MAJ 1013-ADVANCED SOIL MECHANICS

- **STRESS PATHS**
- **CRITICAL STATE SOIL MECHANICS**

Prepared by:

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STRESS PATHS

- Diagrams that represent the successive states of stress during both consolidation and ***shearing*** stage
- Can be plotted for all types of loading (UU, CU & CD)
- Considered for different types of loading condition (i.e. drained & undrained) and either effective or total stress

q/p space

$q = \sigma_1 - \sigma_3$

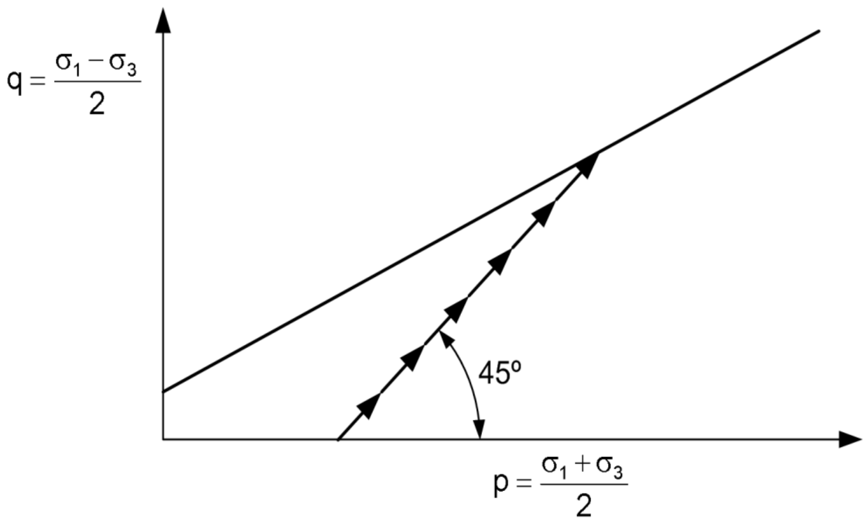
$p = 1/3\ (\sigma_1 + 2\sigma_3)$

t/s space

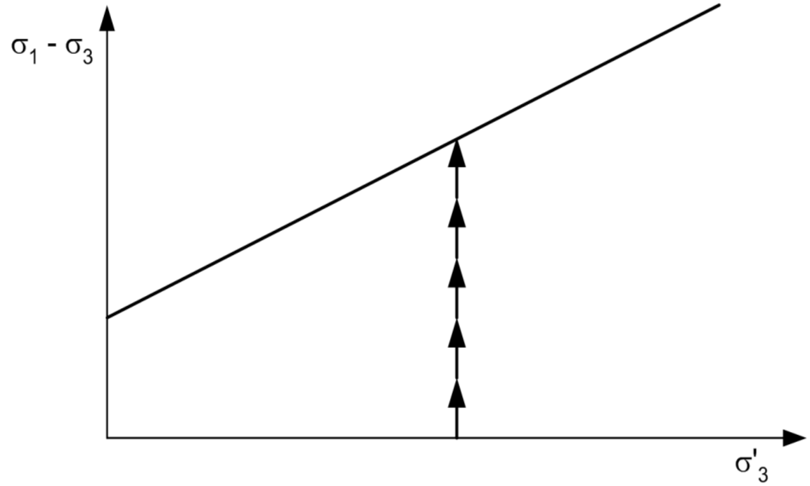
$t = \frac{1}{2}\ (\sigma_1 + \sigma_3)$

$s = \frac{1}{2}\ (\sigma_1 - \sigma_3)$

Total stress paths for shear plotted on p-q diagrams & alternate modified Mohr-Coulomb diagrams – CU TEST

