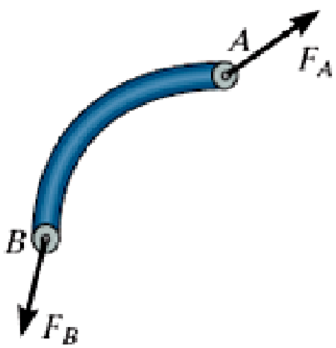
5.4 Two- and Three-Force Members

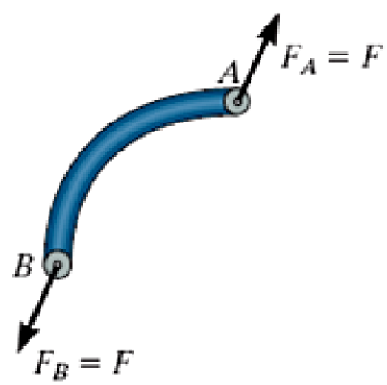
❑ Two-Force Members

- When forces are applied at only two points on a member, the member is called a two-force member

- Only force magnitude must be determined.

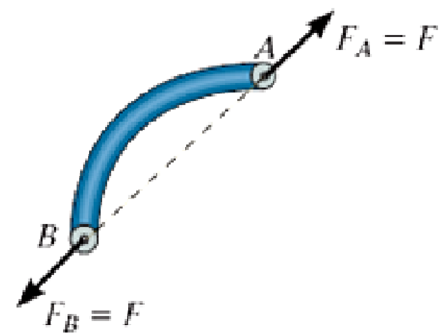


$\sum \mathbf{F} = 0 \quad \Rightarrow$



$\sum \mathbf{M}_A = 0 \quad \Rightarrow$

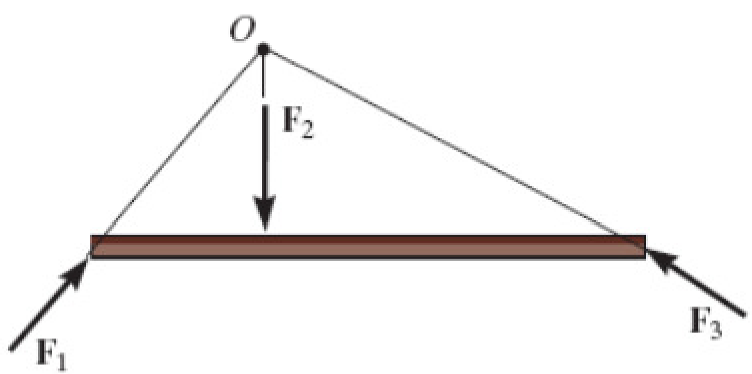
or $\sum \mathbf{M}_B = 0$



- For any two-force member to be in equilibrium, the two forces acting on the member must have
  - *the same magnitude*,
  - *act in opposite directions*,
  - *have the same line of action, directed along the line joining the two points where these forces act.*

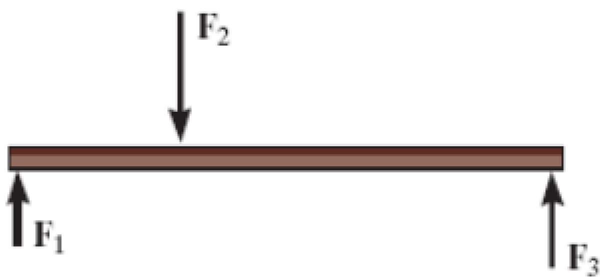
Three-Force Members

- If a member is subjected to only ***three forces***, it is called a *three-force member.*
- The three forces in a *three-force member* form a ***concurrent*** or ***parallel*** force system.



