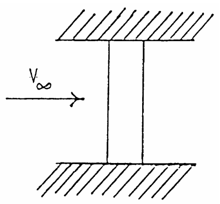
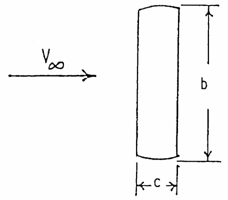
Finite wings

- Infinite wing (2d) versus finite wing (3d)





  - Definition of aspect ratio:

$$AR \equiv \frac{b^2}{S} \Rightarrow AR = \frac{b}{c}$$ For rectangular platform

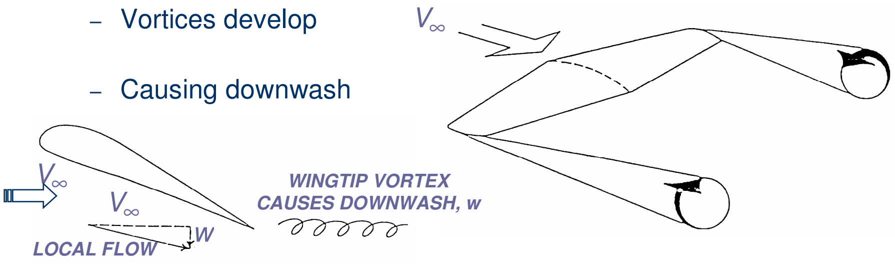
  - Symbol changes:

$$C_l \rightarrow C_L \quad C_d \rightarrow C_D \quad C_m \rightarrow C_M$$

Vortices and wings

- What the third dimension does
  - Difference between upper and lower pressure results in circulatory motion about the wingtips

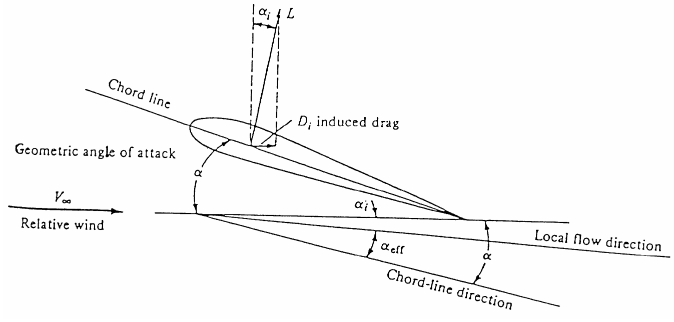
  - Vortices develop
  - Causing downwash



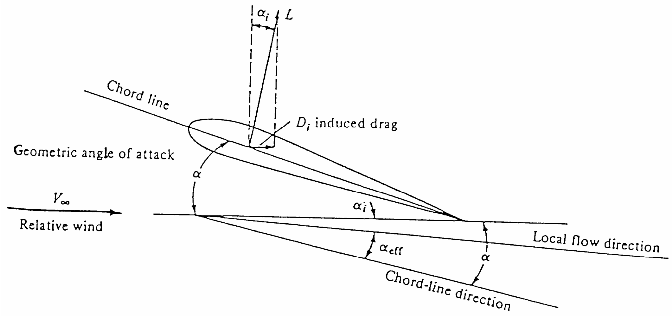
  - Drag is increased by this induced downwash

Origin of induced drag

- Wingtip vortices alter flow field
  - Resulting pressure distribution increases drag
  - Rotational kinetic energy is added to the 2-D flow
  - Lift vector is tilted back
    - AOA is effectively reduced
    - Component of force in drag direction is generated



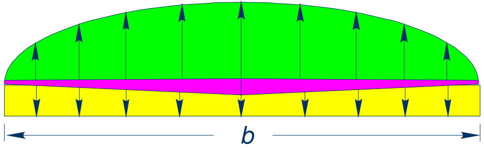
Induced drag



- The sketch shows $D_i = L \sin\alpha_i$
- For small angles of attack $\sin\alpha_i \approx \alpha_i$
- The value of α_i for a given section of a finite wing depends on the distribution of downwash along the span of the wing

