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**Semua Pengarah Ukur dan Pemetaan Negeri**
**Semua Pengarah Ukur Bahagian / Pengarah Bahagian**

**PEKELILING KETUA PENGARAH UKUR DAN PEMETAAN**
**BILANGAN 3 TAHUN 2021**

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# GARIS PANDUAN TEKNIKAL MENGENAI PENUKARAN KOORDINAT, TRANSFORMASI DATUM DAN UNJURAN PETA UNTUK TUJUAN UKUR DAN PEMETAAN

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## 1. TUJUAN

Pekeliling ini bertujuan untuk memberikan garis panduan teknikal mengenai kaedah-kaedah penukaran koordinat, transformasi datum dan unjuran peta bagi kegunaan kerja-kerja ukur dan pemetaan.

## 2. LATAR BELAKANG

2.1. Jabatan Ukur dan Pemetaan Malaysia (JUPEM) bertanggungjawab melaksanakan kerja-kerja ukur hakmilik tanah dan pemetaan asas. Di dalam memenuhi tanggungjawab tersebut, JUPEM telah menyediakan produk dan perkhidmatan yang berasaskan koordinat bagi memenuhi keperluan semasa pelanggan.

2.2. Mutakhir ini, teknologi satelit *Global Navigation Satellite System* (GNSS) semakin giat digunakan di dalam melaksanakan kerja-kerja ukur dan pemetaan bagi mendapatkan koordinat dengan cepat dan tepat di Malaysia.

2.3. Kaedah penentududukan dan penentuan koordinat berasaskan teknologi satelit GNSS telah menggunakan datum geodetik yang berlainan daripada datum-datum geodetik konvensional di Malaysia. Ketidakharmonian di antara koordinat yang berpunca daripada penggunaan datum geodetik yang berlainan ini, jika sekiranya tidak diuruskan dengan baik, boleh dan sering menimbulkan kesilapan di dalam kerja-kerja pengukuran serta produk ukur dan pemetaan.

2.4. Oleh itu, maklumat mengenai kaedah penukaran koordinat, transformasi datum dan unjuran peta yang sedia ada wajar dikemaskini dan didokumentasikan supaya ianya sentiasa relevan dengan kehendak dan tuntutan semasa serta dapat dijadikan panduan dan rujukan kepada para pengguna dalam menguruskan kerja-kerja yang mempunyai kaitan dengan koordinat. Sehubungan dengan itu, Pekeliling ini diharapkan akan dapat membantu pengguna-pengguna produk-produk pemetaan, kadaster, utiliti dan sebagainya untuk memahami kaedah-kaedah tersebut dan seterusnya dapat menggunakannya dengan cara yang betul di dalam urusan kerja mereka, terutamanya di dalam era penggunaan teknologi GNSS yang semakin meluas di Malaysia.

## 3. PERKEMBANGAN PEMBANGUNAN INFRASTRUKTUR GEODETIK DI MALAYSIA

3.1. Sejak penubuhan Jabatan Ukur dan Pemetaan Malaysia (JUPEM) lebih daripada 135 tahun yang lalu, datum geodetik yang digunapakai untuk kegunaan ukur dan pemetaan telah mengalami pelbagai perubahan. Dalam hal ini, sebelum tahun 1990an, Datum Kertau telah pun menjadi tulang belakang kepada *Malayan Revised Triangulation* 1968 (MRT68) di Semenanjung Malaysia, manakala bagi Sabah, Sarawak dan Labuan pula, *Borneo Triangulation* 1968 (BT68) merujuk kepada Datum Timbalai sebagai asasnya.

3.2. Dengan perkembangan teknologi GNSS yang meluas sekitar tahun 1980-an, JUPEM telah membangunkan rangkaian kawalan ukur geodetik yang baharu dengan menggunakan teknologi tersebut untuk menentukan koordinat bagi stesen-stesen kawalan. Bagi Semenanjung Malaysia, rangkaian ini telah ditubuhkan dalam tahun 1994 dan dikenali sebagai *Peninsular Malaysia Geodetic Scientific Network 1994* (PMGSN94). Manakala di Sabah, Sarawak dan Labuan pula, rangkaian kawalan ukur geodetiknya adalah *East Malaysia Geodetic Scientific Network 1997* (EMGSN97), yang ditubuhkan pada tahun 1997.

3.3. Antara tahun 1998 dan 2001 pula, JUPEM telah membangunkan rangkaian *Malaysia Active GPS System* (MASS) dan ini telah diikuti dengan *Malaysia Real-Time Kinematic GNSS Network* (MyRTKnet) antara tahun 2002 dan 2008. Dalam pada itu, pada 26 Ogos 2003, JUPEM telah melancarkan *Geocentric Datum of Malaysia 2000* (GDM2000), iaitu datum rujukan geodetik baharu yang seragam bagi kerja-kerja ukur dan pemetaan di Malaysia.

3.4. Walau bagaimanapun, GDM2000 telah mengalami anjakan yang signifikan, lanjutan daripada berlakunya beberapa siri gempa bumi besar di Indonesia, terutamanya pada tahun 2004 dan 2005. Sehubungan itu, datum ini telah disemak dan dihitung semula dengan mengambil kira penambahan stesen MyRTKnet pada tahun 2006 yang dikenali sebagai GDM2000 (Rev 2006). Pada tahun 2007, telah berlaku satu gempa bumi besar di Sumatera, Indonesia yang juga telah menyebabkan anjakan signifikan dan JUPEM telah melaksanakan semakan dan hitungan semula berdasarkan anjakan ke atas stesen-stesen MyRTKnet dan menghasilkan GDM2000 (Rev 2009).

3.5. Koordinat GDM2000 dan siri-siri kemaskininya diklasifikasikan sebagai datum statik di mana koordinat dianggap tidak berubah dan merujuk kepada *International Terrestrial Reference Frame* (ITRF) 2000 pada epok 1 Januari 2000. Koordinat dalam datum statik perlu sentiasa dikemaskini bagi mengekalkan ketepatan dengan mengambilkira kesan daripada fenomena alam seperti gempa bumi, pergerakan plat tektonik dan deformasi setempat. Pada masa yang sama, ITRF juga telah dikemaskini beberapa kali dan realisasi terkini dikenali sebagai ITRF2014 yang diperkenalkan pada 22 Januari 2016.

3.6. Bagi mengekalkan ketepatan koordinat pada stesen-stesen MyRTKnet, JUPEM telah membangunkan datum baharu berasaskan konsep semi kinematik melalui permodelan pergerakan stesen-stesen yang terlibat dan koordinat dikemaskini mengikut sela masa tertentu.

3.7. Maklumat lebih lanjut mengenai sistem rujukan koordinat terkandung di dalam Pekeliling Ketua Pengarah Ukur dan Pemetaan Malaysia Bilangan 2 Tahun 2021 bertarikh 13 Oktober 2021 bertajuk Garis Panduan Mengenai Sistem Rujukan Koordinat Bagi Tujuan Ukur dan Pemetaan di Malaysia. Ia menyenaraikan semua jenis sistem rujukan koordinat yang telah dibangunkan oleh JUPEM di Malaysia di samping memberi maklumat-maklumat teknikal mengenainya.

## 4. GARIS PANDUAN PENUKARAN KOORDINAT, TRANSFORMASI DATUM DAN UNJURAN PETA UNTUK TUJUAN UKUR DAN PEMETAAN

Penerangan lebih lanjut tentang amalan penggunaan penukaran koordinat, transformasi datum dan unjuran peta terkandung di dalam dokumen *Technical Guide to the Coordinate Conversion, Datum Transformation, Map Projection* seperti di **Lampiran 'A'** yang disertakan. Intisari garis panduan tersebut adalah seperti berikut:

**5. TARIKH BERKUATKUASA**

Pekeliling ini adalah berkuatkuasa mulai tarikh ianya dikeluarkan.

**6. PEMBATALAN**

Dengan berkuatkuasanya Pekeliling ini, maka Pekeliling Ketua Pengarah Ukur dan Pemetaan Bilangan 3 Tahun 2009 bertajuk Garis Panduan Mengenai Penukaran Koordinat, Transformasi Datum dan Unjuran Peta untuk Tujuan Ukur dan Pemetaan adalah dengan ini dibatalkan.

Sekian, terima kasih.

**"WAWASAN KEMAKMURAN BERSAMA 2030"**
**"BERKHIDMAT UNTUK NEGARA"**

**(DATO' Sr DR. AZHARI BIN MOHAMED)**
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Lampiran 'A'

# *Technical Guide to the Coordinate Conversion, Datum Transformation and Map Projection*

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## 1. INTRODUCTION

1.1 The Department of Survey and Mapping Malaysia (JUPEM) defines and maintains the *Coordinate Reference Systems* (CRS) and the Vertical Reference System (VRS) for the whole country. It establishes and manages these geodetic infrastructures for the purpose of the cadastral survey, mapping, engineering and scientific research. The CRS that have been introduced and used since the late 1800s in Malaysia are listed in **Table 1**.

**Table 1**: Coordinate Reference Systems in Malaysia

| No. | Coordinate Reference System | |
|---|---|---|
| | **Reference Frame** | **Geodetic Datum** |
| 1. | Malayan Revised Triangulation 1968 (MRT68) | **KERTAU**<br>Ellipsoid: Modified Everest |
| 2. | Borneo Triangulation 1968 (BT68) | **TIMBALAI**<br>Ellipsoid: Modified Everest |
| 3. | Peninsular Malaysia Geodetic Scientific Network 1994 (PMGSN94) | **WGS84**<br>Ellipsoid: WGS84<br>Reference Frame: WGS84<br>Epoch: 1987.0 |
| 4. | East Malaysia Geodetic Scientific Network 1997 (EMGSN97) | **WGS84**<br>Ellipsoid: WGS84<br>Reference Frame: WGS84 (G783)<br>Epoch: 1997.0 |
| 5. | Malaysia Active GPS System (MASS) | **GDM2000**<br>Ellipsoid: GRS80<br>Reference Frame: ITRF2000<br>Epoch: 2000.0 |
| 6. | Malaysia Primary Geodetic Network 2000 (MPGN2000) | **GDM2000**<br>Ellipsoid: GRS80<br>Reference Frame: ITRF2000<br>Epoch: 2000.0 |
| | | **GDM2000 (Rev 2006)**<br>Ellipsoid: GRS80<br>Reference Frame: ITRF2000<br>Epoch: 2000.0 |

**Table 1**: (*continued*)

<table>
<tr><th rowspan="2">No.</th><th colspan="2">Coordinate Reference System</th></tr>
<tr><th>Reference Frame</th><th>Geodetic Datum</th></tr>
<tr><td rowspan="5">7.</td><td rowspan="5">Malaysia Real-Time Kinematic GNSS Network (MyRTKnet)</td><td>GDM2000<br><br>Ellipsoid: GRS80<br>Reference Frame: ITRF2000<br>Epoch: 2000.0</td></tr>
<tr><td>GDM2000 (Rev 2006)<br><br>Ellipsoid: GRS80<br>Reference Frame: ITRF2000<br>Epoch: 2000.0</td></tr>
<tr><td>GDM2000 (Rev 2009)<br><br>Ellipsoid: GRS80<br>Reference Frame: ITRF2000<br>Epoch: 2000.0</td></tr>
<tr><td>GDM2000 (Rev 2016)<br><br>Ellipsoid: GRS80<br>Reference Frame: ITRF2000<br>Epoch: 2000.0</td></tr>
<tr><td>GDM2020<br><br>Ellipsoid: GRS80<br>Reference Frame: ITRF2014<br>Epoch: 2020.0</td></tr>
</table>

1.2 **Figure 1** represents a schematic diagram to assist users in navigating between different types of coordinates system. All CRS available in Malaysia are shown; some are on the same geodetic datum and others on different ones. Coordinate conversions which do not involve a change of datum are shown as dashed lines. Datum transformations, on the other hand, involve a change of datum and are shown as thick solid lines and map projection as double solid lines.

1.3 This technical guide is produced to assist users in understanding the concept and procedures involved in the process of coordinate conversion, datum transformation and map projection as practised in Malaysia.

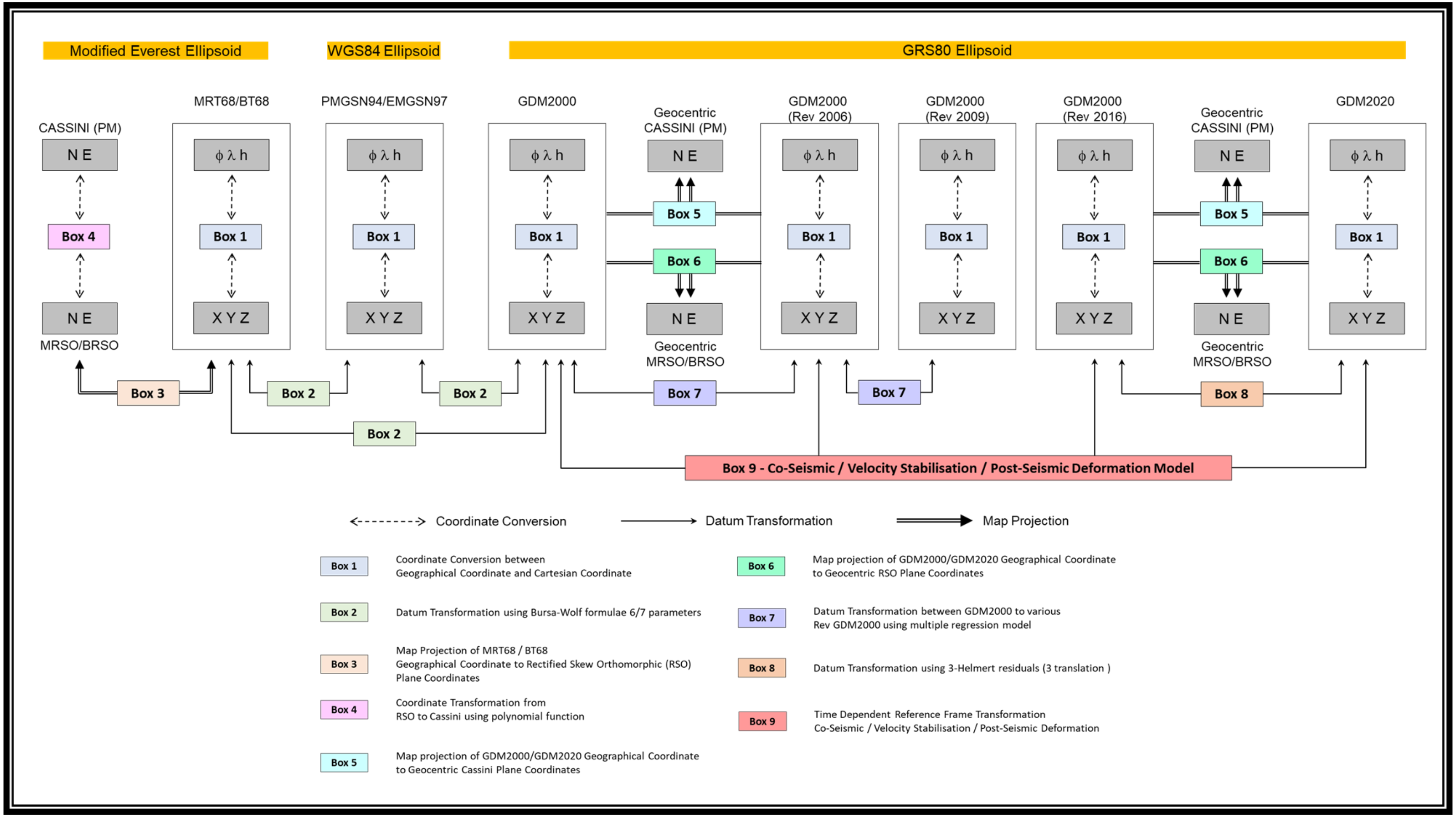



**Figure 1:** Relationship between Various CRS

## 2. COORDINATE CONVERSION

### 2.1 Geographical and Cartesian Coordinates

2.1.1 Three-dimensional geographical coordinates can be defined with respect to an ellipsoid as follows:

Latitude, $\phi$ the angle north or south from the equatorial plane

Longitude, $\lambda$ the angle east or west from the prime meridian

Height, h the distance above the surface of the ellipsoid

2.1.2 A set of cartesian coordinates is defined with the three axes at the origin at the centre of the ellipsoid, such that:

Z-axis: is aligned with the minor (or polar) axis of the ellipsoid

X-axis: is in the equatorial plane and aligned with the prime meridian

Y-axis: forms a right-handed system

2.1.3 In this regard, positions in geographical coordinates of latitude, longitude and height ($\phi$, $\lambda$, h) can be converted into cartesian coordinates (X, Y, Z) and vice-versa as shown in **Table 2** below:

**Table 2**: Geographical and Cartesian Coordinates

| Geographical Coordinates | Cartesian Coordinates |
|---|---|
| Latitude, Longitude and Height ($\phi$, $\lambda$, h) | X Y Z |

### 2.2 Conversion between Geographical Coordinates and Cartesian Coordinates

2.2.1 The conversion of three-dimensional coordinates from geographical to cartesian or vice versa can be carried out through the knowledge of the parameters of an adopted reference ellipsoid (**Figure 2**). The forward conversion from geodetic coordinates ($\phi$, $\lambda$, $h$) to cartesian coordinate (X, Y, Z) is as follows:

