4.9 Reduction of a Simple Distributed Loading

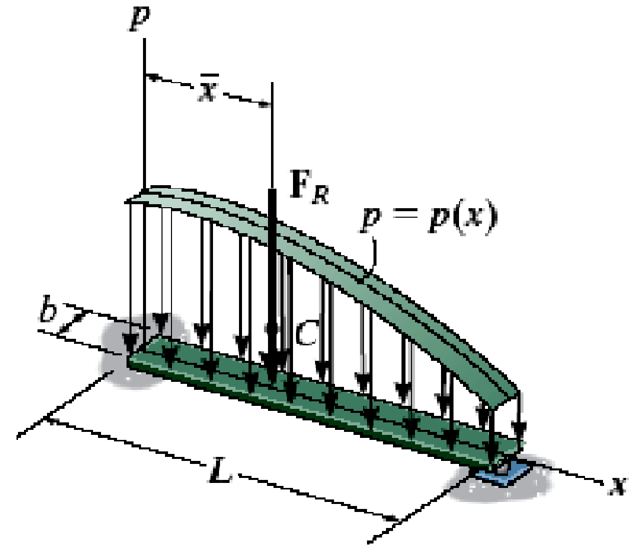
- A body may be subjected to a loading that is distributed over its surface, e.g.,
  - wind pressure on a sign board.
  - water pressure on the tank surface
  - weight of sand on the floor of a storage container
- These types of loading is called *distributed loadings.*
- The unit of measurement for *distributed loadings indicates* the *intensity* of the loadings., e.g., pressure, (measured in N/m^2 or Pa) indicates force per unit area.
- The most common type of distributed loading is uniform loading along a single axis.

Uniform Loading Along a Single Axis

- The beam with a constant width b is subjected to a pressure loading that varies only along the x axis given by

$$p = p(x) \text{ N/m}^2$$

(Note: $p(x)$ is force per unit area)



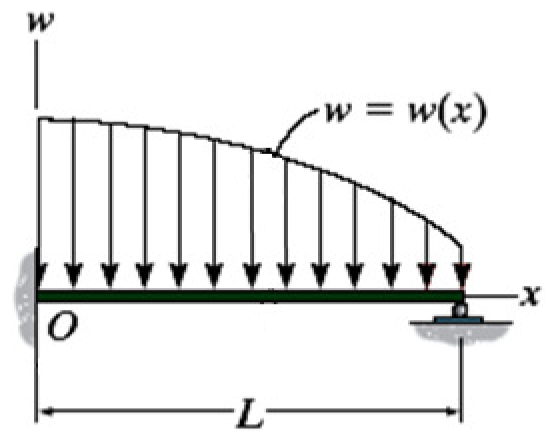
- *The resultant force* $\mathbf{F}_R$ *is equal to the* ***volume*** *under the loading curve* $p = p(x)$.

- *The line of action of* $\mathbf{F}_R$ *passes through the centroid (geometric center) of this volume.*

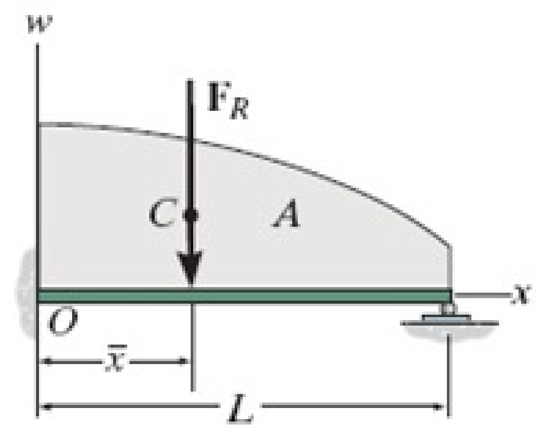
- Since $p(x)$ is a function of one variable only, the distributed loading can be represented as a *coplanar distributed load*,

$$w(x) = p(x)\, b \;\; \text{N/m}$$

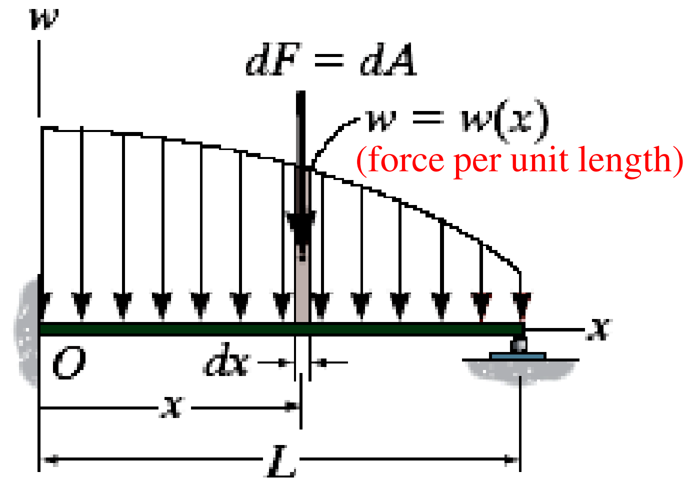
(Note: $w(x)$ is force per unit length)



- The *coplanar distributed load* can be further reduced to a single equivalent resultant force $\mathbf{F}_R$.
  - *The magnitude of* $\mathbf{F}_R$ *is equal to the area under the loading curve* $w = w(x)$.
  - *The line of action of* $\mathbf{F}_R$ *passes through the centroid (geometric center) of this area.*



