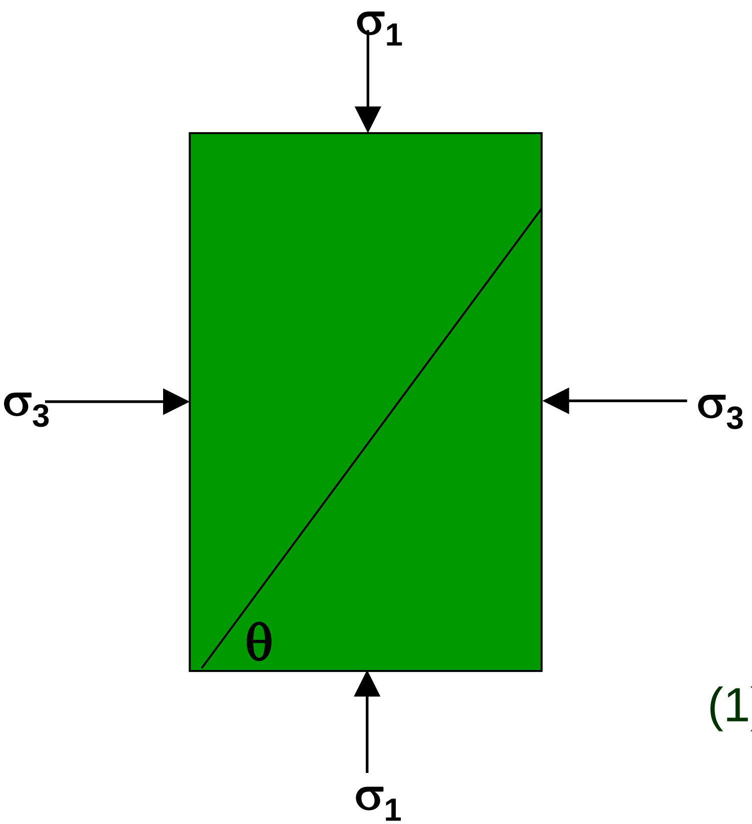
MOHR COULOMB CONCEPT

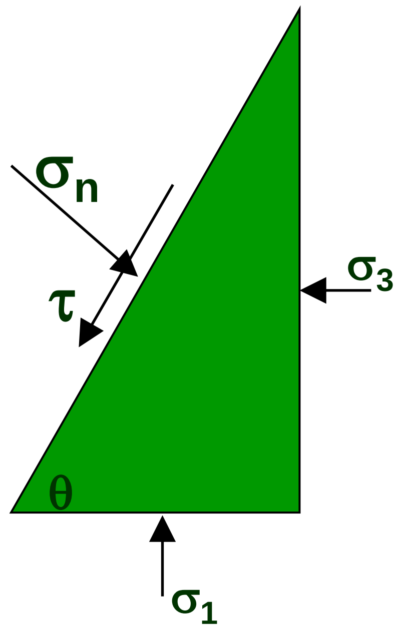
$$\tau = c + \sigma.\tan\phi$$



$$\sigma_1 = \sigma_3 + \Delta\sigma$$

MOHR COULOMB CONCEPT





(1) $$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}.Cos2\theta$$

@

$$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3)kos^2\theta$$

$\sigma_1 > \sigma_3$

(2) $$\tau = \frac{\sigma_1 - \sigma_3}{2}.Sin2\theta$$

MOHR COULOMB CONCEPT

(1) and (2) ⟶ $\tau = c + \sigma_n . \tan\phi$

$$\sigma_1 = \sigma_3 + \frac{(\sigma_3 \ . \ \tan\phi + c)}{(0.5 \ . \ Sin2\theta - Cos^2\theta \ . \ \tan\phi)}$$

The failure occurs when the value of σ_1 is minimum or the value of ($0.5 \ . \ Sin2\theta - Cos^2\theta \ . \ \tan\phi$) maximum

$$\theta = 45^o + \frac{\phi}{2} \longrightarrow \sigma_1 = \sigma_3 . \tan^2(45^o + \phi/2) + 2.c.\tan(45^o + \phi/2)$$

