Chapter 3

# Equilibrium of a Particle

# Chapter Objectives

- Concept of the free-body diagram for a particle
- Solve particle equilibrium problems using the equations of equilibrium

# Chapter Outline

1. Condition for the Equilibrium of a Particle
2. The Free-Body Diagram
3. Coplanar Systems
4. Three-Dimensional Force Systems

# 3.1 Condition for the Equilibrium of a Particle

- A particle is said to be at equilibrium if
  - it remains at rest if originally at rest,
  - it is moving at a constant velocity if originally in motion.

- An object at rest is often said to be in "***static equilibrium***."

- To maintain equilibrium, Newton's first law of motion must be satisfied.

$$\sum \mathbf{F} = 0 \qquad (1)$$

where $\sum \mathbf{F}$ is the vector *sum of all the forces* acting on the particle.

- Consider Newton's second law of motion

$$\sum \mathbf{F} = m\,\mathbf{a} \tag{2}$$

- When the force fulfill Newton's first law of motion, Eq.(2) becomes

$$m\mathbf{a} = 0$$

$$\Rightarrow \quad \mathbf{a} = 0$$

Therefore, the particle is moving in constant velocity or at rest.

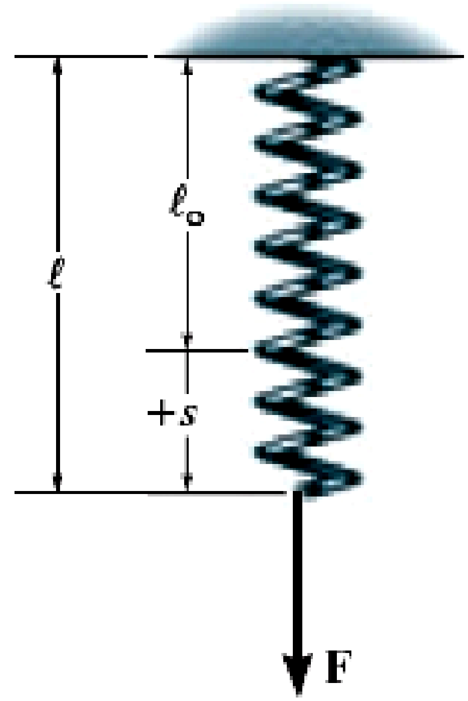
## ❑ Common Connections in Particle Equilibrium Problem

### ❖ Springs

- Let $l_o$ = undeformed length of the spring

  $l$ = length of the spring when a force **F** is acting on it

- If the spring is ***linearly elastic***, the magnitude of the force is related to the change in length of the spring as follows




$$F = ks$$

where $s = l - l_o$ is measured from the unloaded position

$k$ = spring constant or stiffness

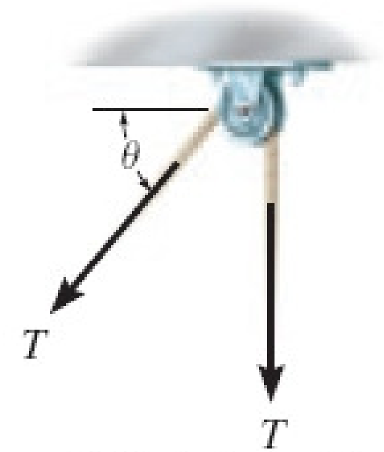
(a characteristic that defines the elasticity of the spring)

- That is, the change in length of a *linearly elastic spring* is proportional to the force **F** acting on it.

- If **F** pulls on the spring, the spring undergoes elongation. Therefore, $s$ is positive.

- If **F** pushes on the spring, the spring undergoes compression. Therefore, $s$ is negative.

## ❖ Cables and Pulleys




- Assumptions:
  - The weight of all cables are negligible.
  - The pulley is frictionless.
  - The cables (or cords) cannot stretch.
  - The cables can support only a tension force.

- Tension force always acts in the direction of the cable.

- Tension force developed in a continuous cable which passes over a frictionless pulley must have a constant magnitude to keep the cable in equilibrium.

  - Hence, for any angle $\theta$, shown in the figure, the cable is subjected to a constant tension $T$ throughout its length.

# 3.2 The Free-Body Diagram

- A drawing that shows the particle with ***all*** the forces that act on it is called a *free-body diagram* (FBD).

