Frame classification

Methods of Analysis

Global frame analysis

- Aims of global frame analysis
  – Determine the distribution of the internal forces
  – Determine the corresponding deformations

- Means
  – Adequate models incorporating assumptions about the behaviour of the structure and its component:

    members and joints

Requirements for analysis

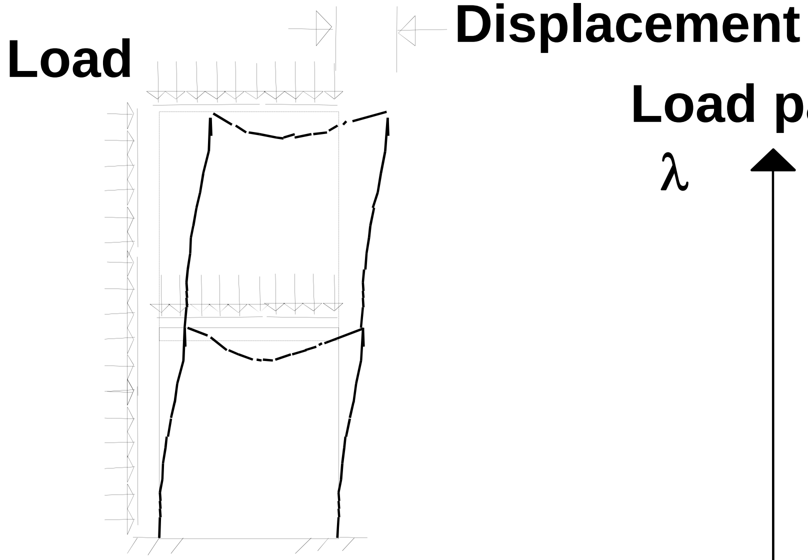
- Basic principles to be satisfied:
  - *Equilibrium* throughout the structure
  - *Compatibility* of deformation between the frame components
  - *Constitutive laws* for the frame components

- Frame model - element model
  - must satisfy the basic principles

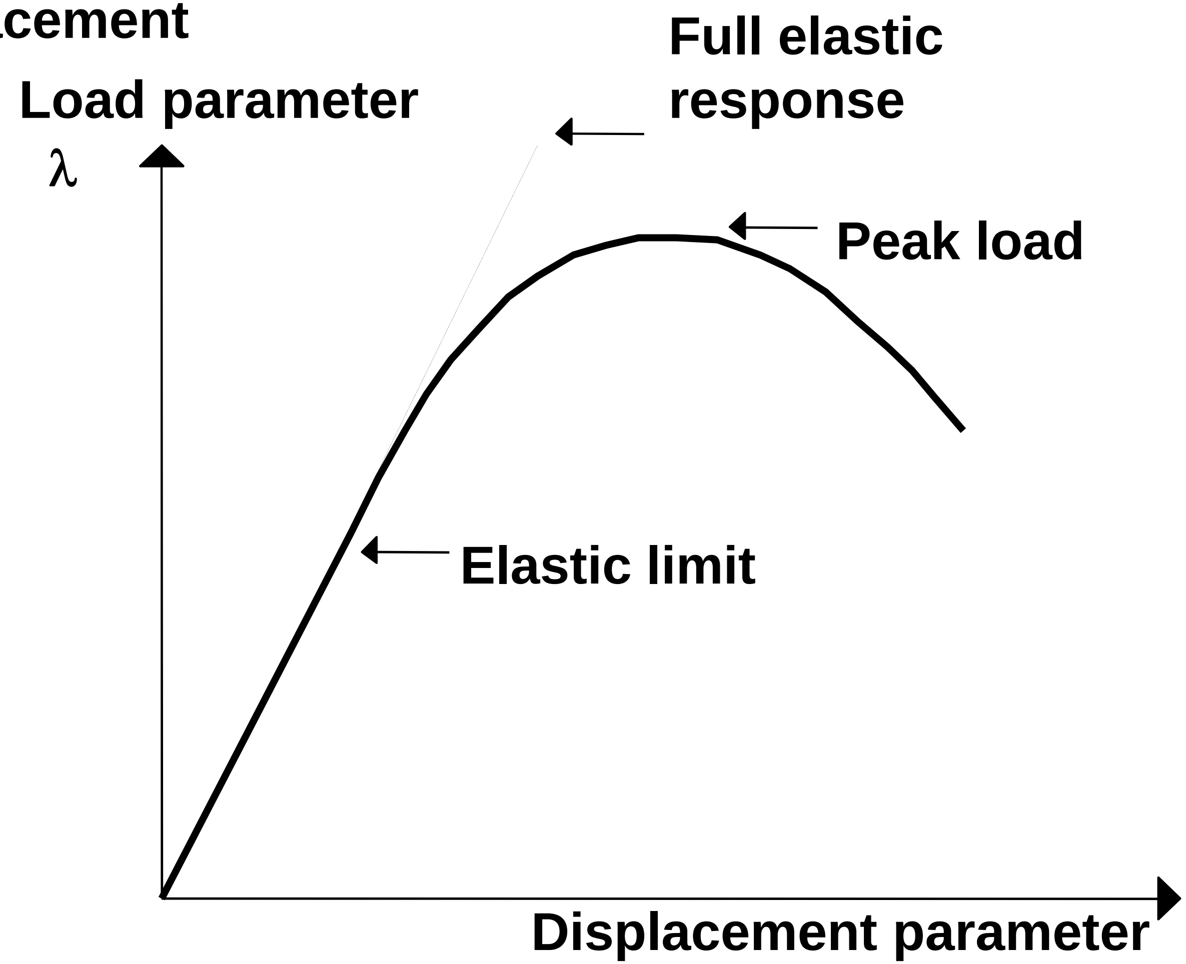
Frame behaviour

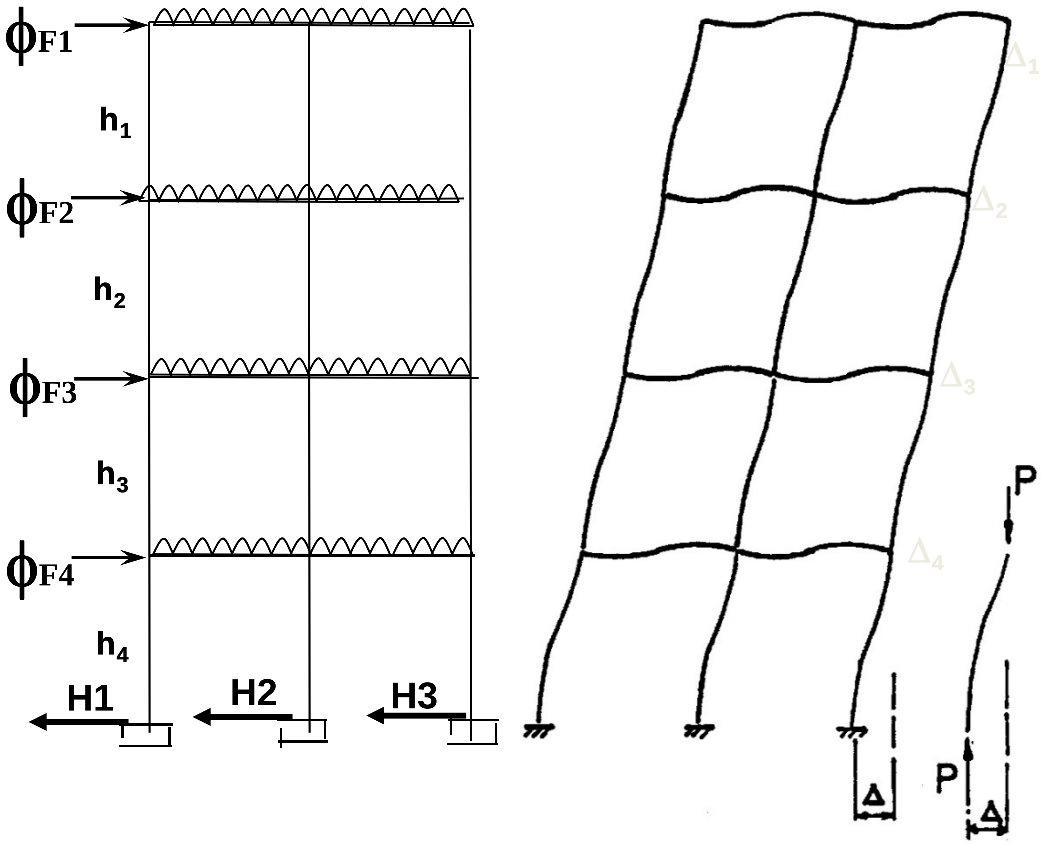
- Actual response of the frame is non linear
  - Linear behaviour limited
  - Non-linear behaviour due to:
    - Geometrical influence of the actual deformed shape (second order effects)
    - Joint behaviour
    - Material yielding