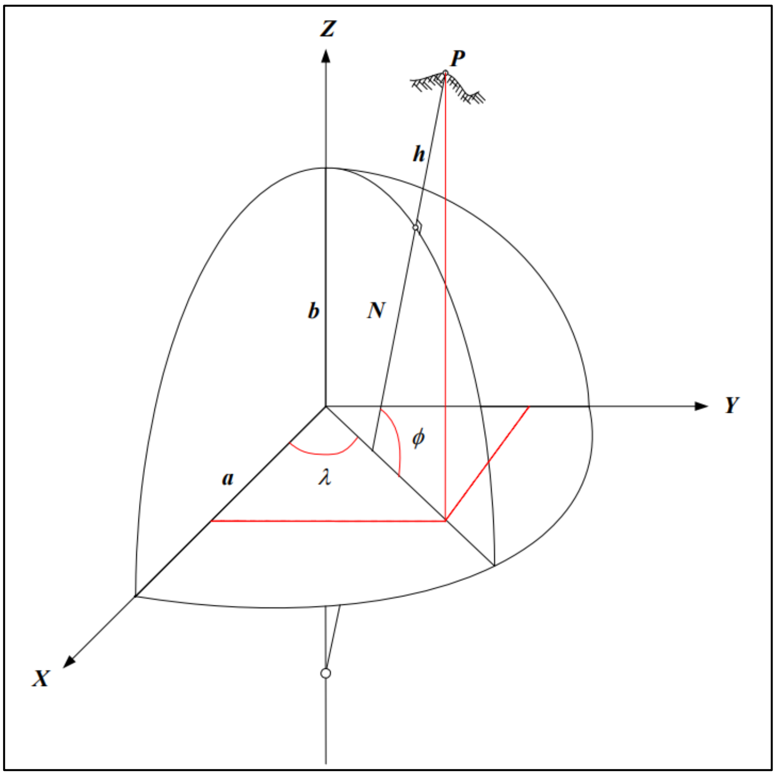



**Figure 2**: Geographical and Cartesian Coordinates

$$X = (N + h)\cos\phi\cos\lambda$$

$$Y = (N + h)\cos\phi\sin\lambda$$

$$Z = \left(\frac{b^2}{a^2}N + h\right)\sin\phi$$

where the prime vertical radius of curvature (N) is:

$$N = \frac{a^2}{\sqrt{\left(a^2\cos^2\phi + b^2\sin^2\phi\right)}}$$

with:

a : the semi-major axis of the reference ellipsoid;

b : the semi-minor axis of the reference ellipsoid;

2.2.2 While the reverse conversion from cartesian coordinates (X, Y, Z) to geodetic coordinates ($\phi, \lambda, h$) is as follows:

$$\phi = tan^{-1} \frac{Z \pm \varepsilon^2 \, b \, sin^3 \, u}{P - e^2 \, a \, cos^3 \, u}$$

$$\lambda = tan^{-1} \frac{y}{x}$$

$$h = P \, cos \, \phi + Z \, sin \, \phi - a\sqrt{1 - e^2 \, sin^2 \, \phi}$$

with:

$$u = tan^{-1} \frac{aZ}{bP}$$

$$P = \sqrt{X^2 + Y^2}$$

$$\varepsilon = \frac{e^2}{1 - e^2}$$

$$e^2 = \frac{a^2 - b^2}{a^2}$$

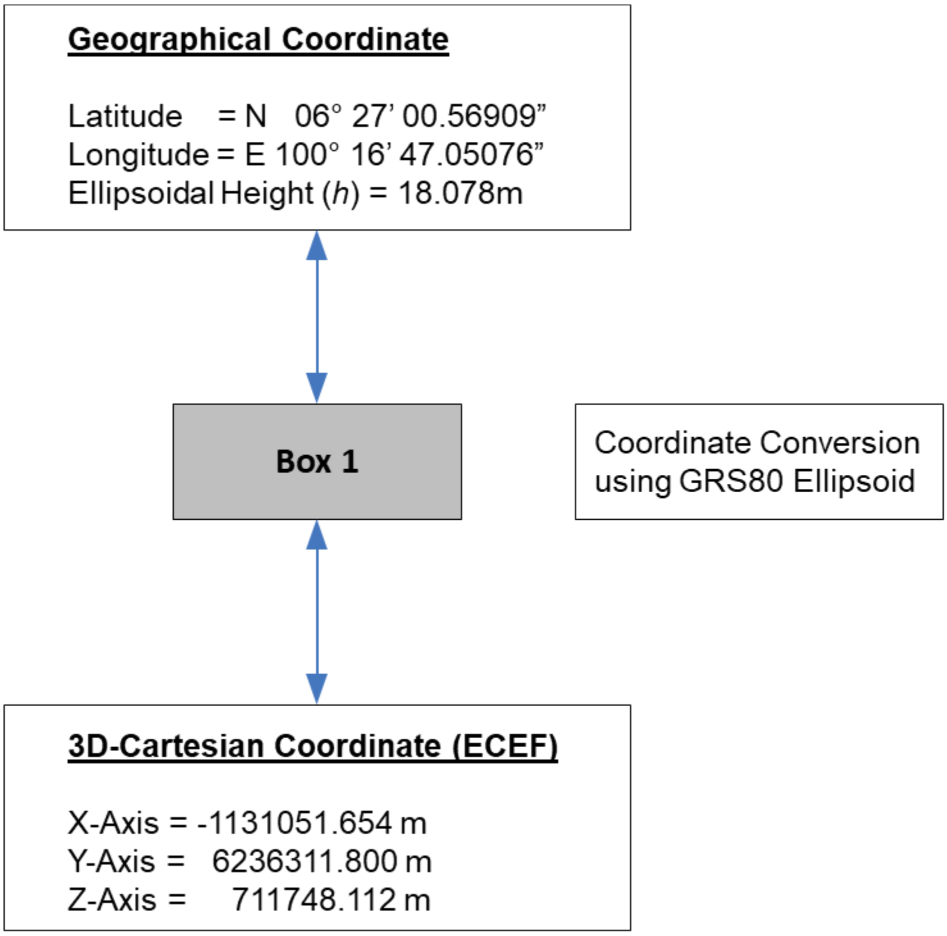
where,

$u$ : the parametric latitude;

$e$ : the first eccentricity of the reference ellipsoid;

$\varepsilon$ : the second eccentricity of the reference ellipsoid.

### 2.3 Test Example



# 3. DATUM TRANSFORMATION

## 3.1 Introduction

3.1.1 Datum transformation is a computational process of converting a position given in one CRS into the corresponding position in another CRS. It requires and uses the parameters of the transformation and the ellipsoids associated with the source and target CRS. For example, the source can be CRS 1, and the target can be CRS 2 as listed in **Table 3**:

**Table 3**: Coordinate Reference Systems

| No. | Coordinate Reference System 1 | Coordinate Reference System 2 |
|---|---|---|
| 1 | Geocentric Datum of Malaysia (GDM2000) | Geocentric Datum of Malaysia GDM2020 |
| | | Geocentric Datum of Malaysia GDM2000 (Rev 2006, Rev 2009, Rev 2016) |
| | | Peninsular Malaysia Geodetic Scientific Network 1994 (PMGSN94) |
| | | East Malaysia Geodetic Scientific Network 1997 (EMGSN97) |
| | | Malayan Revised Triangulation 1968 (MRT68) |
| | | Borneo Triangulation 1968 (BT68) (Sabah) |
| | | Borneo Triangulation 1968 (BT68) (Sarawak) |
| 2 | Peninsular Malaysia Geodetic Scientific Network 1994 (PMGSN94) | Malayan Revised Triangulation 1968 (MRT68) |
| 3 | East Malaysia Geodetic Scientific Network 1997 (EMGSN97) | Borneo Triangulation 1968 (BT68) (Sabah) |
| | | Borneo Triangulation 1968 (BT68) (Sarawak) |

3.1.2 The transformation parameter values associated with the transformation can be determined empirically from a measurement or a calculation process. The parameters are computed based on coordinates of control stations which are common to different datums. They are generated through least square analysis using various models accepted by the global geodetic community.

3.1.3 Datum transformation can be accomplished by many different methods. A simple three-parameter conversion can be accomplished by conversion through Earth-Centred Earth Fixed (ECEF) cartesian coordinates from one reference datum to another by three origin offsets that approximate differences in rotation, translation and scale. A complete datum conversion is usually based on seven parameter transformations, which include three translation parameters, three rotation parameters and a scale.

## 3.2 Bursa-Wolf Datum Transformation Formulae

3.2.1 Bursa-Wolf formulae is a seven-parameter model for transforming three-dimensional cartesian coordinates between two datums (see **Figure 3**). This transformation model is more suitable for satellite datums on a global scale (Krakwisky and Thomson, 1974). The transformation involves three geocentric datum shift parameters ($\Delta X$, $\Delta Y$, $\Delta Z$), three rotation elements ($R_X$, $R_Y$, $R_Z$) and a scale factor ($1+\Delta L$).

3.2.2 The model in its matrix-vector form could be written as (Burford 1985):

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{GDM2000} = \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} + \begin{pmatrix} 1+\Delta L & R_Z & -R_Y \\ -R_Z & 1+\Delta L & R_X \\ R_Y & -R_X & 1+\Delta L \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{MRT68}$$

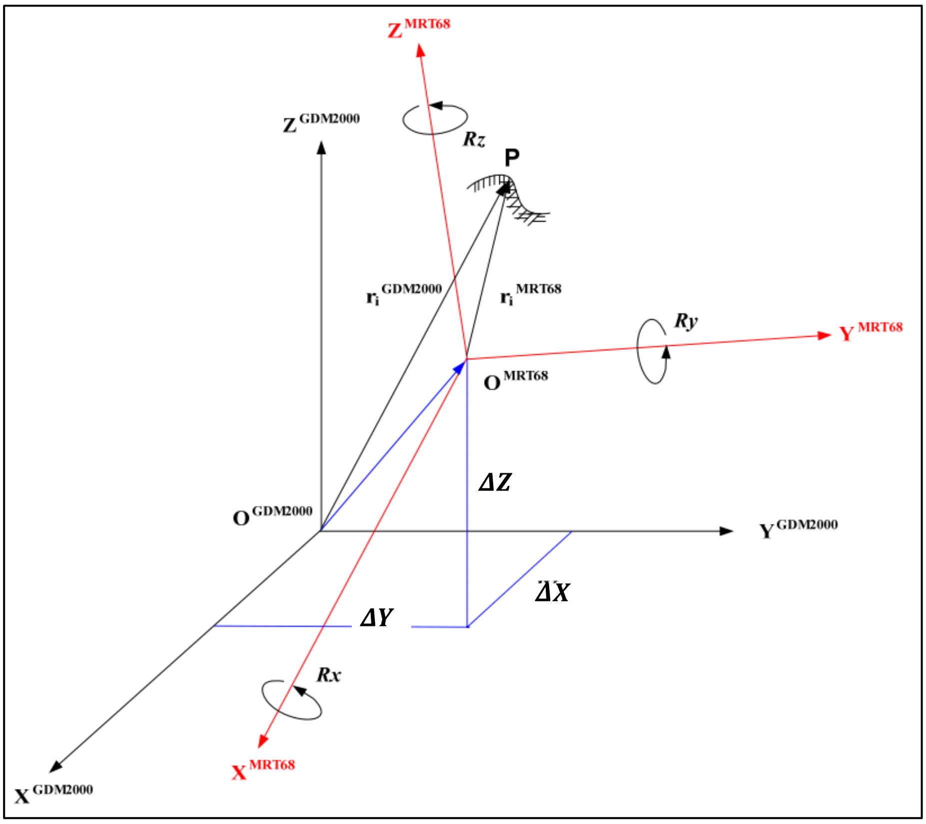



**Figure 3**: Bursa-Wolf 3-D Model Transformation

where;

$(X, Y, Z)_{GDM2000}$ : are the global datum (GDM2000) cartesian coordinates;

$(X, Y, Z)_{MRT68}$ : are the local datum (MRT) cartesian coordinates.

In order to convert the cartesian coordinates of XYZ to the geographical coordinates of $\phi$ $\lambda$ h, ellipsoid properties for the respective datum as listed in **Table 4** below are used:

**Table 4**: Ellipsoid Properties

| No. | Ellipsoid | semi-major axis, a (m) | flattening, f (m) |
|---|---|---|---|
| 1 | Geodetic Reference System 1980 (GRS80) | 6378137.000 | 298.2572221 |
| 2 | World Geodetic System 1984 (WGS84) | 6378137.000 | 298.2572236 |
| 3 | Modified Everest (Peninsular Malaysia) | 6377304.063 | 300.8017 |
| 4 | Modified Everest (East Malaysia) | 6377298.556 | 300.8017 |

## 3.3 Multiple Regression Model

Multiple linear regression, also known as multiple regression, is a model (statistical technique) that uses several explanatory variables to predict the outcome of a response variable. The goal of multiple linear regression is to model the linear relationship (displacement) between the explanatory (independent) variables and response (dependent) variables.

### 3.3.1 Displacement Computation

The computation and modelling of differences in the coordinate between two systems, i.e. GDM2000 and GDM2000 (Rev 2009) are carried out using their respective geographical coordinates in the format of ($\phi$, $\lambda$, h). These differences are then converted to the local geodetic horizon to avoid mathematical errors. The conversion from geographical system to local geodetic system uses the following factor, i.e. 1" = 30 metres.

The differences in the three components are computed separately by using the following formulae:

$$\Delta\text{North (N)} = ( \phi''_{\text{GDM2000}} - \phi''_{\text{GDM2000 (Rev 2009)}} ) \times 30$$

$$\Delta\text{East (E)} = ( \lambda''_{\text{GDM2000}} - \lambda''_{\text{GDM2000 (Rev 2009)}} ) \times 30$$

$$\Delta\text{Height (U)} = ( h_{\text{GDM2000}} - h_{\text{GDM2000 (Rev 2009)}} )$$

### 3.3.2 Displacement Modelling

The differences in coordinate of every GPS station are similarly computed between the two systems, i.e. GDM2000 and GDM2000 (Rev 2009), using their respective coordinates in the format of E and N. The coordinate differences are then gridded to derive the Regression Coefficient. The gridding method uses the polynomial regression with the power to the second order. The surface definition uses the bi-linear saddle regression coefficient with the following formulae:

$$Z(E,N,U) = A_{00} + A_{01}\,N + A_{10}\,E + A_{11}\,EN$$

where,

Z = Value of regression coefficient or the value of displacement correction for each component (i.e. East, North and Up)

E = East coordinate in decimal degree

N = North coordinate in decimal degree

### 3.3.3 Basic Formulae

To convert the coordinate in GDM2000 to GDM2000 (Rev 2009), the following formulae shall be used:

GDM2000 (Rev 2009) = GDM2000 + correction

$$\phi''_{\text{GDM2000 (Rev 2009)}} = \phi''_{\text{GDM2000}} + (Z_N / 30)$$

$$\lambda''_{\text{GDM2000 (Rev 2009)}} = \lambda''_{\text{GDM2000}} + (Z_E / 30)$$

$$h_{\text{GDM2000 (Rev 2009)}} = h_{\text{GDM2000}} + Z_U$$

where,

$\phi''$ = Latitude in second of arc

$\lambda''$ = Longitude in second of arc

h = Ellipsoidal Height in meter

$Z_N$ = Displacement correction in northing

$Z_E$ = Displacement correction in easting

$Z_U$ = Displacement correction in height

$A_{00}$, $A_{01}$, $A_{10}$ and $A_{11}$ are the coefficients of the multiple regression model.

### 3.4 Time-Dependent Reference Frame

#### 3.4.1 Introduction

A more accurate and current Malaysian geodetic reference frame has been determined, fully aligned and compatible with ITRF2014 by taking advantage of the GNSS data availability from MyRTKnet. This reference frame is based on the kinematic concept, where time-dependent elements are modelled as an implicit component of the Continuously Operating Reference Stations (CORS) coordinates. A kinematic datum includes a deformation model consisting of a velocity field that allows the estimation of the plate velocity at any point in the country and patches of modelled displacements to account for substantial ground movements.

Continuously changing coordinates in a kinematic datum present significant challenges for most spatial data users, particularly in the cadastral database acquired at different epochs that need to be integrated harmoniously to meet the legal requirements. On the contrary, a semi-kinematic reference frame enables the time-dependent coordinates to be transformed consistently and accurately to a fixed reference epoch over time; thus, providing a more practical approach in handling spatial databases coordinate systems.

**Table 5**: Types of Geodetic Reference Frames Concerning the Time-Dependent Coordinates

| Reference Frames | Trajectory Models | |
|---|---|---|
| Static | $X(t) = X(t_0)$ | 1 |
| Kinematic | $X(t) = X(t_0) + X^v(t - t_0) + \delta X_{PSD}(t)$ | 2 |
| Semi-Kinematic | $X(t) = X(t_0) + X^v(t - t_0) + \delta X_{PSD}(t)$ | 3 |
| | $X(t_{DB}) = X(t) + X^v(t_{DB} - t) + \delta X_{PSD}(t_{DB}) - \delta X_{PSD}(t)$ | 4 |

**Remarks:**

[1] CORS and databases coordinates are fixed at a specific pre-seismic reference epoch $t_0$

[2] CORS and databases coordinates are continuously updated

[3] CORS coordinates are continuously updated

[4] Quasi-static databases coordinates are fixed at a specific post-seismic reference epoch $t_{DB}$

**Table 5** above provides the description of static, kinematic and semi-kinematic geodetic reference frame with respect to the time-dependent coordinates. A semi-kinematic geodetic reference frame consists of both kinematic and quasi-static coordinates. The quasi-static coordinates of the spatial databases in a semi-kinematic geodetic reference frame always refer to a specific epoch and do not vary until they exceed a certain critical level and the geodetic reference frame is updated while the CORS kinematic coordinates are updated regularly. The coordinates of CORS and spatial databases in a kinematic geodetic reference frame change regularly with time.

