Chapter 2
Force Vectors

Chapter Objectives

- Parallelogram Law
- Cartesian vector form for position and force
- Dot product and angle between 2 vectors

Chapter Outline

1. Scalars and Vectors
2. Vector Operations
3. Vector Addition of Forces
4. Addition of a System of Coplanar Forces
5. Cartesian Vectors
6. Addition and Subtraction of Cartesian Vectors
7. Position Vectors
8. Force Vector Directed along a Line
9. Dot Product

2.1 Scalars and Vectors

Physical quantities are measured using either scalars or vectors.

❑ Scalar

- A quantity characterized by a either positive or negative number.

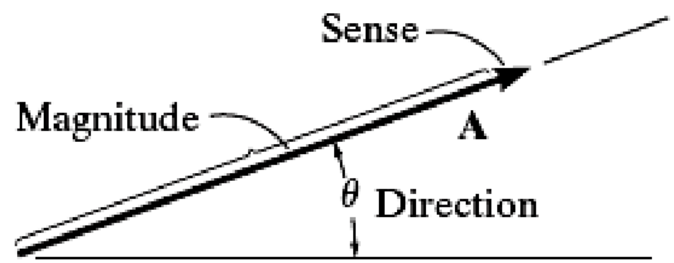
  Examples: mass, volume and length.

- Indicated by letters in italic such as *A*.

❑ Vector

- A quantity that has **magnitude** and **direction**.

  Examples: position, force and moment



- A vector may be represented by :

  (i) A letter with an arrow over it $\vec{A}$

  with its magnitude denoted by $|\vec{A}|$

  or

  (ii) A bold face letter **A** with its magnitude denoted by A.

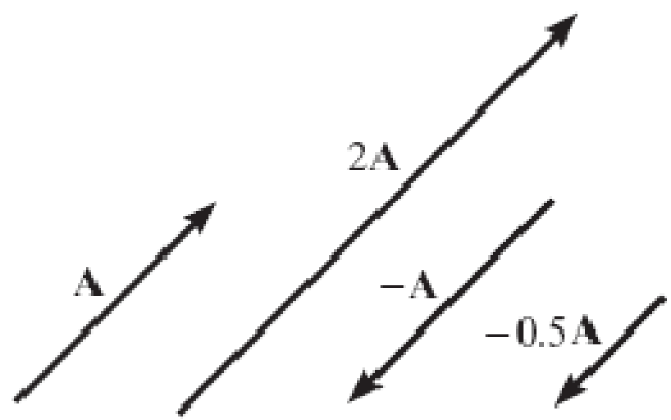
2.2 Vector Operations

I. Multiplication and Division of a Vector by a Scalar

- Product of vector **A** and scalar a is $a\mathbf{A}$.

- Law of multiplication applies.

$$\mathbf{A}/a = (\,1/a\,)\,\mathbf{A}, \quad a \neq 0$$



II. Vector Addition

- Addition of two vectors is ***commutative***

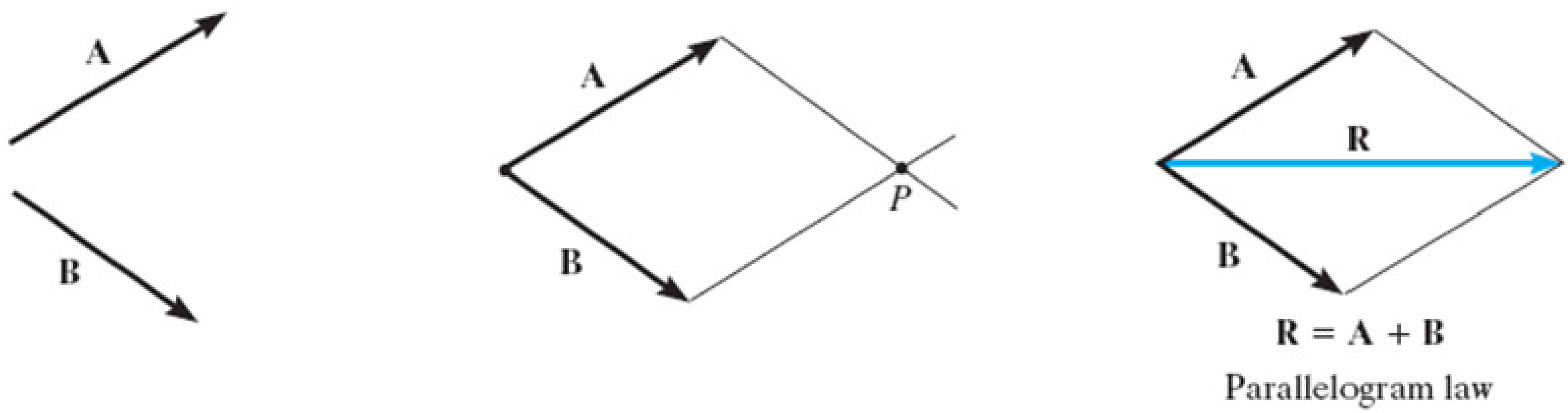
$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