Frame behaviour



Frame





$$\delta_{1Hed} = \frac{h_1}{\Delta_1 - \Delta_2}$$

$$\delta_{2Hed} = \frac{h_2}{\Delta_2 - \Delta_3}$$

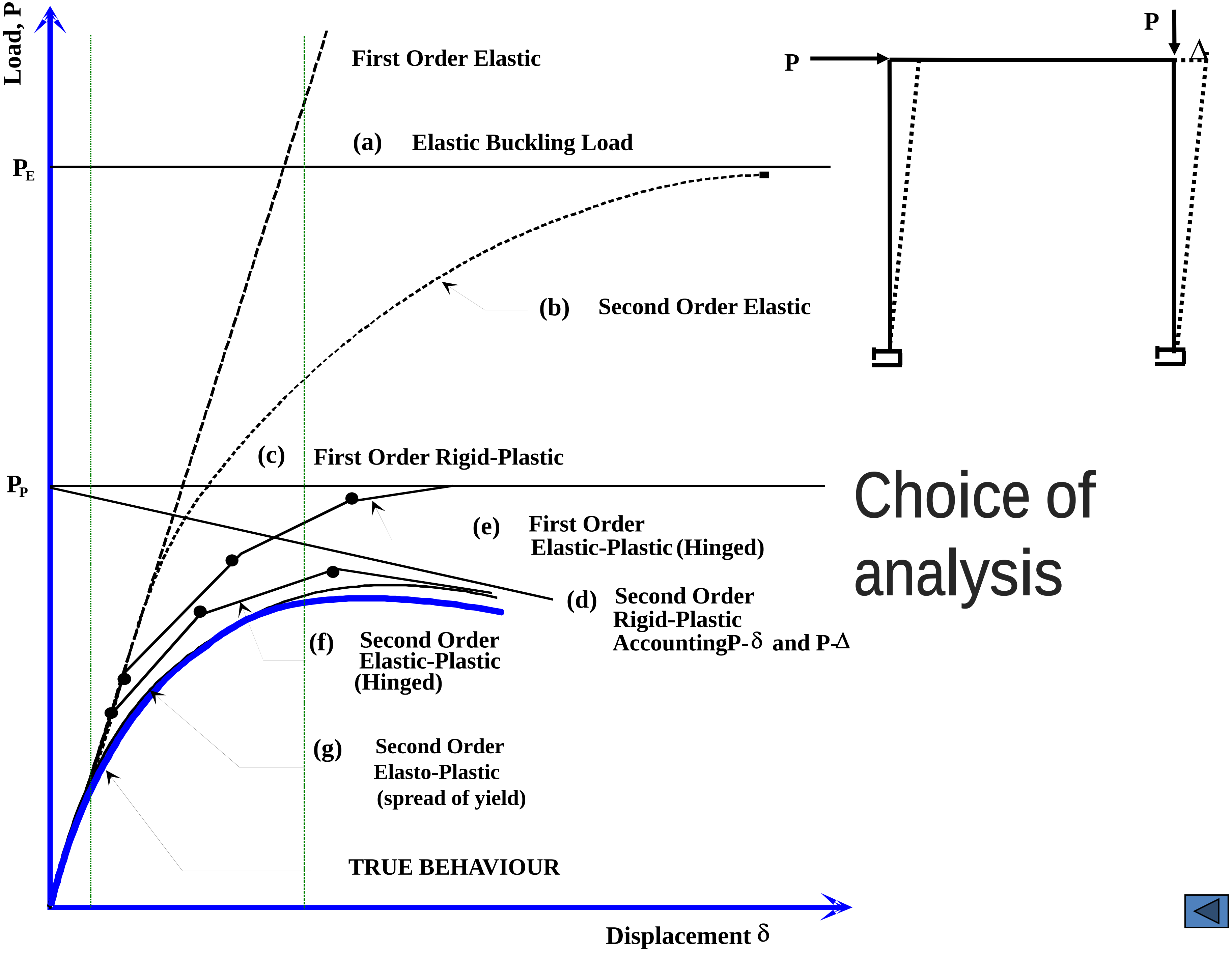
$$\delta_{3Hed} = \frac{h_3}{\Delta_3 - \Delta_4}$$

$$\delta_{4Hed} = \frac{h_4}{\Delta_4}$$

Factors affecting the deformation values;

1. **Material properties**
2. **Geometry of the structure**
3. **Boundary condition**
4. **Loadings**

