

MAJ 1013-ADVANCED SOIL MECHANICS

 **Laboratory Testing**

 **Pwp & Back Pressure Measurements**

*Prepared by:
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Mohr Coulomb failure criterion with Mohr circle of stress

$$\left[c' \cot \phi' + \left(\frac{\sigma'_1 + \sigma'_3}{2} \right) \right] \sin \phi' = \left(\frac{\sigma'_1 - \sigma'_3}{2} \right)$$

$$(\sigma'_1 - \sigma'_3) = (\sigma'_1 + \sigma'_3) \sin \phi' + 2c' \cos \phi'$$

$$\sigma'_1 (1 - \sin \phi') = \sigma'_3 (1 + \sin \phi') + 2c' \cos \phi'$$

$$\sigma'_1 = \sigma'_3 \left(\frac{1 + \sin \phi'}{1 - \sin \phi'} \right) + 2c' \left(\frac{\cos \phi'}{1 - \sin \phi'} \right)$$

$$\sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right) + 2c' \tan \left(45 + \frac{\phi'}{2} \right)$$

Determination of shear strength parameters of soils (c , ϕ or c' , ϕ')

Laboratory tests on specimens taken from representative undisturbed samples

Field tests

Most common laboratory tests to determine the shear strength parameters are,

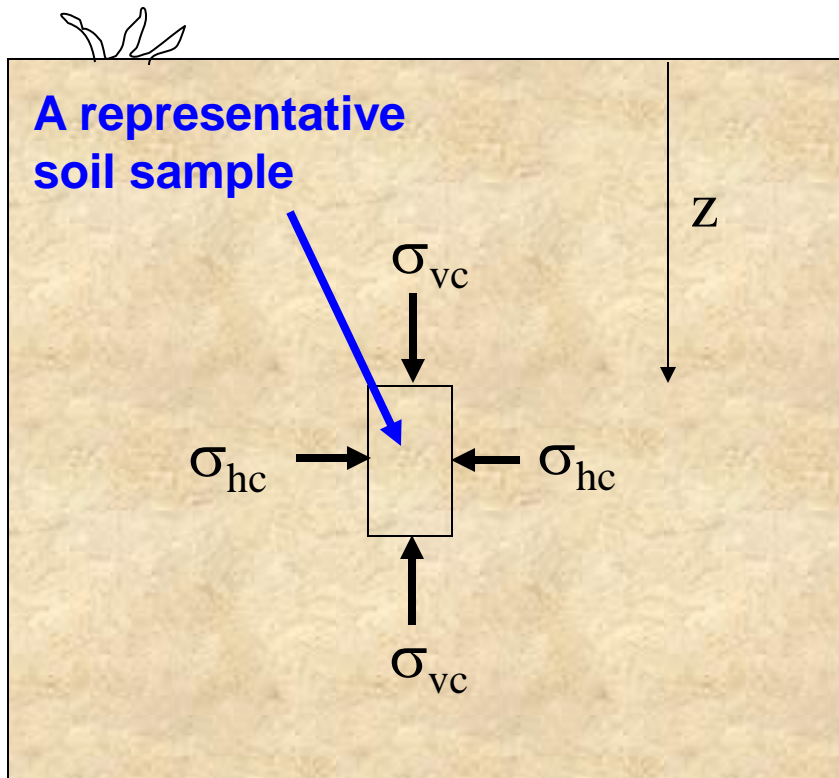
1. Direct shear test
2. Unconfined Compressive Strength test
3. Triaxial shear test

Other laboratory tests include, Direct simple shear test, torsional ring shear test, plane strain triaxial test, laboratory vane shear test, laboratory fall cone test

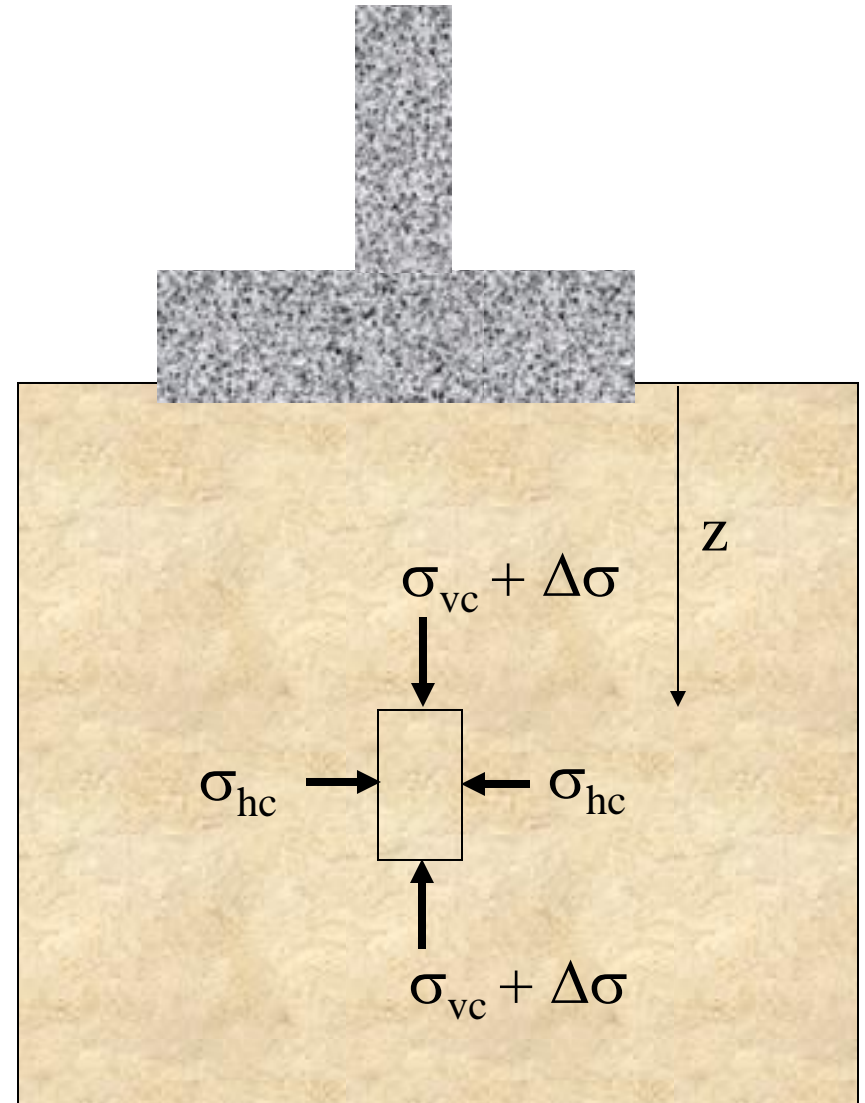
1. Vane shear test
2. Torvane
3. Pocket penetrometer
4. Fall cone
5. Pressuremeter
6. Static cone penetrometer
7. Standard penetration test

Laboratory tests

Field conditions



Before construction



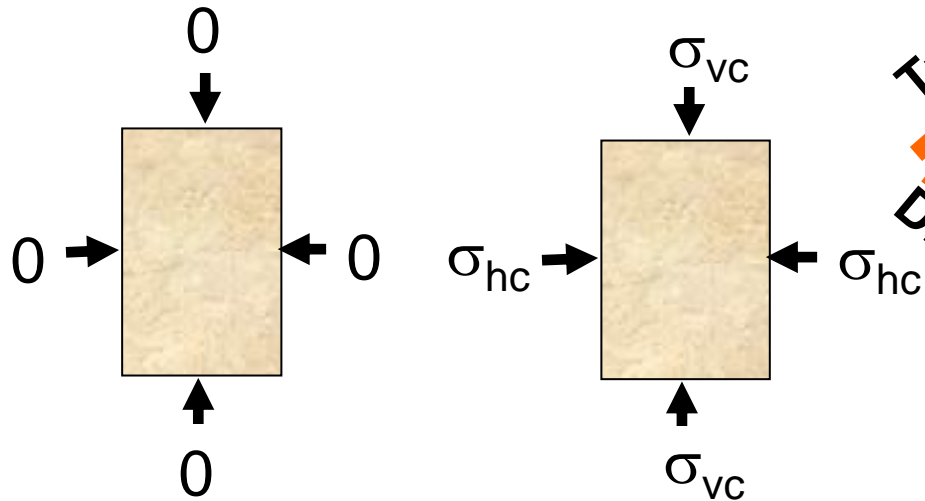
After and during construction

Criteria to determine shear strength

1. Peak deviator stress, $\Delta\sigma$
2. Maximum principal stress, σ_1
3. Minimum principal stress, σ_3
4. Limiting strain
5. Critical state
6. Residual state

Laboratory tests

Simulating field conditions in the laboratory

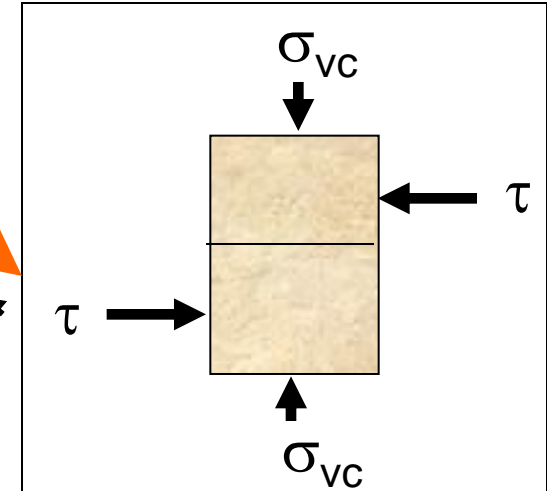
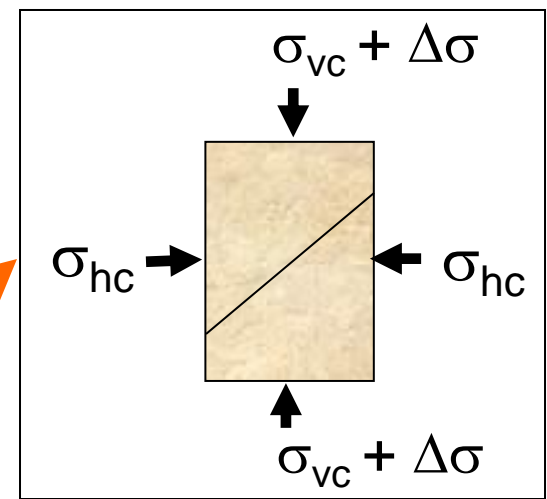


Representative soil sample taken from the site

Step 1

Set the specimen in the apparatus and apply the initial stress condition

Triaxial test
Direct shear test

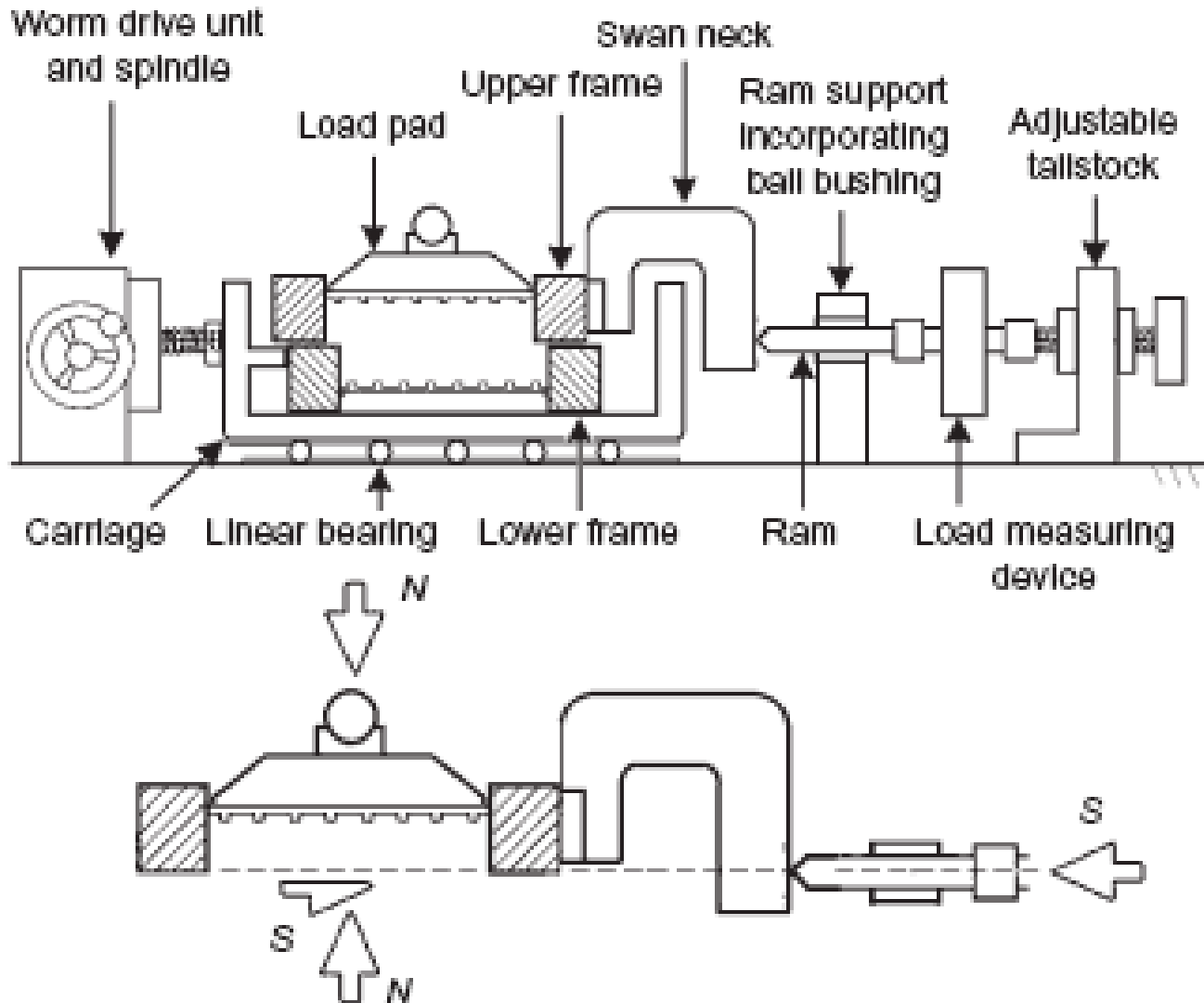


Step 2

Apply the corresponding field stress conditions

Direct shear test

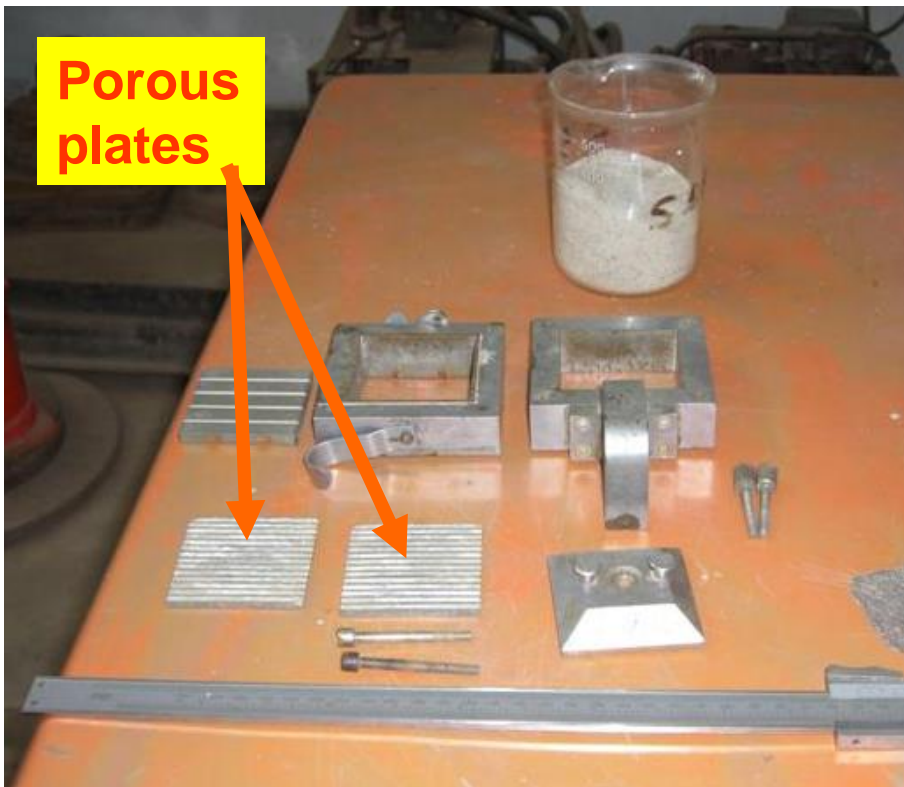
Schematic diagram of the direct shear apparatus



Direct shear test

Direct shear test is most suitable for consolidated drained tests specially on granular soils (e.g.: sand) or stiff clays

