FREDERICK UNIVERSITY CYPRUS

Department of Mechanical Engineering

AMEE 401/ AUTO 400 **Instructor: Marios M. Fyrillas**
Aerodynamics **Email: m.fyrillas@frederick.ac.cy**

HOMEWORK ASSIGNMENT #1 SOLUTION

QUESTION 1

a. Explain the meaning of Drag force and give the expression of the dimensionless Drag coefficient.

The resultant force in the direction of the upstream velocity is termed the Drag Force (*D*). It has two contributions the skin friction, related to shear stress and the pressure drag, related to pressure differential between the front and the back of the body. The dimensionless Drag coefficient is given by:

$$\mathrm{C}_D = \frac{\text{Drag Force}}{\frac{1}{2}\rho U^2 A},$$

where A is usually the frontal area, the projected area seen by a person looking toward the object from a direction parallel to the upstream velocity, U.

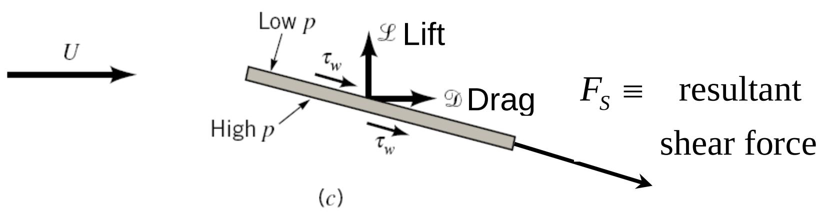
b. Explain the meaning of Lift force and give the expression of the dimensionless Lift coefficient.

The resultant force in the direction normal to the upstream velocity is termed the Lift Force (*L*). Similar to the Drag force it has two contributions however the contribution of the shear stress to the Lift Force is often negligible. The dimensionless Drag coefficient is given by:

$$\mathrm{C}_L = \frac{\text{Lift Force}}{\frac{1}{2}\rho U^2 A},$$

where A is usually the planform area, the projected area seen by a person looking toward the object from a direction normal to the upstream velocity, U.

c. In the Figure below identify the Drag force, the Lift Force, and show the direction of the total shear force.



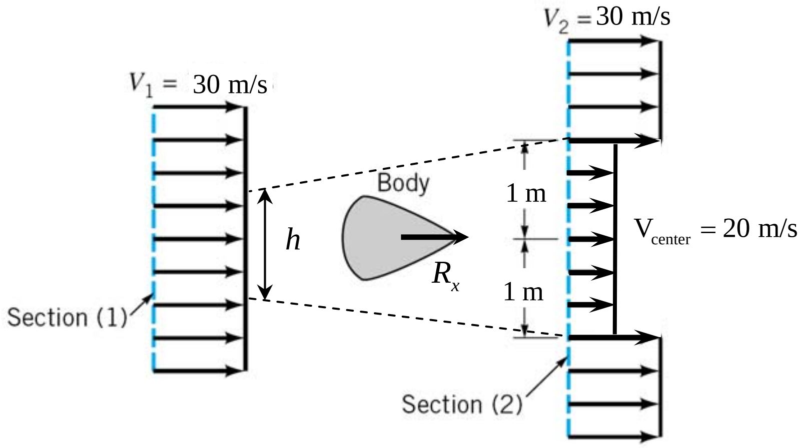
QUESTION 2

The results of a wind tunnel test to determine the drag on a body (see Figure) are summarized below. The upstream [section (1)] velocity is uniform at 30 m/s. The static pressures are given by $p_1 = p_2 = 14.7\text{ psia}$. The downstream velocity distribution [section (2)], which is symmetrical about the centerline, is assumed to be stepwise uniform (see Figure). Assume that the body shape does not change in the direction normal to the paper.

a. Using the mass conservation equation determine the dividing streamlines.

b. Using the momentum equation calculate the drag force (reaction force in x direction) exerted on the air by the body per unit length normal to the plane of the sketch.

c. Find the Drag coefficient C_D.



We choose a control volume within the streamlines ending at the velocity discontinuity on the right-hand side of the control volume and a point, which is going to be determined through mass conservation, on the left-hand side of the control volume.