Choice of analysis

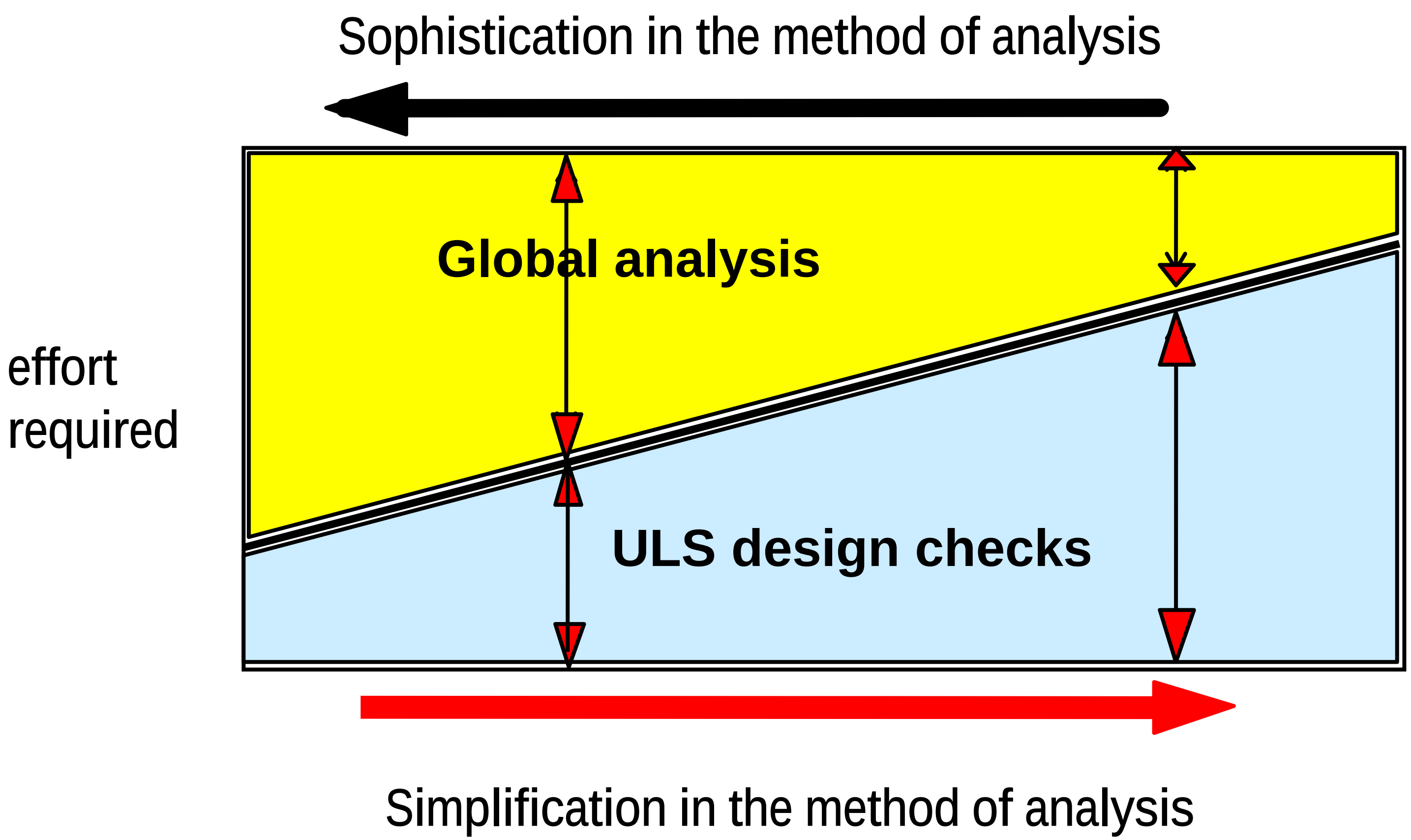


Decisions related to the analysis approach – EC3

Choice between

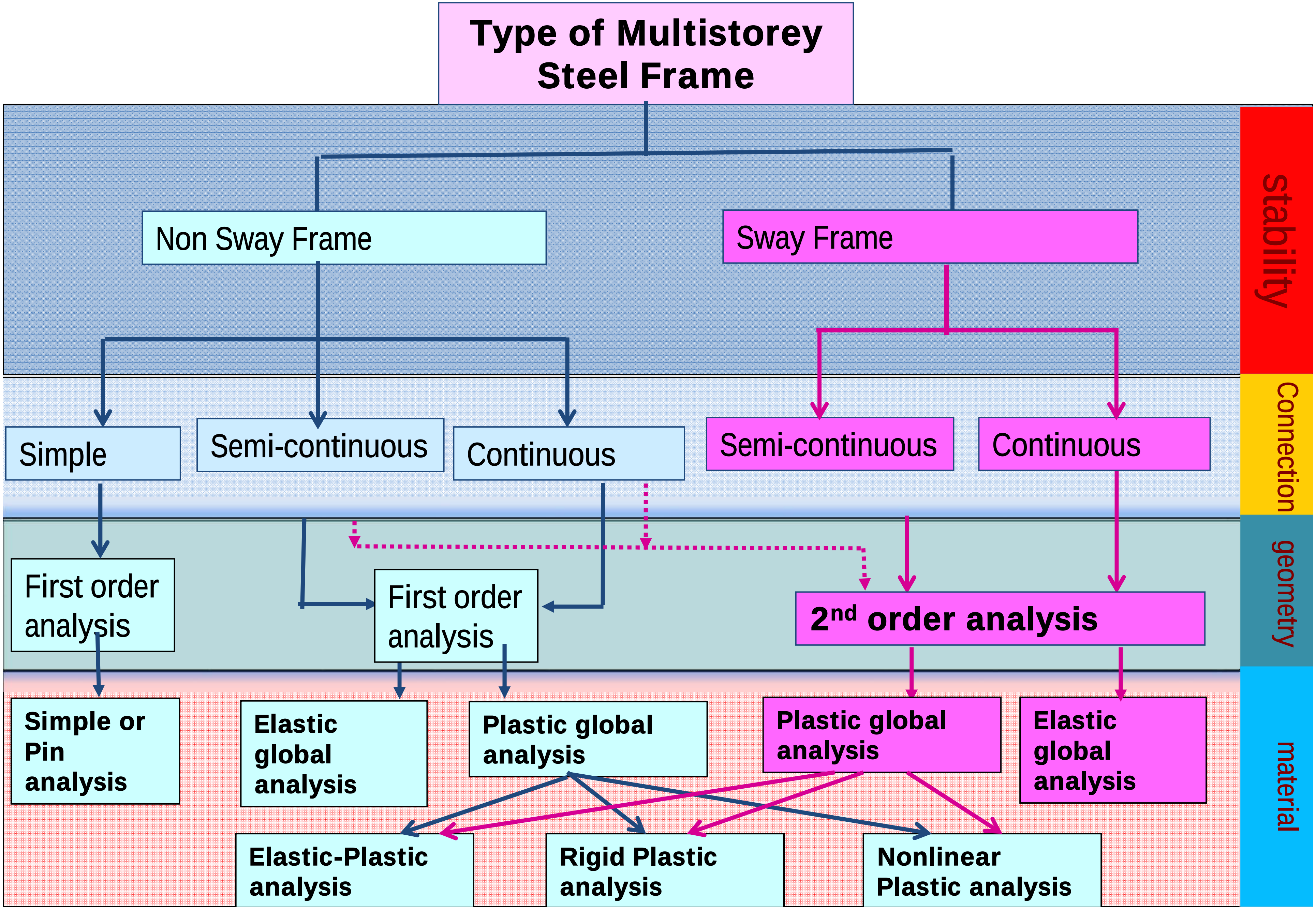
- an elastic and a plastic global analysis
- 1^{st} order and 2^{nd} order analysis
- a traditional approach and a modern approach to connection representation
- Combination of the above

Implications for design of the choice of the global analysis



Overall Design Task= Analysis + Design Checks

Relationship between type of frame, construction and analysis



Sway Stability

A frame is considered to be sway case if:

$$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}} \leq \ 10 \text{ for elastic analysis}$$

$$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}} \leq \ 15 \text{ for plastic analysis}$$

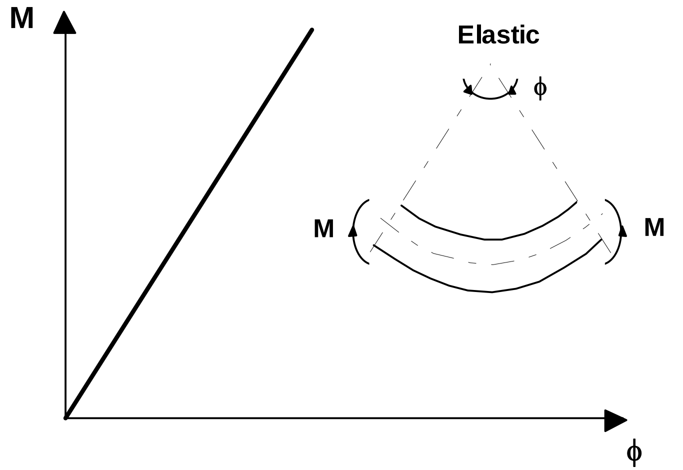
where

α_{cr} is the factor by which the design loading would have to be increased to cause elastic instability in a global mode

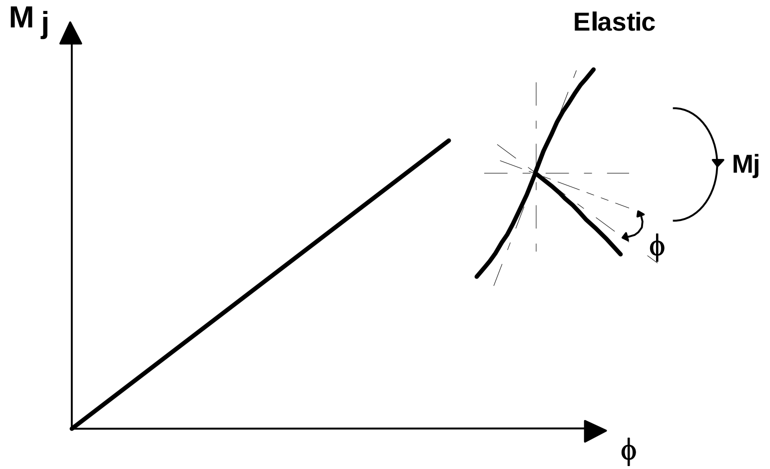
F_{Ed} is the design loading on the structure

F_{cr} is the elastic critical buckling load for global instability mode based on initial elastic stiffnesses

1st-order elastic analysis



Moment rotation characteristic of the section



Moment rotation characteristic of the joint

- Indefinite linear elastic response of member sections and of joints
- Equilibrium established for the *undeformed* structural configuration