- Procedure for Drawing a FBD

1. Imagine the particle to be isolated or cut "free" from its surroundings by drawing its outlined shape.

2. Show all the forces that act on the particle.

   - Active forces: particle in motion
   - Reactive forces: constraints or supports that prevent motion

3. Identify each force.

   - Label known forces with proper magnitude and direction.
   - Represent magnitude and directions of unknown forces with letters.

# Examples

**(a) A bucket held by a cable**

- To find the force in the cable, we need to draw a free-body diagram of the bucket.

- For equilibrium,

$$\sum \mathbf{F} = 0$$

$$\mathbf{T} - \mathbf{W} = 0$$

$$\mathbf{T} = \mathbf{W}$$

## (b) A spool suspended from the crane boon.

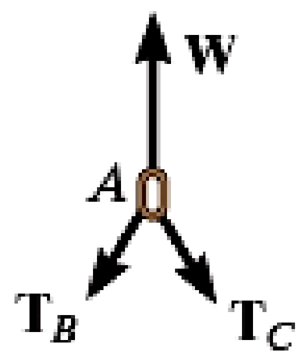

- To find the forces in cables *AB* & *AC*, draw the free-body diagram of the ring at *A*.

- For equilibrium,

$$\sum \mathbf{F} = 0$$

$$\mathbf{W} - \mathbf{T}_B - \mathbf{T}_C = 0$$

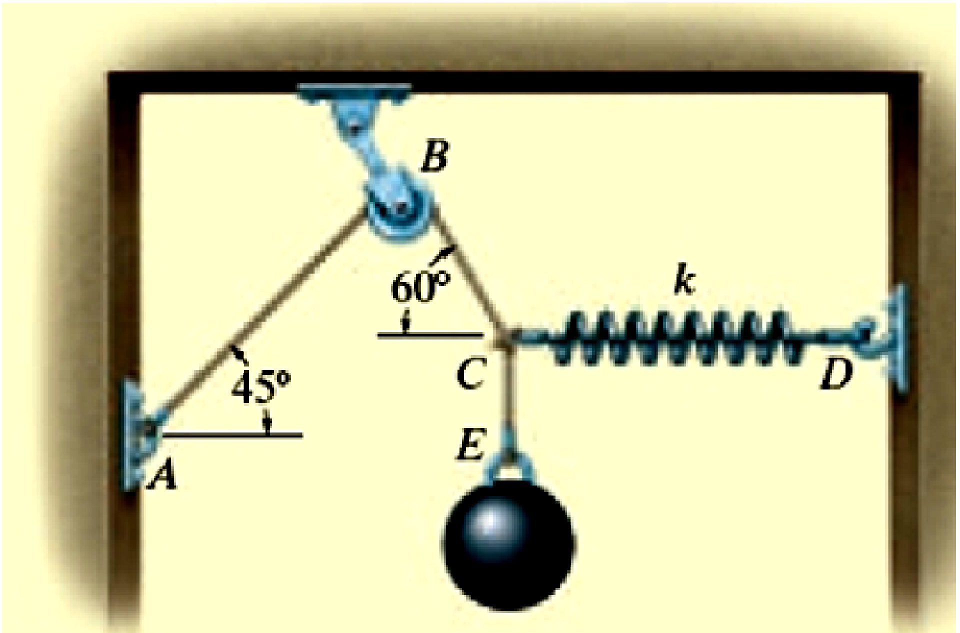
$$\mathbf{T}_B + \mathbf{T}_C = \mathbf{W}$$



# Example 3.1

## Given :

The sphere has a mass of 6kg and is supported as shown.



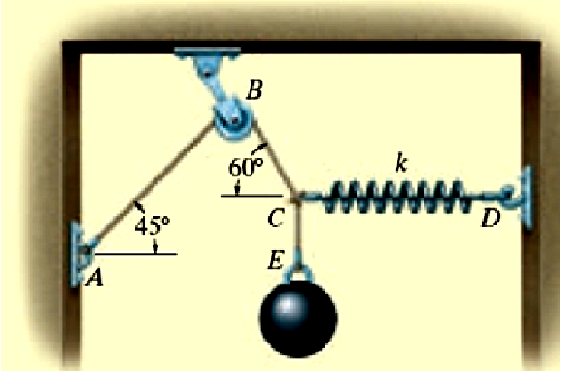

## Find :

Draw a free-body diagram of the sphere, the cord *CE* and the knot at *C*.

