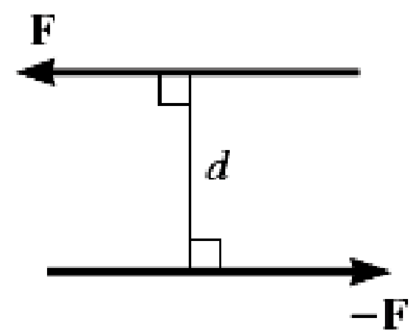
4.6 Moment of a Couple

❑ Couple

- A *couple* is defined as a pair of two parallel forces that
  - ➢ have the same magnitudes, but *opposite* directions
  - ➢ separated by perpendicular distance *d*



- The resultant force in a couple = 0
- Its only effect is to produce a rotation or tendency to rotate in a specified direction

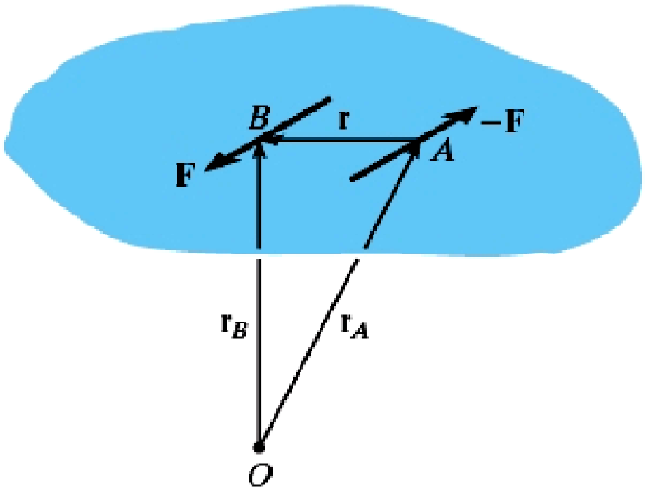
Couple Moment

- The moment produced by a couple is called a *couple moment*.

*Couple moment = sum of moments of both couple forces about **any arbitrary** point*

- The couple moment about O is

$$
\begin{aligned}
\mathbf{M} &= (\mathbf{r}_B \times \mathbf{F}) + (\mathbf{r}_A \times -\mathbf{F}) \\
&= (\mathbf{r}_B \times \mathbf{F}) - (\mathbf{r}_A \times \mathbf{F}) \\
&= (\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{F} \\
&= \mathbf{r} \times \mathbf{F}
\end{aligned}
$$



- Thus, couple moment depends ***only*** upon the position vector $\mathbf{r}$ directed ***between*** the forces and ***not*** the position vector $\mathbf{r}_A$ and $\mathbf{r}_B$ directed from the arbitrary point O to the forces.

- A couple moment is a ***free vector***.

❑ Scalar Formulation

- The magnitude of a couple moment **M** is given by

$$\boxed{M = F\,d}$$

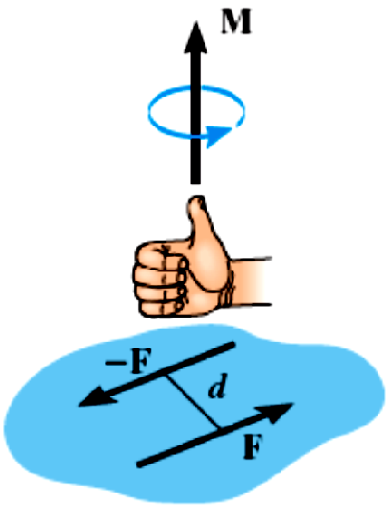
where F = the magnitude of one of the forces.

d = perpendicular between the 2 forces.

- The direction & sense of the couple moment **M** are determined by the right hand rule.