Preparation of a sand specimen



Components of the shear box



Preparation of a sand specimen

Direct shear test

Preparation of a sand specimen

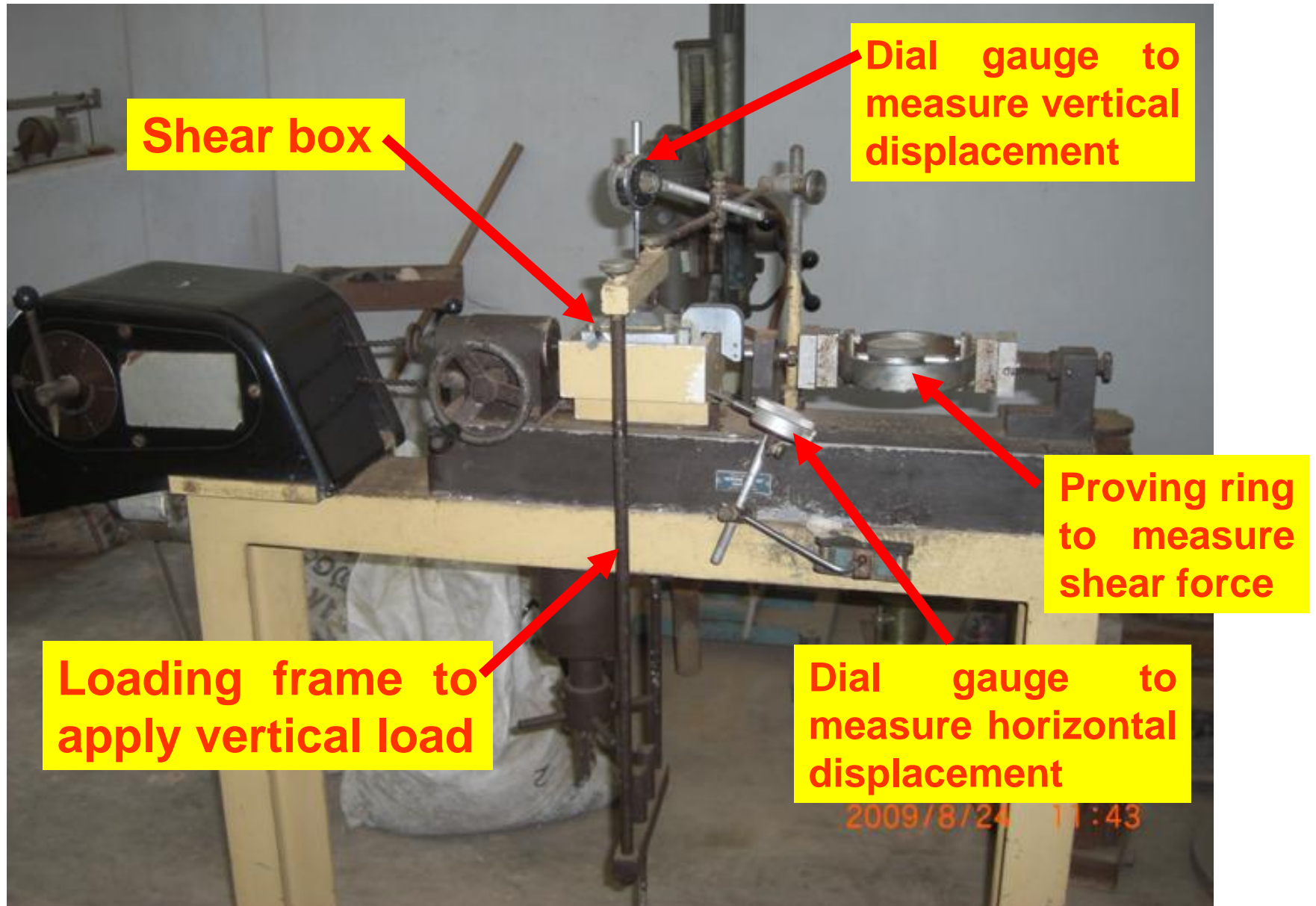


Leveling the top surface
of specimen



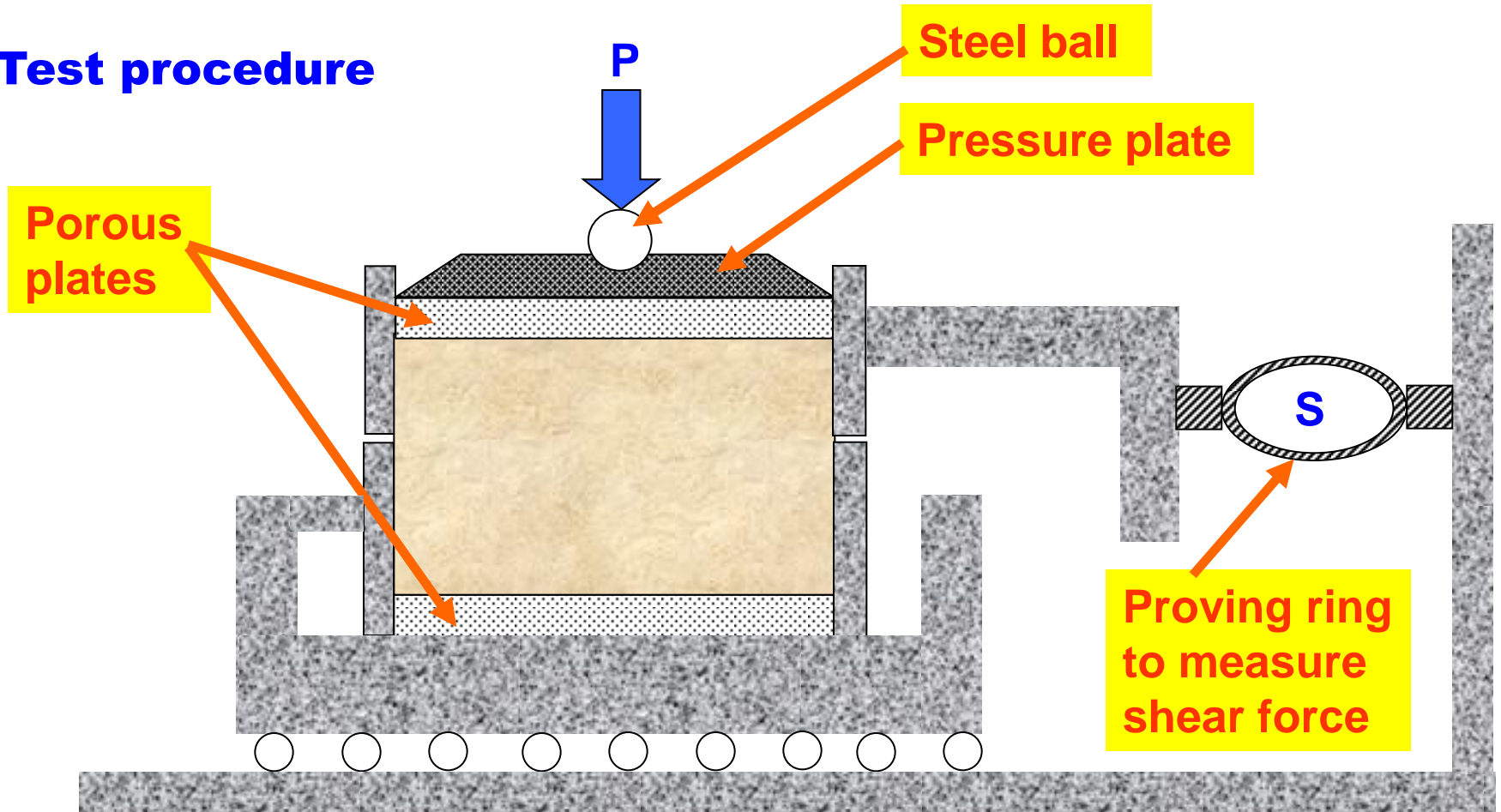
Specimen preparation
completed

Direct shear test



Direct shear test

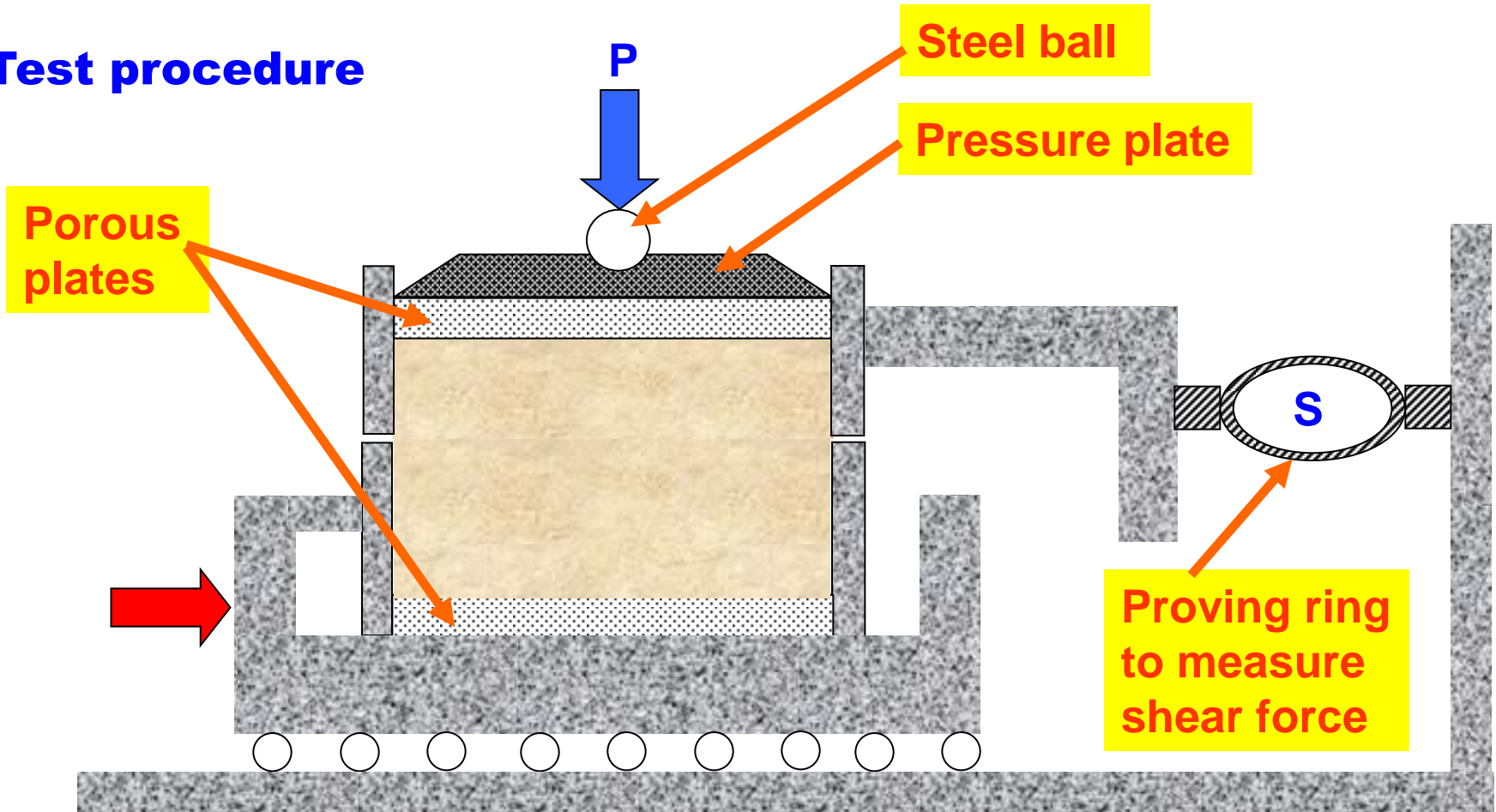
Test procedure



Step 1: Apply a vertical load to the specimen and wait for consolidation

Direct shear test

Test procedure



Step 1: Apply a vertical load to the specimen and wait for consolidation

Step 2: Lower box is subjected to a horizontal displacement at a constant rate

Direct shear test

Analysis of test results

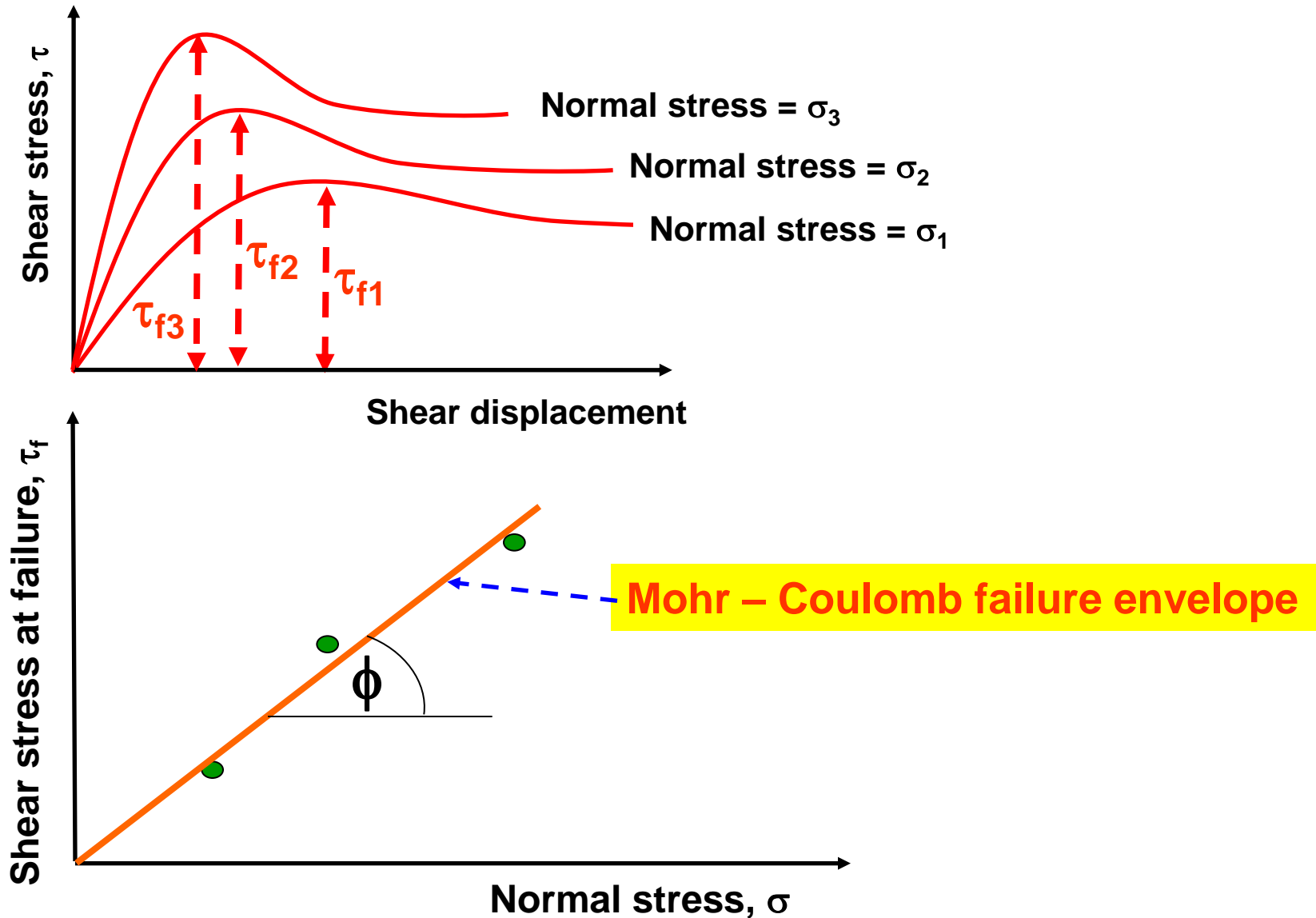
$$\sigma = \text{Normal stress} = \frac{\text{Normal force (P)}}{\text{Area of cross section of the sample}}$$

$$\tau = \text{Shear stress} = \frac{\text{Shear resistance developed at the sliding surface (S)}}{\text{Area of cross section of the sample}}$$

Note: Cross-sectional area of the sample changes with the horizontal displacement

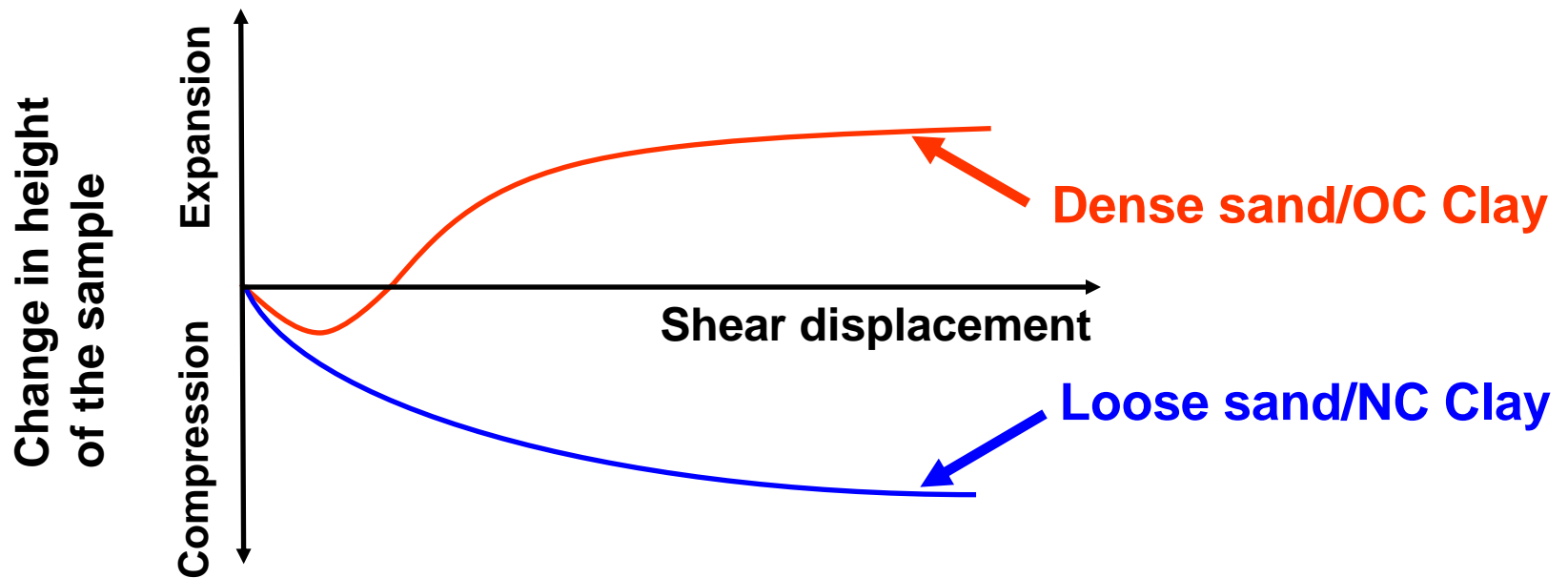
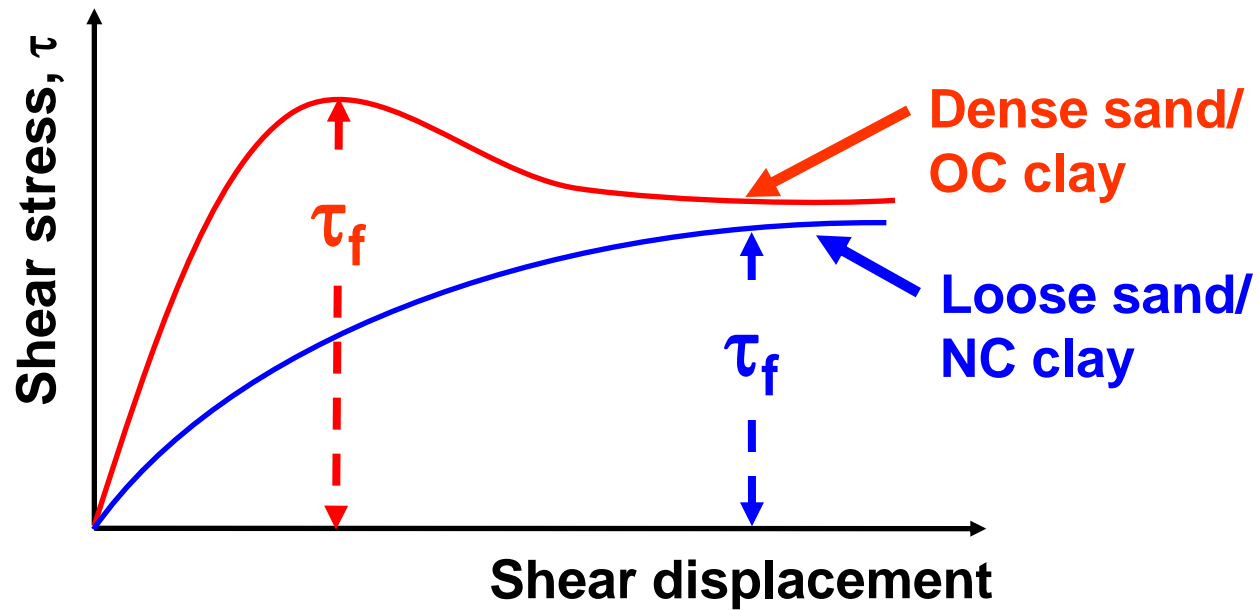
Direct shear tests on sands

How to determine strength parameters c and ϕ



Direct shear tests

Stress-strain relationship



Direct shear tests on sands

Some important facts on strength parameters c and ϕ of sand

Sand is cohesionless
hence $c = 0$

Direct shear tests are
drained and pore water
pressures are
dissipated, hence $u = 0$

Therefore,

$\phi' = \phi$ and $c' = c = 0$



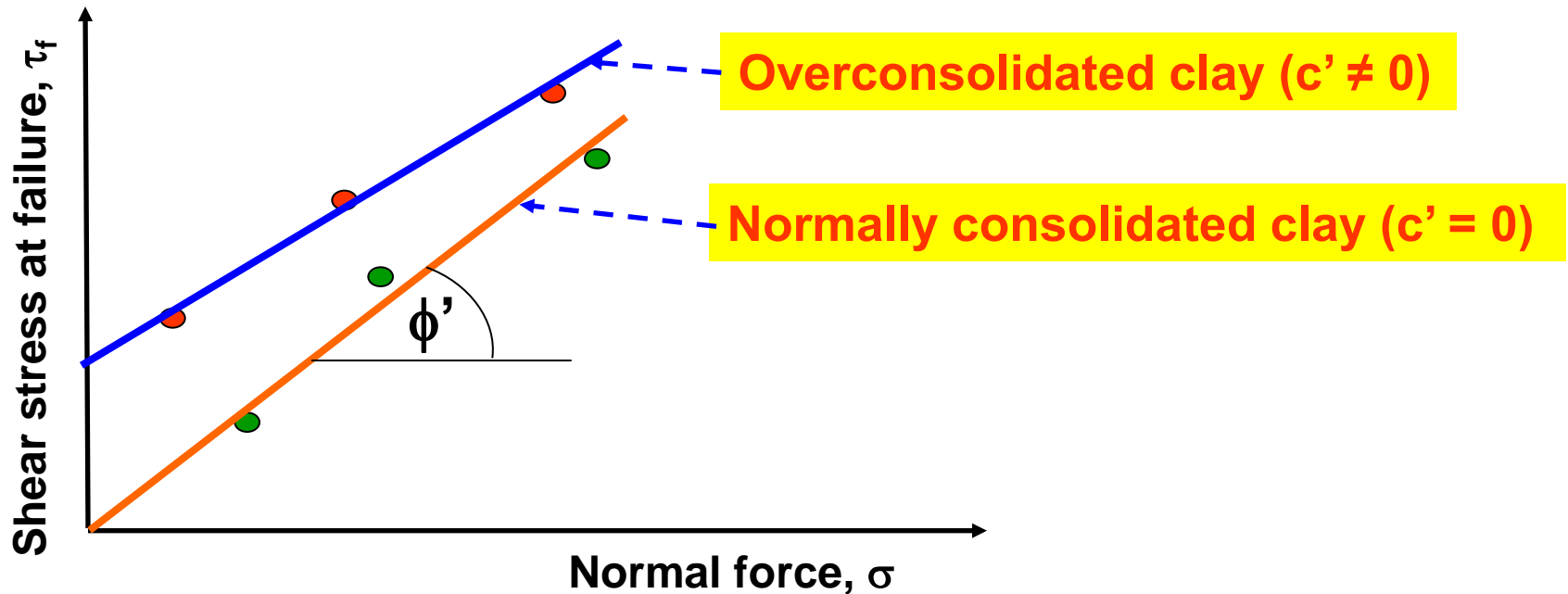
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A GENTLE REMINDER ...

Direct shear tests on clays

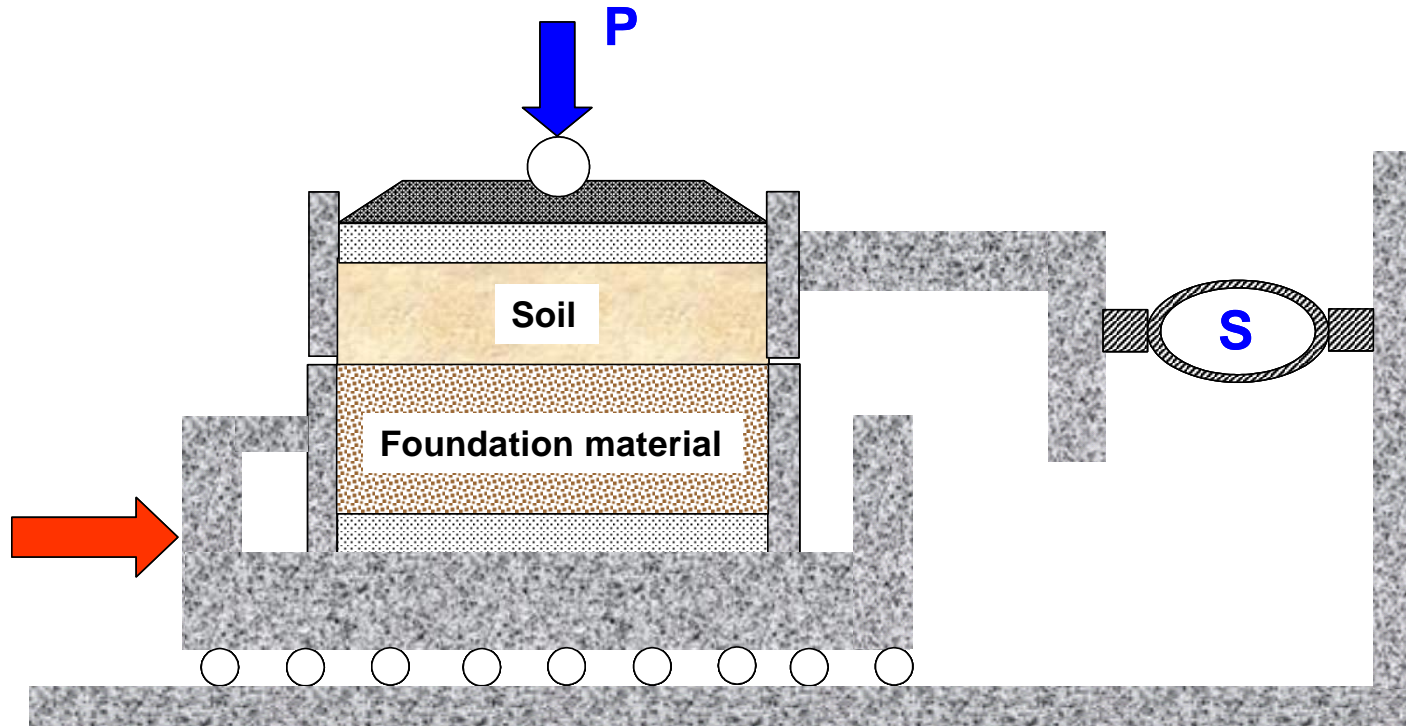
In case of clay, *horizontal displacement* should be applied at a very slow rate to allow dissipation of pore water pressure (therefore, one test would take several days to finish)

Failure envelopes for clay from drained direct shear tests



Interface tests on direct shear apparatus

In many foundation design problems and retaining wall problems, it is required to determine the angle of internal friction between soil and the structural material (concrete, steel or wood)



$$\tau_f = c_a + \sigma' \tan \delta$$

Where,

c_a = adhesion,

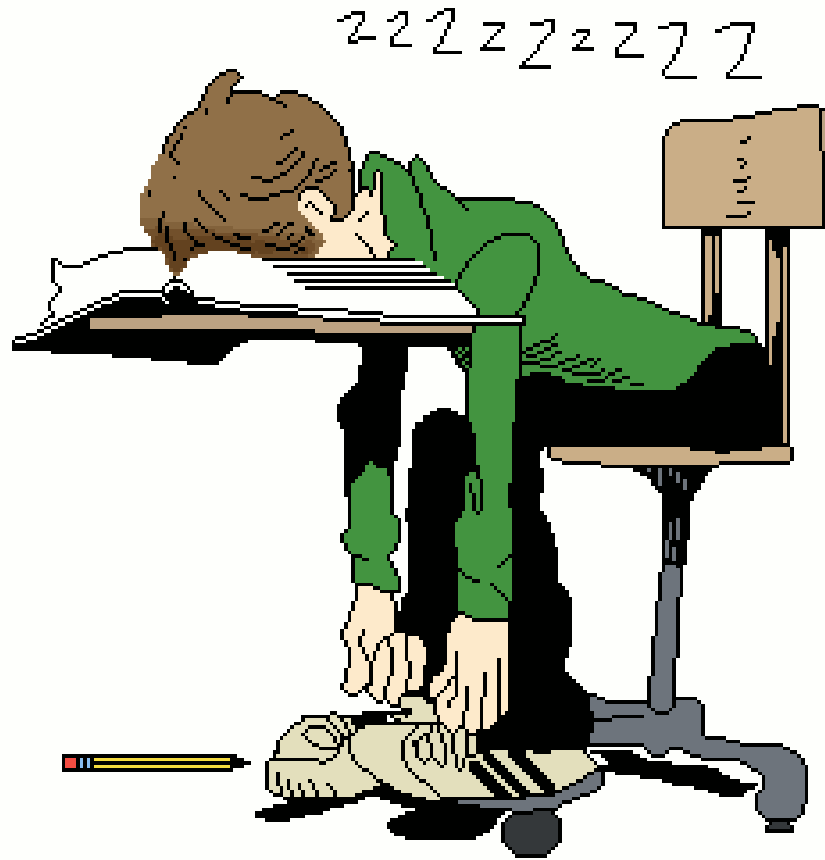
δ = angle of internal friction

Advantages of direct shear apparatus

- ❑ Due to the smaller thickness of the sample, rapid drainage can be achieved
- ❑ Can be used to determine interface strength parameters
- ❑ Clay samples can be oriented along the plane of weakness or an identified failure plane

Disadvantages of direct shear apparatus

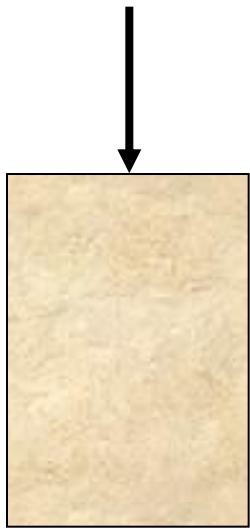
- ❑ Failure occurs along a predetermined failure plane
- ❑ Area of the sliding surface changes as the test progresses
- ❑ Non-uniform distribution of shear stress along the failure surface



Lets do some examples

Unconfined Compression Test (UC Test)

$$\sigma_1 = \sigma_{VC} + \Delta\sigma$$

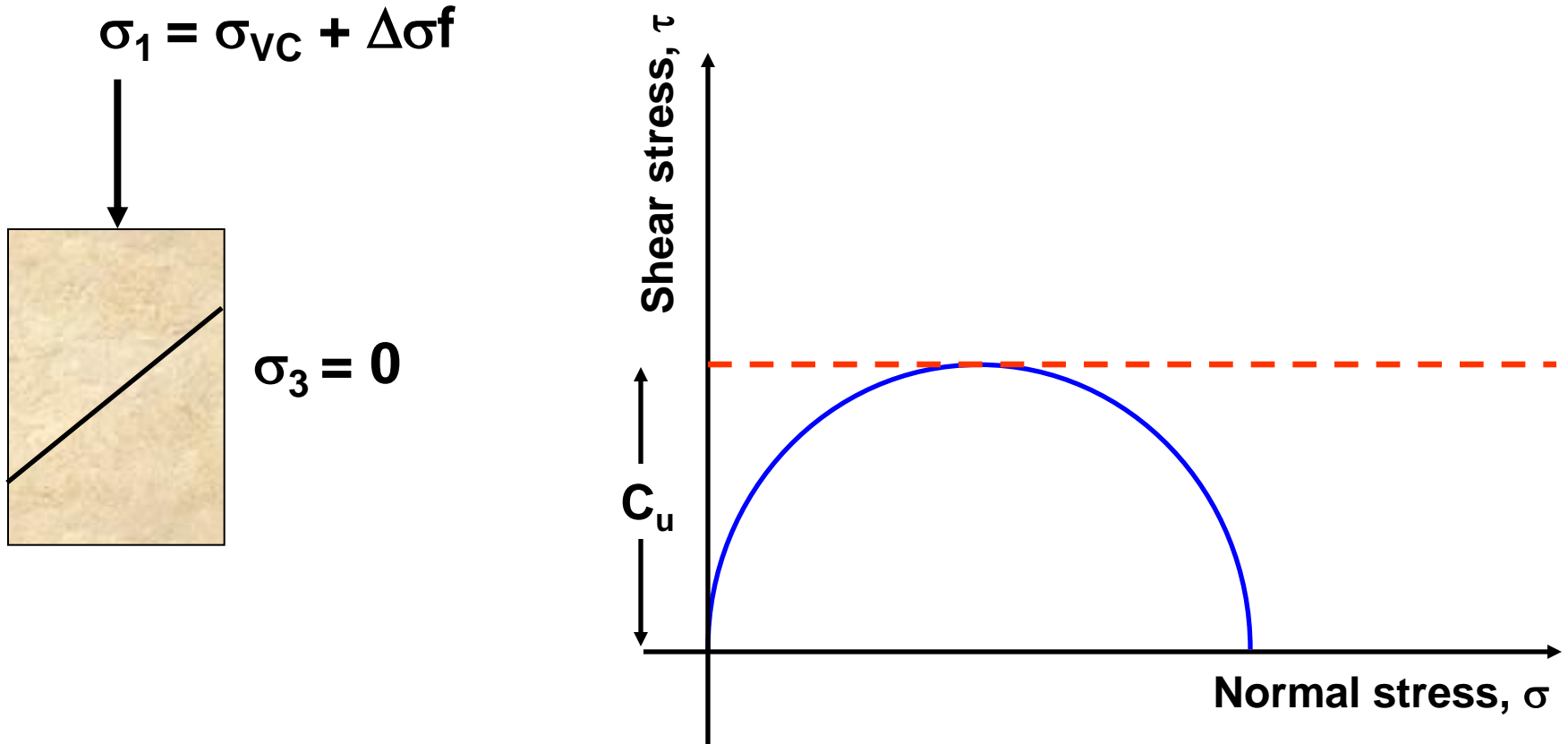


$$\sigma_3 = 0$$



Confining pressure is zero in the UC test

Unconfined Compression Test (UC Test)