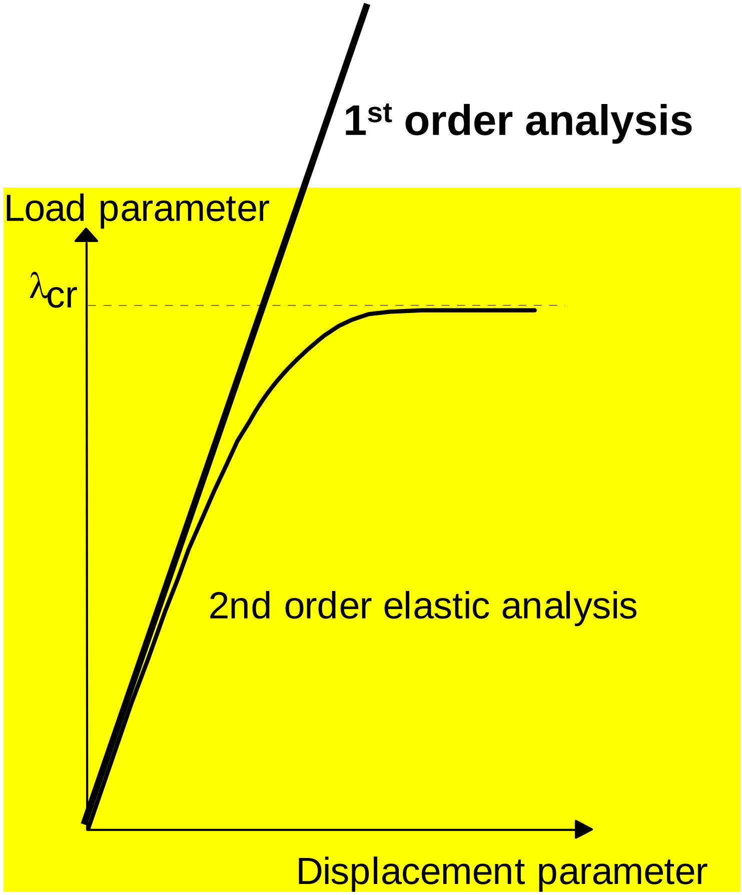
1st and 2nd order analysis

1st order analysis

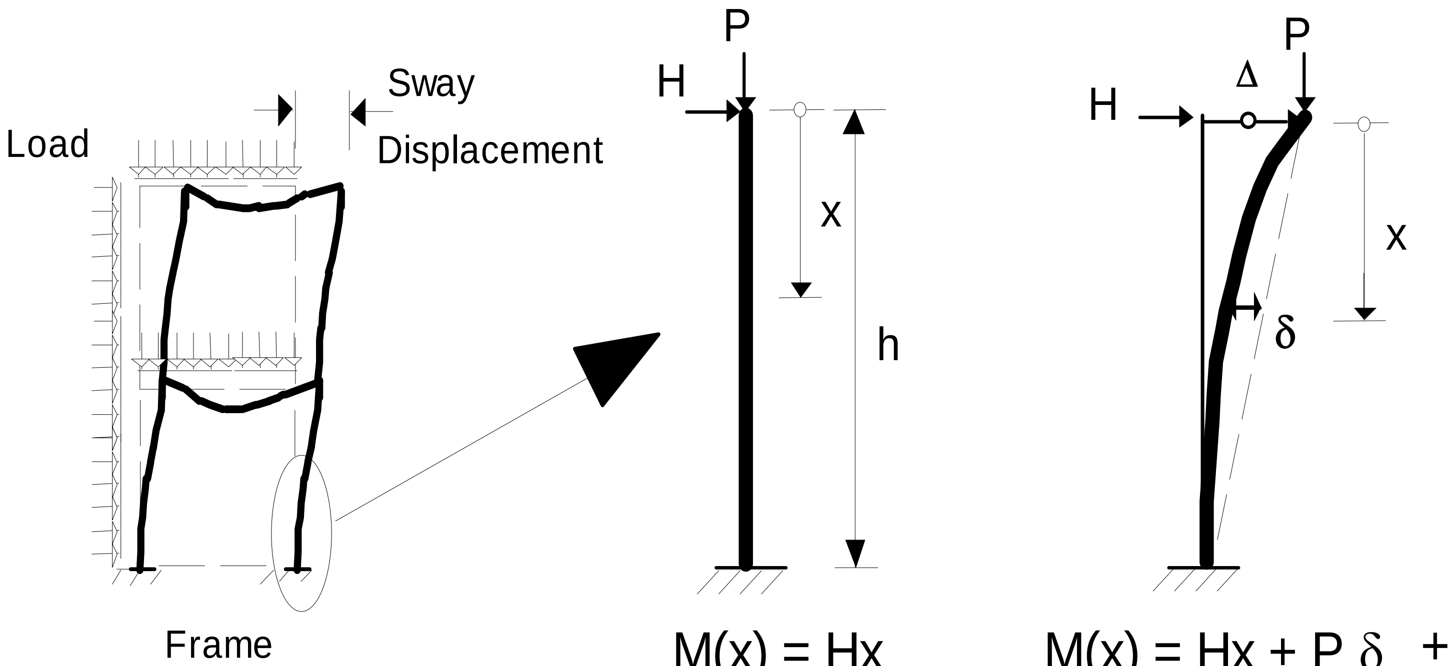
- Indefinite linear
- elastic response of member sections
- geometry and
- connections

2ND order analysis

- Indefinite linear- elastic response of member sections and joints
- Equilibrium established for the *deformed* structure
- Allows for P-D effect and, if necessary, for P-d effect



Second order effects



$M(x) = Hx$

$M(h) = Hh$

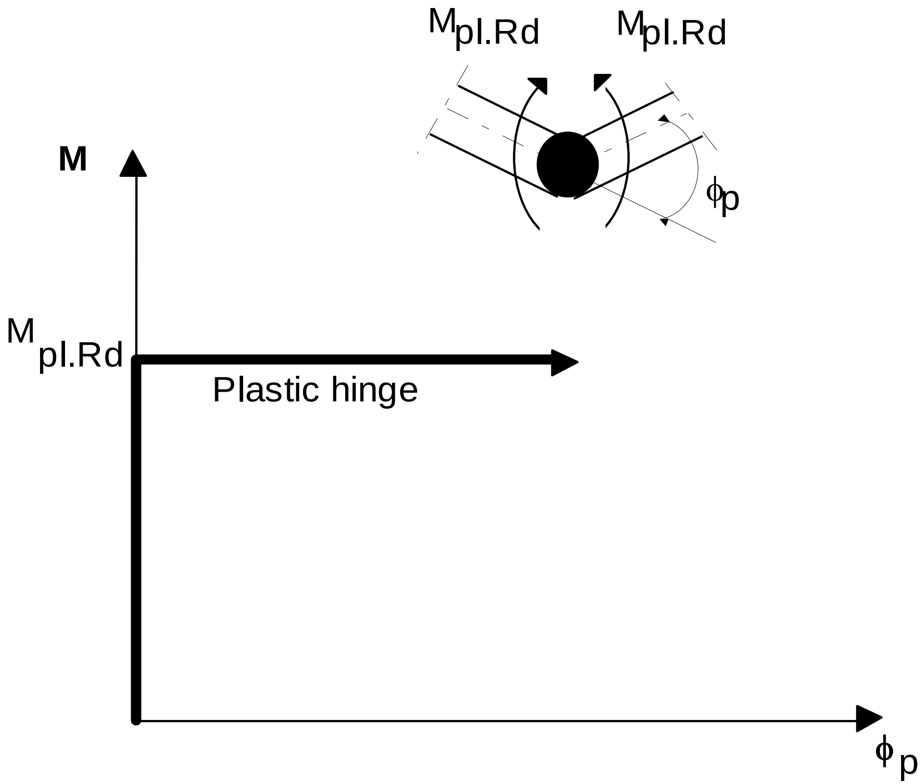
$M(x) = Hx + P\delta + P\Delta\, x/h$

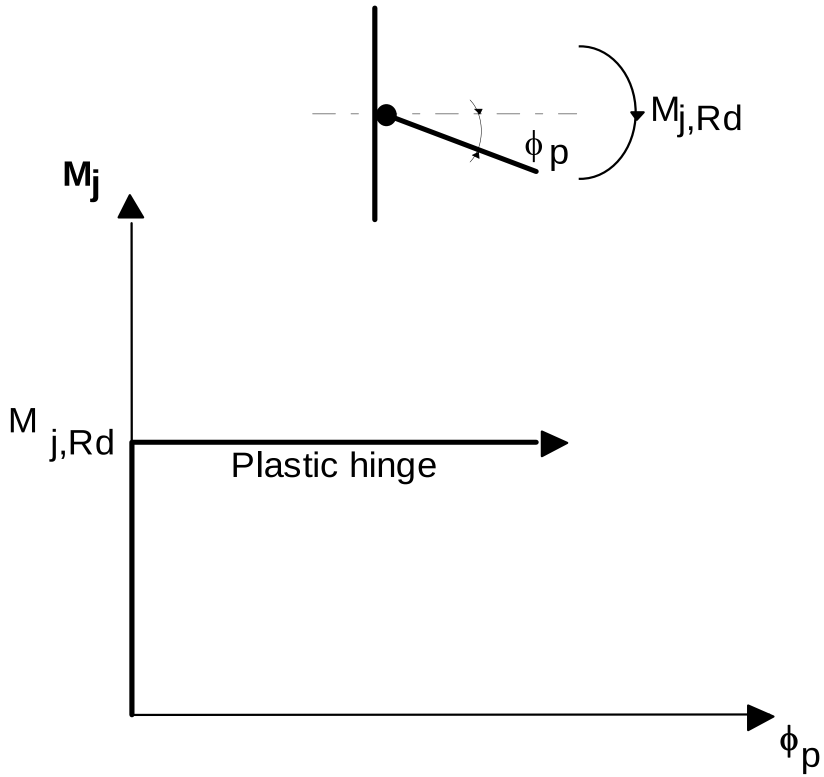
$M(h) = Hh + P\Delta$

Second order effects

- P-D effect :
  – due to floor sway
  – 1st order frame stiffness modified
  – dominant effect
- P-d effect :
  – due to beam-column deflection
  – 1st order member stiffness modified
  – significant only for relatively slender members which is rare

Rigid-plastic global analysis

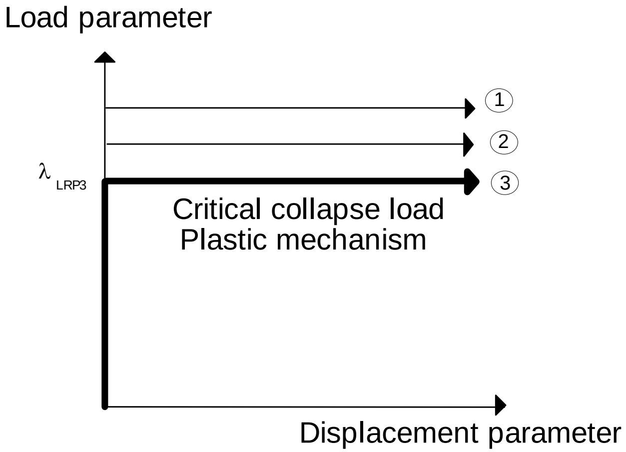


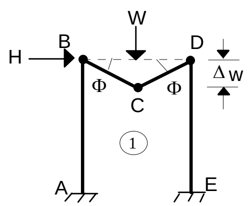


- Rigid-plastic joint behaviour when plastic hinges are allowed there

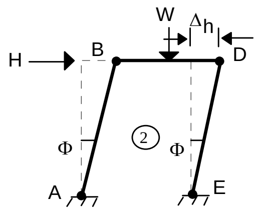
Rigid-plastic global analysis

- Usually a first order analysis
- Find critical mechanism
- Easy application for simple frames e.g. industrial portal frames
- Serviceability deflection check