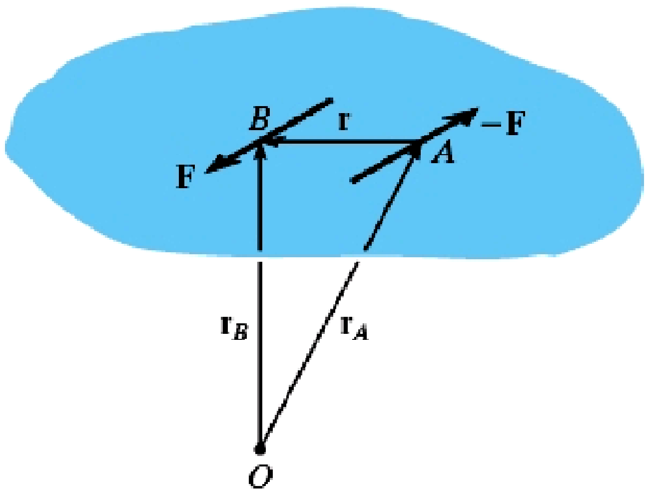
  - *When the right fingers are curled with the sense of rotation caused by the couple forces, the thumb indicates the direction of* ***M.***

  - **M** *will act perpendicular to the plane containing the forces.*



❑ Vector Formulation

- The couple moment **M** can be expressed by the vector cross product as



$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

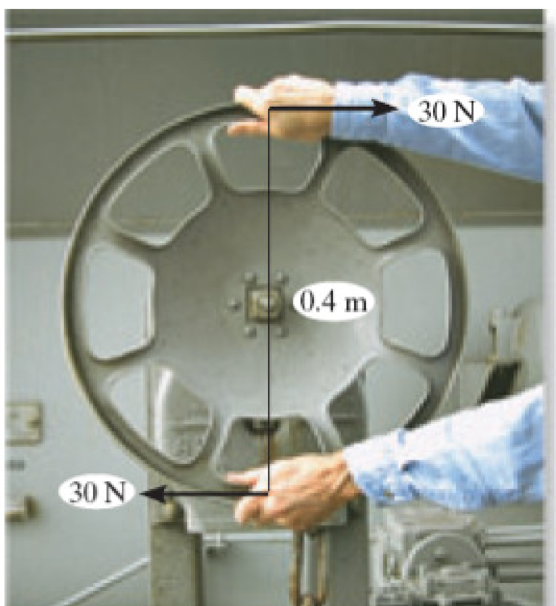
- The moment of the two couple forces can be determined about *any point.*

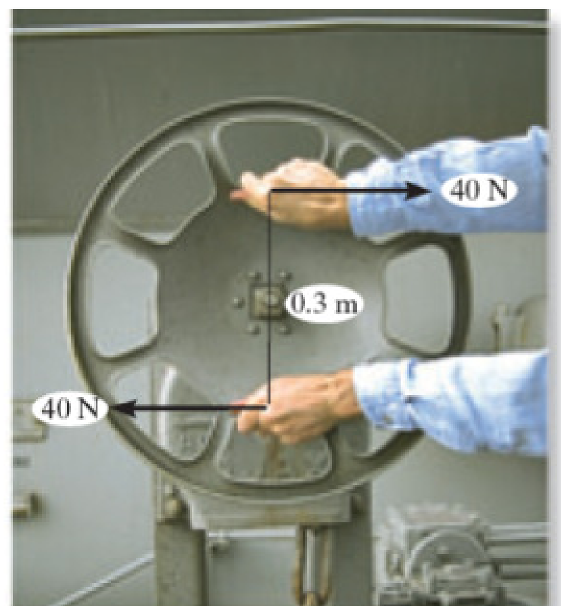
- The couple moment **M** can be determined by taking the moment of both forces about a point lying on the line of action of one of the forces.

❑ Equivalent Couples

- Two couples are ***equivalent*** if they produce a moment with the *same magnitude & direction.*

