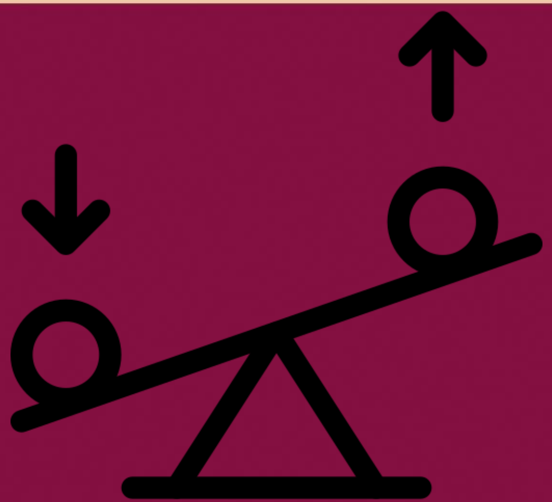
SEEM1113 ENGINEERING MECHANICS

UNIVERSITI TEKNOLOGI MALAYSIA

CH2
Force Vector



Dr. Mohd Saiful Azimi Mahmud (azimi@utm.my)
P19a-04-03-30, School of Electrical Engineering, UTM

OBJECTIVES

At the end of this lesson, you should be able to:

1. Show how to **add forces** and **resolve** them into components using **parallelogram**, **triangle** & **cartesian** methods.
2. Express **force** & **position**, determine their **magnitude** & **direction** (Cartesian Vector)
3. Determine the **angle between** two vectors & the **projection** of one vector onto another using **dot product** operation.

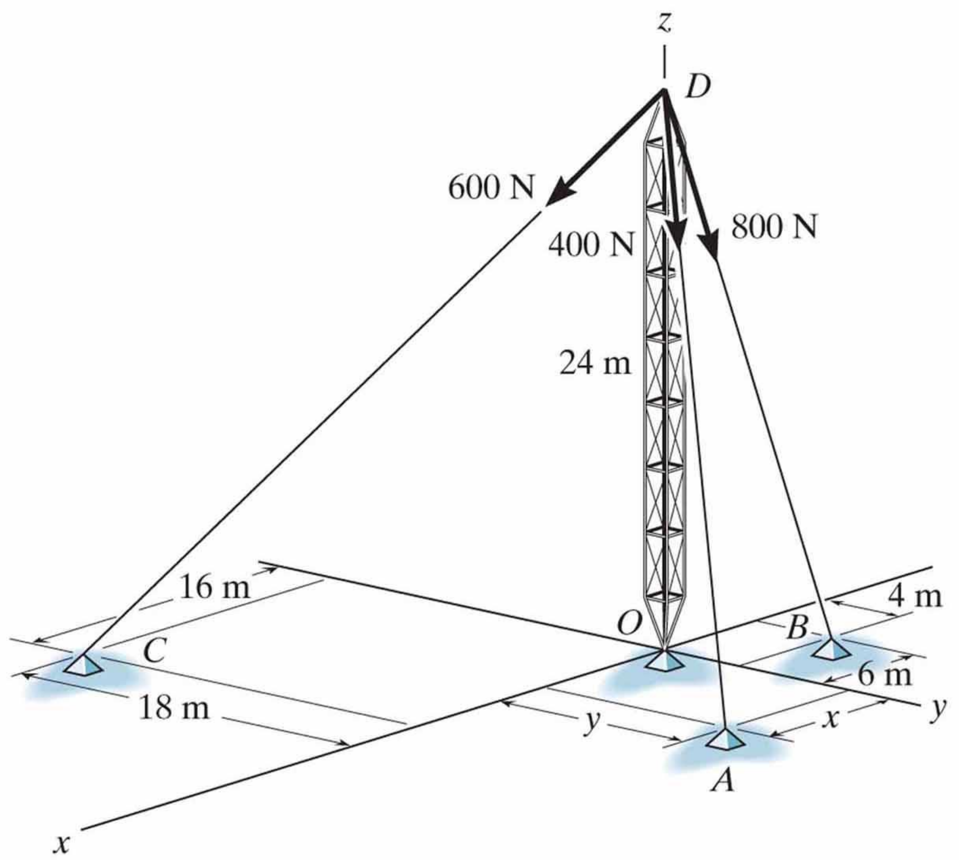
VECTOR OPERATION

GENERAL PRINCIPLES

The bridge tower is stabilized by cables that exert forces at the points of connections. In this chapter we will learn how to express these forces as Cartesian vectors and then determine the resultant force.

VECTOR OPERATION

GENERAL PRINCIPLES



The tower is held in place by three cables.

How to make sure the tower in place?

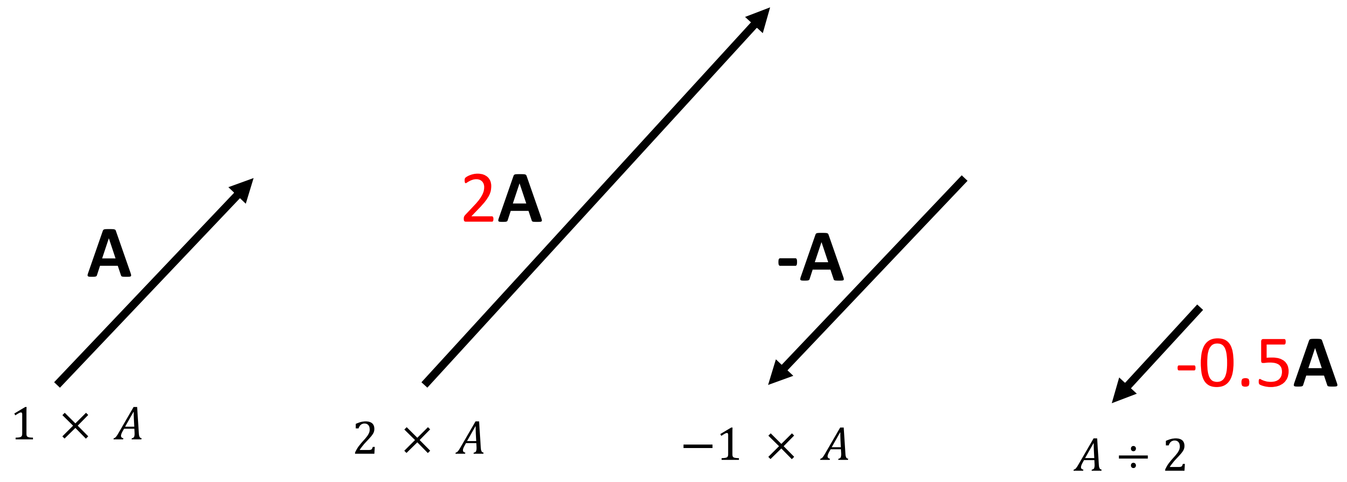
VECTOR OPERATION

SCALAR & VECTORS

	SCALARS	VECTORS
Example	Mass, Volume	Force, Velocity
Characteristics	It only has a magnitude (+ve or -ve)	It has a magnitude and direction
Addition rule	Simple arithmetic	Parallelogram law
Special notation	None	**Bold font**, <u>a line</u>, an arrow $\longrightarrow$

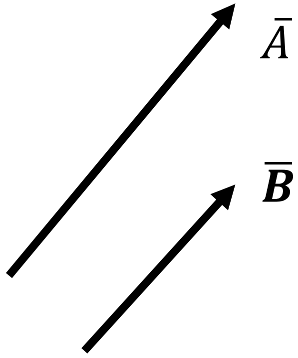
VECTOR OPERATION

MULTIPLICATION & DIVISION OF A VECTOR BY A SCALAR



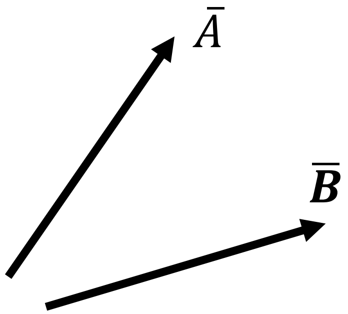
VECTOR OPERATION

VECTOR OPERATION : MAGNITUDE & DIRECTION



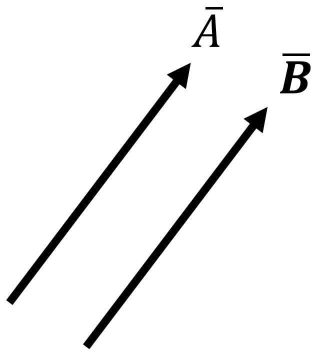
Vector A and Vector B have same direction but different magnitude

$$\bar{A} \neq \bar{B}$$



Vector A and Vector B have same magnitude but different direction

$$\bar{A} \neq \bar{B}$$



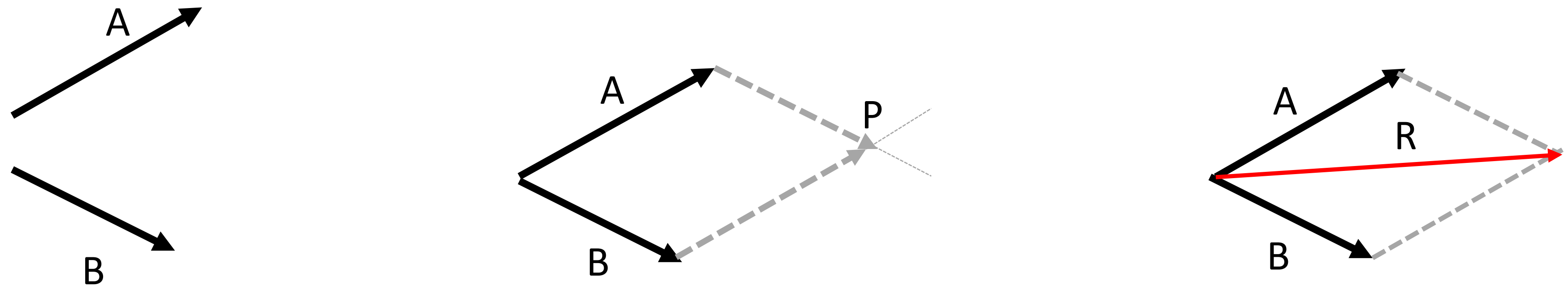
Vector A and Vector B have same direction and magnitude

$$\bar{A} = \bar{B}$$

VECTOR OPERATION

VECTOR OPERATION : ADDITION

- To perform **vector addition**, vector quantities follow the **Parallelogram law of addition**.

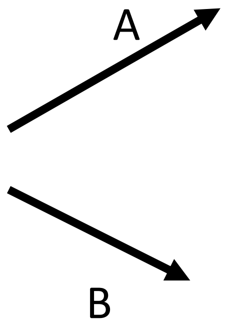


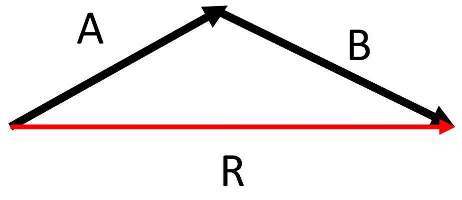
$$R = A + B$$

VECTOR OPERATION

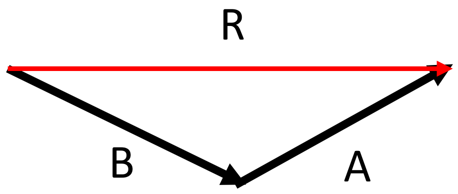
VECTOR OPERATION : ADDITION (SPECIAL CASE)

- In certain cases, **Triangle rule** is used to perform vector addition.



