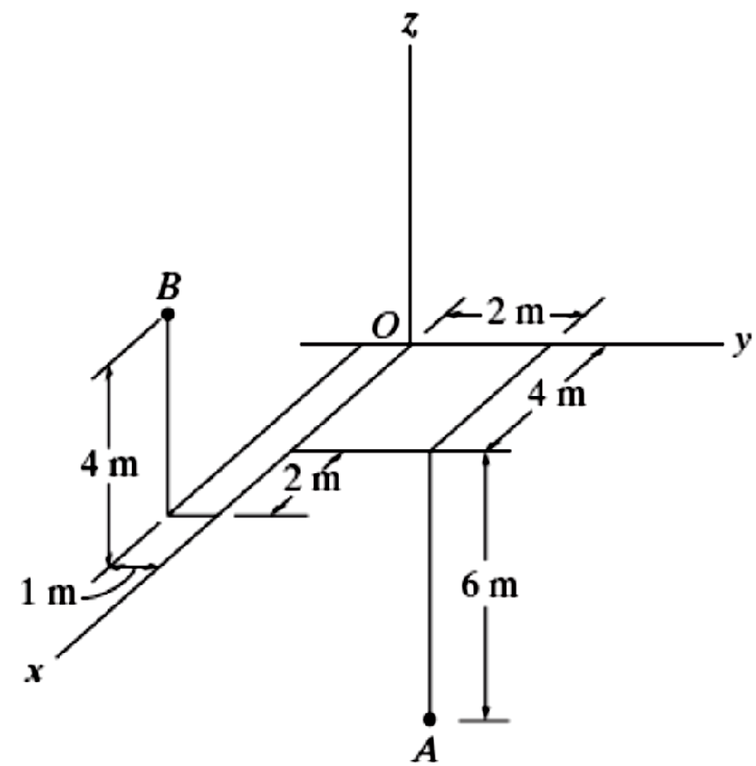
2.7 Position Vectors

❑ *x,y,z* Coordinates

- We will use a right-handed coordinate system in which the positive *z* axis points upwards, and the *x*, *y* axes lie in the horizontal plane.

- Points in space are located relative to the origin of coordinates, *O*, by successive measurements along the *x*, *y*, *z* axes.



E.g. : Coordinates of point *A* is (4 m, 2 m, – 6 m)

Coordinates of point *B* is (6 m, – 1 m, 4 m)

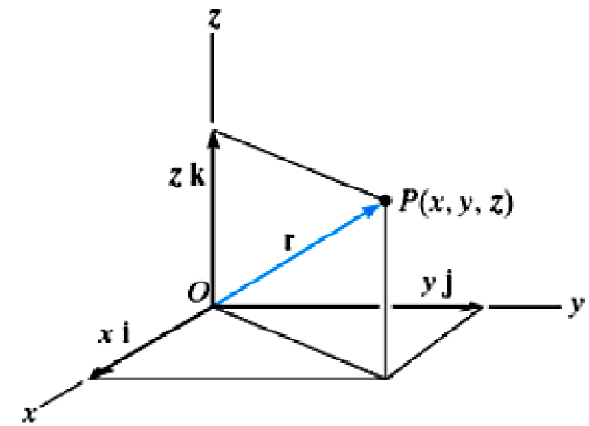
❑ Position Vector

A *position vector* **r** is defined as a fixed vector which locates a point in space relative to another point.

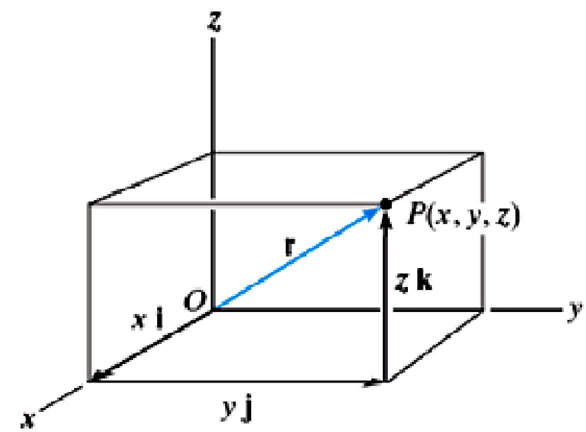
(a) Position vector of a point relative to the origin of coordinates, *O*.

