Formula Book
for
Hydraulics and Pneumatics

Fluid and Mechanical Engineering Systems

Department of Management and Engineering

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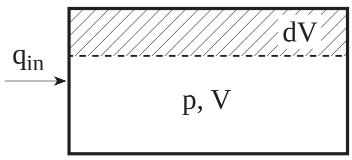
Contents

1 Elementary Equations

1.1 Nomenclature

Re	:Reynold's number	[-]		v	:(mean-) flow velocity	[m/s]
V	:volume	[m^3]		β_e	:effective bulk modulus (reservoir and fluid)	[Pa]
d	:diameter	[m]		Δp	:pressure variation, upstream to downstream	[Pa]
d_h	:hydraulic diameter	[m]		η	:dynamic viscosity	[Ns/m^2]
p	:pressure	[Pa]		ν	:kinematic viscosity ($= \frac{\eta}{\rho}$)	[m^2/s]
q	:flow (volume flow)	[m^3/s]		ρ	:density	[kg/m^3]
q_{in}	:flow **in** into volume	[m^3/s]				
t	:time	[s]				

1.2 The continuity equation



$$\sum q_{in} = \frac{dV}{dt} + \frac{V}{\beta_e}\frac{dp}{dt}$$

Flow out from the volume is counted negative.

1.3 Reynolds number

$$\mathrm{Re} = \frac{v d_h \rho}{\eta} \qquad (d_h \quad \text{hydraulic diameter})$$

$$d_h = \frac{4 \times \text{cross section area}}{\text{circumference}} \qquad (d_h = d \quad \text{at cirkular crosssection})$$

1.4 Flow equations

For flow through fix orifices applies (see also section 3, *Orifices*):

Laminar flow	$q \propto \Delta p$
Turbulent flow	$q \propto \sqrt{\Delta p}$

2 Pipe flow

2.1 Nomenclature

Re	:Reynolds number	[-]
d	:diameter	[m]
d_1	:upstream diameter	[m]
d_2	:downstream diameter	[m]
h	:height	[m]
h_1	:upstream height	[m]
h_2	:downstream height	[m]
g	:gravitation	[m/s^2]
ℓ	:length	[m]
ℓ_x	:distance after disturbance source	[m]
ℓ_s	:distance after disturbance source when the flow profile is completely developed	[m]
p	:pressure	[Pa]
p_1	:upstream pressure	[Pa]
p_2	:downstream pressure	[Pa]
r	:radius	[m]
v	:(mean-) flow velocity	[m/s]
v_1	:upstream (mean-) flow velocity	[m/s]
v_2	:downstream (mean-) flow velocity	[m/s]
α	:correction factor	[-]
Δp_f	:pressure loss, upstream to downstream	[Pa]
φ	:angle	[°]
λ	:friction factor	[-]
ρ	:density	[kg/m^3]
ζ	:single loss factor	[-]
ζ_s	:disturbance source factor	[-]

2.2 Bernoullis extended equation

At stationary incompressible flow

$$p_1 + \frac{\rho v_1^2}{2} + \rho g h_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g h_2 + \Delta p_f$$

2.3 The flow loss, Δp_f

$$\Delta p_f = \begin{cases} \lambda \dfrac{\ell}{d}\dfrac{\rho v^2}{2} & \text{at a straight distance} \\[2ex] \zeta_s \dfrac{\rho v^2}{2} & \text{after a disturbance source} \\[2ex] \zeta \dfrac{\rho v^2}{2} & \text{at a single disturbance source} \end{cases}$$

2.4 The friction factor, λ

$$\lambda = \begin{cases} \dfrac{64}{Re} & \text{Re} < 2300 \quad \text{laminar flow} \\[2ex] \dfrac{0{,}316}{\sqrt[4]{\text{Re}}} & 2300 < \text{Re} < 10^5 \quad \text{turbulent flow in smooth pips} \end{cases}$$

2.5 Disturbance source factor, ζ_s

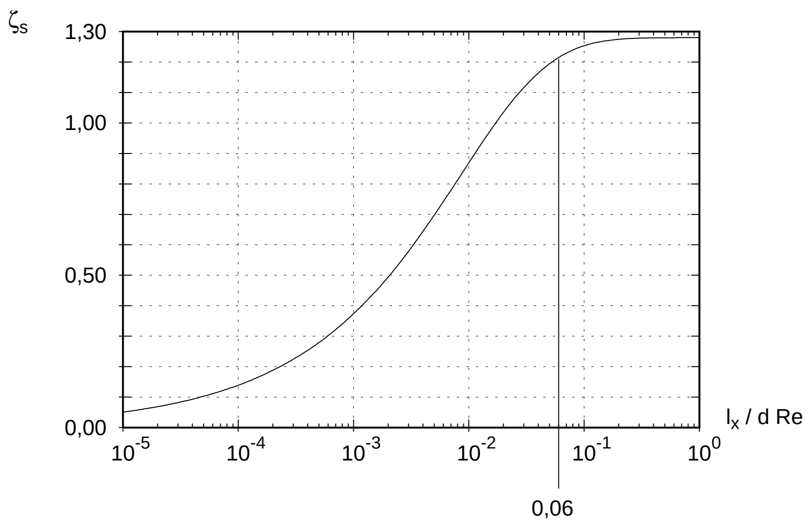
The laminar flow is completely developed at the distance ℓ_s after a disturbance source.

$$\zeta_s \approx 1{,}21 \qquad \ell_s \geq 0{,}06 d\text{Re} \qquad (\text{Re} < 2300),$$

For $\ell < \ell_s$ applies

$$\zeta_s = 1{,}28 \tanh(6{,}28 x^{0{,}44}) \qquad \ell_x < 0{,}06 d\text{Re}, \quad \text{see diagram in figure 1}$$

where $x = \dfrac{\ell_x}{d\text{Re}}$



Figur 1: The disturbance source factor ζ_s as function of $\frac{\ell_x}{d\mathrm{Re}}$.

The turbulant flow is completely developed at the distance ℓ_s after a disturbance source.

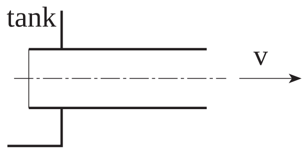
$$\zeta_s \approx 0{,}09 \qquad \ell_s \geq 40d \qquad (\mathrm{Re} > 2300),$$

For $\ell < \ell_s$ applies

$$\zeta_s = 0{,}09\sqrt{\frac{\ell_x}{40d}} \qquad \ell_x < 40d$$

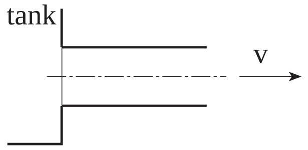
2.6 Single loss factor, ζ

Pipe connections to reservoir



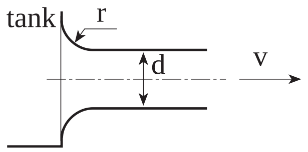
The pipe starts at a given distance inside the reservoir

$$\zeta = \begin{cases} 1 & \text{Sharp edge} \\ 0{,}5 & \text{Slightly rounded edge} \end{cases}$$



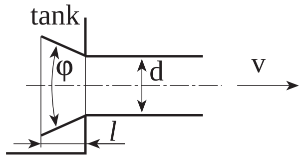
Pipes in the reservoir wall

$\zeta = 0{,}5$



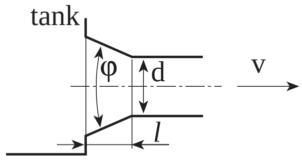
Pipe in the reservoir wall with rounded edge

r/d	0,1	0,15	0,25	0,6
ζ	0,12	0,08	0,05	0,04



The pipe starts at a given distance inside the reservoir with entrance cone: For given ℓ, optimal cone angle and resistance factor is stated.