# Solution

## Free-body diagram of the Sphere

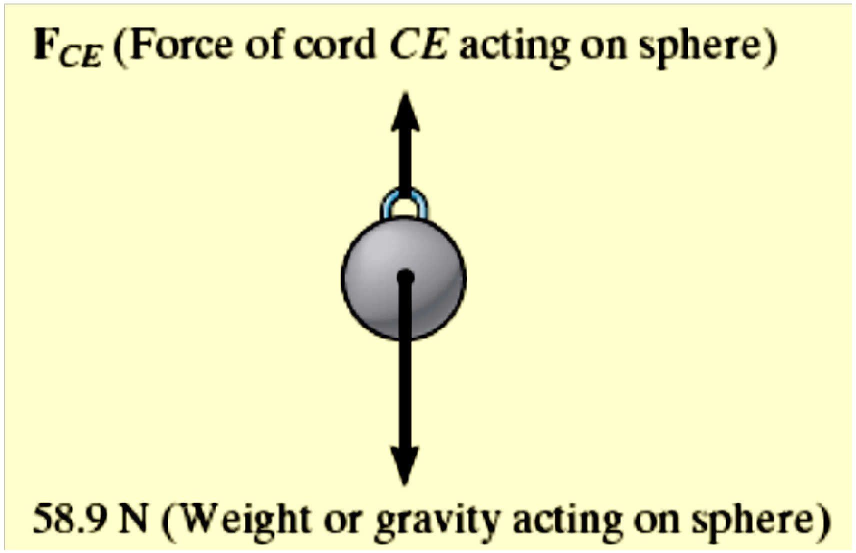
- Two forces acting on the sphere:



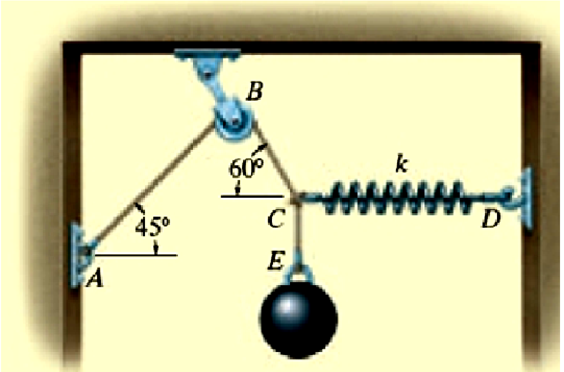

  - Weight of the sphere,

    $W = (6\text{kg})\ (9.81\text{m/s}^2) = 58.9\ \text{N}$

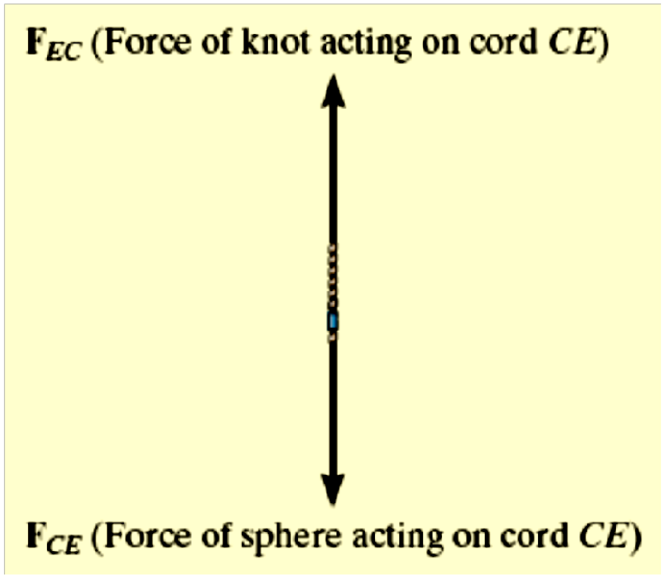
  - Force of cord ($\mathbf{F}_{CE}$).



## Free-body diagram of Cord *CE*




- Two forces acting on the cord *CE*:
  - Force of the sphere ($\mathbf{F}_{CE}$ )
  - Force of the knot ($\mathbf{F}_{EC)}$.
- $\mathbf{F}_{CE}$ on the cord is equal but opposite to $\mathbf{F}_{CE}$ on the sphere (Newton's 3rd Law).