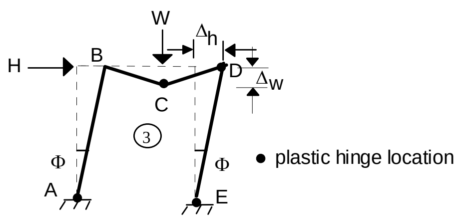




Beam mechanism



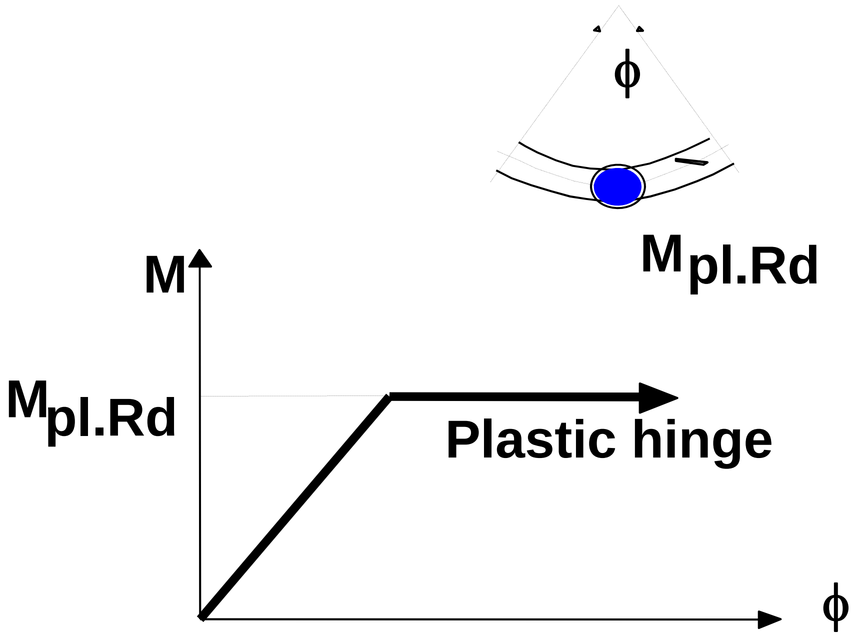
Sway mechanism



Combined mechanism

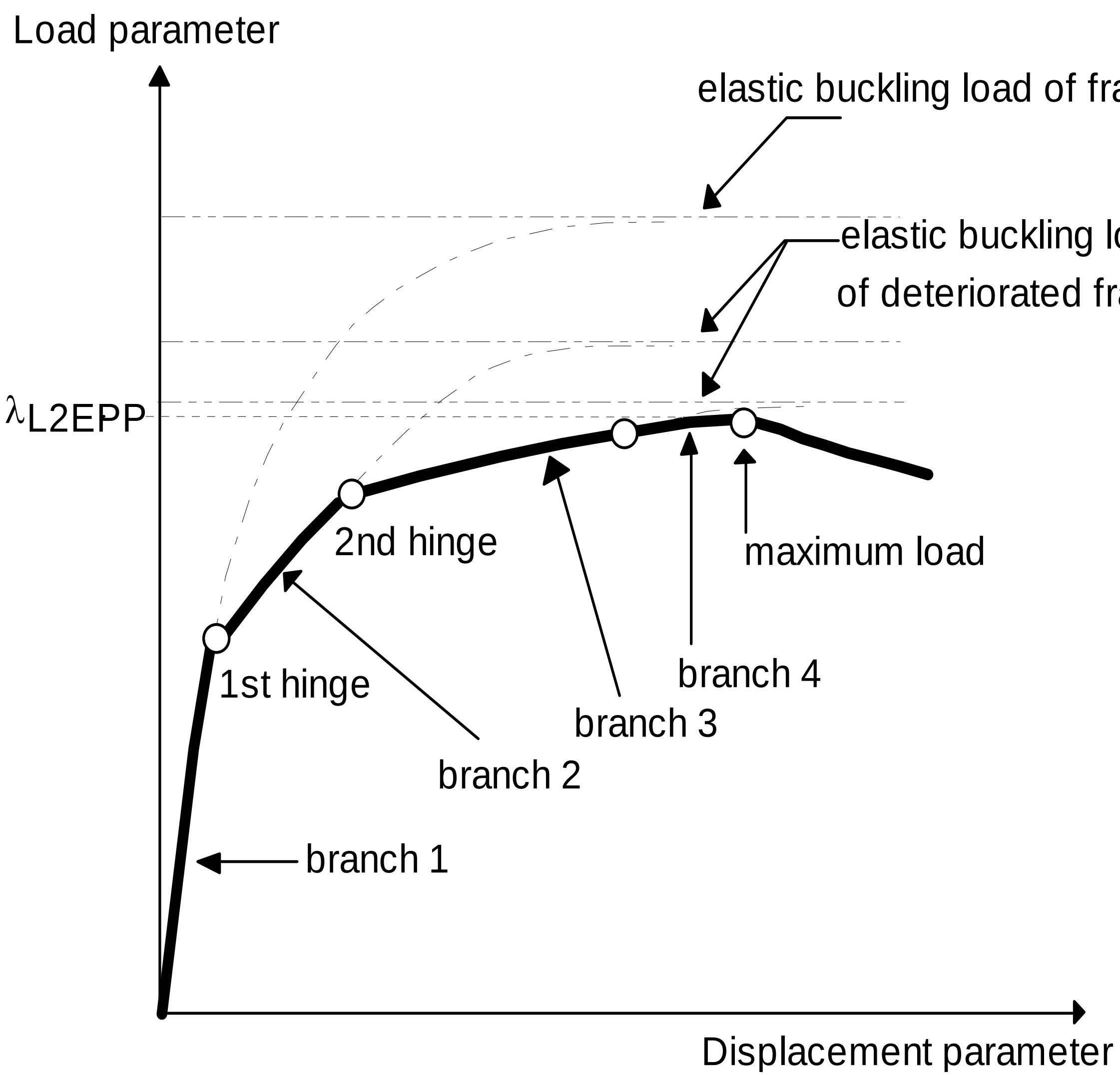
Elastic-perfectly plastic analysis

- Elastic-perfectly plastic response of member sections



Elastic-perfectly plastic analysis

- 2nd-order analysis usually used
- Load applied in increments
- “Deterioration” of frame stability as plastic hinges form



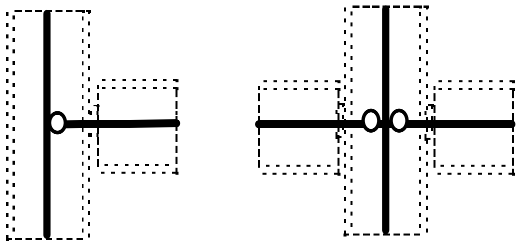
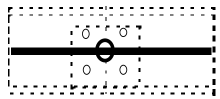
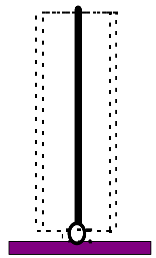
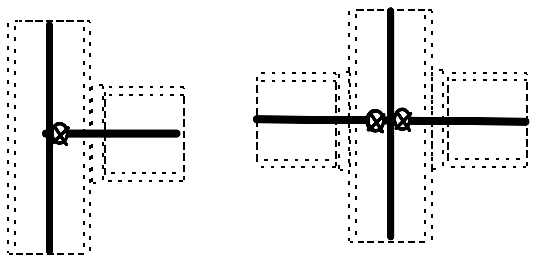
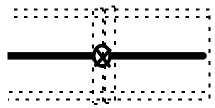
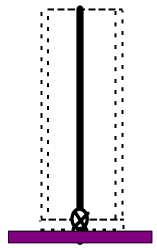
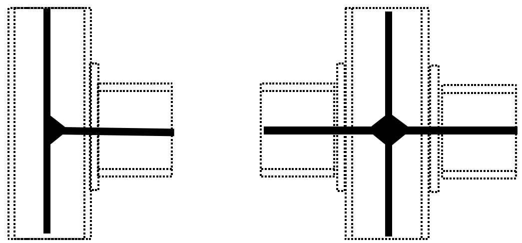
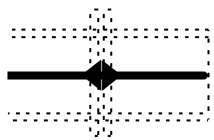
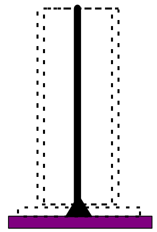
Multistorey Steel Frame

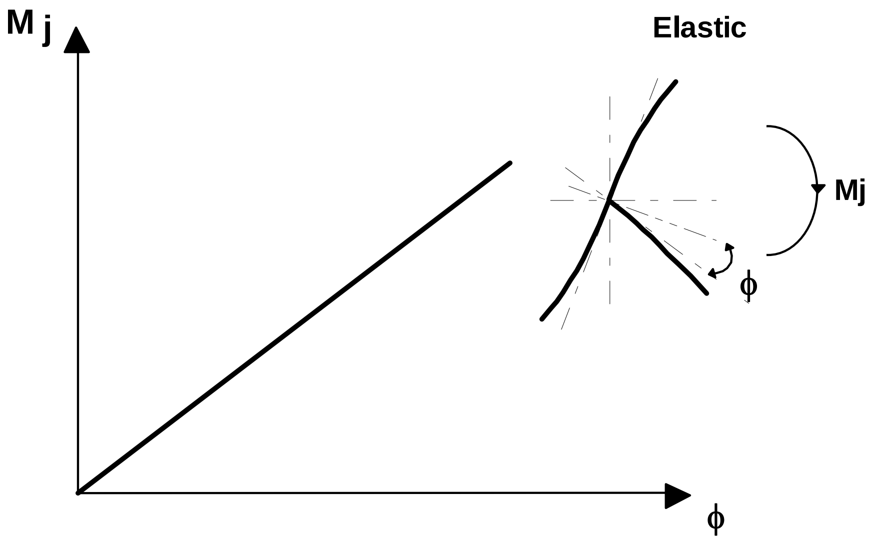
	Non-sway	Sway
Definiton	**Depends on frame geometry and load cases under consideration**	
	Determined by influenced of PΔ effect	
	Horizontal loads are carried by the bracing or by horizontal support	**Horizontal loads are carried by the frame**
	Change of geometry (2nd-order effect) is negligible	**Change of geometry (2nd-order effect) significant**
Method of analysis Geometry and material	**First-order elastic analysis (stifness analysis, moment distribution)**	**First-order elastic analysis with indirect allowance for second order effect (P-Δ and P-δ effect)**
	First-order rigid-plastic analysis	**First-order rigid-plastic analysis with indirect allowance for second order effect (P-Δ and P-δ effect)**
	Second-order elastic analysis	
	Second-order elastic plastic hinged analysis	
	Second-order elasto-plastic analysis	

Connection modelling in frame analysis

- Framing and joints
  - Continuous framing: → rigid joint
  - Simple framing: → pinned joint
  - Semi-continuous framing: → semi-rigid joint
- The main approaches are:
  - the traditional approach in which the joints are considered as (nominally) pinned or rigid
  - the semi-rigid approach in which a more realistic model representing the joint behaviour is used. It is usually introduced as a spiral spring at the extremity of the member it attaches (usually the beam).