Lift per unit span

- Lift per unit span varies
  - Chord may vary in length along the wing span
  - Twist may be added so that each airfoil section is at a different geometric angle of attack
  - The shape of the airfoil section may change along the wing span

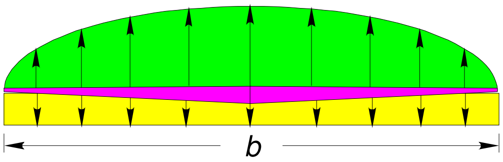


Lift per unit span as a function of distance along the span -- also called the lift distribution

The downwash distribution, w, which results from the lift distribution

Elliptical lift distribution

- An elliptical lift distribution



  - Produces a uniform downwash distribution
  - For a uniform downwash distribution, incompressible theory predicts that

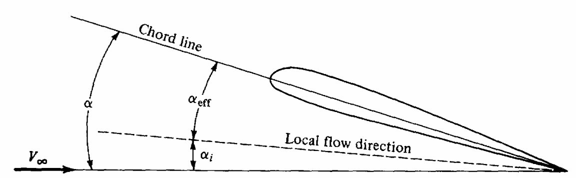
$$\alpha_i = \frac{C_L}{\pi AR}$$

- Where C_L is the finite wing (3d) lift coefficient

$AR = \frac{b^2}{S}$ Aspect Ratio

Lift curve slope

- A finite wing’s lift curve slope is different from its 2D lift curve slope



  - For an elliptical spanwise lift distribution
  - Extending this definition to a general platform

$$\alpha_i = \frac{C_L}{\pi AR}$$

$$\alpha_i = \frac{C_L}{\pi e_1 AR}\left(in\ rad\right) = \frac{180 C_L}{\pi^2 e_1 AR}\left(in\ ^\circ\right)$$

Finite Wing Corrections

- All reference coefficients are not corrected
- Moment coefficients are not corrected
- Lift coefficient due to angle of attack is corrected
  - *AR* is the aspect ratio of the wing
  - *e* is the Oswald Efficiency Factor

$$C_{L0} = C_{l0}$$

$$C_{D0} = C_{d0}$$

$$C_{M0} = C_{m0}$$

$$C_{M\alpha} = C_{m\alpha}$$

$$C_{L\alpha} = \frac{C_{l\alpha}}{1 + \frac{C_{l\alpha}}{\pi e AR}}$$

Note: do not forget **57.3 deg/rad** conversion factor

Finite Wing Corrections – High Aspect Ratio Wings (lifting line theory)

$$a = \frac{a_0}{1 + \frac{a_0}{\pi e AR}}$$

High-aspect-ratio straight wing (incompressible)
Prandtl's lifting line theory

Incompressible lift curve slope

$$a_{0,comp} = \frac{a_0}{\sqrt{1 - M_\infty^2}}$$

Compressibility correction (subsonic flowfield)
Prandtl-Glauert rule (thin airfoil 2D)

$$a_{comp} = \frac{a_{0,comp}}{1 + \frac{a_{0,comp}}{\pi e AR}} = \frac{a_0}{\sqrt{1 - M_\infty^2} + \frac{a_0}{\pi e AR}}$$

High-aspect-ratio straight wing
(subsonic compressible)

$$a_{comp} = \frac{4}{\sqrt{M_\infty^2 - 1}}$$

High-aspect-ratio straight wing
(supersonic compressible)
(obtained from supersonic linear theory)

Effect of Mach Number on the Lift Slope

$$a_{comp} = \frac{a_{0,comp}}{1 + \frac{a_{0,comp}}{\pi e AR}} = \frac{a_0}{\sqrt{1 - M_\infty^2} + \frac{a_0}{\pi e AR}}$$

$$a_{comp} = \frac{4}{\sqrt{M_\infty^2 - 1}}$$

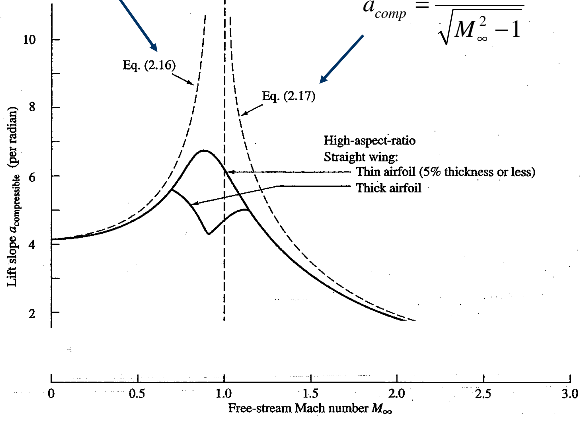


Figure 2.24 Effect of Mach number on the lift slope.