Mass Conservation

Assuming unit depth the mass conservation states:

$$\rho h V_1 = \rho 2 V_{center} \Rightarrow h = 2\frac{20}{30} = 1.333\text{ m}$$

Momentum Conservation in the x - direction

Assuming unit depth the momentum conservation states:

$$\dot{M}_{out_x} - \dot{M}_{in_x} = -R_x$$

$$\Rightarrow R_x = \dot{m}_{in} u_{in_x} - \dot{m}_{out} u_{out_x} = \dot{m}\ (V_1 - V_{center}) = \rho h V_1 (30-20)$$

$$= 1.255 * 1.333 * 30 * 10 = 489\text{ N/m}$$

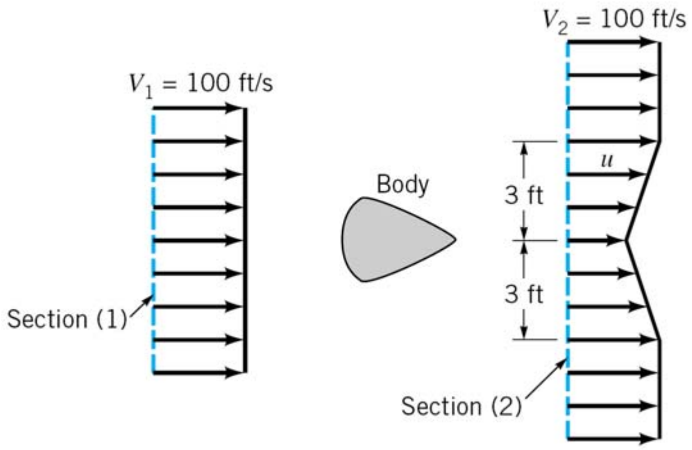
<u>QUESTION 3</u>

The results of a wind tunnel test to determine the drag on a body (see Figure) are summarized below. The upstream [section (1)] velocity is uniform at $100\ \text{ft/s}$**. The static pressures are given by** $p_1 = p_2 = 14.7\ \text{psia}$**. The downstream velocity distribution, which is symmetrical about the centerline, is given by**

$$u = 100 - 30\left(1 - \frac{|y|}{3}\right) \qquad |\text{y}| \leq 3\ \text{ft}$$

$$\text{u=100} \qquad |\text{y}| > 3\ \text{ft}$$

where u is the velocity in ft/s **and** y **is the distance on either side of the centerline in feet (see Figure). Assume that the body shape does not change in the direction normal to the paper. Calculate the drag force (reaction force in x direction) exerted on the air by the body per unit length normal to the plane of the sketch.**



5.23

5.23 The results of a wind tunnel test to determine the drag on a body (see Fig. P5.23) are summarized below. The upstream [section (1)] velocity is uniform at 100 ft/s. The static pressures are given by $p_1 = p_2 = 14.7$ psia. The downstream velocity distribution which is symmetrical about the centerline is given by

$$u = 100 - 30\left(1 - \frac{|y|}{3}\right) \qquad |y| \le 3 \text{ ft}$$

$$u = 100 \qquad |y| > 3 \text{ ft}$$

where u is the velocity in ft/s and y is the distance on either side of the centerline in feet (see Fig. P5.23). Assume that the body shape does not change in the direction normal to the paper. Calculate the drag force (reaction force in x direction) exerted on the air by the body per unit length normal to the plane of the sketch.

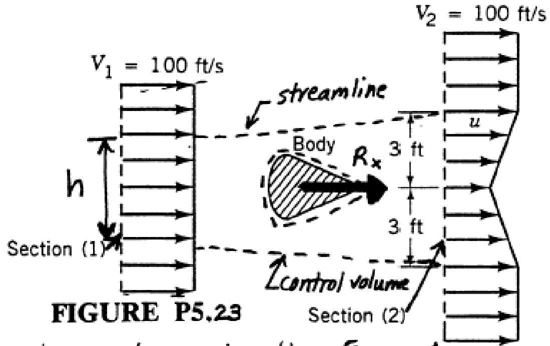


FIGURE P5.23

The control volume containing air only as shown in the figure is used. Application of the x direction component of the linear momentum equation leads to

$$-U_1 \rho U_1 A_1 + 2\int_0^{3\,ft} u \rho u \, dy = -R_x$$

or

$$R_x = \rho U_1^2 h - 2\rho \int_0^{3\,ft} \left[100 - 30\left(1 - \frac{y}{3}\right)\right]^2 dy \qquad (1)$$

To determine the distance h the conservation of mass equation is applied between sections (1) and (2) as follows

$$\rho h U_1 = 2\int_0^{3\,ft} \rho u \, dy$$

Thus

$$h = \frac{2}{U_1}\int_0^{3\,ft}\left[100 - 30\left(1 - \frac{y}{3}\right)\right] dy$$

or

$$h = \frac{(2)\left(255\ \frac{ft^3}{s}\right)}{\left(100\ \frac{ft}{s}\right)(1\ ft)} = 5.1\ ft$$