❑ Magnitude of Resultant Force



- The force acting on an element of length dx is

$$dF = w(x)\,dx$$

- The force acting on the entire length,

$$+\downarrow F_R = \sum F: \qquad F_R = \int dF$$

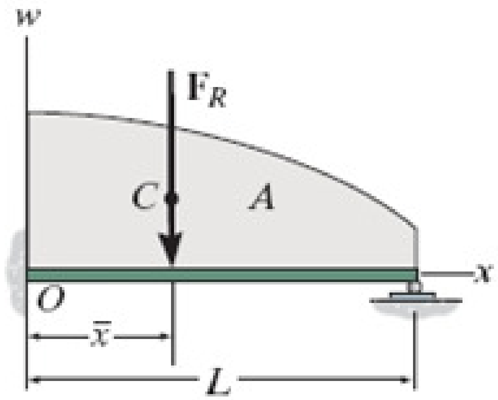
$$= \int_L w(x)\,dx$$

$$= \int_A dA$$

$$= A$$

The magnitude of the resultant force is equal to the total area A under the loading diagram.

❑ Location of Resultant Force



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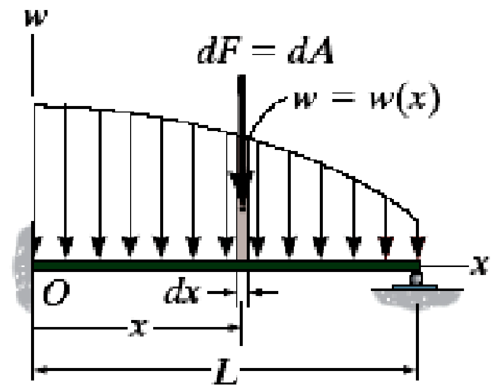


$$\curvearrowright + \quad (M_R)_O = \Sigma M_O:$$

$$-\bar{x}\,F_R = -\int_L x\,dF$$

$$\bar{x}\int_L w(x)\,dx = \int_L x\;w(x)\,dx$$

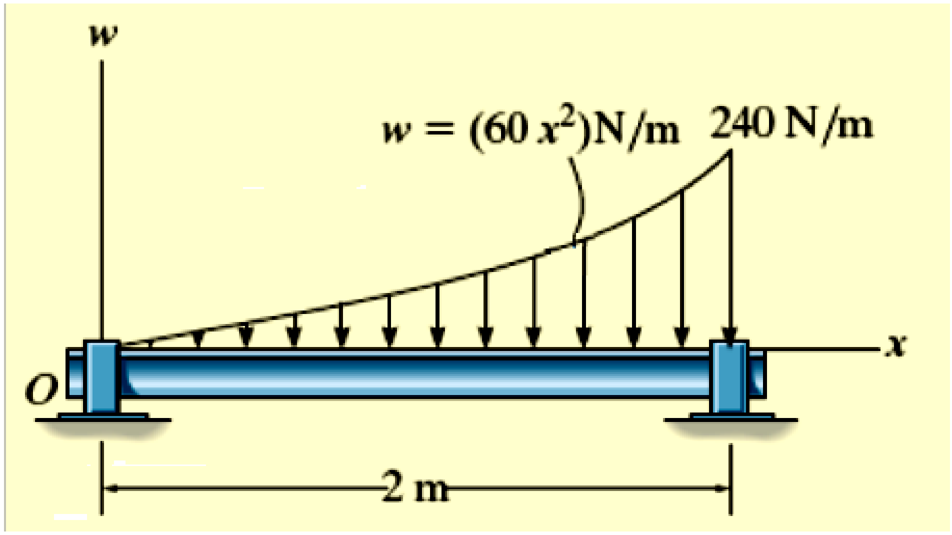
$$\bar{x} = \frac{\int_L x\;w(x)\,dx}{\int_L w(x)\,dx} = \frac{\int_L x\;dA}{\int_L dA}$$

$\bar{x}$ = *the centroid or geometric center of the area under the loading diagram*

Example 4.21

Given :

The shaft is subjected to a distributed loading as shown.



Find :

Determine the magnitude and location of the equivalent resultant force acting on the shaft.