Joint modelling for frame analysis

JOINT MODELLING	BEAM-TO-COLUMN JOINTS MAJOR AXIS BENDING	BEAM SPLICES	COLUMN BASES
SIMPLE			
SEMI-CONTINUOUS			
CONTINUOUS			

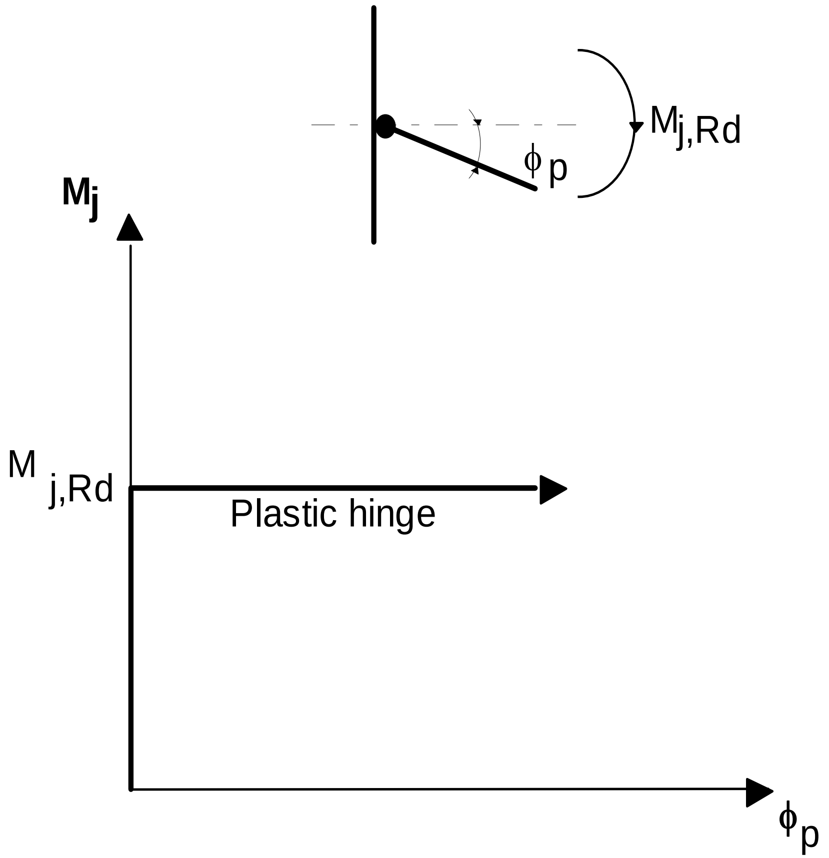


Moment rotation characteristic of the joint

- Indefinite linear elastic response of joints
- Equilibrium established for the *undeformed* structural configuration