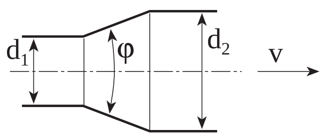
ℓ/d	0,1	0,15	0,25	0,6	1,0
$\varphi[°]$	60–90	60–80	60–70	50–60	50–60
ζ	0,40	0,26	0,17	0,13	0,10



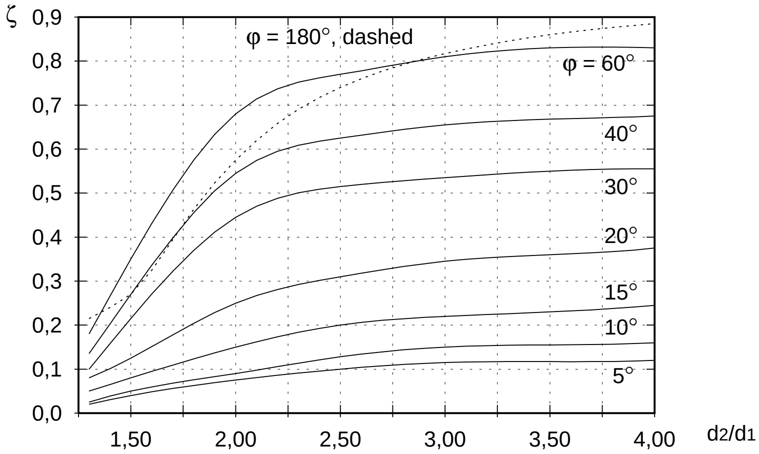
Pipe in reservoir wall with entrance cone: For given ℓ, optimal cone angle and resistance factor is stated.

ℓ/d	0,1	0,15	0,25	0,6
$\varphi[°]$	50–60	50–60	45–55	40–50
ζ	0,18	0,14	0,12	0,10

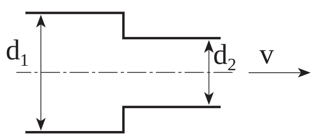
Area variations in the pipe



Increase of area: The loss factor is found in the figure 2



Figur 2: The loss coefficient ζ as function of the area relationship with the angle φ as parameter at increase of area.



Decrease of area: The loss factor is described by $\zeta = \zeta_0\alpha$, where ζ_0 for different geometries is received from section above about *Pipe connection to reservoir*.

According to von Mises applies when

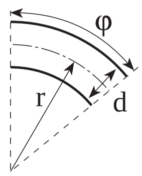
d_2/d_1	0,90	0,80	0,70	0,50	0,30	0,10
α	0,19	0,37	0,51	0,76	0,91	0,99

Disturbance source factor ζ_s applies in a similar way

$$\zeta_s = \zeta_{s0}\left[1 - \left(\frac{d_2}{d_1}\right)^3\right]$$

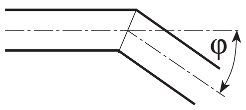
The uncompensated disturbance source factor ζ_{s0} is received from section 2.5, *Disturbance source factor, ζ_s*.

Pipe bend



For pipe bend relates: $\zeta = \zeta_{90}\dfrac{\varphi}{90}$
where ζ_{90} is

d/r	0,20	0,40	0,60	0,80	1,00
ζ_{90}	0,13	0,14	0,16	0,21	0,29

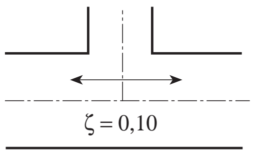


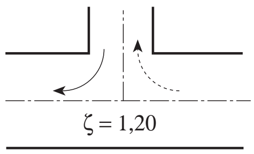
For pipe angle gives ζ directly by

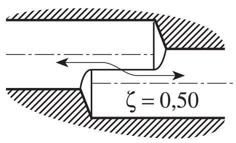
$\varphi[°]$	10	20	30	40	50
ζ	0,04	0,10	0,17	0,27	0,40

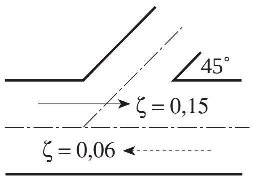
$\varphi[°]$	60	70	80	90
ζ	0,55	0,70	0,90	1,20

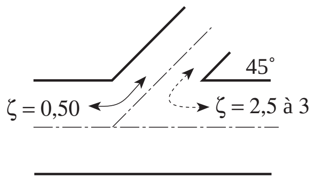
Special geometries

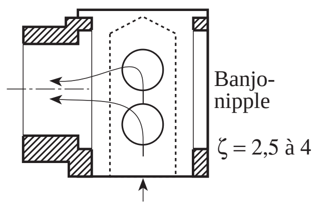








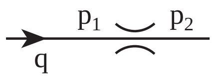




3 Orifices

3.1 Nomenclature

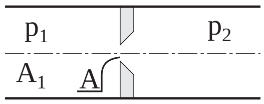
A	:area	$[\mathrm{m}^2]$	ℓ	:length	[m]
A_1	:upstream area	$[\mathrm{m}^2]$	p	:pressure	[Pa]
C_q	:flow coefficient	[-]	p_1	:upstream pressure	[Pa]
Re	:Reynold's number	[-]	p_2	:downstream pressure	[Pa]
d	:diameter	[m]	q	:flow (volume flow)	$[\mathrm{m}^3/\mathrm{s}]$
K	:constant $C_q A\sqrt{2/\rho}$	$[\mathrm{m}^3/\mathrm{s}\sqrt{Pa}]$	ρ	:density	$[\mathrm{kg/m}^3]$



$$q = C_q A\sqrt{\frac{2}{\rho}(p_1 - p_2)}$$

3.2 The flow coefficient, C_q

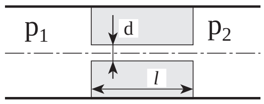
Hole orifice (sharp edged)



$$C_q = C_q(\mathrm{Re}, \frac{A}{A_1})$$

If nothing else is stated $C_q = 0{,}67$ can be used.

Pipe orifice (sharp edged)



Laminar flow in the orifice ($\mathrm{Re} < 2300$)

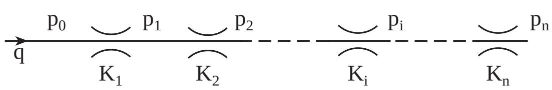
$$C_q = \frac{1}{\sqrt{1{,}5 + 1{,}28\tanh(6{,}28x^{0{,}44}) + 64x}} \qquad \text{där } x = \frac{\ell}{d\mathrm{Re}}$$

The term $1{,}28\tanh(6{,}28x^{0{,}44})$ agrees with ζ_s and can be received from the diagram in figure 1 in section *Pipe flow*. When $x \geq 0{,}06$, the value 1,21 is accepted.