- Forces of equal couples lie on the same plane or plane parallel to one another



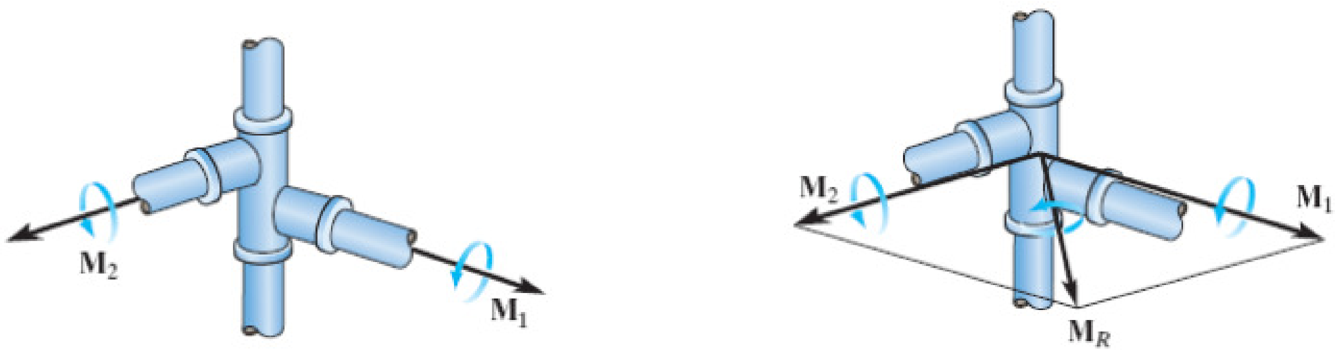


Resultant Couple Moment

- Couple moments are free vectors and may be applied to any point *P* and added vectorially

- For resultant moment of two couples,

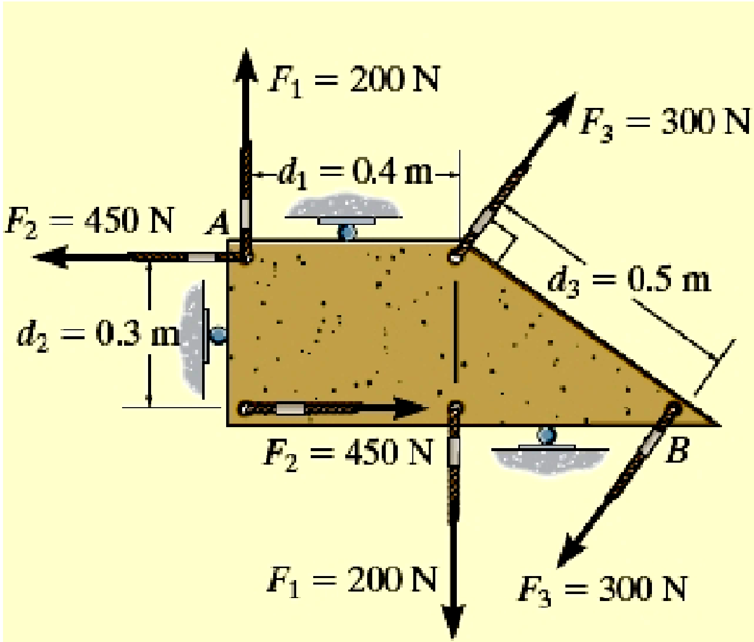
$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$



- For more than 2 couple moments, the resultant couple moment is

$$\mathbf{M}_R = \sum(\mathbf{r} \times \mathbf{F})$$

For example,



$$+ \quad M_R = \Sigma M:$$

$$M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$

$$= (-200 \text{ N})(0.4\text{m}) + (450 \text{ N})(0.3 \text{ m}) - (300 \text{ N})(0.5 \text{ m})$$