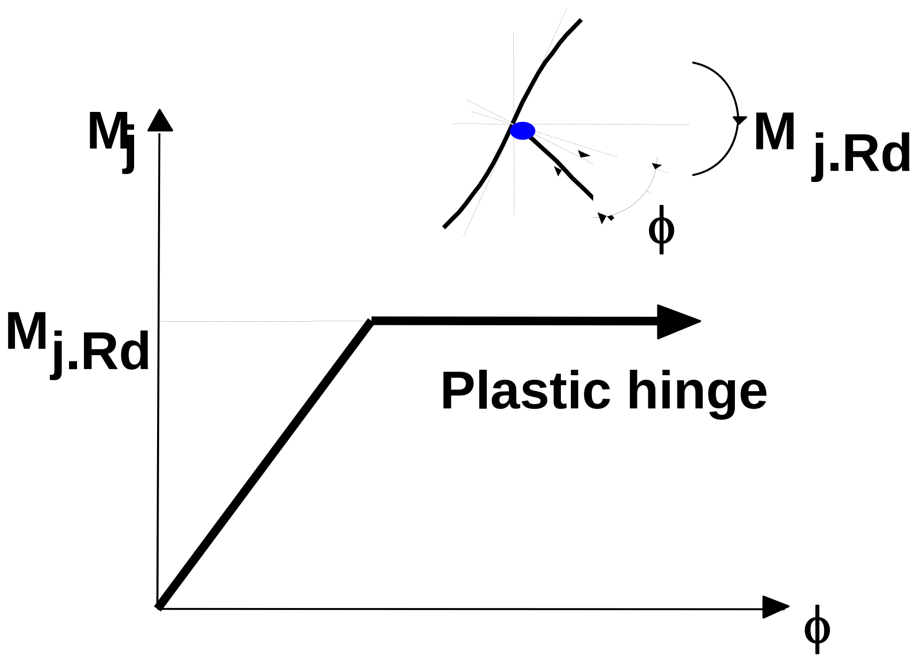
e

Rigid plastic



Moment rotation characteristics of the joint

- Elastic-perfectly plastic response of connections

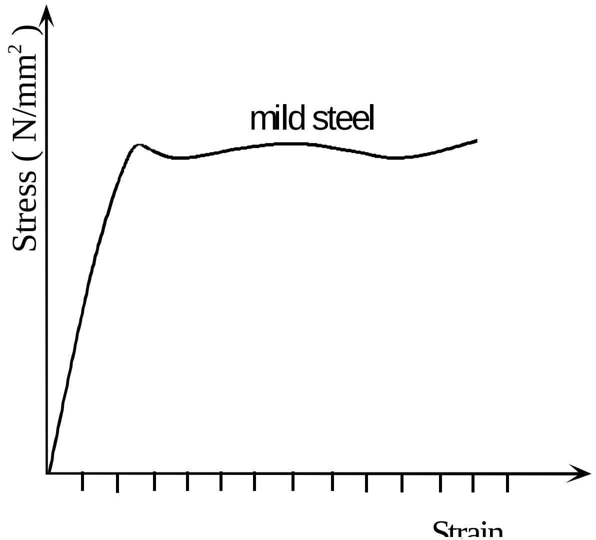


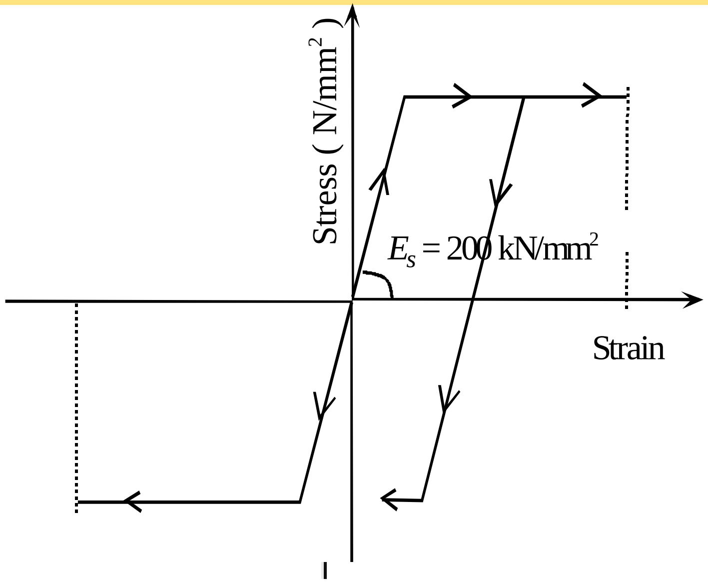
Steel Frame

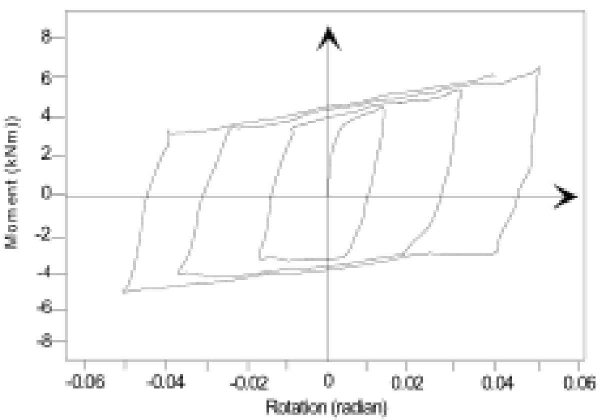
UTM UNIVERSITI TEKNOLOGI MALAYSIA

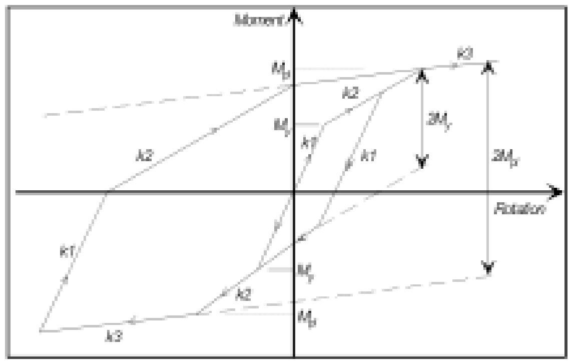
		Non-sway	Sway
Definiton		Depends on frame geometry and load cases under consideration	
		Determined by influenced of $P\Delta$ effect	
		Horizontal loads are carried by the bracing or by horizontal support	Horizontal loads are carried by the frame
		Change of geometry (2nd-order effect) is negligible	Change of geometry (2nd-order effect) significant
Method of analysis	Elastic analysis	First-order elastic analysis (stifness analysis, moment distribution)	First-order elastic analysis with indirect allowance for second order effect (P-Δ and P-δ effect)
		Second-order elastic analysis	
	Plastic analysis	First-order rigid-plastic analysis	First-order rigid-plastic analysis with indirect allowance for second order effect (P-Δ and P-δ effect)
		Second-order elastic plastic hinged analysis	
		Second-order elasto-plastic analysis	

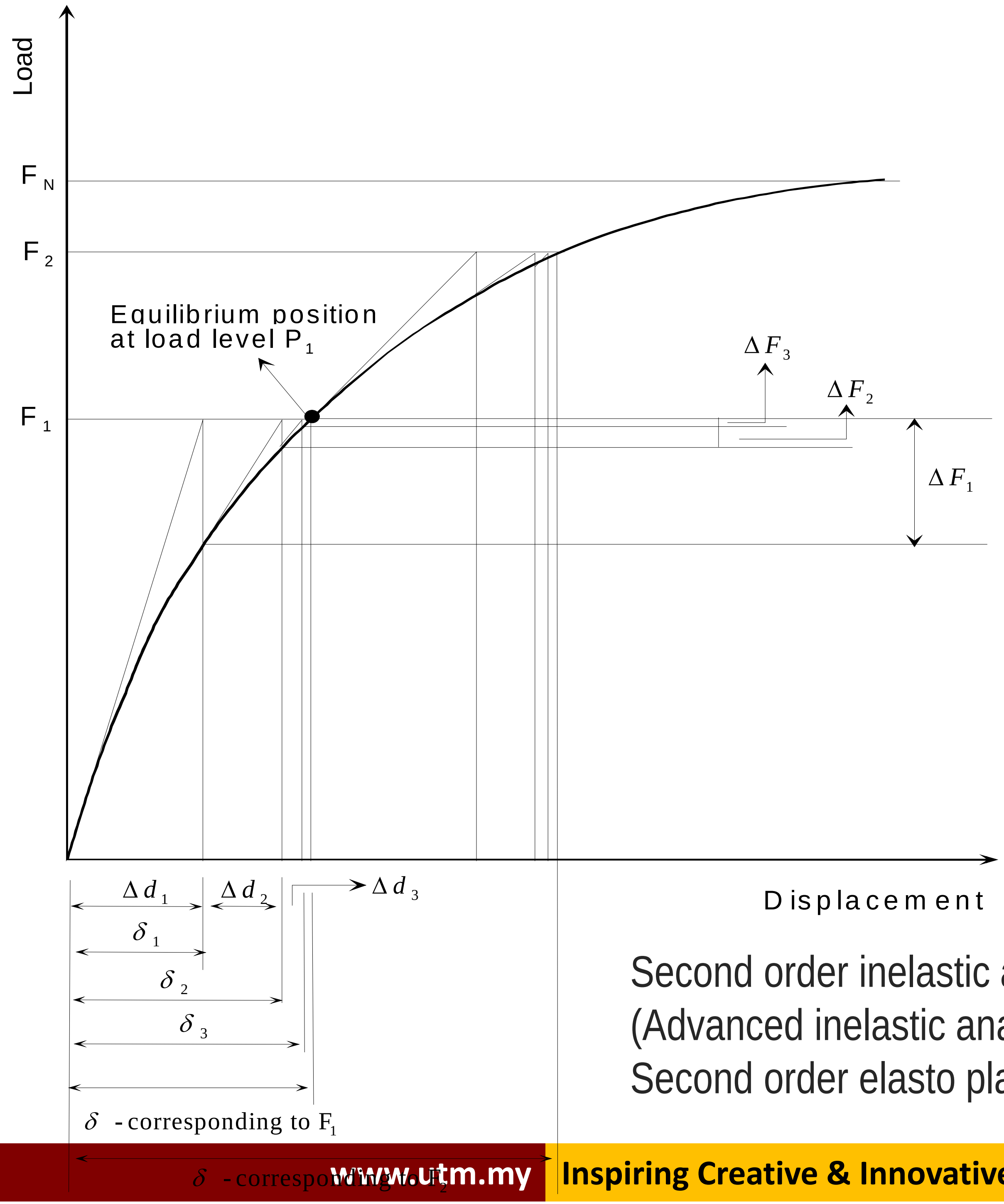
Second order inelastic analysis
(Advanced inelastic analysis)
Second order elasto plastic analysis











Second order inelastic analysis
(Advanced inelastic analysis)
Second order elasto plastic analysis

Second order inelastic analysis for frames (Advanced inelastic analysis)

$$[K] = [K_E] + [K_L] + [K_G]$$

Where

$[K_E]$ represents the elastic, small displacement stiffness matrix.

$[K_L]$ is due to large displacements and is known as initial displacement matrix or the large displacement matrix.

$[K_G]$ is dependent upon the current stress level, and accounts for the effect of axial force on the change of bending stiffness of the element and is known as the initial stress matrix or geometric matrix.

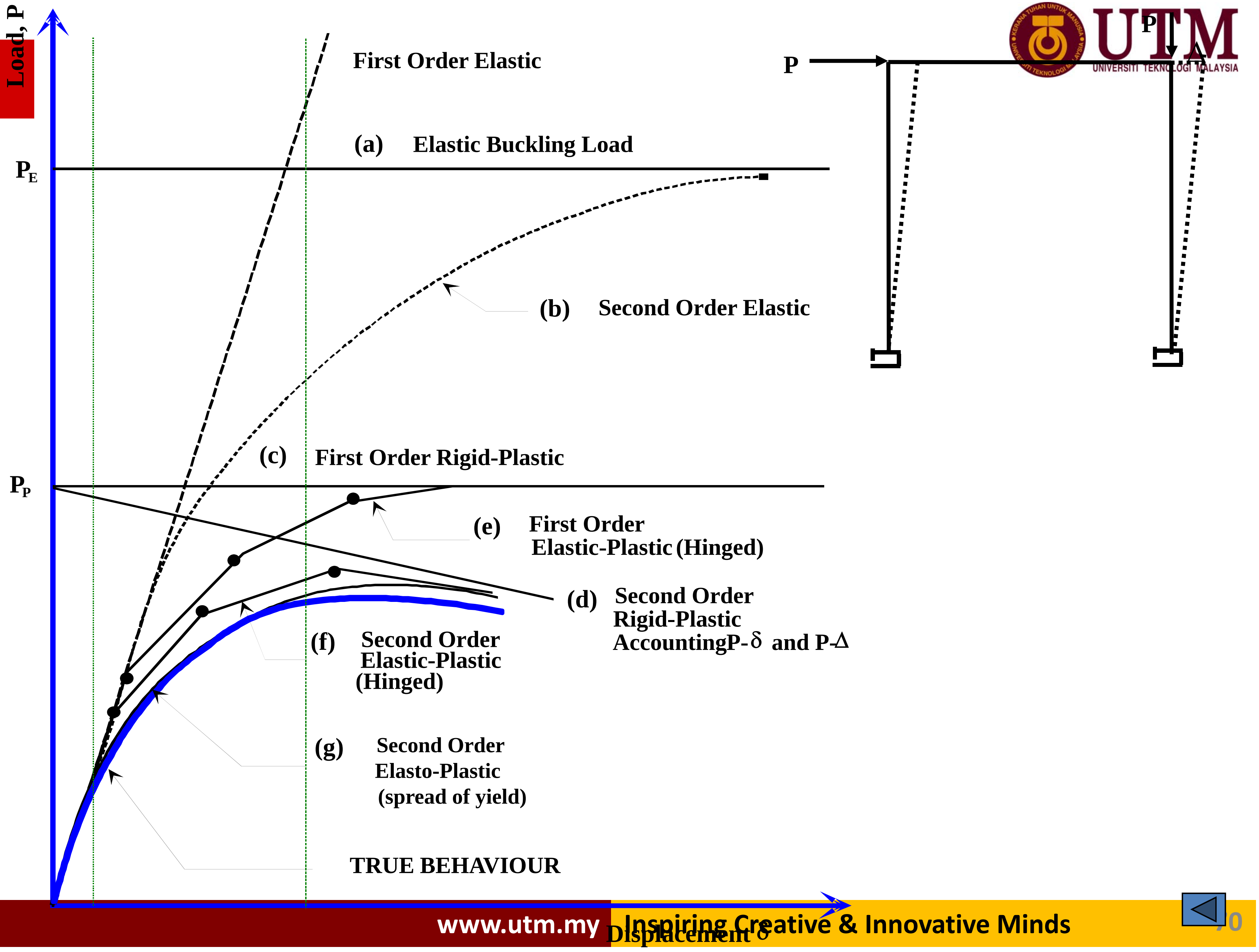
$$
K_E = \int_0^L \begin{bmatrix}
\left(\frac{dN_1}{dx}\frac{dN_1}{dx}EA\right) & 0 & 0 & \left(\frac{dN_1}{dx}\frac{dN_2}{dx}EA\right) & 0 & 0 \\
0 & \left(\frac{d^2N_2}{dx^2}\frac{d^2N_2}{dx^2}EI\right) & \left(\frac{d^2N_2}{dx^2}\frac{d^2N_3}{dx^2}EI\right) & 0 & \left(\frac{d^2N_2}{dx^2}\frac{d^2N_5}{dx^2}EI\right) & \left(\frac{d^2N_2}{dx^2}\frac{d^2N_6}{dx^2}EI\right) \\
0 & \left(\frac{d^2N_3}{dx^2}\frac{d^2N_2}{dx^2}EI\right) & \left(\frac{d^2N_3}{dx^2}\frac{d^2N_3}{dx^2}EI\right) & 0 & \left(\frac{d^2N_3}{dx^2}\frac{d^2N_5}{dx^2}EI\right) & \left(\frac{d^2N_3}{dx^2}\frac{d^2N_6}{dx^2}EI\right) \\
\left(\frac{dN_4}{dx}\frac{dN_1}{dx}EA\right) & 0 & 0 & \left(\frac{dN_4}{dx}\frac{dN_2}{dx}EA\right) & 0 & 0 \\
0 & \left(\frac{d^2N_5}{dx^2}\frac{d^2N_2}{dx^2}EI\right) & \left(\frac{d^2N_5}{dx^2}\frac{d^2N_3}{dx^2}EI\right) & 0 & \left(\frac{d^2N_5}{dx^2}\frac{d^2N_5}{dx^2}EI\right) & \left(\frac{d^2N_5}{dx^2}\frac{d^2N_6}{dx^2}EI\right) \\
0 & \left(\frac{d^2N_6}{dx^2}\frac{d^2N_2}{dx^2}EI\right) & \left(\frac{d^2N_6}{dx^2}\frac{d^2N_3}{dx^2}EI\right) & 0 & \left(\frac{d^2N_6}{dx^2}\frac{d^2N_5}{dx^2}EI\right) & \left(\frac{d^2N_6}{dx^2}\frac{d^2N_6}{dx^2}EI\right)
\end{bmatrix} dx
$$