Then from Eq. 1

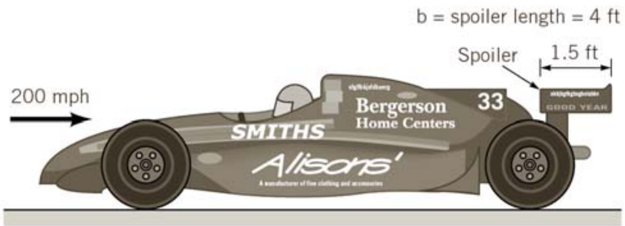
$$R_x = \left(0.00238\frac{slugs}{ft^3}\right)\left(100\ \frac{ft}{s}\right)^2 (5.1\,ft)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)(1\ ft)$$

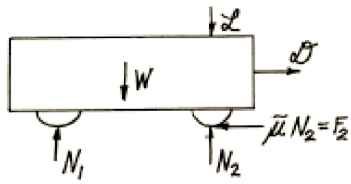
$$- 2\left(0.00238\frac{slugs}{ft^3}\right)\left(21,900\ \frac{ft^4}{s^2}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

$R_x = \underline{\underline{17.1\ lb}}$ per ft of length normal to the plane of the sketch

QUESTION 4

As shown in the Figure below, a spoiler is used on race cars to produce a negative lift, thereby giving a better tractive force. The lift coefficient for the airfoil shown is $C_L = 1.1$, and the coefficient of friction between the wheels and the pavement is 0.6. At a speed of 200 mph, by how much would use of the spoiler increase the maximum tractive force that could be generated between the wheels and ground? Assume the air speed past the spoiler equals the car speed and that the airfoil acts directly over the drive wheels.





Tractive force $= F_2 = \tilde{\mu} N_2$

where $\tilde{\mu}$ = coefficient of friction = 0.6

Thus,

$\Delta F_2 = \tilde{\mu}\,\Delta N_2 = \tilde{\mu}\,\mathscr{L}$, where ΔF_2 is the increase in tractive force due to the (downward) lift.

Hence, with $\mathcal{V} = 200\ mph = 293\ ft/s$,

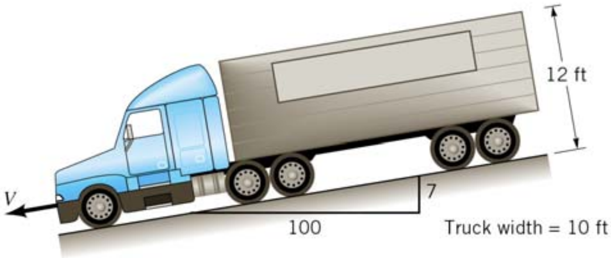
$\mathscr{L} = \frac{1}{2}\rho \mathcal{V}^2 C_L A = \frac{1}{2}(0.00238\ \frac{slugs}{ft^3})(293\ \frac{ft}{s})^2(1.1)(1.5 ft)(4 ft) = 674\ lb$,

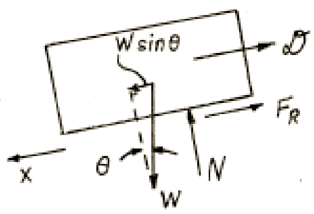
and

$\Delta F_2 = 0.6\,(674 lb) = \underline{\underline{405\ lb}}$

QUESTION 5

A 25-ton (50,000-lb) truck coasts down a steep mountain grade without brakes, as shown in Figure. The truck's ultimate steady-state speed, V, is determined by a balance between weight, rolling resistance, and aerodynamic drag. Assume that the rolling resistance for a truck on concrete is 1.2% of the weight and the drag coefficient is 0.76. Determine V.





For steady state $\Sigma F_x = ma_x = 0$, or $W\sin\theta - F_R - \mathcal{D} = 0$

Thus, with $\sin\theta = 7/[100^2+7^2]^{1/2} = 0.0698$ this becomes

$W\sin\theta = 0.012\,W + \frac{1}{2}\rho V^2 C_D A$, or

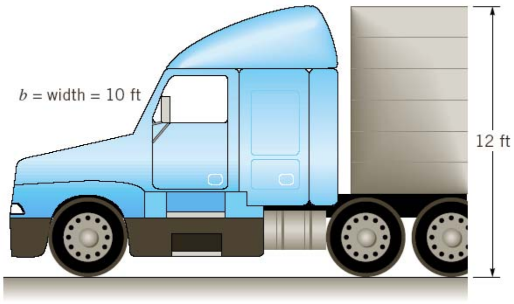
$50{,}000\ lb\ (0.0698) = 0.012\,(50{,}000) + \frac{1}{2}(0.00238\ \frac{slugs}{ft^3})(V\ \frac{ft}{s})^2(0.76)(10\times 12\ ft^2)$