Finite Wing Corrections – Low Aspect Ratio Wings (lifting surface theory)

$$a = \frac{a_0}{\sqrt{1+\left(\frac{a_0}{\pi AR}\right)^2}+\frac{a_0}{\pi AR}}$$

Low-aspect-ratio straight wing (incompressible)
Helmbold’s Equation

$$a_{comp} = \frac{a_0}{\sqrt{1-M_\infty^2+\left(\frac{a_0}{\pi AR}\right)^2}+\frac{a_0}{\pi AR}}$$

Low-aspect-ratio straight wing
(subsonic compressible)

$$a_{comp} = \frac{4}{\sqrt{M_\infty^2-1}}\left(1-\frac{1}{2AR\sqrt{M_\infty^2-1}}\right)$$

Low-aspect-ratio straight wing
(supersonic compressible)
(Hoerner and Borst)

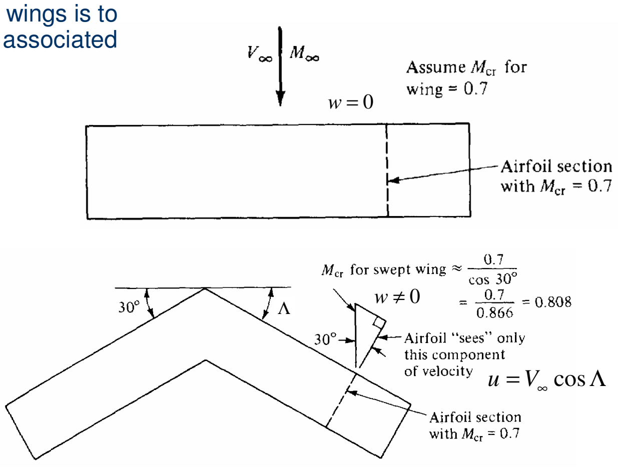
Swept wings

- Subsonically,

  The purpose of swept wings is to delay the drag rise associated with wave drag

For a straight wing

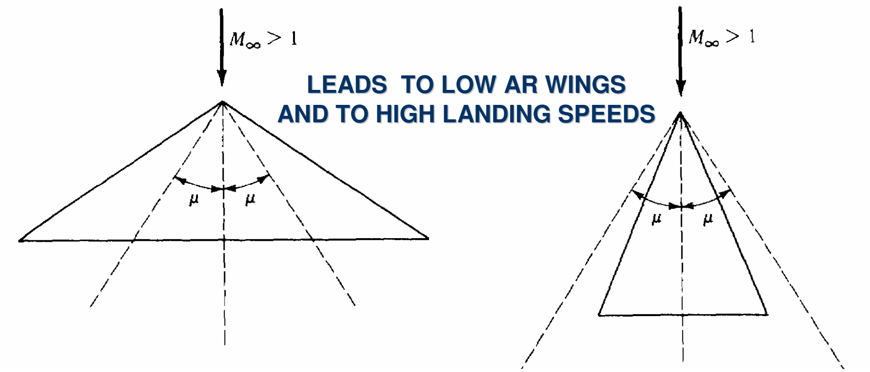
Now, sweep the Wing by 30°



Swept and Delta wings

- Supersonically,

  The goal is to keep wing surfaces inside the mach cone to reduce wave drag

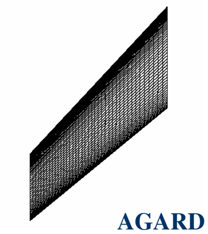


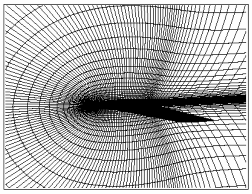
LEADS TO LOW AR WINGS AND TO HIGH LANDING SPEEDS

$$\mu = \arcsin\left(1/M_\infty\right)$$

Computational Fluid Dynamics

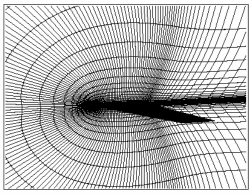
Flow separation from forebody chine and wing leading-edge and roll up to form free vortices