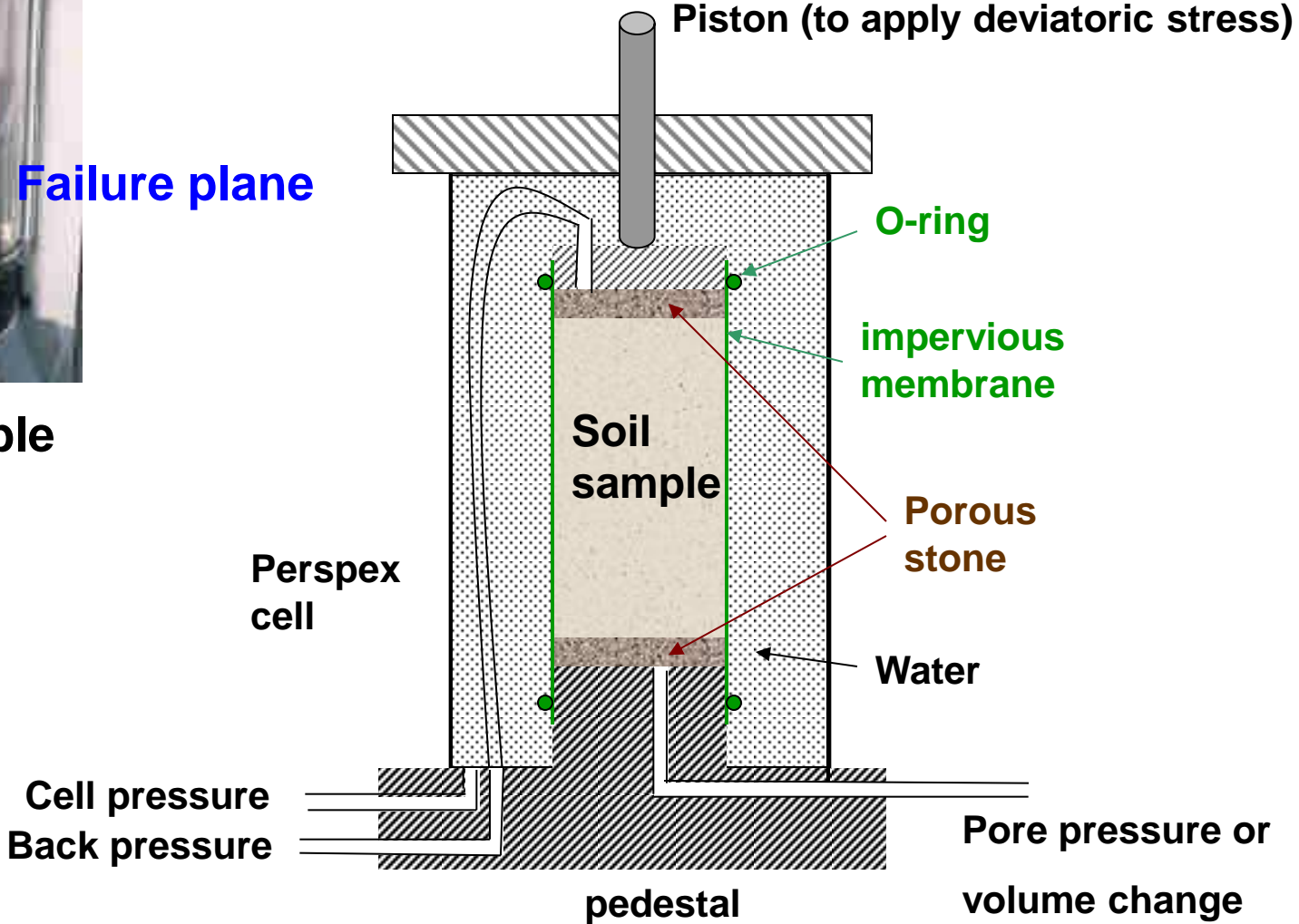
Note: Theoretically $q_u = c_u$, However in the actual case $q_u < c_u$ due to premature failure of the sample

Triaxial Shear Test



Failure plane

Soil sample at failure



Triaxial Shear Test

Specimen preparation (undisturbed sample)



Edges of the sample are carefully trimmed



Setting up the sample in the triaxial cell

Triaxial Shear Test

Specimen preparation (undisturbed sample)



Sample is covered with a rubber membrane and sealed



Cell is completely filled with water

Triaxial Shear Test

Specimen preparation (undisturbed sample)



Proving ring to measure the deviator load

Dial gauge to measure vertical displacement

In some tests

Triaxial Shear Test

 Consists of 3 stages:

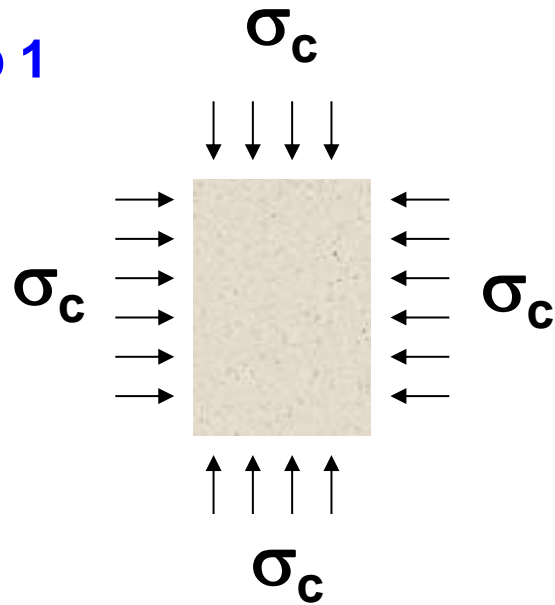
1. Saturation
 2. Consolidation
 3. Shearing
- Main stages*

Triaxial Shear Test

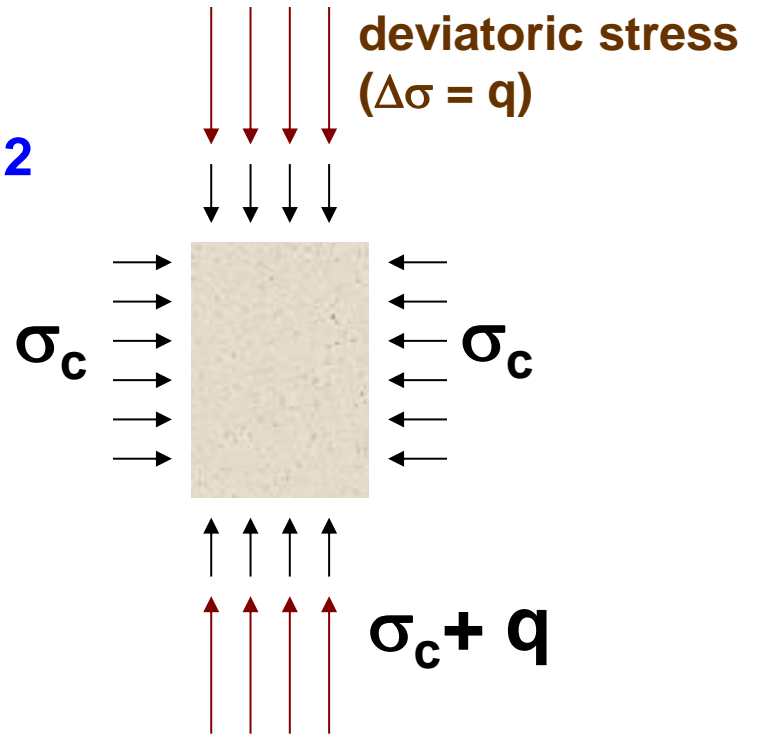
Test Condition	Stage 1	Stage 2
Unconsolidated Undrained (UU)	Apply confining pressure σ_3 while the drainage line from the specimen is kept closed (drainage is not permitted), then the initial pore water pressure ($u=u_0$) is not equal to zero	Apply an added stress $\Delta\sigma$ at axial direction. The drainage line from the specimen is still kept closed (drainage is not permitted) ($u=u_d \neq 0$). At failure state $\Delta\sigma = \Delta\sigma_f$; pore water pressure $u=u_f = u_0 + u_{d(f)}$
Consolidated Undrained (CU)	Apply confining pressure σ_3 while the drainage line from the specimen is opened (drainage is permitted), then the initial pore water pressure ($u=u_0$) is equal to zero	Apply an added stress $\Delta\sigma$ at axial direction. The drainage line from the specimen is kept closed (drainage is not permitted) ($u=u_d \neq 0$). At failure state $\Delta\sigma = \Delta\sigma_f$; pore water pressure $u=u_f = u_0 + u_{d(f)} = u_{d(f)}$
Consolidated Drained (CD)	Apply confining pressure σ_3 while the drainage line from the specimen is opened (drainage is permitted), then the initial pore water pressure ($u=u_0$) is equal to zero	Apply an added stress $\Delta\sigma$ at axial direction. The drainage line from the specimen is opened (drainage is permitted) so the pore water pressure ($u=u_d$) is equal to zero. At failure state $\Delta\sigma = \Delta\sigma_f$; pore water pressure $u=u_f = u_0 + u_{d(f)} = 0$

Types of Triaxial Tests

Step 1



Step 2



Under all-around cell pressure σ_c

Shearing (loading)

Is the drainage valve open?

yes

no

Consolidated
sample

Unconsolidated
sample

Is the drainage valve open?

yes

no

Drained
loading

Undrained
loading

Types of Triaxial Tests

Step 1

Under all-around cell pressure σ_c

Is the drainage valve open?

yes

no

Consolidated
sample

Unconsolidated
sample

Step 2

Shearing (loading)

Is the drainage valve open?

yes

no

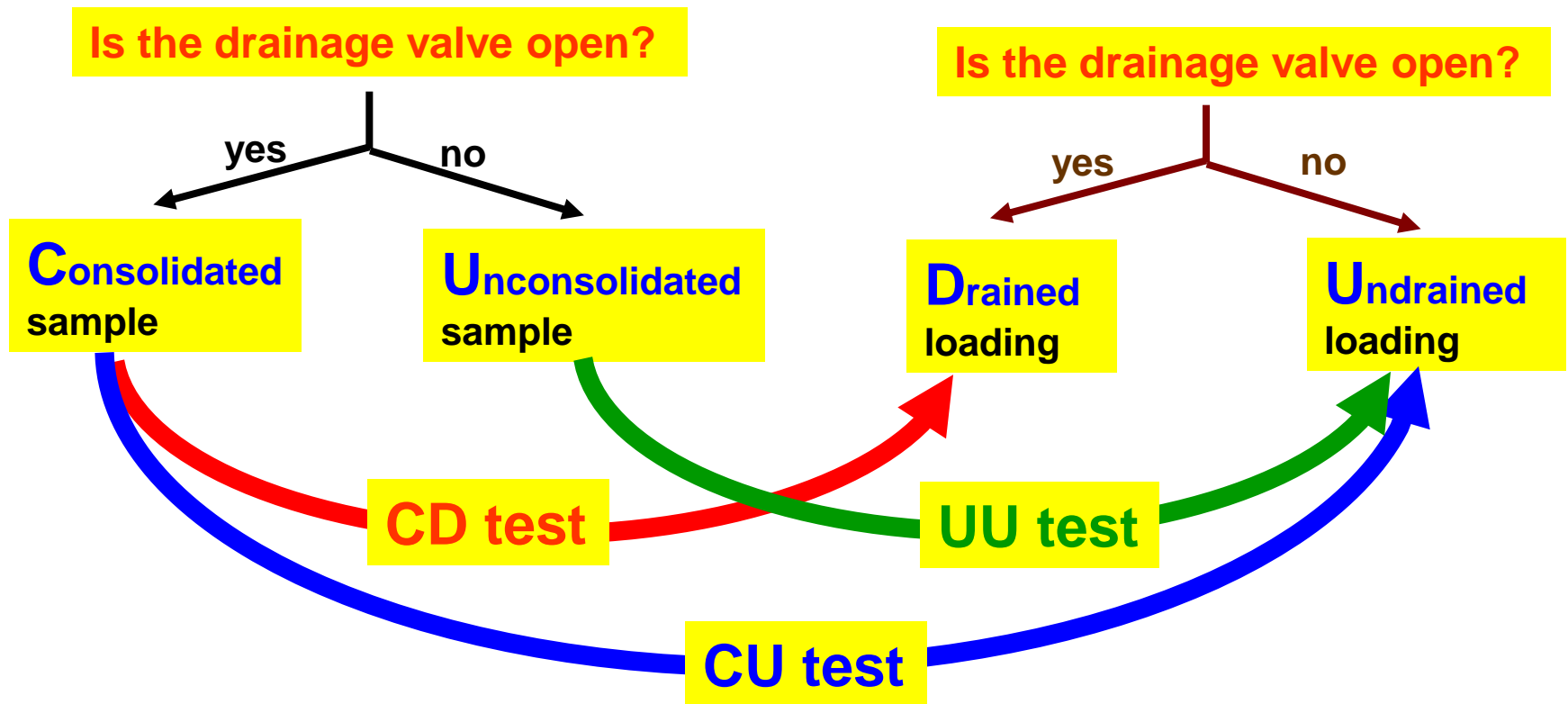
Drained
loading

Undrained
loading

CD test

UU test

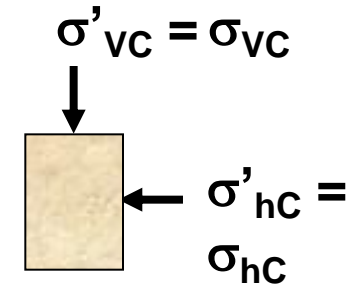
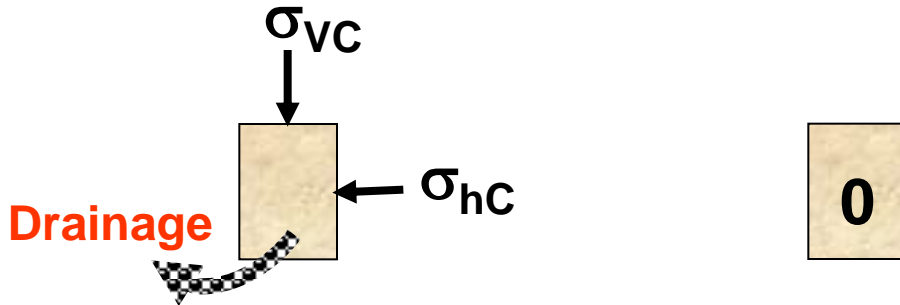
CU test



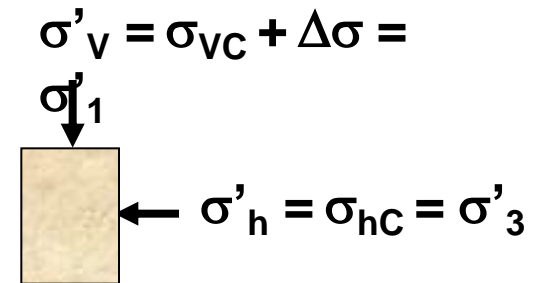
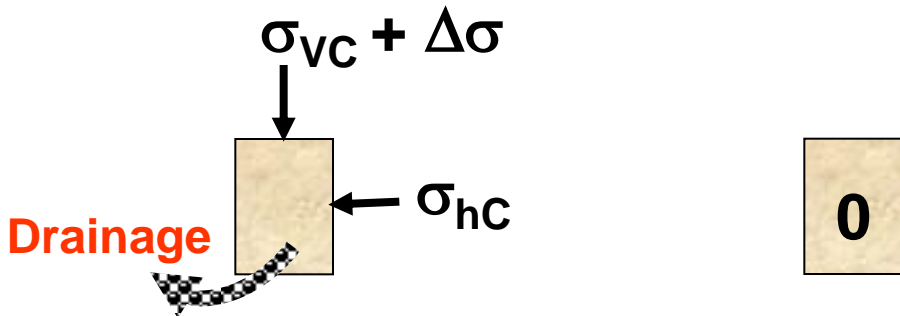
Consolidated- drained test (CD Test)

$$\text{Total, } \sigma = \text{Neutral, } u + \text{Effective, } \sigma'$$

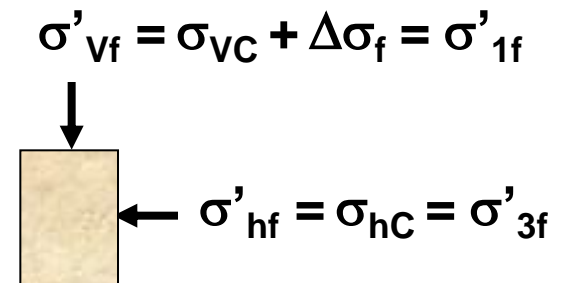
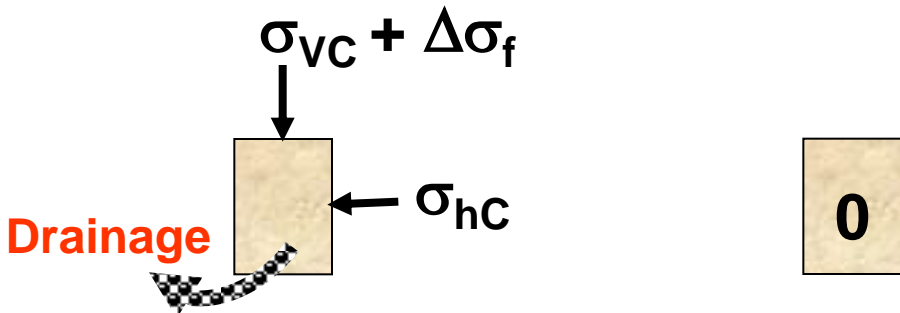
Step 1: At the end of consolidation



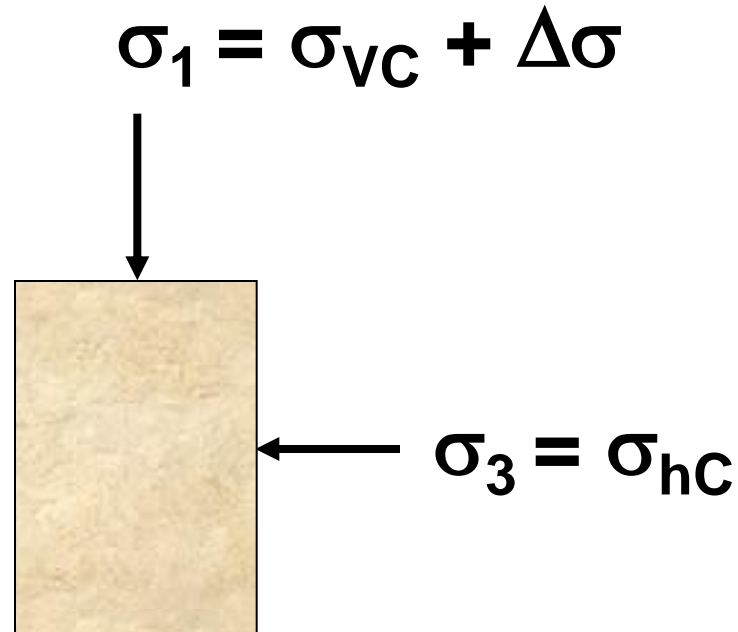
Step 2: During axial stress increase



Step 3: At failure



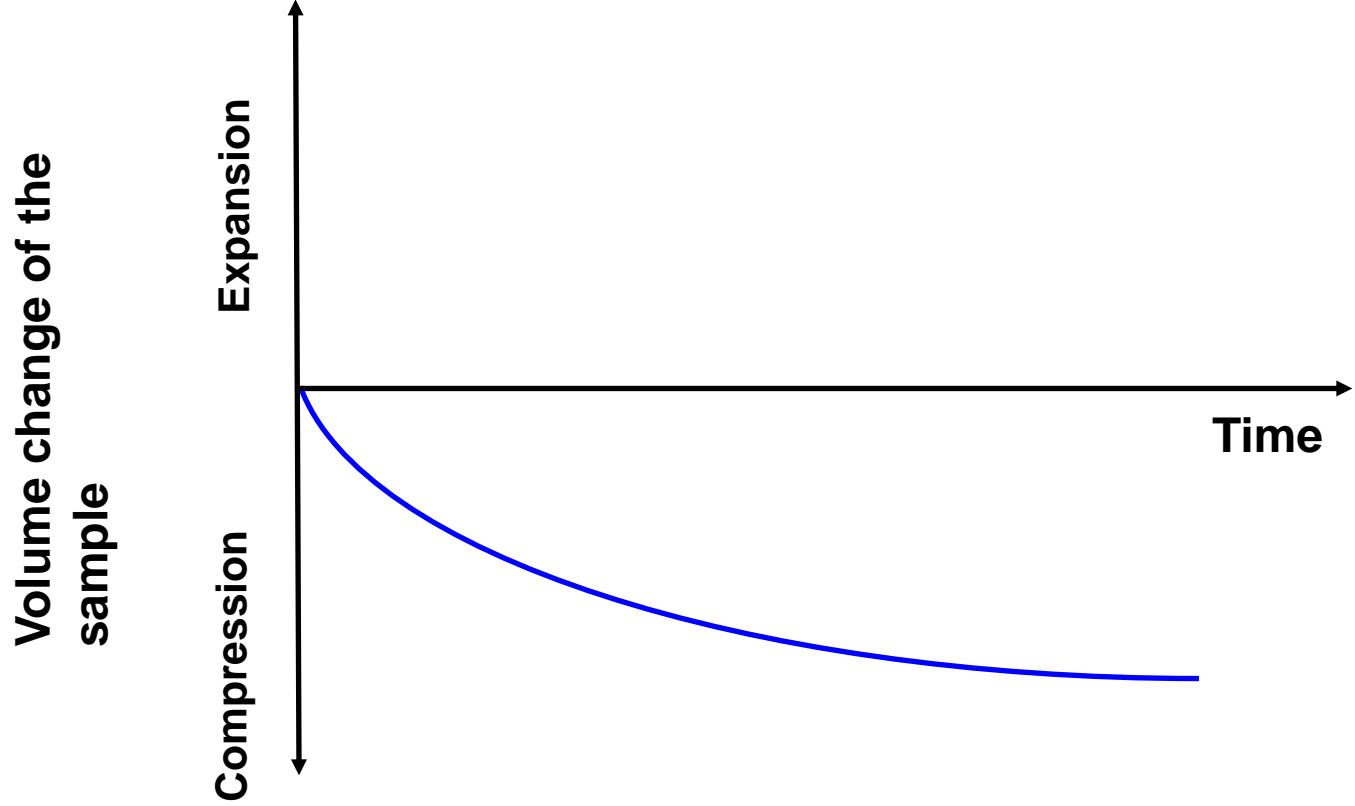
Consolidated- drained test (CD Test)



