Design factors which will influence the lateral stability can be summarized as:

- The slenderness of the member between adequate lateral restraints;
- the shape of cross-section;
- the variation of moment along the beam;
- the form of end restraint provided,
- the manner in which the load is applied, i.e. to tension or compression flange.

# Elastic buckling of beams

Critical Buckling Moment for uniform bending moment diagram is

$$M_{cr} = \frac{\pi^2 EI_z}{L^2}\sqrt{\left[\frac{I_w}{I_z} + \frac{L^2 GI_t}{\pi^2 EI_z}\right]}$$

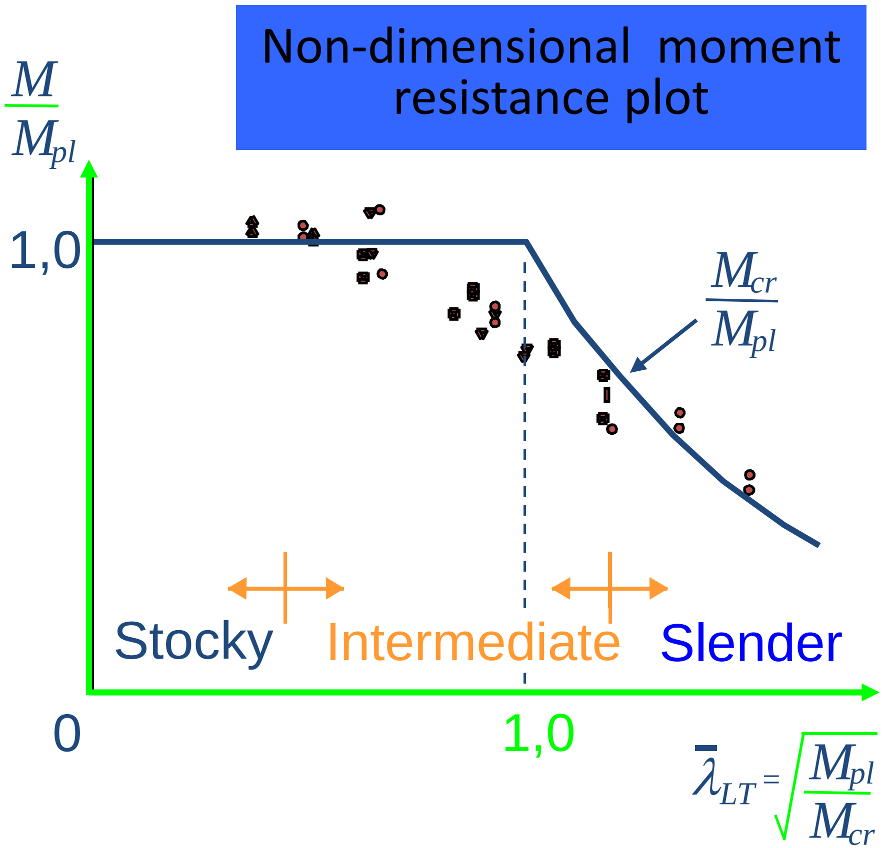
Includes:

- Lateral flexural stiffness $EI_z$
- Torsional and Warping stiffnesses $GI_t$ and $Ei_w$

Their relative importance depends on the type of cross-section used.