Hence,

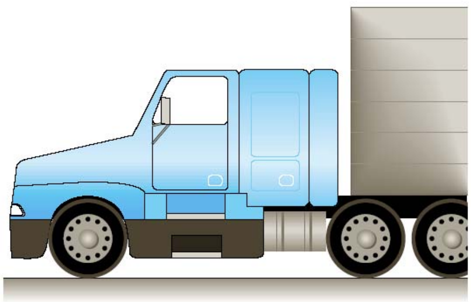
$V = 163\ \frac{ft}{s}\left(\frac{3600\ \frac{s}{hr}}{5280\ \frac{ft}{mi}}\right) = \underline{\underline{111\ mph}}$

QUESTION 6

As shown in the Figure, the aerodynamic drag on a truck can be reduced by the use of appropriate air deflectors. A reduction in drag coefficient from $C_D = 0.96$ to $C_D = 0.7$ corresponds to a reduction of how many horsepower needed at a highway speed of 65 mph?



(a) $C_D = 0.70$



(b) $C_D = 0.96$

$\mathcal{P}$ = power = $\mathcal{D}U$ where

$\mathcal{D} = \frac{1}{2}\rho U^2 C_D A$

Thus, $\Delta\mathcal{P}$ = reduction in power

$= \mathcal{P}_b - \mathcal{P}_a$

$= \frac{1}{2}\rho U^3 A\left[C_{D_b} - C_{D_a}\right]$

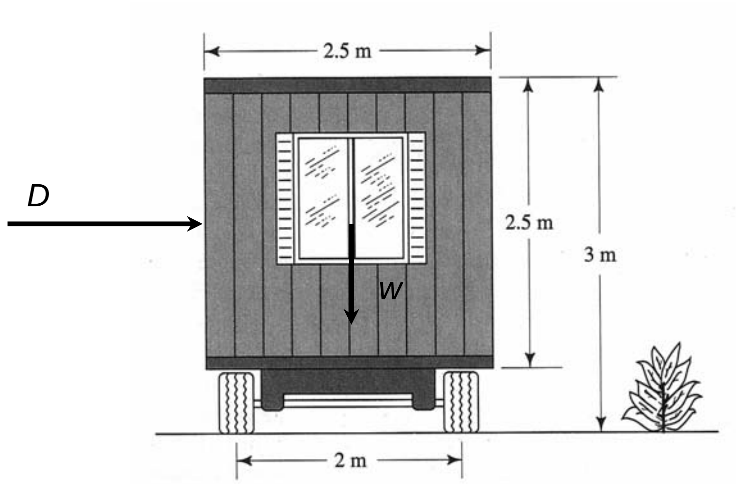
With $U = 65\text{ mph} = 95.3\text{ fps}$,

$$\Delta\mathcal{P} = \frac{1}{2}\left(0.00238\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(95.3\,\frac{\text{ft}}{\text{s}}\right)^3(10\text{ ft})(12\text{ ft})\left[0.96 - 0.70\right]$$

$$= 32{,}100\,\frac{\text{ft}\cdot\text{lb}}{\text{s}}\left(\frac{1\text{ hp}}{550\,\frac{\text{ft}\cdot\text{lb}}{\text{s}}}\right) = \underline{\underline{58.4\text{ hp}}}$$

QUESTION 7

Determine the wind velocity required to overturn the mobile home sketched in the Figure if it is 10m long and weighs 50 kN. Consider it to be a square cylinder.



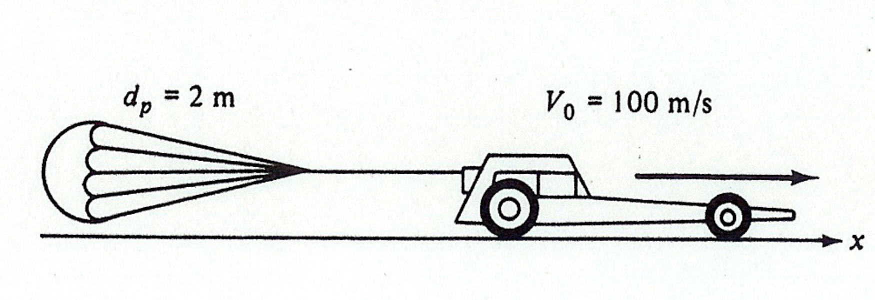
Assume center of gravity is at center of home. When overturning is imminent, no weight is on the right wheel. Then taking moments (M) about the right wheel:

$$\left.\begin{array}{l} \sum M = 0 = w*(1\text{ m}) - F_D\ (1.75\text{ m}) \ \Rightarrow\ F_D = \dfrac{w}{1.75}. \\ \text{But } F_D = C_D\left[\dfrac{1}{2}\rho u^2\right]A;\ \text{Assume } C_D = 2.05 \text{ square cylinder.} \\ \\ A\text{=(2.5 m) (10 m)=25 m}^2 \end{array}\right\} \Rightarrow$$

$$u = \sqrt{\frac{2F_D}{C_D\rho A}} = \sqrt{\frac{2\dfrac{w}{1.75}}{2.05*1.292*25}} = 0.131\sqrt{w}\ \text{m/s} = 29.3\ \text{m/s}$$