Solution

❑ Resultant force

$$+\downarrow F_R = \sum F:$$

$$F_R = \int_A dA$$

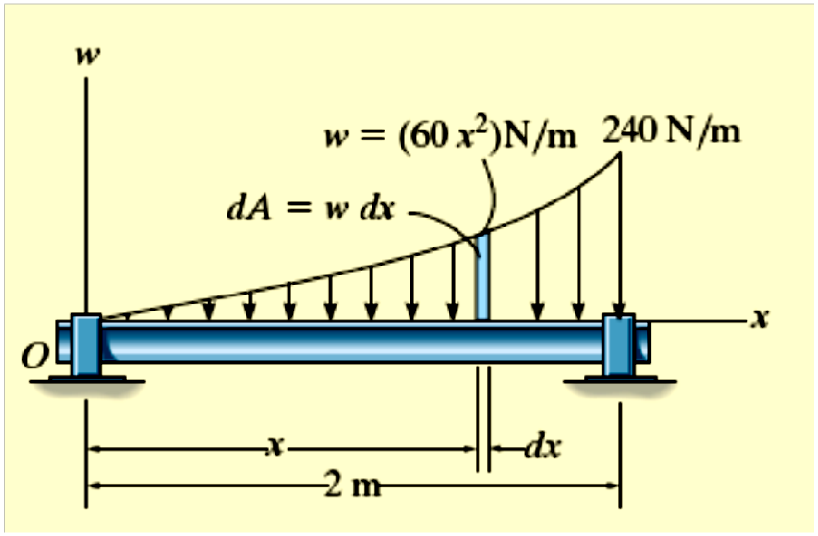
$$= \int_0^{2\text{ m}} w\,dx$$

$$= \int_0^{2\text{ m}} 60x^2\,dx$$

$$= 60\left(\frac{x^3}{3}\right)\Big|_0^{2\text{m}}$$

$$= 60\left(\frac{2^3}{3} - \frac{0^3}{3}\right)$$

$$= 160\text{ N}$$



Location of resultant force

$$\bar{x} = \frac{\int_A x \ dA}{\int_A dA} = \frac{\int_L x \ w \, dx}{F_R}$$

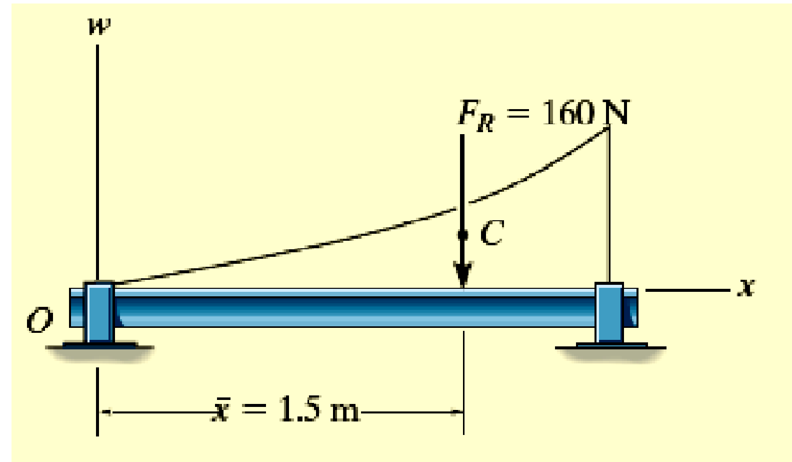
$$= \frac{\int_0^{2m} x\left(60x^2\right)dx}{160\text{N}}$$

$$= \frac{60}{160\text{N}} \int_0^{2m} x^3 \ dx$$

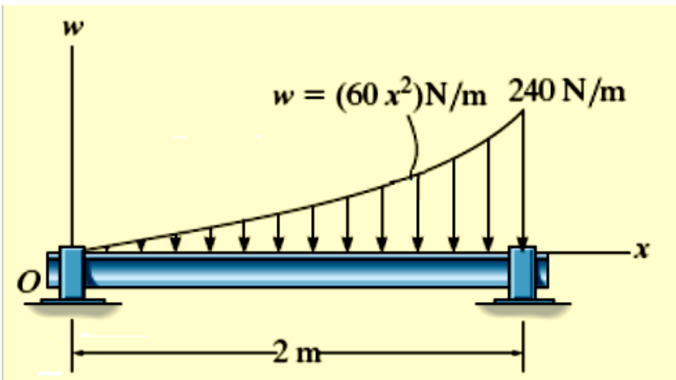
$$= \frac{60}{160\text{N}} \left[\frac{x^4}{4}\right]_0^{2\text{m}}$$