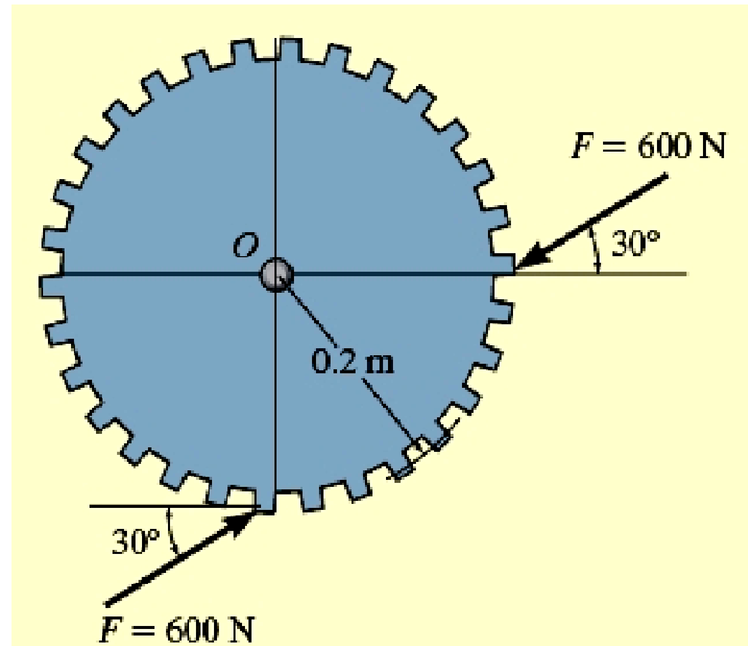
$$= -95 \text{ N}\cdot\text{m}$$

$$= 95 \text{ N}\cdot\text{m}$$

Example 4.11

Given :

The gear is subjected to a couple as shown.



Find :

Determine the magnitude and direction of the couple moment acting on the gear.

Solution

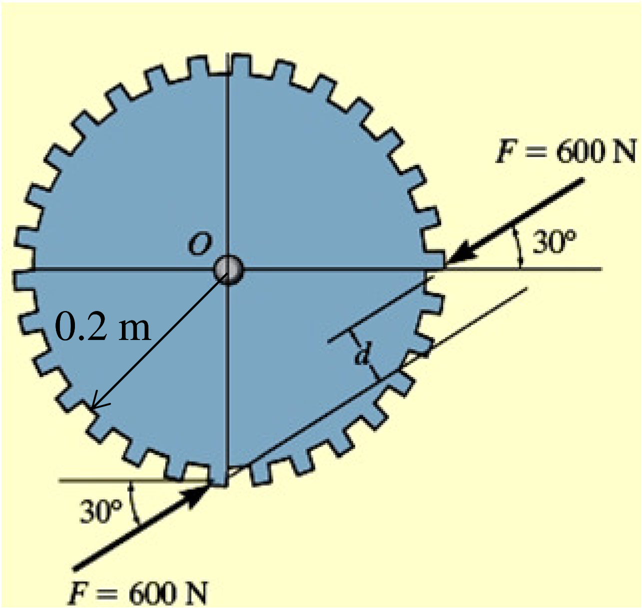
❑ Method I

- By definition, the magnitude of the couple moment **M** is given by

$$M = F\,d$$

where F = the magnitude of one of the forces

d = perpendicular distance between the 2 forces



- Since

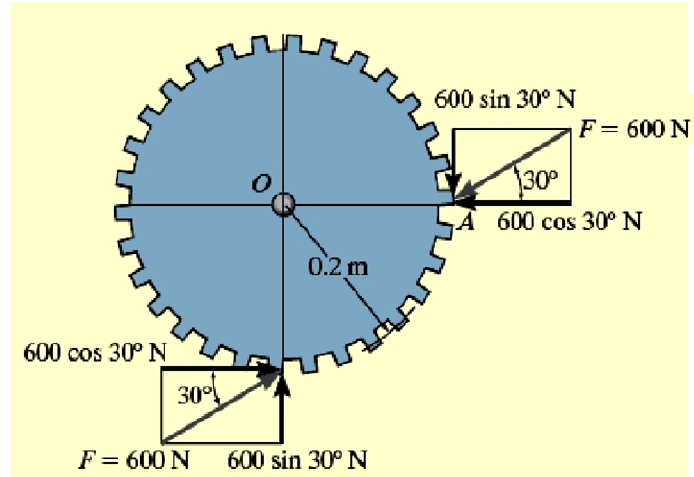
$$d = 0.2 \sin 60^{o} - 0.2 \sin 30^{o}$$
$$= 0.0732 \text{ m}$$

- Therefore,

$$M = (600 \text{ N})\,(0.0732 \text{ m})$$
$$= 43.9 \text{ N·m} \circlearrowleft$$

❑ Method II

- Resolve each force into its components
- The couple moment can be determined by summing the moments of these force components about any point.