Equations…….

$$\tau_f = c + \sigma \tan \phi$$

$$\theta = 45 + \phi/2$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2}.\mathrm{Sin}2\theta$$

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}.Cos2\theta$$

$$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3)\mathrm{kos}^2\theta$$

$$\sigma_1 = \sigma_3.\tan^2\left(45^o + \phi/2\right) + 2.c.\tan\left(45^o + \phi/2\right)$$

Example 1

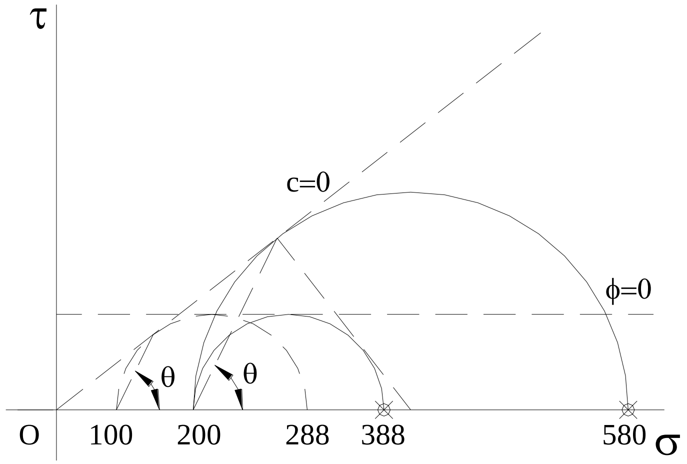
A soil failed at σ_3 = 100 kN/m2 and σ_1 = 288 kN/m^2. If the same soil is given σ_3 = 200 kN/m^2, what is the value of the new σ_1 when the failure is for:

i) Cohesive soil

ii) Cohesionless soil

Solution 1

Graphically



a.From the Mohr's circle,

$\sigma_1 = 388 \text{ kN/m}^2$

b. From the Mohr's circle,

$\sigma_1 = 580 \text{ kN/m}^2$

Solution 1

Analytically

i) Cohesive soil ($\phi = 0°$)

$$\theta = 45 + 0/2 = 45°$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2}.Sin2\theta$$

τ = (288 – 100) / 2 . Sin 2 (45) = 94 kPa

$$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3)kos^2\theta$$

σ_n = 100 + (288 – 100 $_3$)kos^{2}45 = 194 kPa

Solution 1

Analytically

for soil, where σ_3 *= 200 kPa*

$$\sigma_1 = \sigma_3.\tan^2\left(45^o + \phi/2\right) + 2.c.\tan\left(45^o + \phi/2\right)$$

σ_1 =200 $\tan^2$ (45 – 0/2) + 2(94)tan (45)
= ????? kPa

Solution 1

Analytically

i) Cohesionless soil (c = 0)

From graph:

τ_f = 80 kPa & σ_n=145 kPa

$$\tau_f = c + \sigma \tan \phi$$

$\phi = \tan^{-1} (80/145) = 29°$

$$\sigma_1 = \sigma_3 . \tan^2 \left(45^o + \phi / 2\right) + 2.c.\tan\left(45^o + \phi / 2\right)$$

σ_1 =200 $\tan^2$ (45 – 29/2) + 0 = 576 kPa

Example 2

Given:
C= 86 kPa
ϕ= 17°
σ_3= 70 kPa
σ_1= 346 kPa

Determine angle of failure (θ), shear tress at failure (τ_f) and mormal stress at failure (σ_n)

Solution 2

$$\theta = 45 + \phi/2$$

$$\theta = 45 + 17/2 = 53.5°$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2} .Sin 2\theta$$