### 3.4.2 Station Trajectory Model

The kinematic descriptions of station trajectory model or station displacement X(t), can be decomposed to a geocentric cartesian axis system $[\Delta X, \Delta Y, \Delta Z]^t$ or in a local or topocentric cartesian axis system $[\Delta e, \Delta n, \Delta up]^t$ with east, north and up axes. Analysis of station position time series of the CORS network will provide trajectory model parameters such as linear velocity, offsets, e.g. from the instantaneous co-seismic position jumps, seasonal signal, and post-seismic displacements (PSD).

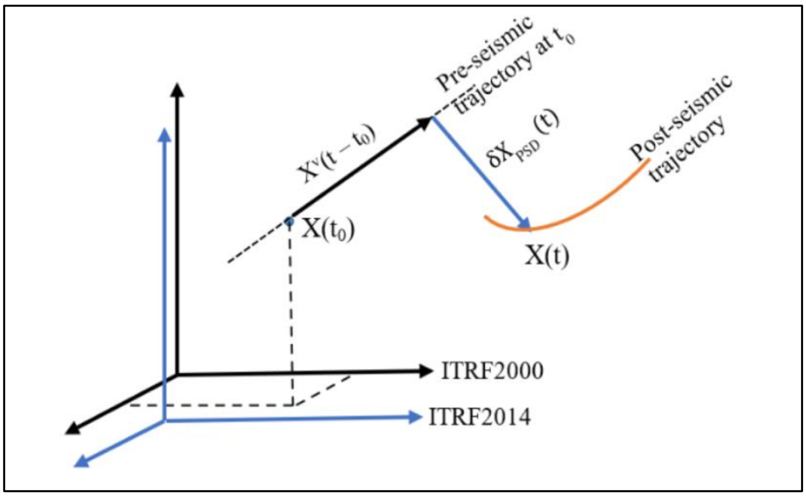



**Figure 4**: Time-Dependent Position of a Station

The time-dependent position of a station during the post-seismic trajectory X at an epoch t is defined as:

$$X(t) = X(t_0) + X^v(t - t_0) + \delta X_{PSD}(t)$$

where,

$t_0$ = the pre-seismic reference epoch

$X^v$ = the inter-seismic linear velocity due to the plate tectonic motion

$\delta X_{PSD}(t)$ = the total sum of the time-dependent non-linear PSD at epoch t, (ΔX, ΔY, ΔZ)

$$= \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD(t)}$$

where,

$\phi$ = Latitude of the station

$\lambda$ = Longitude of the station

$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD(t)}$ = provided in JUPEM's PSD-info log file

### 3.4.3 Transformation between Reference Frame

The transformation from any ITRFyy to ITRF2014 reference frame with data sets at any given epoch t can be given by the time-dependent 14-parameters Helmert transformation (Altamimi et al., 2002) as follows:

$$X_{ITRF2014}(t) = X_{ITRFyy}(t) + \left[T + T^v\left(t - t^{ref}\right)\right] + \left[D + D^v\left(t - t^{ref}\right)\right] \cdot X_{ITRFyy}(t) + \left[R + R^v\left(t - t^{ref}\right)\right] \cdot X_{ITRFyy}(t)$$

where,

$t^{ref}$ = the reference epoch for ITRF2014

$T$ = the translation in meter

$D$ = the unitless scale factor

$R$ = the rotation matrix in arc-second

$T^v$, $D^v$, and $R^v$ = the rates per year of the translation vector, scale factor and rotation matrix as in **Figure 5**.

while the time-dependent position of $X_{ITRFyy}(t)$ concerning a post-seismic reference epoch $t^{ref}$ is derived as follows:

$$X_{ITRFyy}(t) = X_{ITRFyy}\left(t^{ref}\right) + X^v_{ITRFyy}\left(t - t^{ref}\right) + \delta X_{PSD}(t) - \delta X_{PSD}\left(t^{ref}\right)$$

**Figure 5**: Transformation Parameters and Their Rates from ITRF2014 to Past ITRFs

(Source: http://itrf.ign.fr/doc_ITRF/Transfo-ITRF2014_ITRFs.txt)

Transformation parameters from ITRF2014 to past ITRFs.

| SOLUTION | Tx | Ty | Tz | D | Rx | Ry | Rz | EPOCH |
|---|---|---|---|---|---|---|---|---|
| UNITS----------> | mm | mm | mm | ppb | .001" | .001" | .001" | |
| | . | . | . | . | . | . | . | |
| RATES | Tx | Ty | Tz | D | Rx | Ry | Rz | |
| UNITS----------> | mm/y | mm/y | mm/y | ppb/y | .001"/y | .001"/y | .001"/y | |
| ITRF2008 | 1.6 | 1.9 | 2.4 | -0.02 | 0.00 | 0.00 | 0.00 | 2010.0 |
| rates | 0.0 | 0.0 | -0.1 | 0.03 | 0.00 | 0.00 | 0.00 | |
| ITRF2005 | 2.6 | 1.0 | -2.3 | 0.92 | 0.00 | 0.00 | 0.00 | 2010.0 |
| rates | 0.3 | 0.0 | -0.1 | 0.03 | 0.00 | 0.00 | 0.00 | |
| ITRF2000 | 0.7 | 1.2 | -26.1 | 2.12 | 0.00 | 0.00 | 0.00 | 2010.0 |
| rates | 0.1 | 0.1 | -1.9 | 0.11 | 0.00 | 0.00 | 0.00 | |

By propagating GDM2000 coordinates from the pre-seismic reference epoch $t_0 = 2000.0$ to ITRF2014 reference epoch $t^{ref} = 2010.0$, $X_{ITRF2000}\left(t^{ref}\right)$ as follows:

$$X_{ITRF2000}(2010.0) = X_{ITRF2000}(2000.0) + 10yr \cdot X^{v}_{ITRF2000} + \delta X_{PSD}(2010.0)$$

The station velocities in ITRF2000 can be derived from the velocity model in ITRF2014:

$$X^{v}_{ITRF2000} = X^{v}_{ITRF2014} - T^{v} - D^{v} \cdot X_{ITRF2000} - R^{v} \cdot X_{ITRF2000}$$

### 3.4.4 Gridded Velocities

The velocities information generated using CATREF software is only available at the Malaysian CORS. In order for the information to be used regionally, JUPEM has generated gridded velocities for Peninsular and East Malaysia. These gridded velocities files can be obtained from JUPEM.

The raw gridded velocities model for Peninsular Malaysia were generated using 79 CORS, while for East Malaysia, 38 CORS were used. Station velocity values were taken from the latest time series solutions. The data filtering process was done using the kriging / collocation technique. Considering the sparse CORS interval of between 30 to 70 km, the grid mesh was set at 5 arc minutes (~9 km) for Peninsular Malaysia and 10 arc minutes (~18 km) for East Malaysia.

### 3.4.5 Gridded Co-Seismic + PSD Corrections $\delta X_{PSD}(t)$

PSD modelling can be done using four parametric models: (1) logarithmic, (2) exponential, (3) logarithmic + exponential, or (4) a combination of two exponential functions. The PSD parametric models were determined from the MyRTKnet daily position time series at sites where the PSD was judged to be significant. The adjustment of the PSD parametric models was operated separately for the east, north and height components.

The PSD correction for specific epochs was calculated based on the information stored in the PSD's SINEX file, i.e., "psdmodel.snx". CORS affected by the earthquakes have been listed in the file as one of the data inputs, which comprised a total of 55 CORS (active and decommissioned) in Peninsular Malaysia. For stations affected by multiple earthquakes, the PSD corrections computation will iteratively loop through all earthquake events prior to the epoch date. The total cumulative PSD correction is then applied at the observation epoch.

Time-based grid models are formulated to regionally estimate the amount of cumulative co-seismic + PSD correction for the semi-kinematic reference frame. These gridded PSD files can be obtained from JUPEM.

### 3.5 Test Examples

(i) Propagate cartesian coordinate of MyRTKnet station ARAU GDM2020@2020.0 to year 2022.0 with linear velocity and without PSD correction:

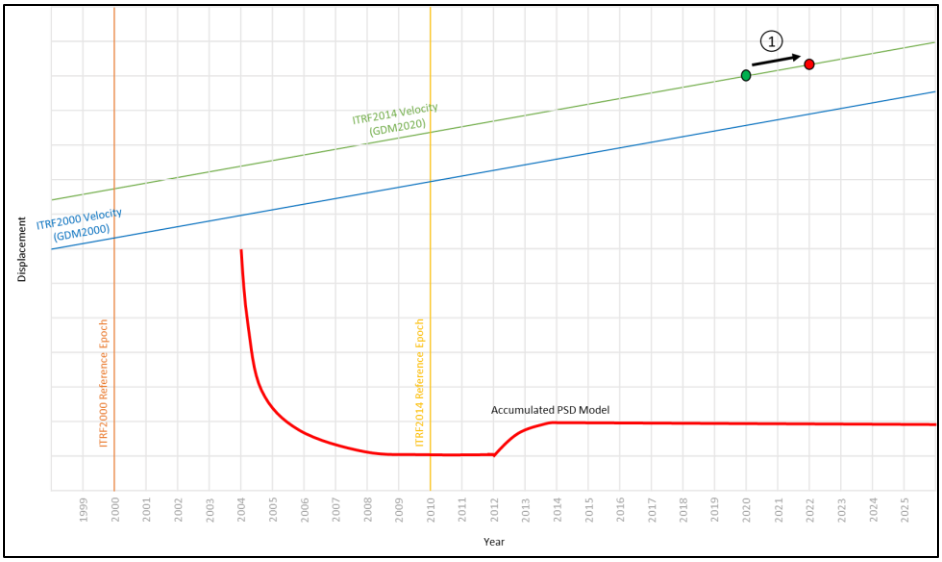



**Figure 6**: Transformation Step from GDM2020@2020.0 to Year 2022.0 With Linear Velocity and Without PSD Correction

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_t = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t_0} + (t - t_0) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{GDM2020}$$

where,

$t$ = 2022

$t_0$ = 2020

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020} = \begin{bmatrix} -1131052.06100 \\ 6236311.72370 \\ 711747.96520 \end{bmatrix} = \begin{bmatrix} \phi & 6.4501567685 \\ \lambda & 100.2797400641 \\ h & 18.05967 \end{bmatrix}$$

Propagate GDM2020@2020.0 to GDM2020@2022.0 coordinates:

Refer MAL2020A.SSC for the velocity of MyRTKnet station:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{GDM2020} = \begin{bmatrix} -0.01867 \\ -0.00155 \\ -0.00487 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2022} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020} + (t - t_0) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{GDM2020}$$

$$= \begin{bmatrix} -1131052.09834 \\ 6236311.72060 \\ 711747.95546 \end{bmatrix}$$

(ii) Propagate cartesian coordinate of MyRTKnet station ARAU GDM2020@2020.0 to year 2022.0 with correction of PSD@2022.0:

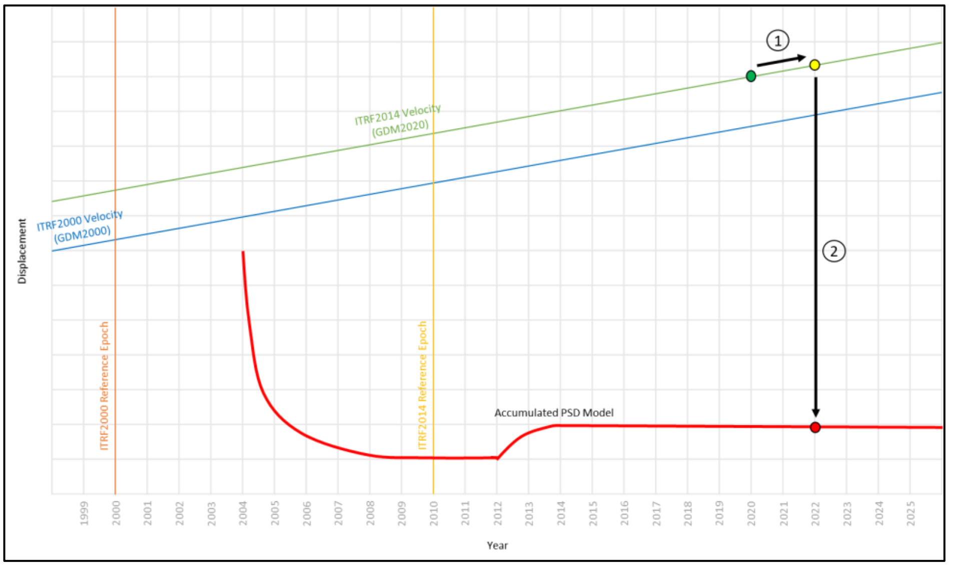



**Figure 7**: Transformation Step from GDM2020@2020.0 to Year 2022.0 With Correction of PSD@2022.0

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_t = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t_0} + (t - t_0)\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{GDM2020@2020} + \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2022.0}$$

where,

$t$ = 2022

$t_0$ = 2020

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020} = \begin{bmatrix} -1131052.06100 \\ 6236311.72370 \\ 711747.96520 \end{bmatrix} = \begin{bmatrix} \phi & 6.4501567685 \\ \lambda & 100.2797400641 \\ h & 18.05967 \end{bmatrix}$$

**<u>Step 1</u>:**

Propagate GDM2020@2020.0 to GDM2020@2022.0 coordinates:

Refer to MAL2020A.SSC for the velocity of MyRTKnet station:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{GDM2020@2020} = \begin{bmatrix} -0.01867 \\ -0.00155 \\ -0.00487 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\mathrm{GDM2020@2022.0}} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\mathrm{GDM2020@2020.0}} + (t - t_0)\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{GDM2020@2020}$$

$$= \begin{bmatrix} -1131052.09834 \\ 6236311.72060 \\ 711747.95546 \end{bmatrix}$$

**<u>Step 2</u>:**

Apply PSD@2022.0 for GDM2020@2022.0 coordinates using geodetic (ϕ, λ, h) coordinates:

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2022.0} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD@2022.0}$$

Refer to 22001psd-info.log

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD@2022.0} = \begin{bmatrix} -0.05148 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2022.0} = \begin{bmatrix} -0.05065 \\ 0.00919 \\ 0.00000 \end{bmatrix}$$

So,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2022} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{GDM2020@2022.0}} + \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2022.0}$$

$$= \begin{bmatrix} -1131052.04769 \\ 6236311.72979 \\ 711747.95546 \end{bmatrix}$$

(iii) Propagate cartesian coordinate of MyRTKnet station ARAU GDM2020@2020.0 with correction of PSD@2020.0 to year 2010.0 and correction of PSD@2010.0:

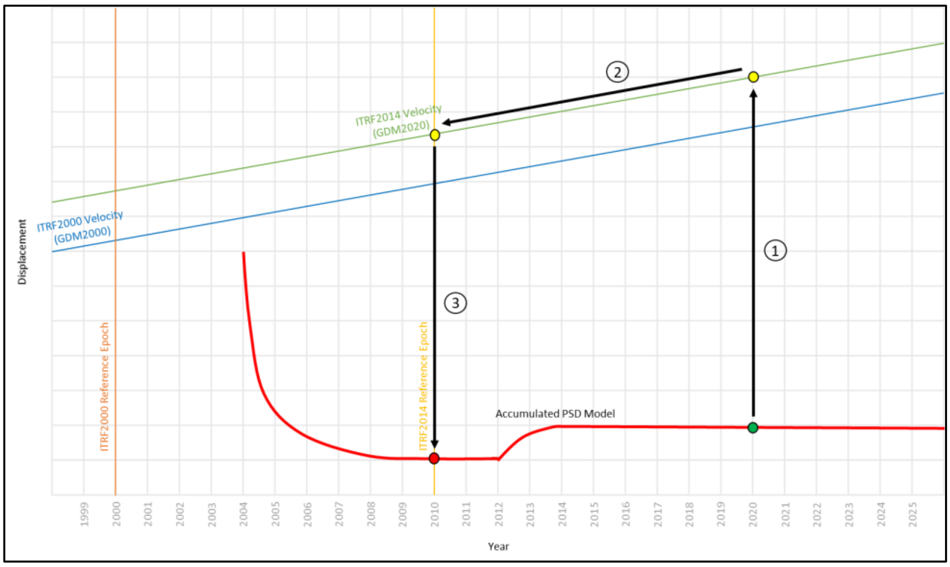



**Figure 8**: Transformation Step from GDM2020@2020.0 With Correction of PSD@2020.0 to Year 2010.0 and Correction of PSD@2010.0

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_t = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{t_0} - \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2020.0} + (t - t_0) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{GDM2020} + \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2010.0}$$

where,

$t$ = 2010

$t_0$ = 2020

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020} = \begin{bmatrix} -1131052.06100 \\ 6236311.72370 \\ 711747.96520 \end{bmatrix} = \begin{bmatrix} \phi & 6.4501567685 \\ \lambda & 100.2797400641 \\ h & 18.05967 \end{bmatrix}$$

<u>**Step1**</u>**:**

Correction of GDM2020@2020 coordinates by removing PSD@2020.0 using geodetic (ϕ, λ, h) coordinates:

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2020.0} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD@2020.0}$$

Refer to 20001psd-info.log

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD@2020.0} = \begin{bmatrix} -0.05147 \\ 0.00000 \\ 0.00000 \end{bmatrix}$$

$$\begin{bmatrix}\delta_x\\ \delta_y\\ \delta_z\end{bmatrix}_{PSD@2020.0} = \begin{bmatrix}0.05064\\ 0.00919\\ 0.00000\end{bmatrix}$$

So,

$$\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_{GDM2020@2020} = \begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_{GDM2020@2020} - \begin{bmatrix}\delta_x\\ \delta_y\\ \delta_z\end{bmatrix}_{PSD@2020.0}$$

$$= \begin{bmatrix}-1131052.11164\\ 6236311.71451\\ 711747.96520\end{bmatrix}$$

**<u>Step 2</u>:**

Propagate corrected GDM2020@2020.0 coordinates to GDM2020@2010.0 coordinates:

Refer to MAL2020A.SSC for the velocity of MyRTKnet station:

$$\begin{bmatrix}V_x\\ V_y\\ V_z\end{bmatrix}_{GDM2020} = \begin{bmatrix}-0.01867\\ -0.00155\\ -0.00487\end{bmatrix}$$

$$\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_{GDM2020@2010.0} = \begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_{GDM2020@2020.0} + (t - t_0)\begin{bmatrix}V_x\\ V_y\\ V_z\end{bmatrix}_{GMD2020}$$

$$= \begin{bmatrix}-1131051.92494\\ 6236311.73001\\ 711748.01390\end{bmatrix}$$

**<u>Step 3</u>:**

Apply PSD@2010.0 for GDM2020@2010.0 coordinates using geodetic ($\phi$, $\lambda$, h) coordinates:

$$\begin{bmatrix}\delta_x\\ \delta_y\\ \delta_z\end{bmatrix}_{PSD@2010.0} = \begin{bmatrix}-\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda\\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda\\ 0 & \cos\phi & \sin\phi\end{bmatrix} * \begin{bmatrix}\Delta e\\ \Delta n\\ \Delta up\end{bmatrix}_{PSD@2010.0}$$

Refer to 10001psd-info.log:

$$\begin{bmatrix}\Delta e\\ \Delta n\\ \Delta up\end{bmatrix}_{PSD@2010.0} = \begin{bmatrix}-0.05906\\ 0.00000\\ 0.00000\end{bmatrix}$$

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2010.0} = \begin{bmatrix} 0.05811 \\ 0.01054 \\ 0 \end{bmatrix}$$

So,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2010} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2010.0} + \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2010.0}$$

$$= \begin{bmatrix} -1131051.86683 \\ 6236311.74055 \\ 711748.01390 \end{bmatrix}$$

(iv) Propagate Cartesian Coordinate GDM2000@2000.0 to GDM2020@2020.0 with PSD correction:

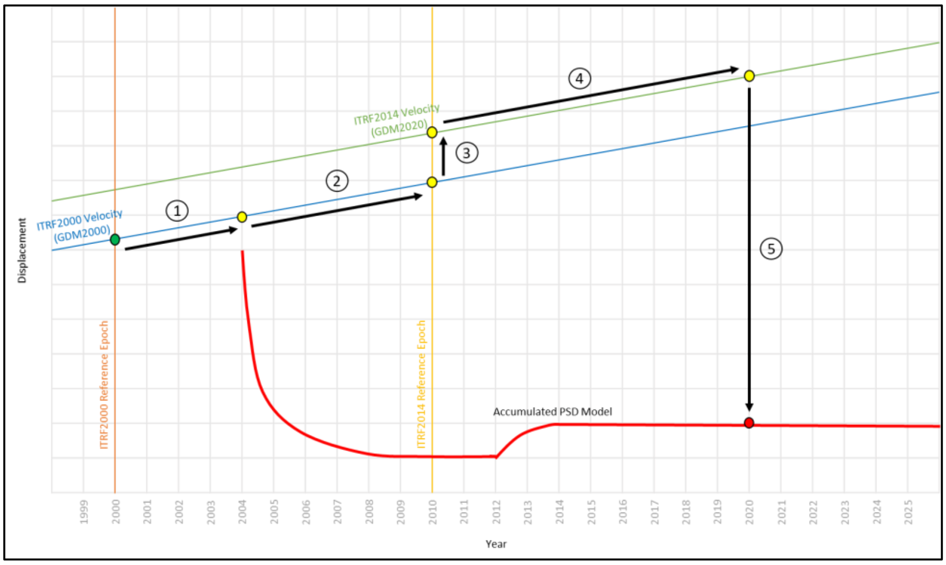



**Figure 9**: Transformation Step from GDM2000@2000.0 to GDM2020@2020.0 With PSD Correction

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000@2000.0} = \begin{bmatrix} -1131051.8664 \\ 6236311.7373 \\ 711748.1627 \end{bmatrix} = \begin{bmatrix} \phi & 6.450185648 \\ \lambda & 100.2797383112 \\ h & 18.060645 \end{bmatrix}$$

**<u>Step 1</u>:**

Correction of GDM2000@2000.0 coordinates with co-seismic and velocity stabilization values for the year 2004 (Stab.2004) using geodetic ($\phi$, $\lambda$, h) coordinates:

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{Stab.2004} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{Stab.2004}$$

Interpolate $[\Delta e, \Delta n, \Delta up]^{Stab.2004}$ using GDM00SW.grd gridded model:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{Stab.2004} = \begin{bmatrix} -0.18079 \\ -0.10098 \\ -0.01687 \end{bmatrix}$$

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{Stab.2004} = \begin{bmatrix} 0.17886 \\ 0.02693 \\ -0.10223 \end{bmatrix}$$

Correct GDM2000@2000.0 with Stab.2004 coordinates:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{GDM2000@2000.0 with Stab.2004}} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000@2000.0} + \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{Stab.2004}$$

$$= \begin{bmatrix} -1131051.68754 \\ 6236311.76423 \\ 711748.06047 \end{bmatrix}$$

**<u>Step 2</u>:**

Interpolate and transform velocity ITRF2014 to velocity ITRF2000 using geodetic (ϕ, λ, h) coordinates:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014}$$

Refer to West_i14.grd gridded model:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014} = \begin{bmatrix} 0.01874 \\ -0.00517 \\ 0.00111 \end{bmatrix}$$

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -0.01874 \\ -0.00169 \\ -0.00502 \end{bmatrix}$$

Refer to **Figure 5** for the transformation parameters and their rates from ITRF2014 to past ITRFs

$$\begin{bmatrix} T_X^v \\ T_Y^v \\ T_Z^v \end{bmatrix}_{ITRF2014\ to\ ITRF2000} = \begin{bmatrix} 0.0001 \\ 0.0001 \\ -0.0019 \end{bmatrix}$$

$$D^v{}_{ITRF2014\ to\ ITRF2000} = 0.11\ ppb$$

So,

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2000} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} + \begin{bmatrix} T_X^v \\ T_Y^v \\ T_Z^v \end{bmatrix}_{ITRF2014\ to\ ITRF2000}$$

$$+ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{GDM2000@2000.0 with Stab.2004}} * D^v{}_{ITRF2014\ to\ ITRF2000}$$

$$= \begin{bmatrix} -0.01877 \\ -0.00090 \\ -0.00684 \end{bmatrix}$$

Then, propagate corrected GDM2000@2000.0 with Stab.2004 to GDM2000@2010.0 coordinates:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000@2010.0}$$

$$= \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000@2000.0\ with\ Stab.2004} + (t - t_0) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2000}$$

$$= \begin{bmatrix} -1131051.87523 \\ 6236311.75523 \\ 711747.99208 \end{bmatrix}$$

**<u>Step 3</u>:**

Transform GDM2000@2010.0 to GDM2020@2010.0 coordinates:

Refer to **Figure 5** for the transformation parameters and their rates from ITRF2014 to past ITRFs

$$\begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}_{ITRF2000\ to\ ITRF2014} = \begin{bmatrix} -0.00070 \\ -0.00120 \\ 0.02610 \end{bmatrix}$$

$$D^v{}_{ITRF2014\ to\ ITRF2000} = -2.12\text{ppb}$$

So,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2010.0} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000@2010.0} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}_{ITRF2000\ to\ ITRF2014}$$

$$+ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000@2010.0} * D_{ITRF2000\ to\ ITRF2014}$$

$$= \begin{bmatrix} -1131051.87354 \\ 6236311.74081 \\ 711748.01667 \end{bmatrix}$$

**<u>Step 4</u>:**

Interpolate velocity ITRF2014 using geodetic (ϕ, λ, h) coordinates:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014}$$

Refer to West_i14.grd gridded model:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014} = \begin{bmatrix} 0.01874 \\ -0.00517 \\ 0.00111 \end{bmatrix}$$

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -0.01874 \\ -0.00169 \\ -0.00502 \end{bmatrix}$$

Propagate GDM2020@2010.0 to GDM2020@2020.0 coordinates:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020.0} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2010.0} + (t - t_0) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014}$$

$$= \begin{bmatrix} -1131052.06098 \\ 6236311.72394 \\ 711747.96650 \end{bmatrix}$$

**<u>Step 5</u>:**

Interpolate PSD@2020.0 using geodetic (ϕ, λ, h) coordinates:

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2020.0} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD@2020.0}$$

Refer to PSD2020.grd gridded model:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD@2020.0} = \begin{bmatrix} -0.05141 \\ -0.01356 \\ 0.00000 \end{bmatrix}$$

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2020.0} = \begin{bmatrix} 0.05031 \\ 0.01067 \\ -0.01348 \end{bmatrix}$$

So,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020.0} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020.0} + \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2020.0}$$

$$= \begin{bmatrix} -1131052.01066 \\ 6236311.73462 \\ 711747.95303 \end{bmatrix}$$

(v) Propagate Cartesian Coordinate of GDM2000 (Rev 2006)@2000.0 to GDM2020@2020.0 with PSD correction:

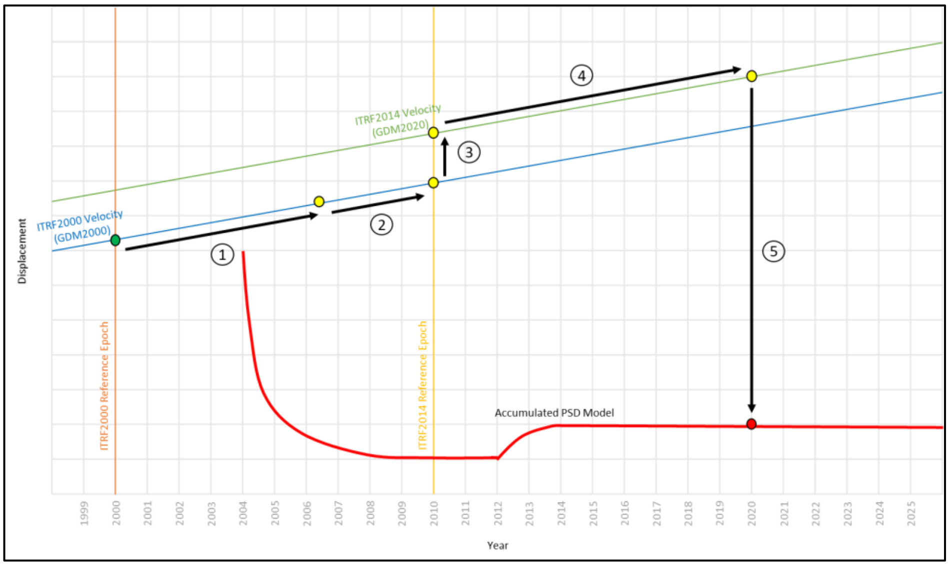



**Figure 10**: Transformation Step from GDM2000 (Rev 2006)@2000.0 to GDM2020@2020.0 With PSD Correction

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000\ (Rev\ 2006)} = \begin{bmatrix} -1131051.65410 \\ 6236311.79950 \\ 711748.11140 \end{bmatrix} = \begin{bmatrix} \phi & 6.4501580802 \\ \lambda & 100.2797363225 \\ h & 18.07805 \end{bmatrix}$$

**<u>Step 1</u>:**

Correction of GDM2000 (Rev 2006) @ 2000.0 coordinates with co-seismic and velocity stabilization values for the year 2006 (Stab.2006) using geodetic (ϕ, λ, h) coordinates:

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{Stab.2006} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{Stab.2006}$$

Refer to GDM06SW.grd gridded model

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{Stab.2006} = \begin{bmatrix} 0.16356 \\ -0.09480 \\ -0.02549 \end{bmatrix}$$

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{Stab.2006} = \begin{bmatrix} -0.15832 \\ -0.04363 \\ -0.09706 \end{bmatrix}$$

Update GDM2000 (Rev 2006)@2000.586.0 coordinates:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{GDM2000(Rev 2006)@2006.586.0}} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000\ (Rev\ 2006)} + \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{Stab.2006}$$

$$= \begin{bmatrix} -1131051.81242 \\ 6236311.75587 \\ 711748.01434 \end{bmatrix}$$

**<u>Step 2</u>:**

Interpolate and transform velocity ITRF2014 to velocity ITRF2000 using geodetic (ϕ, λ, h) coordinates:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014}$$

Refer to West_i14.grd gridded model:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014} = \begin{bmatrix} 0.01874 \\ 0.00517 \\ 0.00111 \end{bmatrix}$$

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -0.01854 \\ -0.00283 \\ 0.00527 \end{bmatrix}$$

Refer to **Figure 5** for the transformation parameters and their rates from ITRF2014 to past ITRFs

$$\begin{bmatrix} T_X^v \\ T_Y^v \\ T_Z^v \end{bmatrix}_{ITRF2014\ to\ ITRF2000} = \begin{bmatrix} 0.0001 \\ 0.0001 \\ -0.0019 \end{bmatrix}$$

$$D^v{}_{ITRF2014\ to\ ITRF2000} = 0.11ppb$$

So,

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2000} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} + \begin{bmatrix} T_X^v \\ T_Y^v \\ T_Z^v \end{bmatrix}_{ITRF2014\ to\ ITRF2000}$$

$$+ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\mathrm{GDM2000@2000.0\ with\ Stab.2006}} * D^v{}_{ITRF2014\ to\ ITRF2000}$$

$$= \begin{bmatrix} -0.01856 \\ -0.00204 \\ -0.00344 \end{bmatrix}$$

Then, propagate GDM2000 (Rev 2006)@2006.586 to GDM2000 (Rev 2006)@2010.0 coodinates:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000(Rev\ 2006)@2010.0}$$

$$= \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000(Rev\ 2006)@2006.586} + (t - t_0) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2000}$$

$$= \begin{bmatrix} -1131051.87578 \\ 6236311.74889 \\ 711748.02610 \end{bmatrix}$$

**<u>Step 3</u>:**

Transform GDM2000@2010.0 to GDM2020@2010.0 coordinates:

Refer to **Figure 5** for the transformation parameters and their rates from ITRF2014 to past ITRFs

$$\begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}_{ITRF2000\ to\ ITRF2014} = \begin{bmatrix} -0.00070 \\ -0.00120 \\ 0.02610 \end{bmatrix}$$

$$D^v{}_{ITRF2000\ to\ ITRF2014} = -2.12\mathrm{ppb}$$

So,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2010.0} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000@2010.0} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}_{ITRF2000\ to\ ITRF2014}$$

$$+ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000@2010.0} * D_{ITRF2000\ to\ ITRF2014}$$

$$= \begin{bmatrix} -1131051.87409 \\ 6236311.73447 \\ 711748.05069 \end{bmatrix}$$

**<u>Step 4</u>:**

Interpolate velocity ITRF2014 using geodetic (ϕ, λ, h) coordinates:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014}$$

Refer to West_i14.grd gridded model:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014} = \begin{bmatrix} 0.01874 \\ -0.00517 \\ 0.00111 \end{bmatrix}$$

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -0.01874 \\ -0.00169 \\ -0.00502 \end{bmatrix}$$

Propagate GDM2020@2010.0 to GDM2020@2020.0 coordinates:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020.0} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2010.0} + (t - t_0) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014}$$

$$= \begin{bmatrix} -1131052.06153 \\ 6236311.71761 \\ 711748.00052 \end{bmatrix}$$

**<u>Step 5</u>:**

Interpolate PSD@2020.0 using geodetic (ϕ, λ, h) coordinates:

$$\begin{bmatrix}\delta_x\\ \delta_y\\ \delta_z\end{bmatrix}_{PSD@2020.0} = \begin{bmatrix}-\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda\\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda\\ 0 & \cos\phi & \sin\phi\end{bmatrix} * \begin{bmatrix}\Delta e\\ \Delta n\\ \Delta up\end{bmatrix}_{PSD@2020.0}$$

Refer to PSD2020.grd gridded model:

$$\begin{bmatrix}\Delta e\\ \Delta n\\ \Delta up\end{bmatrix}_{PSD@2020.0} = \begin{bmatrix}-0.05141\\ -0.01356\\ 0.00000\end{bmatrix}$$

$$\begin{bmatrix}\delta_x\\ \delta_y\\ \delta_z\end{bmatrix}_{PSD@2020.0} = \begin{bmatrix}0.05031\\ 0.01067\\ -0.01348\end{bmatrix}$$

So,

$$\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_{GDM2020@2020.0} = \begin{bmatrix}X\\ Y\\ Z\end{bmatrix}_{GDM2020@2020.0} + \begin{bmatrix}\delta_x\\ \delta_y\\ \delta_z\end{bmatrix}_{PSD@2020.0}$$

$$= \begin{bmatrix}-1131052.01122\\ 6236311.72828\\ 711747.98705\end{bmatrix}$$

(vi) Propagate Cartesian Coordinate of GDM2000 (Rev 2016)@2000.0 to GDM2020@2020.0 with PSD correction:

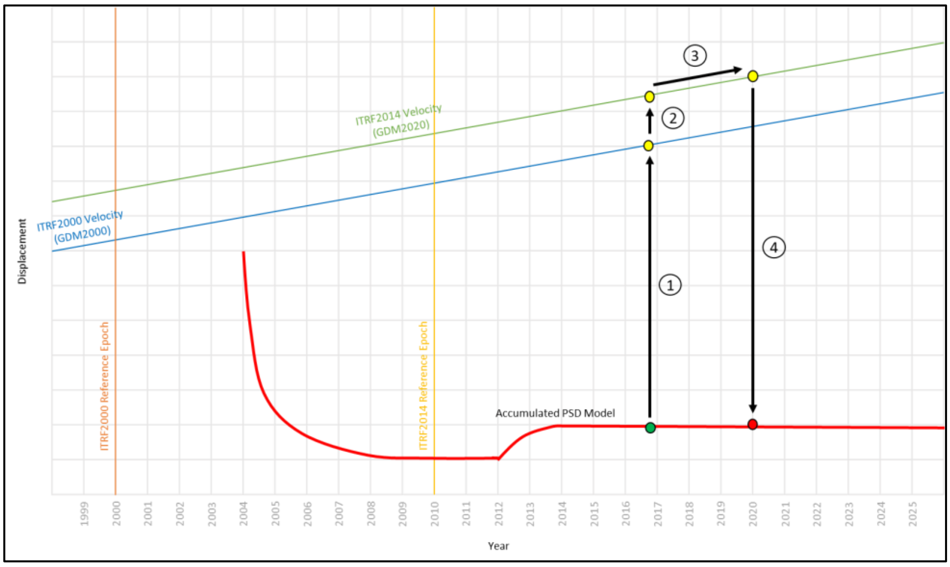



**Figure 11**: Transformation Step from GDM2000 (Rev 2016)@2000.0 to GDM2020@2020.0 With PSD correction

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000\ (Rev\ 2016)} = \begin{bmatrix} -1131051.60124 \\ 6236311.82016 \\ 711748.11244 \end{bmatrix} = \begin{bmatrix} \phi & 6.4501580784 \\ \lambda & 100.2797358190 \\ h & 18.088994 \end{bmatrix}$$

**Step 1**:

Correction of GDM2000 (Rev 2016)@2000.0 coordinates with PSD@2016.5 using geodetic (ϕ, λ, h) coordinate:

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2016.5} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD@2016.5}$$

Interpolate PSD, refer to PSD@2016.5.grd gridded model

$$\begin{bmatrix} \Delta n \\ \Delta e \\ \Delta up \end{bmatrix}_{PSD@2016.5} = \begin{bmatrix} -0.05136 \\ -0.01207 \\ 0.00000 \end{bmatrix}$$

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2016.5} = \begin{bmatrix} 0.05029 \\ 0.01050 \\ -0.01200 \end{bmatrix}$$

Correct GDM2000 (Rev 2016)@2000.0 coordinates:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{GDM2000(Rev 2016)@2000.0}} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000\ (Rev\ 2016)} - \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2016.5}$$

$$= \begin{bmatrix} -1131051.65153 \\ 6236311.80966 \\ 711748.12444 \end{bmatrix}$$

**<u>Step 2</u>:**

Transform GDM2000 (Rev 2016)@2000.0 to GDM2020@2016.425 using 3 parameters:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{3-Par\ \text{(GDM2000 (Rev 2016)\_GDM2020)}} = \begin{bmatrix} 0.34028 \\ 0.07910 \\ 0.13031 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2016.425} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{GDM2000(Rev 2016)@2000.0}} - \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{3-Par}$$

$$= \begin{bmatrix} -1131051.99181 \\ 6236311.73056 \\ 711747.99413 \end{bmatrix}$$

**<u>Step 3</u>:**

Interpolate velocity ITRF2014 using geodetic (ϕ, λ, h) coordinates and propagate GDM2000@2016.425 to GDM2020@2020.0 coordinates:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014}$$

Refer to West_i14.grd gridded model:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014} = \begin{bmatrix} 0.01874 \\ -0.00517 \\ 0.00111 \end{bmatrix}$$

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -0.01874 \\ -0.00169 \\ -0.00502 \end{bmatrix}$$

Then, propagate GDM2000@2016.425 to GDM2020@2020.0 coordinates:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020.0} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2016.425} + (t - t_0) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014}$$

$$= \begin{bmatrix} -1131052.05883 \\ 6236311.72453 \\ 711747.97619 \end{bmatrix}$$

**<u>Step 4</u>:**

Interpolate PSD@2020.0 using geodetic (ϕ, λ, h) coordinates:

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2020.0} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD@2020.0}$$

Refer to PSD2020.grd gridded model:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD@2020.0} = \begin{bmatrix} -0.05141 \\ -0.01356 \\ 0.00000 \end{bmatrix}$$

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2020.0} = \begin{bmatrix} 0.05031 \\ 0.01067 \\ -0.01348 \end{bmatrix}$$

So,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020.0} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020.0} + \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2020.0}$$

$$= \begin{bmatrix} -1131052.00851 \\ 6236311.73521 \\ 711747.96272 \end{bmatrix}$$

**(vii)** Propagate Cartesian Coordinate of MGN@2013.312 to GDM2020@2020.0 with PSD correction:

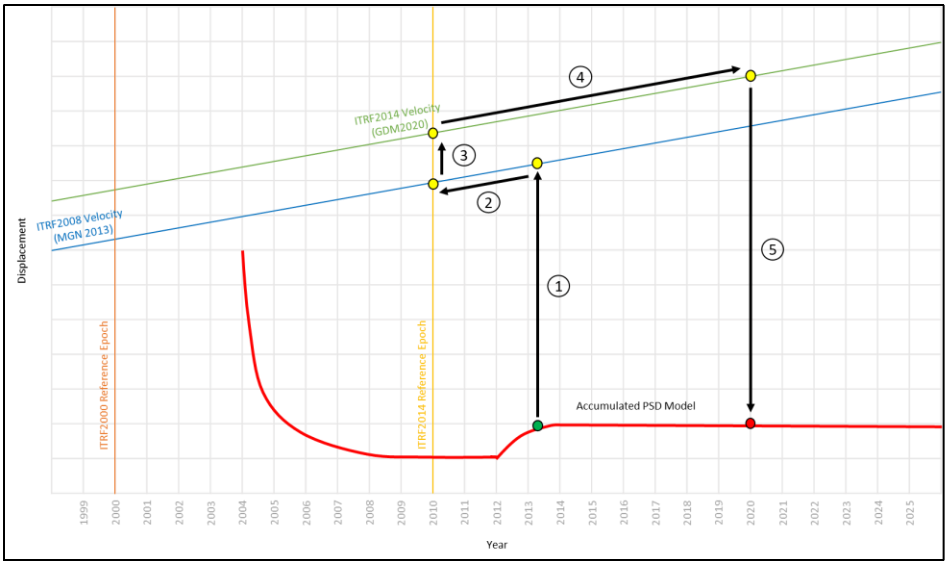



**Figure 12**: Transformation Step from MGN@2013.312 to GDM2020@2020.0 With PSD correction

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ITRF2008(MGN)@2013.312} = \begin{bmatrix} -1131051.88270 \\ 6236311.74320 \\ 711747.99610 \end{bmatrix} = \begin{bmatrix} \phi & 6.4501570590 \\ \lambda & 100.2797384467 \\ h & 18.05059 \end{bmatrix}$$

<u>**Step 1**</u>**:**

Correction of ITRF2008(MGN)@2013.312 coordinates with PSD@2013.312 using geodetic (ϕ, λ, h) coordinate:

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2013.312} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD@2013.312}$$

Interpolate PSD@2013.312, refer to PSD@2013.312.grd gridded model

$$\begin{bmatrix} \Delta n \\ \Delta e \\ \Delta up \end{bmatrix}_{PSD@2013.312} = \begin{bmatrix} -0.05253 \\ -0.01220 \\ 0.00000 \end{bmatrix}$$

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2013.312} = \begin{bmatrix} 0.05144 \\ 0.01072 \\ -0.01213 \end{bmatrix}$$

Correct ITRF2008(MGN)@2013.312 coordinates:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ITRF2008(MGN)@2013.312} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ITRF2008(MGN)@2013.312} - \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2013.312}$$

$$= \begin{bmatrix} -1131051.93414 \\ 6236311.73248 \\ 711748.00823 \end{bmatrix}$$

**<u>Step 2</u>:**

Interpolate velocity ITRF2014, transform to velocity ITRF2008 and propagate MGN@2013.312 to MGN@2010.0 using geodetic (ϕ, λ, h) coordinates:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014}$$

Refer to West_i14.grd gridded model:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014} = \begin{bmatrix} 0.01874 \\ -0.00517 \\ 0.00111 \end{bmatrix}$$

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -0.01874 \\ -0.00169 \\ -0.00502 \end{bmatrix}$$

Refer to **Figure 5** for the transformation parameters and their rates from ITRF2014 to past ITRFs

$$\begin{bmatrix} T_X^v \\ T_Y^v \\ T_Z^v \end{bmatrix}_{ITRF2014\ to\ ITRF2008} = \begin{bmatrix} 0.0000 \\ 0.0000 \\ -0.0001 \end{bmatrix}$$

$$D^v{}_{ITRF2014\ to\ ITRF2008} = 0.03ppb$$

Transform velocity ITRF2014 to velocity ITRF2008

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2008} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} + \begin{bmatrix} T_X^v \\ T_Y^v \\ T_Z^v \end{bmatrix}_{ITRF2014\ to\ ITRF2008}$$

$$+ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ITRF2008(MGN)@2013.312} * D^v{}_{ITRF2014\ to\ ITRF2008}$$

$$= \begin{bmatrix} -0.01878 \\ -0.00150 \\ -0.00510 \end{bmatrix}$$

Then, propagate MGN@2013.312 to MGN@2010.0 coordinates:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ITRF2008(MGN)@2010.0}$$

$$= \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ITRF2008(MGN)@2013.312} + (t - t_0) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2008}$$

$$= \begin{bmatrix} -1131051.87195 \\ 6236311.73744 \\ 711748.02510 \end{bmatrix}$$

**Step 3:**

Transform ITRF2008 MGN@2010.0 to GDM2020@2010.0 coordinates:

Refer to **Figure 5** for the transformation parameters and their rates from ITRF2014 to past ITRFs

$$\begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}_{ITRF2008\ to\ ITRF2014} = \begin{bmatrix} -0.00160 \\ -0.00190 \\ -0.00240 \end{bmatrix}$$

$$D^v{}_{ITRF2008\ to\ ITRF2014} = 0.02\text{ppb}$$

So,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ITRF2014@2010.0} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ITRF2008@2010.0} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}_{ITRF2008\ to\ ITRF2014}$$

$$+ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2000@2010.0} * D_{ITRF2008\ to\ ITRF2014}$$

$$= \begin{bmatrix} -1131051.87357 \\ 6236311.73567 \\ 711748.02272 \end{bmatrix}$$

**Step 4**:

Interpolate velocity ITRF2014 using geodetic (ϕ, λ, h) coordinates:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014}$$

Refer to West_i14.grd gridded model:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{ITRF2014} = \begin{bmatrix} 0.01874 \\ -0.00517 \\ 0.00111 \end{bmatrix}$$

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014} = \begin{bmatrix} -0.01874 \\ -0.00169 \\ -0.00502 \end{bmatrix}$$

Propagate GDM2020@2010.0 to GDM2020@2020.0 coordinates:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020.0} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2010.0} + (t - t_0) \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ITRF2014}$$

$$= \begin{bmatrix} -1131052.06102 \\ 6236311.71880 \\ 711747.97255 \end{bmatrix}$$

**Step 5**:

Interpolate PSD@2020.0 using geodetic (ϕ, λ, h) coordinates:

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2020.0} = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} * \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD@2020.0}$$

Refer to PSD2020.grd gridded model:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta up \end{bmatrix}_{PSD@2020.0} = \begin{bmatrix} -0.05141 \\ -0.01356 \\ 0.00000 \end{bmatrix}$$

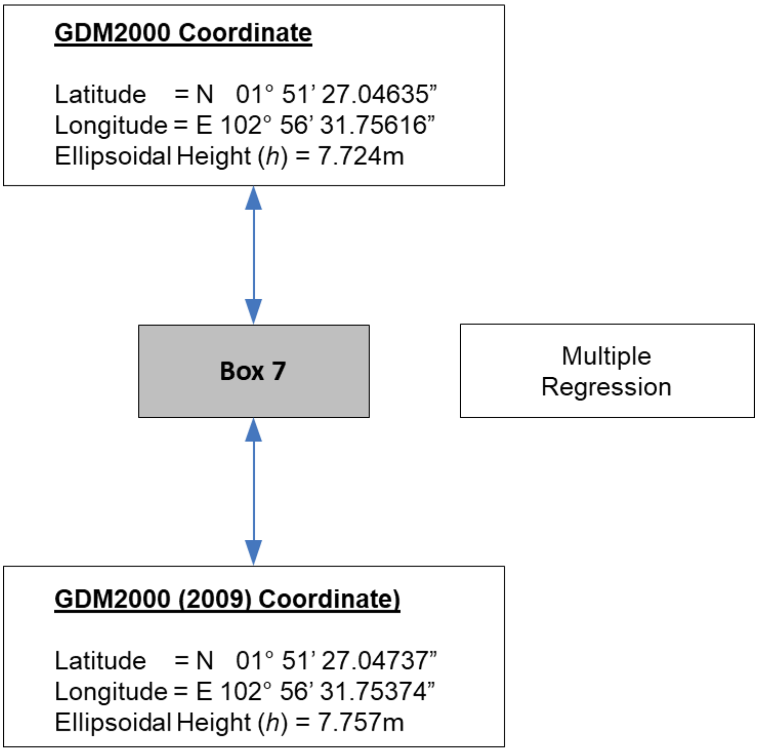
$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2020.0} = \begin{bmatrix} 0.05031 \\ 0.01067 \\ -0.01348 \end{bmatrix}$$

So,

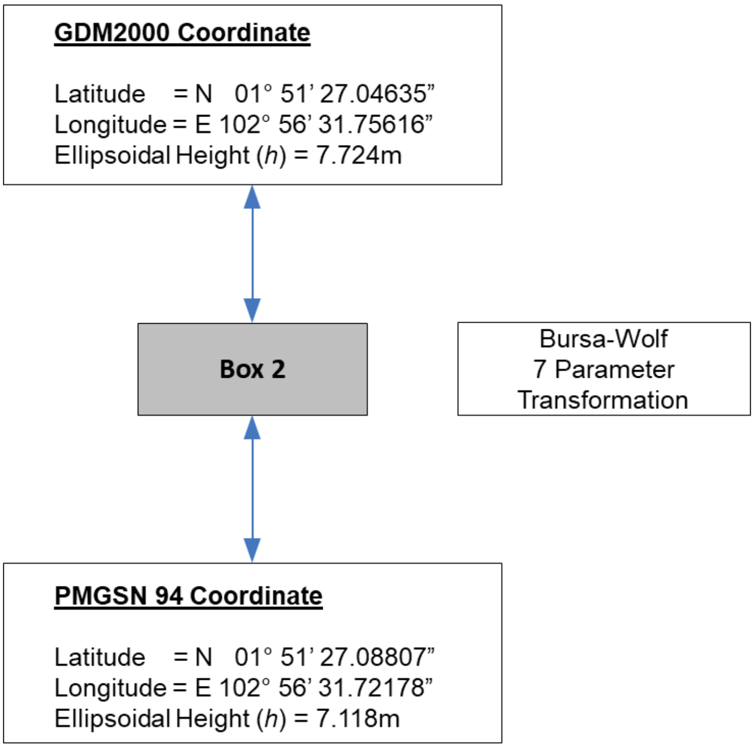
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020.0} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{GDM2020@2020.0} + \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}_{PSD@2020.0}$$

$$= \begin{bmatrix} -1131052.01070 \\ 6236311.72948 \\ 711747.95907 \end{bmatrix}$$

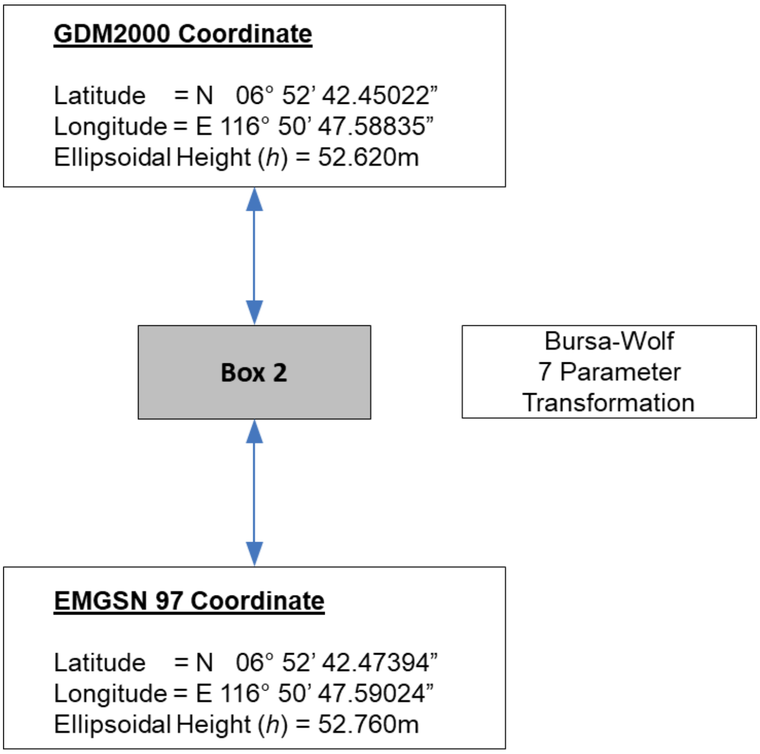
**(viii) GDM2000 to GDM2000 (2009)**



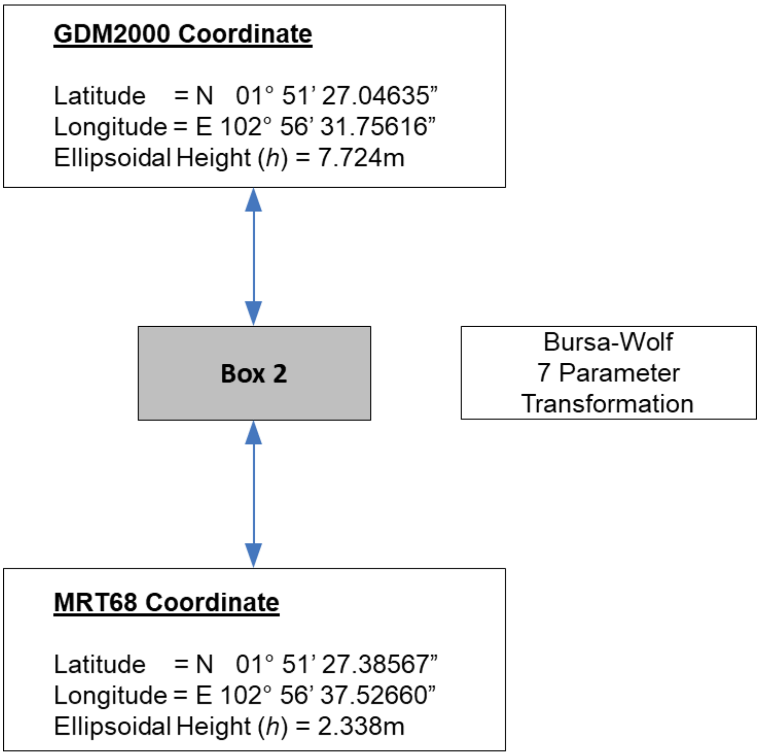

**(ix) GDM2000 to PMGSN94**



**(x) GDM2000 to EMGSN97**




**(xi) GDM2000 to MRT68**



### (xii) GDM2000 to BT68 (Sabah)

**GDM2000 Coordinate**

Latitude = N 06° 52' 42.45022"
Longitude = E 116° 50' 47.58835"
Ellipsoidal Height (*h*) = 52.620m