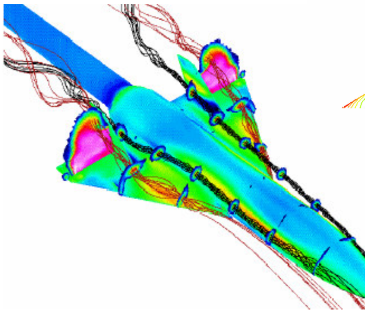


(a) Euler.

AGARD WING 445.6



(b) Navier-Stokes.



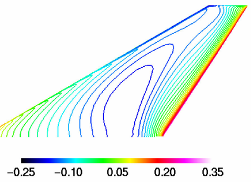
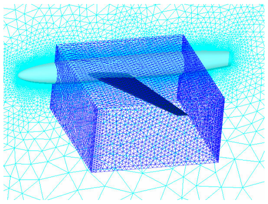


Figure 5.16: C_p Variation on the Upper Surface of a Rigid Wing



Finite Wing Corrections

a_0 Lift curve slope airfoil section perpendicular to the leading edge

Lift curve slope for an infinite swept wing

$$a = \frac{\boxed{a_0 \cos\Lambda}}{\sqrt{1+\left(\frac{a_0\cos\Lambda}{\pi AR}\right)^2}+\frac{a_0\cos\Lambda}{\pi AR}}$$

Swept wing (incompressible)

Kuchemann approach

$a_0 \Rightarrow a_0/\beta$ $\quad M_{\infty,n} = M_\infty \cos\Lambda$

$$\beta = \sqrt{1-M_{\infty,n}^2} = \sqrt{1-M_\infty^2\cos^2\Lambda}$$

$$a_{comp} = \frac{a_0\cos\Lambda}{\sqrt{1-M_\infty^2\cos^2\Lambda+\left(\frac{a_0\cos\Lambda}{\pi AR}\right)^2}+\frac{a_0\cos\Lambda}{\pi AR}}$$

Swept wing (subsonic compressible)

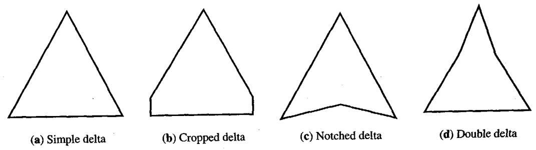

(a) Simple delta (b) Cropped delta (c) Notched delta (d) Double delta

Figure 2.32 Four versions of a delta-wing planform. (After Loftin, Ref. 13.)

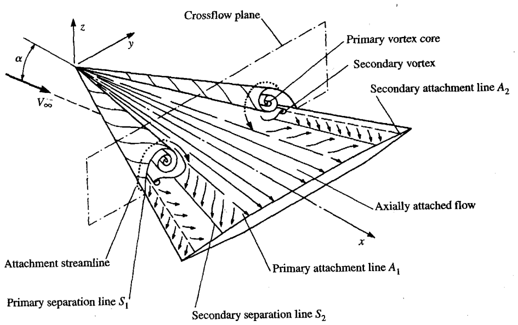


Figure 2.33 Schematic of the subsonic flow over the top of a delta wing at angle of attack. (Courtesy of John Stollery, Cranfield Institute of Technology, England.)