QUESTION 8

A high speed car with $m = 2000\text{kg}$, $C_D = 0.3$, and $A = 1\text{m}^2$, deploys a 2-m parachute to slow down form an initial velocity of 100 m/s. Assuming constant C_D, brakes free, and no rolling resistance, calculate the distance and velocity of the car after 1, 10, 100, and 1000 s. Neglect interference between the wake of the car and the parachute.



Newton's law applied in the direction of motion gives

$$F_x = m\frac{\mathrm{d}V}{\mathrm{d}t} = -F_c - F_p = -\frac{1}{2}\rho V^2 (C_{D_c} A_c + C_{D_p} A_p)$$

where subscript c is the car and subscript p the parachute.

This is of the form $\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{K}{m}V^2$ where $K = \frac{1}{2}\rho (C_{D_c} A_c + C_{D_p} A_p)$.

Separate the variables and integrate

$$\int_{V_0}^{V} \frac{\mathrm{d}V}{V^2}\mathrm{d}t = -\frac{K}{m}\int_0^t \mathrm{d}t \Rightarrow V_0^{-1} - V^{-1} = -\frac{K}{m}t \Rightarrow V^{-1} = V_0^{-1} + \frac{K}{m}t$$

$$\Rightarrow \boxed{V = \frac{1}{V_0^{-1} + Kt/m}}$$

Integrate once more to obtain the distance travelled $\frac{\mathrm{d}s}{\mathrm{d}t} = V = \frac{1}{V_0^{-1} + Kt/m}$

$$\Rightarrow \boxed{s - s_0 = \frac{V_0}{\alpha}\ \ln(1+\alpha t) \text{ where } \alpha = (K/m)\ V_0}$$

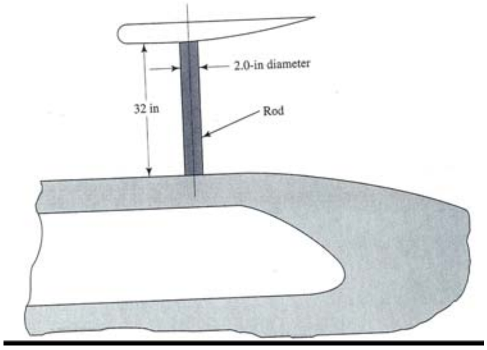
$C_{D_p} \approx 1.2$; Hence $C_{D_c} A_c + C_{D_p} A_p = 0.3*1 + 1.2*(\pi/4)*2^2 = 4.07\text{m}^2$

$\alpha = 0.122\ \text{s}^{-1}$

t, s	1	10	100	1000
V, m/s	89	45	7.6	0.8
S, m	94	654	2100	3940

QUESTION 9

A wing on a race car is supported by two cylindrical rods, as shown in the Figure. Compute the drag force exerted on the car due to these rods when the car is traveling through still air at a speed of 150 mph. Compute the drag if the cylindrical rods are replaced by elongated elliptical cylinders having a length to breath ration of 8:1.



$$\mathrm{Re} = \frac{\rho u D}{\mu} = 3.15\times10^5 \Rightarrow C_D = 0.8$$

$$A = DL = 0.0826\ \mathrm{m}^2$$

$$F_D = C_D \frac{1}{2}\rho u^2 A = 214.4\ \mathrm{N}$$

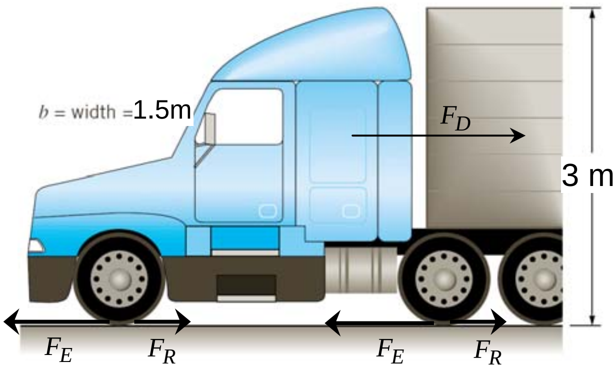
For elliptical cylinder

$$\mathrm{Re} = \frac{\rho u L}{\mu} = 2.5\times10^6 \Rightarrow C_D = 0.2 \Rightarrow F_D = 53.6\ \mathrm{N}$$

So there is 75% reduction

QUESTION 10

A 25000 kg truck coasts in a highway at 60 km/hr. Determine the power required assuming that the rolling resistance for a truck on concrete is 1.2% of the weight and the drag coefficient is 0.96.