- The position vector of point $P(x, y, z)$ relative to the origin of coordinates, O, is

$$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$$

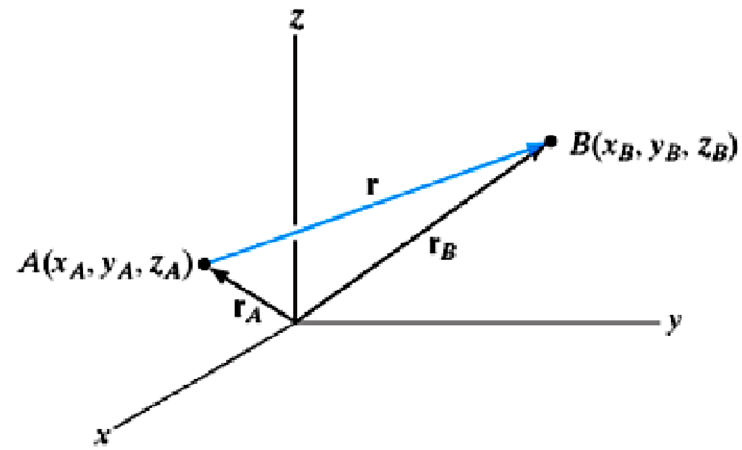


- The position vector **r** with respect to the origin of coordinates, O, is formed by the head-to-tail vector addition of its 3 components



(b) Position vector of a point relative to another point.

- Let $\mathbf{r}_A$ = position vector of point A relative to the origin.
  $\mathbf{r}_B$ = position vector of point B relative to the origin.
  $\mathbf{r}$ (or $\mathbf{r}_{AB}$) = position vector of point B relative to point A



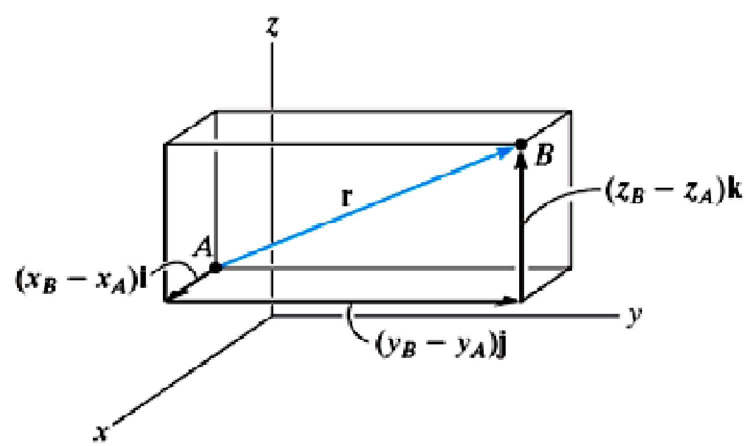
- From the figure,

$$\mathbf{r}_A + \mathbf{r} = \mathbf{r}_B$$

- Therefore,

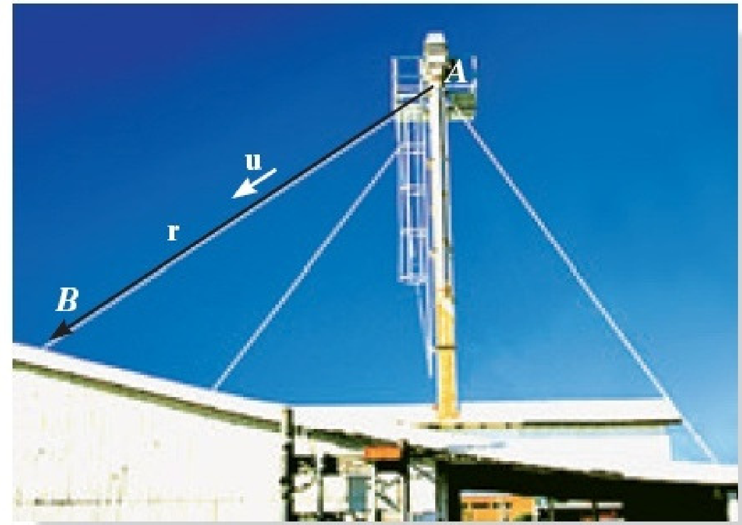
$$\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$$

$$= (x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k}) - (x_A\mathbf{i} + y_A\mathbf{j} + z_A\mathbf{k})$$



$$\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$

(b) Application



Length and direction of cable *AB* can be found as follows.

- Determine the coordinates of *A* and *B* in a Cartesian coordinate system.