Turbulent flow in the orifice with $2 \leq \dfrac{\ell}{d} \leq 20$

$$C_q = \begin{cases} \dfrac{1}{\sqrt{1{,}46 + 0{,}088\dfrac{\ell}{d} + \dfrac{0{,}316}{\sqrt[4]{\mathrm{Re}}}\dfrac{\ell}{d}}} & 2300 \leq \mathrm{Re} \leq 2\cdot 10^4 \\ \dfrac{1}{\sqrt{1{,}46 + 0{,}115\dfrac{\ell}{d}}} & 2\cdot 10^4 \leq \mathrm{Re} \end{cases}$$

3.3 Series connection of turbulent orifices



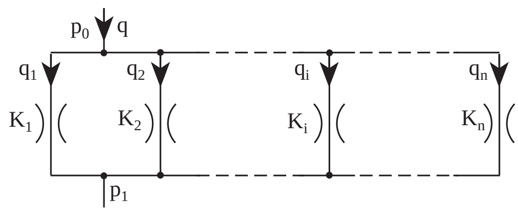
The sum of the orifices applies:

$$q = K\sqrt{p_0 - p_n} \qquad \text{där} \quad K = \frac{1}{\sqrt{\sum_{i=1}^{n} \frac{1}{K_i^2}}} \qquad K_i = C_{qi} A_i \sqrt{\frac{2}{\rho}}$$

The pressure after the j:th orifice is given by

$$p_j = p_0 - (p_0 - p_n) K^2 \sum_{i=1}^{j} \frac{1}{K_i^2}$$

3.4 Parallel connection of turbulent orifices



The sum of the orifices applies:

$$q = K\sqrt{p_0 - p_1} \qquad \text{där} \quad K = \sum_{i=1}^{n} K_i \qquad K_i = C_{qi} A_i \sqrt{\frac{2}{\rho}}$$

The flow through the i:th orifice is given by

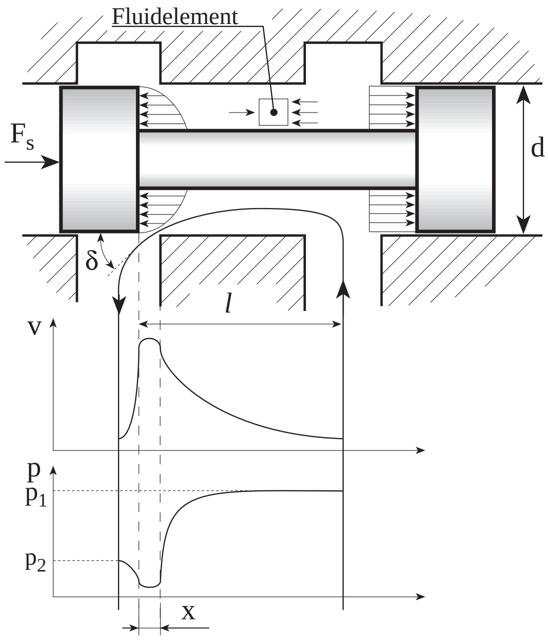
$$q_i = \frac{K_i}{K} q$$

4 Flow forces

4.1 Nomenclature

F_s	:flow force	[N]	q	:flow (volume flow)	[m^3/s]
d	:spool diameter	[m]	v	:(mean-)flow velocity	[m/s]
ℓ	:length	[m]	w	:area gradient	[m]
p	:pressure	[Pa]	x	:spool opening	[m]
p_1	:upstream pressure	[Pa]	δ	:jet angle	[°]
p_2	:downstream pressure	[Pa]	ρ	:density	[kg/m^3]

4.2 Spool



For this valve, without pressure relief grooves, is the area gradient:

$$w = \pi d$$

$$F_s = |2C_q wx(p_1 - p_2)\cos(\delta)| + \rho\ell\dot{q}$$

The term with absolute value is the static part of the flow force and has a closing effect.
If the spool and bushing have sharp and right angle edges and if the gap between the spool and the bushing is small and also $x \ll d$, then are:

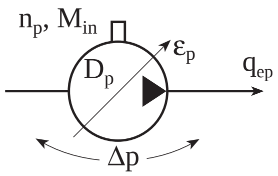
$$0{,}62 \leq C_q \leq 0{,}67 \qquad \text{and} \qquad \delta = 69°$$

5 Rotational transmissions

5.1 Nomenclature

C_v	:laminar leakage losses	[-]		q_e	:effective flow	[m^3/s]
D	:displacement	[m^3/rev]		Δp	:pressure difference	[Pa]
M_{in}	:driving torque pump	[Nm]		ε	:displacement setting	[-]
M_{ut}	:output torque motor	[Nm]		η_{hm}	:hydraulic mechanical efficiency	[-]
k_p	:Coulomb friction	[-]		η_{vol}	:volumetric efficiency	[-]
k_v	:viscous friction losses	[-]		**Sub index**		
k_ε	:displacement coefficient	[-]		p	pump	
n	:revs	[rev/s]		m	motor	

Pump



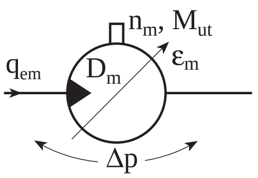
Effective flow

$$q_{ep} = \varepsilon_p D_p n_p \eta_{volp}$$

Torque

$$M_{in} = \frac{\varepsilon_p D_p}{2\pi} \Delta p \frac{1}{\eta_{hmp}}$$

Motor



Effective flow

$$q_{em} = \varepsilon_m D_m n_m \frac{1}{\eta_{volm}}$$

Torque

$$M_{out} = \frac{\varepsilon_m D_m}{2\pi} \Delta p \eta_{hmm}$$

5.2 Efficiency models

Pump

Volumetric efficiency

$$\eta_{volp} = 1 - C_v \frac{\Delta p}{|\varepsilon_p| n_p \eta}$$

Hydraulic mechanical efficiency

$$\eta_{hmp} = \frac{1}{1 + (k_p + k_v \frac{n_p \eta}{\Delta p}) e^{(k_\varepsilon (1 - |\varepsilon_p|))}}$$

Motor

Volumetric efficiency

$$\eta_{volm} = \frac{1}{1 + C_v \frac{\Delta p}{|\varepsilon_m| n_m \eta}}$$

Hydraulic mechanical efficiency

$$\eta_{hmm} = 1 - (k_p + k_v \frac{n_m \eta}{\Delta p}) e^{(k_\varepsilon (1 - |\varepsilon_m|))}$$

6 Accumulators

6.1 Nomenclature

V_0	:accumulator volume	[m^3]	p_1	:minimum working pressure (absolute)	[Pa]
n	:polytrophic exponent	[-]	p_2	:maximum working pressure (absolute)	[Pa]
p_0	:pre-charged pressure (absolute pressure, normally $\approx$ 90 % av p_1)	[Pa]	ΔV	:working volume	[m^3]

6.2 Calculating of the accumulator volume, V_0

A. Both the charging and discharging is either adiabatic or isotherm process

$$V_0 = \frac{\Delta V \dfrac{p_1}{p_0}}{1 - \left(\dfrac{p_1}{p_2}\right)^{\frac{1}{n}}} \qquad n = \begin{cases} 1 & \text{isotherm process} \\ 1{,}4(1{,}5) & \text{adiabatic process} \end{cases}$$

B. Isotherm charging and adiabatic discharging

$$V_0 = \frac{\Delta V \dfrac{p_2}{p_0}}{\left(\dfrac{p_2}{p_1}\right)^{\frac{1}{n}} - 1} \qquad n = 1{,}4(1{,}5)$$