↕

**Box 2** — Bursa-Wolf 7 Parameter Transformation

↕

**BT68 (Sabah) Coordinate**

Latitude = N 06° 52' 46.04669"
Longitude = E 116° 50' 37.60565"
Ellipsoidal Height (*h*) = 1.693m

### (xiii) GDM2000 to BT68 (Sarawak)

**GDM2000 Coordinate**

Latitude = N 01° 54' 39.28066"
Longitude = E 112° 52' 01.24460"
Ellipsoidal Height (*h*) = 976.993m

↕

**Box 2** — Bursa-Wolf 7 Parameter Transformation

↕

**BT68 (Sarawak) Coordinate**

Latitude = N 01° 54' 41.51240"
Longitude = E 112° 51' 49.44225"
Ellipsoidal Height (*h*) = 933.784m

### (xiv) PMGSN94 to MRT68

**PMGSN 94 Coordinate**

Latitude = N 01° 51' 27.08807"
Longitude = E 102° 56' 31.72178"
Ellipsoidal Height (*h*) = 7.118m

↕

**Box 2** — Bursa-Wolf 6 Parameter Transformation

↕

**MRT68 Coordinate**

Latitude = N 01° 51' 27.38567"
Longitude = E 102° 56' 37.52660"
Ellipsoidal Height (*h*) = 2.338m

### (xv) EMGSN97 to BT68 (Sabah)

**EMGSN 97 Coordinate**

Latitude = N 06° 52' 42.47394"
Longitude = E 116° 50' 47.59024"
Ellipsoidal Height (*h*) = 52.760m

↕

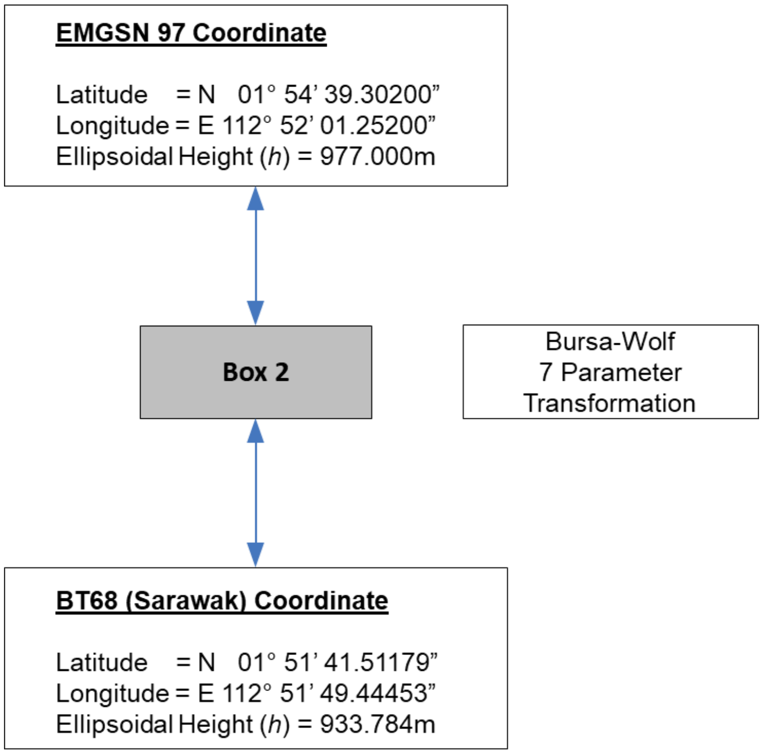
**Box 2** — Bursa-Wolf 7 Parameter Transformation

↕

**BT68 (Sabah) Coordinate**

Latitude = N 06° 52' 46.04669"
Longitude = E 116° 50' 37.60565"
Ellipsoidal Height (*h*) = 1.693m

### (xvi) EMGSN97 to BT68 (Sarawak)



## 4. MAP PROJECTION

### 4.1 Introduction

4.1.1 Typically, there are many different methods for projecting longitude and latitude in coordinate reference system 1 onto a flat map in the same system as shown in **Table 6**:

**Table 6**: Coordinate Reference and Map Projection Systems

| No. | Coordinate Reference System 1 | Map Projection System 1 |
|---|---|---|
| 1 | Geocentric Datum of Malaysia (GDM2000)<br><br>GDM2000 (Rev 2006 Rev 2009 Rev 2016)<br><br>Geocentric Datum of Malaysia 2020 (GDM2020) | Geocentric Cassini-Soldner |
| 2 | GDM2000<br><br>GDM2000 (Rev 2006 Rev 2009 Rev 2016)<br><br>GDM2020 | Geocentric Malayan Rectified Skew Orthomorphic (Peninsular Malaysia) |
| 3 | GDM2000<br><br>GDM2000 (Rev 2006 Rev 2009 Rev 2016)<br><br>GDM2020 | Geocentric Borneo Rectified Skew Orthomorphic (Sabah and Sarawak) |
| 4 | Malayan Revised Triangulation 1968 (MRT68) | Malayan Rectified Skew Orthomorphic (Peninsular Malaysia) |
| 5 | Rectified Skew Orthomorphic (Peninsular Malaysia) | Cassini-Soldner |
| 6 | Borneo Triangulation 1968 (BT68) | Borneo Rectified Skew Orthomorphic (Sabah and Sarawak) |

## 4.2 Rectified Skew Orthomorphic (RSO) Map Projection

4.2.1 The rectified skew orthomorphic (RSO) map projection (**Figure 13**) is an oblique Mercator projection developed by Hotine in 1947 (Snyder, 1984). This projection is orthomorphic (conformal) and cylindrical. All meridians and parallel are complex curves. Scale is approximately true along a chosen central line (exactly true along a great circle in its spherical form). It is thus a suitable projection for an area like Switzerland, Italy, New Zealand, Madagascar and Malaysia as well.

4.2.2 The RSO provides an optimum solution in the sense of minimizing distortion whilst remaining conformal for Malaysia. **Figure 14** shows the new geocentric RSO and old RSO parameters for Peninsular Malaysia and East Malaysia.

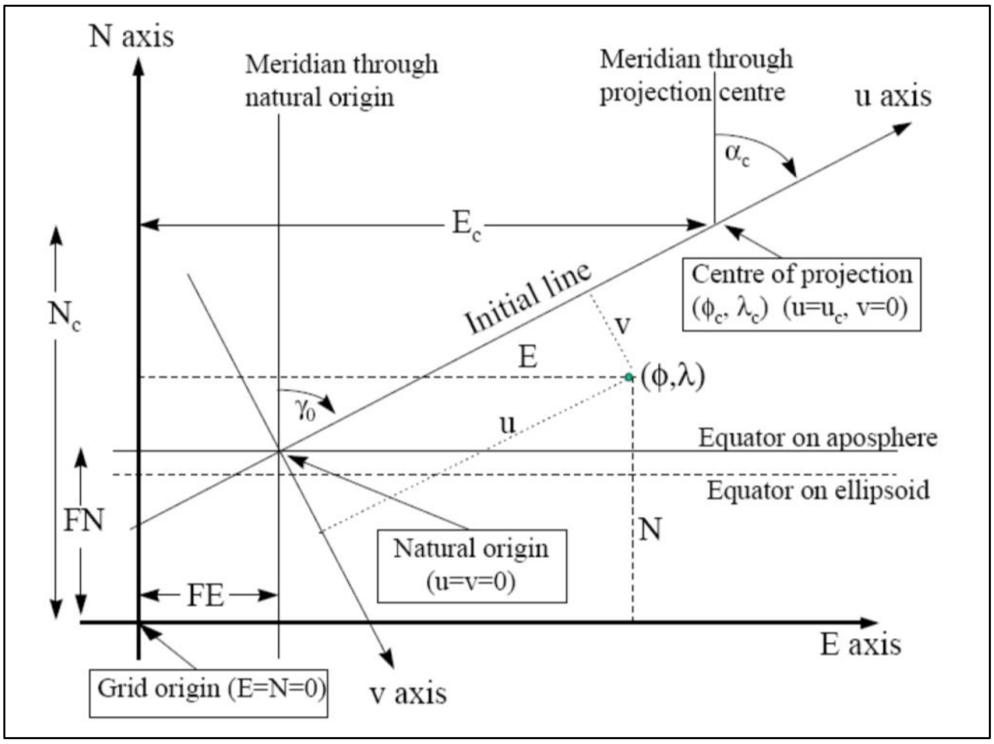



**Figure 13**: Hotine, 1947 (Snyder, 1984), Oblique Mercator (Source: EPSG)

| | Peninsular M'sia | | | | | | | | Sabah & Sarawak | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | MRSO | | | | Geocentric MRSO | | | | BRSO | | | | Geocentric BRSO | | | |
| | Ellipsoid Parameters | | | | | | | | | | | | | | | |
| Ellipsoid | Mod. Everest (Pen. M'sia) | | | | GRS80 | | | | Mod. Everest (East M'sia) | | | | GRS80 | | | |
| Semi Major, a | 6377304.063000000 | | | | 6378137.000000000 | | | | 6377298.556000000 | | | | 6378137.000000000 | | | |
| Flattening, 1/f | 1/300.8017 | | | | 1/298.2572221 | | | | 1/300.8017 | | | | 1/298.2572221 | | | |
| | Defined Parameters | | | | | | | | | | | | | | | |
| | N/S | Deg | Min | Sec | N/S | Deg | Min | Sec | N/S | Deg | Min | Sec | N/S | Deg | Min | Sec |
| Latitude of Projection Center, $\Phi_c$ | | | | | | | | | | | | | | | | |
| Longitude of Projection Center, $\lambda_c$ | | | | | | | | | | | | | | | | |
| Rectified to Skew Grid, $\gamma_c$ | | | | | | | | | | | | | | | | |
| Azimuth of Central Line, $\alpha_c$ | | | | | | | | | | | | | | | | |
| Scale Factor, k | | | | | | | | | | | | | | | | |
| False Origin Northing, FN | | | | | | | | | | | | | | | | |
| False Origin Easting, FE | | | | | | | | | | | | | | | | |
| Chain to Meter | | | | | | | | | | | | | | | | |

Defined parameters can be obtained from JUPEM

**Figure 14**: The RSO Projection Parameters

4.2.3 The notation adopted for use in this section is as follows:

$\phi_c$ = latitude of center of the projection

$\lambda_c$ = longitude of center of the projection

$\alpha_c$ = azimuth (true) of the center line passing through the center of the projection

$\gamma_c$ = rectified bearing of the center line

k = scale factor at the center of the projection

$\phi$ = geographical latitude

$\lambda$ = geographical longitude

$t_o$ = isometric latitude for $\theta = 4°$

$$t_o = \frac{\operatorname{Tan}\left(\frac{\pi}{4} - \frac{\phi_c}{2}\right)}{\left(\sqrt{\frac{(1 - e * \operatorname{Sin}\phi_c)}{(1 + e * \operatorname{Sin}\phi_c)}}\right)^{e}}$$

t = isometric latitude

$\lambda_o$ = basic longitude

a = semi major axis of ellipsoid

b = semi minor axis of ellipsoid

f = flattening of ellipsoid

$$f = \frac{(a - b)}{a}$$

e = eccentricity of the ellipsoid

$$e = \sqrt{\frac{(a^2 - b^2)}{a^2}} \qquad e' = \sqrt{\frac{(a^2 - b^2)}{b^2}}$$

u = skew coordinate parallel to initial line

v = skew coordinate at right angles to initial line

$\gamma$ = skew convergence at meridians

$\gamma_R$ = rectified convergence of meridians

$$\gamma_R = \gamma + 36° 52' 11.6314''$$

N = Northing map coordinate

E = Easting map coordinate

FE= False Easting at the natural origin

FN= False Northing at the natural origin

where,

The Constants of the Projection are given as follows:

$$B = \sqrt{1 + \frac{(e^2 \operatorname{Cos}^4 \phi_c)}{(1 - e^2)}}$$

$$A = \frac{a * B * k \sqrt{(1 - e^2)}}{(1 - e^2 * \operatorname{Sin}^2 \phi_c)}$$

$$D = \frac{B * \sqrt{(1 - e^2)}}{\operatorname{Cos} \phi_c * \sqrt{(1 - e^2 \operatorname{Sin}^2 \phi_c)}}$$

To avoid problems with computation of F, if D<1, make $D^2 = 1$

$$F = D + \operatorname{Sign} \phi_c * \sqrt{(D^2 - 1)}$$

$$H = F * t_o^B$$

$$G = \frac{F - \frac{1}{F}}{2}$$

$$\gamma_o = \operatorname{Sin}^{-1}\left(\frac{\operatorname{Sin} \alpha_c}{D}\right)$$

Basic Longitude:

$$\lambda_o = \lambda_c - \frac{\sin^{-1}(G * \tan \gamma_o)}{B}$$

4.2.4 The projection formulae are as follows:

(a) Conversion of Geographical to Rectangular and vice versa

Forward Case: To compute (E, N) from a given ($\phi$, $\lambda$):

$$t = \frac{\tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}{\left(\sqrt{\frac{(1 - e * \sin\phi)}{(1 + e * \sin\phi)}}\right)^e}$$

$$Q = \frac{H}{t^B}$$

$$S = \frac{Q - \frac{1}{Q}}{2}$$

$$T = \frac{Q + \frac{1}{Q}}{2}$$

$$V = \sin[B(\lambda - \lambda_o)]$$

$$U = \frac{S \sin\gamma_o - V \cos\gamma_o}{T}$$

$$v = \frac{A \ln\left(\frac{1 - U}{1 + U}\right)}{2B}$$

For the Hotine Oblique Mercator (where the FE and FN values have been specified with respect to the origin of the (u, v) axes):

$$u = \frac{A}{B} \tan^{-1}\left(\frac{S \cos\gamma_o + V \sin\gamma_o}{\cos[B(\lambda - \lambda_o)]}\right)$$

The rectified skew coordinates are then derived from:

$$E = v \cos\gamma_c + u \sin\gamma_c + FE$$

$$N = u \cos\gamma_c - v \sin\gamma_c + FN$$

Reverse Case: Compute ($\phi$, $\lambda$) from a given (E, N):

For the Hotine Oblique Mercator:

$$v' = (E - FE) * \text{Cos}\,\gamma_c - (N - FN) * \text{Sin}\,\gamma_c$$

$$u' = (N - FN) * \text{Cos}\,\gamma_c + (E - FE) * \text{Sin}\,\gamma_c$$

Then the other parameters can be calculated.

$$Q' = \exp\left(-\frac{B\,v'}{A}\right)$$

$$S' = \frac{Q' - \frac{1}{Q'}}{2}$$

$$T' = \frac{Q' + \frac{1}{Q'}}{2}$$

$$V' = \text{Sin}\left(\frac{B\,u'}{A}\right)$$

$$U' = \frac{V'\ \text{Cos}\,\gamma_c + S'\ \text{Sin}\,\gamma_c}{T'}$$

$$t' = \left[\frac{H}{\sqrt{\frac{(1 + U')}{(1 - U')}}}\right]^{\frac{1}{B}}$$

$$\chi = \left(\frac{\pi}{2}\right) - 2 * \left(\text{Tan}^{-1}\ t'\right)$$

$$\begin{aligned}\phi = \chi + (\text{Sin}\ 2\chi) * &\left(\frac{e^2}{2} + \frac{5\,e^4}{24} + \frac{e^6}{12} + \frac{13\,e^8}{360}\right) \\ + (\text{Sin}\ 4\chi) * &\left(\frac{7\,e^4}{48} + \frac{29\,e^6}{240} + \frac{811\,e^8}{11520}\right) \\ + (\text{Sin}\ 6\chi) * &\left(\frac{7\,e^6}{120} + \frac{81\,e^8}{1120}\right) \\ + (\text{Sin}\ 8\chi) * &\left(\frac{4279\,e^4}{161280}\right)\end{aligned}$$

$$\lambda = \lambda_o - \frac{\text{Tan}^{-1}\left(\frac{S'\ \text{Cos}\,\gamma_c - V'\ \text{Sin}\,\gamma_c}{\text{Cos}\ \frac{B\,u'}{A}}\right)}{B}$$

(b) Convergence of Map Meridians

The convergence of the map meridians is defined as the angle measured clockwise from True North to the Rectified Grid North, and is denoted $\gamma_R$:

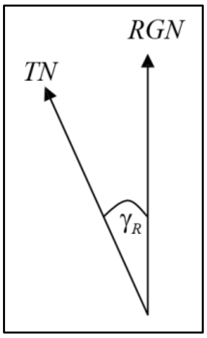



**Figure 15**: Convergence of Map Meridians

$$\gamma_R = \gamma - \sin^{-1}(-0.6)$$

$$= \gamma + 36° \, 52' \, 11.6314''$$

where,

$$\tan\gamma = \frac{\tan\gamma_0 - \sin B \, (\lambda_o - \lambda) \sinh \, (Bt + C)}{\cos B \, (\lambda_o - \lambda) \cosh \, (Bt + C)}$$

$$= \frac{\sin\frac{Bv}{A'm_0} \sinh\frac{Bu}{A'm} + \tan\gamma_0}{\cos\frac{Bv}{A'm_0} \cosh\frac{Bu}{A'm_0}}$$

(c) Scale Factor at any Point

The formulas giving the scale fact factor “m” at any point in terms of isometric latitude and longitude and coordinate (v,u) are:

$$m = \frac{A'm_0}{p} \, \frac{\cosh\frac{Bu}{A'm_0}}{\cosh \, (Bt + C)}$$

$$= \frac{A'm_0}{p} \, \frac{\cos\frac{Bv}{A'm_0}}{\cos B \, (\lambda_o - \lambda)}$$

It can be shown that the initial line of the projection has a scale factor that is nearly constant throughout its length.