$$= \frac{60}{160\text{N}} \left(\frac{2^4}{4} - 0\right)$$

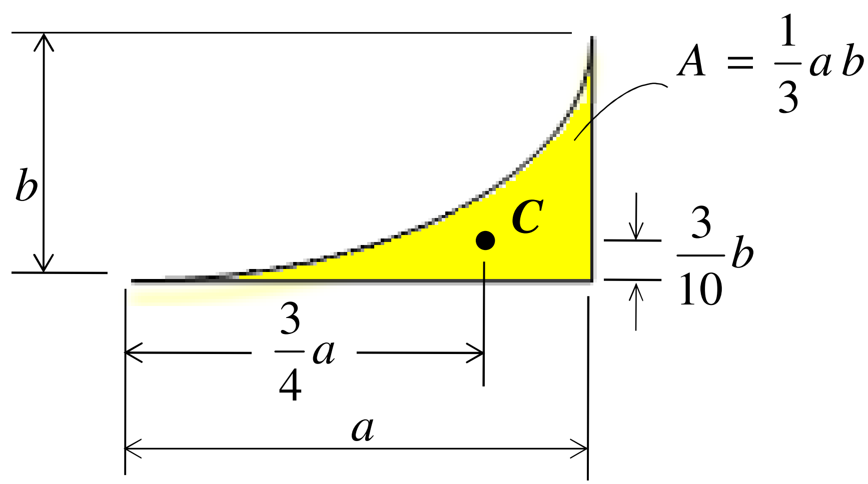
$$= 1.5 \text{ m}$$



❑ Checking



Exparabolic area



- From the loading diagram,

$$a = 2\text{ m}, \qquad b = 240\text{ N/m}$$

- Thus, using the formula for exparabolic curve, we have

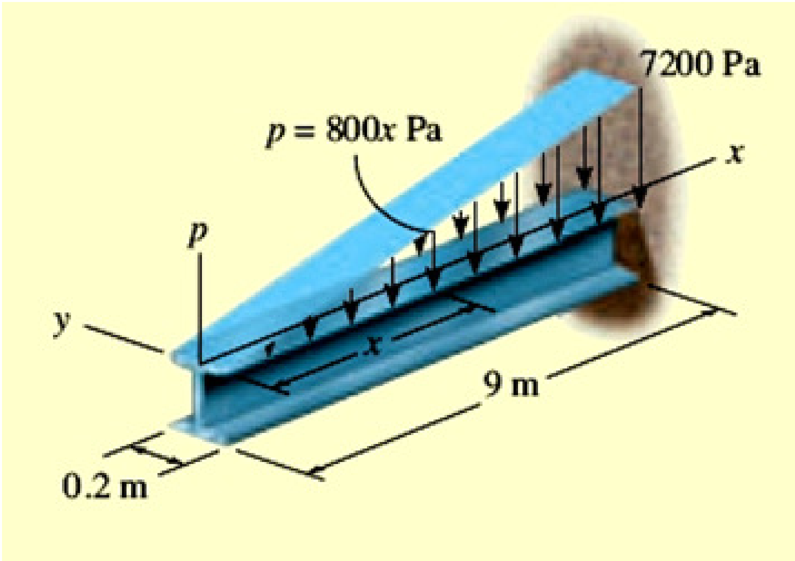
$$F_R = A = \frac{ab}{3} = \frac{(2\text{ m})(240\text{ N/m})}{3} = 160\text{ N}$$

$$\bar{x} = \frac{3}{4}a = \frac{3}{4}(2\text{ m}) = 1.5\text{ m}$$

Example 4.21

Given :

A distributed loading $p = (800x)$ Pa acts over the top surface of the beam shown in the figure.



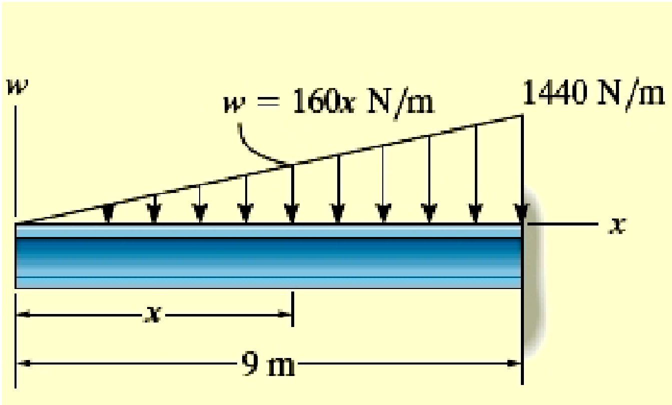
Find :

Determine the magnitude and location of the equivalent resultant force.

Solution

❑ Method I

- Since the loading is uniform across the width (b = 0.2m) of the beam (the y axis), the loading can be represented as a *coplanar distributed load*,