- Addition of two vectors can be obtained using

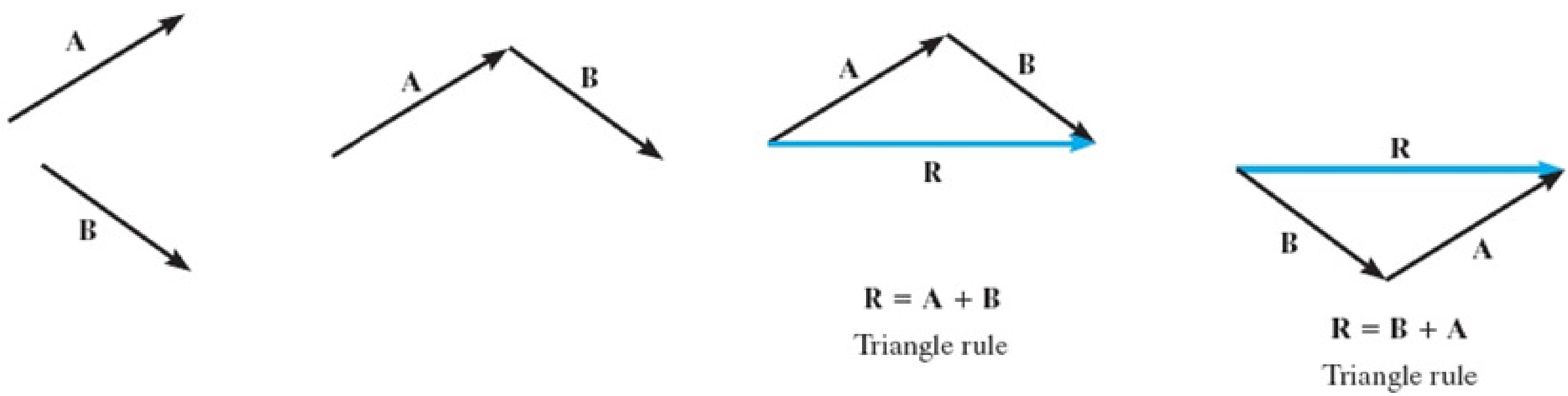
  - Parallelogram law

  - Triangle rule.

- *Parallelogram law*



- *Triangle rule*



Note :

- If two vectors are ***collinear***, the parallelogram law reduces to an algebraic or scalar addition

$$R = A + B$$

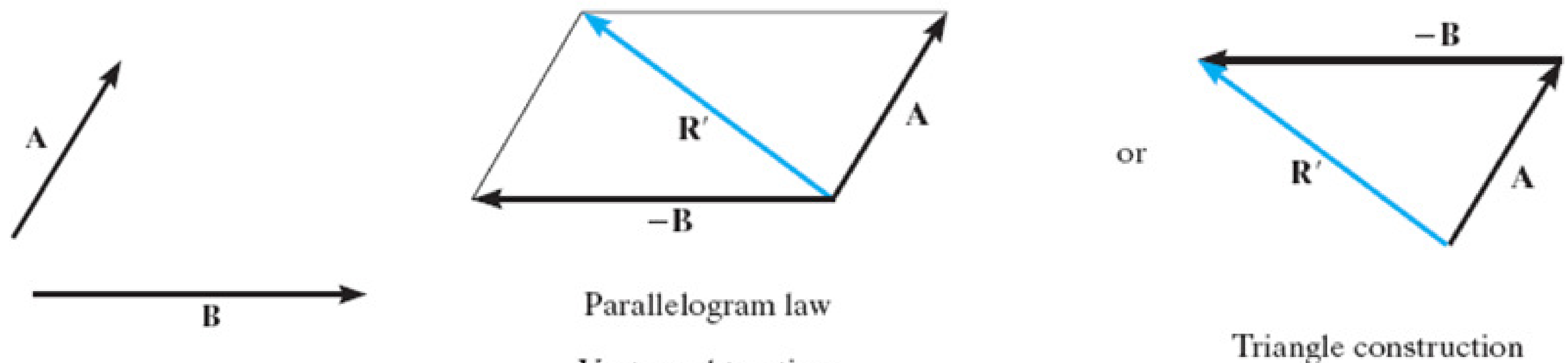
Addition of collinear vectors

III. Vector Subtraction

- A special case of addition

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

- Rules of vector addition applies.