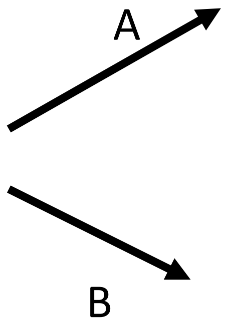


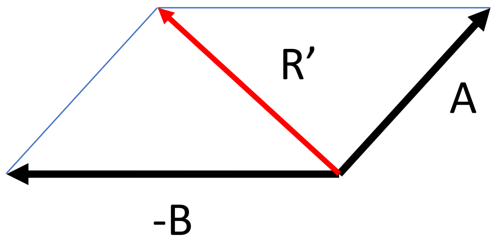
$$R = A + B$$



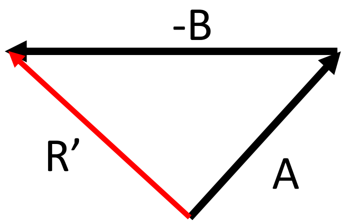
$$R = B + A$$

VECTOR OPERATION : SUBTRACTION





Parallelogram law vector subtraction

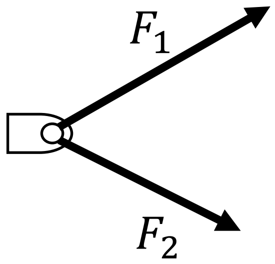


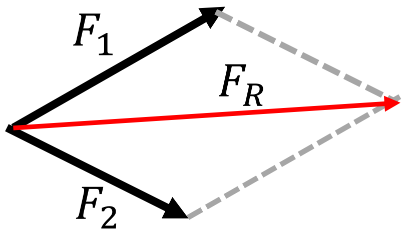
Triangle construction

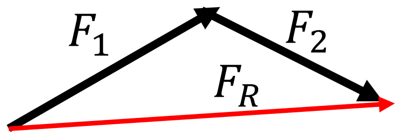
$$\boldsymbol{R}' = \boldsymbol{A} - \boldsymbol{B} = \boldsymbol{A} + (-\boldsymbol{B})$$

VECTOR OPERATION

VECTOR ADDITION OF FORCES: FINDING A RESULTANT FORCE



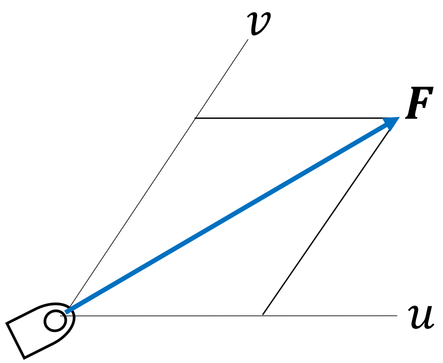


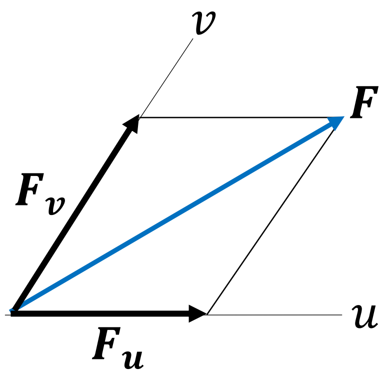


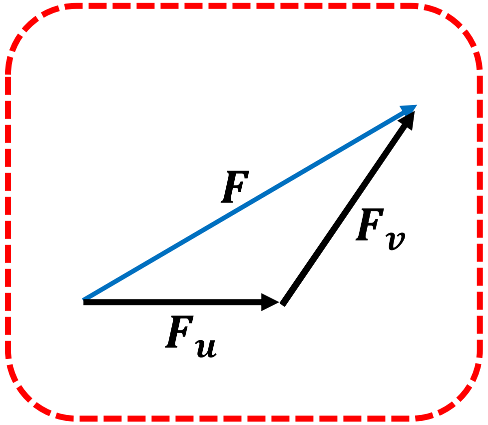
$$\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2}$$

Law of sines or law of cosines to the triangle can be used to find the magnitude and direction of the resultant force

VECTOR ADDITION OF FORCES: FINDING A COMPONENT OF A FORCE



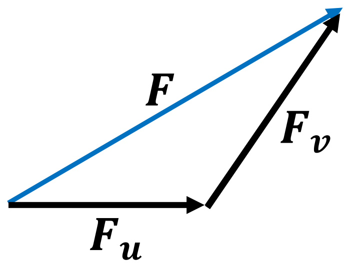


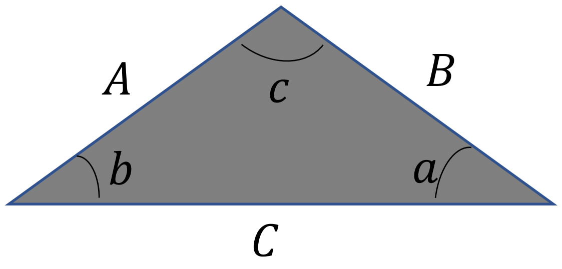


Law of sines can be applied to find the magnitude

VECTOR OPERATION

VECTOR ADDITION OF FORCES: FINDING A COMPONENT OF A FORCE



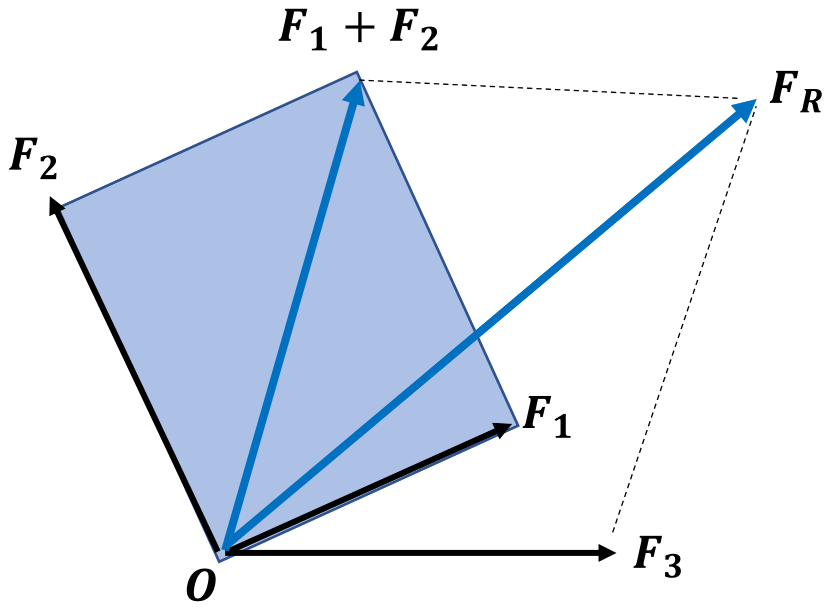


$$Cosine\ law:\ C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

$$Sine\ law:\ \frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

VECTOR OPERATION

VECTOR ADDITION OF FORCES: ADDITION OF SEVERAL FORCES



$$F_R = (F_1 + F_2) + F_3$$

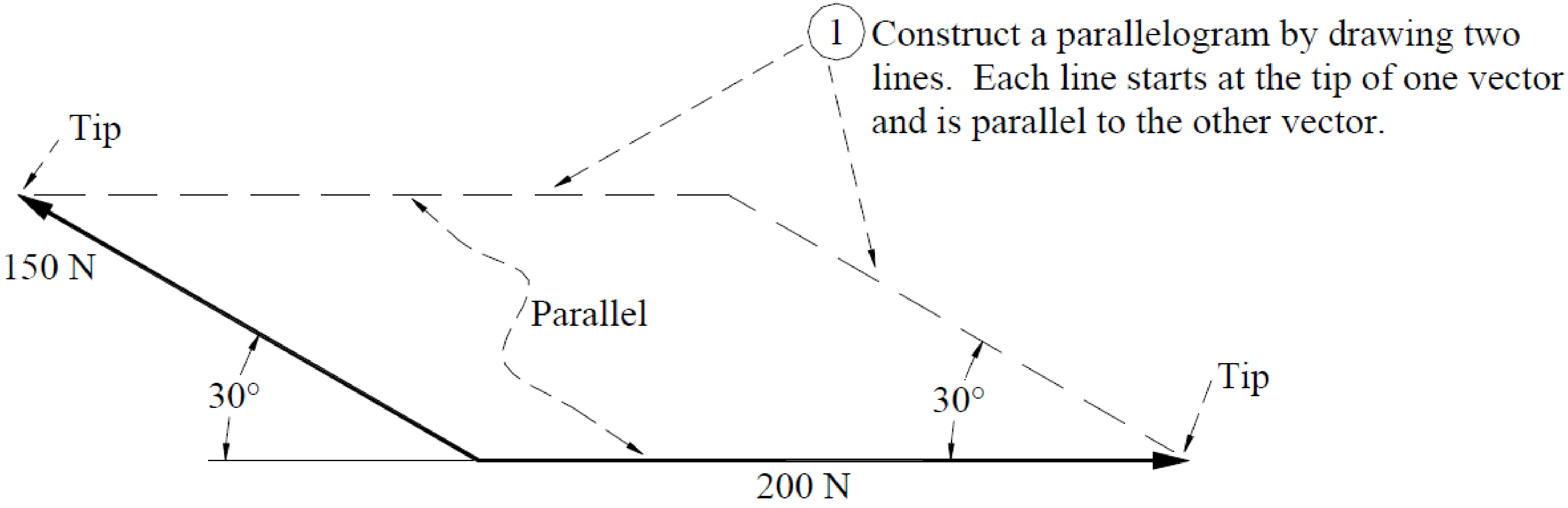
VECTOR OPERATION

STEP BY STEP EXAMPLE

Determine the magnitude and direction of resultant of the forces shown.