$$w(x) = p\,b$$

$$= (800\,x\ \mathrm{N/m^2})\ (\,0.2\ \mathrm{m})$$

$$= 160\,x\ \mathrm{N/m}$$

- **Resultant Force**

$$+\downarrow\ F_R = \sum F:$$

$$F_R = A$$

$$= \tfrac{1}{2}\,(9\ \mathrm{m})\ (1440\ \mathrm{N/m})$$

$$= 6480\ \mathrm{N} = 6.480\ \mathrm{kN}$$

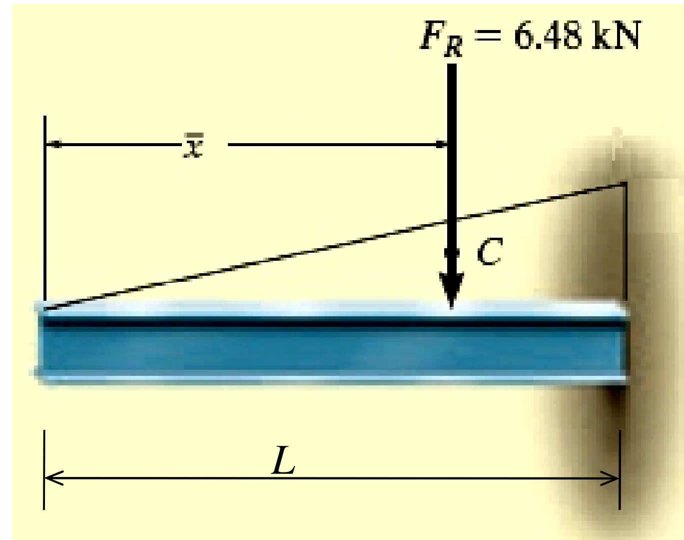
- **Location of resultant force**

The centroid of the triangle is given by

$$\bar{x} = \frac{2}{3}L$$

$$= \frac{2}{3}(9\text{ m})$$

$$= 6\text{ m}$$



❑ Method II

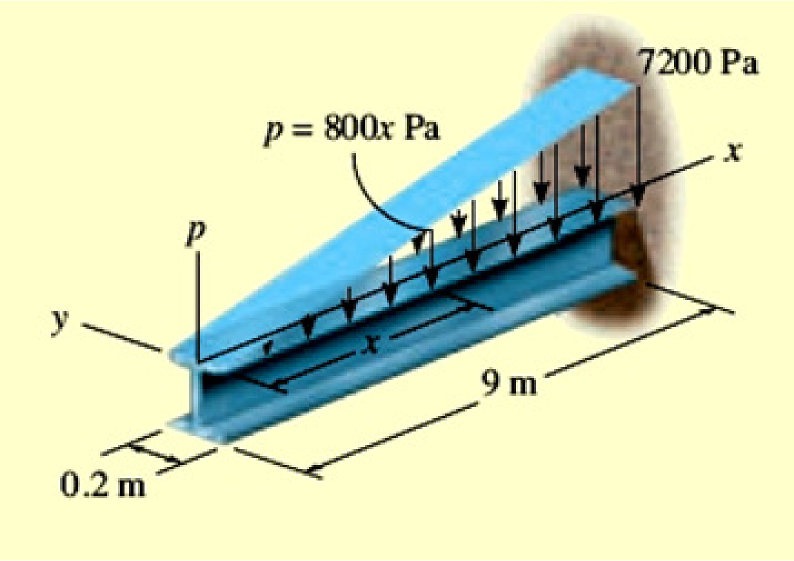
- The resultant force $\mathbf{F}_R$ has a magnitude equal to the *volume* under the loading curve $p = (800\,x)$ and a line of action which passes through the *centroid* (*geometric center*) of this volume.

$$F_R = V$$

$$= (A)(b)$$

$$= [\tfrac{1}{2}\,(9)\,(7200)]\,(0.2)$$

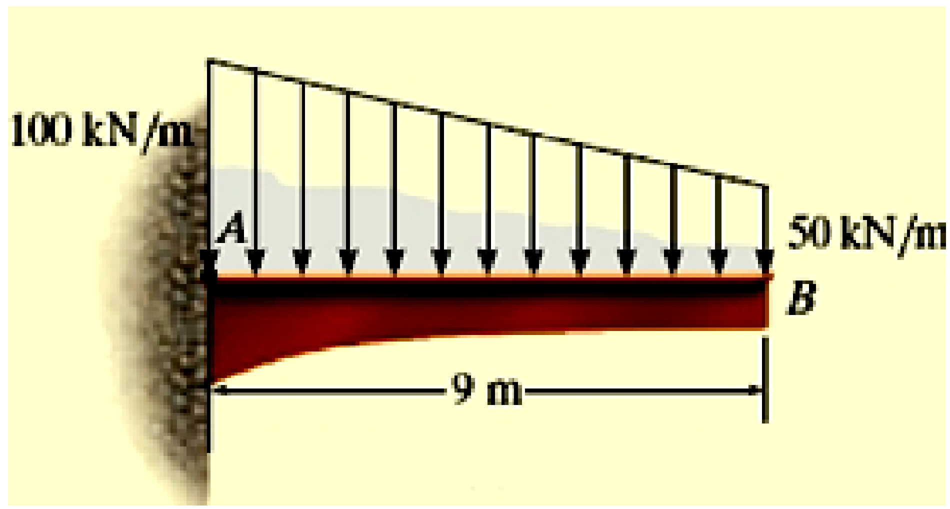
$$= 6480 \text{ N}$$



Example 4.23

Given :

The granular material exerts the distributed loading on the beam as shown in the figure.



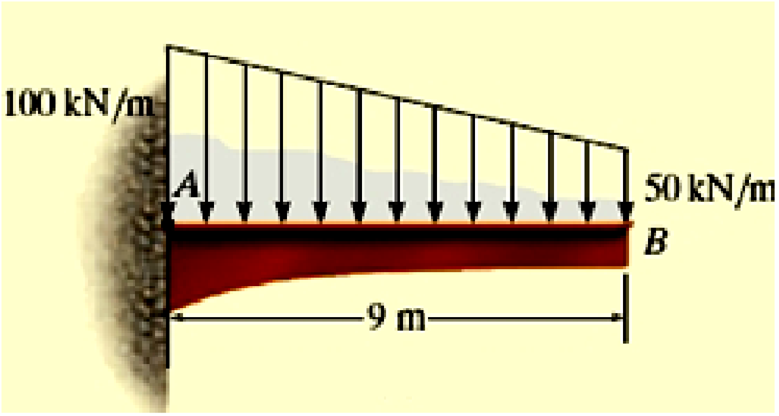
Find :

Determine the magnitude and location of the equivalent resultant of this load.

Solution

❑ Method I

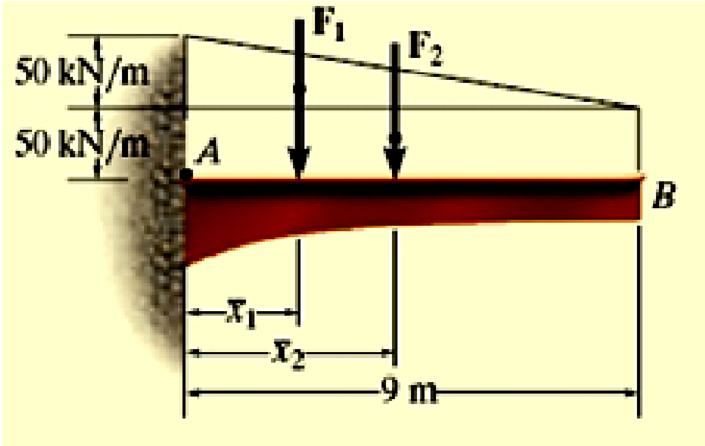
- Divide the trapezoidal loading into a rectangular and triangular loading.