Deviator stress (q or $\Delta\sigma_d$) = $\sigma_1 - \sigma_3$

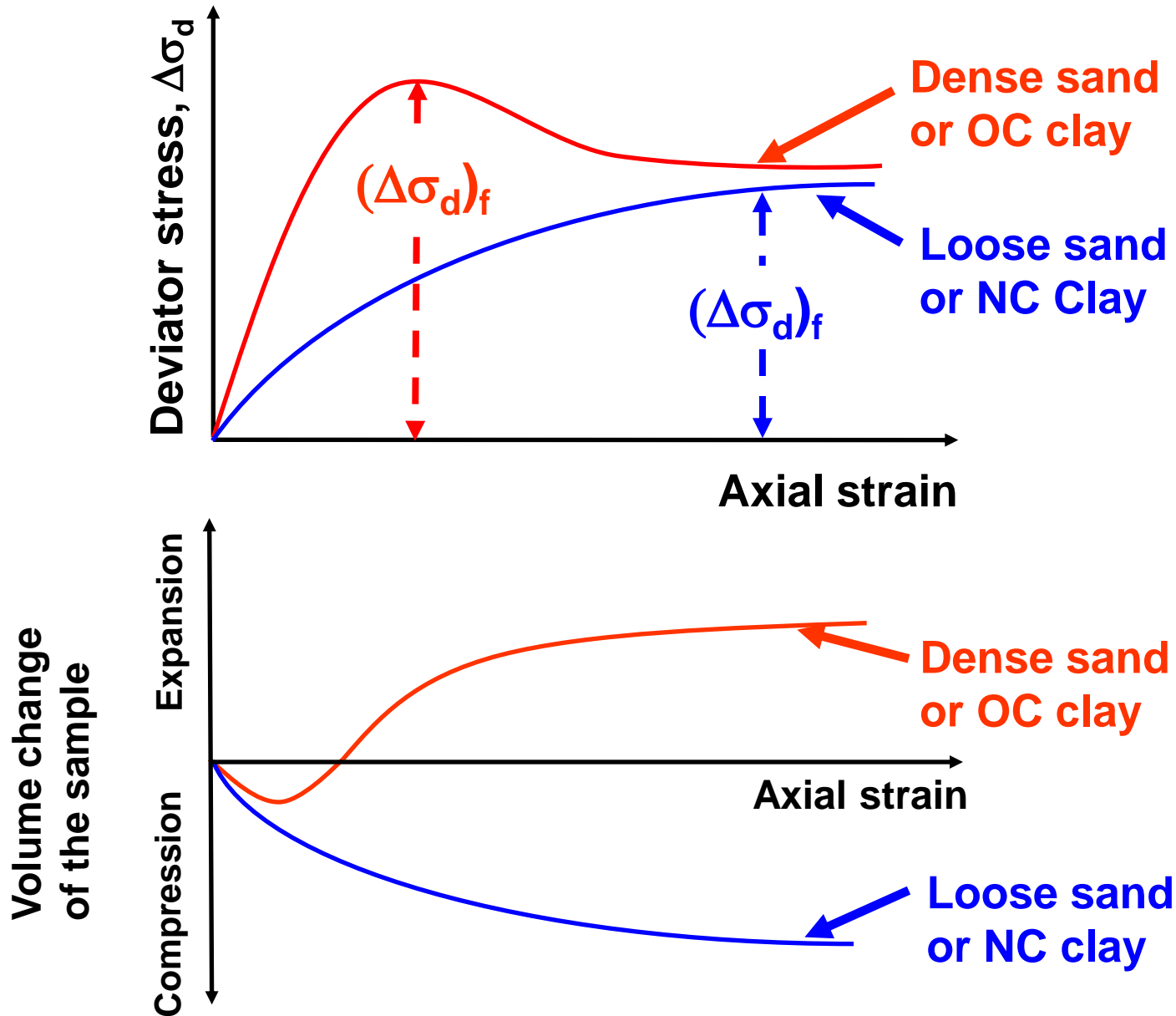
Consolidated- drained test (CD Test)

Volume change of sample during consolidation

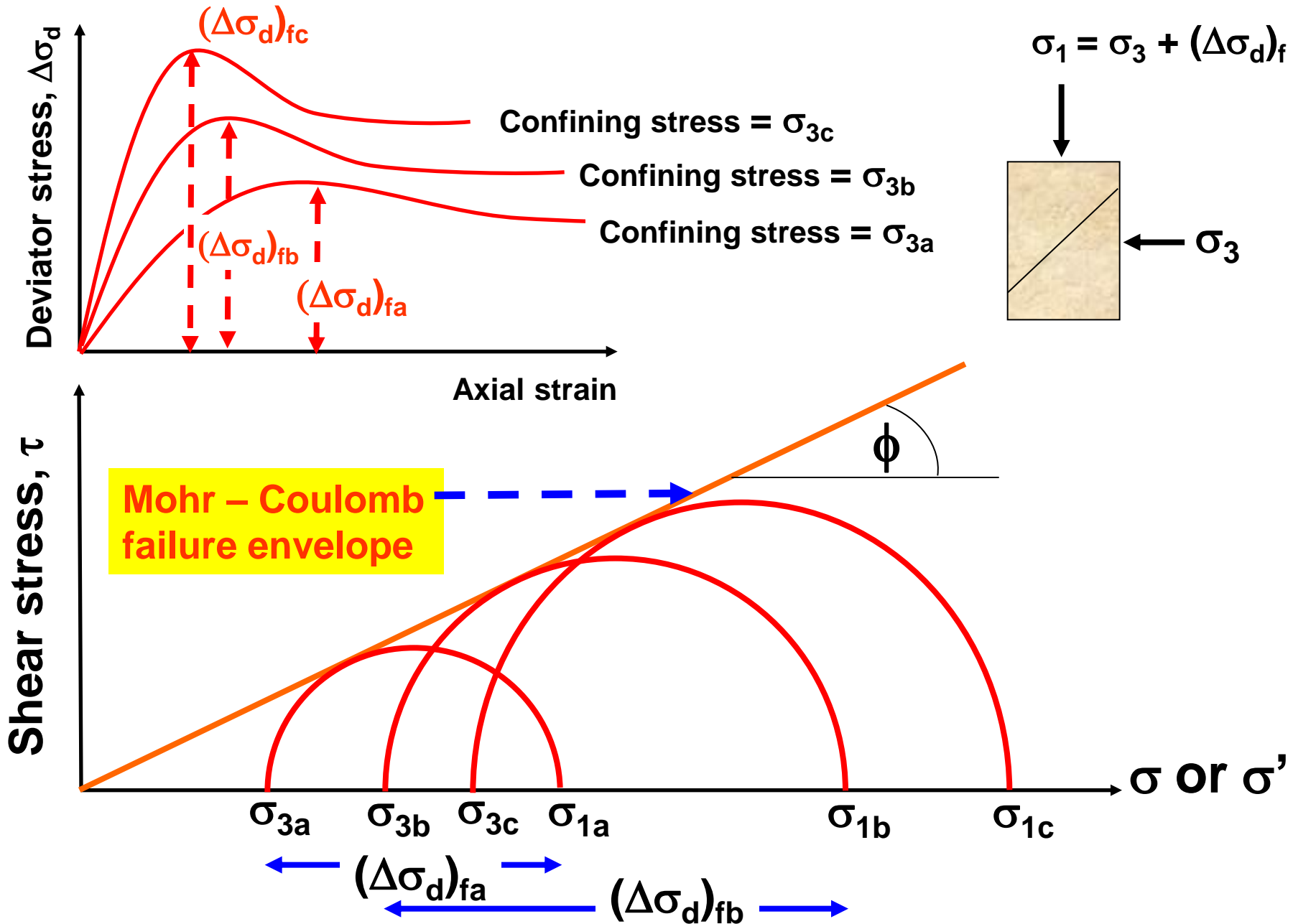


Consolidated- drained test (CD Test)

Stress-strain relationship during shearing



CD tests How to determine strength parameters c and ϕ



CD tests

Strength parameters c and ϕ obtained from CD tests

Since $u = 0$ in CD tests, $\sigma = \sigma'$

Therefore, $c = c'$ and $\phi = \phi'$

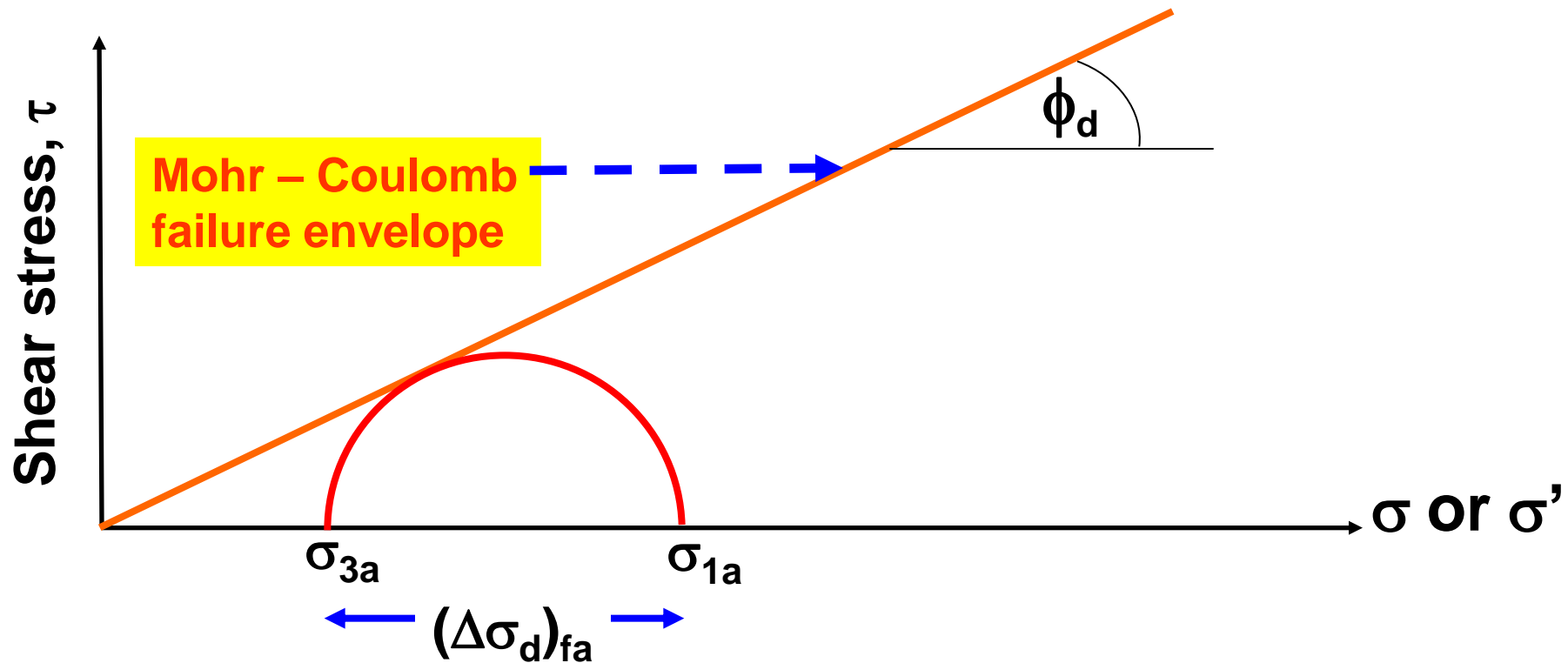
c_d and ϕ_d are used to denote them



A GENTLE REMINDER ...

CD tests Failure envelopes

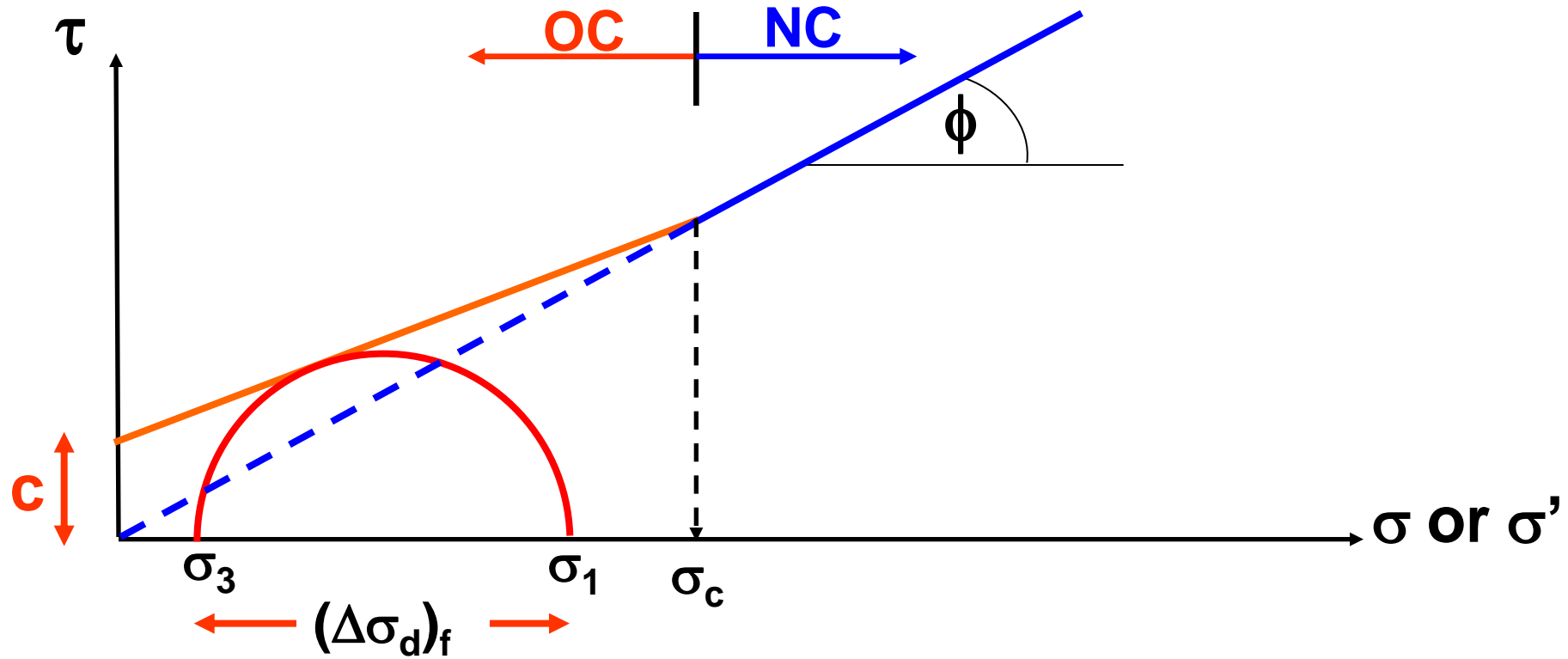
For sand and NC Clay, $c_d = 0$



Therefore, one CD test would be sufficient to determine ϕ_d of sand or NC clay

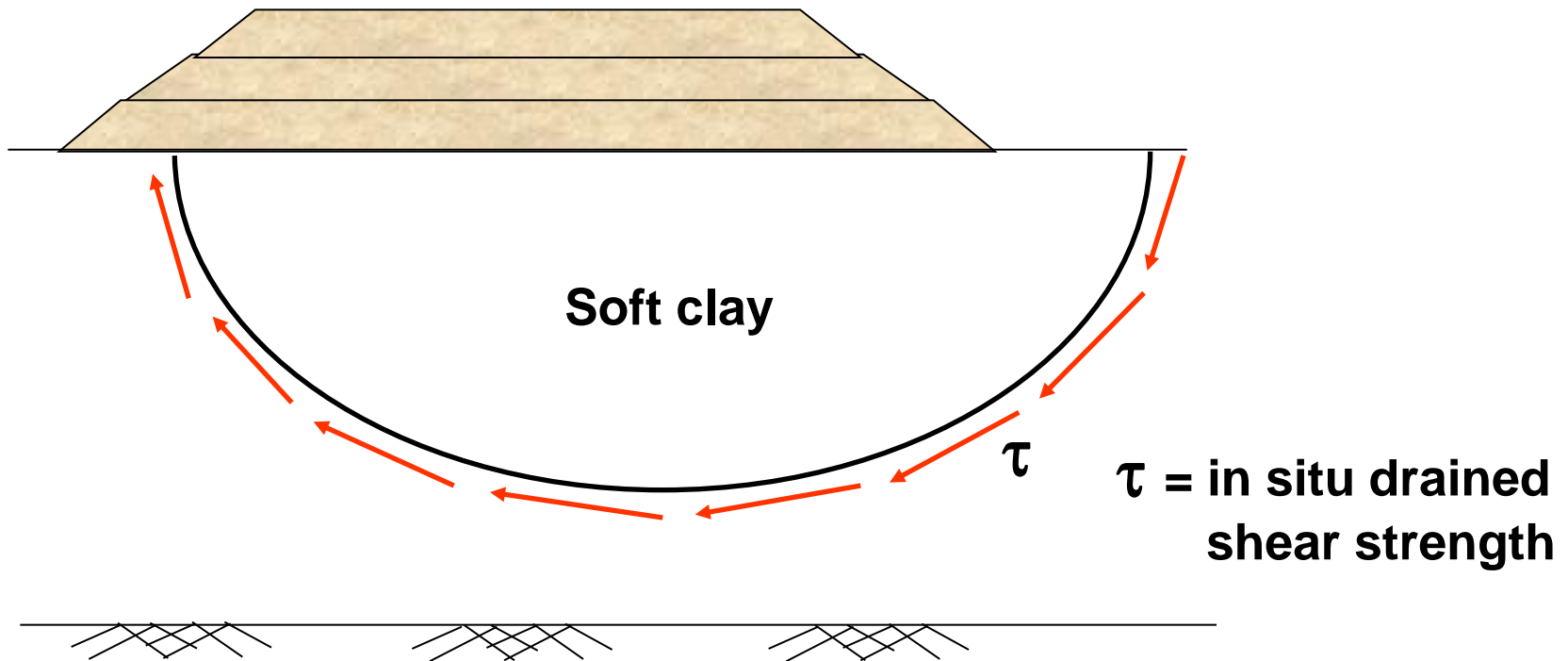
CD tests Failure envelopes

For OC Clay, $c_d \neq 0$



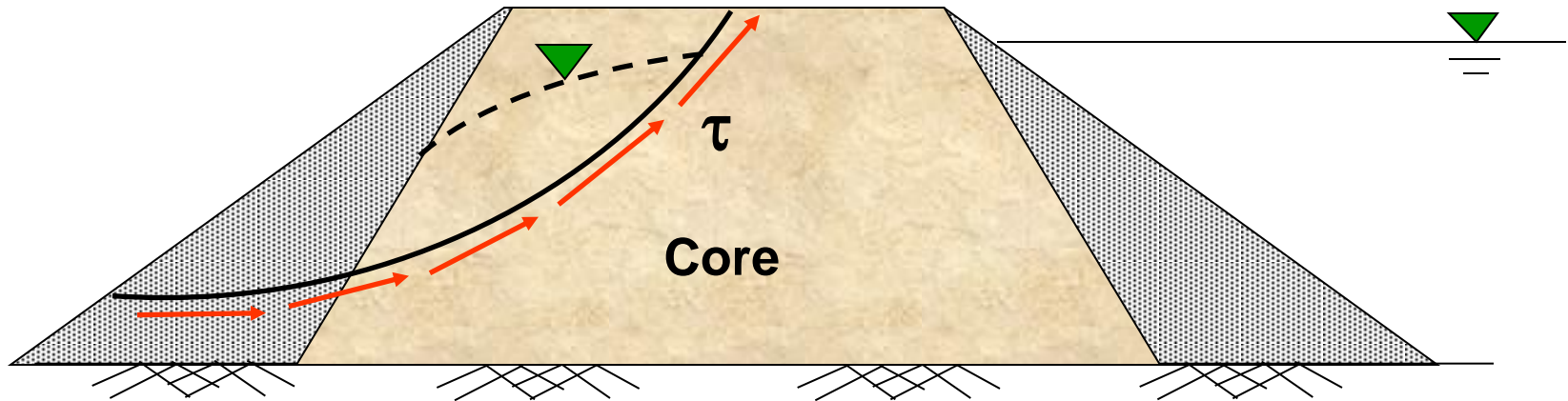
Some practical applications of CD analysis for clays

1. Embankment constructed very slowly, in layers over a soft clay deposit



Some practical applications of CD analysis for clays

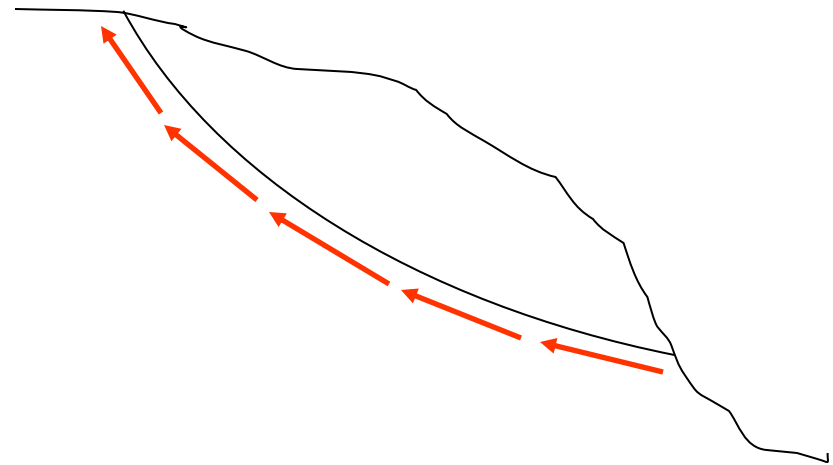
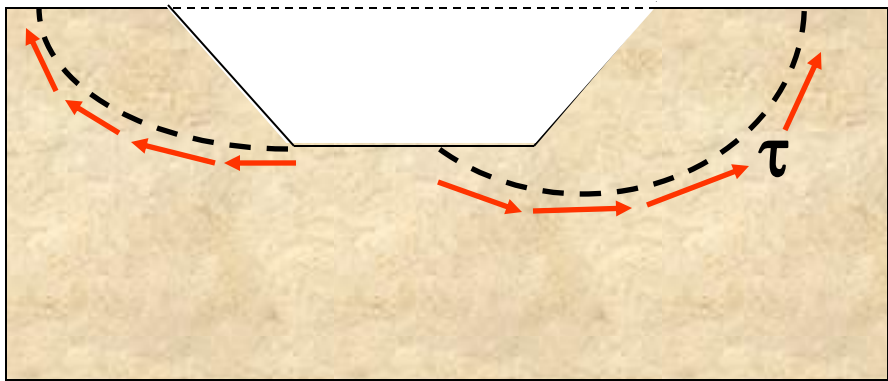
2. Earth dam with steady state seepage



τ = drained shear
strength of clay core

Some practical applications of CD analysis for clays

3. Excavation or natural slope in clay



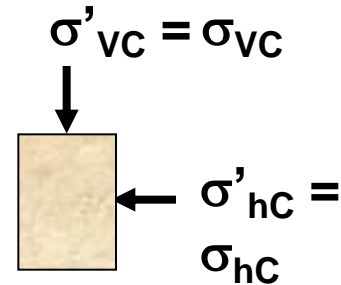
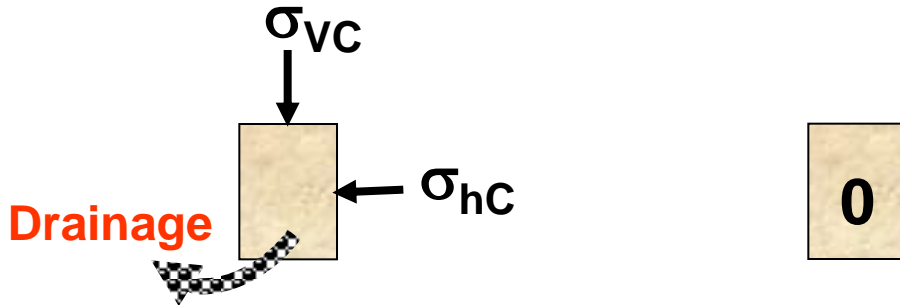
τ = In situ drained shear strength

Note: CD test simulates the long term condition in the field. Thus, c_d and ϕ_d should be used to evaluate the long term behavior of soils

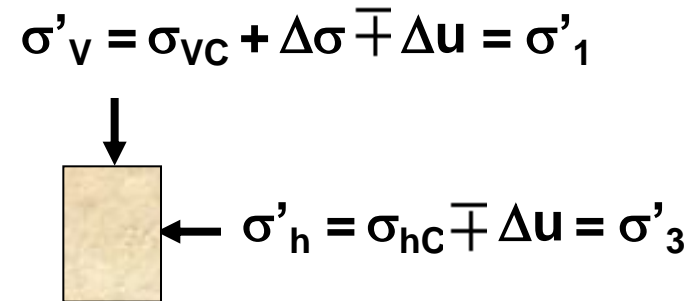
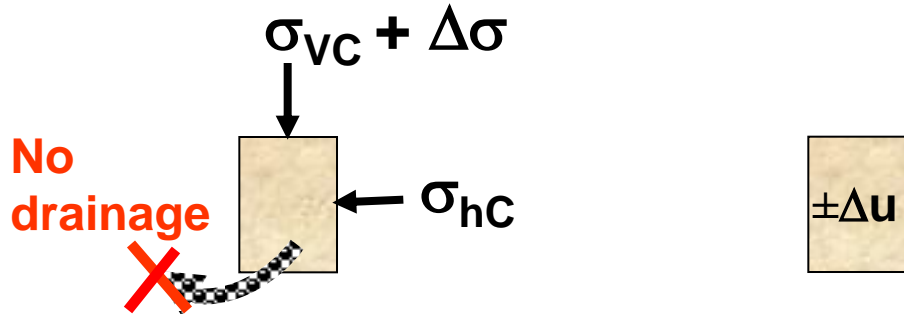
Consolidated- Undrained test (CU Test)

$$\text{Total, } \sigma = \text{Neutral, } u + \text{Effective, } \sigma'$$

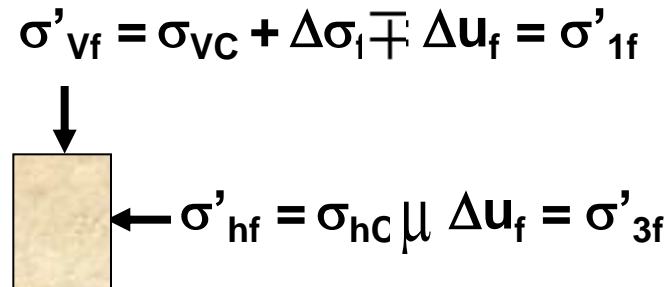
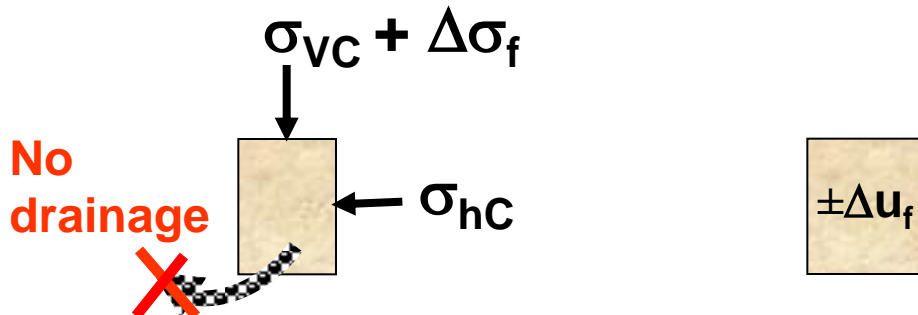
Step 1: At the end of consolidation