If $\mathbf{F}_1$ *&* $\mathbf{F}_2$ intersect at *O, the line of action of* $\mathbf{F}_3$ *must also pass through O so that the forces satisfy*

$$\sum \mathbf{M}_O = \mathbf{0}$$

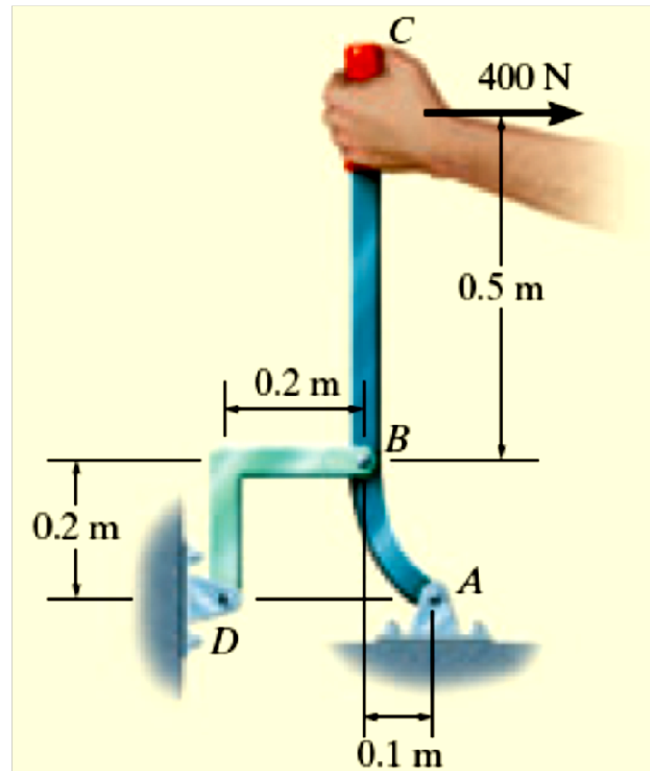


The location of the point of intersection of the three parallel forces will approach infinity.

Example 5.13

Given :

- The lever *ABC* is pin-supported at *A* and connected to a short link *BD*.
- The weight of the members are negligible.



Find :

Determine the force of the pin on the lever at *A*.

Solution

Method I

Free-Body Diagram

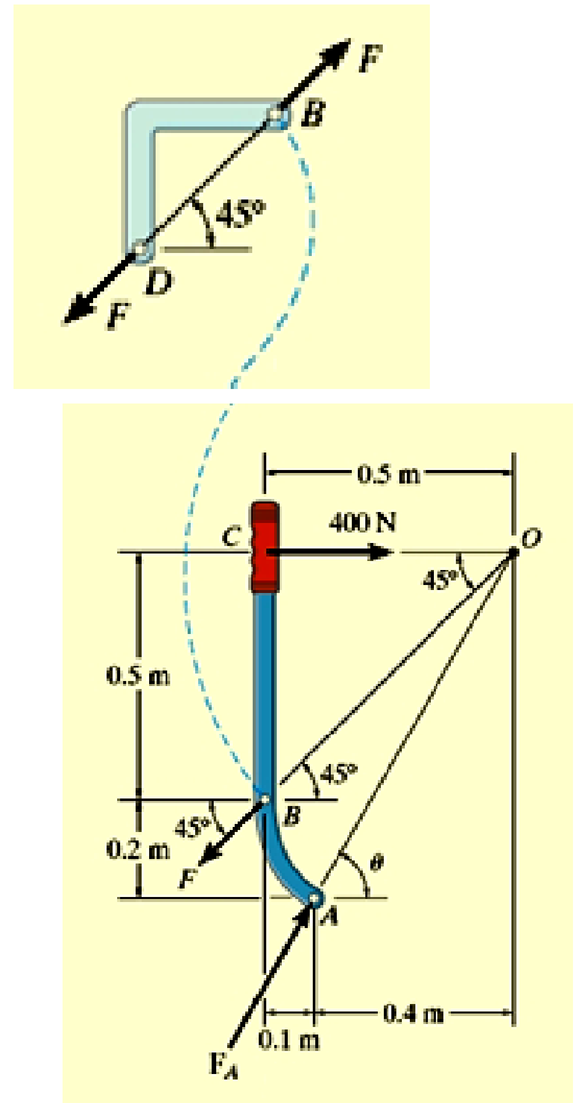
- **Link *BD***

  Since BD is a two-force member, the resultant force at D & B
  - *are equal & opposite.*
  - *lie on same line that passes through B & D.*

- **Lever *ABC* is a three-force member**
  - *The lines of action of the forces at A, B & C must intersect at O.*