Triangular loading:

$$F_1 = ½ (9m) (50 kN/m) = 225 kN$$

$$\bar{x}_1 = ⅓ (9m) = 3m$$



Rectangular loading:

$$F_2 = (9m) (50 kN/m) = 450 kN$$

$$\bar{x}_2 = ½ (9m) = 4.5 m$$

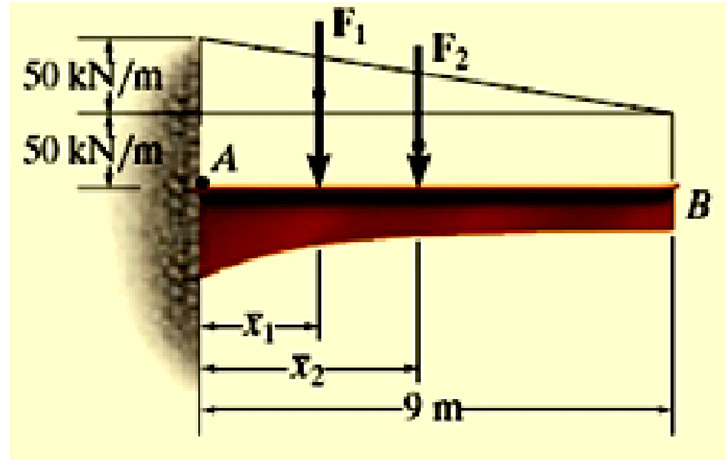
- **Resultant force**

$$+\downarrow \; F_R = \sum F:$$

$$F_R = F_1 + F_2$$

$$= 225 \text{ kN} + 450 \text{ kN}$$

$$= 675 \text{ kN}$$



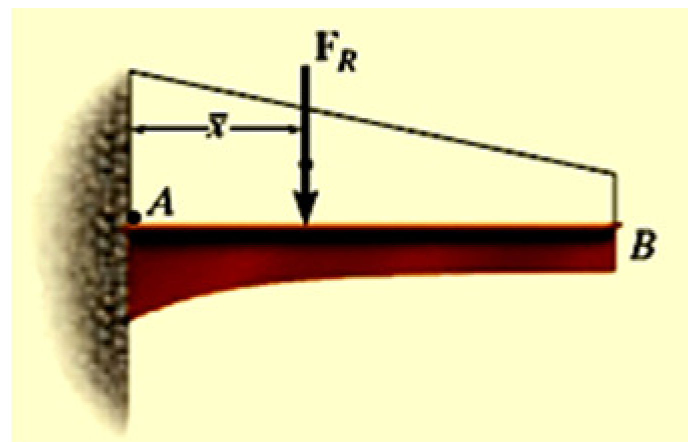
- **Location of resultant force**

$$\circlearrowleft + \; (M_R)_A = \Sigma M_A:$$

$$F_R \, \bar{x} = F_1 \, \bar{x}_1 + F_2 \, \bar{x}_2$$

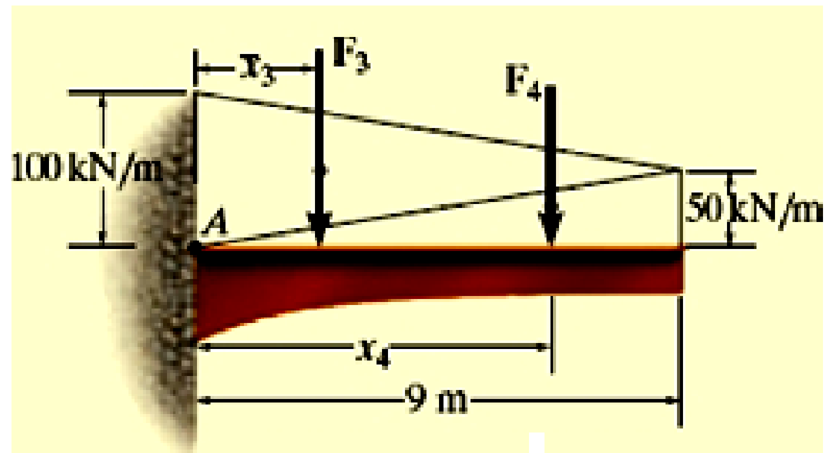
$$675 \; \bar{x} = (225)(3) + (450)(4.5)$$

$$\bar{x} = 4 \text{ m}$$



❑ Method II

- Divide the trapezoidal loading into 2 triangular loadings.