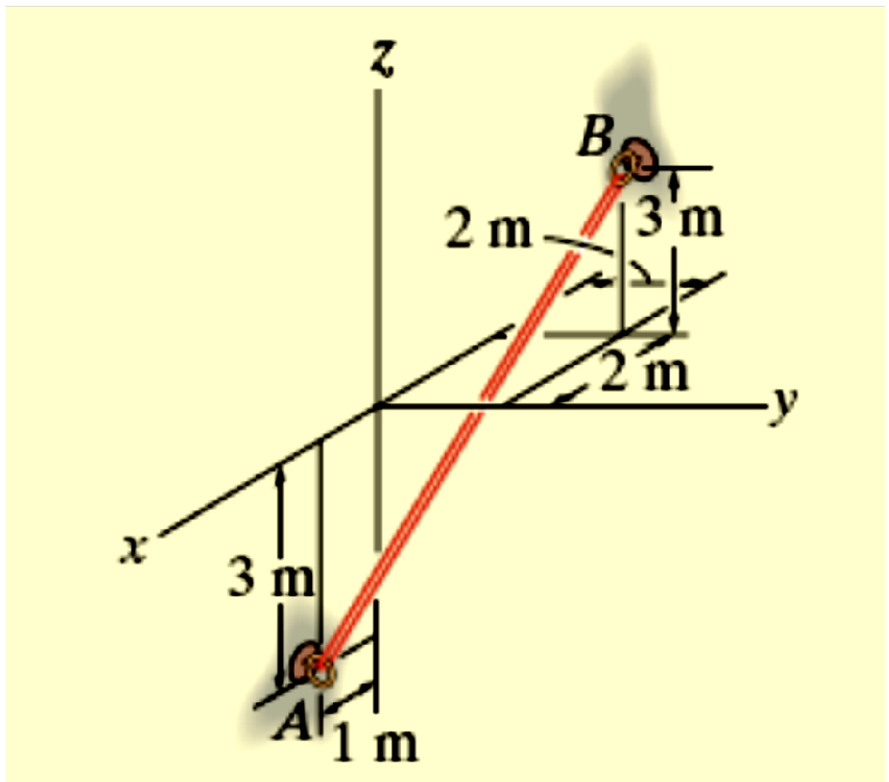
- Determine the position vector **r** of point *B* relative to point *A*.
  - *Magnitude of* **r** *represents the length of the cable.*

- Determine the unit vector **u**= **r**/*r*
  - *The components of the unit vector* **u** *give the coordinate direction angles α, β and γ.*

Example 2.12

Given :

An elastic rubber band is attached to points *A* and *B* as shown.



Find :

Determine its length and its direction measured from *A* toward *B*.

Solution

- Coordinates : A (1 m, 0, –3 m)

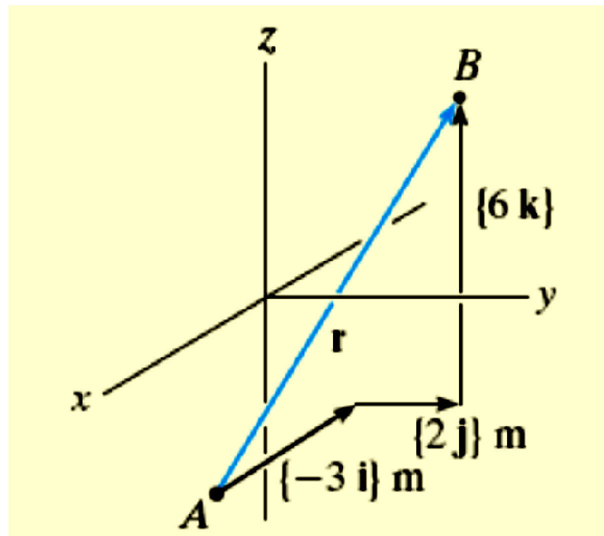
  B (– 2 m, 2m , 3 m)

- Position vector:

$$\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$

$$\mathbf{r} = \{[-2 - 1]\mathbf{i} + [2 - 0]\mathbf{j} + [3 - (-3)]\mathbf{k}\}\text{ m}$$

$$= \{-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}\text{ m}$$



- Length of the rubber band:

$$r = \sqrt{(-3)^2 + (2)^2 + (6)^2} = 7\text{ m}$$

- Unit vector in the direction of **r**

$$\mathbf{u} = \frac{\mathbf{r}}{r} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

- The components of the unit vector **u** give the coordinate direction angles.

$$\alpha = \cos^{-1}\left(-\frac{3}{7}\right) = 115^{\circ}$$

$$\beta = \cos^{-1}\left(\frac{2}{7}\right) = 73.4^{\circ}$$

$$\gamma = \cos^{-1}\left(\frac{6}{7}\right) = 31.0^{\circ}$$