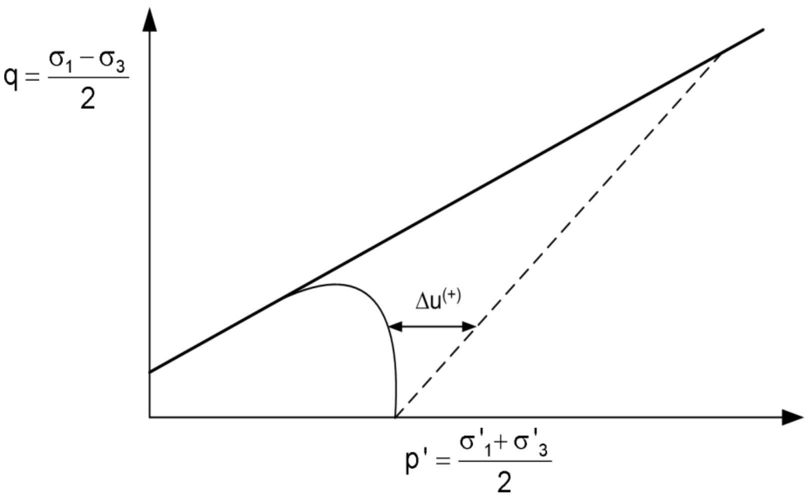


a. p-q Diagram

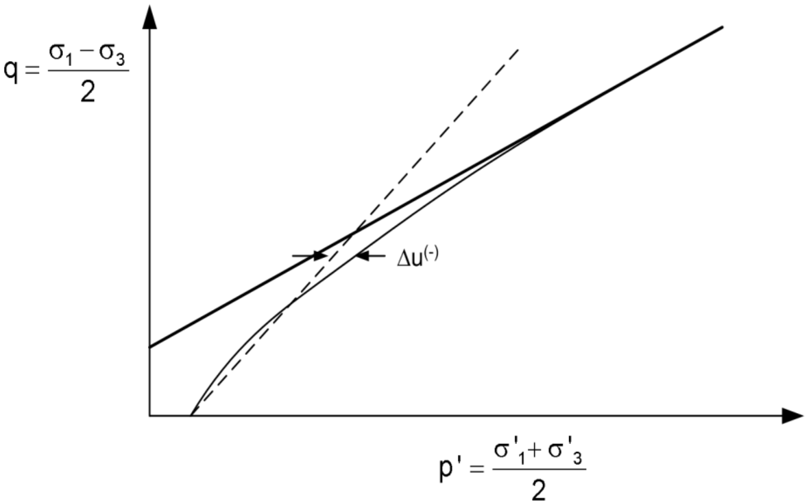


b. Alternate modified Mohr-Coulomb diagram

Effective stress paths for shear plotted on p-q diagrams – CU TEST

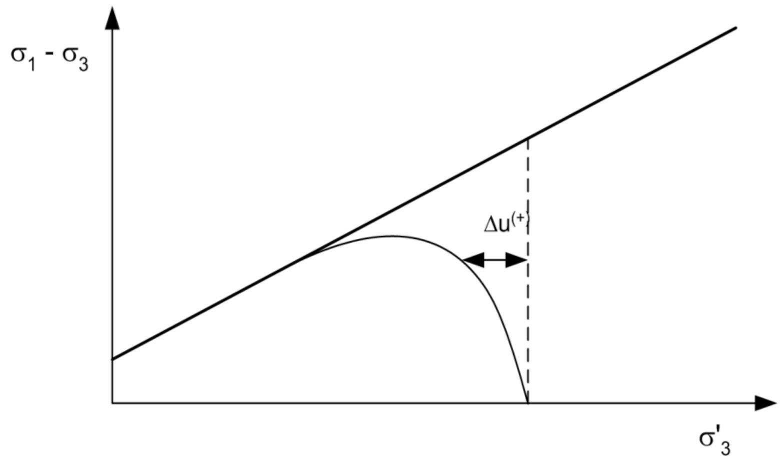


a. Normally consolidated soil

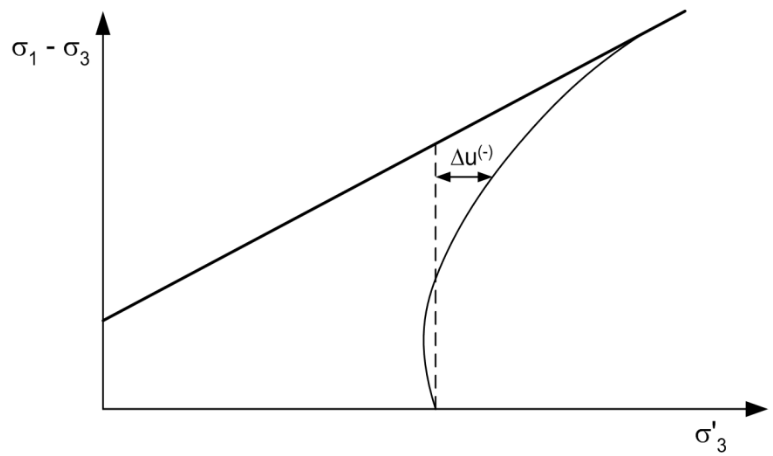


b. Heavily overconsolidated soil

Effective stress paths for shear plotted on alternate modified Mohr-Coulomb diagrams – CU TEST

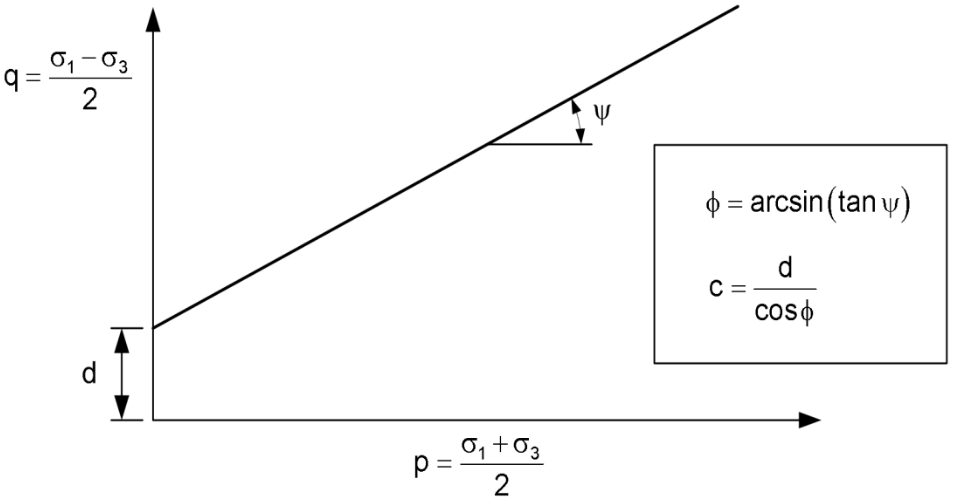


a. Normally consolidated soil

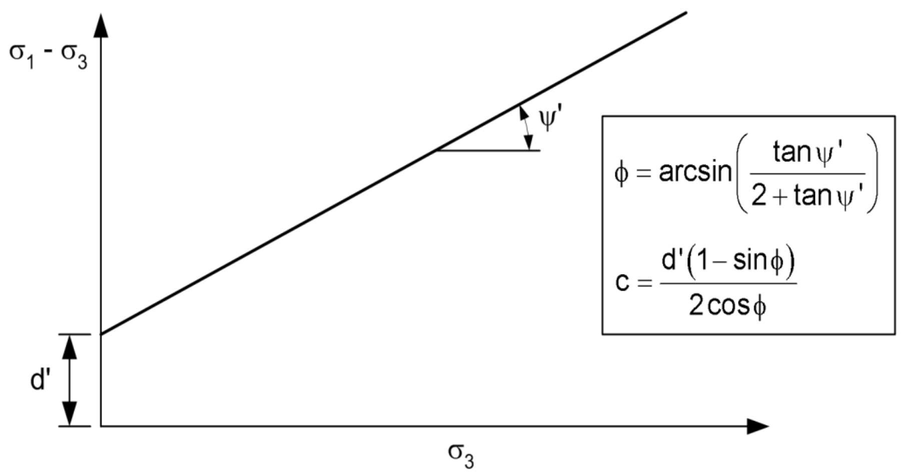


b. Heavily overconsolidated soil

Shear strength parameters (c & ϕ)



a. p-q diagram

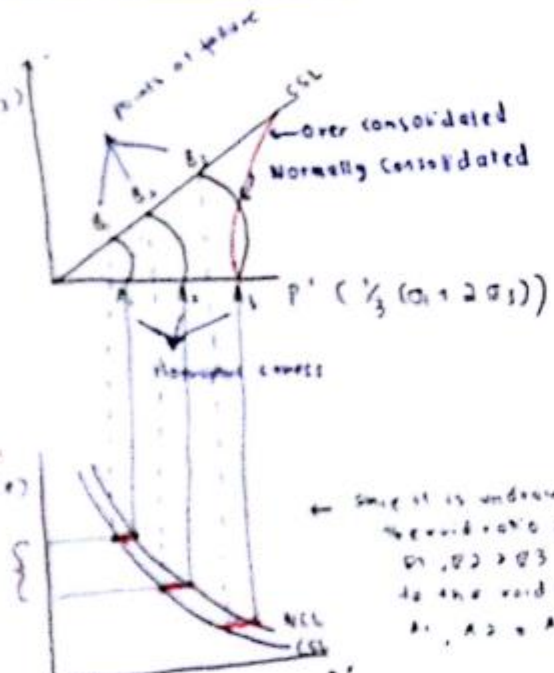


b. Alternate modified Mohr-Coulomb diagram

CU

* as similar in undrained case, the failure point are also falls into similar curve line as Normal compression line (NCL)

Effective Stress Path



← since it is undrained the void ratio at points B1, B2 & B3 are similar to the void ratio at A1, A2 & A3