➢ **Taking the moments about *O***

$$\circlearrowleft + \quad M = \Sigma M_O:$$

$$M = (600 \cos 30^{o}\ \mathrm{N})(0.2\mathrm{m}) - (600 \sin 30^{o}\ \mathrm{N})\ (0.2\ \mathrm{m})$$

$$= 43.9\ \mathrm{N{\cdot}m} \ \circlearrowleft$$

➢ **Taking the moments about *A***

$$\circlearrowleft + \quad M = \Sigma M_A:$$

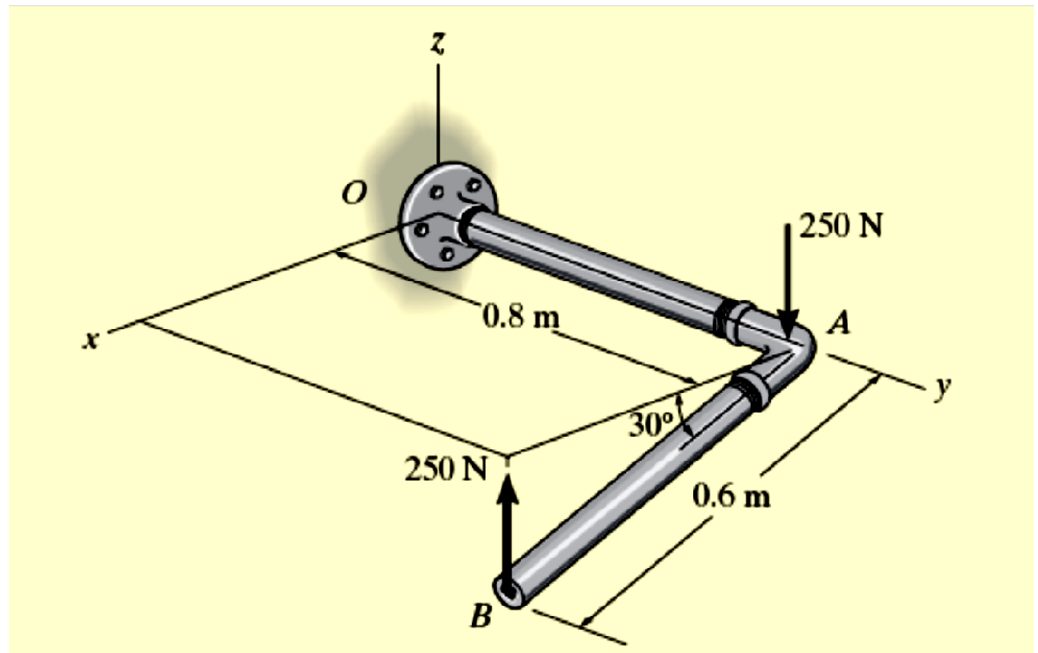
$$M = (600 \cos 30^{o}\ \mathrm{N})(0.2\mathrm{m}) - (600 \sin 30^{o}\ \mathrm{N})\ (0.2\ \mathrm{m})$$

$$= 43.9\ \mathrm{N{\cdot}m} \ \circlearrowleft$$

Example 4.12

Given :

Segment *AB* of the pipe is directed 30° below the *x*-*y* plane.



Find :

Determine the couple moment acting on the pipe.

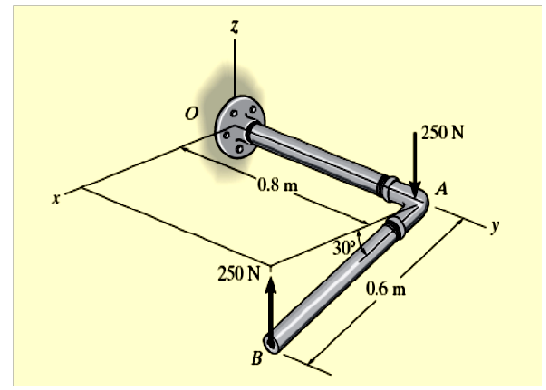
Solution

❑ Method I (Vector Analysis)

The moment of the 2 couple forces can be found about *any point*.