# Effect of Slenderness

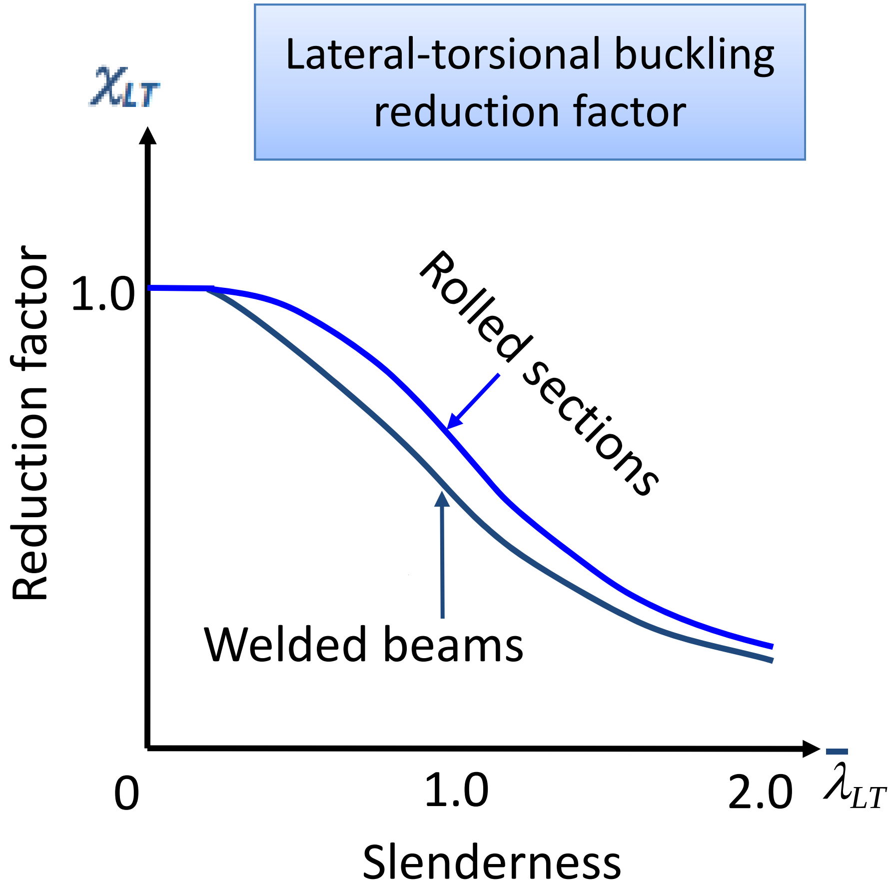





**Non-dimensional plot permits results from different test series to be compared**

- **Stocky beams** ( $\bar{\lambda}_{LT}$ < 0,4) unaffected by lateral torsional buckling
- **Slender beams** ( $\bar{\lambda}_{LT}$>1,2) resistance close to theoretical elastic critical moment $M_{cr}$
- Intermediate slenderness - adversely affected by inelasticity and geometric imperfections
- EC3 uses a reduction factor $\chi_{LT}$ on plastic resistance moment to cover the whole slenderness range