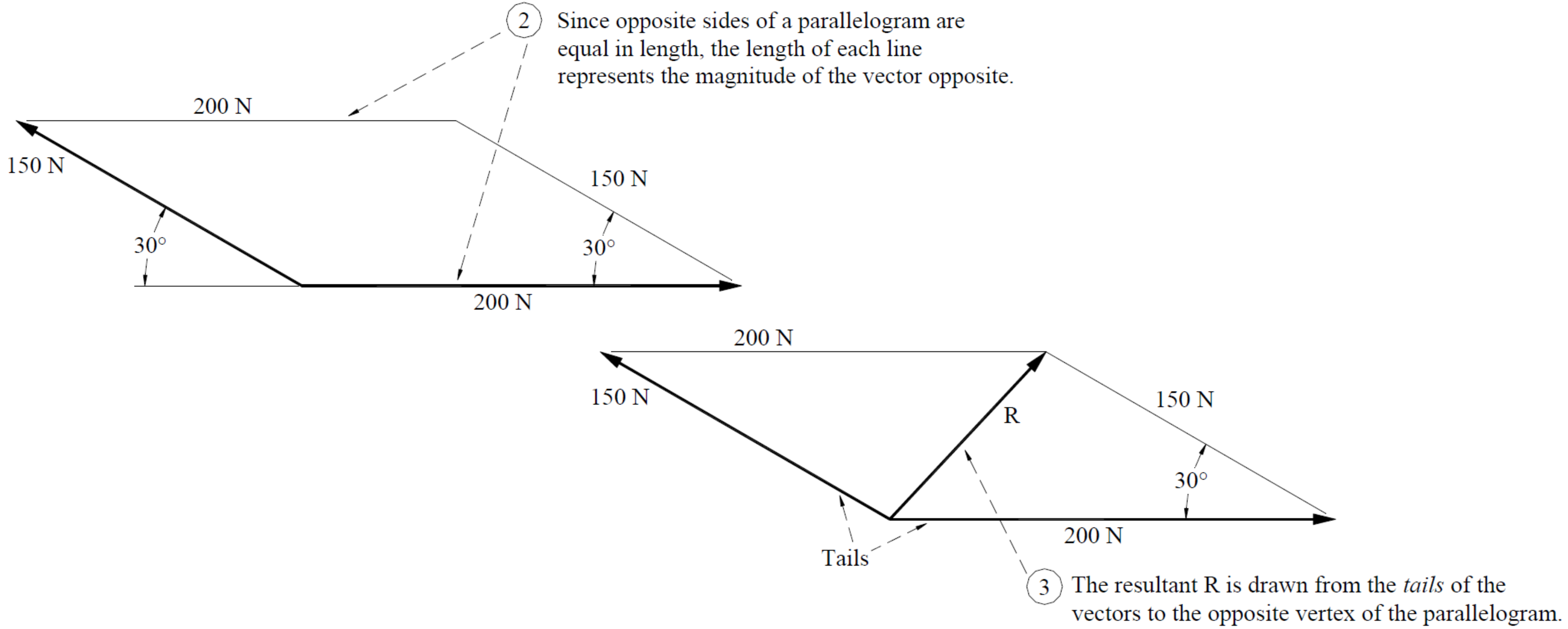
VECTOR OPERATION

STEP BY STEP EXAMPLE



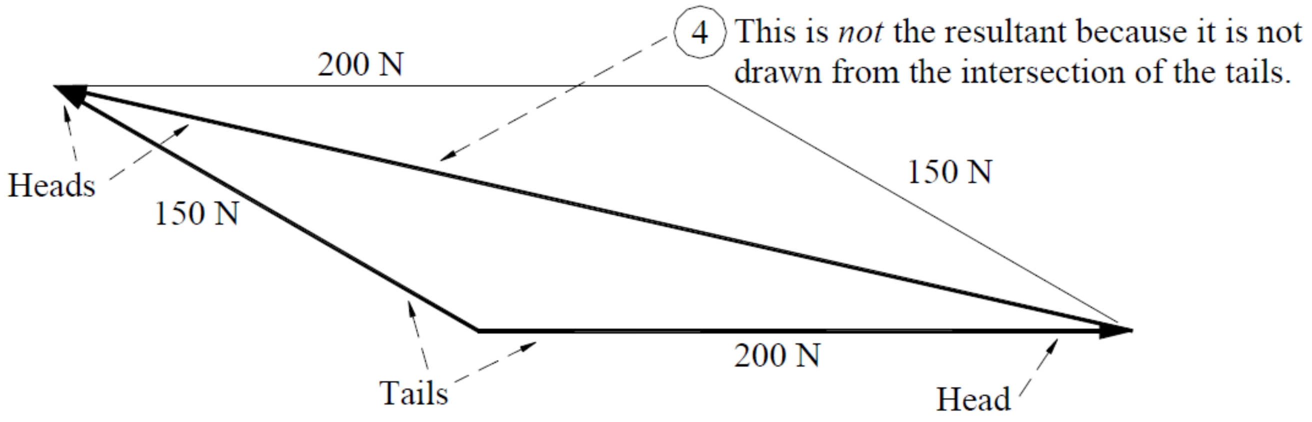
VECTOR OPERATION

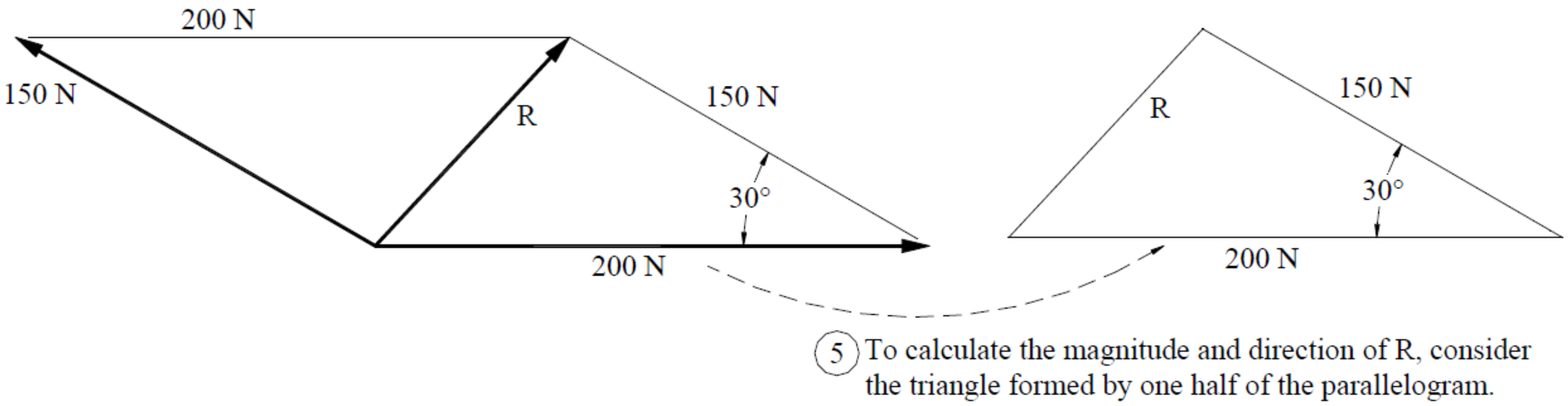
STEP BY STEP EXAMPLE



VECTOR OPERATION

STEP BY STEP EXAMPLE

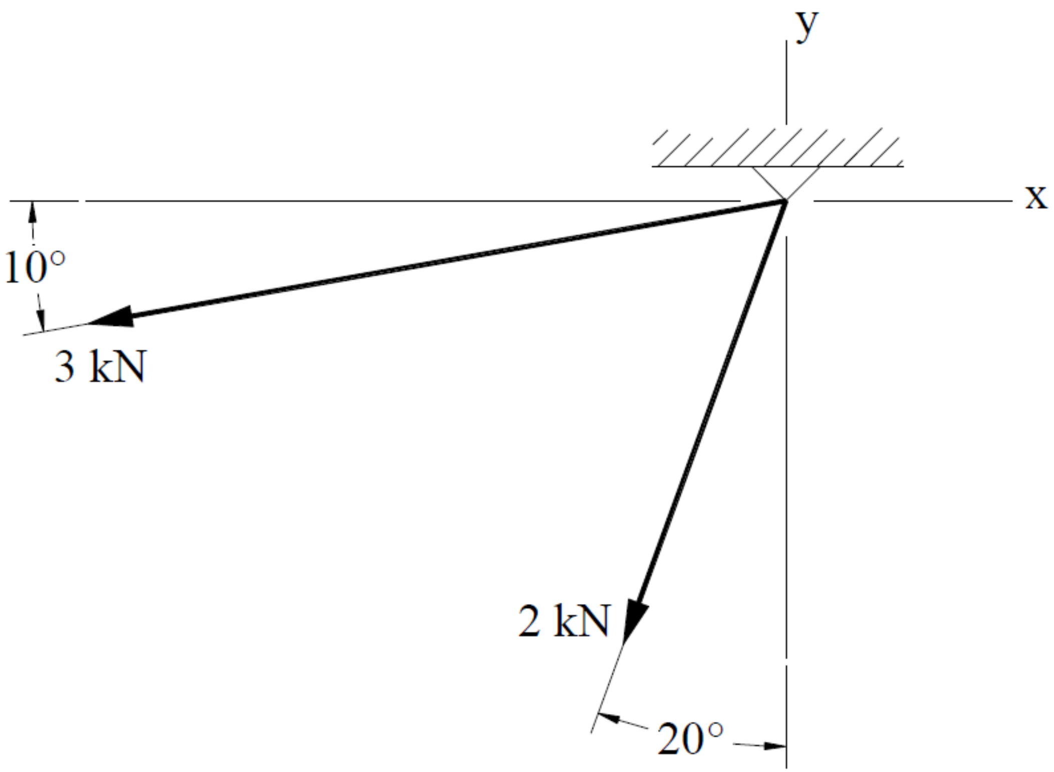




VECTOR OPERATION

Example 1

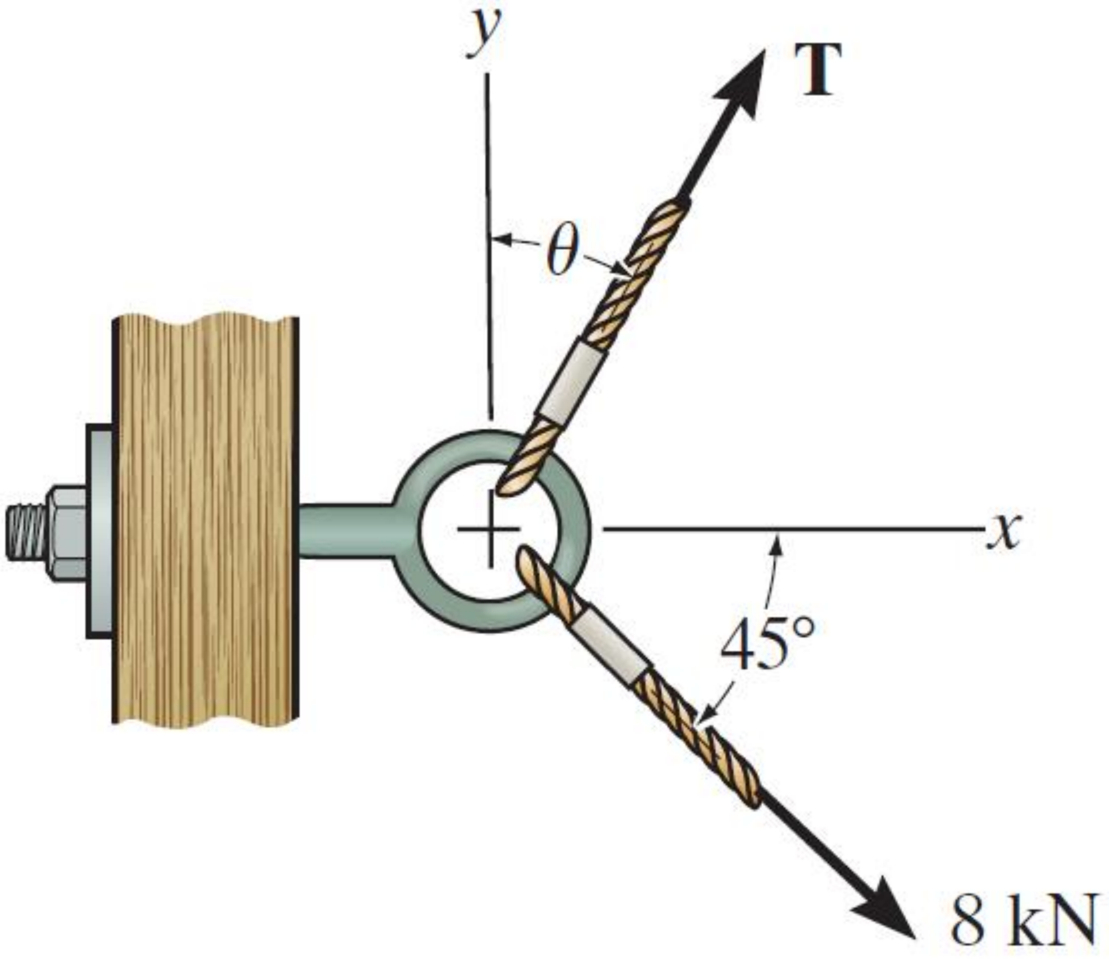
Find the magnitude and direction of resultant of the forces shown below:



VECTOR OPERATION

Example 2

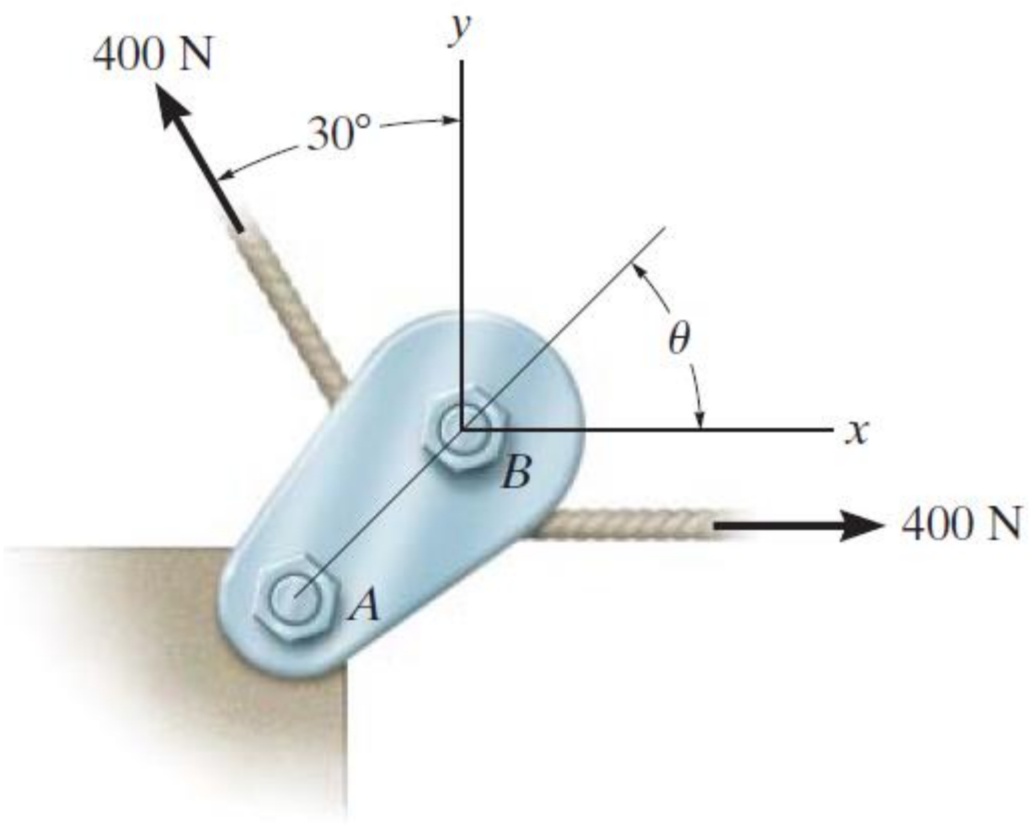
If $\theta = 30°$ and $T = 6\ kN$, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x-axis



VECTOR OPERATION

Example 3

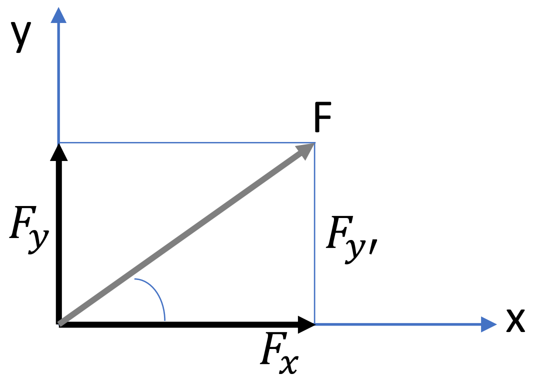
If the tension of the cable is 400 N, determine the magnitude and direction of the resultant force acting on the pulley. This angle is the same angle θ from line AB on the tailboard block.