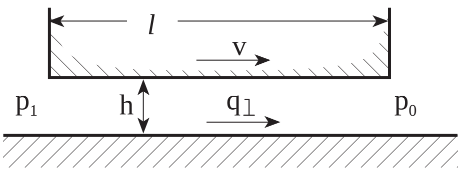
7 Gap theory

7.1 Nomenclature

F_f	:friction force	[N]
M_f	:friction torque	[Nm]
P_f	:power loss	[W]
b	:gap width perpendicular to flow direction	[m]
e	:eccentricity	[m]
h	:gap height	[m]
h_0	mean gap height $(\frac{h_1+h_2}{2})$	[m]
ℓ	:length	[m]

p	:pressure	[Pa]
q_ℓ	:leakage flow	[m^3/s]
r	:radius	[m]
r_1	:inner radius	[m]
r_2	:outer radius	[m]
v	:relative velocity	[m/s]
Δp	:pressure difference through the gap $(p_1 - p_0)$	[Pa]
γ	:angle	[rad]
η	:dynamic viscosity	[Ns/m^2]
ω	:angular speed	[rad/s]

7.2 Plane parallel gap



Leakage flow relative to the fix wall

$$q_\ell = \frac{vbh}{2} + \frac{bh^3}{12\eta}\frac{\Delta p}{\ell}$$

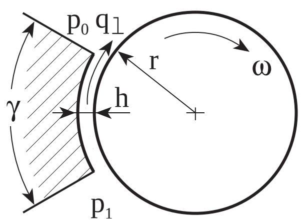
Friction force

$$F_f = \frac{\eta v b \ell}{h} - \frac{bh}{2}\Delta p$$

Effect losses (flow and frictional losses)

$$P_f = \frac{bh^3}{12\eta}\frac{\Delta p^2}{\ell} + \frac{\eta v^2 b \ell}{h}$$

7.3 Radial gap

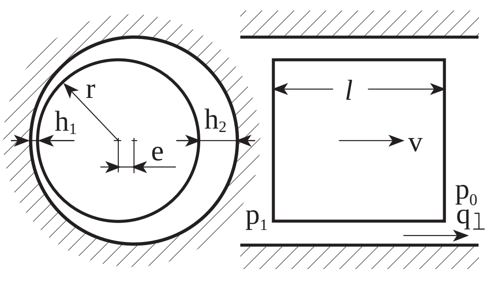


When $h \ll r$ can the equations for plane parallel gap be used with following substitution:

$v = r\omega$

$\ell = \gamma r$

7.4 Gap between cylindrical piston and cylinder



Leakage flow relative to the fix gap wall

$$q_\ell = \pi v r h_0 + \frac{\pi r h_0^3}{6\eta}\frac{\Delta p}{\ell}\left[1 + 1{,}5\left(\frac{e}{h_0}\right)^2\right]$$

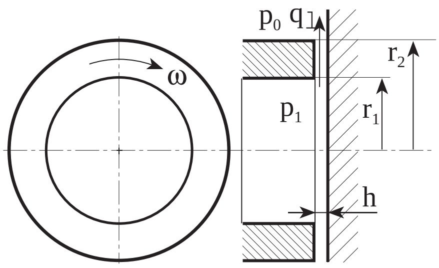
Frictional force

$$F_f = \frac{2\pi r \eta v \ell}{h_0\sqrt{1-\left(\frac{e}{h_0}\right)^2}} - \pi r h_0 \Delta p$$

Effect losses (flow and frictional losses)

$$P_f = \frac{\pi r h_0^3}{6\eta}\frac{\Delta p^2}{\ell}\left[1 + 1{,}5\left(\frac{e}{h_0}\right)^2\right] + \frac{2\pi r \eta v^2 \ell}{h_0\sqrt{1-\left(\frac{e}{h_0}\right)^2}}$$

7.5 Axial, annular gap



Leakage flow

$$q_\ell = \frac{\Delta p \pi h^3}{6\eta \ln\left(\frac{r_2}{r_1}\right)}$$

Pressure as function of radius

$$p_r = p_1 - (p_1 - p_0)\frac{\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)}$$

Friction moment

$$M_f = \frac{\pi}{2}\frac{\eta\omega}{h}(r_2^4 - r_1^4)$$

Effect losses (flow and frictional losses)

$$P_f = \frac{\pi h^3}{6\eta}\frac{\Delta p^2}{\ln\left(\frac{r_2}{r_1}\right)} + \frac{\pi}{2}\frac{\eta\omega^2}{h}(r_2^4 - r_1^4)$$

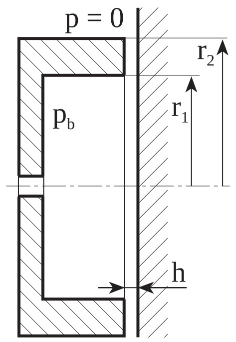
8 Hydrostatic bearings

8.1 Nomenclature

A_e	:effective area	[m^2]	k_b	:constant	[Ns/m^2]
B	:bearing chamber length	[m]	k_2	:constant	[Nms]
F	:load	[N]	p	:pressure	[Pa]
K_1	:constant	[Ns]	p_b	:pressure in the bearing chamber	[Pa]
K_2	:constant	[Nm^{2}s]	q_s	:flow through the bearing	[m^3/s]
L	:bearing surface length	[m]	q_{sB}	:flow through the bearing/width	[m^2/s]
a_e	:effective area/width	[m]	r_1	:inner radius	[m]
f	:load/width	[N/m]	r_2	:outer radius	[m]
h	:gap height	[m]	η	:dynamic viscosity	[Ns/m^2]
ℓ	:length	[m]			

8.2 Circled block

h = constant



Flow

$$q_s = \frac{h^3}{k_b} p_b$$

Load

$$F = A_e p_b$$

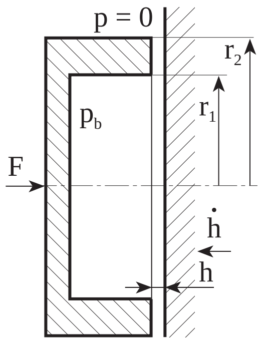
where

$$k_b = \frac{6\eta}{\pi} \ln\left(\frac{r_2}{r_1}\right)$$

and

$$A_e = \frac{\pi}{2} \frac{\left(r_2^2 - r_1^2\right)}{\ln\left(\frac{r_2}{r_1}\right)}$$

Squeeze



Pressure

$$p_b = -\frac{K_1}{h^3}\dot{h}$$

Load

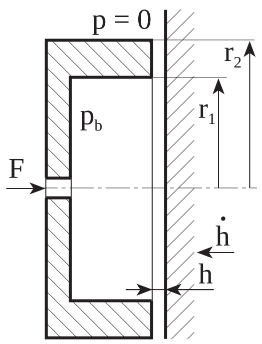
$$F = -\frac{K_2}{h^3}\dot{h}$$

where

$$K_1 = 3\eta r_2^2 \left[1 - \left(\frac{r_1}{r_2}\right)^2\right]$$

and

$$K_2 = \frac{3\pi}{2}\eta r_2^4 \left[1 - \left(\frac{r_1}{r_2}\right)^4\right]$$



If $p_b = 0$

Load

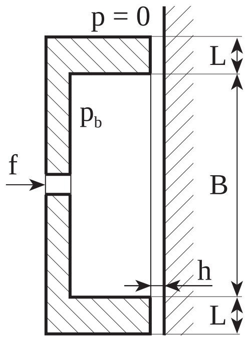
$$F = -\frac{K_2}{h^3}\dot{h}$$

where

$$K_2 = \frac{3\pi}{2}\eta r_2^4 \left[1 - 2\frac{r_1}{r_2} + 2\left(\frac{r_1}{r_2}\right)^3 - \left(\frac{r_1}{r_2}\right)^4\right]$$