The track is moving at constant speed, hence the sum of forces acting on the track must be zero. There are three forces acting on the track, the Drag Force F_D, theRolling Resistance F_R and the Forward Force from the engine through the wheels F_E.

$\sum F = 0 \Rightarrow F_D + F_R = F_E$.

The power (P) required to overcome F_D and F_R is equal to

$$P = (F_D + F_R)U = \left(C_D \frac{1}{2} \rho U^2 A + 0.012\ W \right) U =$$

$$= \left(0.96\ \ \frac{1}{2}\ \ 1.225\ \left(\frac{60\ 10^3}{3600} \right)^2\ \ 1.5\ 3\ +\ 0.012\ 25000\ 9.81 \right) \frac{60\ 10^3}{3600}$$

$$= 61.3\text{kW}$$

QUESTION 11

A small spherical air-bubble is rising in still water (see Figure). Show that if the density of the air in the bubble is neglected from the calculations, then the size of the bubble can be estimated through the equation:

$$D = \sqrt{\frac{18\,U\mu}{\rho g}}\,.$$

The drag coefficient of a sphere is $C_D = \dfrac{24}{\mathrm{Re}}$ **for** $\mathrm{Re} < 1$**.**

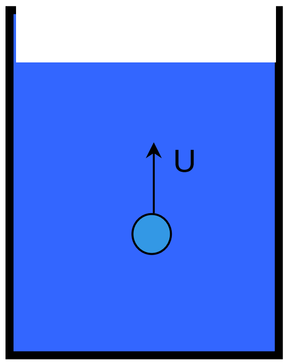
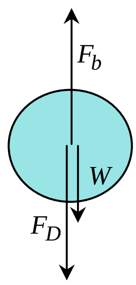


Figure: Rising air-bubble in a container



The forces acting on the bubble are the drag force F_D, the bouyancy force F_b and the weight W. If the bubble is moving at a constant velocity the forces must balance. If we neglect the weight then:

$$F_D = F_b \Rightarrow \frac{1}{2}\rho U^2 A\, C_D = \cancel{V}\rho g \Rightarrow \frac{1}{2}\rho U^2 \frac{\pi D^2}{4}\,\frac{24}{\mathrm{Re}} = \frac{\pi D^3}{6}\rho g$$

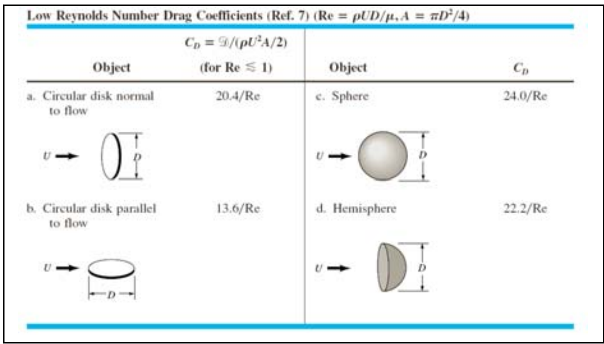
$$\Rightarrow \frac{1}{2}\rho U^2 \frac{\pi D^2}{4}\,\frac{24}{\dfrac{\rho U D}{\mu}} = \frac{\pi D^3}{6}\rho g$$

Solving above equation for D

$$D = \sqrt{\frac{18\,U\mu}{\rho g}}$$

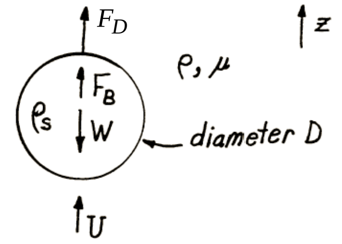
QUESTION 12

A sphere of diameter D and density ρ_s falls at a steady rate through a liquid of density ρ_s and viscosity μ. If the Reynolds number is less than 1, show that the viscosity can be determined from $\mu = \dfrac{gD^2(\rho_s - \rho)}{18\,U}$.

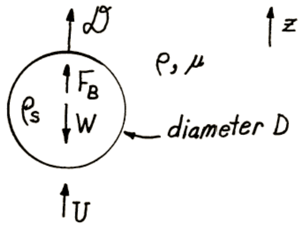


Low Reynolds Number Drag Coefficients (Ref. 7) (Re = $\rho UD/\mu$, $A = \pi D^2/4$)

Object	$C_D = \mathscr{D}/(\rho U^2 A/2)$ (for Re $\lesssim$ 1)	Object	C_D
a. Circular disk normal to flow	20.4/Re	c. Sphere	24.0/Re
b. Circular disk parallel to flow	13.6/Re	d. Hemisphere	22.2/Re



9.22 A sphere of diameter D and density ρ_s falls at a steady rate through a liquid of density ρ and viscosity μ. If the Reynolds number, Re $= \rho DU/\mu$, is less than 1, show that the viscosity can be determined from $\mu = gD^2(\rho_s - \rho)/18\,U$.