VECTOR OPERATION

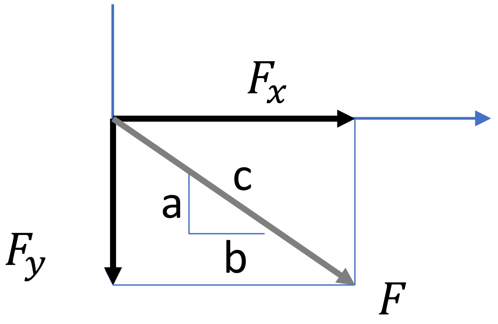
ADDITION OF A SYSTEM OF COPLANAR FORCES : SCALAR NOTATION

- Rectangular components : When force is resolved into two components along the x and y axes.
- Can be represent using scalar or Cartesian vector



Paralellogram law : $F = F_x + F_{y'}$

$F_x = F\cos\theta \; and \; F_y = F\sin\theta$



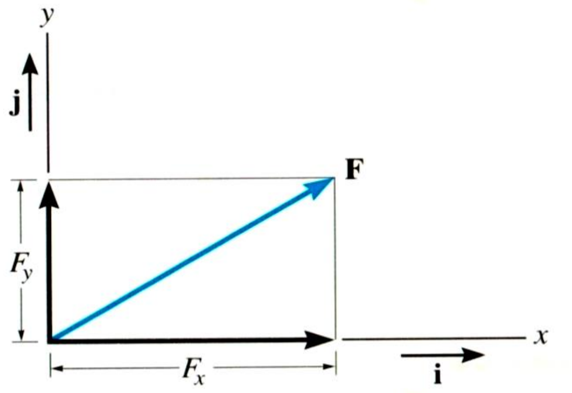
using small slope triangle

$$F_x = (b/c)F$$

$$F_y = -(a/c)F$$

VECTOR OPERATION

ADDITION OF A SYSTEM OF COPLANAR FORCES : CARTESIAN VECTOR NOTATION

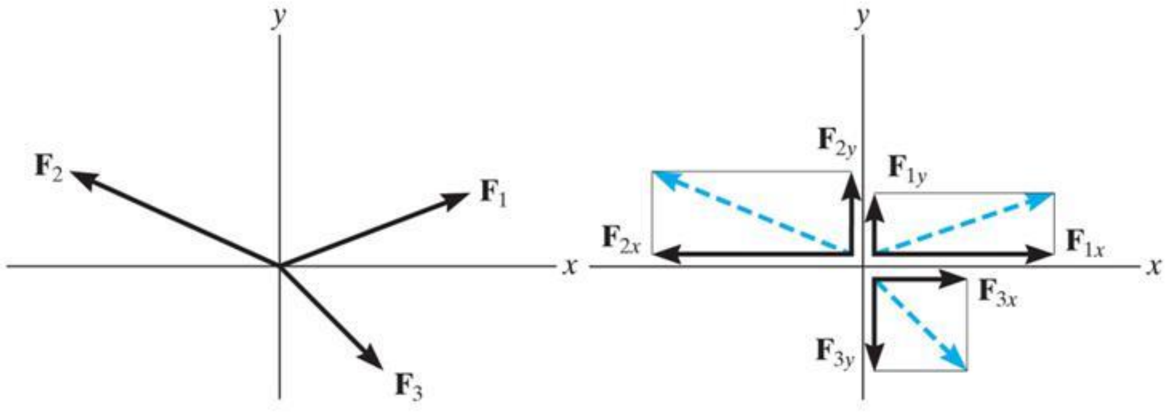


(a)

x and *y* components represented in unit vector *i* and *j.*

VECTOR OPERATION

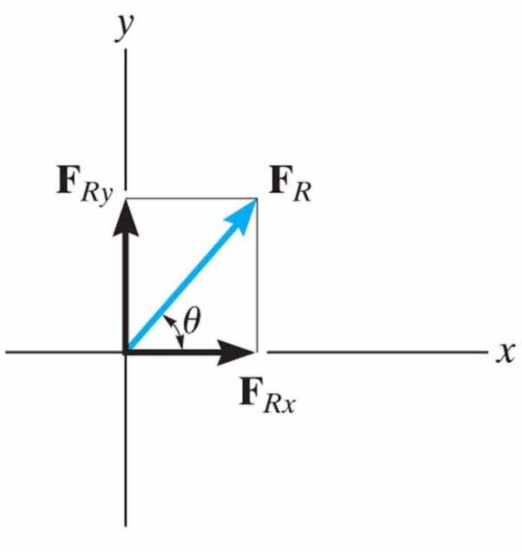
ADDITION OF A SYSTEM OF COPLANAR FORCES : COPLANAR FORCE RESULTANTS



$$F_1 = F_{1x}i + F_{1y}j$$
$$F_2 = -F_{2x}i + F_{2y}j$$
$$F_3 = F_{3x}i - F_{3y}j$$

VECTOR OPERATION

ADDITION OF A SYSTEM OF COPLANAR FORCES : COPLANAR FORCE RESULTANTS



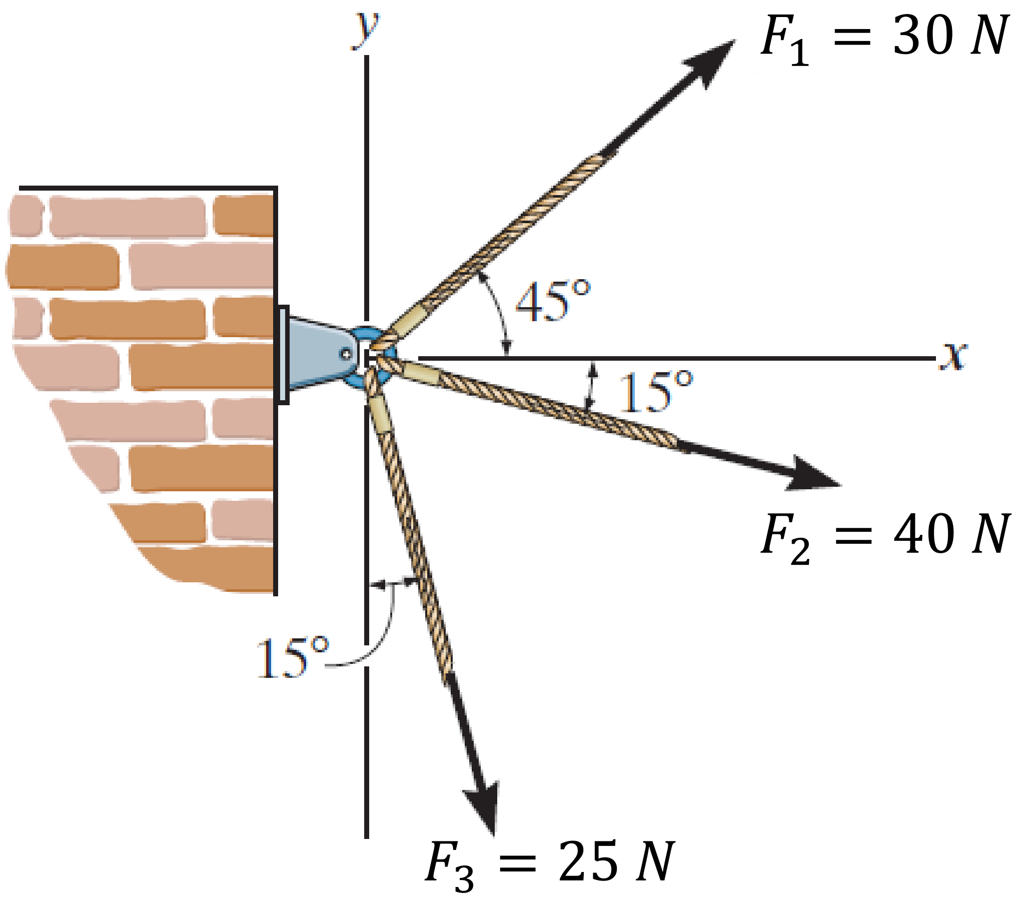
$$F_R = \sqrt{\left(F_{RX}^2 + F_{RY}^2\right)}$$

$$\theta = \tan^{-1}\left|\frac{F_{RY}}{F_{RX}}\right|$$

VECTOR OPERATION

Example 4

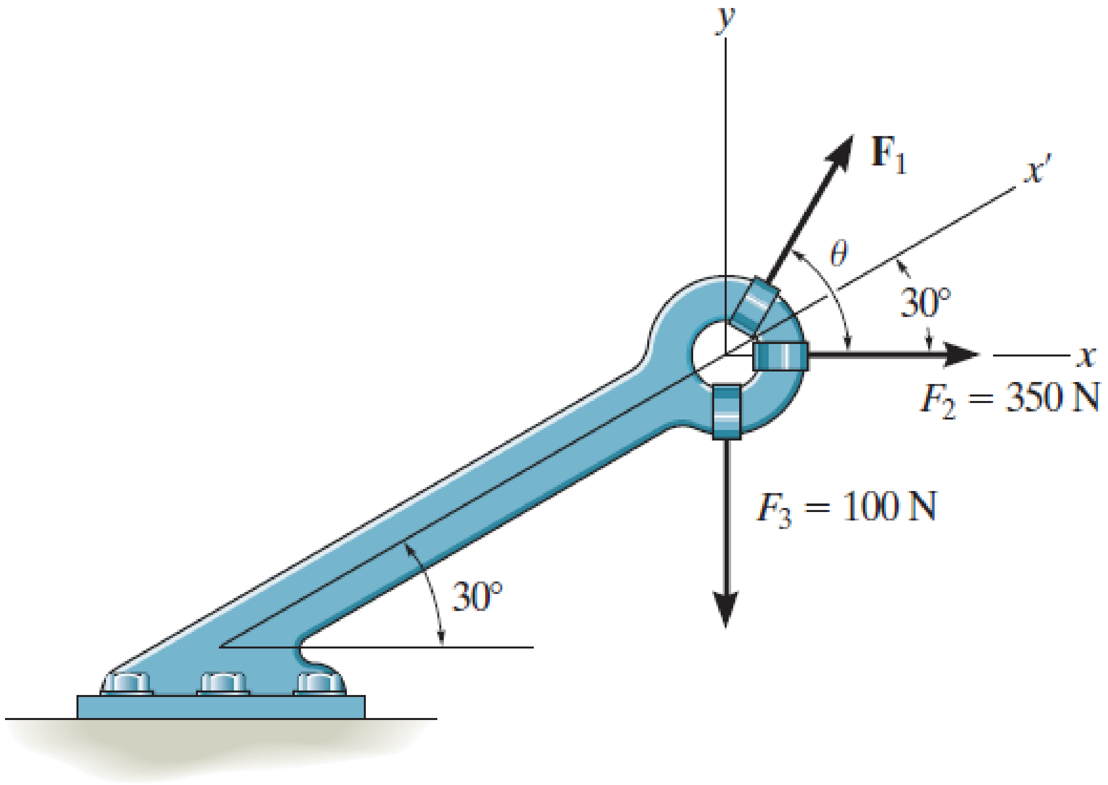
Determine the magnitude of the resultant force acting on the pin and its direction measured clockwise from the positive x axis



VECTOR OPERATION

Example 5

Express each of the three forces acting on the bracket in Cartesian Vector form with respect to the x and y axes. Determine the magnitude and direction θ of $\boldsymbol{F_1}$ so that the resultant force is directed along the positive x' axis and has a magnitude of $F_R = 600\ N$



VECTOR OPERATION

Example 6

PAST YEAR QUESTION (20182019-2)

Q.1

(a) List two differences between scalar and vector.

(2 marks)

(b) Referring to Figure Q.1(b), sketch the parallelogram diagram for resultant force:

i) $F_{R12} = (F_1 + F_2)$

(2 marks)

ii) $F_R = (F_{R12} + F_3)$

(2 marks)

(c) Determine the magnitude of the resultant force acting on the plate and its direction, measured counter clockwise from positive x axis using parallelogram law in (b).