8.3 Rectangular block

$h =$ **constant**



Flow

$$q_{sB} = \frac{h^3}{k_b} p_b$$

Load

$$f = a_e p_b$$

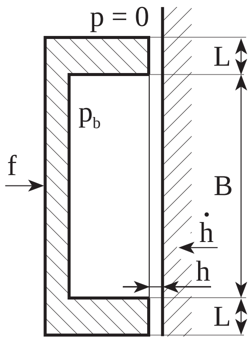
where

$$k_b = 6\eta L$$

and

$$a_e = B + L$$

Squeeze



Pressure

$$p_b = -\frac{K_1}{h^3}\dot{h}$$

Load

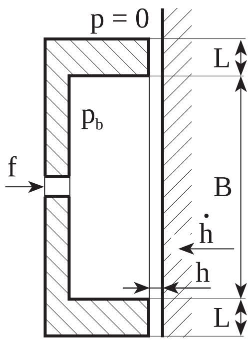
$$f = -\frac{k_2}{h^3}\dot{h}$$

where

$$K_1 = 6\eta L(B + L)$$

and

$$k_2 = 6\eta L \left(B^2 + 2BL + \frac{4}{3}L^2 \right)$$



If $p_b = 0$

Load

$$f = -\frac{k_2}{h^3}\dot{h}$$

where

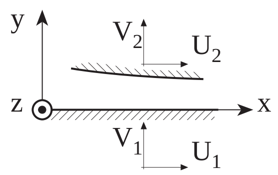
$$k_2 = 2\eta L^3$$

9 Hydrodynamic bearing theory

9.1 Nomenclature

U_1	:velocity in x-axis surface 1	[m/s]
U_2	:velocity in x-axis surface 2	[m/s]
V_1	:velocity in y-axis surface 1	[m/s]
V_2	:velocity in y-axis surface 2	[m/s]
T	:temperature	[K]
c	:specific heat	[kJ/kg K]
h	:gap height	[m]
p	:pressure	[Pa]
q_x	:flow/width unit in x-axis	[m^2/s]
q_z	:flow/width unit in z-axis	[m^2/s]
q_r	:flow/width unit in r-axis	[m^2/s]
q_θ	:flow/width unit in θ-axis	[m^2/s]
r	:radius	[m]
t	:time	[s]
u	:flow velocity in x-axis	[m/s]
w	:flow velocity in z-axis	[m/s]
η	:dynamic viscosity	[Ns/m^2]
τ_x	:shear stress in x-axis	[N/m^2]
τ_θ	:shear stress in θ-axis	[N/m^2]
ρ	:density	[kg/m^3]
ω_1	:velocity in θ-axis surface 1	[rad/s]
ω_2	:velocity in θ-axis surface 2	[rad/s]
θ	:angle	[rad]

9.2 Cartesian coordinate



Speeeds

$$u = \frac{1}{2\eta}\frac{\partial p}{\partial x}[y(y-h)] + U_1\left(1-\frac{y}{h}\right) + U_2\frac{y}{h} \qquad w = \frac{1}{2\eta}\frac{\partial p}{\partial z}[y(y-h)]$$

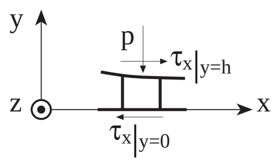
Flows

$$q_x = -\frac{h^3}{12\eta}\frac{\partial p}{\partial x} + (U_1+U_2)\frac{h}{2} \qquad q_z = -\frac{h^3}{12\eta}\frac{\partial p}{\partial z}$$

Shear stresses

$$\tau_x|_{y=0} = -\frac{h}{2}\frac{\partial p}{\partial x} + \eta\frac{U_2-U_1}{h}$$

$$\tau_x|_{y=h} = \frac{h}{2}\frac{\partial p}{\partial x} + \eta\frac{U_2-U_1}{h}$$



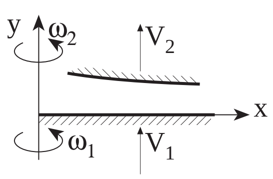
Reynolds equation

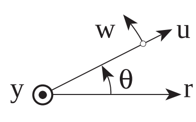
$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial z}\right) = 6(U_1-U_2)\frac{\partial}{\partial x}(\rho h) + 12\rho(V_2-V_1)$$

Adiabatic energy equation

$$\rho c\left[q_x\frac{\partial T}{\partial x} + q_z\frac{\partial T}{\partial z} + h\frac{\partial T}{\partial t}\right] = \frac{\eta}{h}(U_1-U_2)^2 + \frac{h^3}{12\eta}\left[\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial z}\right)^2\right]$$

9.3 Polar coordinates





Velocities

$$u = \frac{1}{2\eta}\frac{\partial p}{\partial r}[y(y-h)] \qquad w = \frac{1}{2\eta}\frac{\partial p}{r\partial\theta}[y(y-h)] + r\omega_1\left(1-\frac{y}{h}\right) + r\omega_2\frac{y}{h}$$

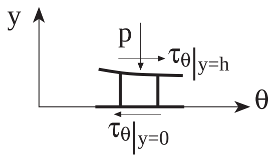
Flows

$$q_r = -\frac{h^3}{12\eta}\frac{\partial p}{\partial r} \qquad q_\theta = -\frac{h^3}{12\eta}\frac{\partial p}{r\partial\theta} + (\omega_1+\omega_2)\frac{rh}{2}$$

Shear stresses

$$\tau_{\theta|_{y=0}} = -\frac{h}{2}\frac{\partial p}{r\partial\theta} + \eta\frac{r(\omega_2-\omega_1)}{h}$$

$$\tau_{\theta|_{y=h}} = \frac{h}{2}\frac{\partial p}{r\partial\theta} + \eta\frac{r(\omega_2-\omega_1)}{h}$$



Reynolds equation

$$\frac{\partial}{\partial r}\left(\frac{\rho h^3 r}{\eta}\frac{\partial p}{\partial r}\right) + \frac{\partial}{r\partial\theta}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial\theta}\right) = 6r(\omega_1-\omega_2)\frac{\partial}{\partial\theta}(\rho h) + 12\rho r(V_2-V_1)$$

Adiabatic energy equation

$$\rho c\left[q_r\frac{\partial T}{\partial r} + q_\theta\frac{\partial T}{\partial\theta} + h\frac{\partial T}{\partial t}\right] = \frac{r^2\eta}{h}(\omega_1-\omega_2)^2 + \frac{h^3}{12\eta}\left[\left(\frac{\partial p}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial p}{\partial\theta}\right)^2 + \frac{p}{r}\frac{\partial p}{\partial r}\right]$$

10 Non-stationary flow

10.1 Nomenclature

A	:line sectional area	[m^2]
B	:constant (L_0a)	[kg/s m^4]
C_H	:conc. hydr. capacitance	[m^5/N]
C_0	:conc. hydr. capacitance/l.enh.	[m^4/N]
F	:force	[N]
L_H	:conc. hydr. inductance	[kg/m^4]
L_0	:conc. hydr. inductance/l.enh.	[kg/m^5]
P	:pressure (frequency dependent)	[Pa]
Q	:flow (volume flow) (frequency dependent)	[m^3/s]
$R_{H\ell}$	:conc. hydr. resistance (lam.)	[Ns/m^5]
R_{Ht}	:conc. hydr. resistance (turb.)	[Ns/m^5]
$R_{0\ell}$	:conc. hydr. res./l.unit. (lam.)	[Ns/m^6]
R_{0t}	:conc. hydr. res./l.unit. (turb.)	[Ns/m^6]
V_1	:volume	[m^3]
V_2	:volume	[m^3]
Z_0	:impedance	[Ns/m^5]
a	:speed of sound	[m/s]
d	:diameter	[m]
ℓ	:length	[m]
m	:mass	[kg]

p	:pressure	[Pa]
p_0	:pressure in point of operation	[Pa]
p_1	:upstream pressure	[Pa]
p_2	:downstream pressure	[Pa]
p_s	:supply pressure	[Pa]
q	:flow (volume flow)	[m^3/s]
q_0	:flow in point of operation	[m^3/s]
s	:Laplace operator ($i\omega$)	[1/s]
t	:time	[s]
t_v	:valve closing time	[s]
v	:flow velocity	[m/s]
v_0	:velocity of cylinder	[m/s]
α	:dimensionless area	[-]
β_e	:effective bulk modulus	[Pa]
Δp	:change in pressure due to pressure peek	[Pa]
η	:dynamic viscosity	[Ns/m^2]
λ	:friction coefficient	[-]
	:parameter	[1/m]
ρ	:density	[kg/m^3]
ζ	:single resistant loss	[-]