➢ <u>(a) Taking moments about point *O*</u>

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F}_A + \mathbf{r}_B \times \mathbf{F}_B$$

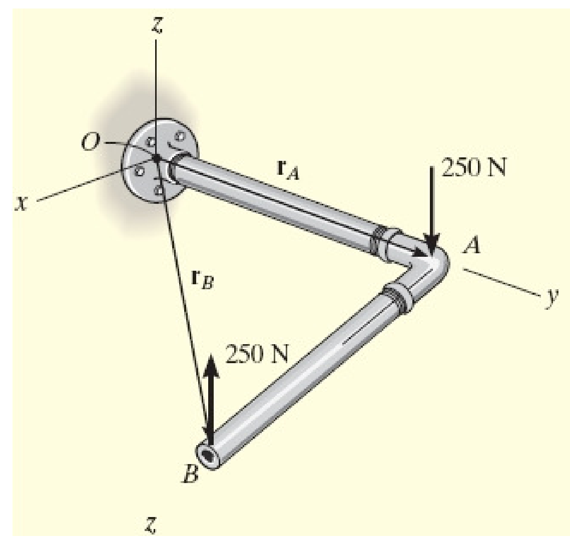


- Since

$\mathbf{r}_A = \{\, 0.8\mathbf{j}\}$ m

$\mathbf{r}_B = \{0.6 \cos 30^{\circ}\, \mathbf{i} + 0.8\mathbf{j} - 0.6 \sin 30^{\circ}\, \mathbf{k}\}$ m

$= \{0.5196\, \mathbf{i} + 0.8\mathbf{j} - 0.3\, \mathbf{k}\}$ m



$\mathbf{F}_A = \{-250\, \mathbf{k}\}$ N, $\quad \mathbf{F}_B = \{\, 250\, \mathbf{k}\}$ N

- Therefore,

$$\begin{aligned}\mathbf{M} &= (0.8\mathbf{j}) \times (-250\, \mathbf{k}) \\ &\quad + (0.5196\, \mathbf{i} + 0.8\mathbf{j} - 0.3\, \mathbf{k}) \times (250\, \mathbf{k}) \\ &= \{-130\mathbf{j}\,\}\ \text{N·m}\end{aligned}$$

➢ (b) Taking moments about point *A*

$$\mathbf{M} = \mathbf{r}_{AB} \times \mathbf{F}_A$$

- Since

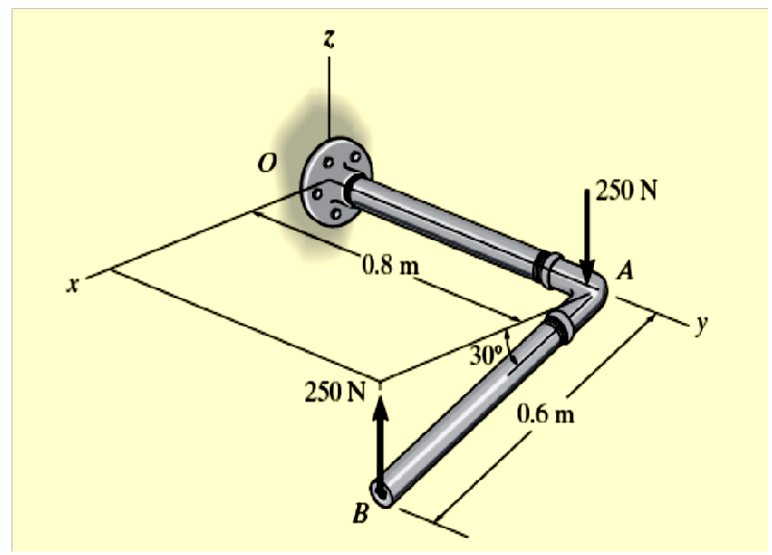
$$\mathbf{r}_{AB} = \{0.6 \cos 30^{\circ}\, \mathbf{i} - 0.6 \sin 30^{\circ}\, \mathbf{k}\} \text{ m}$$

$$= \{0.5196\, \mathbf{i} - 0.3\, \mathbf{k}\} \text{ m}$$

- Therefore,

$$\mathbf{M} = (0.5196\, \mathbf{i} - 0.3\, \mathbf{k}) \times (250\, \mathbf{k})$$

$$= \{-130\mathbf{j}\} \text{ N·m}$$



❑ Method II (Scalar Analysis)