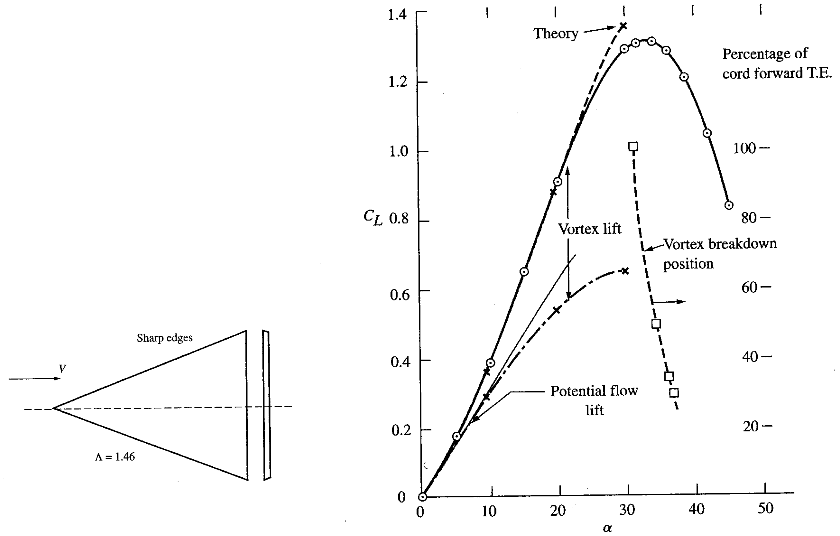


Figure 2.34 Lift coefficient curve for a delta wing in low-speed subsonic flow. (After Hoerner and Borst, Ref. 18.)

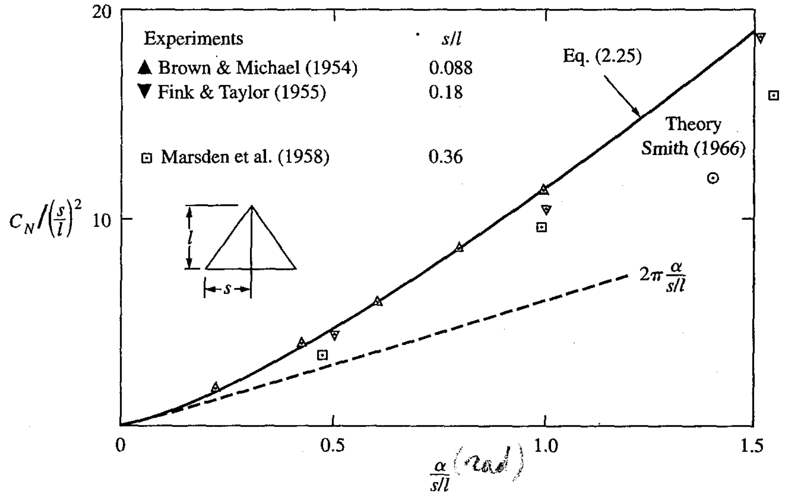
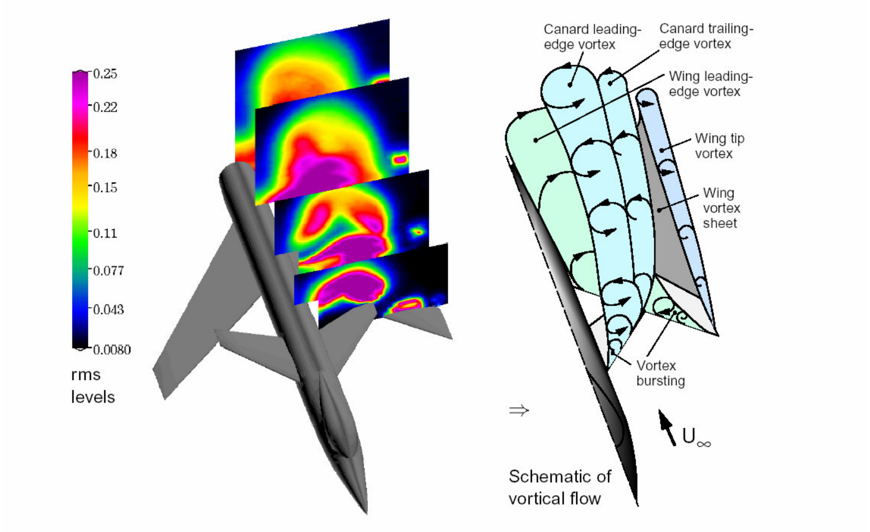


Figure 2.35 Normal forces on slender delta wings. (After Kuchemann, Ref. 24.)