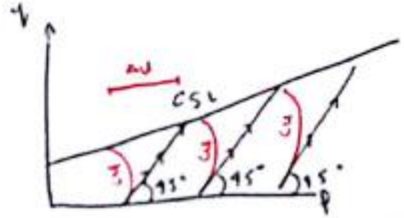
* NC

SP tends to lies to the left of 45° indicate an increase in pwp, thus soil will compress during sheared

* OC

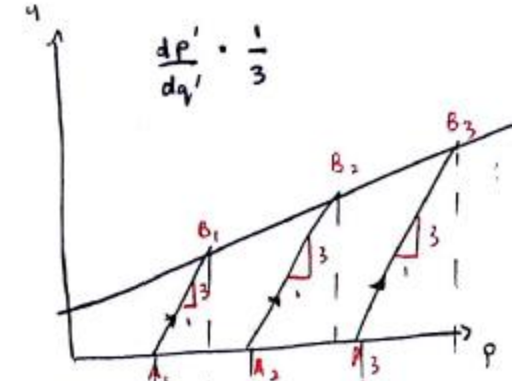
SP approach lies to the right of 45° at failure indicate a [illegible] in pwp, thus soil [illegible]

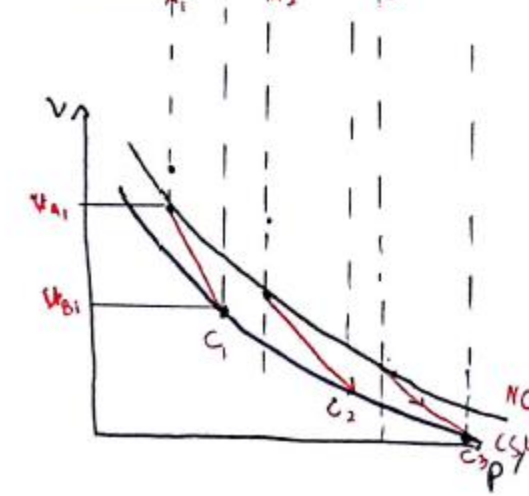
Total Stress Path



- The total stress path is along 45° degree line extending from horizontal to the failure envelope (CSL).
- If the stress decrease after failure, the stress path will move back along the initial path
- The graph is a straight line

CD





- for q:p' plane, the graph is a straight line that sloping to the horizontal at $\tan^{-1} 3$ y
- Points C_1, C_2 & C_3 represents the failure point after sheared in drained void ratio at failure is less than [illegible]

Example 1 (Stress path)

Results for undrained test for normally consolidated clay as shown in table. Given Gs = 2.65. Calculate pf' and v and plot:

i) q vs p'

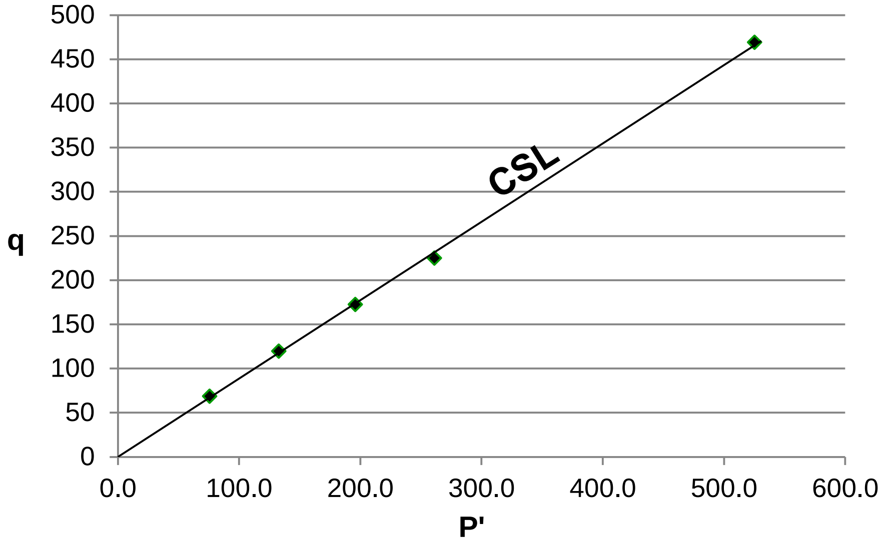
ii) v vs p'

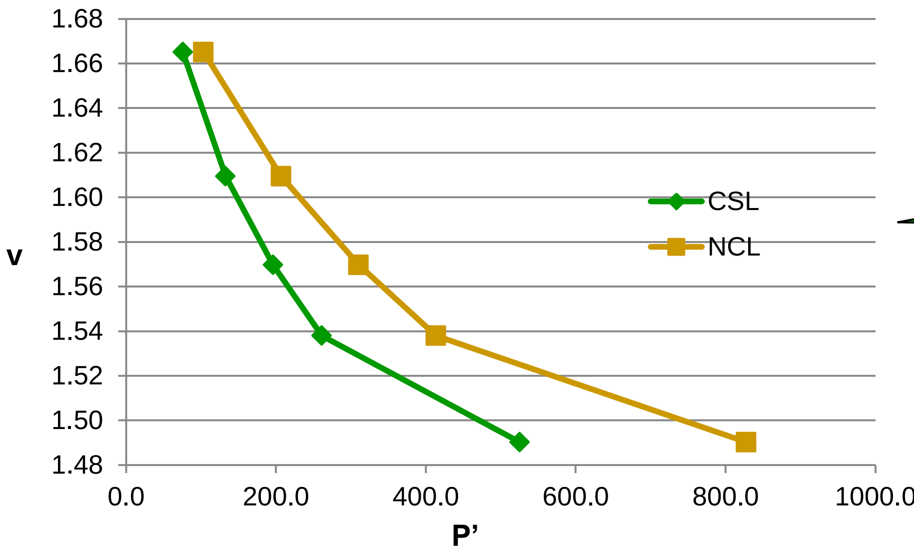
iii) Show NCL and CSL

Solution….

Calculate σ_1, σ_1', σ_3' for each sample, then calculate p_f' and v

confining pressure (σ_3)	axial stress at failure, q ($\sigma_1 - \sigma_3$)	pore water pressure at failure (u_f)	moisture contents at failure (w_f)	σ_1	σ_1'	σ_3'	pf'	e = wGs	v = 1 + e
103.4	68.3	50.3	25.1	171.7	121.4	53.1	75.9	0.67	1.67
206.9	119.3	113.8	23	326.2	212.4	93.1	132.9	0.61	1.61
310.3	172.4	171.7	21.5	482.7	311	138.6	196.1	0.57	1.57
413.7	224.8	227.5	20.3	638.5	411	186.2	261.1	0.54	1.54
827.4	468.9	458.5	18.5	1296.3	837.8	368.9	525.2	0.49	1.49





CSL = V vs p’

NCL = V vs σ3

CRITICAL STATE SOIL MECHANICS (CSSM)

- Is a tool to estimate soil responses when complete characterisation of soil at site is limited (to predict soil's response from changes in loading during and after construction)
- In corporates volume changes in its failure criterion (Mohr coulomb only defines failure as the attainment of the maximum stress. Failure stress state only is not sufficient to guarantee failure)
- Is an attempt to get a correlation between the shear strength and the void ratio in term of a model that can be applied to all types of soils.
- The state of soil sample is characterized by 3 parameters:
    - Effective mean stress p'
    - Deviatoric (shear stress) q, and
    - Specific volume V.