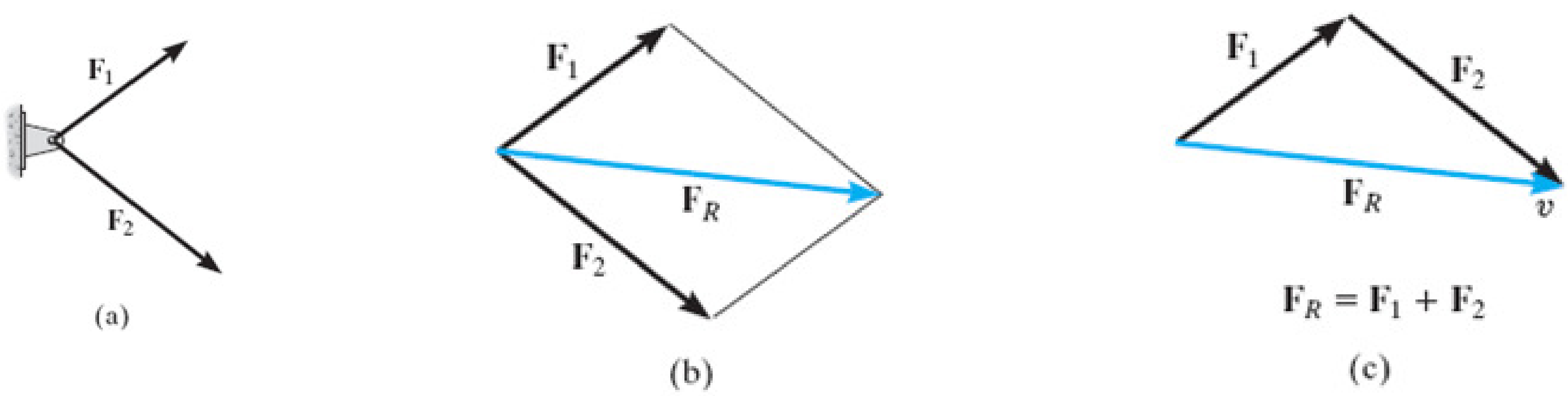


Vector subtraction

2.3 Vector Addition of Forces

I. Finding a Resultant Force

- The method of vector addition is used to find the resultant force $\mathbf{F}_R$ of adding two forces $\mathbf{F}_1 + \mathbf{F}_2$.

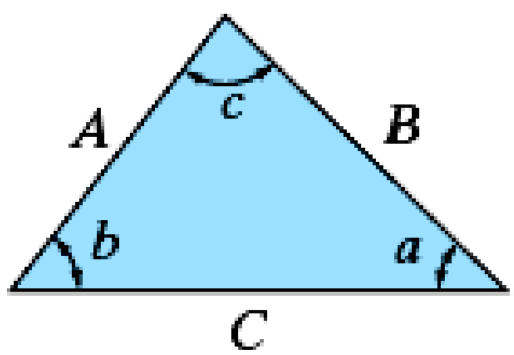


Parallelogram law

Triangle rule

- Magnitude of the resultant force can be determined by the ***law of cosines***.

- Direction of the resultant force can be determined by the ***law of sines***



Cosine law:

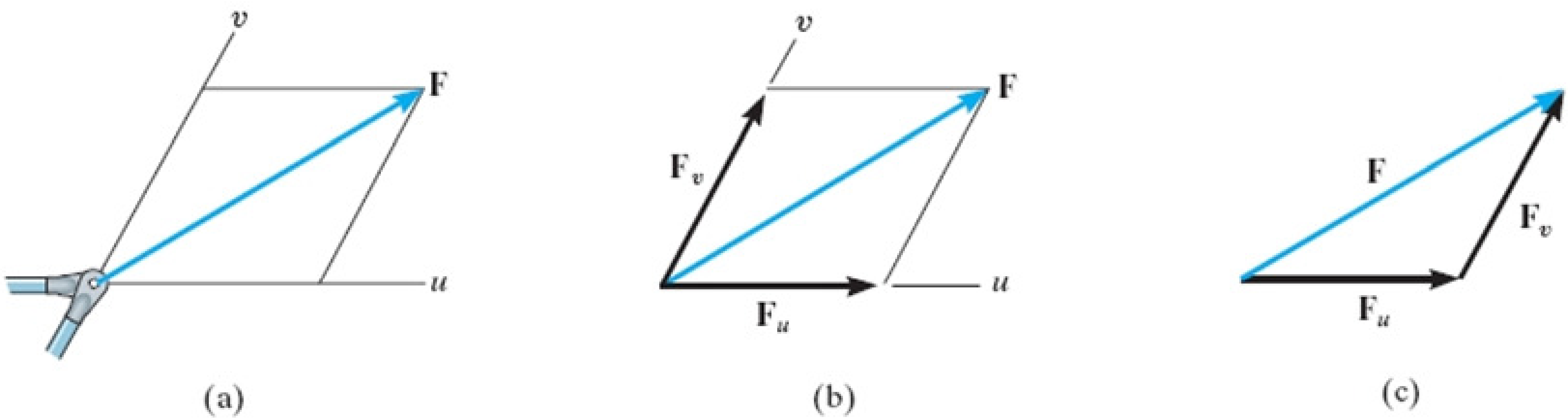
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