- From trigonometry

$$\theta = \tan^{-1}\left(\frac{0.7}{0.4}\right) = 60.3^\circ$$



Equations of Equilibrium

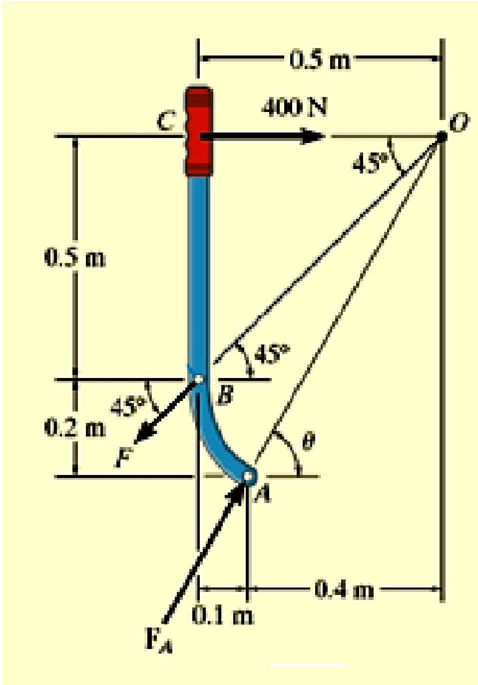
$$\overset{+}{\rightarrow} \sum F_x = 0: \quad F_A \cos 60.3^o - F\cos 45^o + 400\text{ N} = 0 \qquad (1)$$

$$+\uparrow \sum F_y = 0: \quad F_A \sin 60.3^o - F \sin 45^o = 0 \qquad (2)$$

- Eq(1) × sin 45° – Eq(2) × cos 45°:

$$F_A (\sin 45^o \cos 60.3^o - \cos 45^o \sin 60.3^o) + 400 \sin 45^o \text{ N} = 0$$

$$\Rightarrow \quad F_A = 1071.9 \text{ N} = 1.07 \text{ kN}$$



- Substituting F_A into Eq. (2), we get

$$F = F_A \frac{\sin 60.3^\circ}{\sin 45^\circ} = 1071.9 \frac{\sin 60.3^\circ}{\sin 45^\circ}$$