# Design buckling resistance



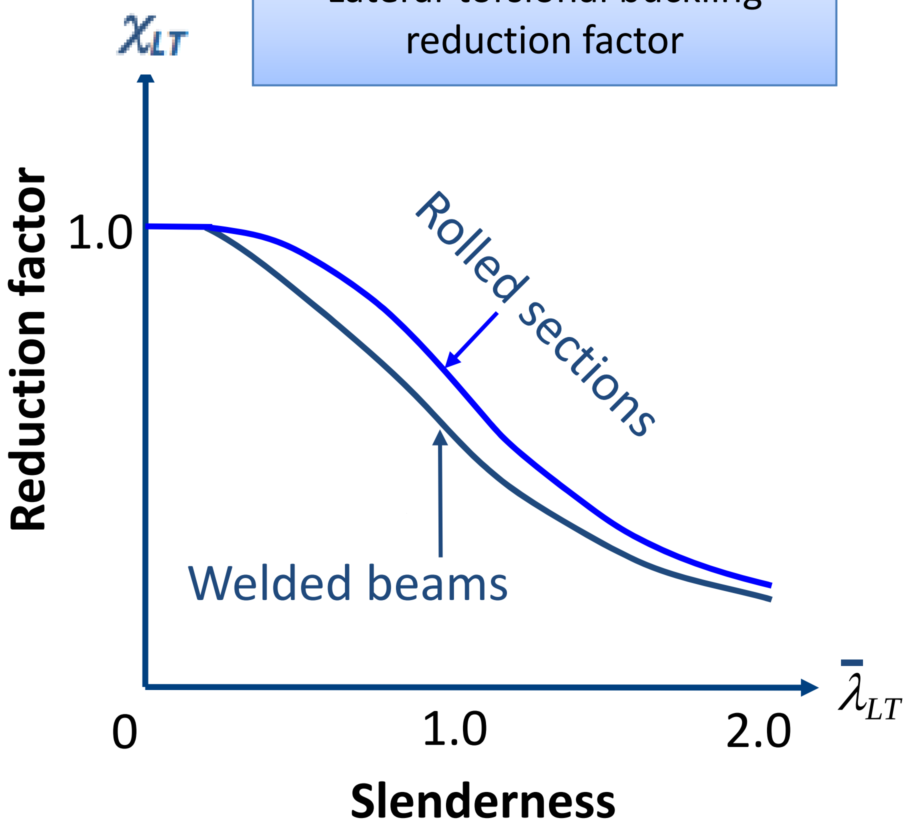

The design buckling resistance moment $M_{b.Rd}$ of a laterally unrestrained beam is calculated as

$$M_{b.Rd} = \chi_{LT} \beta_w W_{pl.y} f_y / \gamma_{M1}$$

which is effectively the plastic resistance of the section multiplied by the reduction factor $\chi_{LT}$

# Reduction factor for LTB






Lateral-torsional buckling reduction factor

$$\chi_{LT} = \frac{1}{\varphi_{LT} + \left[\varphi_{LT}^2 - \bar{\lambda}_{LT}^2\right]^{0.5}}$$

**where**

$$\varphi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right]$$

**and**

$\alpha_{LT} = 0.21$ for rolled sections

$\alpha_{LT} = 0.49$ for welded sections

# Determining $\bar{\lambda}_{LT}$

The non-dimensional slenderness

$$\bar{\lambda}_{LT} = \sqrt{M_{pl.Rd} / M_{cr}}$$

calculated by calculating the plastic resistance moment $M_{pl.Rd}$ and elastic critical moment $M_{cr}$ from first principles

or using

$$\bar{\lambda}_{LT} = \left[\frac{\lambda_{LT}}{\lambda_1}\right] \beta_w^{\,0.5}$$

where

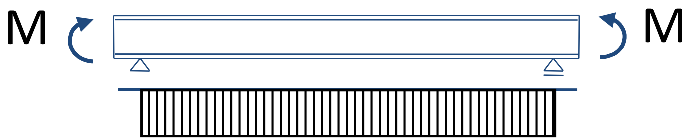
$$\lambda_1 = \pi \left[\frac{E}{fy}\right]^{0.5}$$

For any plain **I** or **H** section with equal flanges, under uniform moment with simple end restraints

$$\lambda_{LT} = \frac{L / i_z}{\left[1 + \frac{1}{20}\left[\frac{L / i_z}{h / t_f}\right]^2\right]^{0.25}}$$