Step 2: During axial stress increase

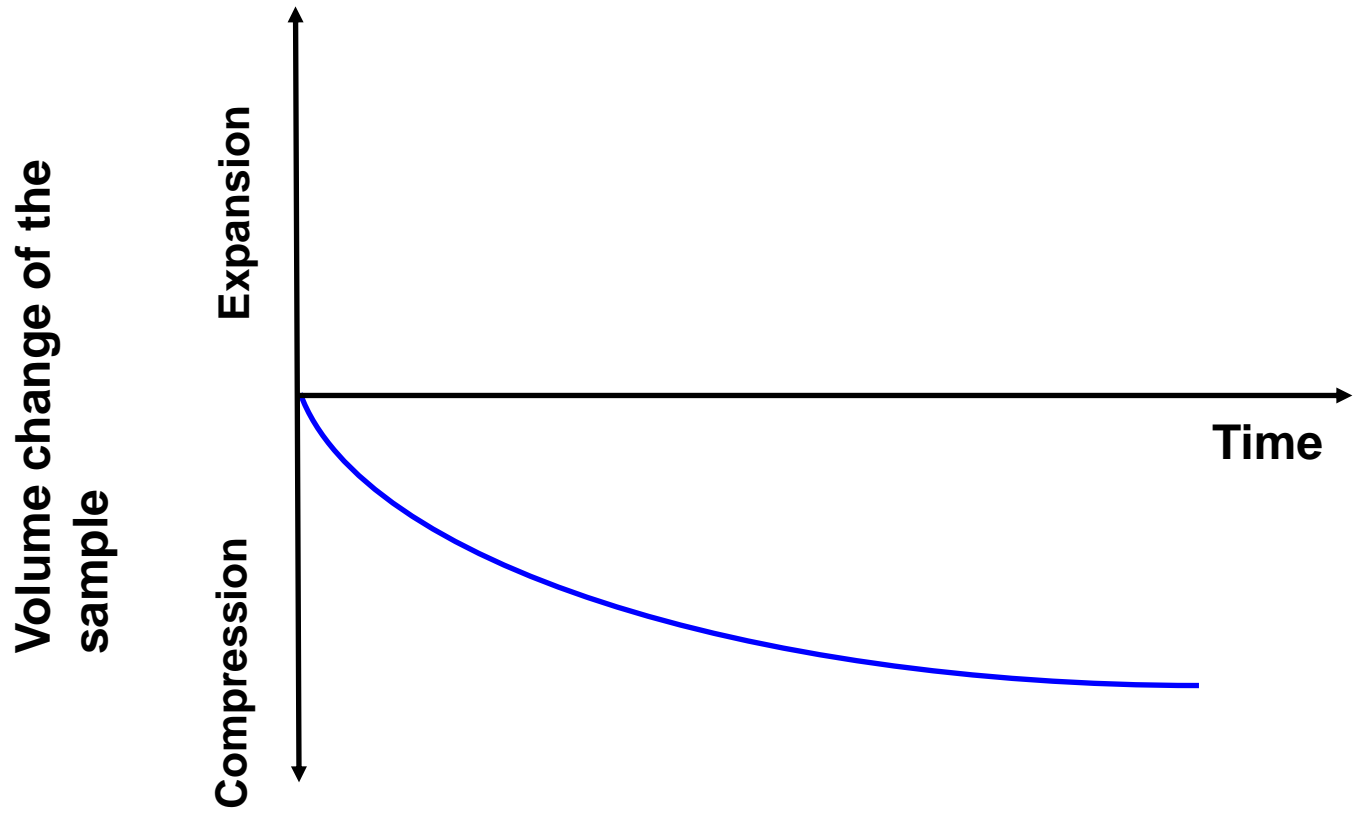


Step 3: At failure



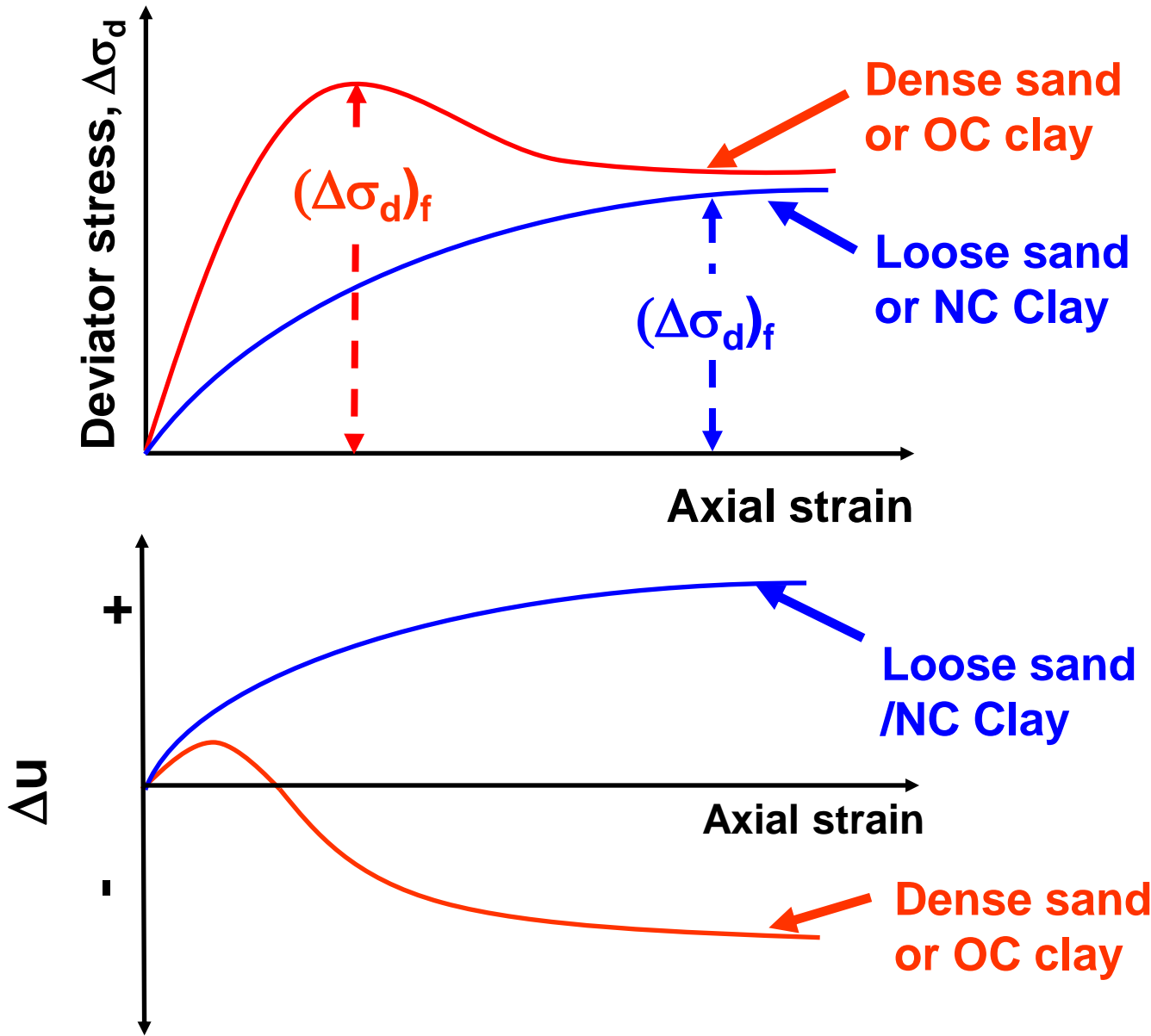
Consolidated- Undrained test (CU Test)

Volume change of sample during consolidation

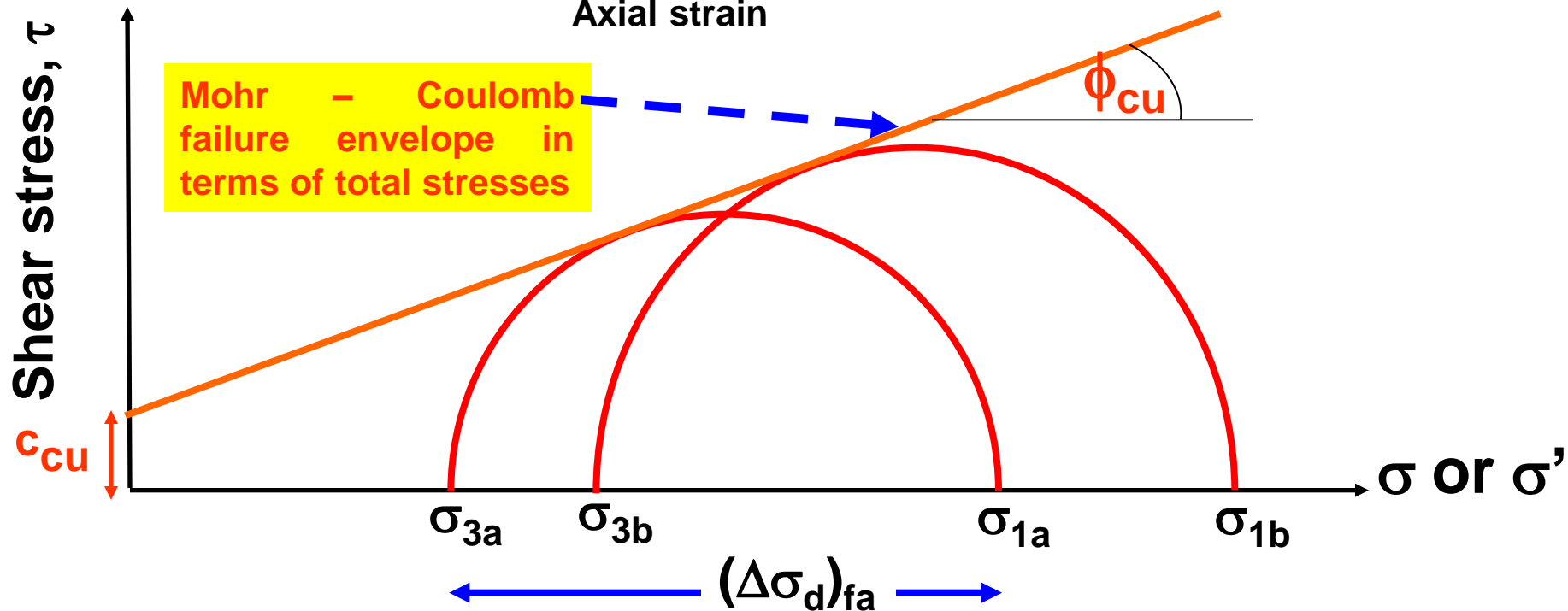
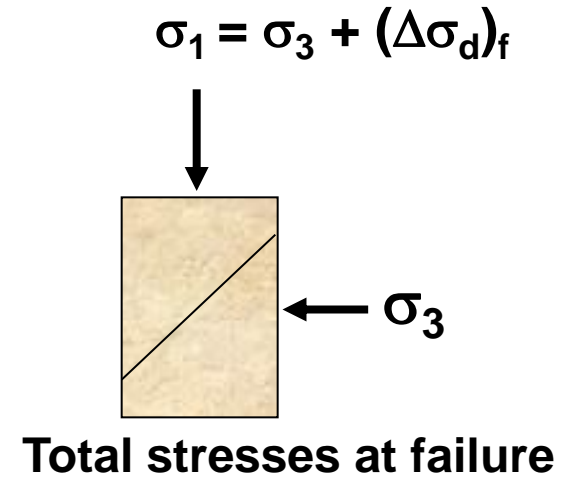
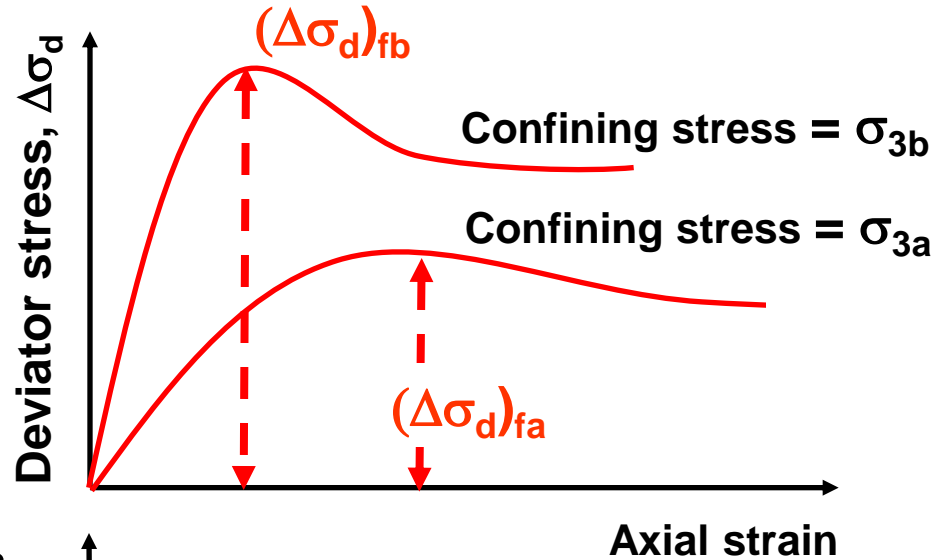


Consolidated- Undrained test (CU Test)

Stress-strain relationship during shearing

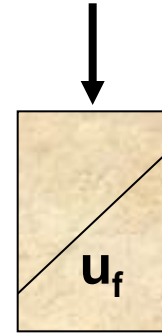


CU tests How to determine strength parameters c and ϕ



CU tests How to determine strength parameters c and ϕ

$$\sigma'_1 = \sigma_3 + (\Delta\sigma_d)_f - u_f$$

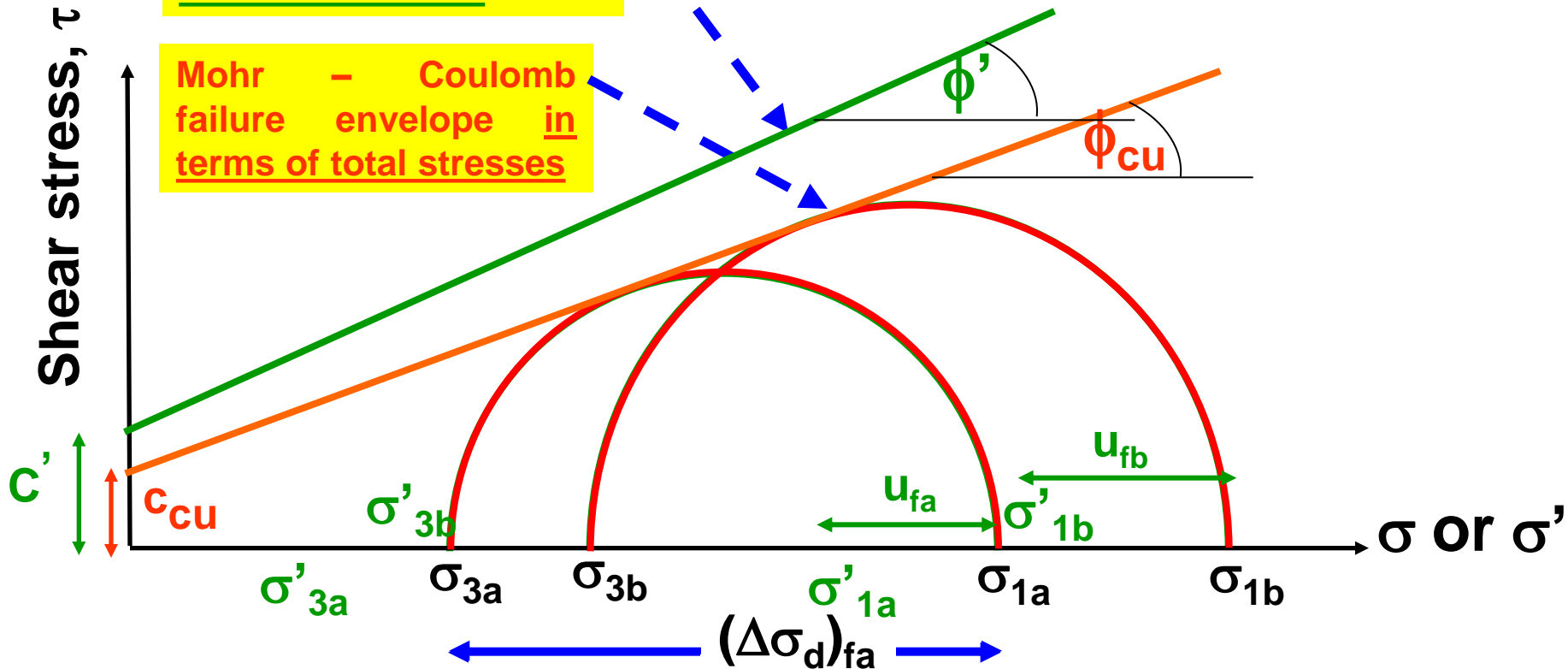


$$\sigma'_3 = \sigma_3 - u_f$$

Effective stresses at failure

Mohr - Coulomb failure envelope in terms of effective stresses

Mohr - Coulomb failure envelope in terms of total stresses



CU tests

Strength parameters c and ϕ obtained from CD tests

Shear strength parameters in terms of total stresses are c_{cu} and ϕ_{cu}

Shear strength parameters in terms of effective stresses are c' and ϕ'

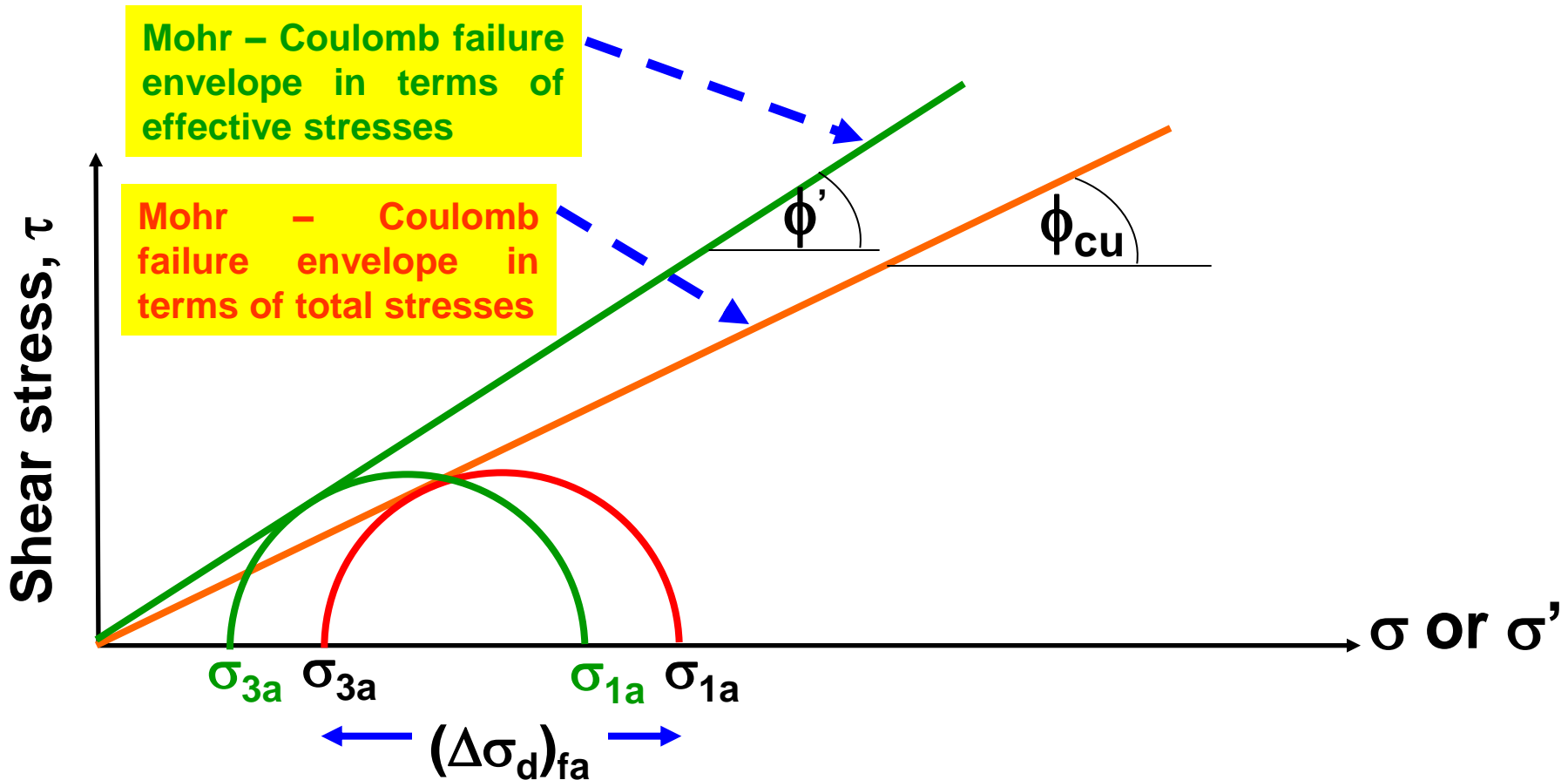
$c' = c_d$ and $\phi' = \phi_d$



A GENTLE REMINDER ...

CU tests Failure envelopes

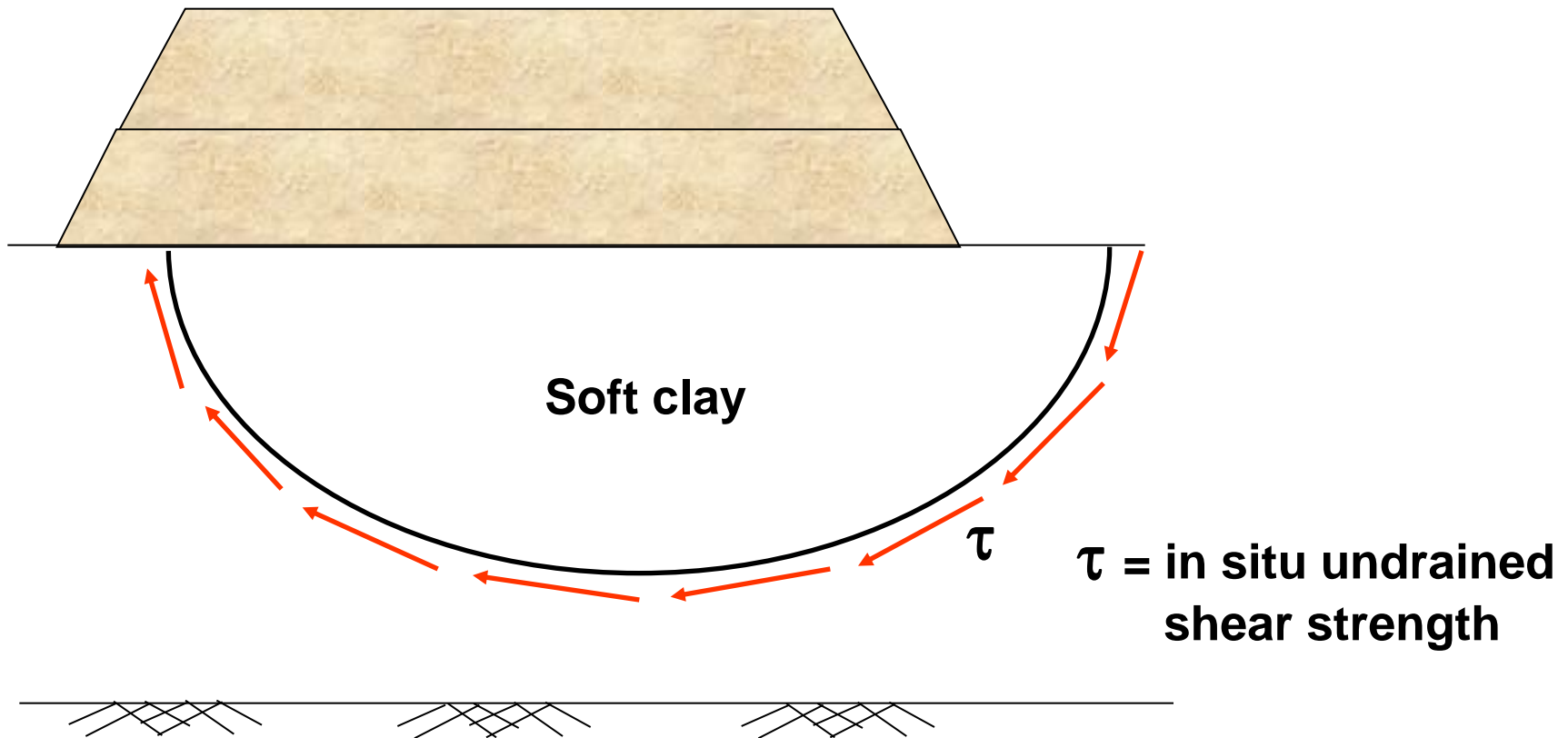
For sand and NC Clay, c_{cu} and $c' = 0$



Therefore, one CU test would be sufficient to determine ϕ_{cu} and ϕ' ($= \phi_d$) of sand or NC clay