Top Triangular loading:

F_3 =½ (100 kN/m) (9m) = 450 kN

$\bar{x}_3$ =⅓ (9m) = 3m

Bottom Triangular loading

F_4 = ½ (9m) (50 kN/m) = 225 kN

$\bar{x}_4$ = ⅔ (9m) = 6 m

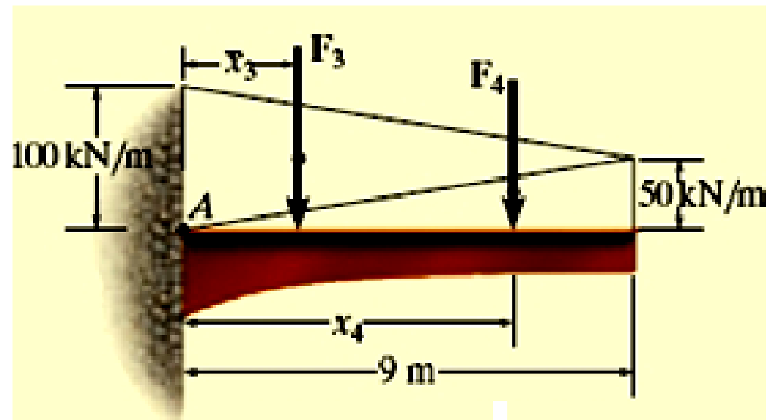
- **Resultant force**

$$+\downarrow\ F_R = \sum F:$$

$$F_R = F_3 + F_4$$

$$= 450 \text{ kN} + 225 \text{ kN}$$

$$= 675 \text{ kN}$$



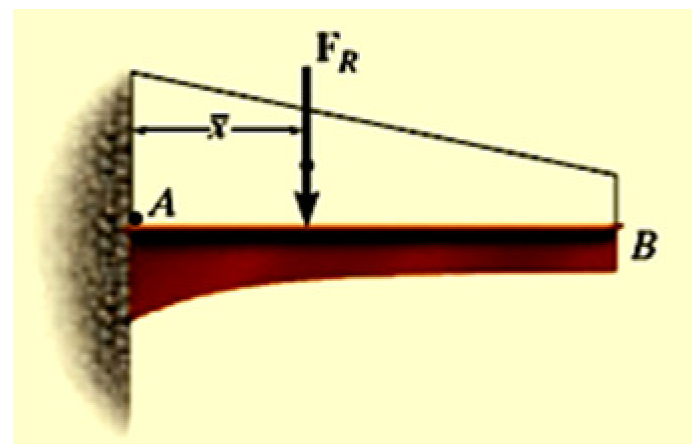
- **Location of resultant force**

$$\circlearrowleft + \ (M_R)_A = \Sigma M_A:$$

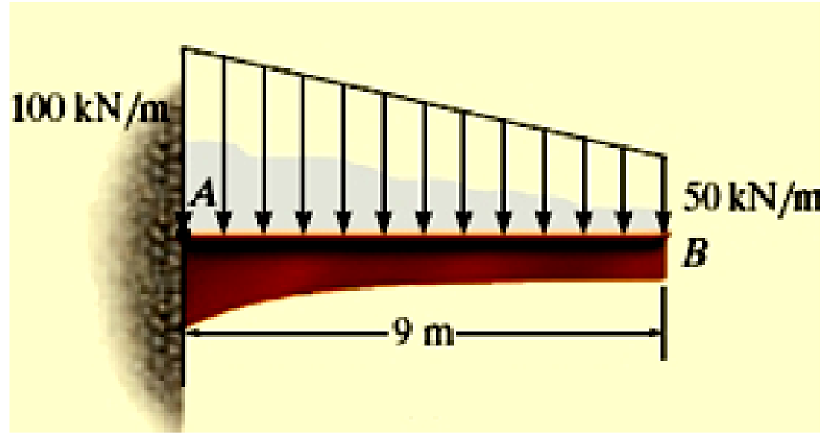
$$F_R\,\bar{x} = F_3\,\bar{x}_3 + F_4\,\bar{x}_4$$

$$675\ \bar{x} = (450)(3) + (225)(6)$$

$$\bar{x} = 4 \text{ m}$$



❑ Method III



The area under the loading diagram is a trapezoid

- **Resultant Force**

$$+\downarrow \; F_R = \sum F:$$

$$F_R = A$$

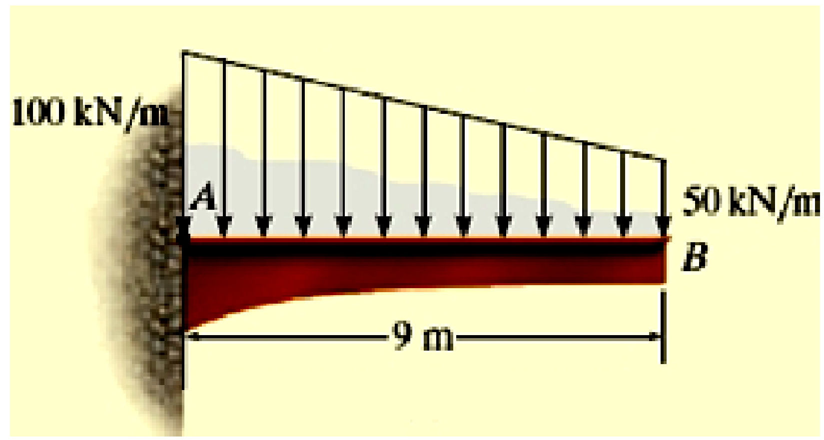
$$= \tfrac{1}{2}\,(100 + 50\ \text{kN/m})\,(9\ \text{m})$$