(d) Scale Factor for a line

The Scale factor for a line can be computed from the formula:

$$m = \frac{1}{6}\,(m_1 + 4m_3 + m_2)$$

where $m_1$ , $m_2$ are the scale factors at the ends of the line and $m_3$ is the scale factor at its mid-point. The scale factor for a line also may be evaluated from the following formula:

$$m = \frac{A' m_0}{v_m \ \cos\varphi_m \ \cosh(B\varphi_m + C)}\left[1 + \frac{B^2}{6\,A'^2\,m_0^2}\,(u_1^2 + u_1u_2 + u_2^2)\right]$$

where

$\varphi_m$ are evaluated for the mid-latitude of the line

$u_1$ and $u_2$ are the u-coordinate of the points:

(e) Arc-to-Chord Correction

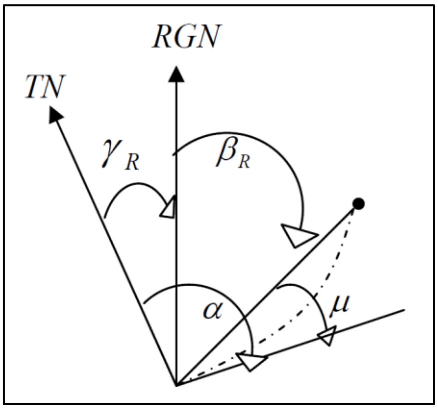



**Figure 16**: Arc-to-Chord

In **Figure 16**, if is the true azimuth of a line, $\beta_R$ is the rectified grid bearing, then

$$\alpha = \beta_R + \gamma_R + \mu$$

where $\mu$ is given in seconds by the formulae

$$\mu = \frac{B}{2A' m_0 \sin 1''}\,(v_2 - v_1)\,\tanh\left\{\frac{B}{2A' m_0}\,\frac{2u_1 + u_2}{3}\right\}$$
$$+ k_1^2\,\frac{\rho_0}{v_0}\,\sin\phi_0\left(\sin\phi_3 - \sin\phi_0\right)^2(\lambda_1 - \lambda_2)$$

where,

$\phi_3 = \frac{1}{3}(2\phi_1 + \phi_2)$ and $(\lambda_1 - \lambda_2)$ is measured in seconds.

For a line not exceeding 113 km (70 miles) in length, the maximum value of the second term of the formula is 0.007"; it can therefore bsafely neglected.

### 4.3 Cassini-Soldner Map Projection

4.3.1 There are nine state origins used in the coordinate projection in the cadastral system of Peninsular Malaysia. The Cassini-Soldner map projection has been used for over one hundred years and shall continue to be used for cadastral surveys in the new geodetic frame.

4.3.2 The mapping equations are as given in Richardus and Adler, 1974 and the formulas to derive projected Easting and Northing coordinates are as follows:

(a) <u>Forward</u> Computation

$$E = FE + v[A - T(A3/6) - (8 - T + 8C)T * A5/120]$$

$$N = FN + M - M_0 + v*\tan\phi\,[A2/2 + (5 - T + 6C)A4/24]$$

where,

| | | |
|---|---|---|
| N, E | = | Computed Cassini coordinate |
| FE, FN | = | Cassini State origin coordinate |
| A | = | $(\lambda - \lambda_0)$ Cos $\phi$ |
| $\lambda$ | = | Longitude of computation point |
| $\lambda_0$ | = | Longitude of state origin |
| $\phi$ | = | Latitude of computation point |
| T | = | $\tan^2 \phi$ |
| $C$ | = | $\frac{e^2}{(1 - e^2)} \cos^2 \phi$ |
| V | = | Radius of curvature in prime vertical |
| | = | $\frac{a}{\sqrt{(1 - e^2 \sin^2 \phi)}}$ |
| M | = | Meridianal arc distance |
| | = | $a[1 - e^2/4 - 3e^4/64 - 5e^6/256 - ..)\,\phi - (3e^2/8 + 3e^4/32 + 45e^6/1024 + \ldots)\sin^2\phi + (15e^4/256 + 45e^6/1024 + \ldots)\sin^4\phi - (35e^6/3072 + \ldots)\sin^6\phi ..]$ |

with $\phi$ is in radians.

$M_0$ = the value of M calculated for the latitude of the chosen origin.

(b) Reverse Computation

$$\phi = \phi_1 - \frac{v_1 \tan\phi_1}{\rho_1}\left[\frac{D^2}{2} - (1+3T_1)\frac{D^4}{24}\right]$$

$$\lambda = \lambda_0 + \frac{D - T_1\frac{D^3}{3} + (1+3T_1)\,T_1\,\frac{D^5}{15}}{\cos\phi_1}$$

where,

$\phi_1$ = $\mu_1 + (3e_1/2 - 27e_1^3/32 + ..)\ \mathrm{Sin}^2\,\mu_1 + (21e_1^2/16 - 55e_1^4/32 + ..)\ \mathrm{Sin}^4\,\mu_1 + (151e_1^3/96 + \ldots)\ \mathrm{Sin}^6\,\mu_1 + (1097e_1^4/512 - \ldots)\ \mathrm{Sin}^8\,\mu_1 + \ldots$

$\rho_1$ = $\dfrac{a\,(1-e^2)}{\sqrt{(1-e^2\sin^2\phi_1)^3}}$

$v_1$ = $\dfrac{a}{\sqrt{(1-e^2\sin^2\phi_1)}}$

$e_1$ = $\dfrac{1-\sqrt{(1-e^2)}}{1+\sqrt{(1-e^2)}}$

$\mu_1$ = $\dfrac{M_1}{a\,(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ..)}$

$M_1$ = $M_0 + (N - FN)$

= $M_0$ is the value of M calculated for latitude of the origin

$T_1$ = $\tan^2\phi_1$

D = $(E - FE) / v_1$

(c) Scale and Arc-to-Chord Correction for Cassini Projection

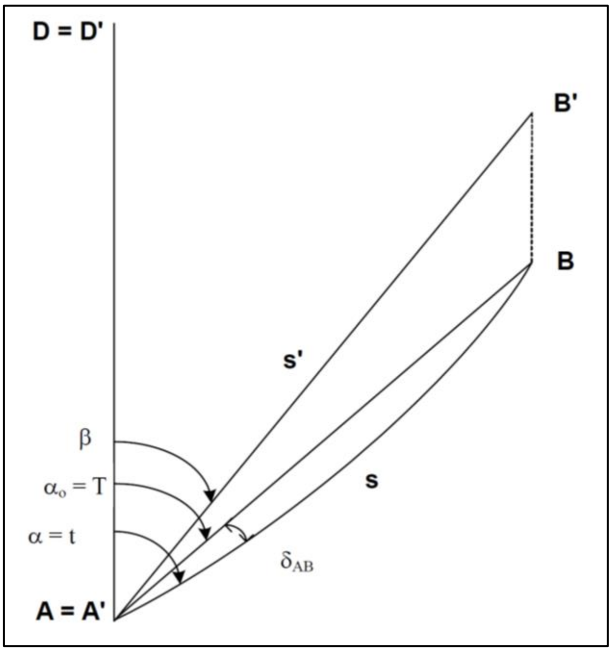



**Figure 17**: Scale and Arc-to-Chord

Refer to **Figure 17**,

$\delta$AB = $(t - T)'' = ((N_b - N_a)(E_b + 2E_a)) / (6R^2.\text{Sin } 1'')$

where,

| | | |
|---|---|---|
| Na, Nb, Ea, Eb | = | Cassini Coordinate |
| (t – T)" | = | Arc-to-Chord |

Bearing correction:-

$(\beta - \alpha)''$ = $- ((\text{Sin } \alpha_0 \text{ Cos } \alpha_0) / (6R^2 \text{ Sin } 1'')) E^2_\mu$

$E^2_\mu$ = $(E^2a + EaEb + E^2b)$

Linear correction:-

s' = $s + [ ((\text{Cos2 } \alpha_0) / (6R^2)) E^2_\mu ]s$

Projected Coordinate on Cassini:-

Eb = Ea + s' . Sin β

Nb = Na + s' . Cos β

### 4.4 Polynomial Function

The relationship between the MRSO coordinate and the Cassini SoldnerOld is defined by a series of polynomial function which make use of the coordinate of the origin of both projections for each state in Peninsular Malaysia. The following are the formulae used:

N(MRSO) = ΔN(CAS) + N(OMRSO) + (R1+XA1+YA2+XYA3+X2A4+Y2A5)

E(MRSO) = ΔE(CAS) + E(OMRSO) + (R2+XB1+YB2+XYB3+X2B4+Y2B5)

N(CAS) = ΔN(MRSO) + N(OCAS) - (R1+XA1+YA2+XYA3+X2A4+Y2A5)

E(CAS) = ΔE(MRSO) + E(OCAS) - (R2+XB1+YB2+XYB3+X2B4+Y2B5)

where,

| | |
|---|---|
| N(MRSO) | : MRSO Coordinate in North Component |
| E(MRSO) | : MRSO Coordinate in East Component |
| N(CAS) | : Cassini Coordinate in North Component |
| E(CAS) | : Cassini Coordinate in East Component |
| ΔN(CAS) | : Cassini Coordinate of Origin (North) |
| ΔE(CAS) | : Cassini Coordinate of Origin (East) |
| ΔN(MRSO) | : RSO Coordinate of Origin (North) |
| ΔE(MRSO) | : RSO Coordinate of Origin (East) |

X : ΔN(CAS) / 10000 or ΔN(MRSO) / 10000

Y : ΔE(CAS) / 10000 or ΔE(MRSO) / 10000

All values of the coordinates must be in unit chains (use 0.11678249 as the multiplying factor to convert from metres)

### 4.5 Redefinition of State Origins in GDM2000 to GDM2020

In order to update the geodetic infrastructure for the use of e-Kadaster, a redefinition of the nine (9) state cadastral origins has been carried out, and the results are given in **Table 7**.

**Table 7**: Redefinition of State Cadastral Origins in GDM2000 and GDM2020 Coordinates

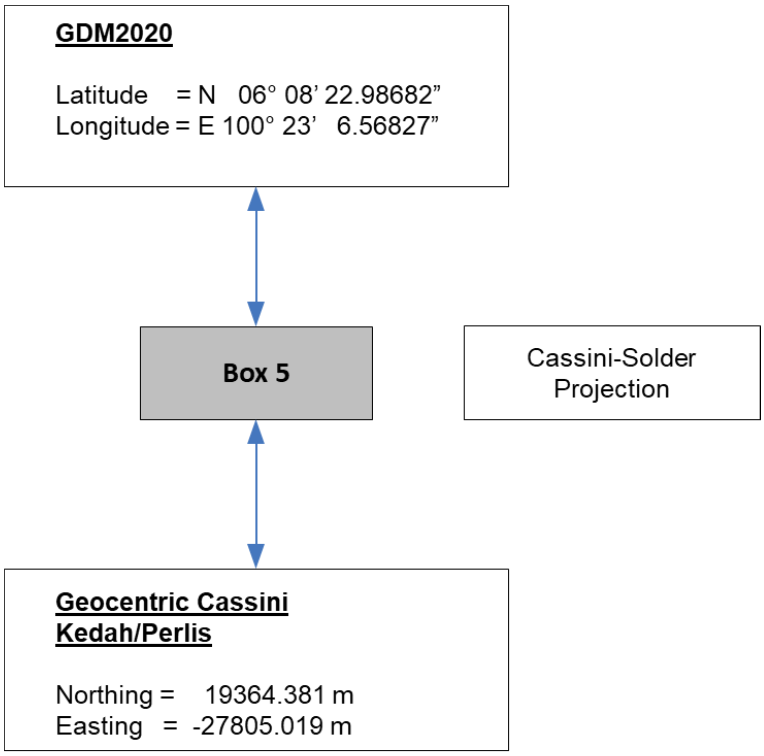
| State | Station Location | GDM2020 / GDM2000 (Rev 2009) / GDM2000 | | Cassini-Soldner | |
|---|---|---|---|---|---|
| | | Latitude (North) | Longitude (East) | Northing | Easting |
| Johor | Gunung Belumut | 2° 02’33.19581” | 103° 33’ 39.85196” | 0.000 | 0.000 |
| | | 2° 02’ 33.20279” | 103° 33’ 39.83599” | | |
| | | 2° 02’ 33.20196” | 103° 33’ 39.83730” | | |
| Negeri Sembilan Melaka | Gun Hill | 2° 42’ 43.62944” | 101° 56’ 22.94446” | 0.000 | 0.000 |
| | | 2° 42’ 43.63412” | 101° 56’ 22.92628” | | |
| | | 2° 42’ 43.63383” | 101° 56’ 22.92969” | | |
| Pahang | Gunung Sinyum | 3° 42’ 38.68785” | 102° 26’ 04.62219” | 0.000 | 0.000 |
| | | 3° 42’ 38.69308” | 102° 26’ 04.60447” | | |
| | | 3° 42’ 38.69263” | 102° 26’ 04.60772” | | |
| Selangor | Bukit Asa | 3° 40’ 48.37310” | 101° 30’ 24.49970” | 0.000 | 0.000 |
| | | 3° 40’ 48.37751” | 101° 30’ 24.48130” | | |
| | | 3° 40’ 48.37778” | 101° 30’ 24.48581” | | |
| Terengganu | Gunung Gajah Trom | 4° 56’ 44.96715” | 102° 53’ 37.01899” | 0.000 | 0.000 |
| | | 4° 56’ 44.97144” | 102° 53’ 37.00068” | | |
| | | 4° 56’ 44.97184” | 102° 53’ 37.00496” | | |

**Table 7**: (*continued*)

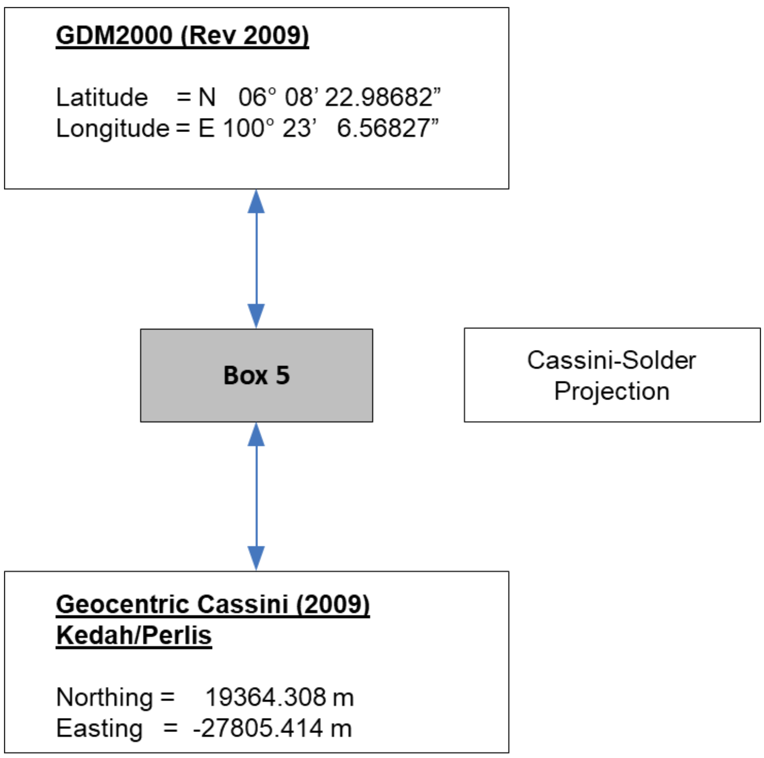
| State | Station Location | GDM2020 / GDM2000 (Rev 2009) / GDM2000 | | Cassini-Soldner | |
|---|---|---|---|---|---|
| | | Latitude (North) | Longitude (East) | Northing | Easting |
| Pulau Pinang<br>Seberang Perai | Fort Cornwallis | 5° 25’ 15.19941” | 100° 20’ 40.77228” | 0.000 | 0.000 |
| | | 5° 25’ 15.20204” | 100° 20’ 40.75188” | | |
| | | 5° 25’ 15.20433” | 100° 20’ 40.76024” | | |
| Kedah<br>Perlis | Gunung Perak | 5° 57’ 52.81746” | 100° 38’ 10.94996” | 0.000 | 0.000 |
| | | 5° 57’ 52.81981” | 100° 38’ 10.93028” | | |
| | | 5° 57’ 52.82155” | 100° 38’ 10.93860” | | |
| Perak | Gunung Hijau Larut | 4° 51’ 32.64021” | 100° 48’ 55.48363” | 0.000 | 0.000 |
| | | 4° 51’ 32.64361” | 100° 48’ 55.46334” | | |
| | | 4° 51’ 32.64488” | 100° 48’ 55.47038” | | |
| Kelantan | Bukit Panau (Baru) | 5° 53’ 37.07511” | 102° 10’ 32.25823” | 0.000 | 0.000 |
| | | 5° 53’ 37.07908” | 102° 10’ 32.24004” | | |
| | | 5° 53’ 37.07975” | 102° 10’ 32.24529” | | |