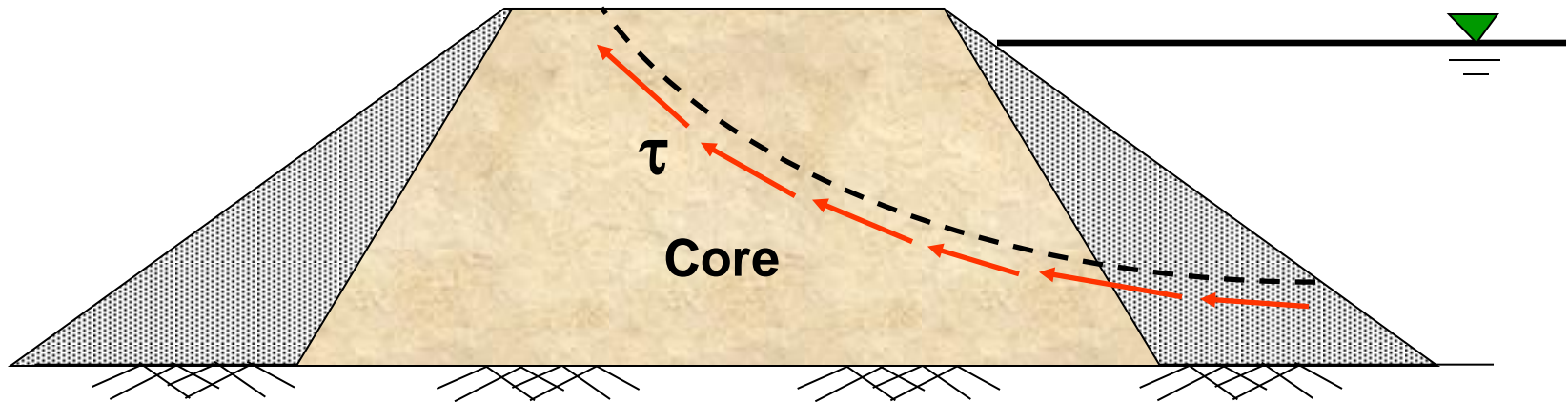
Some practical applications of CU analysis for clays

1. Embankment constructed rapidly over a soft clay deposit



Some practical applications of CU analysis for clays

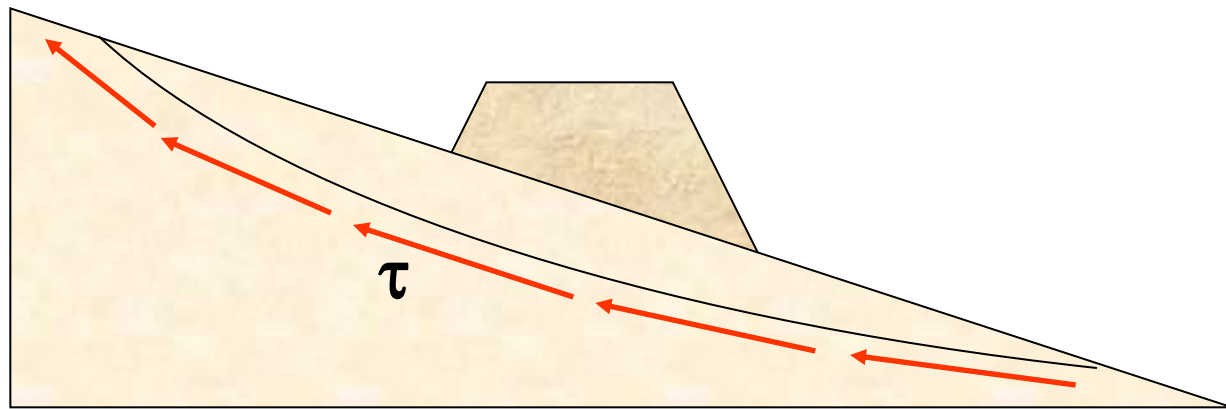
2. Rapid drawdown behind an earth dam



τ = Undrained shear strength of clay core

Some practical applications of CU analysis for clays

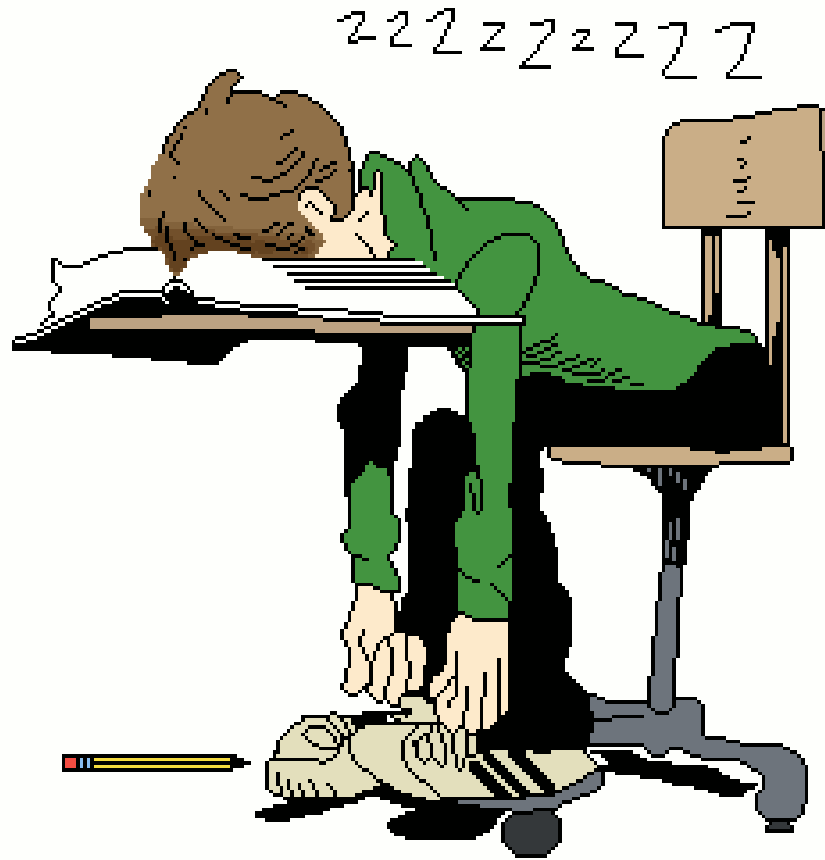
3. Rapid construction of an embankment on a natural slope



τ = In situ undrained shear strength

Note: Total stress parameters from CU test (c_{cu} and ϕ_{cu}) can be used for stability problems where,

Soil have become fully consolidated and are at equilibrium with the existing stress state; Then for some reason additional stresses are applied quickly with no drainage occurring

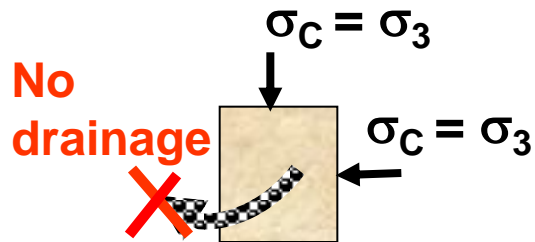


Lets do an example

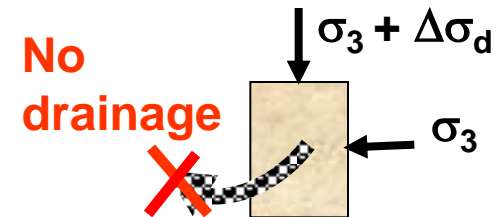
Unconsolidated- Undrained test (UU Test)

Data analysis

Initial specimen condition



Specimen condition during shearing



Initial volume of the sample = $A_0 \times H_0$

Volume of the sample during shearing = $A \times H$

Since the test is conducted under undrained condition,

$$A \times H = A_0 \times H_0$$

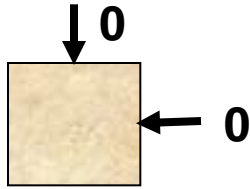
$$A \times (H_0 - \Delta H) = A_0 \times H_0$$

$$A \times (1 - \Delta H/H_0) = A_0$$

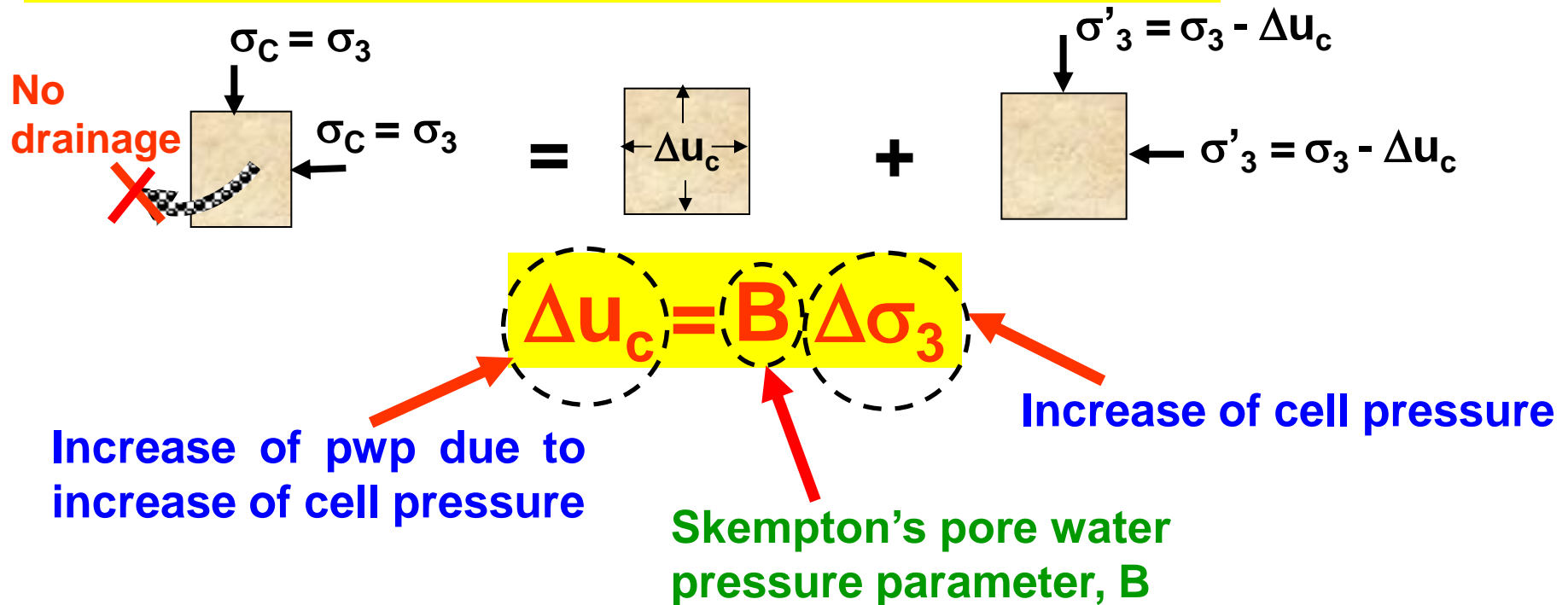
$$A = \frac{A_0}{1 - \varepsilon_z}$$

Unconsolidated- Undrained test (UU Test)

Step 1: Immediately after sampling



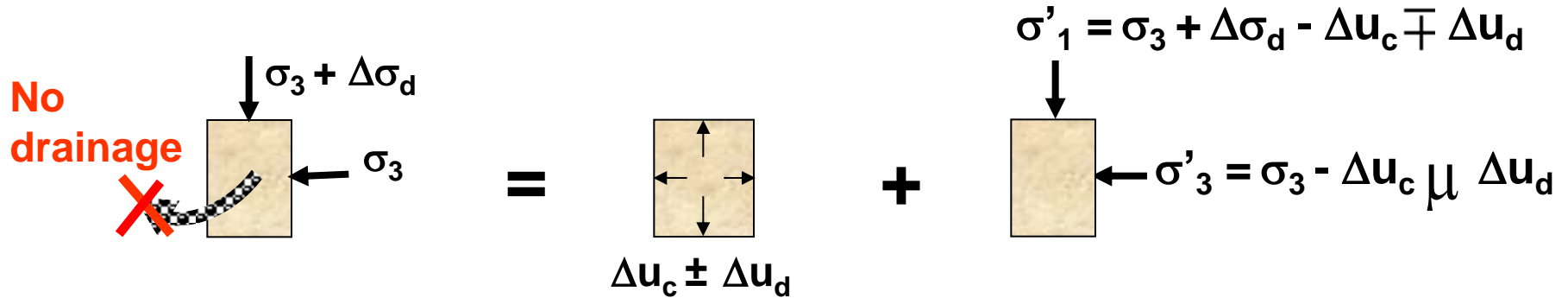
Step 2: After application of hydrostatic cell pressure



Note: If soil is fully saturated, then $B = 1$ (hence, $\Delta u_c = \Delta \sigma_3$)

Unconsolidated- Undrained test (UU Test)

Step 3: During application of axial load



Increase of pwp due to increase of deviator stress

Increase of deviator stress

Skempton's pore water pressure parameter, A

Unconsolidated- Undrained test (UU Test)

Combining steps 2 and 3,

$$\Delta u_c = B \Delta \sigma_3$$

$$\Delta u_d = AB \Delta \sigma_d$$

Total pore water pressure increment at any stage, Δu

$$\Delta u = \Delta u_c + \Delta u_d$$

$$\Delta u = B [\Delta \sigma_3 + A \Delta \sigma_d]$$

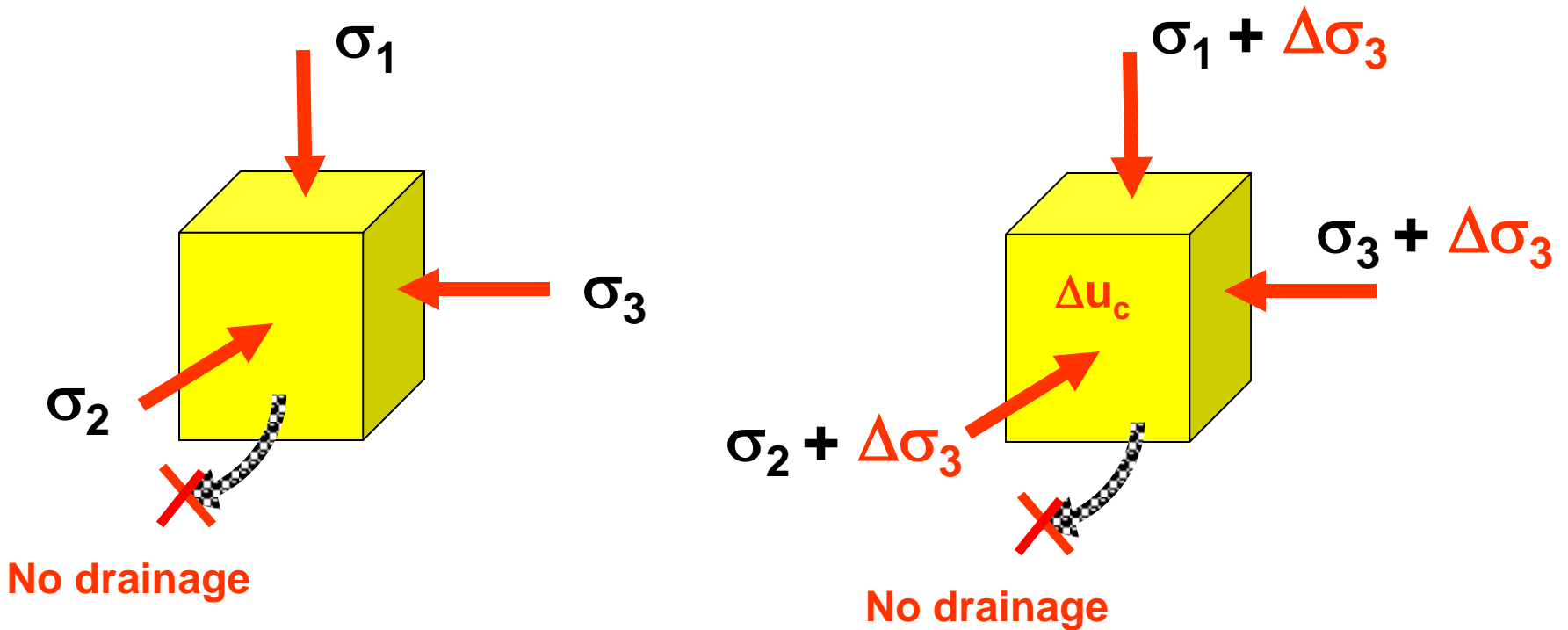
$$\Delta u = B [\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)]$$

Skempton's pore water pressure equation

Unconsolidated- Undrained test (UU Test)

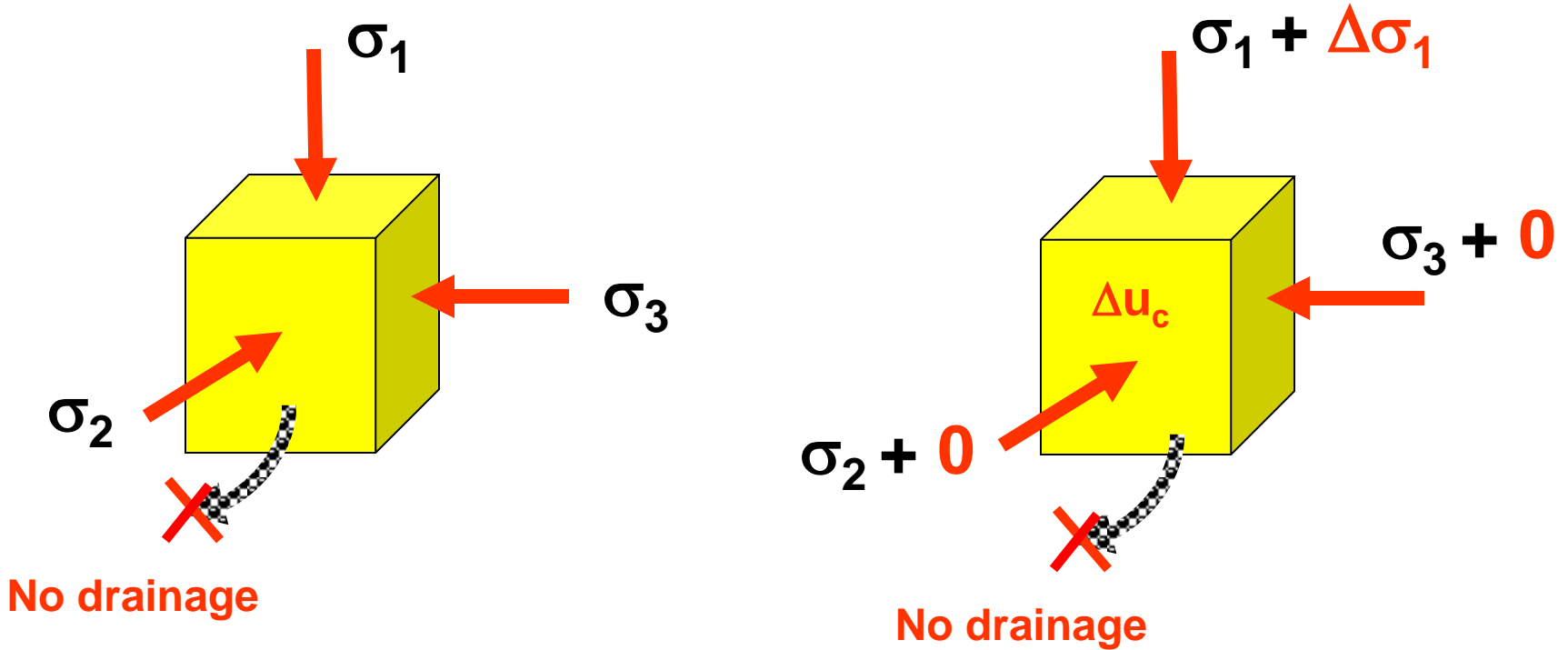


Step 1 : Increment of isotropic stress



Increase in effective stress in each direction = $\Delta\sigma_3 - \Delta u_c$

Step 2 : Increment of major principal stress

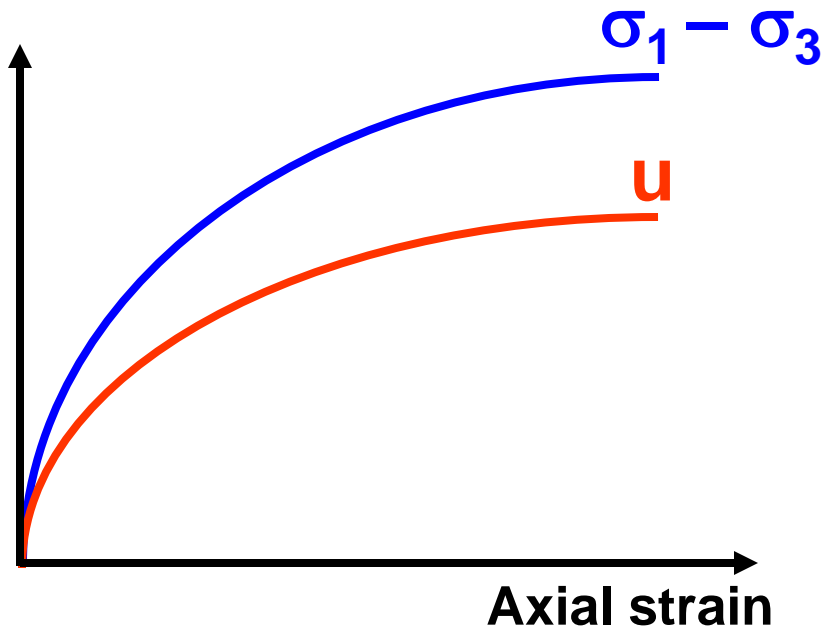


Increase in effective stress in σ_1 direction = $\Delta\sigma_1 - \Delta u_d$

Increase in effective stress in σ_2 and σ_3 directions = $0 - \Delta u_d$

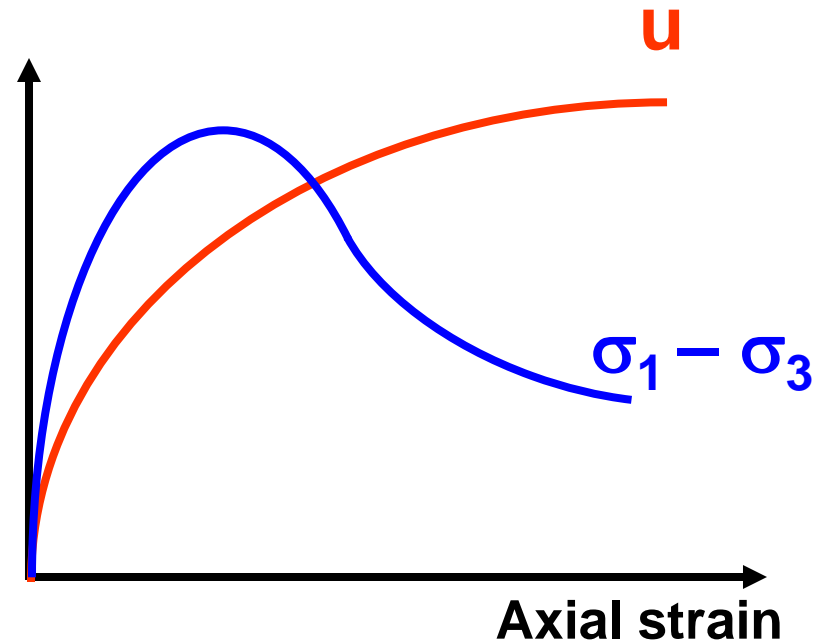
Average Increase in effective stress = $(\Delta\sigma_1 - \Delta u_d - \Delta u_d - \Delta u_d)/3$

Typical values for parameter A



NC Clay (low sensitivity)

($A = 0.5 - 1.0$)



NC Clay (High sensitivity)

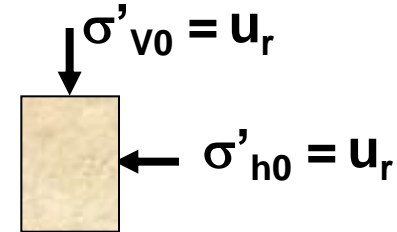
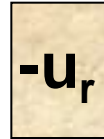
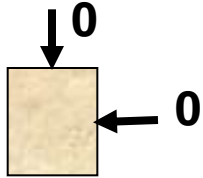
($A > 1.0$)

Collapse of soil structure may occur in high sensitivity clays due to very high pore water pressure generation

Unconsolidated- Undrained test (UU Test)

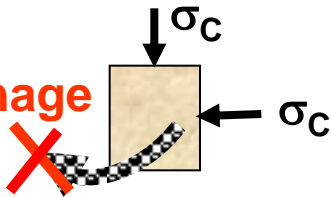
$$\text{Total, } \sigma = \text{Neutral, } u + \text{Effective, } \sigma'$$

Step 1: Immediately after sampling



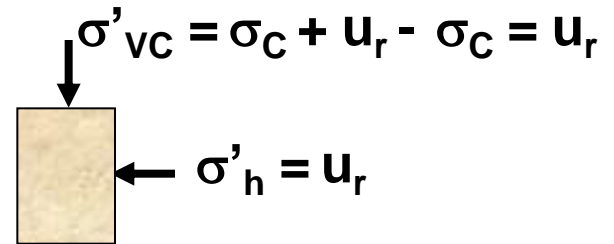
Step 2: After application of hydrostatic cell pressure

No drainage



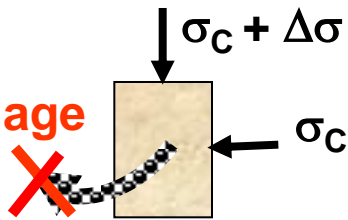
$$-u_r + \Delta u_c = -u_r + \sigma_c$$

($S_r = 100\%$; $B = 1$)