The specific volume is defined as $V = 1 + e$, where e is the void ratio.

CRITICAL STATE SOIL MECHANICS (CSSM)

Critical State

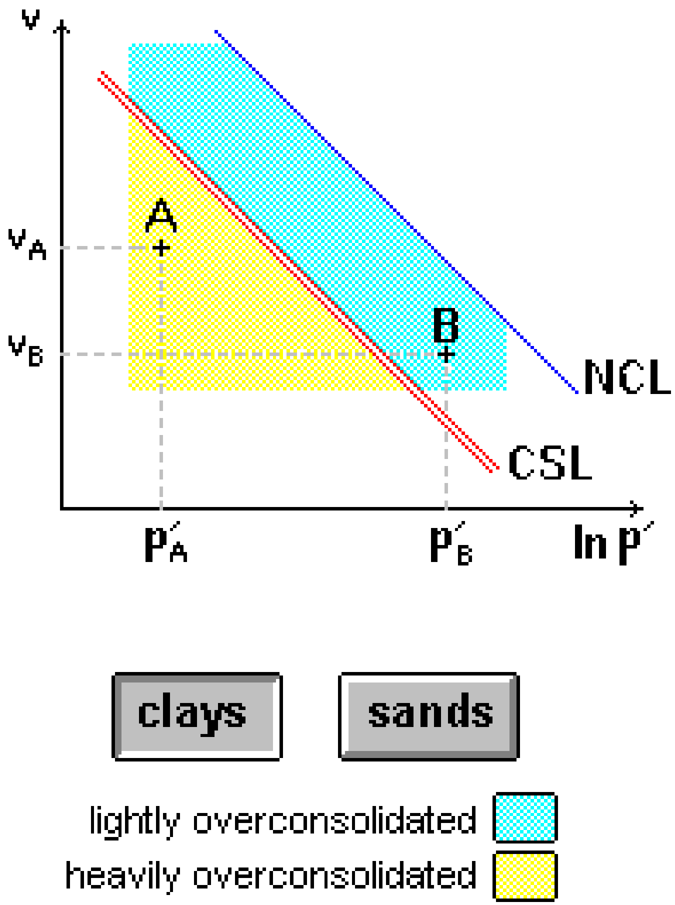
- The condition in which a soil has reached *a critical void ratio* (deformation occurs under *constant stress* and *constant volume*)

Critical Void Ratio

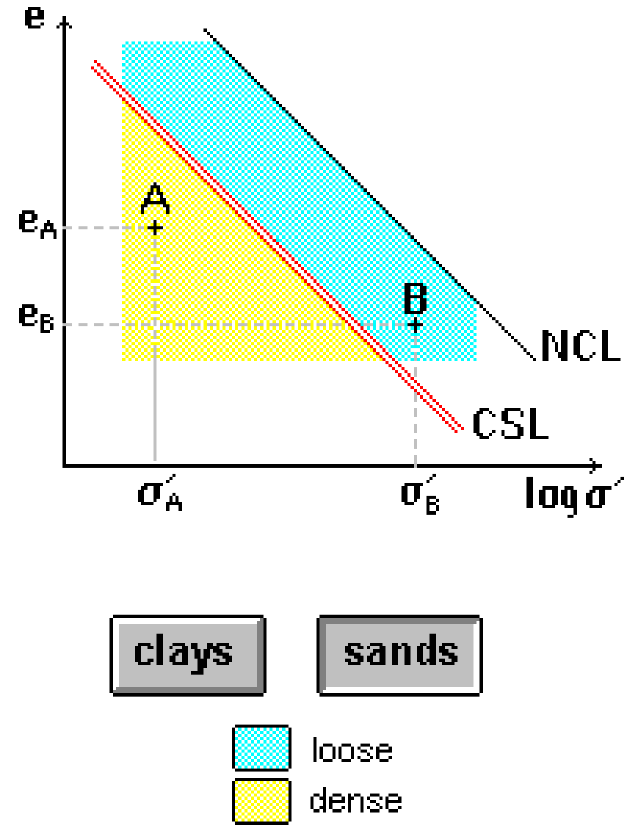
- The value of the void ratio for a particular state of compaction of a granular material below which *dense*r material tends to increase in volume when sheared and above which *looser* material tends to decrease in volume (thus the material will neither expand nor contract when disturb)

Critical State Line (CSL)

- The graph of critical void ratio (or specific volume) plotted against the effective stress under which that void ratio is achieved.
- The CSL lies parallel to the virgin compression line (NCL) and slightly below it.



v
v_A
v_B
A
B
NCL
CSL
p'_A
p'_B
$\ln p'$
clays
sands
lightly overconsolidated
heavily overconsolidated



e
e_A
e_B
A
B
NCL
CSL
σ'_A
σ'_B
$\log \sigma'$
clays
sands
loose
dense

CRITICAL STATE SOIL MECHANICS (CSSM)