II. Finding the Components of a Force

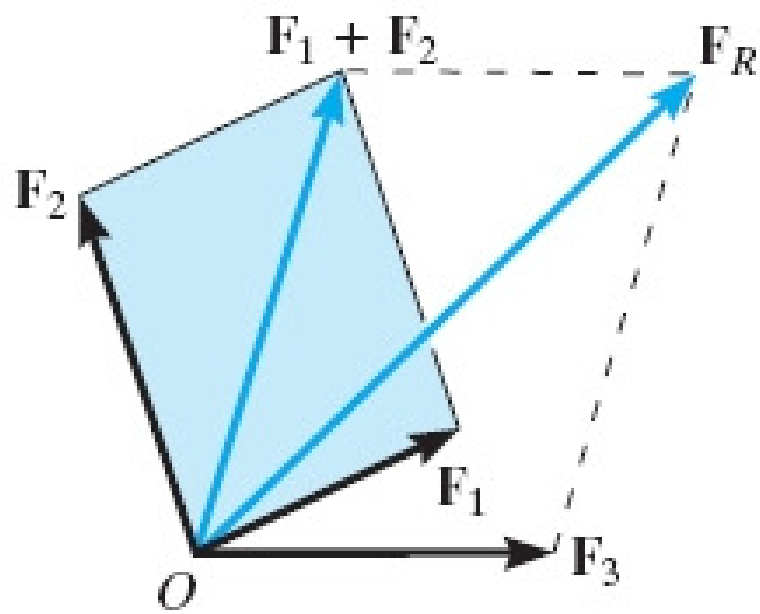
- If a force **F** is to be resolved into components along two axes *u* & *v*, a parallelogram is constructed.



- The sides of the parallelogram represent the components $\boldsymbol{F}_u$ and $\boldsymbol{F}_v$.
- Magnitude of the two components can be determined by the ***law of sines***.

III. Addition of Several Forces

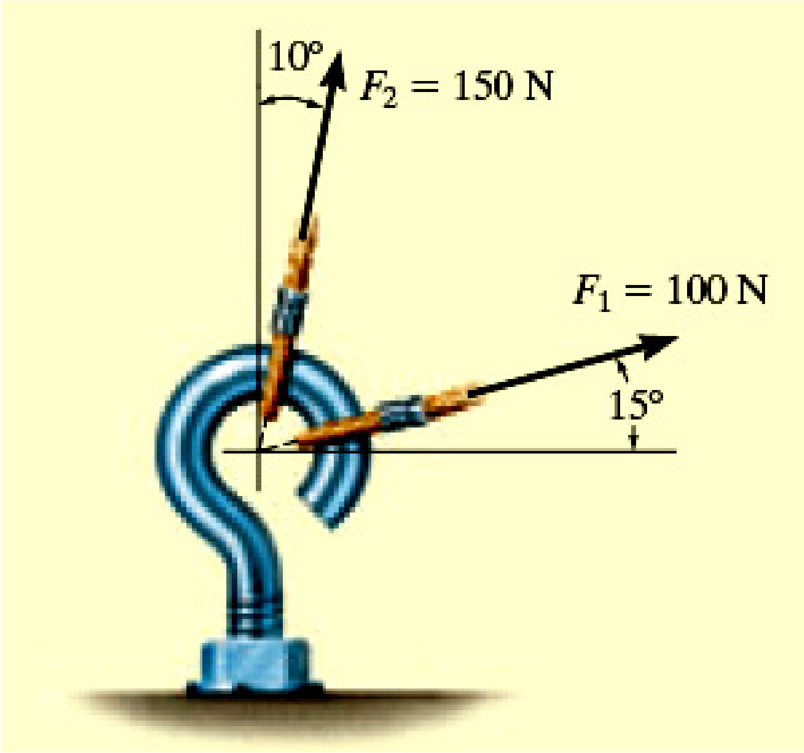
- If more than 2 forces are to be added, successive applications of parallelogram law can be carried out.



Example 2.1

Given :

The screw eye is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$.