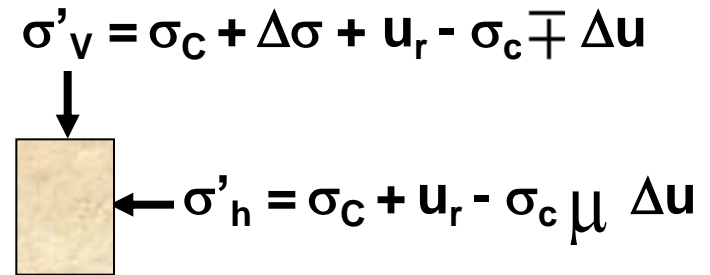


Step 3: During application of axial load

No drainage

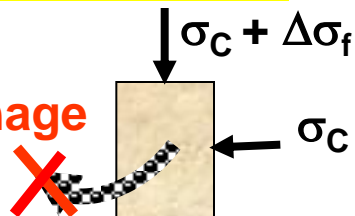


$$-u_r + \sigma_c \pm \Delta u$$

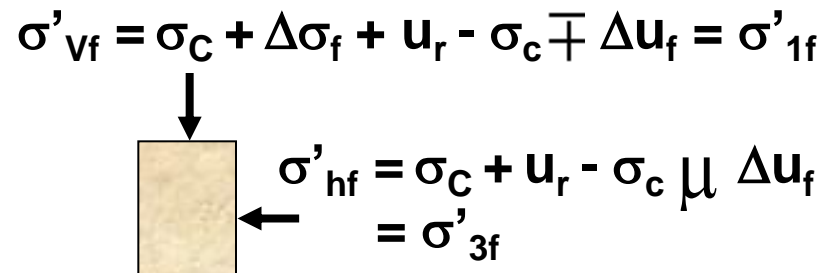


Step 3: At failure

No drainage



$$-u_r + \sigma_c \pm \Delta u_f$$



Unconsolidated- Undrained test (UU Test)

Total, σ

=

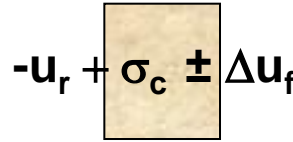
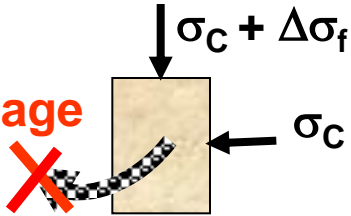
Neutral, u

+

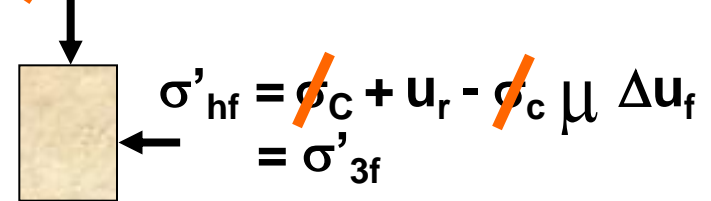
Effective, σ'

Step 3: At failure

No drainage

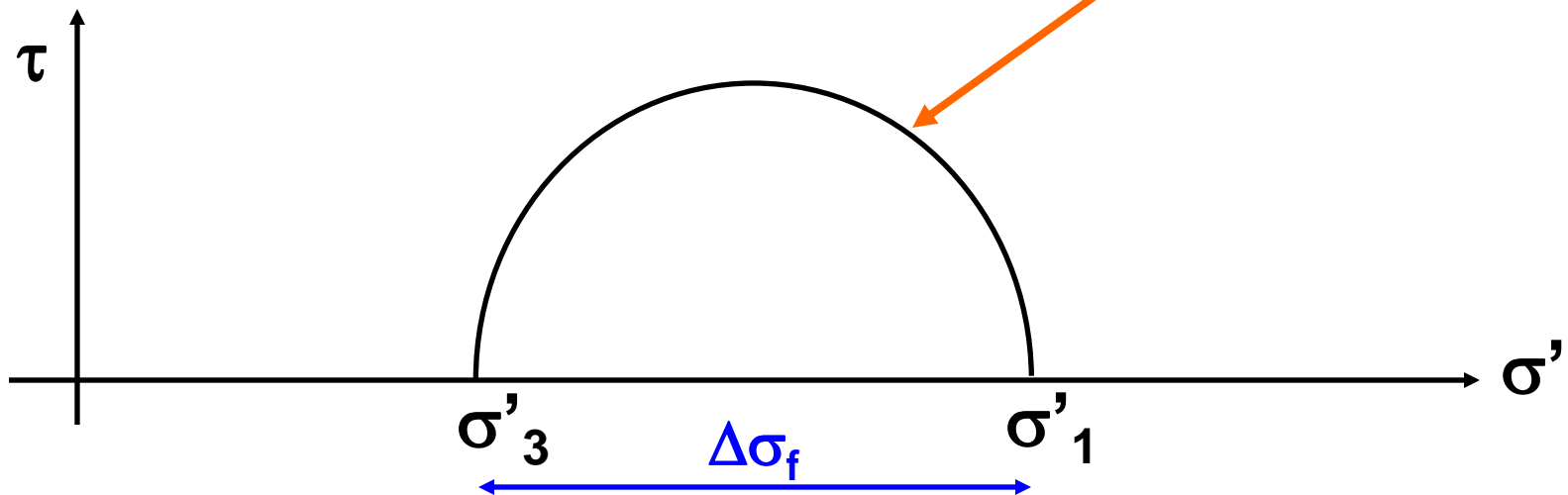


$$\sigma'_{vf} = \cancel{\sigma_c} + \Delta\sigma_f + u_r - \cancel{\sigma_c} \mp \Delta u_f = \sigma'_{1f}$$



Mohr circle in terms of effective stresses do not depend on the cell pressure.

Therefore, we get only one Mohr circle in terms of effective stress for different cell pressures

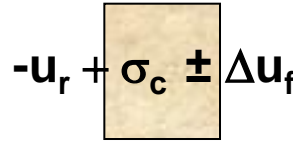
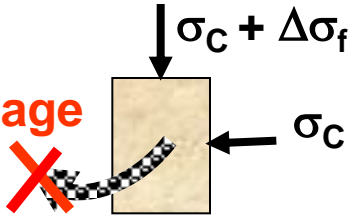


Unconsolidated- Undrained test (UU Test)

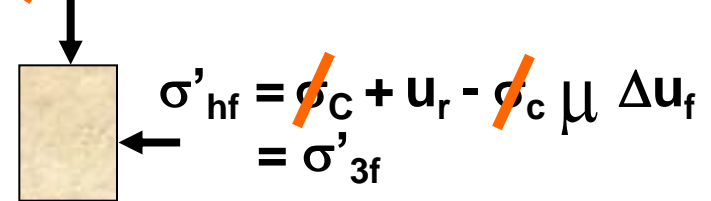
$$\text{Total, } \sigma = \text{Neutral, } u + \text{Effective, } \sigma'$$

Step 3: At failure

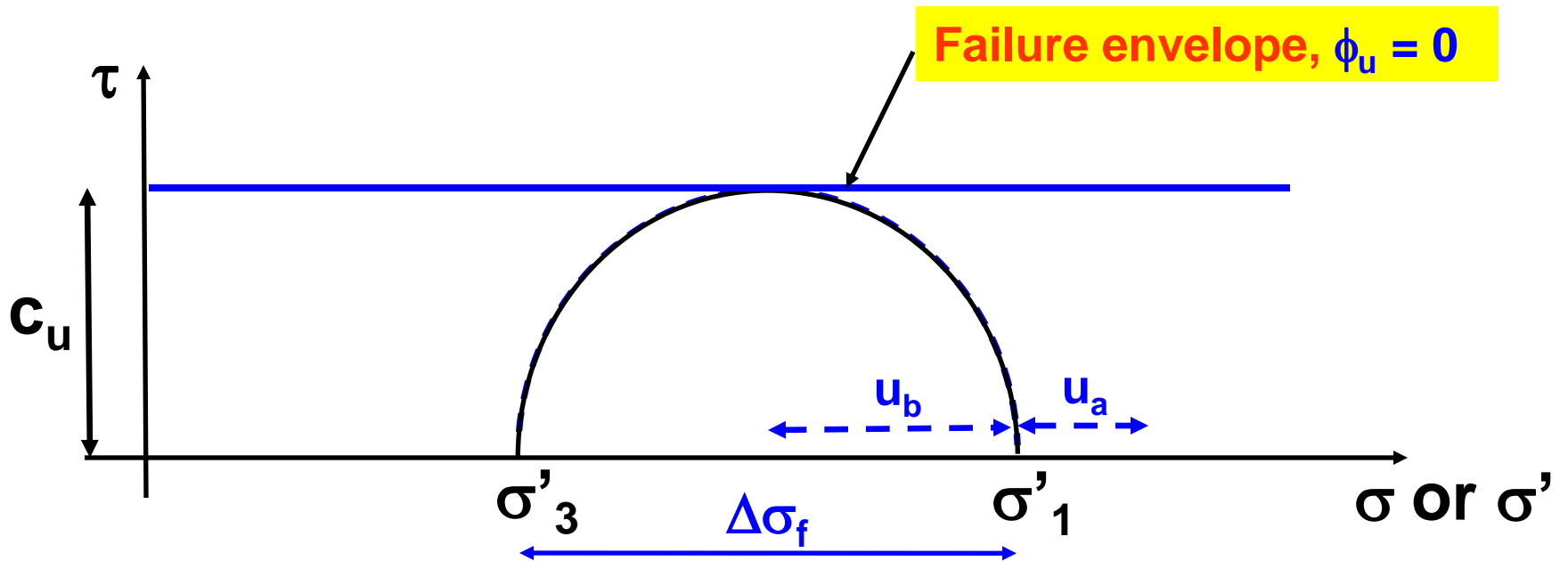
No drainage



$$\sigma'_{vf} = \cancel{\sigma_c} + \Delta\sigma_f + u_r - \cancel{\sigma_c} \mp \Delta u_f = \sigma'_{1f}$$

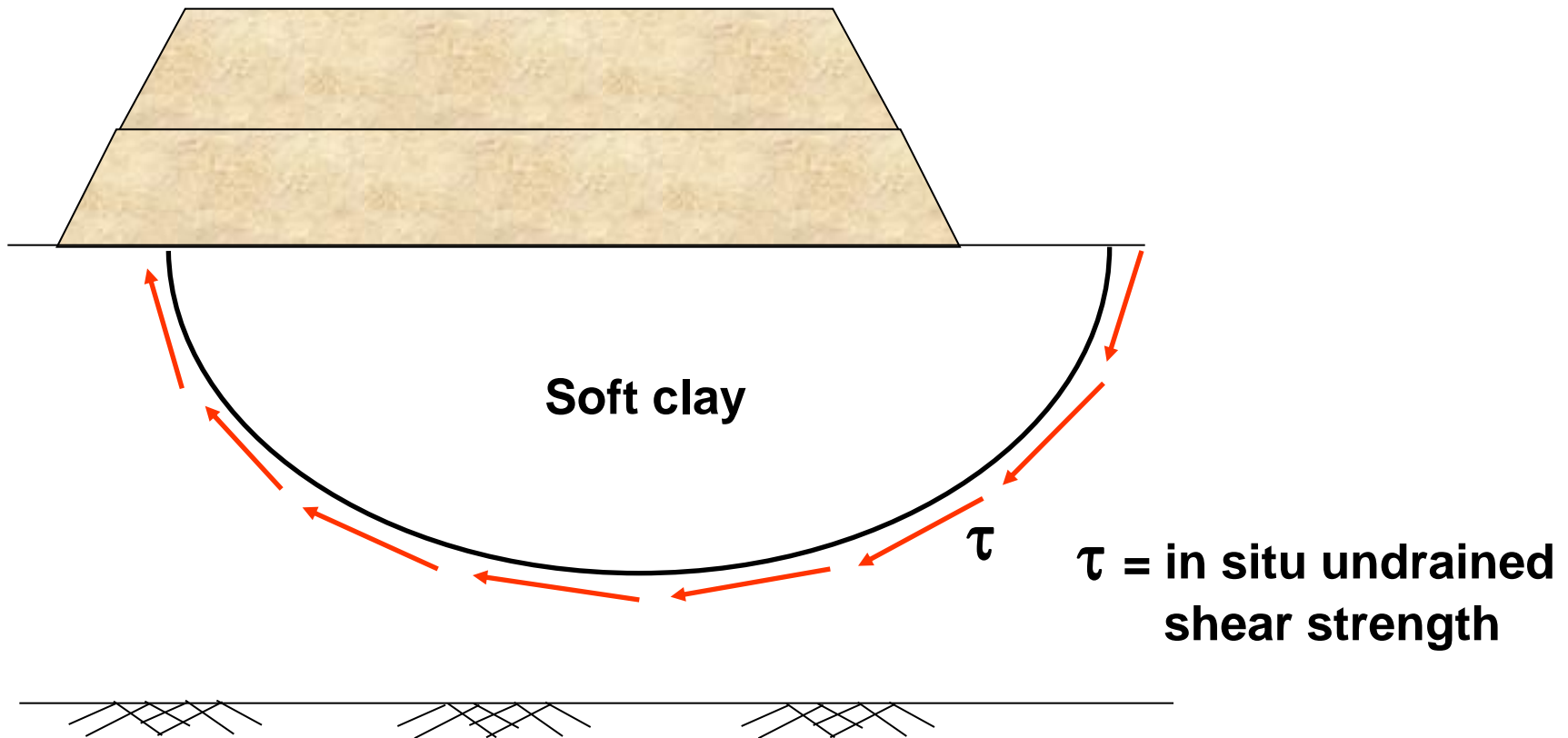


Mohr circles in terms of total stresses



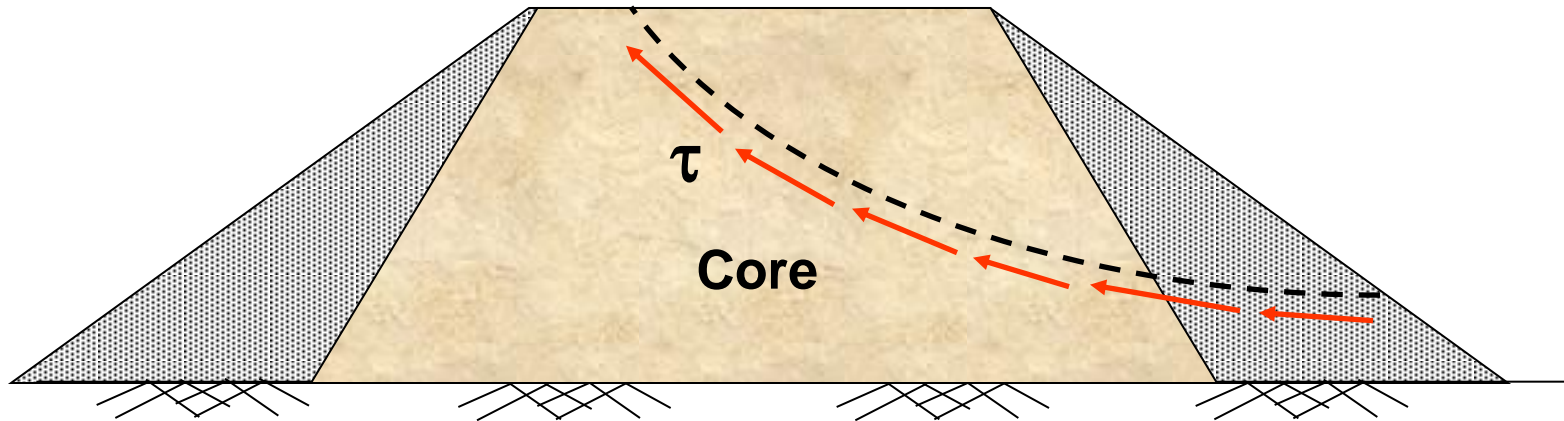
Some practical applications of UU analysis for clays

1. Embankment constructed rapidly over a soft clay deposit



Some practical applications of UU analysis for clays

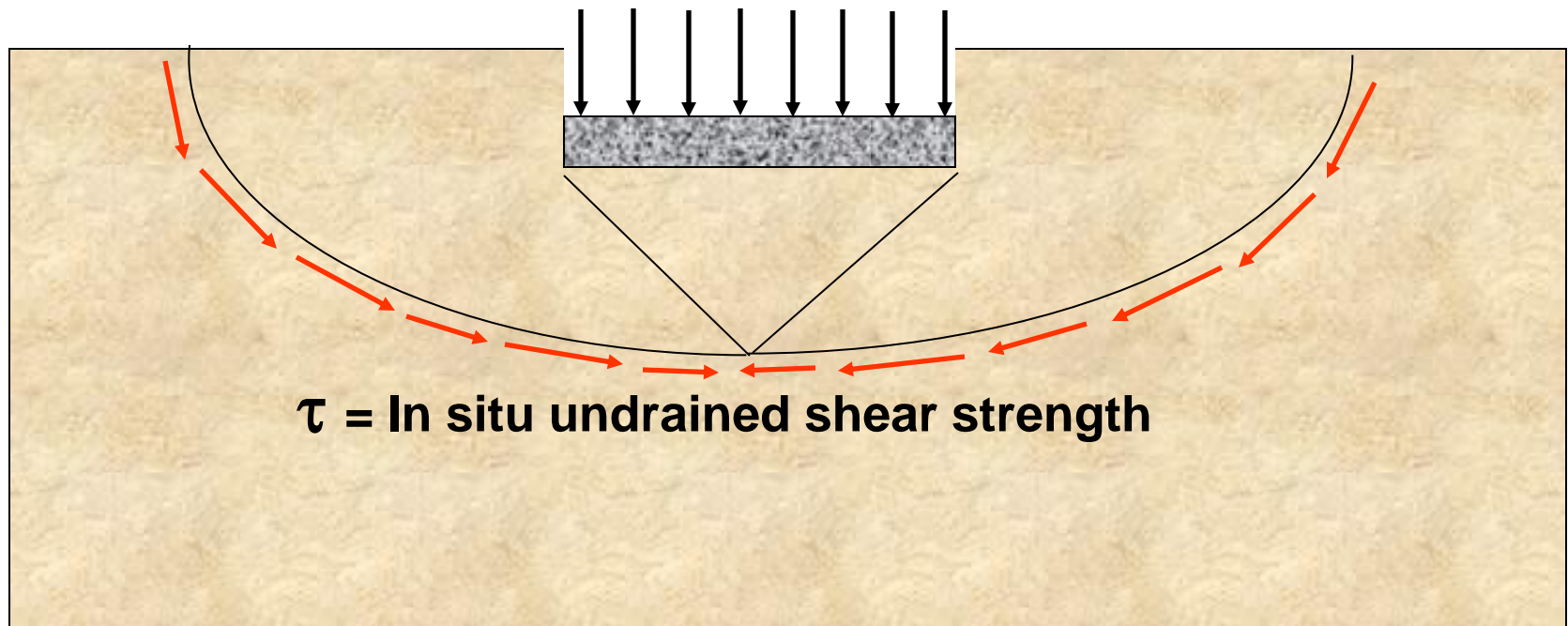
2. Large earth dam constructed rapidly with no change in water content of soft clay



τ = Undrained shear strength of clay core

Some practical applications of UU analysis for clays

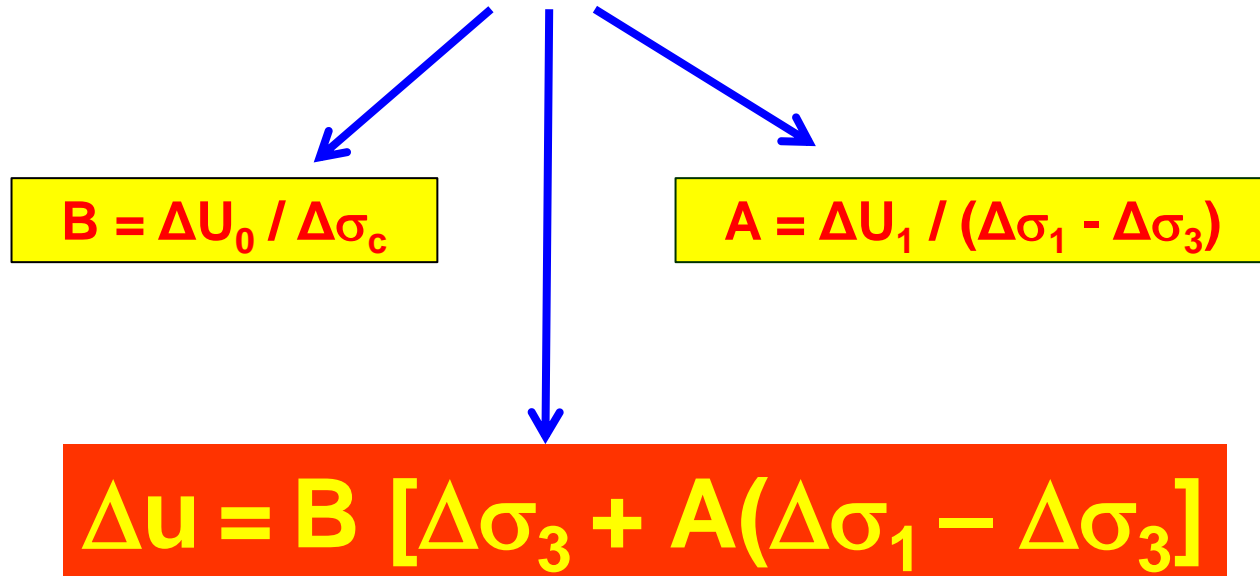
3. Footing placed rapidly on clay deposit



Note: UU test simulates the short term condition in the field. Thus, c_u can be used to analyze the short term behavior of soils

Pore water pressure measurement

Skempton's pore water pressure equation



Derivation of Skempton's pore water pressure equation

Saturation & use of back pressure

1. Reason for saturation
2. Principle of saturation
3. Maintaining saturation
4. Advantages of saturation

Reason for saturation

- 📁 deal with partly saturated soils
- 📁 complexity while measuring pore air pressure
- 📁 To measure the shear strength at failure

Principle of saturation

- raised the pwp sufficiently enough for water to absorb into the solution
- At the same time, σ_c is raised to maintain a small positive σ' in the sample
- Reasonable magnitude = 50 kPa to 100 kPa
- Check B value:

$$B = \Delta U_0 / \Delta \sigma_c$$

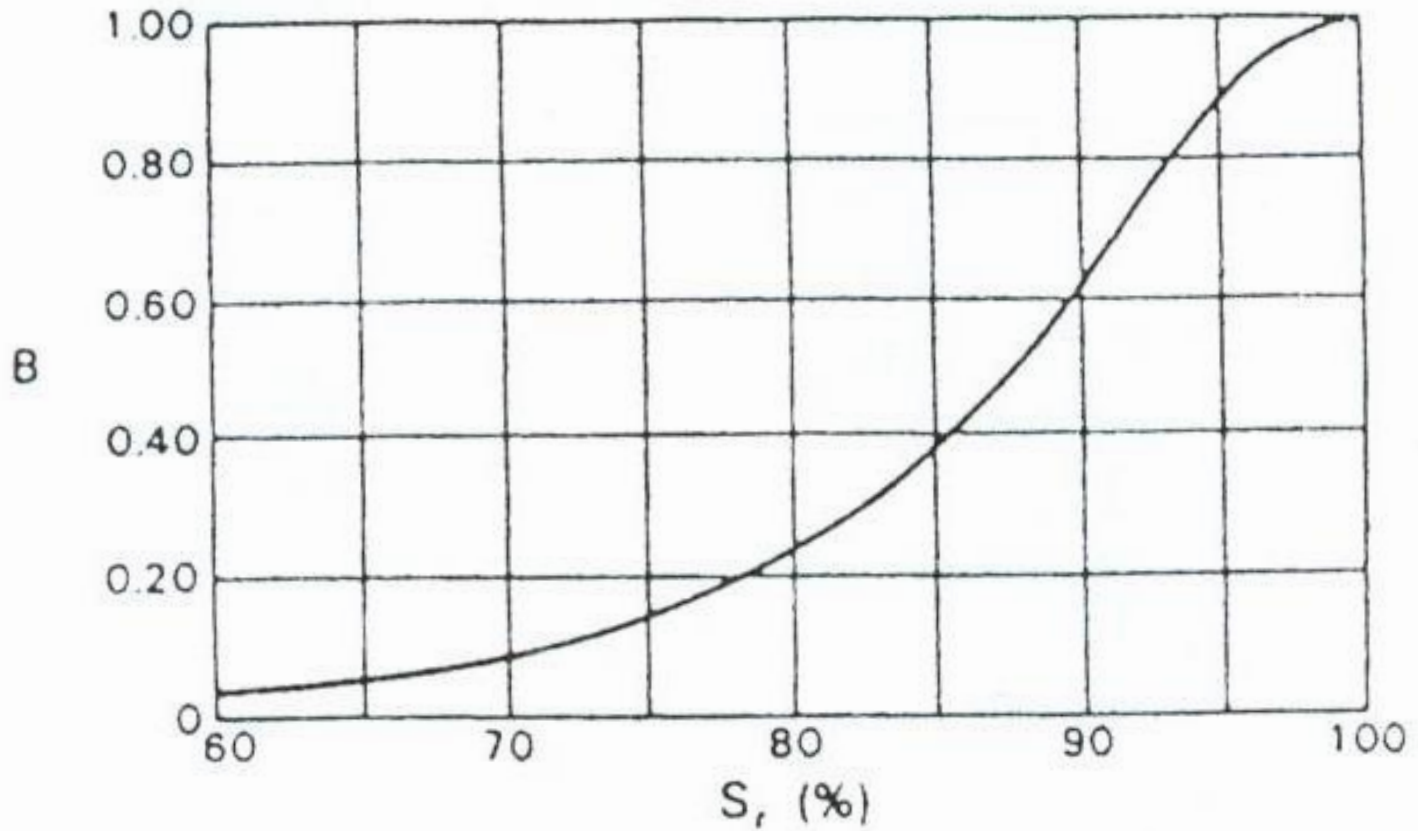
Maintaining saturation

- 📄 elevated pwp should if possible be maintained
- 📄 Pwp should not be less than 150 kPa

Advantages of saturation

- ❏ Can dissolved any air trapped between the membrane and the sample
- ❏ Any air bubbles remained in the pore pressure and back pressure systems are eliminated
- ❏ Reliable measurement of permeability can be made on soils that are initially partly saturated

Typical values for parameter **B**



Typical relationship between B and degree of saturation.