### 4.6 Test Examples

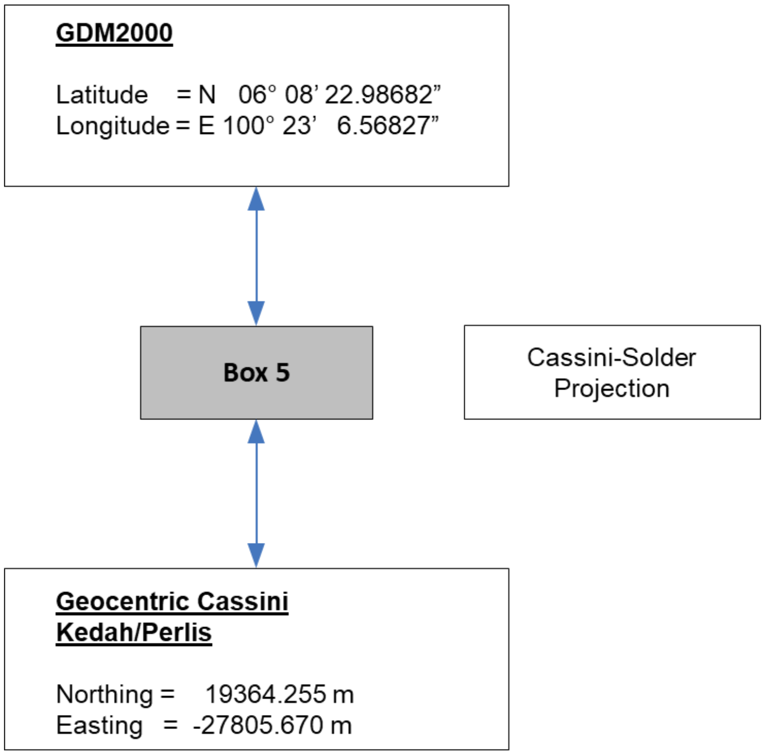
**(i) GDM2020 to Geocentric Cassini (GDM2020)**



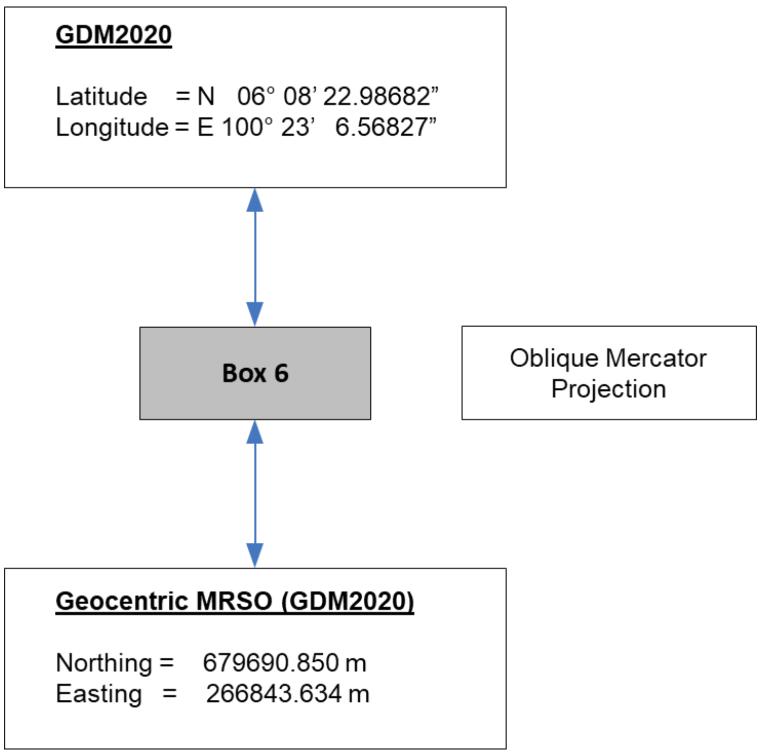

**(ii) GDM2000 (Rev 2009) to Geocentric Cassini (2009)**



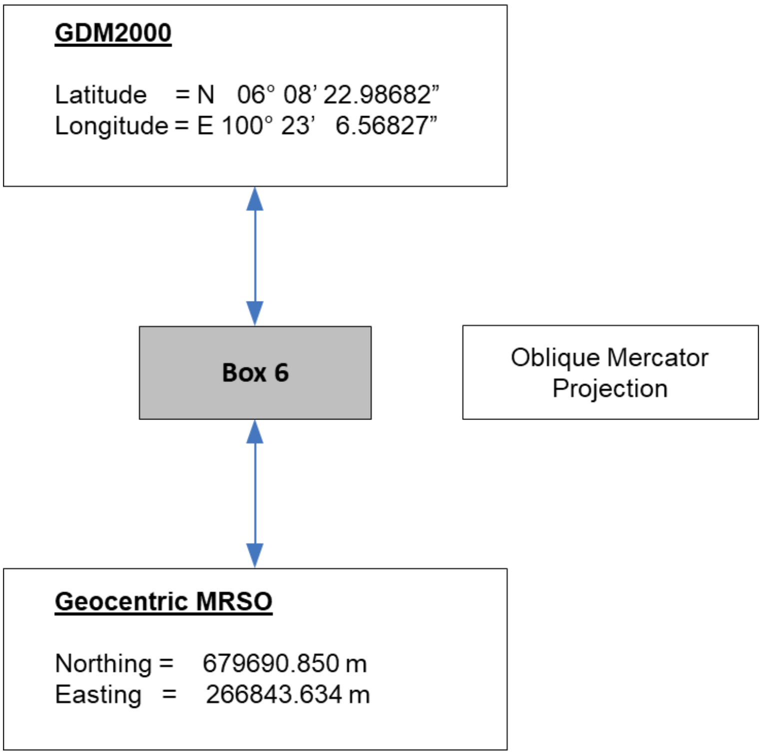
**(iii) GDM2000 to Geocentric Cassini**



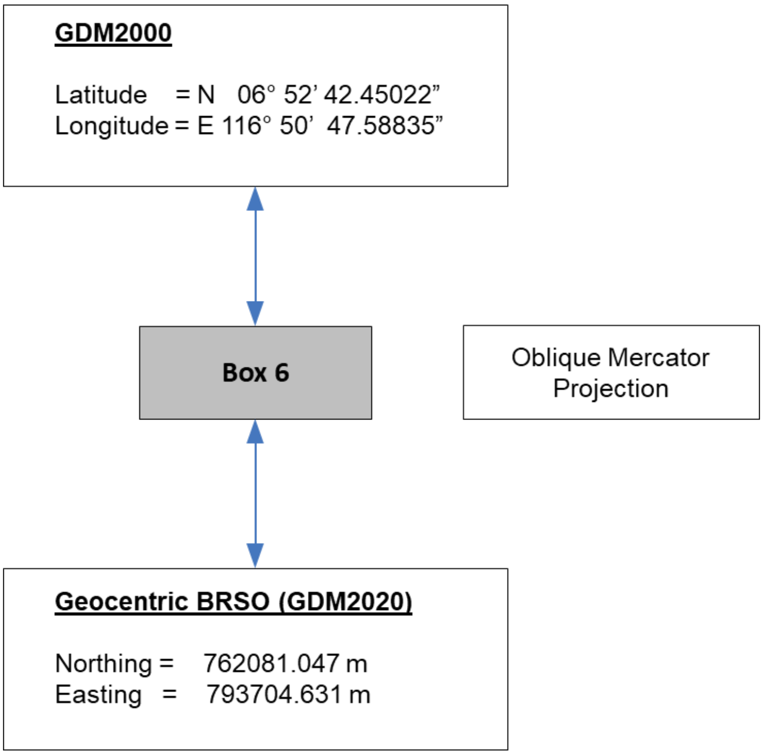

**(iv) GDM2020 to Geocentric MRSO (GDM2020)**



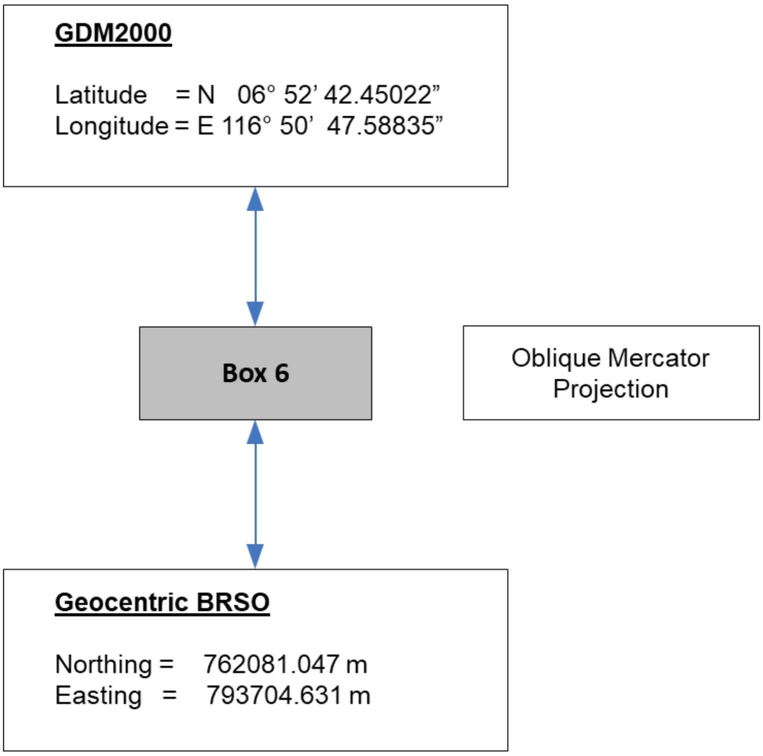
**(v) GDM2000 to Geocentric MRSO**



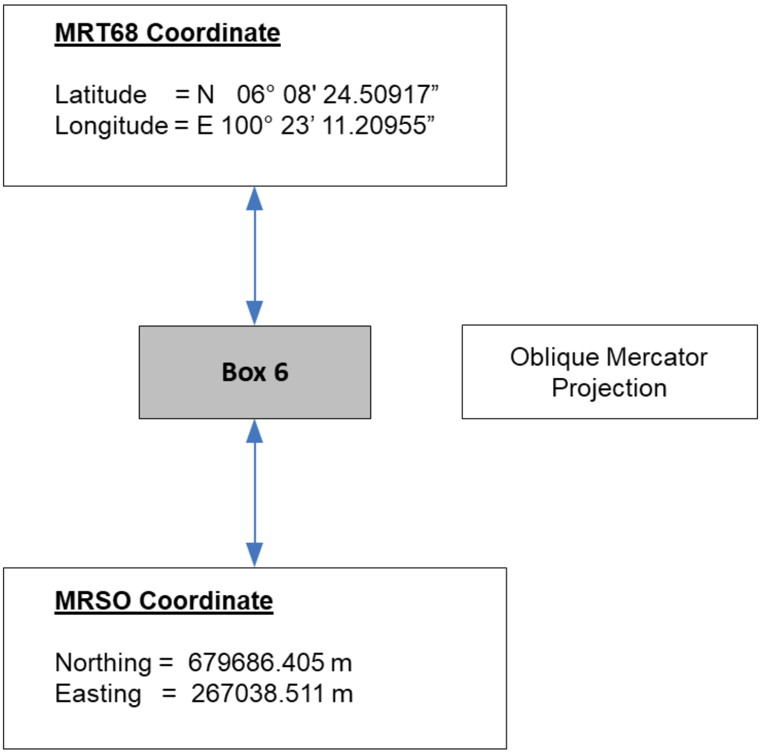

**(vi) GDM2020 to Geocentric BRSO (GDM2020)**



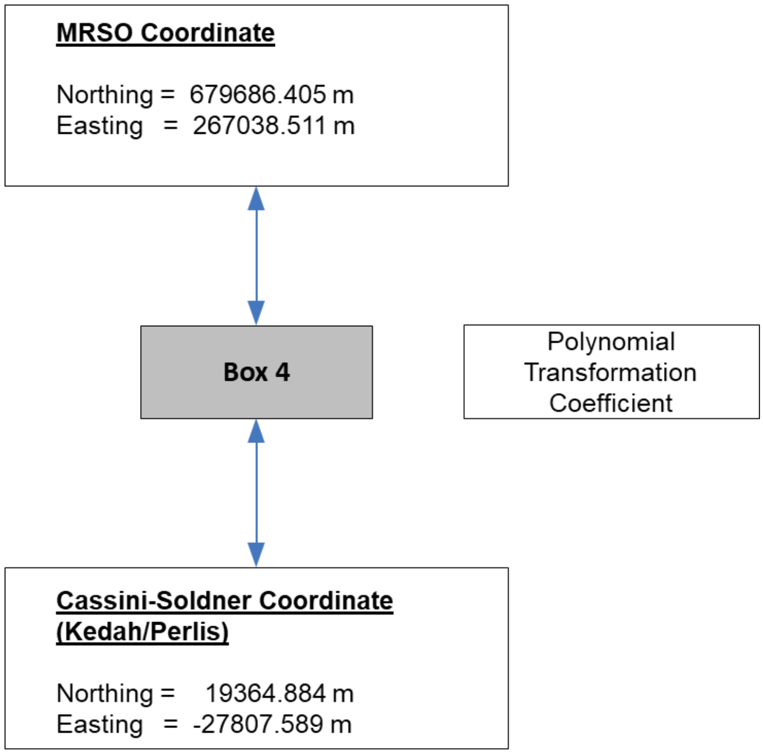
**(vii) GDM2000 to Geocentric BRSO**



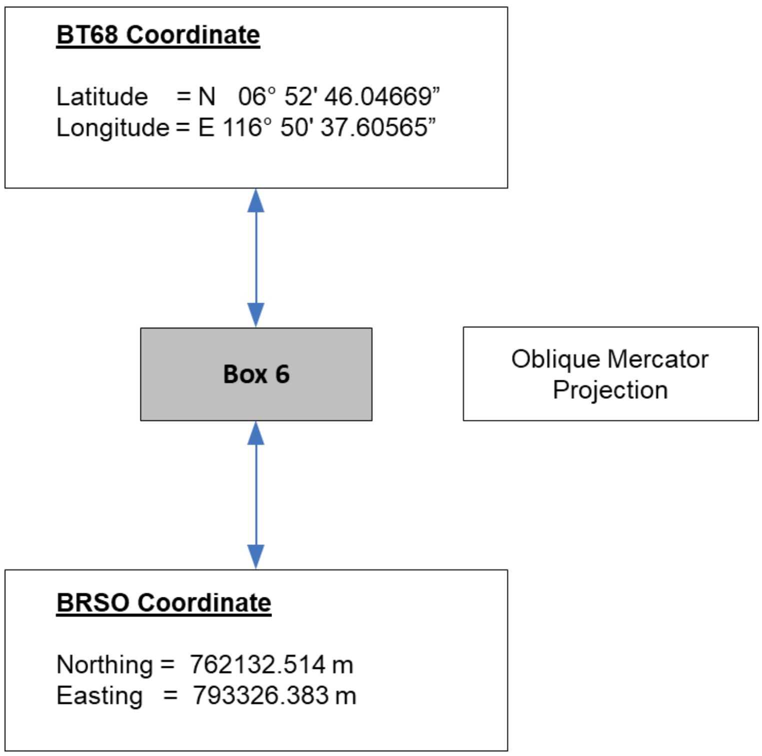

**(viii) MRT68 to MRSO**



## (ix) MRSO to Cassini Soldner




## (x) BT68 to BRSO



## 5. CONCLUSION

5.1 The determination of a position requires the choice of a coordinate reference system. A situation now exists whereby it is common for a user acquiring data in a coordinate system that is completely different to which the data will be ultimately required.

5.2 In order to facilitate the conversion of various coordinates system in Malaysia, JUPEM has produced numerous sets of datum transformation and map projection parameters to relate the different types of coordinate system. This technical guide is designed as a practical solution to those who encountered datums transformation and map projections problems.

5.3 The parameter values relating to different coordinate reference systems are derived from standard coordinate conversion formulae, Bursa-Wolf transformation formulae and multiple regression model.

5.4 In addressing the effects of plate tectonic motion due to natural disasters such as earthquakes on the coordinates reference system and vertical datum systems for the whole country, JUPEM has successfully established a more accurate, precise and contemporary GDM2020. This newly derived geodetic datum system is fully aligned to ITRF2014, where velocities and PSD are modelled as an intrinsic component of the kinematic/ semi-kinematic concept of the CORS coordinates.

## REFERENCES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