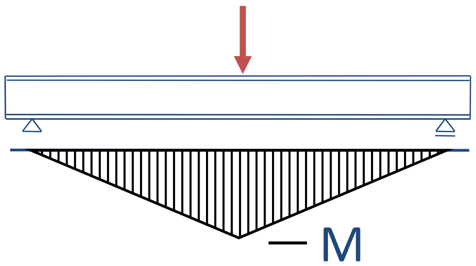
# Effect of load pattern on LTB

The elastic critical moment for a beam under uniform bending moment is




$$M_{cr} = \frac{\pi}{L}\sqrt{EI_z GI_t}\sqrt{1+\frac{\pi^2 EI_w}{L^2 GI_t}}$$

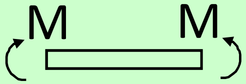
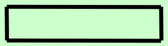
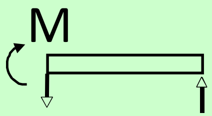
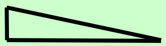
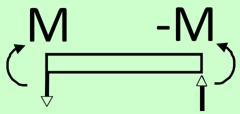
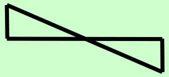
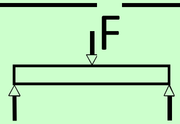
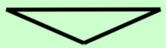
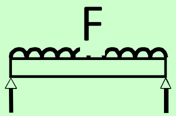
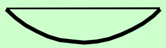
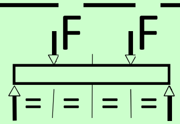
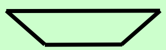
The elastic critical moment (mid-span moment) for a beam with a central point load is




$$M_{cr} = \frac{4,24}{L}\sqrt{EI_z GI_t}\sqrt{1+\frac{\pi^2 EI_w}{L^2 GI_t}}$$

… which is increased from the basic (uniform moment) case by a factor $C_1 = 4.24/\pi = 1.365$

# $C_1$ factor

| Loads | Bending moment | $M_{max}$ | $C_1$ |
|---|---|---|---|
|  |  | M | 1.00 |
|  |  | M | 1.879 |
|  |  | M | 2.752 |
|  |  | FL/4 | 1.365 |
|  |  | FL/8 | 1.132 |
|  |  | FL/4 | 1.046 |

EC3 expresses the elastic critical moment $M_{cr}$ for a particular loading case as

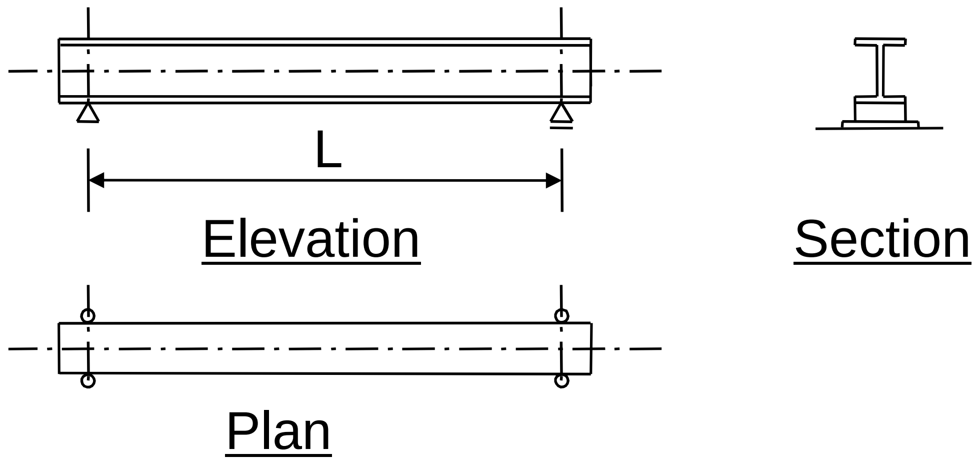
$$M_{cr} = C_1 \frac{\pi}{L}\sqrt{EI_z GI_t}\sqrt{1+\frac{\pi^2 EI_w}{L^2 GI_t}}$$

$C_1$ appears:

- as a simple multiplier in expressions for Mcr
- as 1/ $C_1^{0.5}$ in expressions for □LT.

# End support conditions

- Basic case assumes end conditions which prevent lateral movement and twist but permit rotation on plan.



# End support conditions

- **End conditions which prevent rotation on plan enhance the elastic buckling resistance**
- **Can include the effect of different support conditions by redefining the unrestrained length as an effective length**
- **Two effective length factors, k and $k_w$.**
- **Reflect the two possible types of end fixity, lateral bending restraint and warping restraint.**
- **Note: it is recommended that $k_w$ be taken as 1.0 unless special provision for warping fixing is made.**
- **EC3 recommends k values of 0,5 for fully fixed ends, 0,7 for one free and one fixed end and of course 1,0 for two free ends.**

**Choice of k is at the designer's discretion**