For steady flow $\sum F_z = 0$

or $\mathscr{D} + F_B = W$, where F_B = buoyant force $= \rho g \forall = \rho g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$

W = weight $= \rho_s g \forall = \rho_s g \left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$

and $\mathscr{D}$ = drag $= C_D \frac{1}{2}\rho \frac{\pi}{4} D^2$, or since Re<1

$\mathscr{D} = 3\pi D U \mu$

Thus,

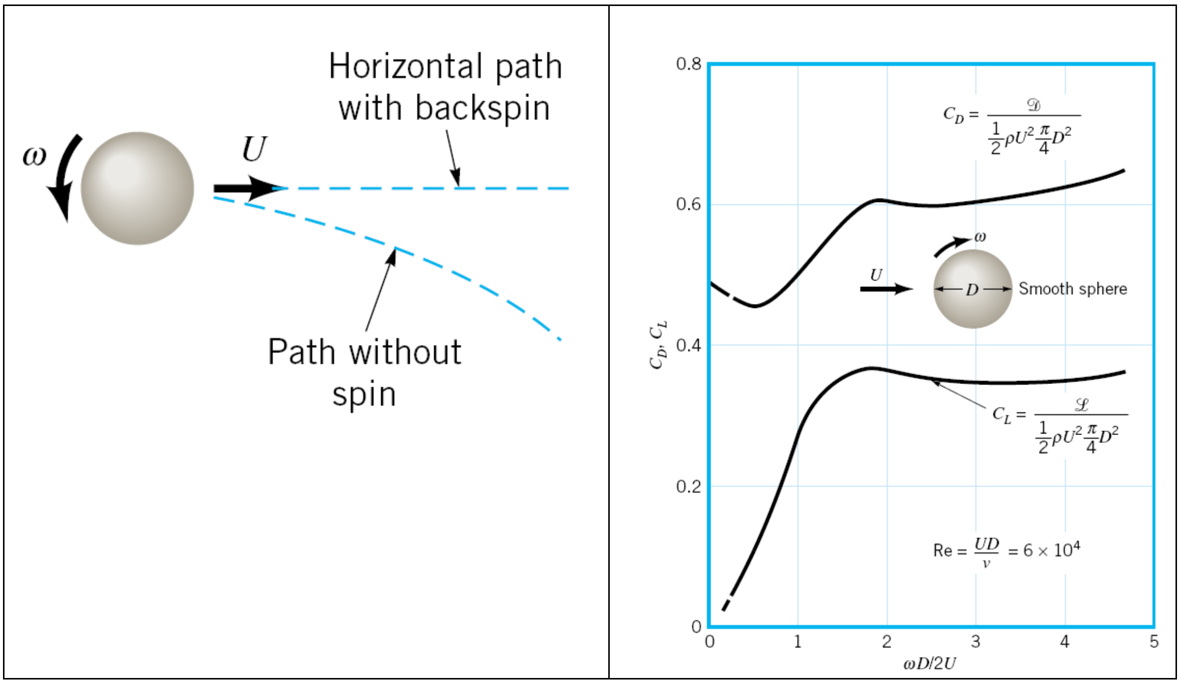
$$3\pi D U\mu + \rho g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3 = \rho_s g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$$

which can be rearranged to give

$$\mu = \frac{g D^2(\rho_s - \rho)}{18\,U}$$

QUESTION 13

A table tennis ball weighing $2.45\times10^{-2}\mathrm{N}$ with diameter $D=3.8\times10^{-2}\mathrm{m}$ is hit at a velocity of $U=12\mathrm{m/s}$ with a back spin of angular velocity ω as is shown in the figure. What is the value of ω if the ball is to travel on a horizontal path, not dropping due to the acceleration of gravity?



