5.7 Constraints and Statical Determinacy

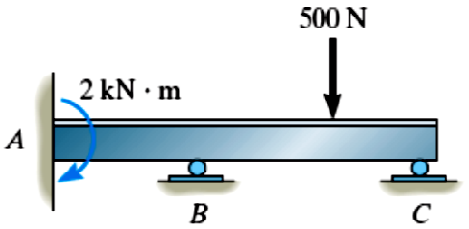
- To ensure the equilibrium of a rigid body, the body must also be properly held or constrained by its supports.

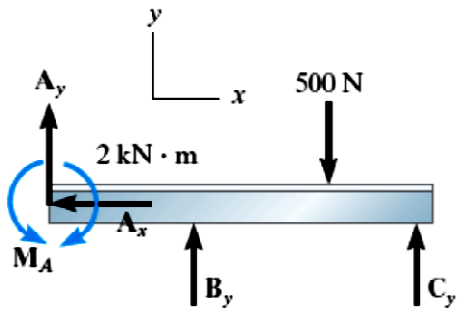
❑ Redundant Constraints

- A body is said to have redundant constraints if it has more supports than are necessary to hold it in equilibrium.
- A body with redundant constraints is ***statically indeterminate*** as there are more unknown loadings than equations of equilibrium.

- **Examples of statically indeterminate problems.**

(i)



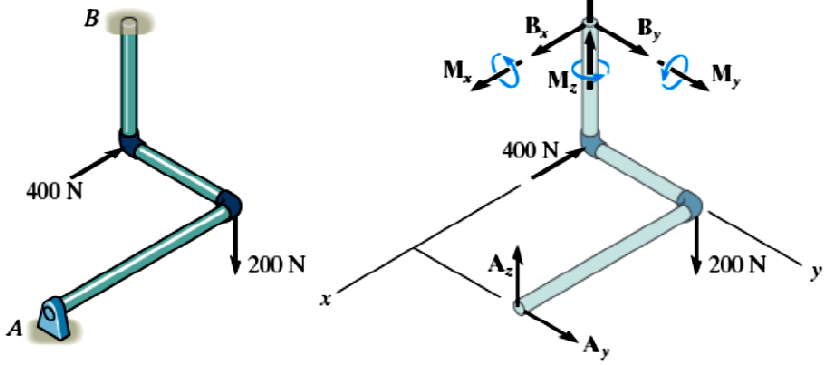


(a)

No. of equilibrium equations = 3

No. of unknowns = 5

(ii)



No. of equilibrium equations = 6

No. of unknowns = 8

❑ Improper Constraints

- Having the same number of unknowns as the available equations of equilibrium does not always guarantee that a body will be stable when subjected to a particular loading.

- Instability may occur in a rigid body if it is *improperly constrained* by its supports.

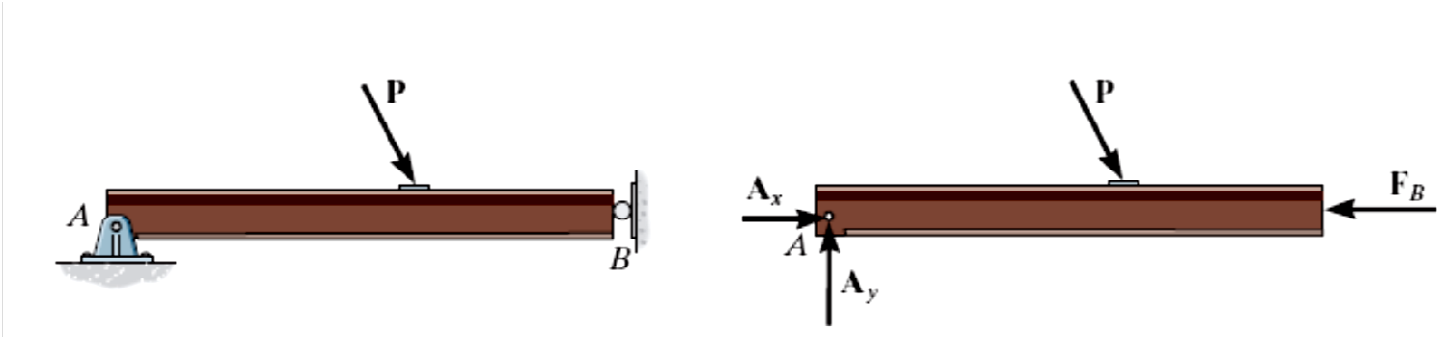
- A body is considered *improperly constrained* if

(1) all the reactive forces intersect at a common point (2-D case) or pass through a common axis (3-D case),

(2) all the reactive forces are parallel.