Isotropic Consolidation

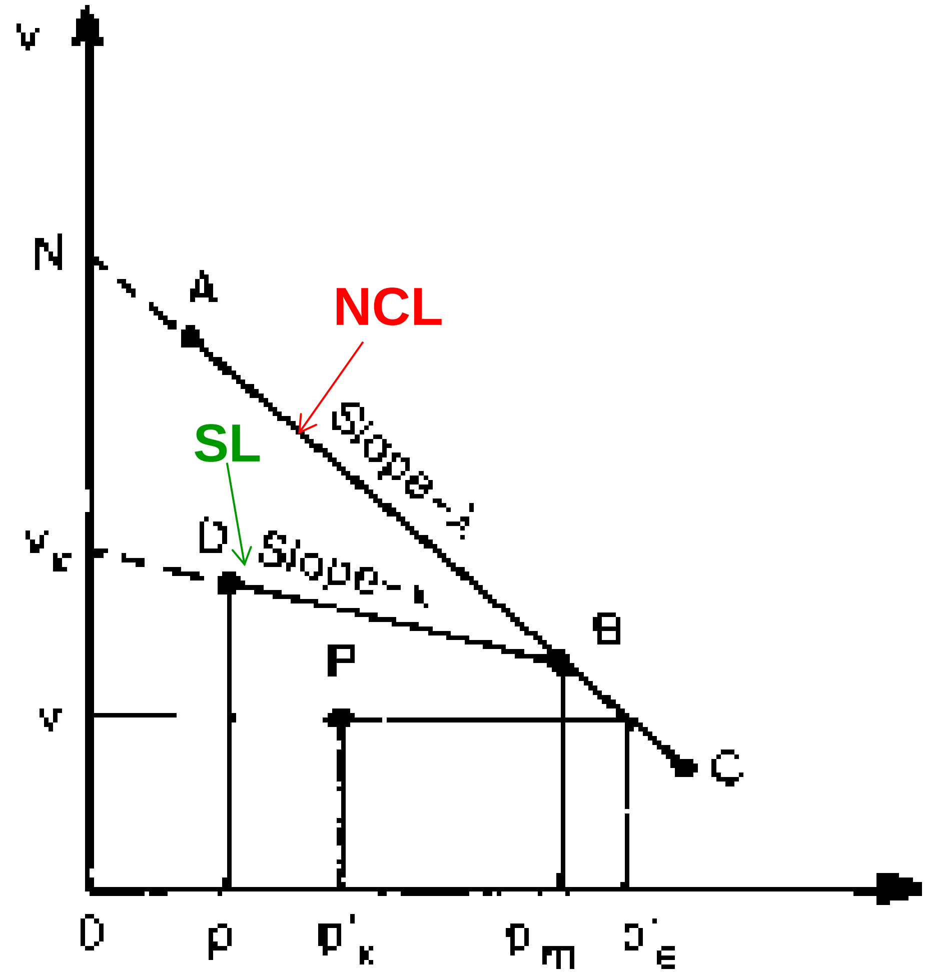
- Samples that *consolidated under hydrostatic pressure*, before the samples are sheared until failure.

- Consist of 2 lines:

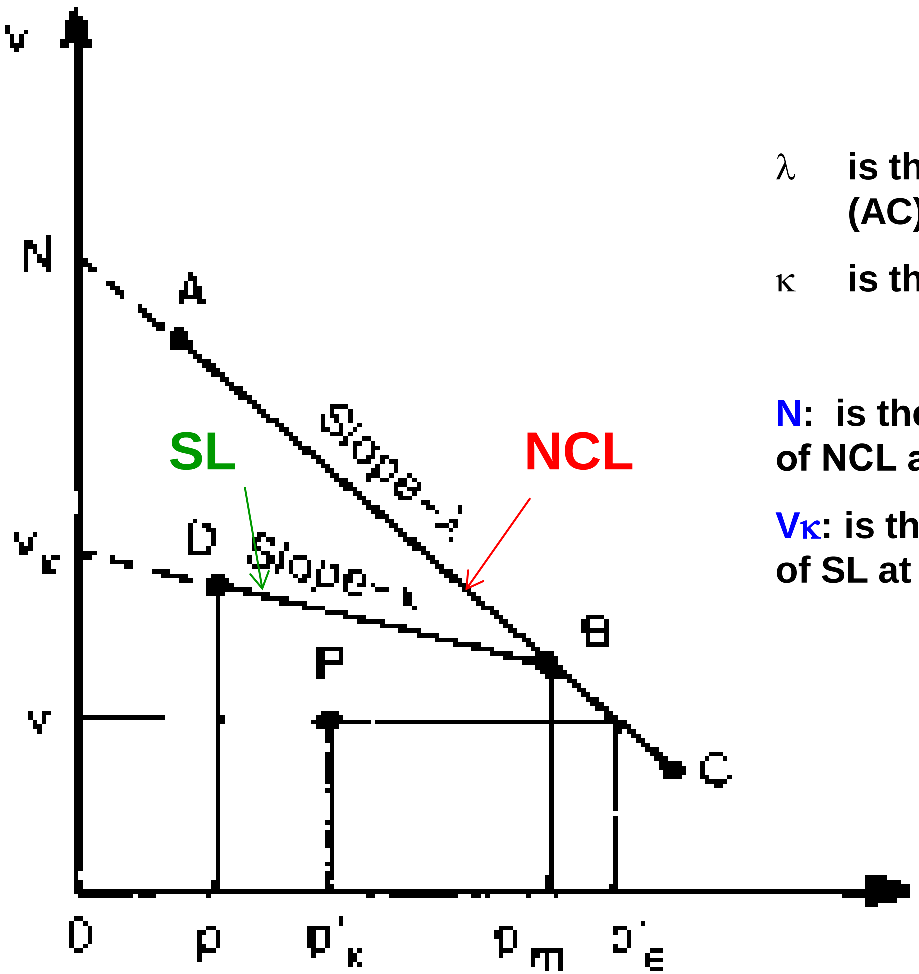
❖ Virgin compression line / Normal compression line (NCL)

❖ Swelling line (SL) (unloading and re-loading lines)

- Any point at line ABC represent the *'normal consolidation'* while any point at line BD, or below the line ABC represents *'overconsolidation'*.



CRITICAL STATE SOIL MECHANICS (CSSM)



λ is the slope of NCL (AC)

κ is the slope of SL (BD)

N: is the specific volume of NCL at p' =0

Vκ: is the specific volume of SL at p' =0

$V = N - \lambda \ln p'$	**For line AC**	isotropic NCL
$V = V_\kappa - \kappa \ln p'$	**For line BD**	swelling and recompression

Example 2 (CSSM)

p' (kN/m^2)	25	50	100	200	300	400	600	25
Change in volume (ml)	0	0.67	1.39	2.33	4.75	6.54	8.92	5.69

Physical properties of the specimen at $p' = 25$ kN/m^2:

$G_s = 2.72$ $\gamma = 18.6$ kN/m^3 $w = 34\%$ *Volume* $= 86.19$ ml

(a) *Plot the* v/ln p′ *curves and hence determine values for the parameters* λ, κ, N *and* v_κ.

Solution….

It is first necessary to establish the specific volume corresponding to $p' = 25$ kN/m^2.