$$\Rightarrow \quad F = 1316.8 \text{ N} = 1.32 \text{ kN}$$

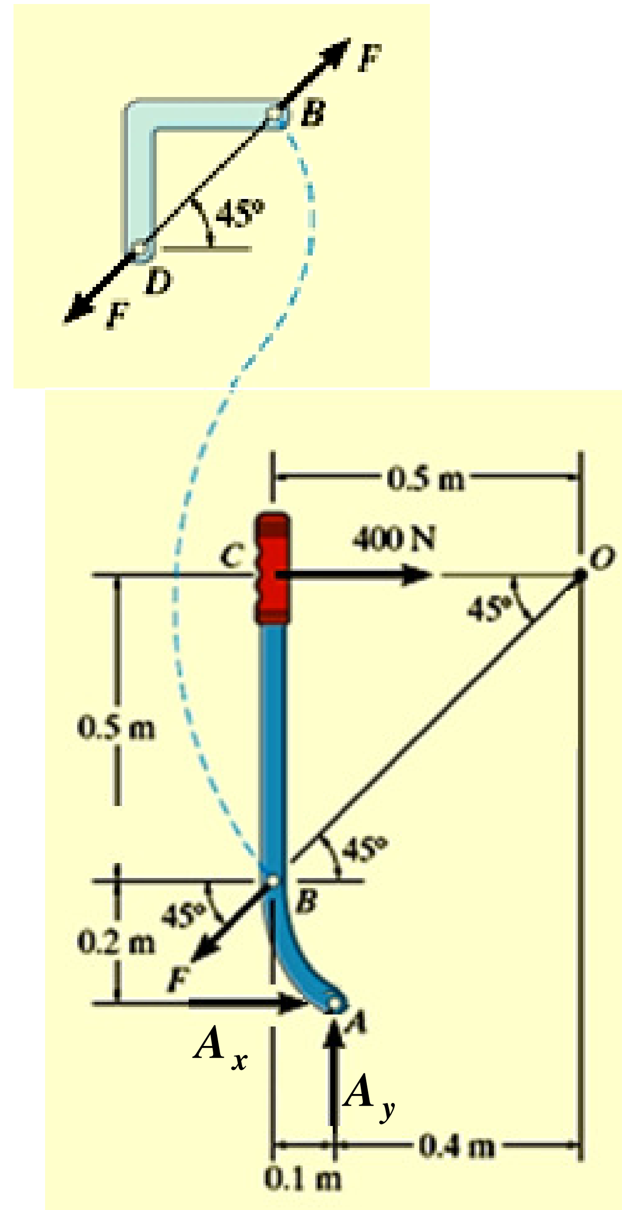
Method II

Free-Body Diagram

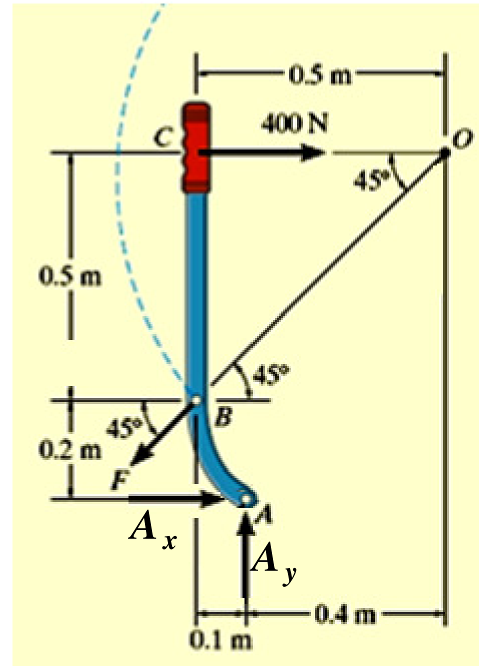
- **Link *BD***

  Since BD is a two-force member, the resultant force at D & B must
  - *be equal & opposite.*
  - *lie on same line that passes through B & D.*

- **Level ABC**
  - Point *A* is pin-supported. Therefore, the reaction at *A* has *x* & *y* components.



Equations of Equilibrium



$$\overset{+}{\rightarrow} \sum F_x = 0: \quad A_x - F\cos 45^\circ + 400\text{ N} = 0 \quad (1)$$