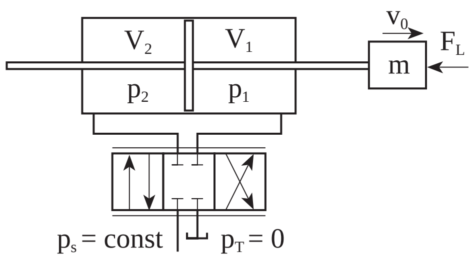
10.2 Joukowskis equation

$$\Delta p = \rho a v_0$$

Reduction due to valve closing time.

$$\Delta p_{red} = \Delta p \frac{2\ell}{a t_v} \qquad \text{for} \quad t_v > \frac{2\ell}{a}$$

10.3 Retardation of cylinder with inertia load



Pressure difference for retardation ($v_0 \rightarrow 0$)

$$p_{1_{max}} - p_{2_{min}} = v_0 \sqrt{\frac{2\beta_e m}{V_1}}$$

10.4 Concentrated hydraulic inductance

$$p_1 - p_2 = L_H \frac{dq}{dt} \qquad \text{were} \quad L_H = \frac{\rho\ell}{A}$$

10.5 Concentrated hydraulic capacitance

$$q_1 - q_2 = C_H \frac{dp}{dt} \qquad \text{were} \quad C_H = \frac{A\ell}{\beta_e}$$

10.6 Concentrated hydraulic resistance

$$p_1 - p_2 = R_H q \qquad \text{were} \quad R_H = \begin{cases} R_{H\ell} = \dfrac{128\eta\ell}{\pi d^4} & \text{For laminar flow} \\[2ex] R_{Ht} = \lambda \dfrac{\ell}{d}\dfrac{\rho q_0}{A^2} & \text{For turbulent flow, with linearization around the working point with the flow } q_0 \end{cases}$$

10.7 Basic differential equations on flow systems with parameter distribution in space

$$\frac{\partial p}{\partial x} + L_0 \frac{\partial q}{\partial t} + R_0 q|q|^m = 0 \qquad \frac{\partial q}{\partial x} + C_0 \frac{\partial p}{\partial t} = 0$$

Parameter values (per length unit)
independent of flow regime

$$C_0 = \frac{A}{\beta_e} \qquad L_0 = \frac{\rho}{A}$$

with laminar flow

$$R_{0\ell} = \frac{128\eta}{\pi d^4} \qquad m = 0$$

with turbulent flow

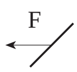
$$R_{0t} = \frac{0{,}1582\,\eta^{0{,}25}\,\rho^{0{,}75}}{d^{1{,}25}\,A^{1{,}75}} \qquad m = 0{,}75$$

10.8 Speed of waves in pipes filled with liquid

$$a = \frac{1}{\sqrt{L_0 C_0}} = \sqrt{\frac{\beta_e}{\rho}}$$

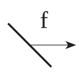
Graphical solution

F-wave



$$\Delta p = B \Delta q$$

f-wave



$$\Delta p = -B \Delta q$$

were $B = L_0 a$

Boundary conditions:

At a valve

$$\frac{q}{q_0} = \alpha\sqrt{\frac{p}{p_0}}$$

were α = dimensionless area and p_0, q_0 is stationary state.
At a pressure source with concentrated friction loss

$$p = p_0 - \left(\zeta + \lambda\frac{\ell}{d}\right)\frac{\rho q^2}{2A^2}$$

Solution with impedance method

Transfer matrices

$$\begin{bmatrix} P(s,\ell) \\ Q(s,\ell) \end{bmatrix} = \begin{bmatrix} \cosh(\lambda\ell) & -Z_0\sinh(\lambda\ell) \\ -\frac{1}{Z_0}\sinh(\lambda\ell) & \cosh(\lambda\ell) \end{bmatrix} \begin{bmatrix} P(s,0) \\ Q(s,0) \end{bmatrix}$$

$$\begin{bmatrix} P(s,0) \\ Q(s,0) \end{bmatrix} = \begin{bmatrix} \cosh(\lambda\ell) & Z_0\sinh(\lambda\ell) \\ \frac{1}{Z_0}\sinh(\lambda\ell) & \cosh(\lambda\ell) \end{bmatrix} \begin{bmatrix} P(s,\ell) \\ Q(s,\ell) \end{bmatrix}$$

$$\begin{bmatrix} P(s,x) \\ Q(s,x) \end{bmatrix} = \begin{bmatrix} \cosh(\lambda x) & -Z_0\sinh(\lambda x) \\ -\frac{1}{Z_0}\sinh(\lambda x) & \cosh(\lambda x) \end{bmatrix} \begin{bmatrix} P(s,0) \\ Q(s,0) \end{bmatrix}$$

$$\begin{bmatrix} P(s,x) \\ Q(s,x) \end{bmatrix} = \begin{bmatrix} \cosh(\lambda(\ell-x)) & Z_0\sinh(\lambda(\ell-x)) \\ \frac{1}{Z_0}\sinh(\lambda(\ell-x)) & \cosh(\lambda(\ell-x)) \end{bmatrix} \begin{bmatrix} P(s,\ell) \\ Q(s,\ell) \end{bmatrix}$$

$$\text{were} \qquad \lambda = \sqrt{(L_0 s + R_0)C_0 s} \qquad\qquad Z_0 = \sqrt{\frac{L_0 s + R_0}{C_0 s}}$$

11 Pump pulsations

11.1 Nomenclature

A	:the pipe's cross-sectional area	[m^2]	p_s	:static pressure level	[Pa]
T	:wave propagation time	[s]	z	:the pump's piston number	[-]
D	:pump displacement	[m^3/varv]	η	:dynamic viscosity	[Ns/m^2]
L	:pipe length	[m]	α_p	:dim.free cylinder volume	[-]
P	:pulsation amplitude	[N/m^2]	β	:bulk modulus	[Pa]
V	:volume	[m^3]	ε	:the pump's displacement	[-]
a	:wave propagation speed	[m/s]	γ	:dim.free dead volume	[-]
d	:pipe diameter	[m]	ρ	:density	[kg/m^3]
f_p	:dim. free flow spectrum	[-]	τ	:dim.free charging time	[-]
n	:pump speed	[rev/s]	ω	:angular frequency	[rad/s]

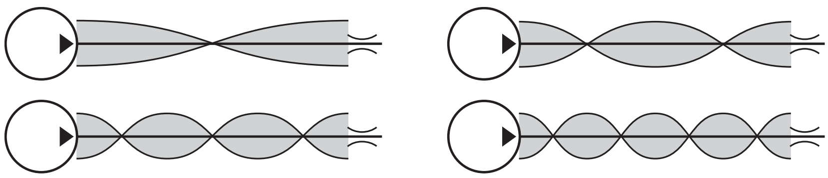
11.2 System with closed end

Resonances in a pipe system with closed end are obtained at following frequency/-ies:

$$\omega = \frac{\pi k}{T} \qquad k = 1,\ 2,\ 3,\ \ldots$$

where

$$T = \frac{L}{a} \qquad a = \sqrt{\frac{\beta}{\rho}}$$



Figur 3: Amplitude for the resonances $k = 1,\ 2,\ 3$ and 4 in a system with closed end.