Computational Fluid Dynamics for Wing - Body combinations



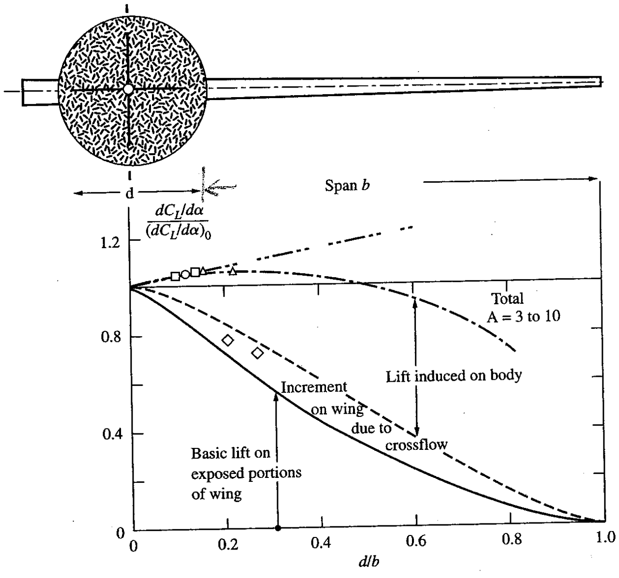


Figure 2.36 The lift-curve slope of wing-fuselage combinations as a function of the diameter ratio d/b. (After Hoerner and Borst, Ref. 18.)

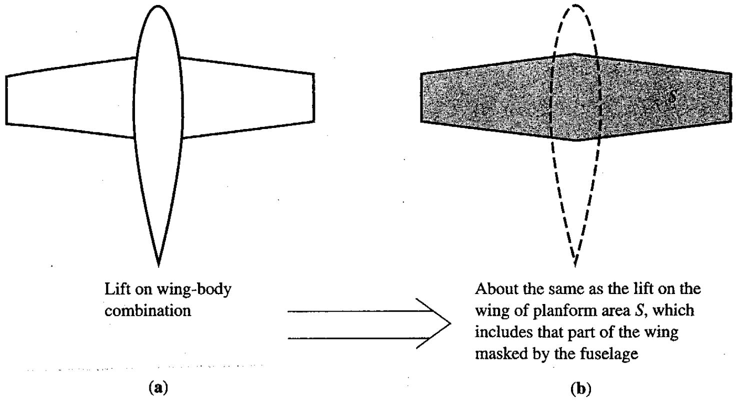
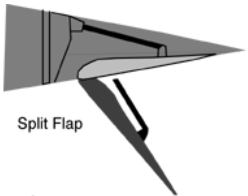
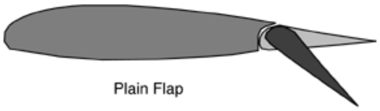


Figure 2.37 Significance of the conventional definition of wing planform area.