$$+\uparrow \sum F_y = 0: \quad A_y - F\sin 45^\circ = 0 \quad (2)$$

$$\circlearrowright + \quad \Sigma M_A = 0:$$

$$-400(0.7) + (F\cos 45^\circ)(0.2) + (F\sin 45^\circ)(0.1) = 0$$

$$\Rightarrow F = 1319.9\text{ N} = 1.32\text{ kN}$$

- Substituting *F* into Eqs. (1) & (2), we get

$$A_x = 533.33\text{N}$$

$$A_y = 933.33\text{N}$$

- Magnitude of the resultant force at *A*

$$F_A = \sqrt{(A_x)^2 + (A_y)^2}$$

$$= \sqrt{(533.33)^2 + (933.33)^2} \text{ N}$$

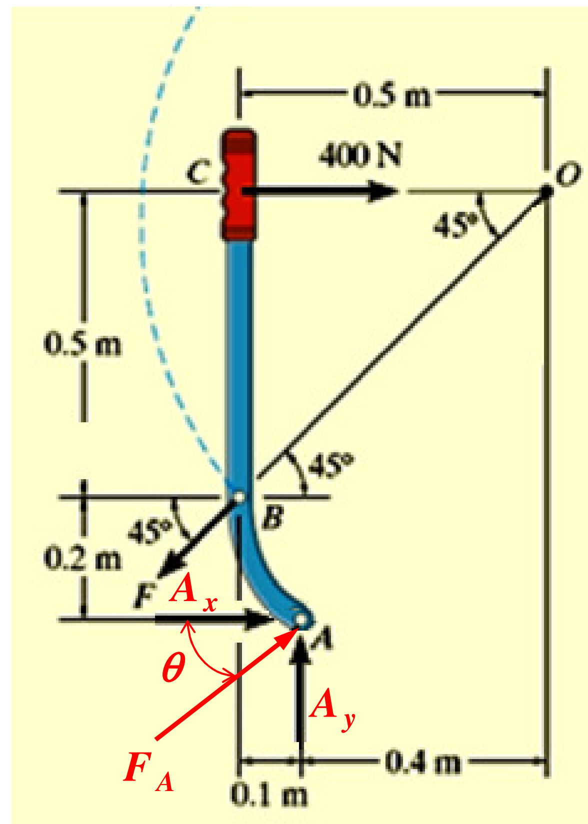
$$= 1.07 \text{ kN}$$

- Direction of the resultant force at *A*

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

$$= \tan^{-1}\left(\frac{933.33}{533.33}\right)$$

$$= 60.3^{o}$$



EQUILIBRIUM IN THREE DIMENSIONS

5.5 Free-Body Diagrams

- As in the 2-D case, the first step in solving 3-D equilibrium problems is to draw a free-body diagram.
- Only external forces are required to be shown on the FBD.
- The most important external forces are **support reactions**.
  - A force is developed by a support that restricts the translation of its attached member.
  - A couple moment is developed when rotation of the attached member is prevented

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

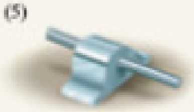
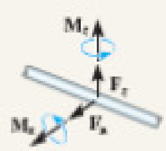
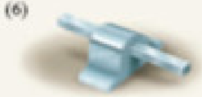
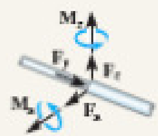
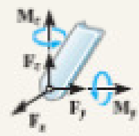
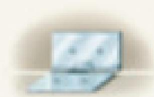
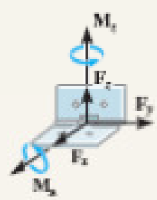
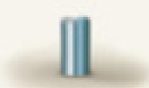
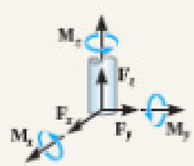
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are generally not applied if the body is supported elsewhere. See the examples.

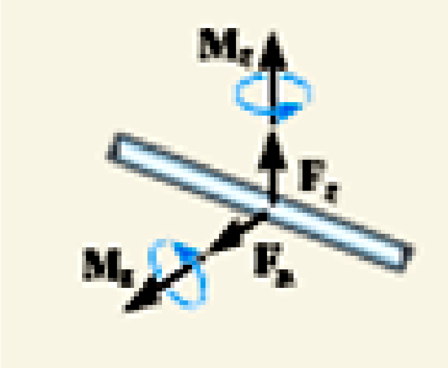
TABLE 5–2 Continued

Types of Connection	Reaction	Number of Unknowns
(6)  single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(7)  single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(8)  single smooth pin		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(9)  single hinge		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(10)  fixed support		Six unknowns. The reactions are three force and three couple-moment components.