Flow disturbance from the pump rises at following frequencies:

$$\omega = 2\pi nzj \qquad j = 1,\ 2,\ 3,\ \ldots$$

Under condition that the pumps flow disturbance frequencies coincide with the pipe system's resonances, can the resulting pressure amplitude at this frequency be calculated with following equation:

$$\left|\frac{P}{p_s}\right|_{max} = \frac{2f_p\alpha_p D}{dL}\sqrt{\frac{\rho n}{\pi^3\eta zj}}$$

where

$$f_p = \frac{2}{1+(\tau j)^3} \qquad \alpha_p = \frac{1+\varepsilon}{2} + \gamma$$

The dimensionless charging time τ is the relationship between the time you, due to the oil's compressibility, receive a back flow into a cylinder, and the total time period, i.e. the time between two volumes in succession are charged. This parameter is very difficult to decide, a typical value of τ is between 0,05 till 0,3. The higher values are referred to pumps designed for low flow pulsations for example pumps with pressure relief grooves.

The dead volume γ has often the magnitude of 0,2, that is 20% of the effective cylinder volume.

Anti-resonances are received in a pipe system with closed end at following frequencies:

$$\omega = \frac{\pi(k+\frac{1}{2})}{T} \qquad k = 0,\ 1,\ 2,\ \ldots$$

Under condition that the pump flow disturbance frequencies coincide with the pipe system's anti-resonances, can the resulting maximum pressure amplitude at this frequency be calculated with following equation:

$$\left|\frac{P}{p_s}\right|_{max} = \frac{4f_p\alpha_p Dn}{\pi d^2 a}$$

Example: The above equations are used for following example. A constant pressure pump presumed work against a closed valve. Following data is obtained:

$d = 38 \cdot 10^{-3}$	[m]	$\gamma = 0{,}2$	
$n = 25$	[rev/s]	$\varepsilon = 0$	
$z = 9$		$\eta = 0{,}02$	[Ns/m^2]
$D = 220 \cdot 10^{-6}$	[m^3/rev]	$\rho = 900$	[kg/m^3]
$\beta_e = 1{,}5 \cdot 10^9$	[Pa]	$\tau = 0{,}25$	

Four different pipe lengths between pump and valve is analysed:

$L = 1{,}45$ m	Gives resonances for $j = 2,\ 4,\ 6,\ \ldots$	anti-resonances for $j = 1,\ 3,\ 5,\ \ldots$
$L = 1{,}91$ m	Gives resonance for $j = 3,\ 6,\ \ldots$	no anti-resonance
$L = 2{,}15$ m	Gives resonance for $j = 4,\ \ldots$	anti-resonances for $j = 2,\ 6,\ \ldots$
$L = 2{,}90$ m	Gives resonance for $j = 1,\ 2,\ 3,\ \ldots$	no anti-resonance

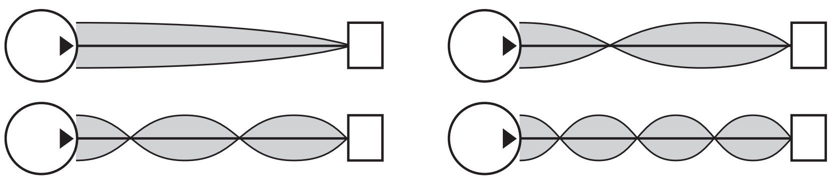
In the table below shows the obtained relationship between pulsation's pressure amplitude and the system's pressure level for respective disturbance harmonic. Note, for $L = 1{,}91$ m and $L = 2{,}15$ m can some disturbance harmonics not be analysed with the equations above, since they don't coincide with any of the pipe's resonances or anti-resonances. However, the amplitudes at these frequencies are relative small because they don't coincide with any of the pipe's resonances. In the table below, these values are in parenthesis.

		Relative pulsation amplitude $\lvert P/p_s \rvert$			
j	f [Hz]	$L = 1{,}45$ m	$L = 1{,}91$ m	$L = 2{,}15$ m	$L = 2{,}90$ m
1	225	0,01	(0,01)	(0,01)	0,35
2	450	0,45	(0,01)	0,00	0,22
3	675	0,00	0,22	(0,01)	0,14
4	900	0,18	(0,00)	0,12	0,09
5	1125	0,00	(0,00)	(0,01)	0,05
6	1350	0,07	0,05	0,00	0,03

11.3 Systems with low end impedance (e.g. volume)

Resonances in a pipe system with low end impedance is obtained at following frequencies:

$$\omega = \frac{\pi(k + \frac{1}{2})}{T} \qquad k = 0,\ 1,\ 2,\ \ldots$$



Figur 4: Amplitude for the resonances $k = 0,\ 1,\ 2$ and 3 for a system with low end impedance.

Anti-resonances in a pipe system with low end impedance is obtained at following frequencies:

$$\omega = \frac{\pi k}{T} \qquad k = 1,\ 2,\ 3,\ \ldots$$

The same equations as in previous section can be used here for calculation of maximum pressure amplitude for the pulsations.

If a volume, which size is not infinite, is connected to the pipe system is a dislocation of the line's resonances from the values in above equations obtained. This dislocation can be calculated according to following equation

$$\Delta\omega = \frac{1}{T}\left[\frac{\pi}{2} - \arctan\left(\frac{V\omega}{Aa}\right)\right]$$

I.e. if a finite volume is used the resonance frequency is increased.

As example on this section, a high pressure filter is placed before the valve in the example with the constant pressure pump. The valve is closed in this example too. The volume of the filter is 2 liters; other parameters are the same as the previous example except for the length of the line. The following line lengths are analysed:

$L = 1{,}47$ m	Gives resonance for $j = 3,\ 5,\ \ldots$	anti-resonance for $j = 2,\ 4,\ 6,\ \ldots$
$L = 1{,}87$ m	Gives resonance for $j = 1,\ 4,\ \ldots$	anti-resonance for $j = 3,\ 6,\ \ldots$
$L = 3{,}00$ m	Gives no resonance	anti-resonance for $j = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ \ldots$

Note, the volume is relative small and therefore the dislocation equation has to be used. When the new resonance frequency is calculated, "'passningsräkning"' has to be used. The method is shown bellow for $L = 1{,}47$ m and $k = 1$.

$$\omega = \frac{\pi\left(k + \frac{1}{2}\right)}{T} = 1350\text{rad/s} \Rightarrow \quad \begin{array}{ll} \Delta\omega = 430 \text{ rad/s} & \omega = 1810 \text{ rad/s} \\ \Delta\omega = 340 \text{ rad/s} & \omega = 1720 \text{ rad/s} \\ \Delta\omega = 350 \text{ rad/s} & \omega = 1730 \text{ rad/s} \end{array}$$

The size of the pulsation amplitude in relation to the static system pressure is shown in the table below. The values in parenthesis show, as in previous example, a more correct analyze of the disturbance harmonic which can not be calculated with the equation given in this handbook.