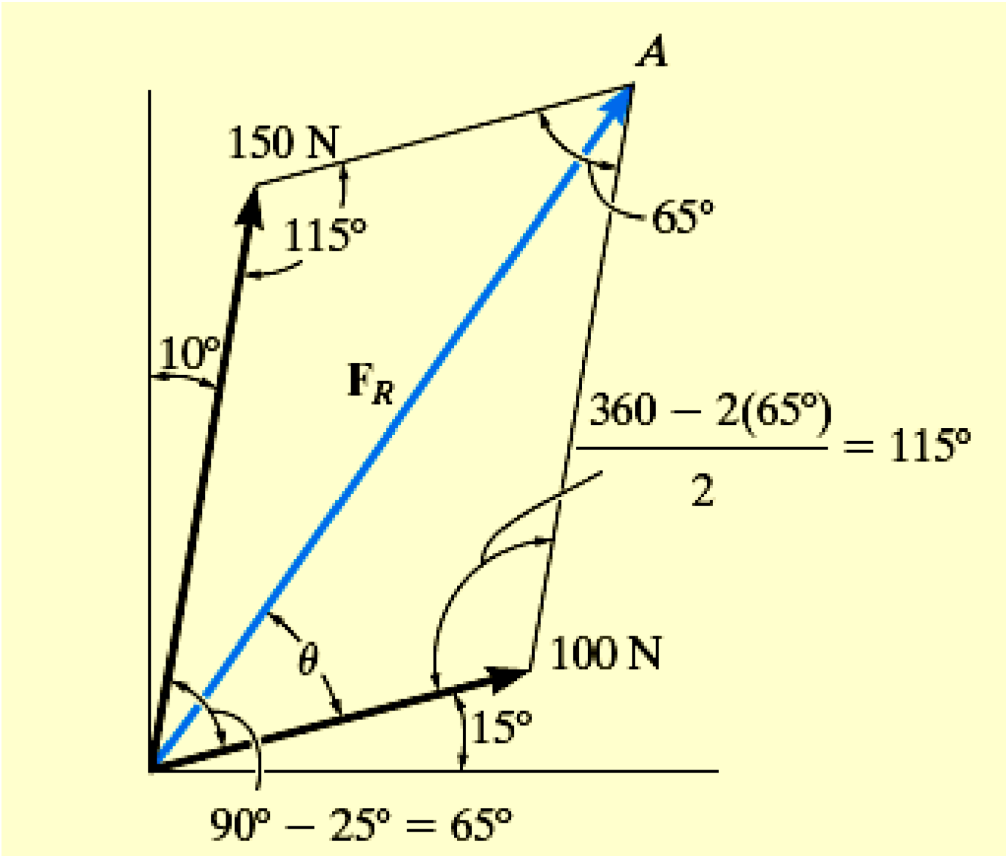


Find :

Determine the magnitude and direction of the resultant force.

Solution

- Construct a parallelogram with sides parallel to the given forces.

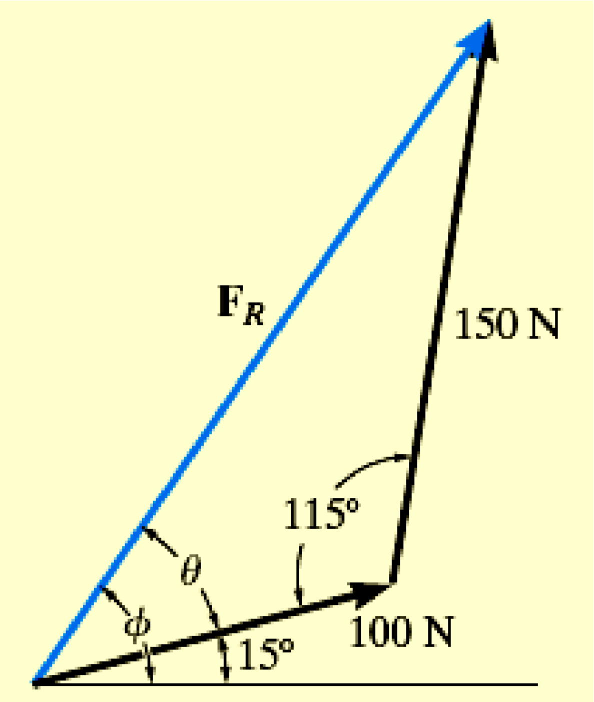


Unknown: magnitude of $\mathbf{F}_R$ and angle θ

Magnitude of resultant force:

Law of Cosines

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{N})^2 - 2(100\text{ N})(150\text{ N})\cos 115^\circ}$$
$$= \sqrt{10000 + 22500 - 30000(-0.4226)}$$
$$= 212.6N = 213N$$



Direction resultant force:

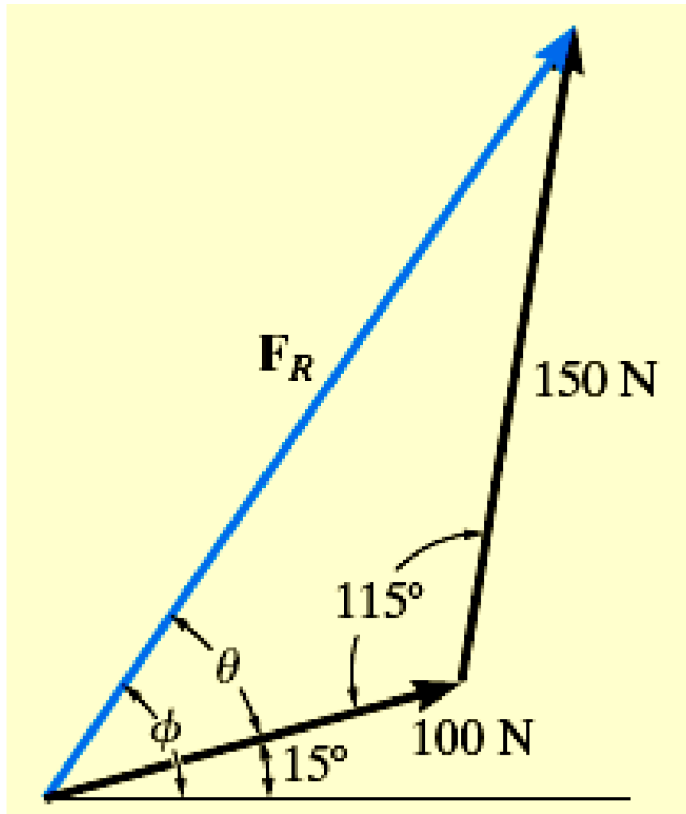
Law of Sines

$$\frac{150\text{N}}{\sin\theta} = \frac{212.6\text{N}}{\sin 115^\circ}$$
$$\sin\theta = \frac{150\text{N}}{212.6\text{N}}\sin 115^\circ \quad \Rightarrow \quad \theta = 39.8^\circ$$

Direction of $\mathbf{F}_R$ measured from the horizontal is

$$\phi = 39.8^\circ + 15^\circ$$

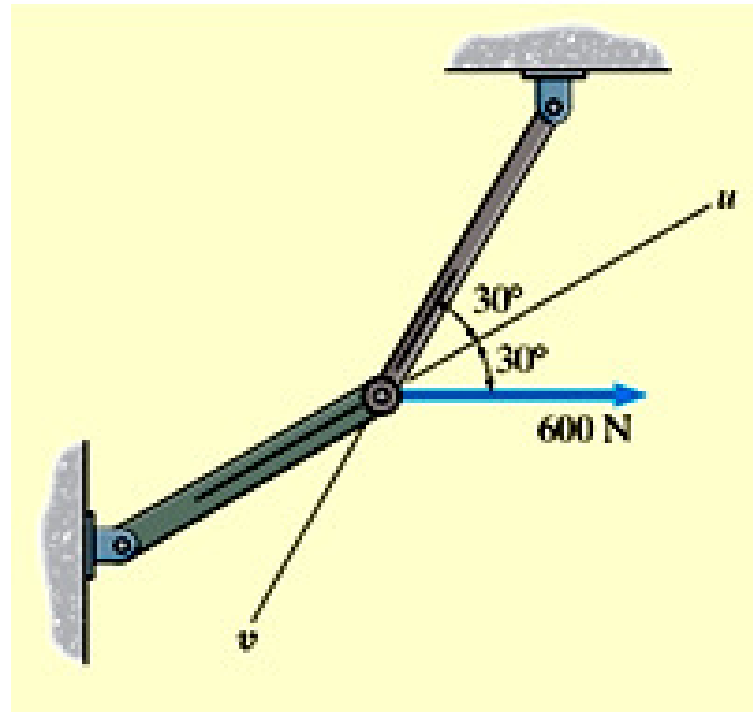
$$= 54.8^\circ$$



Example 2.2

Given :

- A horizontal 600-N force is to be resolved along the *u* and *v* axes.

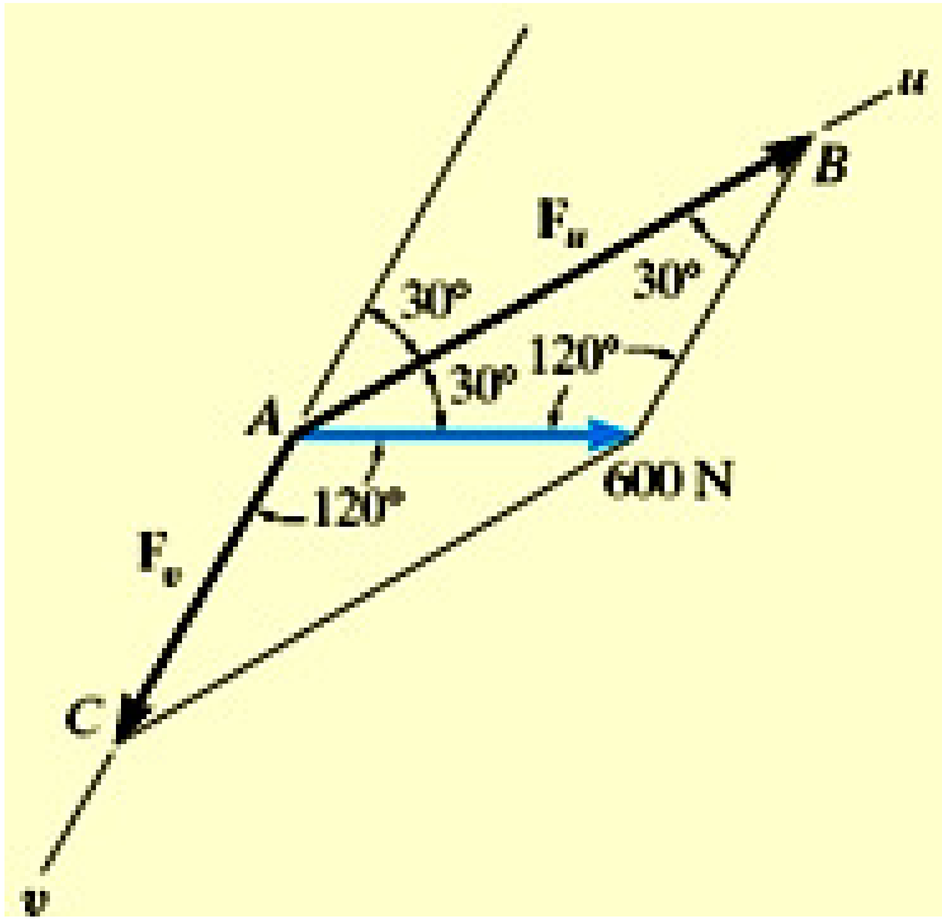


Find :

- Determine the magnitude of the resolved components.