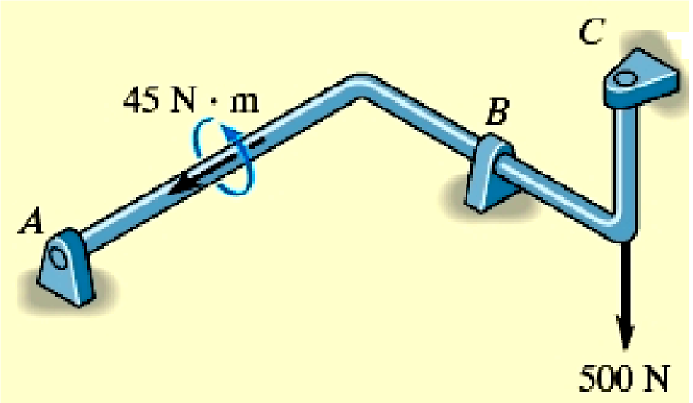
❑ Examples : Supports for Rigid Bodies

(a) Journal Bearing

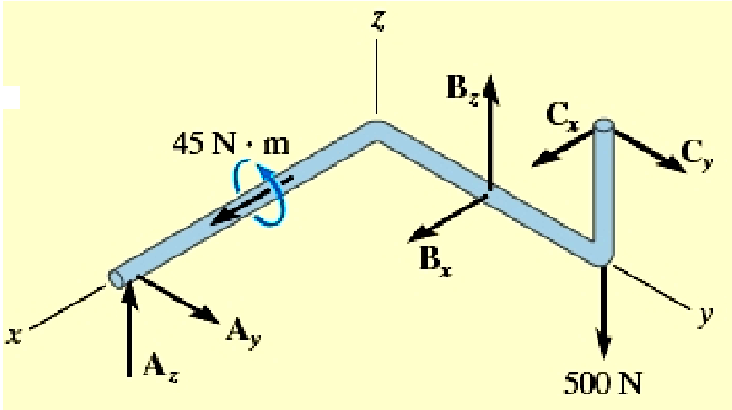
FBD →



Rigid body supported by single journal bearing



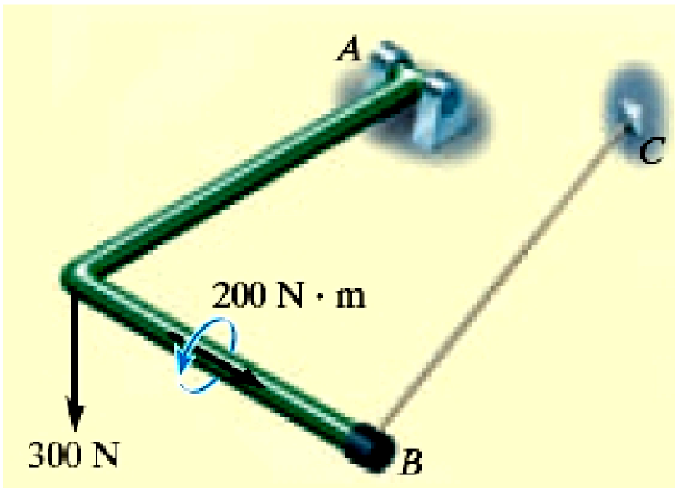
FBD →



Properly aligned journal bearings at *A*, *B*, *C*.

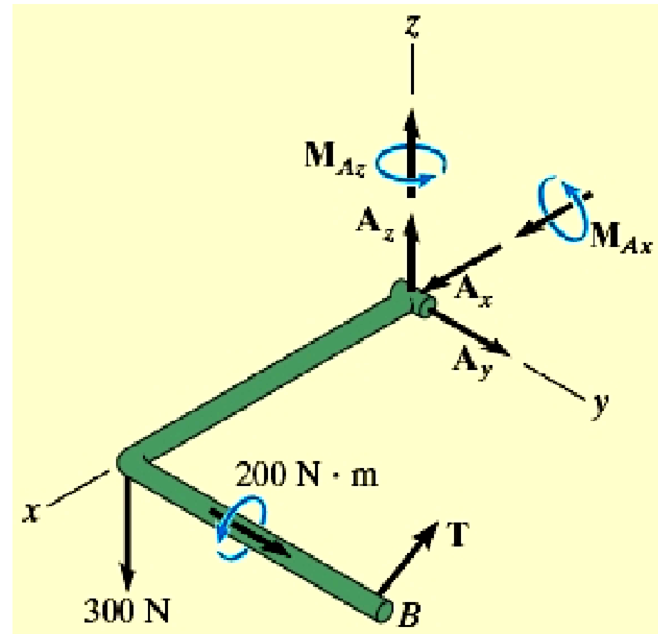
The force reactions developed by the bearings are sufficient for equilibrium since they prevent the shaft from rotating about each of the coordinate axes.

(b) Pin



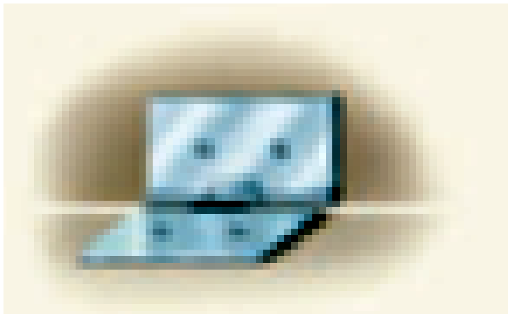
Pin at *A* and cable *BC*.

FBD →



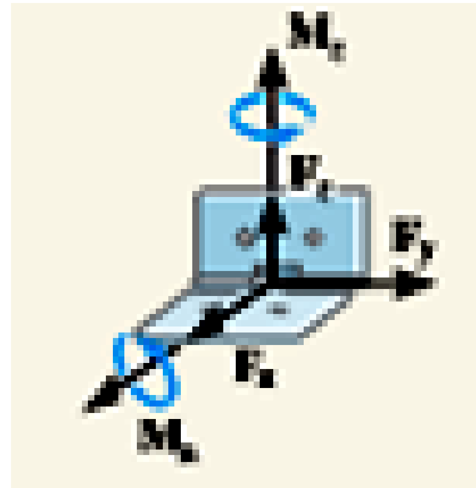
Moment components are developed by the pin on the rod to prevent rotation about the *x* and *z* axes.