$$
+ \quad \left(N_{j1}{}^T C_{j1} N_{j1} + N_{j2}{}^T C_{j2} N_{j2}\right)
$$

$$K_L = \int_0^L \begin{bmatrix} (0) & \left(\frac{dN_1}{dx}EA\frac{dN_3}{dx}\{X\}\right) & \left(\frac{dN_1}{dx}EA\frac{dN_4}{dx}\{X\}\right) & 0 & \left(\frac{dN_1}{dx}EA\frac{dN_5}{dx}\{X\}\right) & \left(\frac{dN_1}{dx}EA\frac{dN_6}{dx}\{X\}\right) \\ \left(\frac{dN_1}{dx}EA\frac{dN_3}{dx}\{X\}\right) & \left(\frac{dN_3}{dx}\{X\}EA\frac{dN_3}{dx}\{X\}\right) & \left(\frac{dN_3}{dx}\{X\}EA\frac{dN_4}{dx}\{X\}\right) & \left(\frac{dN_2}{dx}EA\frac{dN_3}{dx}\{X\}\right) & \left(\frac{dN_3}{dx}\{X\}EA\frac{dN_5}{dx}\{X\}\right) & \left(\frac{dN_3}{dx}\{X\}EA\frac{dN_6}{dx}\{X\}\right) \\ \left(\frac{dN_1}{dx}EA\frac{dN_4}{dx}\{X\}\right) & \left(\frac{dN_4}{dx}\{X\}EA\frac{dN_3}{dx}\{X\}\right) & \left(\frac{dN_4}{dx}\{X\}EA\frac{dN_4}{dx}\{X\}\right) & \left(\frac{dN_2}{dx}EA\frac{dN_4}{dx}\{X\}\right) & \left(\frac{dN_4}{dx}\{X\}EA\frac{dN_5}{dx}\{X\}\right) & \left(\frac{dN_4}{dx}\{X\}EA\frac{dN_6}{dx}\{X\}\right) \\ 0 & \left(\frac{dN_2}{dx}EA\frac{dN_3}{dx}\{X\}\right) & \left(\frac{dN_2}{dx}EA\frac{dN_4}{dx}\{X\}\right) & 0 & \left(\frac{dN_2}{dx}EA\frac{dN_5}{dx}\{X\}\right) & \left(\frac{dN_2}{dx}EA\frac{dN_6}{dx}\{X\}\right) \\ \left(\frac{dN_1}{dx}EA\frac{dN_5}{dx}\{X\}\right) & \left(\frac{dN_5}{dx}\{X\}EA\frac{dN_3}{dx}\{X\}\right) & \left(\frac{dN_5}{dx}\{X\}EA\frac{dN_4}{dx}\{X\}\right) & \left(\frac{dN_2}{dx}EA\frac{dN_5}{dx}\{X\}\right) & \left(\frac{dN_5}{dx}\{X\}EA\frac{dN_5}{dx}\{X\}\right) & \left(\frac{dN_5}{dx}\{X\}EA\frac{dN_6}{dx}\{X\}\right) \\ \left(\frac{dN_1}{dx}EA\frac{dN_6}{dx}\{X\}\right) & \left(\frac{dN_6}{dx}\{X\}EA\frac{dN_3}{dx}\{X\}\right) & \left(\frac{dN_6}{dx}\{X\}EA\frac{dN_4}{dx}\{X\}\right) & \left(\frac{dN_2}{dx}EA\frac{dN_6}{dx}\{X\}\right) & \left(\frac{dN_6}{dx}\{X\}EA\frac{dN_5}{dx}\{X\}\right) & \left(\frac{dN_6}{dx}\{X\}EA\frac{dN_6}{dx}\{X\}\right) \end{bmatrix} dx$$

$$\{X\} = \left[\frac{dN_3}{dx}v_1 + \frac{dN_4}{dx}\theta_1 + \frac{dN_5}{dx}v_2 + \frac{dN_6}{dx}\theta_2\right]$$

$$K_G = \int_0^L \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \left[\frac{dN_3}{dx} P \frac{dN_3}{dx}\right] & \left[\frac{dN_3}{dx} P \frac{dN_4}{dx}\right] & 0 & \left[\frac{dN_3}{dx} P \frac{dN_5}{dx}\right] & \left[\frac{dN_3}{dx} P \frac{dN_6}{dx}\right] \\ 0 & \left[\frac{dN_4}{dx} P \frac{dN_3}{dx}\right] & \left[\frac{dN_4}{dx} P \frac{dN_4}{dx}\right] & 0 & \left[\frac{dN_4}{dx} P \frac{dN_5}{dx}\right] & \left[\frac{dN_4}{dx} P \frac{dN_6}{dx}\right] \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \left[\frac{dN_5}{dx} P \frac{dN_3}{dx}\right] & \left[\frac{dN_5}{dx} P \frac{dN_4}{dx}\right] & 0 & \left[\frac{dN_5}{dx} P \frac{dN_5}{dx}\right] & \left[\frac{dN_5}{dx} P \frac{dN_6}{dx}\right] \\ 0 & \left[\frac{dN_6}{dx} P \frac{dN_3}{dx}\right] & \left[\frac{dN_6}{dx} P \frac{dN_4}{dx}\right] & 0 & \left[\frac{dN_6}{dx} P \frac{dN_5}{dx}\right] & \left[\frac{dN_6}{dx} P \frac{dN_6}{dx}\right] \end{bmatrix} dx$$



Load, P
First Order Elastic
(a) Elastic Buckling Load
P_E
(b) Second Order Elastic
(c) First Order Rigid-Plastic
P_P
(e) First Order Elastic-Plastic (Hinged)
(d) Second Order Rigid-Plastic Accounting P-δ and P-Δ
(f) Second Order Elastic-Plastic (Hinged)
(g) Second Order Elasto-Plastic (spread of yield)
TRUE BEHAVIOUR
Displacement δ
P
P
Δ

Summary

- The frame has first to be idealised
- Then a frame classification is carried out

  ⇒ sway-non sway / braced-unbraced
- On the basis of the frame class (and the type of steel and profiles), the type of frame analysis is finally selected

- Choice of type analysis/design: depends on type of structure, available tools , EC3 requirements, etc.
- The more sophisticated the analysis tool used, the lesser the design ULS checks