(12 marks)

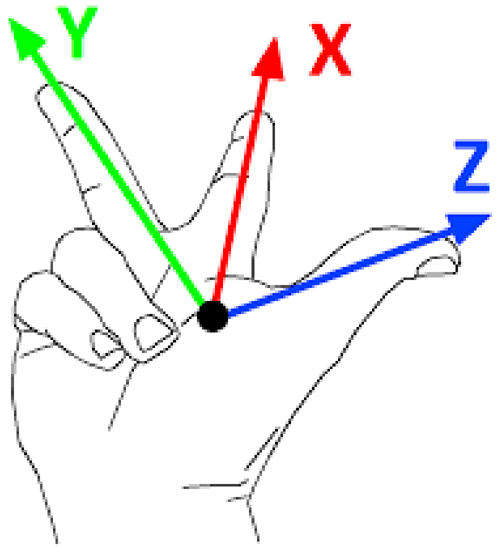
UTM

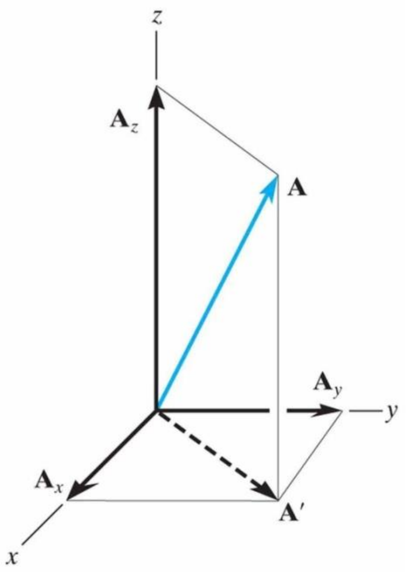
CARTESIAN VECTOR

CARTESIAN VECTOR

3D SPACE CARTESIAN VECTOR

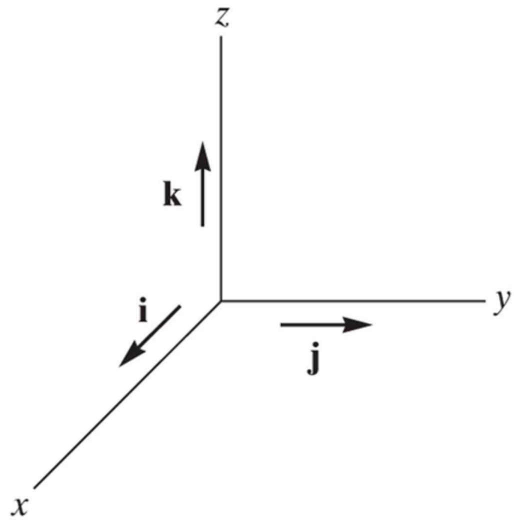
- Use right-handed coordinate system:
- Thumb points to the direction of positive z axis
- Fingers are curled about z axis and directed from the positive x towards y axis





A can be expressed as:

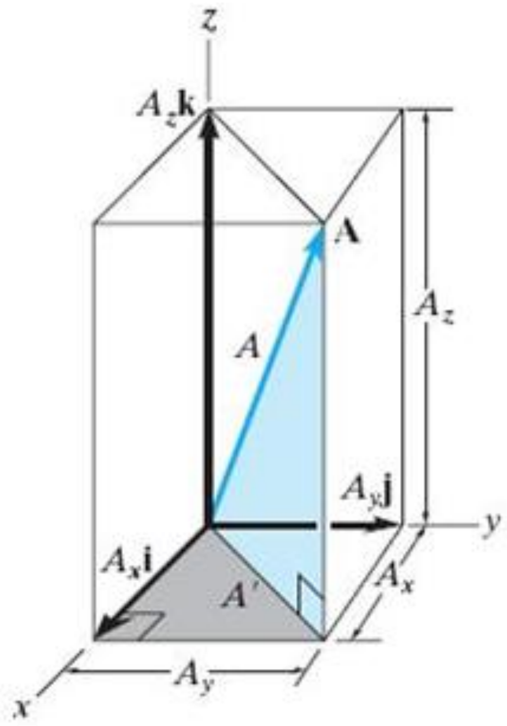
$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z$$



Unit vector

CARTESIAN VECTOR

3D SPACE CARTESIAN VECTOR REPRESENTATION



Consider a box with sides A_X, A_Y, and A_Z meters long.

The vector ***A*** can be defined as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

The projection of vector ***A*** in the *x-y* plane is A´. The magnitude of A´:

$$A' = \sqrt{{A_x}^2 + {A_y}^2}$$

Thus the magnitude of the position vector ***A*** can be obtained as

$$A = \sqrt{{A_x}^2 + {A_y}^2 + {A_z}^2}$$

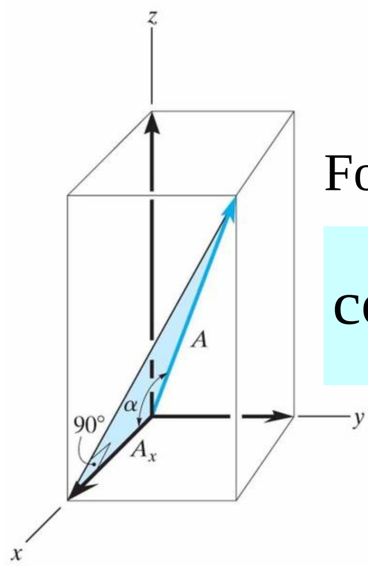
CARTESIAN VECTOR

DIRECTION OF A CARTESIAN VECTOR

- The orientation of A is defined by the coordinate direction angles, α (alpha), β(beta), and γ(gamma).
- These angles are measured between the vector and the positive X, Y and Z axes, respectively. Their range of values are from 0° to 180°.

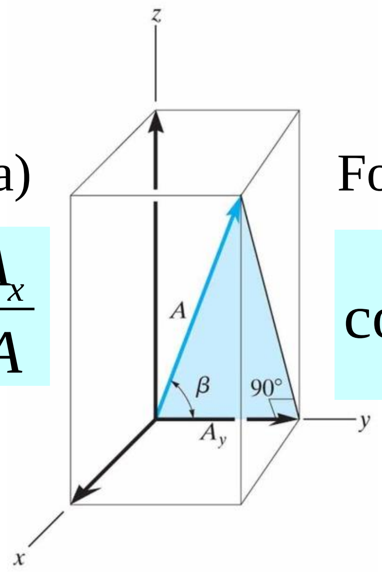
Unit vector $\mathbf{u}_A$ $$\mathbf{u}_A = \frac{\mathbf{A}}{A} =$$

Direction cosines of ***A***:



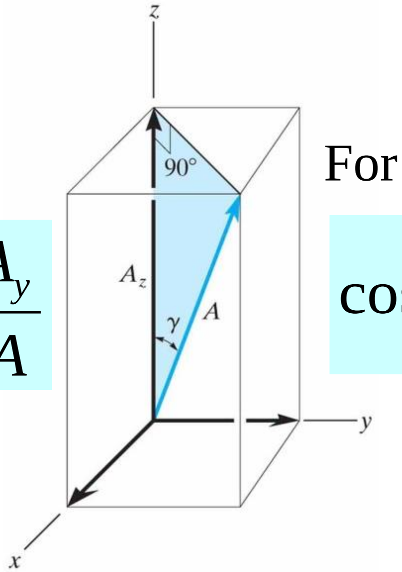
For α (alpha)

$$\cos\alpha = \frac{A_x}{A}$$



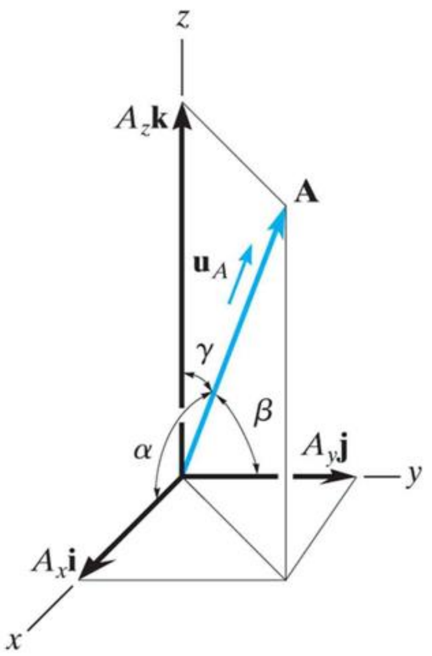
For β(beta)

$$\cos\beta = \frac{A_y}{A}$$



For γ(gamma).

$$\cos\gamma = \frac{A_z}{A}$$



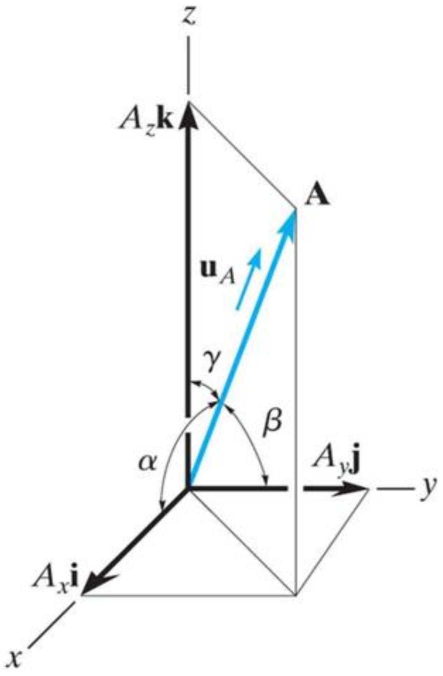
CARTESIAN VECTOR

DIRECTION OF A CARTESIAN VECTOR

- The orientation of A is defined by the coordinate direction angles, α (alpha), β(beta), and γ(gamma).

Important cosines formula

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$



A in Cartesian vector form

$$\begin{aligned}\mathbf{A} &= A\mathbf{u}_A \\ &= A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \\ &= A\cos\alpha\,\mathbf{i} + A\cos\beta\,\mathbf{j} + A\cos\gamma\,\mathbf{k}\end{aligned}$$

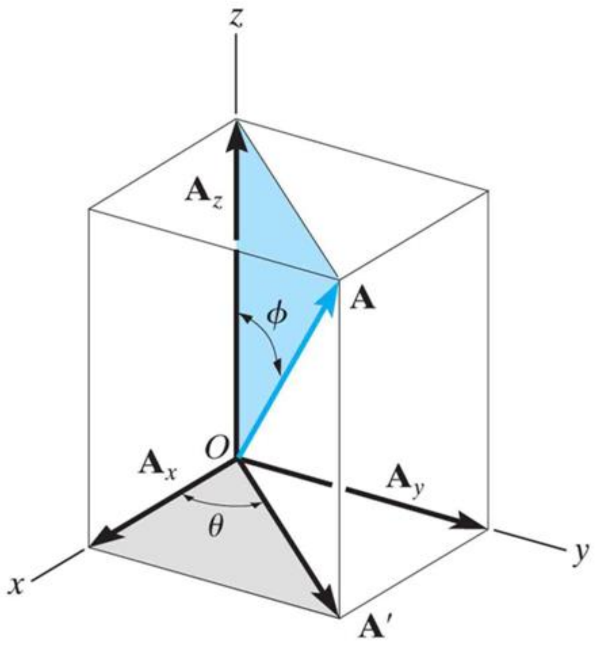
CARTESIAN VECTOR

CARTESIAN VECTOR – 2 ANGLES

- The direction of A is specified by two angle angles, θ and φ (phi).

Component of A

$$A_z = A\cos\emptyset$$
$$A' = A\sin\emptyset$$



Component of A’

$$A_x = A'\cos\theta = A\sin\emptyset\cos\theta$$
$$A_y = A'\sin\theta = A\sin\emptyset\sin\theta$$

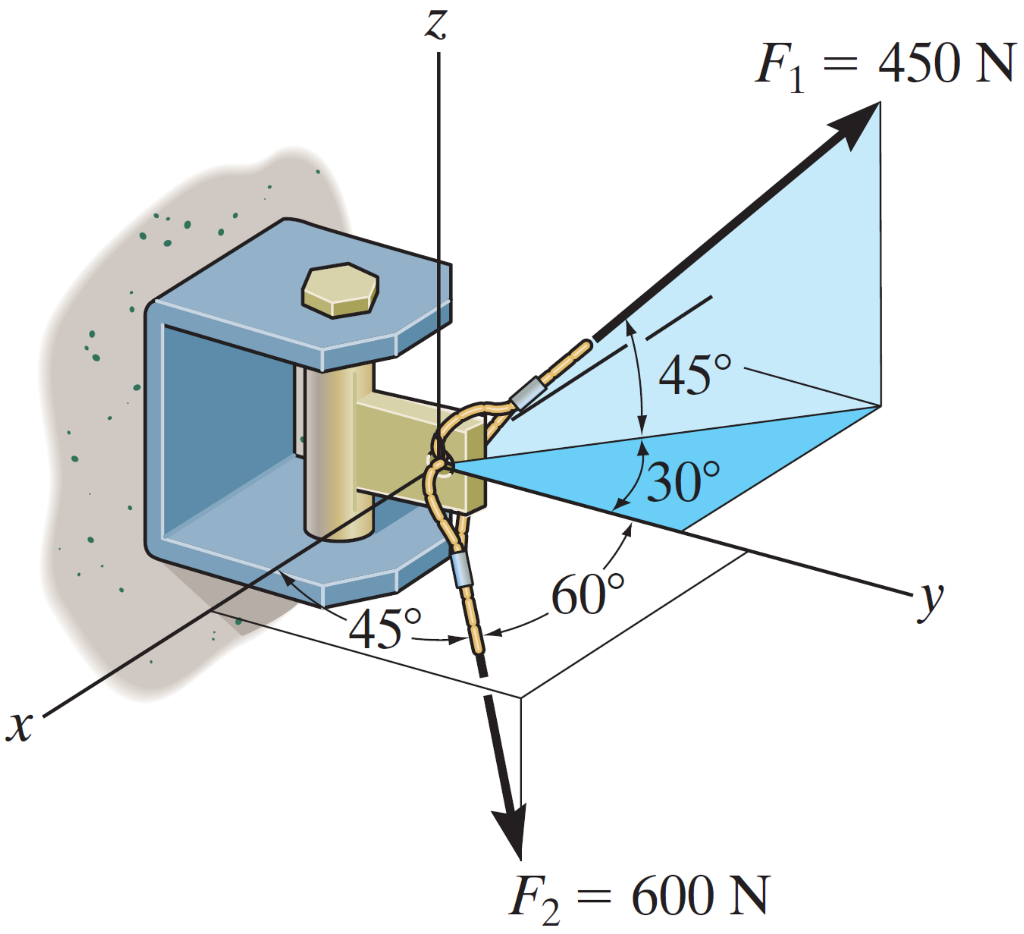
Vector A in Cartesian vector form

$$\boldsymbol{A} = A\sin\emptyset\cos\theta\,\boldsymbol{i} + A\sin\emptyset\sin\theta\,\boldsymbol{j} + A\cos\emptyset\,\boldsymbol{k}$$