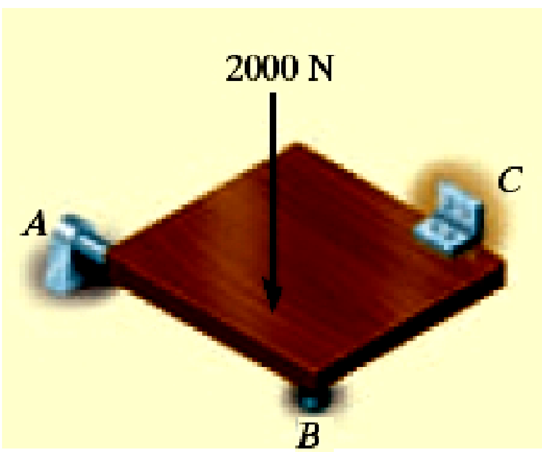
(c) Hinge



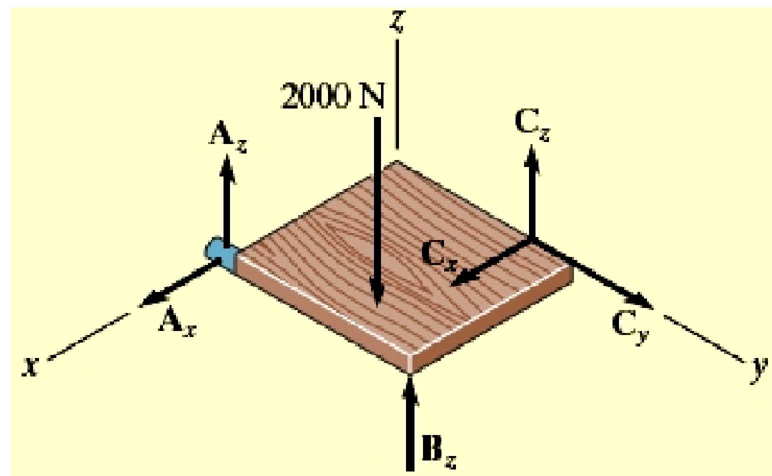
Single hinge

FBD →





FBD →



Properly aligned journal bearing at *A* and hinge at *C*. Roller at *B*.

Only force reactions are developed by the bearing and hinge on the plate to prevent rotation about each coordinate axis. No moments at the hinge are developed.

5.6 Equations of Equilibrium in Three Dimensions

❑ Vector Equations of Equilibrium

- The necessary and sufficient conditions for equilibrium are

$$\sum \mathbf{F} = \mathbf{0}$$

$$\sum \mathbf{M}_O = \mathbf{0}$$

where

$\sum \mathbf{F}$ = vector sum of all the external forces acting on the body.

$\sum \mathbf{M}_O$ = vector sum of the couple moments and the moments of all the forces about any point O located either on or off the body.

Scalar Equations of Equilibrium

- The external forces and couple moments can be expressed in Cartesian vector form as

$$\sum \mathbf{F} = \sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k} = \mathbf{0}$$

$$\sum \mathbf{M}_O = \sum M_x \mathbf{i} + \sum M_y \mathbf{j} + \sum M_z \mathbf{k} = \mathbf{0}$$

- Since the **i**, **j**, and **k** components are independent from one another, the above equations are satisfied provided

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

and

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

Note: *These 6 scalar equilibrium equations may be used to solve at most 6 unknowns on the free-body diagram.*

Procedure for Analysis of 3-D Problems

❑ Free Body Diagram

- "Isolating" the body by drawing its outlined shape.
- Establish an x, y, z coordinate system.
- Show all the forces and couple moments acting on the body.
- Show all the unknown components having a positive sense.
- Indicate the dimensions of the body necessary for computing the moments of forces.

Equations of Equilibrium

- Apply the six scalar equations of equilibrium or vector equations
- The set of axes chosen for force summation need not be coincided with the set of axes chosen for moment summation.
- Choose the direction of an axis for moment summation such that it intersects the lines of action of as many unknown forces as possible.