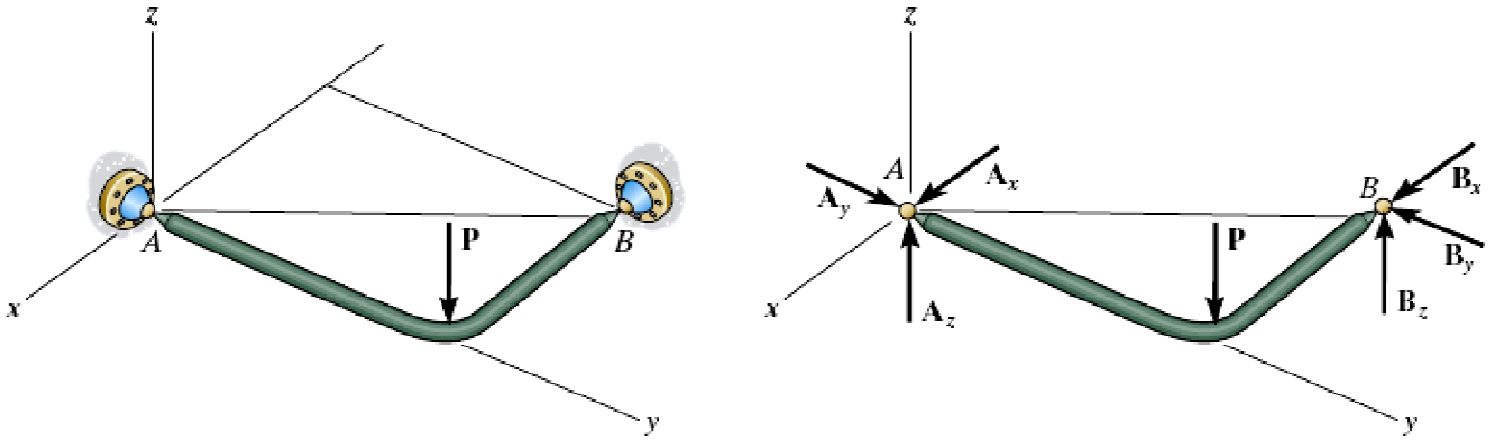
(3) the body is partially constrained.

(1.a) 2-D Case: All reactive forces intersect at a common point



- The reactive forces A_x, A_y, and F_B are concurrent at point A. Therefore, the moments of these forces about A are zero.
- However, the presence of **P** causes $\sum M_A \neq 0$.
- Consequently, the beam will rotate about A.
- So, the beam is improperly constrained.

(1.b) 3-D Case: : All reactive forces pass through a common axis

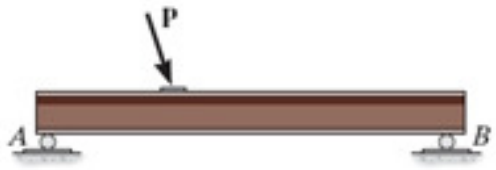


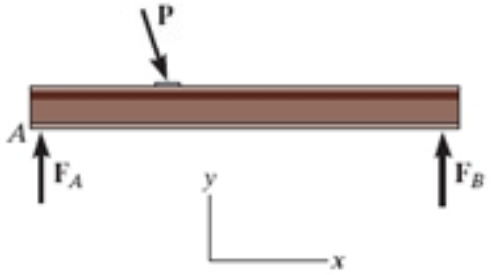
- The reactive forces at the ball-and socket supports A_x, A_y, A_z, B_x, B_y, and B_z, pass through the common axis AB.

  Therefore, the moments of these forces about A & B are all zero.

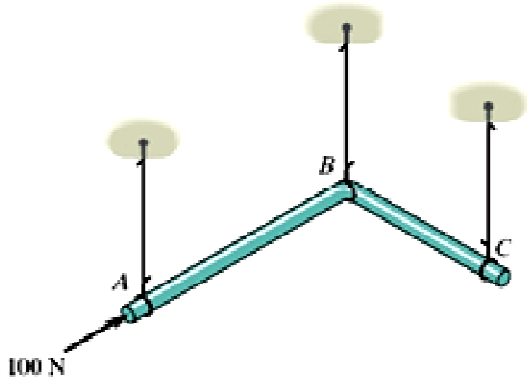
- However, the presence of **P** causes $\sum M_{AB} \neq 0$.
- Consequently, the member will rotate about the AB axis.
- So, the memeber is improperly constrained.

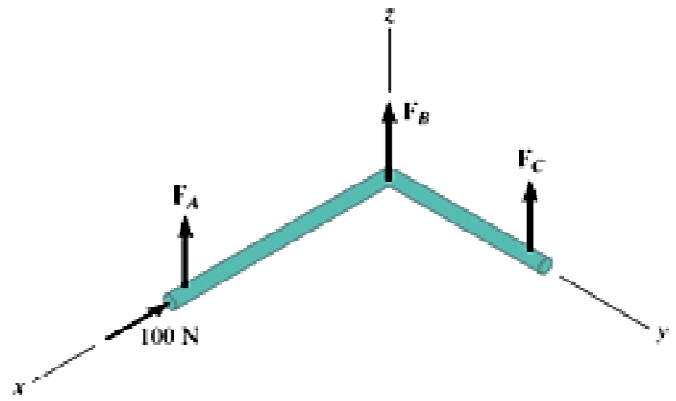
(2) All reactive forces are parallel





(a) 2 D case



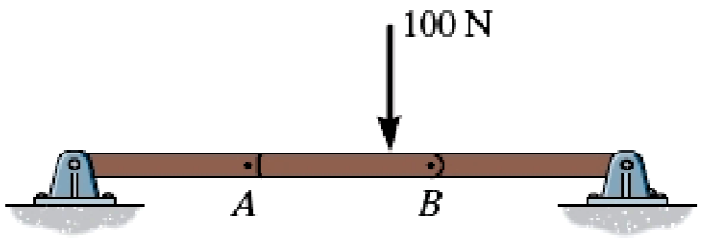


(b) 3-D case

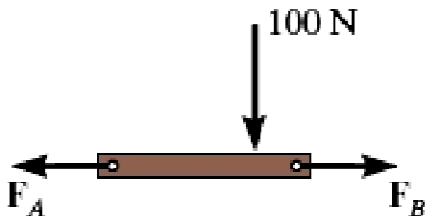
$$\sum F_x \neq 0.$$

(3) A Partially Constrained Body

A body is partially constrained if it has fewer reactive forces than equations of equilibrium that must be satisfied.



(a)



(b)

$$\sum F_x = 0$$

$$\sum F_y \neq 0$$