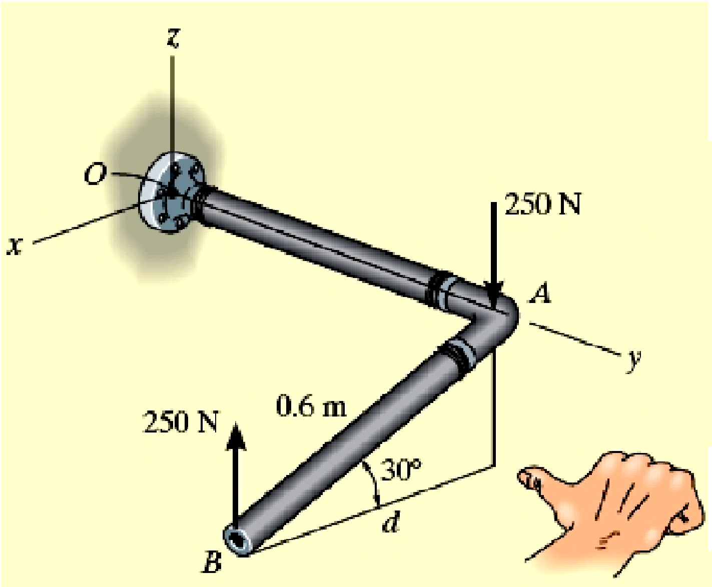
➢ **Taking moments about either point *A* or point *B***

- The magnitude of the couple moment is

$$M = F\,d$$

$$= (250\text{N})\,(0.6 \cos 30^{o}\ \text{m})$$

$$= 130\ \text{N}\cdot\text{m}$$



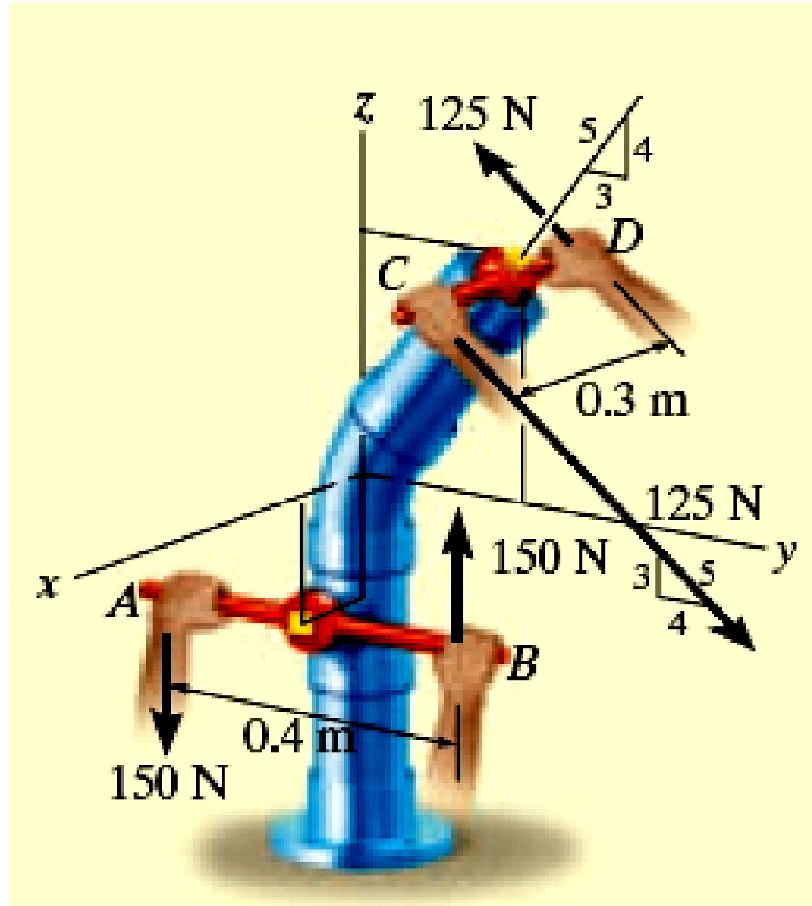
- Apply the right hand rule, **M** acts in the –**j** direction

$$\mathbf{M} = \{-\,130\mathbf{j}\}\ \text{N}\cdot\text{m}$$

Example 4.13

Given :

Two couples acting on the pipe column as shown.