Dry density, $\rho_d = \dfrac{18.6}{9.81(1+0.34)} = 1.415$ Mg/m^3

Specific volume, $v = 2.72/1.415 = 1.922$

Now, volumetric strain, $\varepsilon_v = \dfrac{\Delta V}{V} = \dfrac{\Delta v}{v}$

Therefore, $\Delta v = (1.922/86.19)\Delta V$

Initial specific volume, ν o

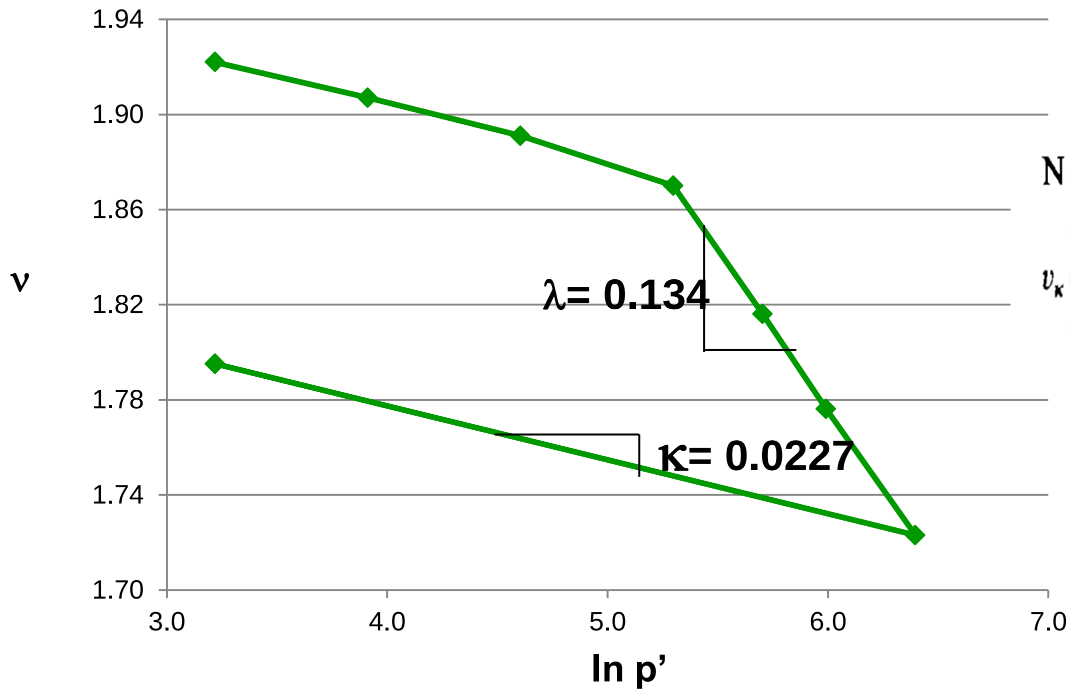
Initial volume, V

Change in specific volume,Δν

Solution….

mean normal stress, p' (kPa)	25	50	100	200	300	400	600	25	25
change in volume, Δv (ml)	0	0.67	1.39	2.33	4.75	6.54	8.92	5.69	5.69
change in specific volume, Δν	0.000	0.015	0.031	0.052	0.106	0.146	0.199	0.127	0.127
specific volume, ν	1.922	1.907	1.891	1.870	1.816	1.776	1.723	1.795	1.795
ln p'	3.2	3.9	4.6	5.3	5.7	6.0	6.4	3.2	3.2

ν corresponding to p'



N = the intercept of the isotropic NCL at $p' = 1.0\ \mathrm{kN/m^2}$

$$= 1.870 + 0.134 \times \ln 200 = 2.580$$

v_κ = the intercept of the swelling-reloading curve at $p' = 1.0\ \mathrm{kN/m^2}$

$$= 1.922 + 0.224 \times \ln 25 = 1.994$$

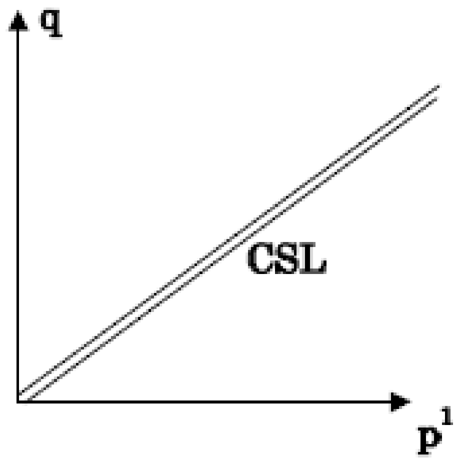
The Critical State Line (CSL)

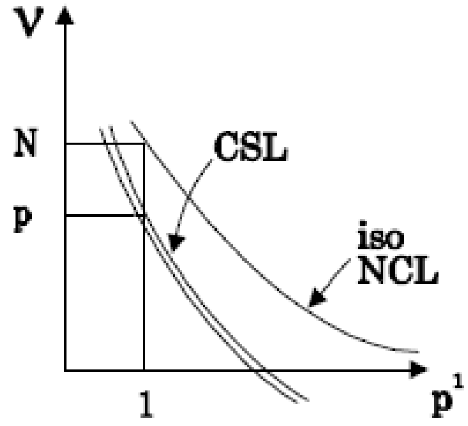
- If a soil is continuously sheared it will eventually reach a critical state in which further *shear strains can occur* with *no changes in effective stresses or volume .*
- When a soil is at the critical state:

$$q = Mp'$$

$$V = \Gamma - \lambda \ln p'$$

M and τ are constant for particular soil





M, N, Γ, κ, λ are soil constants

p', q, V (and V_κ) vary during a test.

The Equation of Critical State Line (CSL)

$$q = Mp'$$

$$V = \Gamma - \lambda \ln p'$$

The equation may be written as:

$$\log_e p' = \frac{\Gamma - \nu}{\lambda}$$

or

$$p' = \text{exponen} \frac{\Gamma - \nu}{\lambda}$$

Hence the CSL is the line that fulfils both equations:

$$q = Me^{\frac{\Gamma - \nu}{\lambda}}$$

$$M = \frac{6 \sin \phi'_c}{3 - \sin \phi'_c}$$

$$q_f = Mp'_f = \frac{3Mp'_o}{3 - M}$$

The Equation of Critical State Line – Undrained Test

- No volume change occurs, thus $\Delta V = 0$ → $\varepsilon_p = 0$; $\Delta e = 0$

- The void ratio at failure, e_f is similar after consolidation

$$V_0 = V_f$$

The equation may be written as:

$$e_f = e_o = e_\Gamma - \lambda \ln p'_f$$

$$p'_f = \exp\left(\frac{e_\Gamma - e_o}{\lambda}\right) \rightarrow p' = \text{exponen}\,\frac{\Gamma - v_o}{\lambda}$$

$$q_f = M \exp\left(\frac{e_\Gamma - e_o}{\lambda}\right) \rightarrow q = Mp' = M\,\text{exponen}\,\frac{\Gamma - v_o}{\lambda}$$

Example 3 (CSSM- Undrained test)

A sample of weald clay was consolidated in a triaxial cell with cell pressure of 200 kPa, then sheared in undrained condition. Determine the values of q, p' and ν at failure. Given M= 0.85, τ = 2.09, N = 2.13, λ = 0.10

Solution….