CARTESIAN VECTOR

Example 7

Determine the magnitude and coordinate direction angles of the resultant force acting on the bracket



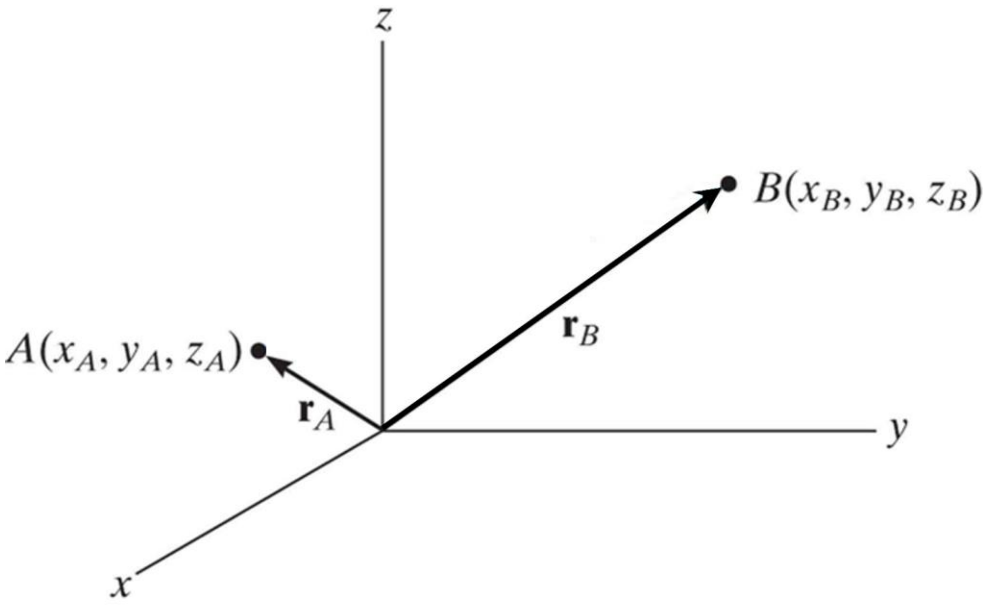
POSITION & FORCE DIRECTED VECTORS

POSITION & FORCE DIRECTED VECTORS

This awning is held up by three chains. What are the forces in the chains and how do we find their directions? Why would we want to know these things?

POSITION & FORCE DIRECTED VECTORS

WHAT IS POSITION VECTOR?

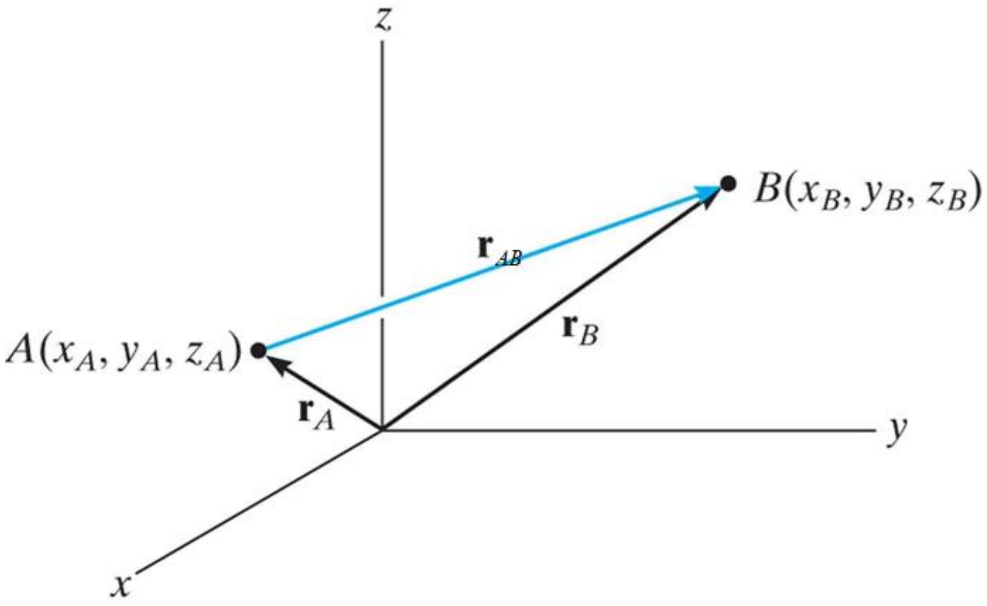


A **position vector** is defined as a fixed **vector** that **locates a point** in **space** relative to another point.

POSITION & FORCE DIRECTED VECTORS

POSITION VECTOR

Consider two points, A and B, with coordinates be (x_A, y_A, z_A) and (x_B, y_B, z_B), respectively.



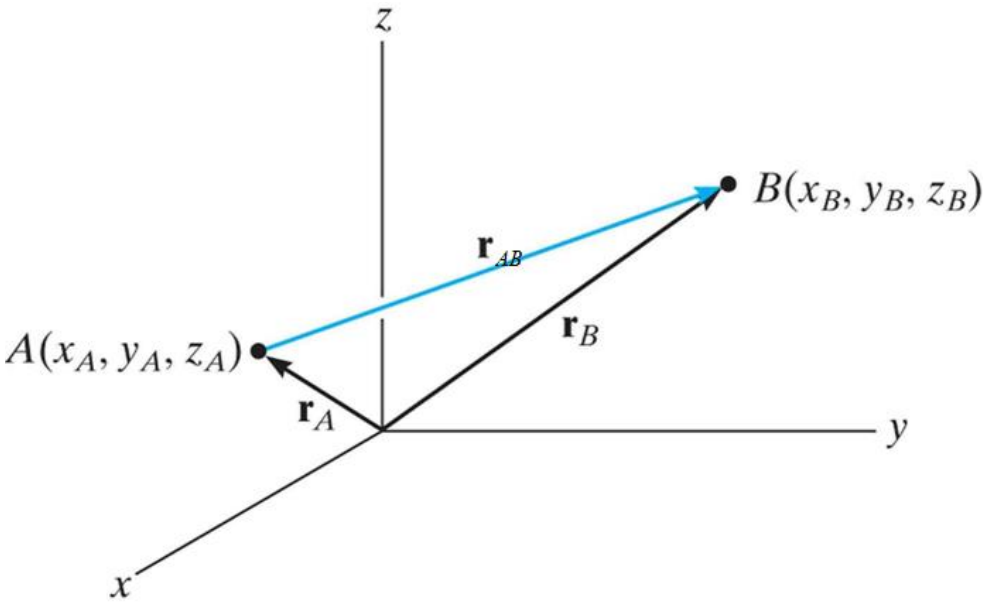
Consider three position vectors $\boldsymbol{r}_A$, $\boldsymbol{r}_B$ & $\boldsymbol{r}_{AB}$

Vector addition gives : $\boldsymbol{r}_A + \boldsymbol{r}_{AB} = \boldsymbol{r}_B$

POSITION & FORCE DIRECTED VECTORS

POSITION VECTOR

Consider two points, A and B, with coordinates be (x_A, y_A, z_A) and (x_B, y_B, z_B), respectively.



Solving $\boldsymbol{r_{AB}}$ in Cartesian vector form yield:

$$\boldsymbol{r} = \boldsymbol{r_B} - \boldsymbol{r_A} = (x_B\boldsymbol{i} + y_B\boldsymbol{j} + z_B\boldsymbol{k}) - (x_A\boldsymbol{i} + y_A\boldsymbol{j} + z_A\boldsymbol{k})$$

or

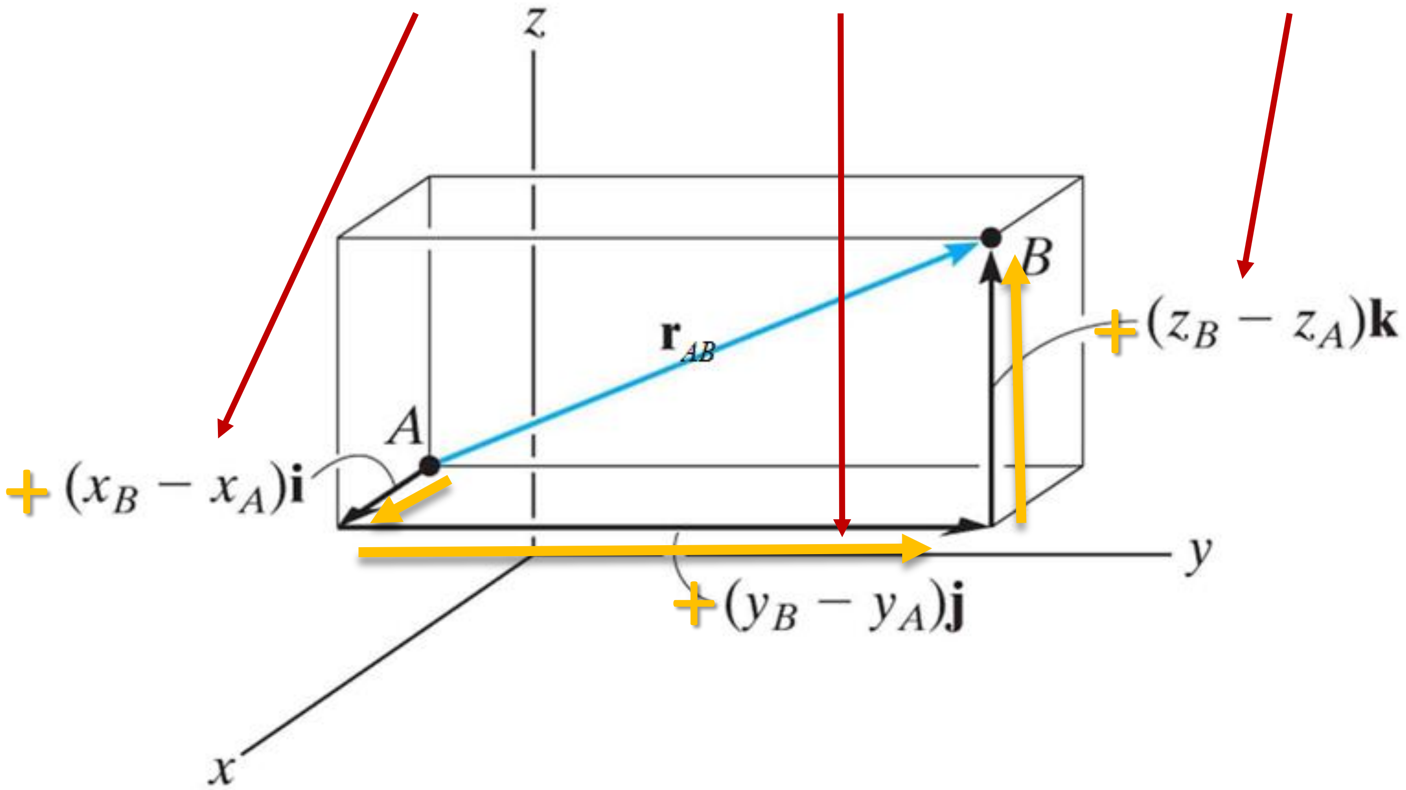
$$\boldsymbol{r} = (x_B - x_A)\boldsymbol{i} + (y_B - y_A)\boldsymbol{j} + (z_B - z_A)\boldsymbol{k}$$

Unit : **meter (m)**

POSITION & FORCE DIRECTED VECTORS

POSITION VECTOR

$$\mathbf{r}_{AB} = \underbrace{(x_B - x_A)}\mathbf{i} + \underbrace{(y_B - y_A)}\mathbf{j} + \underbrace{(z_B - z_A)}\mathbf{k}$$