Find :

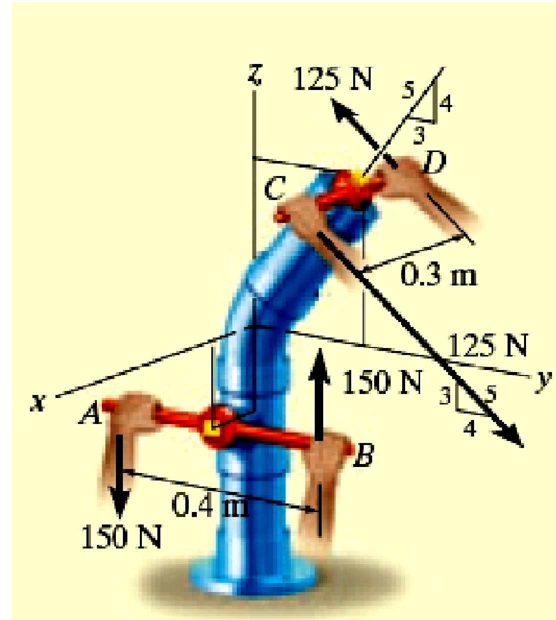
Replace the two couples by a resultant couple moment.

Solution

- **Find $\mathbf{M}_1$**

- The magnitude of the couple moment due to the forces at *A* & *B* is

$$M_1 = F\,d$$
$$= (150 \text{ N} (0.4 \text{ m})$$
$$= 60 \text{ N}\cdot\text{m}$$



- By the right hand rule, $\mathbf{M}_1$ acts in the $+\mathbf{i}$ direction.

Hence,

$$\mathbf{M}_1 = \{60\mathbf{i}\} \text{ N}\cdot\text{m}$$

Find $\mathbf{M}_2$

- The couple moment due to the forces at *C* & *D* is obtained by taking the moment about point *D*

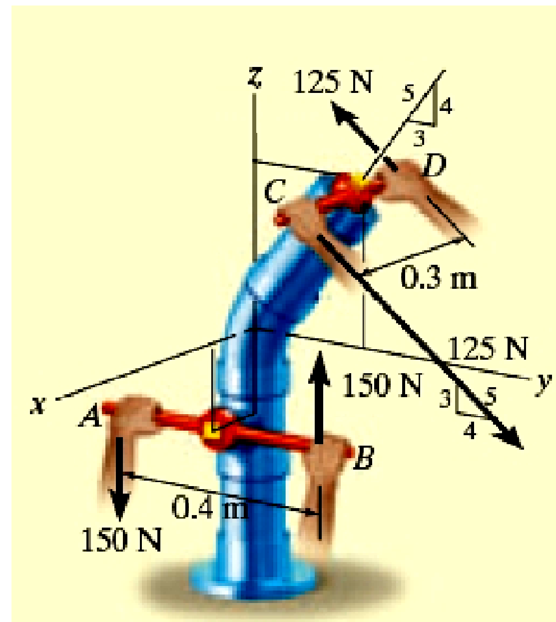
$$\mathbf{M}_2 = \mathbf{r}_{DC} \times \mathbf{F}_C$$

$$= (0.3\mathbf{i}) \times [125(4/5)\mathbf{j} - 125(3/5)\mathbf{k}]$$

$$= (0.3\mathbf{i}) \times [100\mathbf{j} - 75\mathbf{k}]$$

$$= 30 (\mathbf{i} \times \mathbf{j}) - 22.5 (\mathbf{i} \times \mathbf{k})$$

$$= \{22.5\mathbf{j} + 30\mathbf{k}\} \text{ N·m}$$



Resultant couple moment $\mathbf{M}_R$

$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$

$$= \{60\mathbf{i} + 22.5\mathbf{j} + 30\mathbf{k}\} \text{ N·m}$$