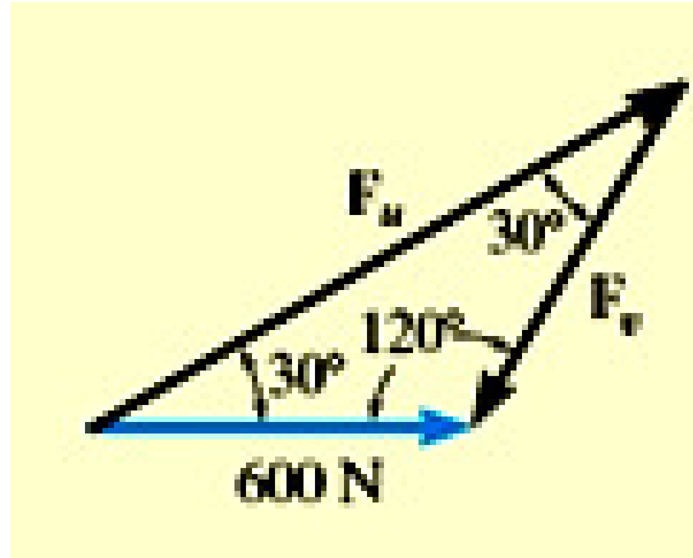
Solution

- Construct a parallelogram with sides parallel to the *u* and *v* axes.



- Consider half portion of the parallelogram.

- Applying law of sines,



$$\frac{F_u}{\sin 120^\circ} = \frac{600\ \text{N}}{\sin 30^\circ}$$

$$\Rightarrow \quad F_u = 1039\ \text{N}$$

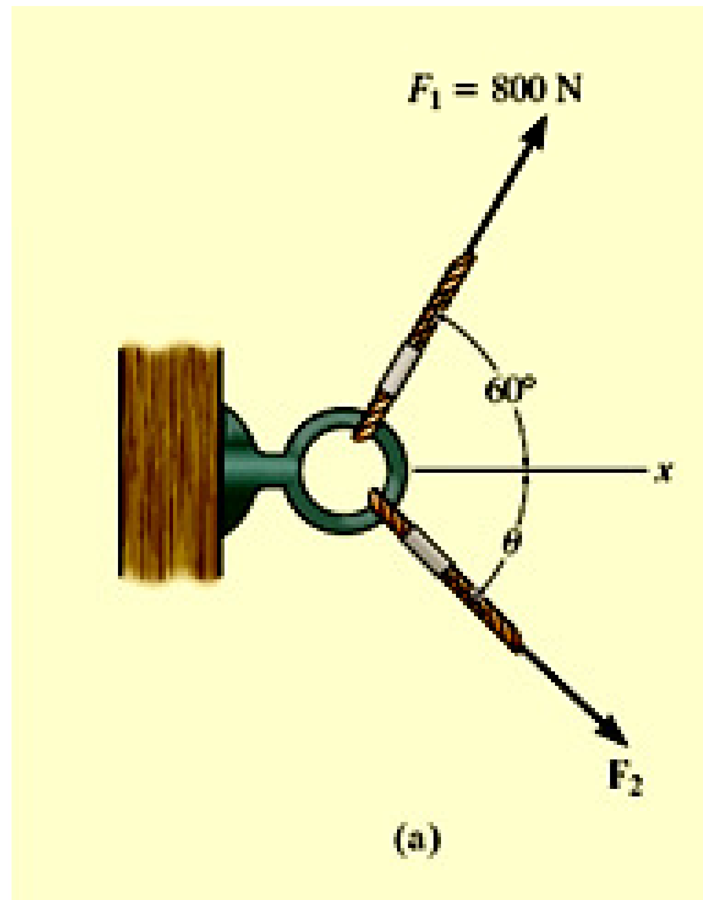
$$\frac{F_v}{\sin 30^\circ} = \frac{600\ \text{N}}{\sin 30^\circ}$$

$$\Rightarrow \quad F_v = 600\ \text{N}$$

Example 2.4

Given :

• The resultant force acting on the eyebolt is directed along the positive *x* axis.