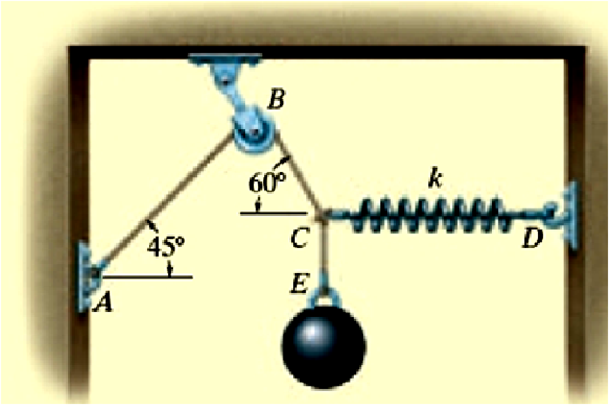


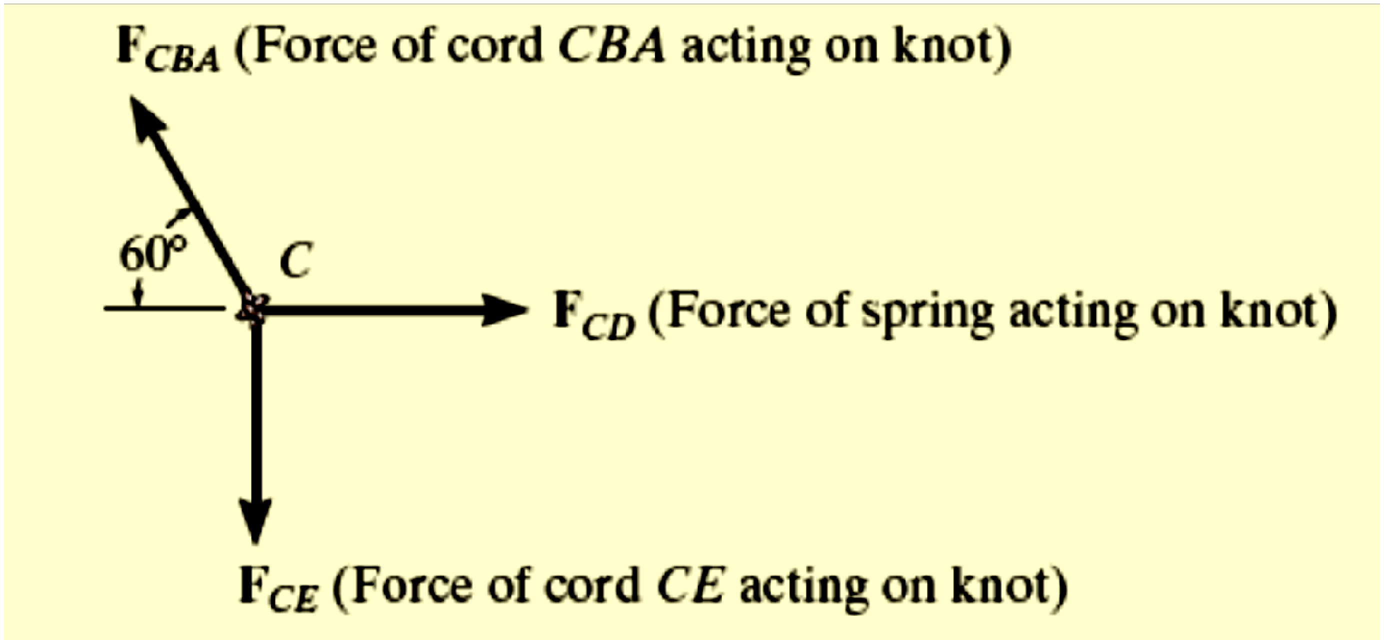

**Note:** (1) $\mathbf{F}_{CE}$ and $\mathbf{F}_{EC}$ pull on the cord and keep it in tension.

(2) For equilibrium, $\mathbf{F}_{CE} = \mathbf{F}_{EC}$

# Free-body diagram of the knot at *C*

- The knot at *C* is subjected to 3 forces.
  - Force caused by cord *CBA*
  - Force caused by cord *CE*
  - Force caused by the spring *CD*




Note : The weight of the sphere does not act directly on the knot. The cord *CE* subject the knot to this force.