Undrained → $V_o = V_f$

NCL → $V_o = N - \lambda \ln p_o'$ → take $\sigma_3 = 200\,kPa$ as NCL represent the point of isotropic stress

CSL → $V_f = \Gamma - \lambda \ln p_f'$

as $V_o = V_f$

from NCL ; $V_o = V_f = N - \lambda \ln p_o'$

$= 2.13 - 0.10 \ln 200$

$V_f = 1.6$ //

so from CSL ; $V_f = \Gamma - \lambda \ln p_f'$

$1.6 = 2.09 - 0.10 (\ln p_f')$

$\ln p_f' = 4.9$

$\therefore p_f' = \exp 4.9$

$= 134\ kPa$ //

$q_f = M p_f'$

$= 0.85 (134)$

$= 114\ kPa$ //

The Equation of Critical State Line – Drained Test

- It is known that the projection of drained path at q:p plane is a straight line which sloping to *tan⁻¹ 3* to horizontal. Thus:

$$q_f = Mp'_f \quad = \quad q_f = 3(p'_f - p'_o)$$

$$p'_f = \frac{3p'_o}{3 - M}$$

The equation may be written as:

$$q_f = Mp'_f = \frac{3Mp'_o}{3 - M}$$

Also the specific volume at failure (ν_f), could be calculated as:

$$v_f = \Gamma - \lambda \log_e \frac{3p'_o}{3 - M}$$

Example 4 (CSSM- Drained test)

A clay sample was isotropically consolidated to a pressure of 350 kPa. The sample is then sheared in drained condition. Determine the values of q, p' and ν at failure if the characteristics of the soil are:

M= 0.89, τ = 2.76, N = 2.87, λ = 0.16

Solution….

$P_o = 350 \text{ kPa}$

Drained Test:

$$q_f = Mp_f = 3(p_f - p_o')$$

i) $\therefore p_f' = \dfrac{3p_o'}{3-M}$

$$= \frac{3(350)}{3-0.89}$$

$$= 498 \text{ kPa}$$

ii) $q_f = Mp_f'$

$$= 0.89(498)$$

$$= 443 \text{ kPa}$$

iii) $v_f = \Gamma - \lambda \log_e \left(\dfrac{3p_o'}{3-M}\right)$

$$= 2.76 - 0.16 \log_e 498$$

$$= 2.33$$

Summary of Input Parameters for Cam-Clay and Modified Cam-Clay Materials

Specification of Cam-Clay and Modified Cam-Clay models requires five material parameters. These parameters are outlined below.

1. λ – the slope of the normal compression (virgin consolidation) line and critical state line (CSL) in $v - \ln p'$ space

2. κ – the slope of a swelling (reloading-unloading) line in $v - \ln p'$ space

3. M – the slope of the CSL in $q - p'$ space

4. N – the specific volume of the normal compression line at unit pressure
   or
   Γ – the specific volume of the CSL at unit pressure

5. μ – Poisson's ratio
   or
   G – shear modulus.

The initial state of consolidation of such materials must also be specified. This is accomplished by indicating

OCR – the overconsolidation ratio: the ratio of the previous maximum mean stress to the current mean stress
  or
  p_o – the preconsolidation pressure.

Example 5 (CSSM)

A series of drained and undrained triaxial compression tests yielded the following results at point of failure:

Test No. (D = drained, U = undrained)	D1	U1	D2	U2	D3	U3
Cell pressure, σ_r (kPa)	120	120	200	200	400	400
Total axial stress, σ_a (kPa)	284	194	493	320	979	645
Pore pressure at failure, u_f (kPa)	0	69	0	117	0	230
Specific volume, v_f	1.80	1.97	1.70	1.86	1.54	1.72

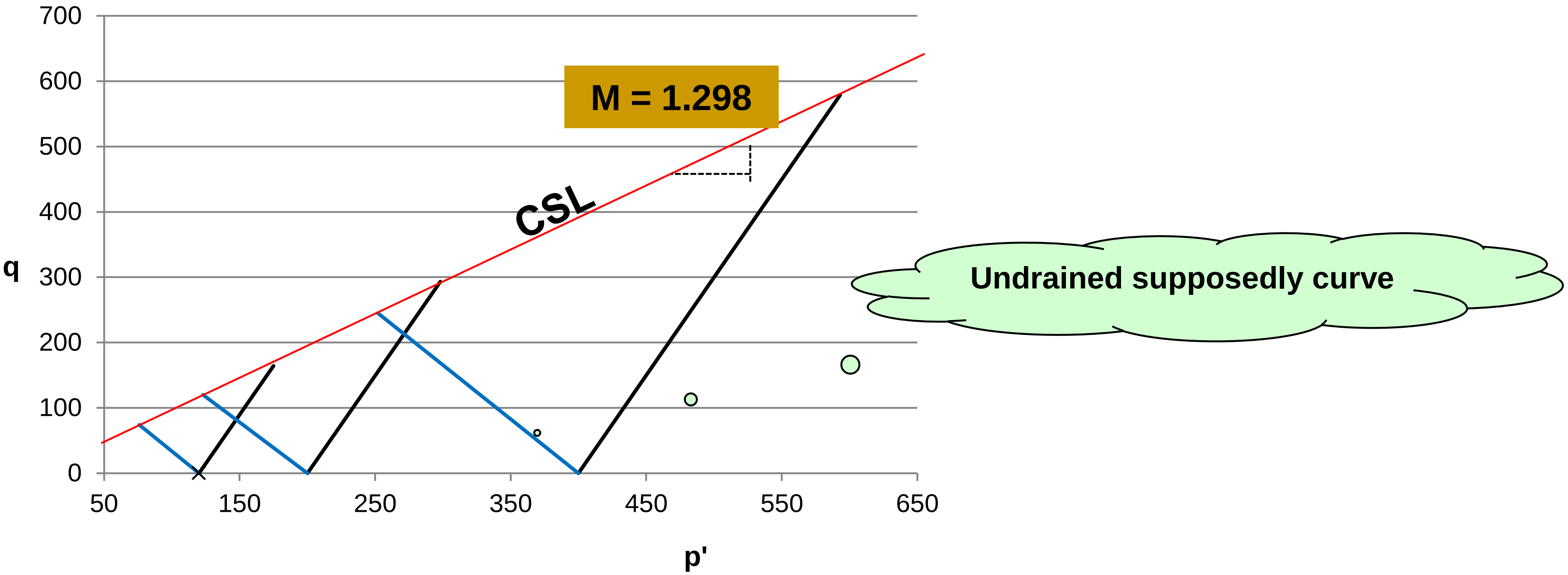
Plot the critical state line and obtain the critical state parameters M, Γ and λ.

Solution….