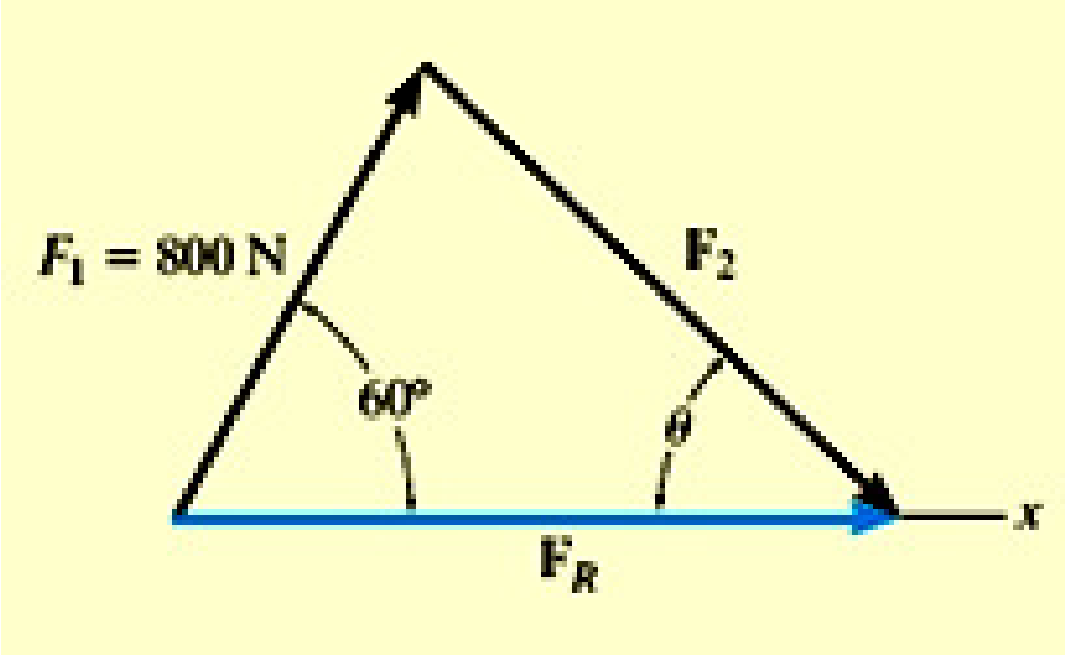


Find :

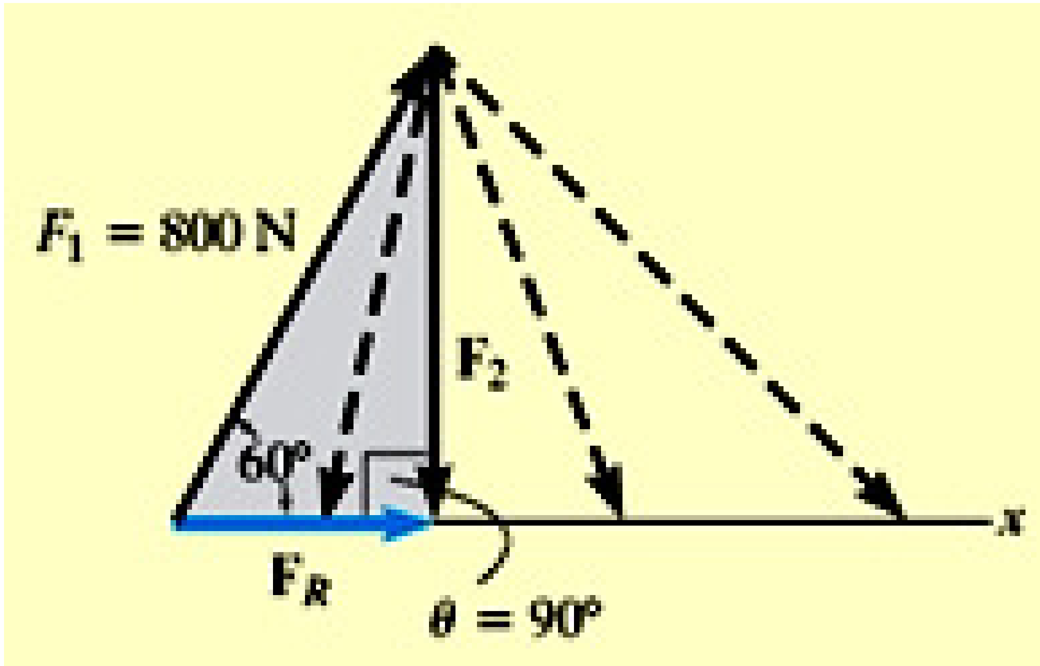
- the minimum magnitude of $\mathbf{F}_2$
- the angle θ.
- the corresponding resultant force.

Solution

- Unknown: magnitudes of $\mathbf{F}_R$ and $\mathbf{F}_2$, angle θ.
- Using the triangle rule, we have



- As shown in the figure, F_2 is minimum if $\theta = 90°$



- Therefore,

$$F_2 = (800\ \text{N}) \sin 60° = 693\ \text{N}$$

$$F_R = (800\ \text{N}) \cos 60° = 400\ \text{N}$$