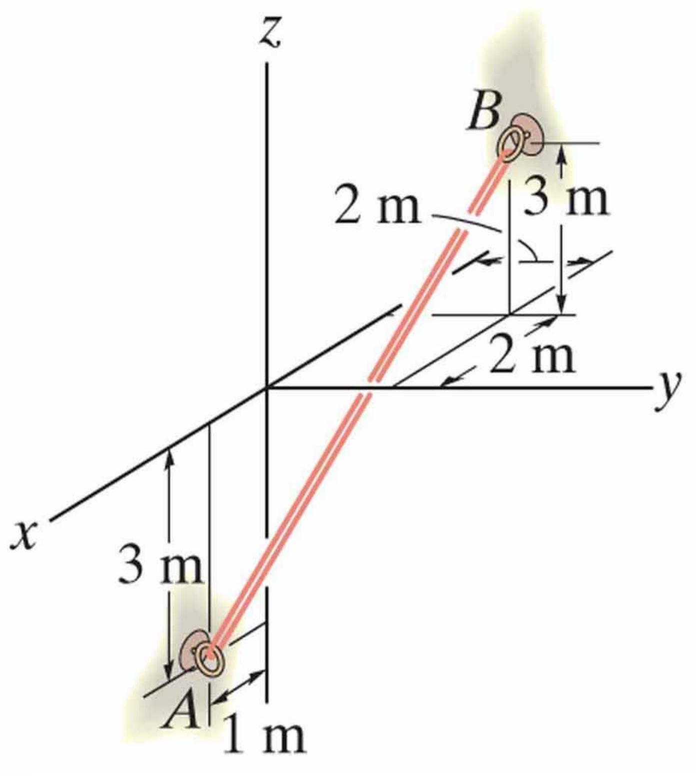


POSITION & FORCE DIRECTED VECTORS

FORCE VECTOR

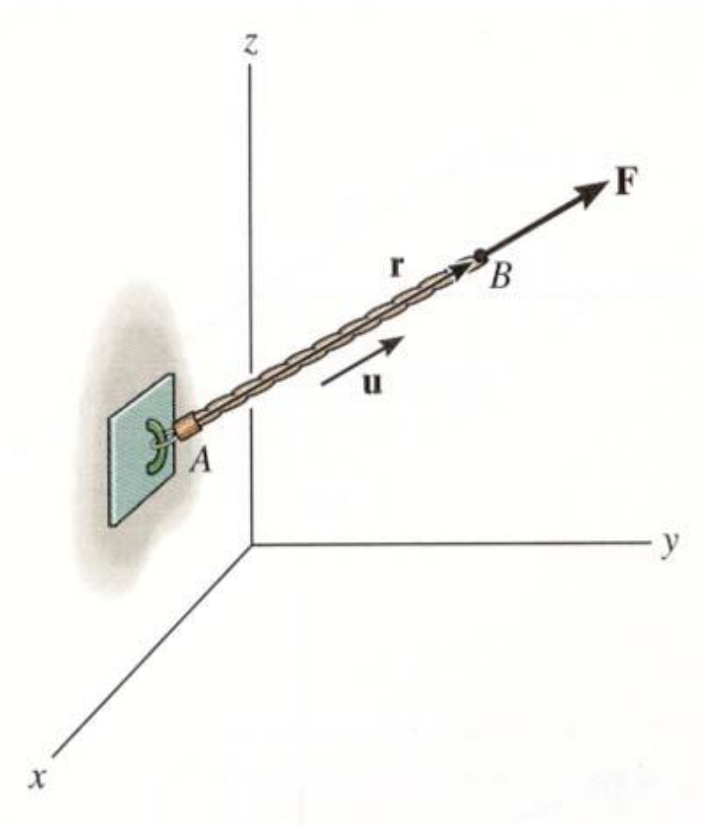
Example 8

An elastic rubber band is attached to points *A* and *B* as shown. Determine its length and direction (3 angle coordinate system) measured from A toward B.



POSITION & FORCE DIRECTED VECTORS

HOW FORCE VECTOR DIRECTED ALONG THE LINE?



$\boldsymbol{r}$ = *Position vector*
r = *Length of the position vector*
F = *Magnitude of the Force*

- If a force is directed along a line, then we can represent the force vector in Cartesian coordinates by using a unit vector and the magnitude of force. So we need to:

a) Find the position vector, $\boldsymbol{r}_{AB}$, along two points on that line.

b) Find the unit vector describing the line's direction,

$$\boldsymbol{u}_{AB} = \frac{\boldsymbol{r}}{r} = \frac{(x_B - x_A)\boldsymbol{i} + (y_B - y_A)\boldsymbol{j} + (z_B - z_A)\boldsymbol{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}$$

c) Find force vector by multiplying the unit vector by the magnitude of the force

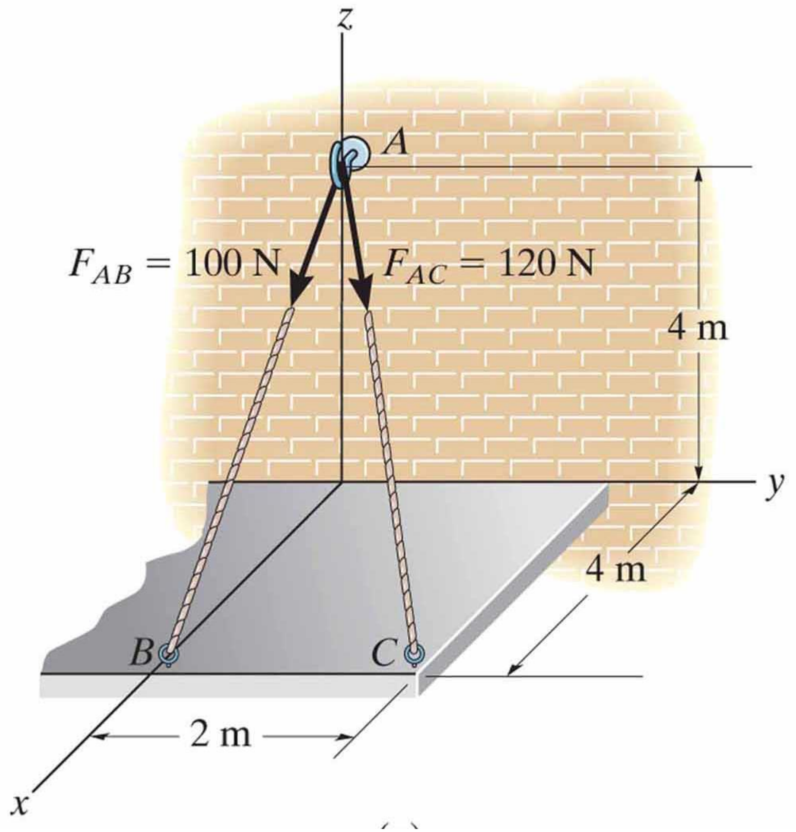
$$\boldsymbol{F} = F\boldsymbol{u}_{AB}$$

POSITION & FORCE DIRECTED VECTORS

FORCE VECTOR

Example 9

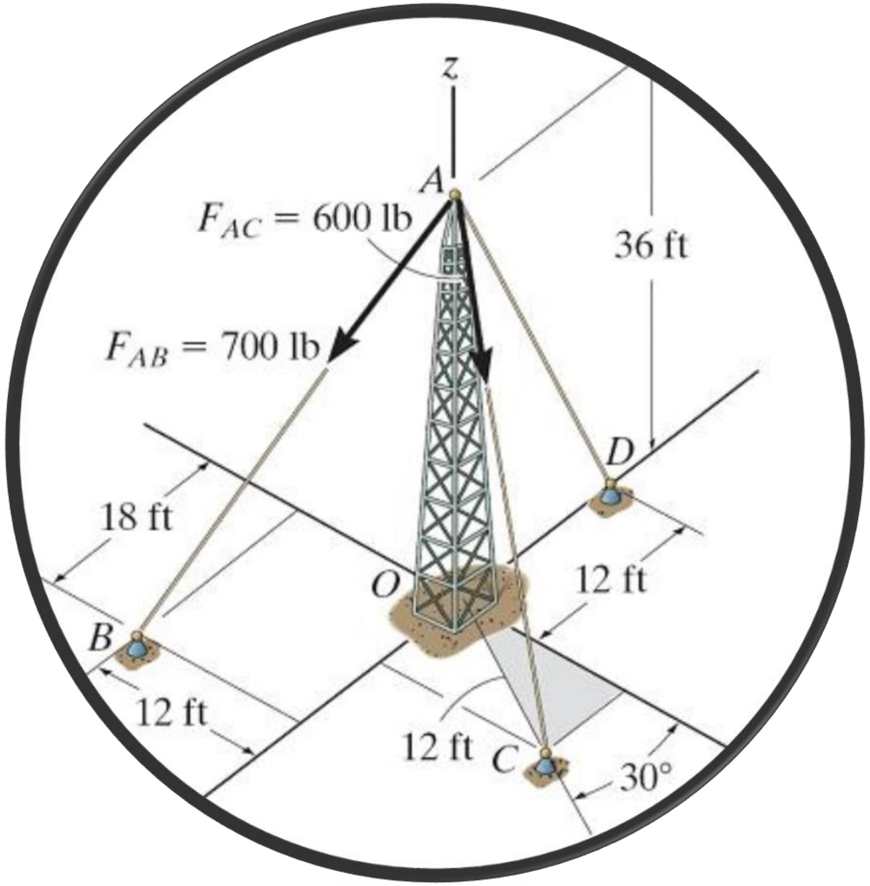
The roof is supported by cables as shown. If the cables exert forces F_{AB} = 100 N and F_{AC} = 120 N on the wall hook at A, determine the resulting force acting at A. Express the result as a Cartesian vector.



DOT PRODUCT

DOT PRODUCT

APPLICATION



Why points B, C, D are important?

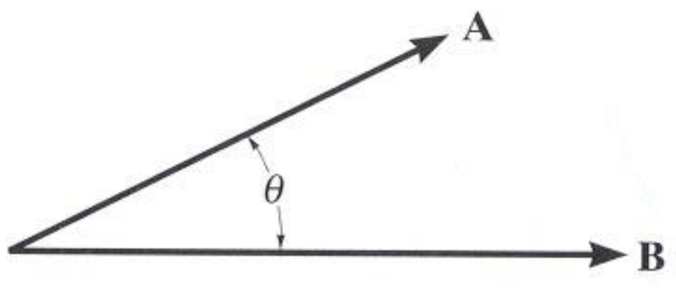
Are angles between the forces important?

How would you check this installation to make sure the angles were correct?

DOT PRODUCT

DEFINITION

- The dot product is used to determine:
  - The angle between two vectors
  - The projection of a vector in a specified direction
- The dot product of vectors A and B is defined as $A \bullet B \ = \ ABcos\theta$
- The angle θ is the smallest angle between the two vectors and is always in a range of $0°$ to $180°$.



Commutative law $\boldsymbol{A} \bullet \boldsymbol{B} = \boldsymbol{B} \bullet \boldsymbol{A}$

Multiplication by a scalar $a(\boldsymbol{A} \bullet \boldsymbol{B}) = (a\boldsymbol{A}) \bullet \boldsymbol{B} = \boldsymbol{A} \bullet (a\boldsymbol{B})$

Distribution law $\boldsymbol{A} \bullet (\boldsymbol{B} + \boldsymbol{D}) = (\boldsymbol{A} \bullet \boldsymbol{B}) + (\boldsymbol{A} \bullet \boldsymbol{D})$

DOT PRODUCT

DEFINITION

- Dot Product Characteristics:
  - The result of the dot product is a scalar (a positive or negative number).
  - The units of the dot product will be the product of the units of the A and B vectors.
- Dot Product of Cartesian unit vectors, A and B

$$\boldsymbol{A} \bullet \boldsymbol{B} \;=\; (A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}) \bullet (B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k})$$

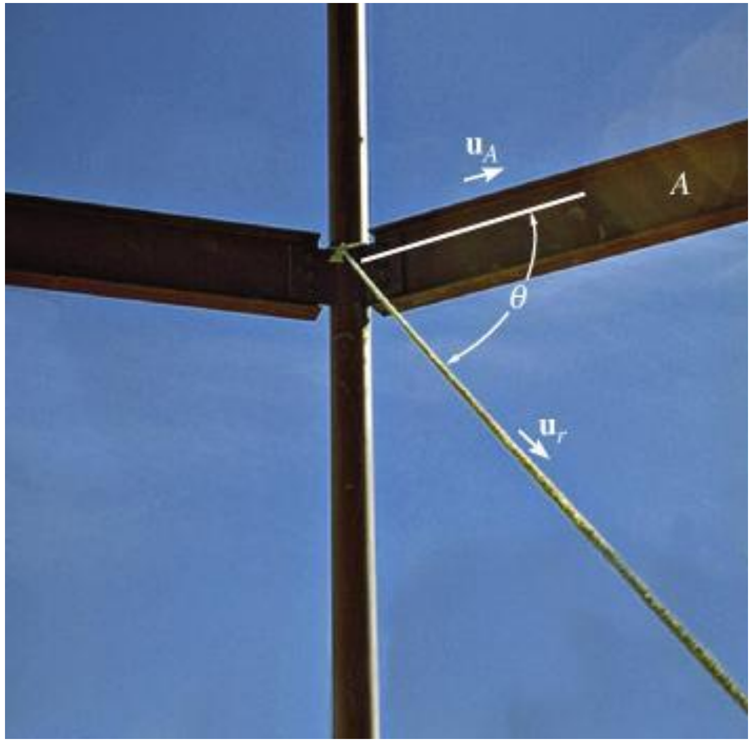
- By applying dot product, this will be produced:

$$= A_xB_x + A_yB_y + A_zB_z$$

DOT PRODUCT

APPLICATION-ANGLE

The angle formed between two vectors of intersecting lines

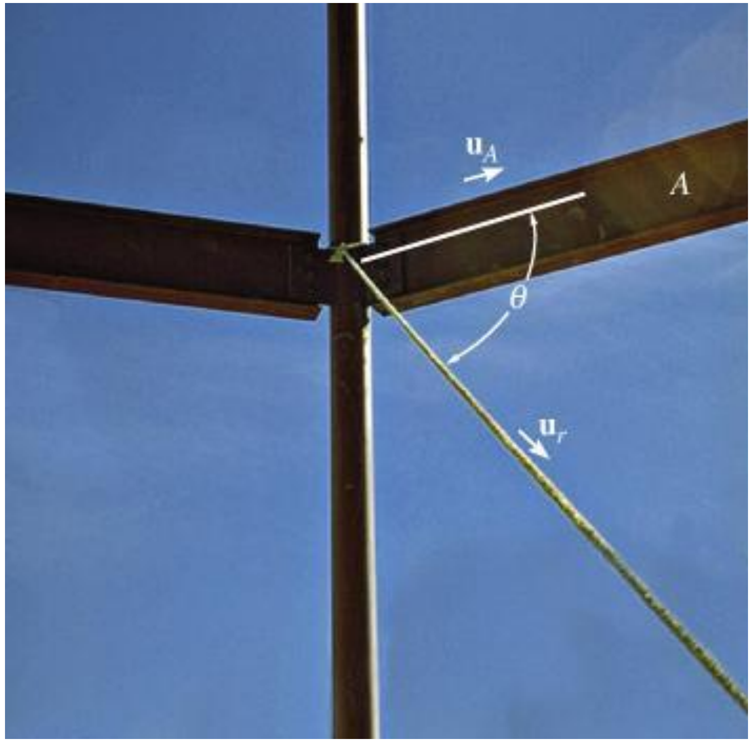


The angle θ between the rope and the beam can be determined by formulating unit vectors along the beam and rope and then using the dot product.