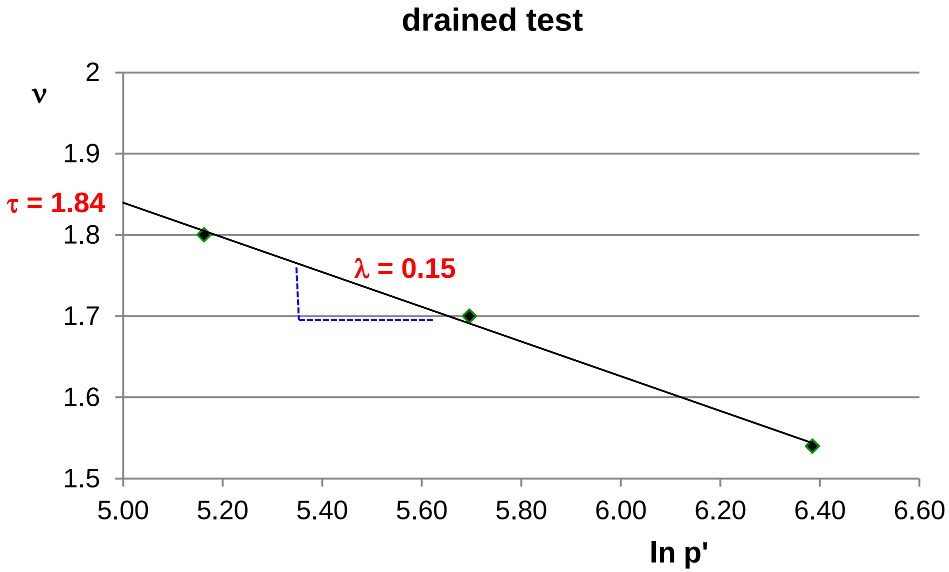
$P_0 = \sigma_3$

test	D1	U1	D2	U2	D3	U3
cell pressure, σ_3 (kPa)	120	120	200	200	400	400
total axial stress, σ_1 (kPa)	284	194	493	320	979	645
pore pressure at failure, u_f (kPa)	0	69	0	117	0	230
specific volume, v_f	1.8	1.97	1.7	1.86	1.54	1.72
effective cell pressure, σ_3' (kPa)	120	51	200	83	400	170
effective total axial stress, σ_1' (kPa)	284	125	493	203	979	415
mean effective stress, p'	175	76	298	123	593	252
ln p'	5.16	4.33	5.70	4.81	6.39	5.53
difference in stress, q (kPa)	164	74	293	120	579	245

Solution….



M = 1.298
CSL
Undrained supposedly curve
q
p'
0
100
200
300
400
500
600
700
50
150
250
350
450
550
650



drained test
ν
τ = 1.84
λ = 0.15
ln p'
2
1.9
1.8
1.7
1.6
1.5
5.00
5.20
5.40
5.60
5.80
6.00
6.20
6.40
6.60

Determination of critical state soil parameters from
- OEDOMETER TEST-

According to Cam-clay theory, one-dimensional loading also gives a "λ - line" in a (V, ln p') plot.

Also, it is easy to show that:

$$\lambda = \frac{C_c}{2.3}$$

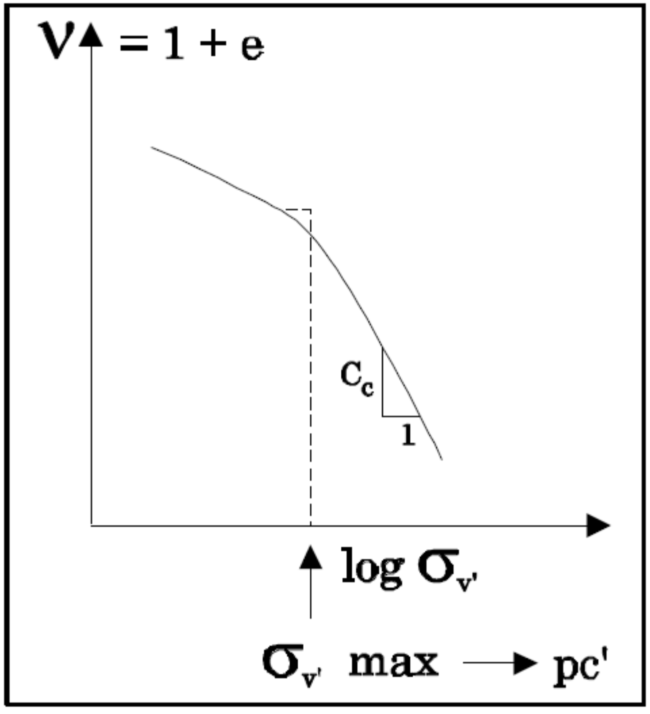
$$\kappa = \frac{C_s}{2.3}$$

where C_c and C_c are obtained by the standard interpretation of the oedometer test.

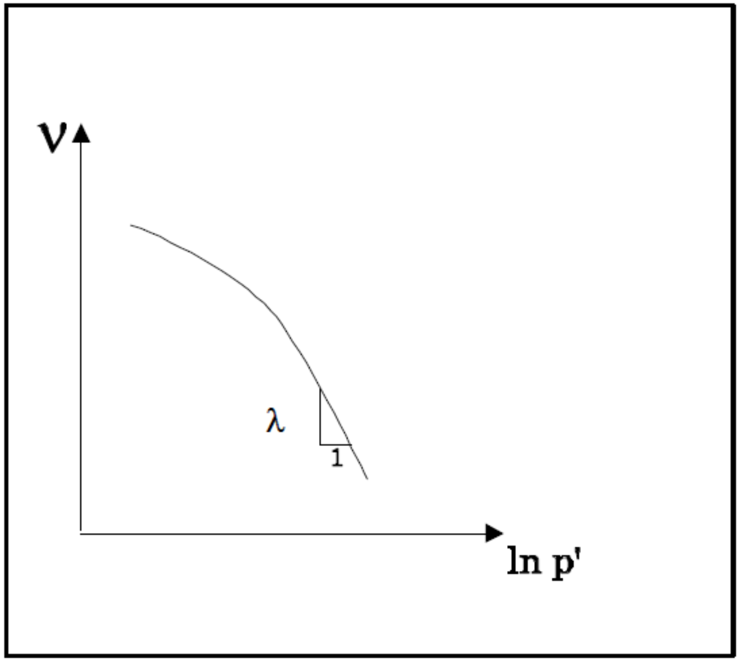
(NB: ln (10) = 2.303)

Determination of critical state soil parameters from - OEDOMETER TEST-

"conventional" plot e, log(σ'$_v$)



CSSM plot v, ln p'



Determination of critical state soil parameters from - INDEX TEST-

These are done to establish the moisture contents corresponding to the Plastic Limit (PL) and Liquid Limit (LL) of the soil.

If we assume that the strength of the soil at the Plastic Limit is 100 times the strength at the Liquid Limit, then:

$$\lambda = \frac{V_L - V_p}{\ln 100} = \frac{(w_L - w_p)G_s}{\ln 100}$$

or

$$\lambda = \frac{\text{PI x } G_s}{160}$$

where PI is % Plasticity Index.

THE END