$\tau = (346 – 70) / 2 . \text{Sin } 2 (53.5) = 132$ kPa

$$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3) kos^2\theta$$

$\sigma_n = 70 + (346 – 70) kos^2 53.5 = 167$ kPa

Example 3

A proposed structure will cause the vertical stress to increase by 60 kN/m^2 at the 4m depth. Samples taken from a uniform deposit of granular soil are found to have a unit weight of 19.6 kN/m^3 and an angle of internal friction of 35°. Assume that the weight of the structure also causes the shearing stress to increase to 52 kN/m^2 on a horizontal plane at this depth. Does this shearing stress exceed the shearing strength of the soil? *(consider also when the water table rise to the ground surface)*

Solution 3

Solution

The total vertical pressure due to the structure and soil overburden is:

$$60\ \text{kN/m}^2 + 78.4\ \text{kN/m}^2\ \text{overburden pressure} = 138.4\ \text{kN/m}^2$$

The shearing strength that can be developed by the soil at this depth is:

$$\tau = (138.4\ \text{kN/m}^2)(\tan 35°) = 96.9\ \text{kN/m}^2$$

This would indicate that the shear strength of the soil is greater than the imposed shear stress; therefore, a shear failure does not occur (96.9 kN/m^2 > 52 kN/m^2).

If the water table rose to the ground surface, the effective soil overburden pressure would be reduced to about:

$$(\tfrac{1}{2} \times 19.6\ \text{kN/m}^3)(4\ \text{m}) = 39.2\ \text{kN/m}^2$$

[This value represents the effective vertical stress, i.e., $\bar{\sigma} = \sigma_t - u = \gamma_{sub} Z = (\gamma_t - \gamma_w)Z$.]

The total vertical stress would be:

$$39.2\ \text{kN/m}^2 + 60\ \text{kN/m}^2 = 99.2\ \text{kN/m}^2$$

The shear strength available is:

$$\tau = (99.2\ \text{kN/m}^2)(\tan 35°) = 69.46\ \text{kN/m}^2$$

This is still greater than the shear stress resulting from the loading conditions; that is, 69.46 kN/m^2 > 52 kN/m^2.

Example 4

c) A normally consolidated clay is consolidated under a stress of 150 kPa, then sheared undrained in a axial compression. The principle stress difference at failure is 100 kPa and the induced pore water pressure at failure is 88 kPa.

   i) Determine the Mohr Coulomb strength parameters in terms of both total & effective stresses

   ii) Compute the principle stress ratio for both total & effective stresses

   iii) Determine the theoretical angle of failure plane in the specimen

Solution 4

a. To use these equations, we need σ_{1f}, σ'_{1f}, σ_{3f}, and σ'_{3f}. We know $\sigma_{3f} = 150$ kPa and $(\sigma_1 - \sigma_3)_f = 100$ kPa. Therefore

$$\sigma_{1f} = (\sigma_1 - \sigma_3)_f + \sigma_{3f} = 100 + 150 = 250 \text{ kPa}$$
$$\sigma'_{1f} = \sigma_{1f} - u_f = 250 - 88 = 162 \text{ kPa}$$
$$\sigma'_{3f} = \sigma_{3f} - u_f = 150 - 88 = 62 \text{ kPa}$$

From Eq. (11.13),

$$\phi' = \arcsin\frac{100}{224} = 26.5°$$
$$\phi_T = \arcsin\frac{100}{400} = 14.5°$$

b. For the graphical solution, we need to plot the total and effective Mohr circles, and to do this we need to calculate σ_{1f}, σ'_{1f}, and σ'_{3f}. The centers of the circles are at (200, 0) for total stresses and at (112, 0) for effective stresses. The graphical solution including the failure envelopes is shown in Fig. Ex. 12.9.

c. The stress ratios at failure are

$$\frac{\sigma'_1}{\sigma'_3} = \frac{162}{62} = 2.61$$
$$\frac{\sigma_1}{\sigma_3} = \frac{250}{150} = 1.67$$