DOT PRODUCT

APPLICATION-ANGLE

The angle formed between two vectors of intersecting lines



$$\boldsymbol{A}\cdot\boldsymbol{B} = AB\cos\theta$$

$$\text{then } \theta = \cos^{-1}\left(\frac{\boldsymbol{A}\cdot\boldsymbol{B}}{AB}\right)$$

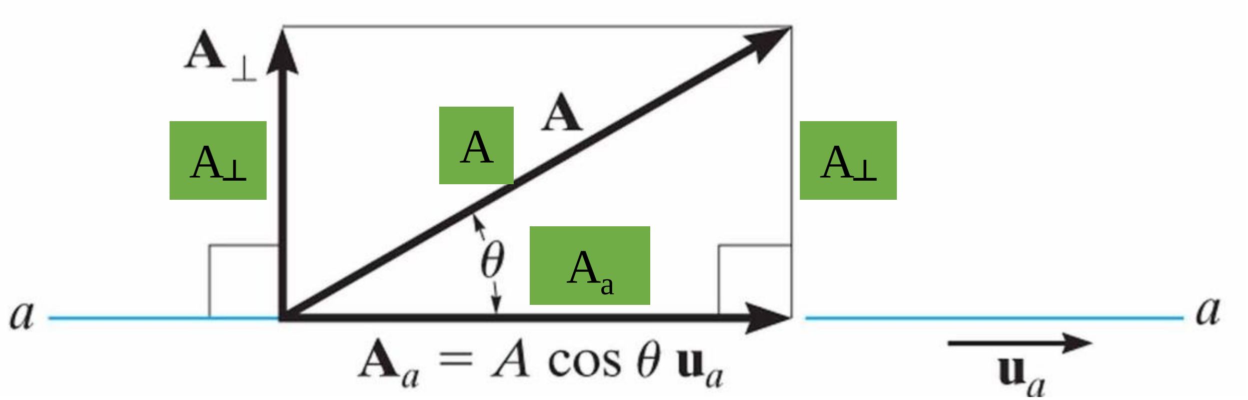
- Similarly

$$\boldsymbol{u_A}\cdot\boldsymbol{u_r} = (1)(1)\cos\theta$$

$$\text{then } \theta = \cos^{-1}(\boldsymbol{u_A}\cdot\boldsymbol{u_r})$$

DOT PRODUCT

APPLICATION-PROJECTION



Recap

- The scalars are A, A⊥, and A_a
- Cosine formula

$$\cos\theta = \frac{A_a}{A}$$

- Thus,

$$A_a = A\cos\theta$$

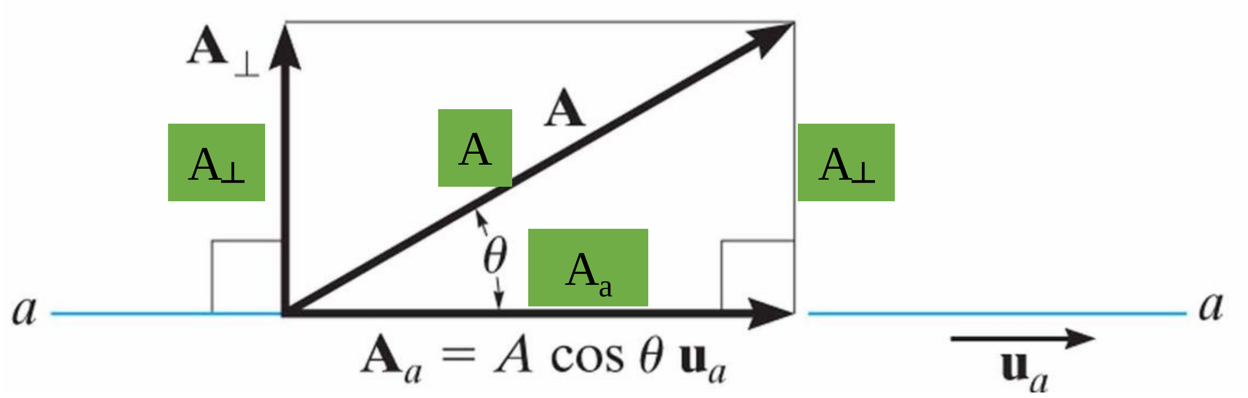
- Dot product to unit vector

$$\boldsymbol{A}\cdot\boldsymbol{u_a} = A u_a \cos\theta$$
$$= A(1)\cos\theta$$
$$= A\cos\theta$$
$$\boxed{\boldsymbol{A}\cdot\boldsymbol{u_a} = A_a} \quad Magnitude\ of\ A_a$$

$$A_a\boldsymbol{u_a} = Vector\ of\, A_a\ (\boldsymbol{A_a})$$

DOT PRODUCT

APPLICATION-PROJECTION



The components (scalar) of a vector ***A***

- parallel to a line

$$A_a = A - A_\perp$$

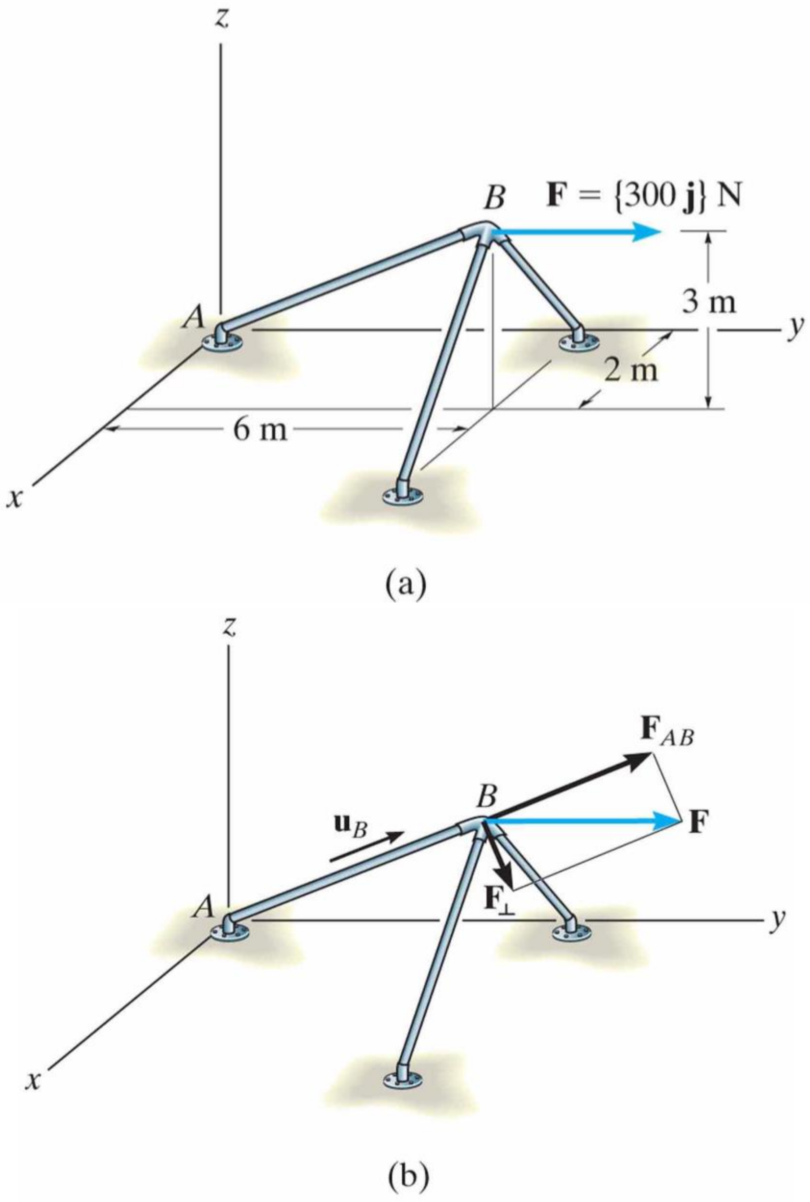
- Perpendicular to a line

$$A_\perp = \mathrm{A} - A_a$$

DOT PRODUCT

Example 10

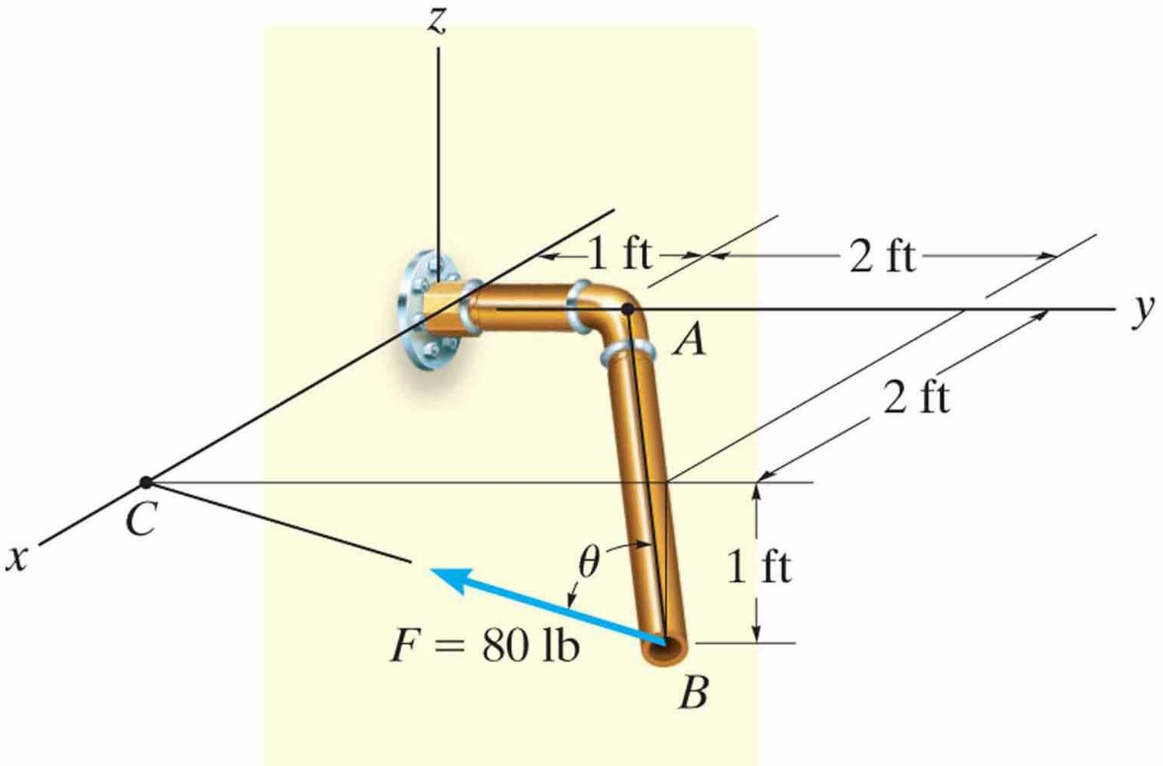
The frame shown is subjected to a horizontal force $\boldsymbol{F} = \{300\}\boldsymbol{j}$. Determine the magnitude of the components of this force parallel and perpendicular to member AB



DOT PRODUCT

Example 11

The pipe is subjected to the force of $F = 80\ N$. Determine the angle θ between $\boldsymbol{F}$ and the pipe segment BA and the projection of $\boldsymbol{F}$ along this segment.



(a)

UNIVERSITI TEKNOLOGI MALAYSIA

univteknologimalaysia

utm_my

utmofficial

Thank You

www.utm.my

innovative ● entrepreneurial ● global