For horizontal flight, the lift generated by the spinning of the ball must exactly balance the weight, $\mathcal{W}$, of the ball so that

$$\mathcal{W} = \mathcal{L} = \tfrac{1}{2}\rho U^2 A C_L$$

or

$$C_L = \frac{2\mathcal{W}}{\rho U^2(\pi/4)D^2}$$

where the lift coefficient, C_L, can be obtained from Fig. 9.39. For standard atmospheric conditions with $\rho = 1.23\ \mathrm{kg/m^3}$ we obtain

$$C_L = \frac{2(2.45 \times 10^{-2}\ \mathrm{N})}{(1.23\ \mathrm{kg/m^3})(12\ \mathrm{m/s})^2(\pi/4)(3.8 \times 10^{-2}\ \mathrm{m})^2} = 0.244$$

which, according to Fig. 9.39, can be achieved if

$$\frac{\omega D}{2U} = 0.9$$

or

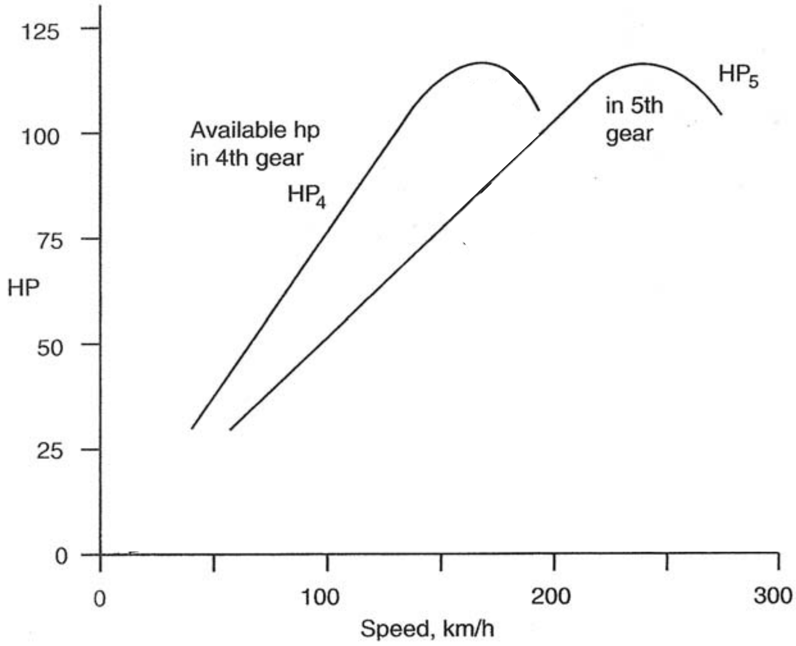
$$\omega = \frac{2U(0.9)}{D} = \frac{2(12\ \mathrm{m/s})(0.9)}{3.8 \times 10^{-2}\ \mathrm{m}} = 568\ \mathrm{rad/s}$$

Thus,

$$\omega = (568\ \mathrm{rad/s})(60\ \mathrm{s/min})(1\ \mathrm{rev}/2\pi\ \mathrm{rad}) = 5420\ \mathrm{rpm} \qquad \text{(Ans)}$$

QUESTION 14

In the figure, we show two curves representing the maximum engine output (in terms of horsepower, HP) of a vehicle for two different gear ratios, HP4 and HP5. Determine the maximum speed given that the drag coefficient $C_D = 0.5$, the frontal area of the car is $A = 2\ \mathrm{m}^2$, the rolling resistance is 2% of the vehicles weight and the mass of the vehicle is $1000\ \mathrm{kg}$.



The maximum steady-state speed of a vehicle is achieved when the maximum available driving force at that speed equals the resistance force due to aerodynamic drag and tire resistance.
Typically, the maximum engine output, initially increases with engine RPM, and the maximum engine output can be related, through the overall gear ratio, to the vehicle's speed. This maximum available driving power is shown schematically for two gear ratios, HP4 and HP5, corresponding to fourth and fifth gears of a passenger car.

The total opposing force, which is a sum of tire rolling resistance and the aerodynamic drag), increases rapidly with speed, and its power requirement is obtained through

$$P = F_{tot} \times V = \left(F_{Drag} + F_{Rolling}\right) \times V = \left(C_D \frac{1}{2} \rho V^2 A + C_R \ W\right) \times V$$

$$= \left(0.5 \ \frac{1}{2} \ 1.225 \ V^2 \ 2 + 0.02 \ 1000 \ 9.81\right) \times V = (2.45 \ V^2 + 196.2) \times V$$

Usually, the increase in tire rolling resistance is marginal compared to the rapid increase in the aerodynamic drag.
At a specific gear, the speed attained at maximum driving power is a balance between the available power and the power required, i.e. the intersection of the HP4 and HP5 curves with the power curve. Lower velocities are achieved by partial throttle.
Careful gear selection allows a higher maximum speed in the lower gear, while the highest gear ratio is usually designed for good fuel economy. The latter avoids reaching the maximum output range of the engine, rather it operates at maximum efficiency of the engine.
The primary conclusion is that total resistance increases rapidly with speed, and selection of the proper gear ratio for maximum vehicle speed requires engine, tire and aerodynamic data.