$$= 675\ \text{kN}$$

- **Location of resultant force**

The resultant force is acting through the centroid of the trapezoid given by

$$\bar{x} = \frac{h(2a+b)}{3(a+b)}$$



where $a =$ 50 kN/m,
$b =$ 100 kN/m,
$h =$ 9 m

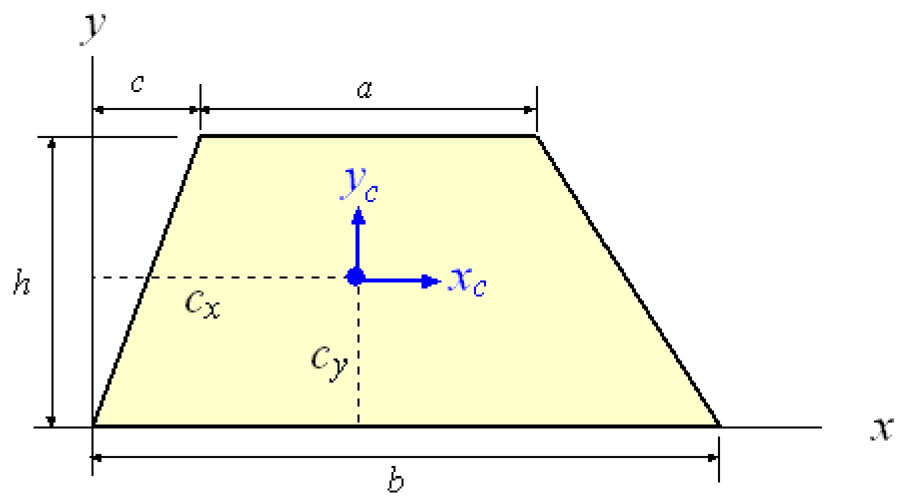
Thus,

$$\bar{x} = \frac{9[2(50)+(100)]}{3(50+100)}$$

$$= 4 \text{ m}$$

Geometric Properties

- Trapezoidal Area



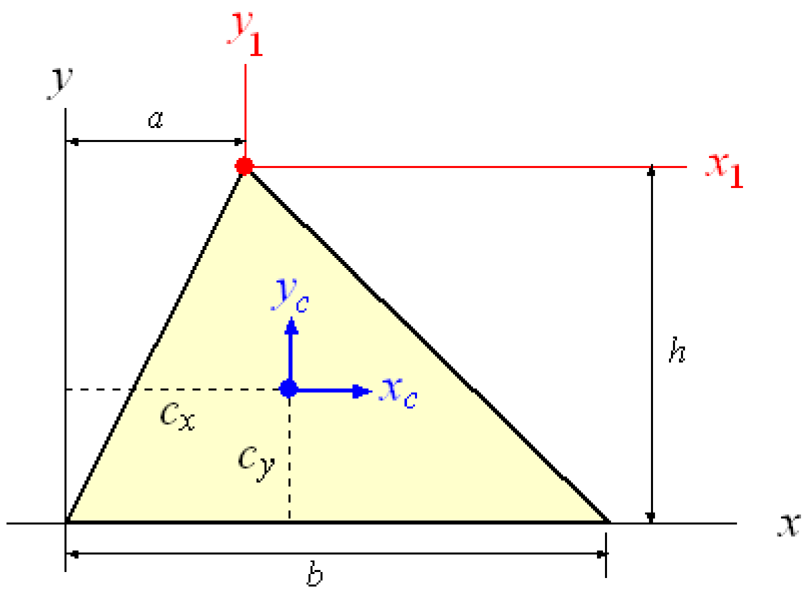
Area:

$$A = \frac{h(a+b)}{2}$$

Centroid :

$$C_x = \frac{2ac + a^2 + cb + ab + b^2}{3(a+b)}, \qquad C_y = \frac{h(2a+b)}{3(a+b)}$$

❑ Triangular Area



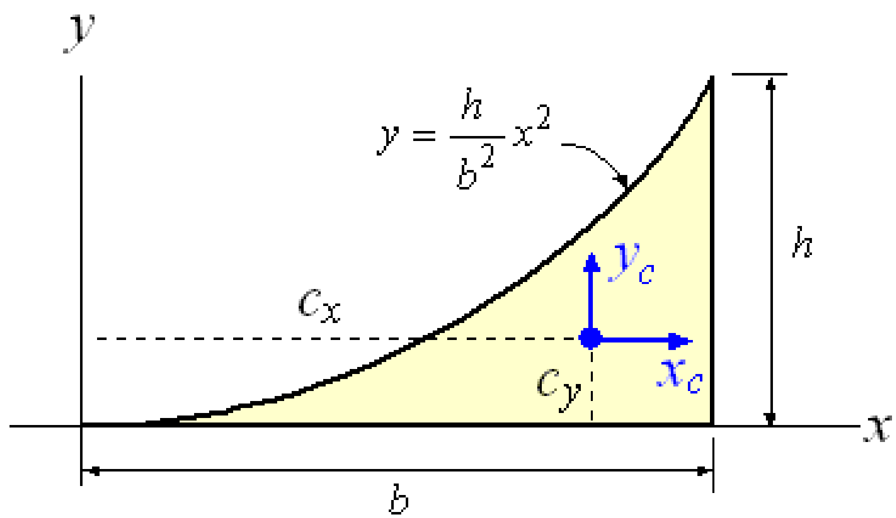
Area:

$$A = \frac{bh}{2}$$

Centroid :

$$C_x = \frac{(a+b)}{3}, \qquad C_y = \frac{h}{3}$$

❑ Exparabolic Area



Area:

$$A = \frac{bh}{3}$$

Centroid :

$$C_x = \frac{3b}{4}, \qquad C_y = \frac{3h}{10}$$