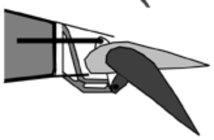
Flaps



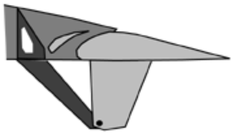
Split Flap



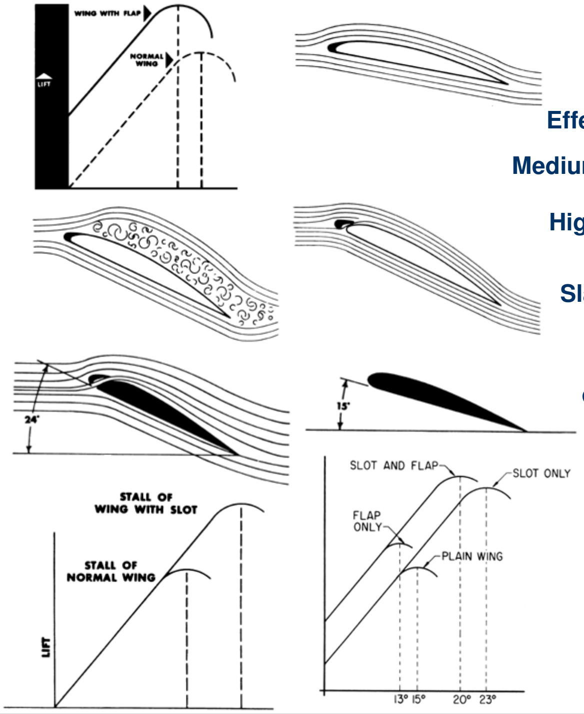
Plain Flap



with Fixed Hinge



Double-Slotted Flap with Fixed Hinge
and Fixed Vane (DC-9)



Effect of flaps on lift

Medium angle of attack

High angle of attack

Slat opened at high angle of attack

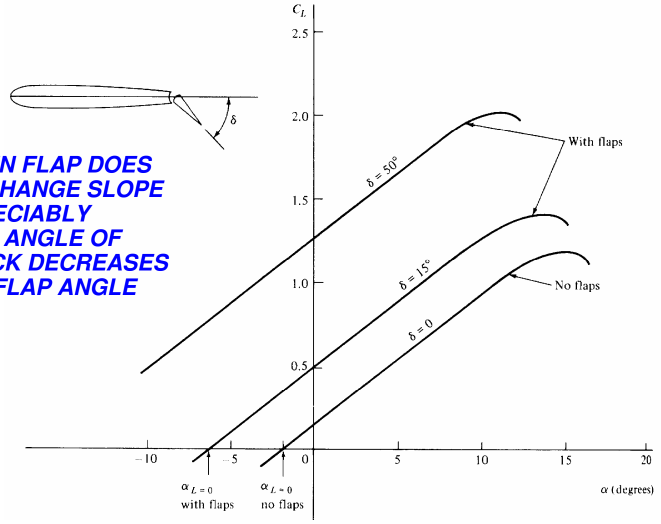
With slats 24 degree angle of attack

Without slats 15 degree angle of attack

Angle of attack

High lift devices

- Flaps are the most common high lift device



- ***A PLAIN FLAP DOES NOT CHANGE SLOPE APPRECIABLY***
- ***STALL ANGLE OF ATTACK DECREASES WITH FLAP ANGLE***

High lift devices

- The lift equation $L = q_\infty S C_L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L$
  - Solving for V_∞ gives the true airspeed in unaccelerated level flight for a particular C_L

$$V_\infty = \sqrt{\frac{2L}{\rho_\infty S C_L}} = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

  - In level unaccelerated flight stall speed occurs when maximum C_L occurs

$$V_{stall} = \sqrt{\frac{2W}{\rho_\infty S C_{L_{max}}}}$$

- Flaps are not the only high lift device

High lift devices

- Other high lift devices include

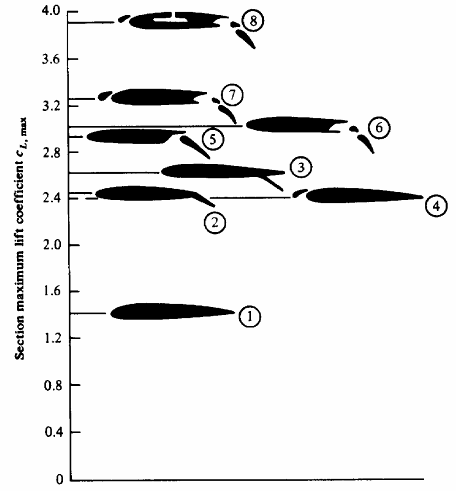


Figure 5.44 Typical values of airfoil maximum lift coefficient for various types of high-lift devices: (1) airfoil only, (2) plain flap, (3) split flap, (4) leading-edge slat, (5) single-slotted flap, (6) double-slotted flap, (7) double-slotted flap in combination with a leading-edge slat, (8) addition of boundary-layer suction at the top of the airfoil. (*From Loftin, NASA SP 468, 1985.*)