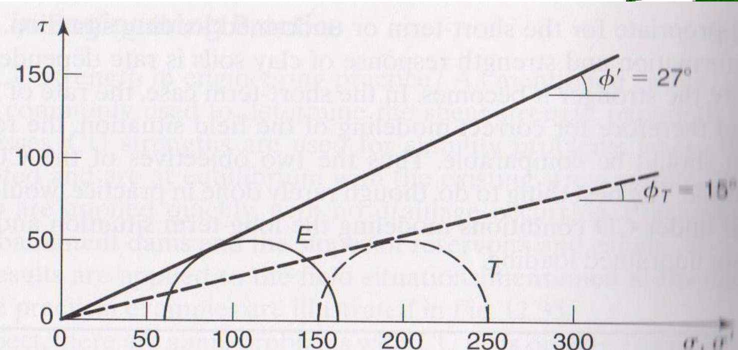


FIGURE Ex. 12.9

Another way to get these values would be to use Eq. (11.14).

$$\frac{\sigma'_1}{\sigma'_3} = \frac{1 + \sin 26.5°}{1 - \sin 26.5°} = \frac{1.45}{0.55} = 2.61$$
$$\frac{\sigma_1}{\sigma_3} = \frac{1 + \sin 14.5°}{1 - \sin 14.5°} = \frac{1.25}{0.75} = 1.67$$

d. Use Eq. (11.10), in terms of *effective* stresses:

$$\alpha_f = 45° + \frac{\phi'}{2} = 58° \text{ from the horizontal}$$

SIVA

Example 5

a) A consolidated undrained (CU) triaxial test was performed on a specimen of saturated clay with a chamber pressure, $\sigma_3 = 2.0$ kg/cm^2. At failure,

$\sigma 1 - \sigma_3 = 2.8$ kg/cm^2

$u = 1.8$ kg/cm^2

$\theta = 57°$ (angle of failure plane)

Calculate:

i) Normal stress, σ on the failure surface

ii) Shear stress, τ on the failure surface

iii) Maximum shear stress on the specimen

iv) If $\phi = 24°$, $c' = 0.80$ kg/cm^2, show why the sample failed at 57° instead of at the plane of maximum shear stress ($\theta = 45°$)

Solution 5

$\sigma_1 = 2.8 + 2.0 = 4.8 \text{ kg/cm}^2$

$\sigma_3 = 2.0 \text{ kg/cm}^2$

i) $\sigma_n = \left(\frac{\sigma_1 + \sigma_3}{2}\right) + \left(\frac{\sigma_1 - \sigma_3}{2}\right) \cos 2\theta$

$= \left(\frac{4.8 + 2.0}{2}\right) + \left(\frac{4.8 - 2.0}{2}\right) \cos 114^\circ$

$= 2.83 \text{ kg/cm}^2$

ii) $\tau = \left(\frac{\sigma_1 - \sigma_3}{2}\right) \sin 2\theta = \left(\frac{4.8 - 2.0}{2}\right) \sin 114^\circ = 1.27 \text{ kg/cm}^2$

iii) $\tau_{max} = \left(\frac{\sigma_1 - \sigma_3}{2}\right) \sin 2\theta = \left(\frac{4.8 - 2.0}{2}\right) \sin 90 = 1.4 \text{ kg/cm}^2$ (at $\theta = 45^\circ$)

iv) $S_{57^\circ} = c' + \sigma' \tan\phi = 0.8 + (2.8\bar{3}) \tan 24^\circ$

$= 1.27 \text{ kg/cm}^2$ (similar to previous τ)

Now at the plane of maximum shear stress $\theta = 45^\circ$

$\sigma = \left(\frac{4.8 + 2}{2}\right) + \left(\frac{4.8 - 2}{2}\right) \cos 90^\circ = 3.4 \text{ kg/cm}^2$

$\sigma' = (3.4 - 1.8) = 1.6 \text{ kg/cm}^2$

$S_{45^\circ} = c' + \sigma' \tan\phi = 0.8 + (1.6 \tan 24^\circ) = 1.51 \text{ kg/cm}^2$

$\therefore$ The shear strength at 45° is larger than at 57°, therefore failure does not occur at 57°