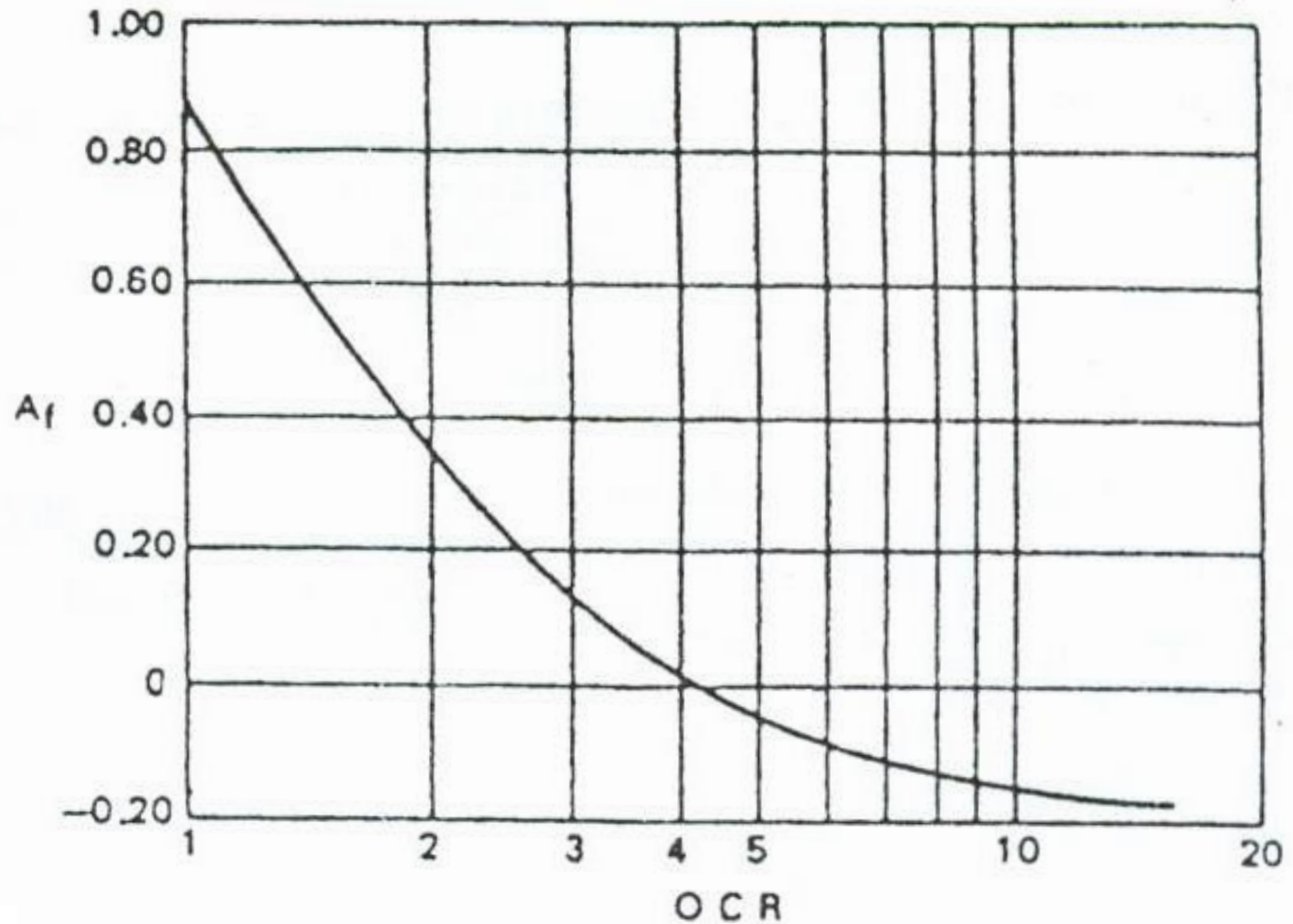
Typical values for parameter A_f

Table 15.2. TYPICAL VALUES OF PORE PRESSURE COEFFICIENT A_f

<i>Type of soil</i>	<i>Volume change due to shear</i>	A_f
Highly sensitive clay	large contraction	+0.75 to +1.5
Normally consolidated clay	contraction	+0.5 to +1
Compacted sandy clay	slight contraction	+0.25 to +0.75
Lightly overconsolidated clay	none	0 to +0.5
Compacted clay gravel	expansion	-0.25 to +0.25
Heavily overconsolidated clay	expansion	-0.5 to 0

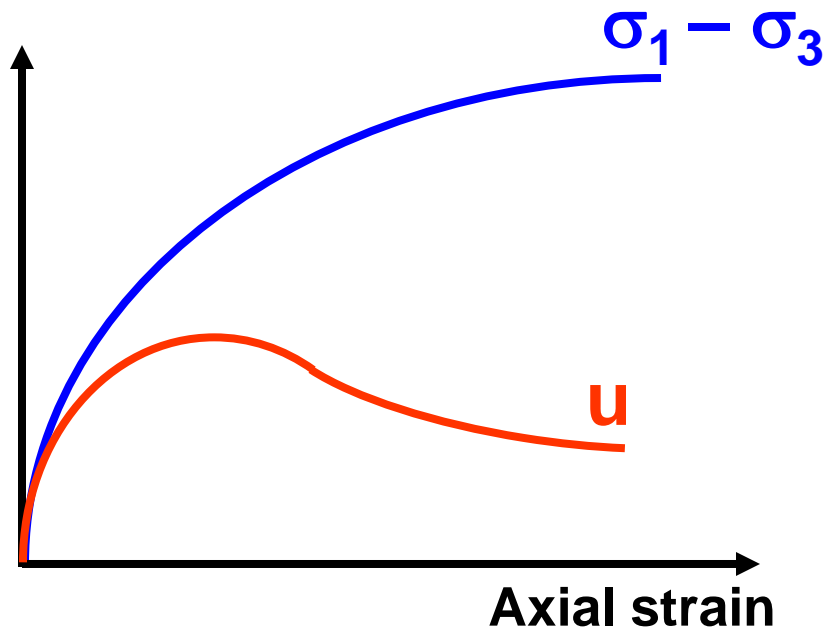
(After Skempton, 1954)

Typical values for parameter **A**



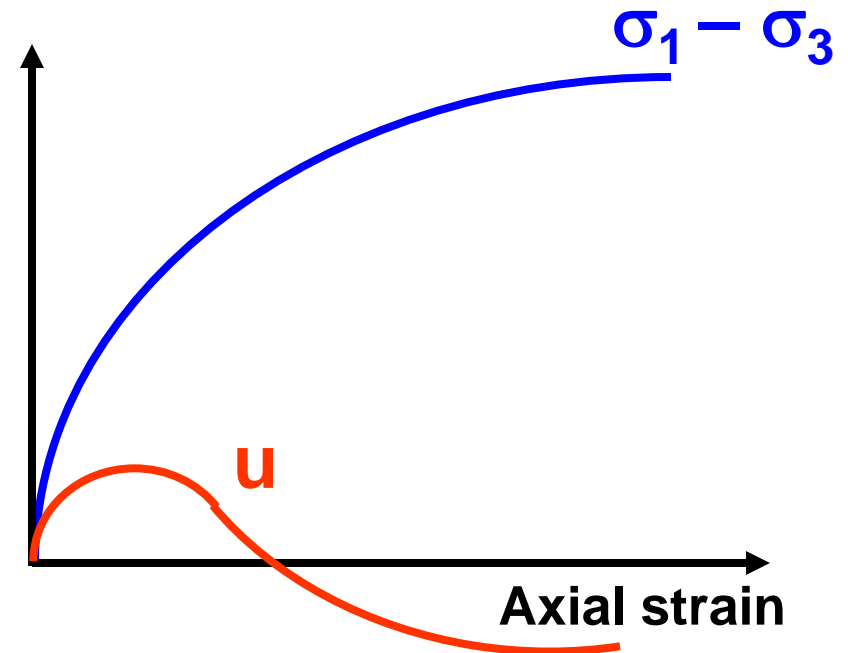
Typical relationship between A at failure and overconsolidation

Typical values for parameter A



OC Clay (Lightly overconsolidated)

($A = 0.0 - 0.5$)

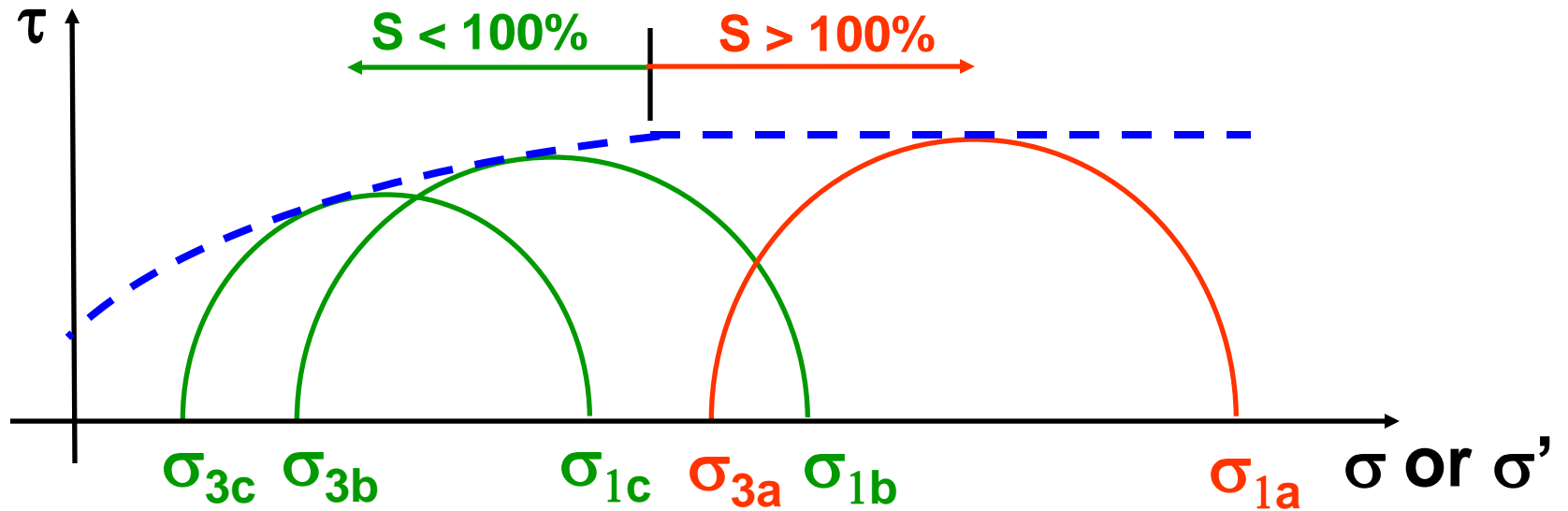


OC Clay (Heavily overconsolidated)

($A = -0.5 - 0.0$)

During the increase of major principal stress pore water pressure can become negative in heavily overconsolidated clays due to dilation of specimen

Effect of degree of saturation on failure envelope



THE END

The text "THE END" is rendered in a bold, sans-serif font. Each letter is filled with a different color from a rainbow spectrum: 'T' is magenta, 'H' is red, 'E' is orange, the second 'E' is green, 'N' is blue, and 'D' is purple. The letters have a slight 3D effect, with soft grey shadows cast to their left and slightly behind them, giving them a sense of depth against the plain white background.

Example 1

An unconfined compression test has given a UCS value of 126.6 kPa. The effective shear strength parameters are: $c' = 25$ kPa, $\phi' = 30^\circ$. Assuming the pore pressure parameter $A = -0.09$, calculate the initial pore pressure in the samples (use the failure criterion consideration)

Solution 1

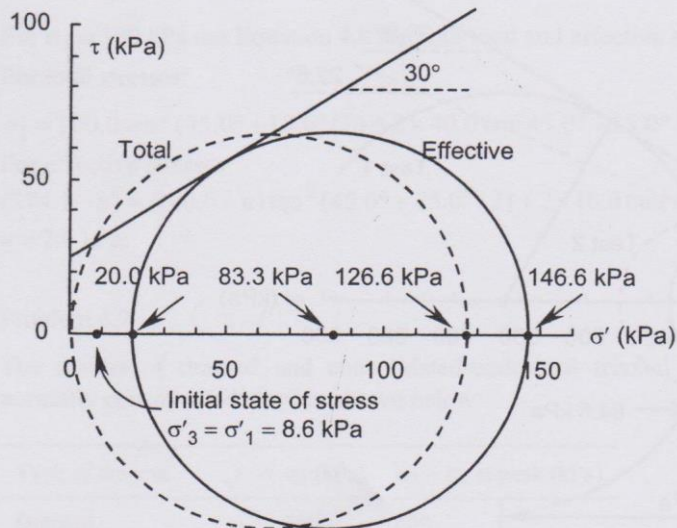
Solution:

Using the failure criterion in terms of effective stresses (Equation 4.8):

$$\sigma'_1 = \sigma'_3 \tan^2(45.0^\circ + 30.0^\circ/2) + 2 \times 25.0 \tan(45.0^\circ + 30.0^\circ/2),$$

$$\sigma'_1 = 3\sigma'_3 + 86.6.$$

The unconfined compression strength represents:



$$\sigma_1 - \sigma_3 = \sigma_1 - 0.0 = \sigma'_1 - \sigma'_3. \text{ Thus}$$

$$\sigma'_1 - \sigma'_3 = 126.6 \rightarrow \sigma'_1 = \sigma'_3 + 126.6.$$

Substituting σ'_1 in the equation of the failure criterion:

$$\sigma'_3 + 126.6 = 3\sigma'_3 + 86.6 \rightarrow \sigma'_3 = 20.0 \text{ kPa.}$$

The effective major principal stress is:

$$\sigma'_1 = \sigma'_3 + 126.6 = 20.0 + 126.6 = 146.6 \text{ kPa.}$$

In unconfined compression test the confining pressure σ_3 is zero:

$$\sigma_3 = \sigma'_3 + u = 20.0 + u = 0 \rightarrow u = -20.0 \text{ kPa.}$$

The excess pore pressure in a traditional triaxial compression test is determined from the following equation where A and B are termed pore pressure coefficients:

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)] \quad (4.11)$$

In an unconfined compression test $\Delta\sigma_3 = 0$. Furthermore the incremental stresses can be replaced by σ_1 and σ_3 for evaluation of the average values for A and B . Assuming $B = 1$ and knowing that:

$$\sigma_1 - \sigma_3 = \sigma'_1 - \sigma'_3,$$

the pore pressure coefficient A at failure is calculated from:

$$A = \frac{\Delta u}{\sigma_1 - \sigma_3} = \frac{\Delta u}{\sigma'_1 - \sigma'_3}, \text{ thus}$$

$$A = -0.09 = \frac{\Delta u}{\sigma_1 - \sigma_3} = \frac{\Delta u}{126.6} \rightarrow \Delta u = -11.4 \text{ kPa.}$$

The initial pore pressure is:

$$u_0 = -20.0 - (-11.4) = -8.6 \text{ kPa.}$$

Example 2

The results of CU test on identical specimens are given in a table below:

Based on Mohr's circle diagram, determine the total and effective shear strength parameters. In addition, what would be the expected pore pressure (at failure if $\sigma_3 = 100$ kPa).

Test no.	σ_3 (kPa)	$\sigma_1 - \sigma_3$ (kPa)	U_f (kPa)
1	200	244	55
2	300	314	107
3	400	384	159

Solution 2

total stress

Test no.	σ_3 (kPa)	σ_1 (kPa)
1	200	444 (1/2)
2	300	614 (1/2)
3	400	784 (1/2)

effective stress

Test no.	σ'_3 (kPa)	σ'_1 (kPa)
1	145 (1/2)	389 (1/2)
2	193 (1/2)	507 (1/2)
3	241 (1/2)	625 (1/2)

Solution 1

Based on Mohr's circle diagram:

$$c = 40 \text{ kPa}; \phi = 15^\circ \quad (3)$$

$$c' = 10 \text{ kPa}; \phi' = 25^\circ \quad (3)$$

If $\sigma_3 = 100 \text{ kPa}$;

Based on either this equation

$$\sigma_1 = \sigma_3 \tan^2 (45 + \phi/2) + (2c \tan (45 + \phi/2)) \quad (\text{Eq. 1})$$

@

$$\sigma_3 = \sigma_1 \tan^2 (45 + \phi/2) - (2c \tan (45 + \phi/2)) \quad (\text{Eq. 2})$$

Based on Eq. 1

$$\begin{aligned} \sigma_1 &= 100 \tan^2 (45 + 15/2) + [(2 \cdot 40) \tan (45 + 15/2)] \\ &= 274.1 \text{ kPa} \end{aligned} \quad (2)$$

$$\therefore (\sigma'_1 - u) = (100 - u) \tan^2 (45 + \phi'/2) + (2c' \tan (45 + \phi'/2))$$

$$(274.1 - u) = (100 - u) \tan^2 (45 + 25/2) + [(2 \cdot 10) \tan (45 + 25/2)]$$

$$u = 2.25 \text{ kPa} \quad (2.5)$$

Example 3

A soil has the following properties, n (porosity) = 0.38, E_s (Modulus of Elasticity) = 10 Mpa, $\mu = 0.3$. The bulk modulus of the pore water is 2200 Mpa. Estimate the pore pressure coefficient B

Solution 3

Solution:

If an element of saturated soil is subjected to isotropic compression $\Delta\sigma$, the instant increase in pore pressure Δu (excess pore pressure) is calculated from:

$$\Delta u = \frac{1}{1 + \frac{nE_s}{3K(1-2\mu)}} \Delta\sigma = B\Delta\sigma \quad (4.10)$$

where n is the porosity, E_s is the Modulus of Elasticity of soil, μ is the Poisson's ratio, K is the bulk modulus of the pore water and B is termed the pore pressure coefficient. Substituting the given data in the above equation:

$$B = \frac{1}{1 + \frac{nE_s}{3K(1-2\mu)}} = \frac{1}{1 + \frac{0.38 \times 10.0}{3 \times 2200.0(1-2 \times 0.3)}} = 0.9986.$$

Example 4

The shear strength of a normally consolidated clay can be given by the equation $\tau = \sigma' \tan 27^\circ$. Following are the results of a consolidated undrained test on clay:

Chamber confining pressure = 150 kPa

Deviator stress at failure = 120 kPa

Determine the consolidated undrained friction angle

Pore water pressure developed in the specimen at failure

If the sample was carried out in drained compression loading, what would have been the expected pore water pressure at failure?

Solution 4

$$\tau_f = \sigma' \tan 27^\circ$$

$$\sigma_3 = 150 \text{ kPa}$$

$$\Delta\sigma_f = 120 \text{ kPa}$$

$$\begin{aligned}\therefore \sigma_1 &= \sigma_3 + \Delta\sigma_f \\ &= (150 + 120) \text{ kPa} \\ &= 270 \text{ kPa}\end{aligned}$$

$$\begin{aligned}\text{a) } \phi_y &= \sin^{-1} \left[\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} \right] \\ &= \sin^{-1} \left[\frac{270 - 150}{270 + 150} \right] \\ &= 16.6^\circ\end{aligned}$$

$$\text{b) } \phi' = \sin^{-1} \left[\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2(\Delta u_d)} \right]$$

$$27^\circ = \sin^{-1} \left[\frac{270 - 150}{270 + 150 - 2(\Delta u_d)} \right]$$

By try & error

$$\Delta u_d = 77 \text{ kPa}$$

Example 5

Undisturbed samples from a normally consolidated clay layer were collected during a field exploration. Drained triaxial test showed that the effective friction angle, $\phi' = 28^\circ$. The unconfined compressive strength, q_u , of a similar specimen was found to be 148 kN/m^2 . Determine the pore pressure at failure of the sample.

Solution 5

$$\sigma_3 = 0 \text{ (for UCS sample)}$$

$$\sigma_1 = \sigma_3 + \Delta\sigma$$

$$= 148 \text{ kPa}$$

$$\sigma_1 = \sigma_3 \cdot \tan^2(45^\circ + \phi/2) + 2c \cdot \tan(45^\circ + \phi/2)$$

$$(148 - u) = (0 - u) \tan^2(45 + 28/2)$$

$$(148 - u) = (0 - u) 2.7698$$

$$(148 - u) = -2.7698 u$$

$$u = -83.6 \text{ kPa}$$

Example 6

A consolidated undrained triaxial test was conducted on dense sand with a chamber confining pressure of 138 kN/m². Results showed that $\phi' = 24^\circ$ and $\phi = 31^\circ$. Determine the deviator stress and the pore water pressure at failure. If the sand were loose, what would have been the expected behavior? .

Solution 6

CU test

$$\sigma_3 = 138 \text{ kN/m}^2$$
$$\phi' = 24^\circ$$
$$\phi = 31^\circ$$
$$\sigma_1 = \sigma_3 + 9n^2 \left(45 + \frac{\phi}{2} \right)$$
$$= 138 + 9n^2 \left(45 + \frac{31}{2} \right)$$
$$= 431 \text{ kPa}$$
$$\Delta\sigma_f = \sigma_1 - \sigma_3$$
$$= 431 - 138$$
$$= 293 \text{ kPa} \downarrow$$

Example 7

A specimen of saturated sand was consolidated under all-around pressure of 105 kN/m^2 . The axial stress was then increased and drainage was prevented. The specimen failed when the axial deviator stress reached 70 kN/m^2 . The pore pressure at failure was 50 kN/m^2 . Determine:

- Consolidated undrained angle of shearing resistance, ϕ
- Drained friction angle, ϕ' .

Solution 7

$$i) \sigma_3 = 105 \text{ kN/m}^2, (\Delta u_d)_f = 50 \text{ kN/m}^2$$

$$\sigma_1 = \sigma_3 + \Delta\sigma$$

$$= 105 \text{ kN/m}^2 + 70 \text{ kN/m}^2$$

$$= 175 \text{ kN/m}^2$$

$$\phi = \sin^{-1} \left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} \right) = \sin^{-1} \left(\frac{175 - 105}{175 + 105} \right)$$

$$\approx 14.5^\circ$$

$$ii) \phi' = \sin^{-1} \left(\frac{[\sigma_1 - (\Delta u_d)_f] - [\sigma_3 - (\Delta u_d)_f]}{[\sigma_1 - (\Delta u_d)_f] + [\sigma_3 - (\Delta u_d)_f]} \right)$$

$$= \sin^{-1} \left[\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2(\Delta u_d)_f} \right]$$

$$= \sin^{-1} \left[\frac{175 - 105}{175 + 105 - 2(50)} \right]$$

$$= 22.9^\circ$$

Example 8

(b) Undisturbed sample from normally consolidated clay layer were collected during a field exploration program. Drained triaxial test showed that the effective friction angle $\phi' = 28^\circ$. The value of A_f for the insitu conditions is estimated at 0.8. The unconfined compressive strength, q_u , of a similar specimen was found to be 148 kN/m^2 . Determine the pore pressure at failure, u_f , for the unconfined compression test.

Solution 8

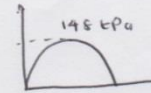
$$c' = 0 \text{ (Normally Consolidated Clay)}$$

$$\phi' = 28^\circ$$

$$A_f = 0.8$$

$$q_u = \sigma_1' - \sigma_3' = 148 \text{ kPa} \quad (\Delta\sigma_d)$$

$$\sigma_3 = 0$$



$$A_f = \frac{\Delta u}{\Delta\sigma_d}$$

$$\therefore \Delta u = A_f \times \Delta\sigma_d$$

$$= 0.8 \times 148 \text{ kPa}$$

$$= 118.4 \text{ kPa}$$

$$u_f = u_0 + \Delta u = u_0 + 118.4 \text{ kPa}$$

$$\sigma_3' = \sigma_3 - u_f$$

$$= -(u_0 + 118.4 \text{ kPa})$$

$$\sigma_1' = \sigma_3' + \Delta\sigma_d$$

$$= -u_0 - 118.4 + 148$$

$$= -u_0 + 29.6$$

$$\sigma_1' = \sigma_3' \tan^2(45 + \phi'/2) + 2c \tan(45 + \phi'/2)$$

$$-u_0 + 29.6 = (-u_0 - 118.4) \tan^2(45 + 28/2)$$

$$-u_0 + 29.6 = -2.7698 u_0 - 327.95$$

$$1.7698 u_0 = -357.55 \text{ kPa}$$

$$\therefore u_0 = -202 \text{ kPa}$$

$$\therefore u_f = u_0 + \Delta u = -202 + 118.4 \text{ kPa}$$

$$= -83.6 \text{ kPa}$$