		Relative pulsation amplitude $\lvert P/p_s \rvert$		
j	f [Hz]	$L = 1{,}47$ m	$L = 1{,}87$ m	$L = 3{,}00$ m
1	225	(0,01)	(0,54)	0,01
2	450	0,00	(0,01)	0,00
3	675	0,28	0,00	0,00
4	900	0,00	0,14	0,00
5	1125	0,11	(0,00)	0,00
6	1350	0,00	0,00	0,00

12 Hydraulic servo systems

12.1 Nomenclature

A	:piston area	[m^2]
A_h	:control piston area (3-port valve)	[m^2]
A_m	:amplitude margin	[dB]
A_r	:piston area, rod side (3-port valve)	[m^2]
A_u	:the open system's transfer function	
B_m	:viscous friction coeff. (motor)	[Nms/rad]
B_p	:viscous friction coeff. (cylinder)	[Ns/m]
C_e	:external leakage flow coeff. (cylinder/motor/pump)	[m^5/Ns]
C_i	:internal leakage flow coeff. (cylinder/motor/pump)	[m^5/Ns]
C_t	:total leakage flow coeff.	[m^5/Ns]
C_q	:leakage flow coeff.	[-]
D	:displacement (motor/pump)	[m^3/rad]
F_L	:external (load-) force on cylinder	[N]
G_c	:the closed system's transfer function	
G_o	:the open system's transfer function	
G_{reg}	:controller transfer function	
J_t	:total moment of inertia (motor and load)	[kg m^2]
K_c	:flowpressure coeff. (servo valve)	[m^5/Ns]
K_p	:pressure gain (servo valve)	[Pa/m]
K_q	:flow gain (servo valve)	[m^2/s]
K_{reg}	:control gain	
K_v	:loop gain	
M_t	:total mass (cylinder piston and load)	[kg]
N_p	:pump speed	[rad/s]
S	:stiffness	
T_L	:external (load-)moment on motor	[Nm]
U	:under lap	[m]
V	:volume	[m^3]
V_h	:control volume (3-port valve)	[m^3]
V_t	:total volume	[m^3]
e	:control error	
k_p	:displacement gradient (pump)	[m^3/rad^2]
p	:pressure	[Pa]
p_c	:control pressure (3-port valve)	[Pa]
p_s	:supply pressure	[Pa]
q	:flow (volume flow)	[m^3/s]
q_c	:centre flow	[m^3/s]
s	:Laplace operator ($i\omega$)	[rad/s]
t	:time	[s]
x_v	:position (servo valve)	[m]
x_p	:position (cylinder piston)	[m]
w	:area gradient	[m]
β_e	:effective bulk modulus	[Pa]
δ_h	:hydraulic damping	[-]
ε_0	:control error (stationary)	
ρ	:density	[kg/m^3]
θ_m	:angular position (motor)	[rad]
ϕ_p	:displacement angle (pump)	[rad]
φ_m	:phase margin	[°]
ω	:angular frequency	[rad/s]
ω_b	:bandwidth	[rad/s]
ω_c	:crossing-out frequency	[rad/s]
ω_h	:hydraulic eigen frequency	[rad/s]
Re	:real part	
Im	:imaginary part	

Tilläggsindex

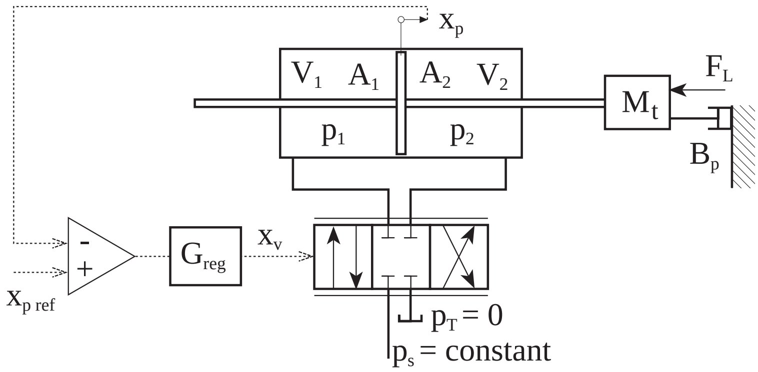
0	:working point
e	:effective
m	:motor
p	:cylinder (piston), pump
v	:valve
t	:total
L	:load

12.2 Introduction

The servo technical section discusses following system:

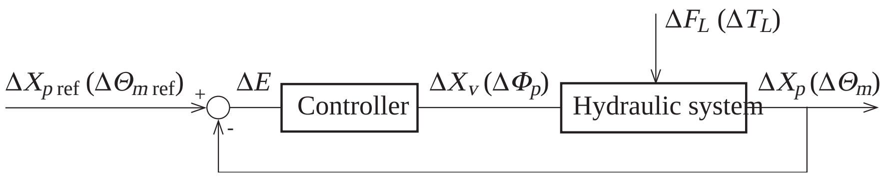
- valve controlled cylinder
- valve controlled motor
- pump controlled cylinder
- pump controlled motor

As example in this section will a position servo of the type valve controlled cylinder in a constant pressure system be used. In this example is the servo valve a 4-port valve with negligible dynamic and the cylinder is symmetric. See figure 5.

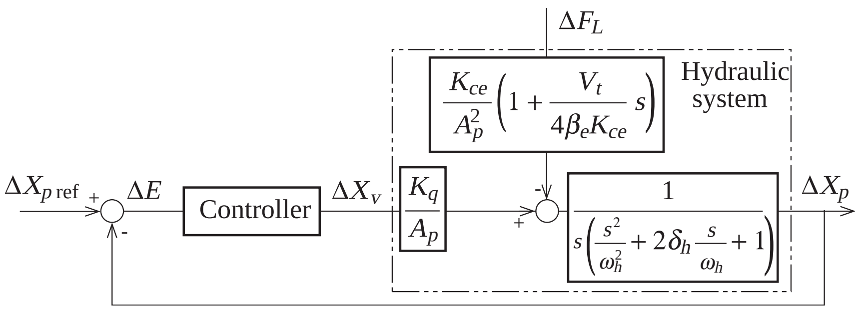


Figur 5: Lägesservo: ventilstyrd cylinder i konstanttryckssystem.

A position servo looks, in general, out as follow:



The special case when the hydraulic system consists of a valve controlled cylinder becomes the block diagram as:



12.3 The servo valve's transfer function

All the transfer functions in this section are applied when the load spring constant K is negligible and its viscous friction B_p and B_m respectively is small (can often be set to 0). The direction dependent friction coefficient C_f is neglected also in the motor case.

Valve controlled systems

The dynamic of the servo valve in the valve controlled systems is assumed to be negligible compared to the system. Following is valid for the servo valve (see also section 12.4):

4-ports servo valve	3-ports servo valve	
$K_q = \frac{\partial q_L}{\partial x_v}$	$K_q = \frac{\partial q_L}{\partial x_v}$	the servo valve's flow gain
$K_c = -\frac{\partial q_L}{\partial p_L}$	$K_c = -\frac{\partial q_L}{\partial p_c}$	the servo valve's flowpressure coefficient
$K_p = \frac{K_q}{K_c} = \frac{\partial p_L}{\partial x_v}$	$K_p = \frac{K_q}{K_c} = \frac{\partial p_c}{\partial x_v}$	the servo valve's pressure gain

The load flow and load pressure through a 4-port valve controlled symmetric cylinder/motor is defined according to following expression:

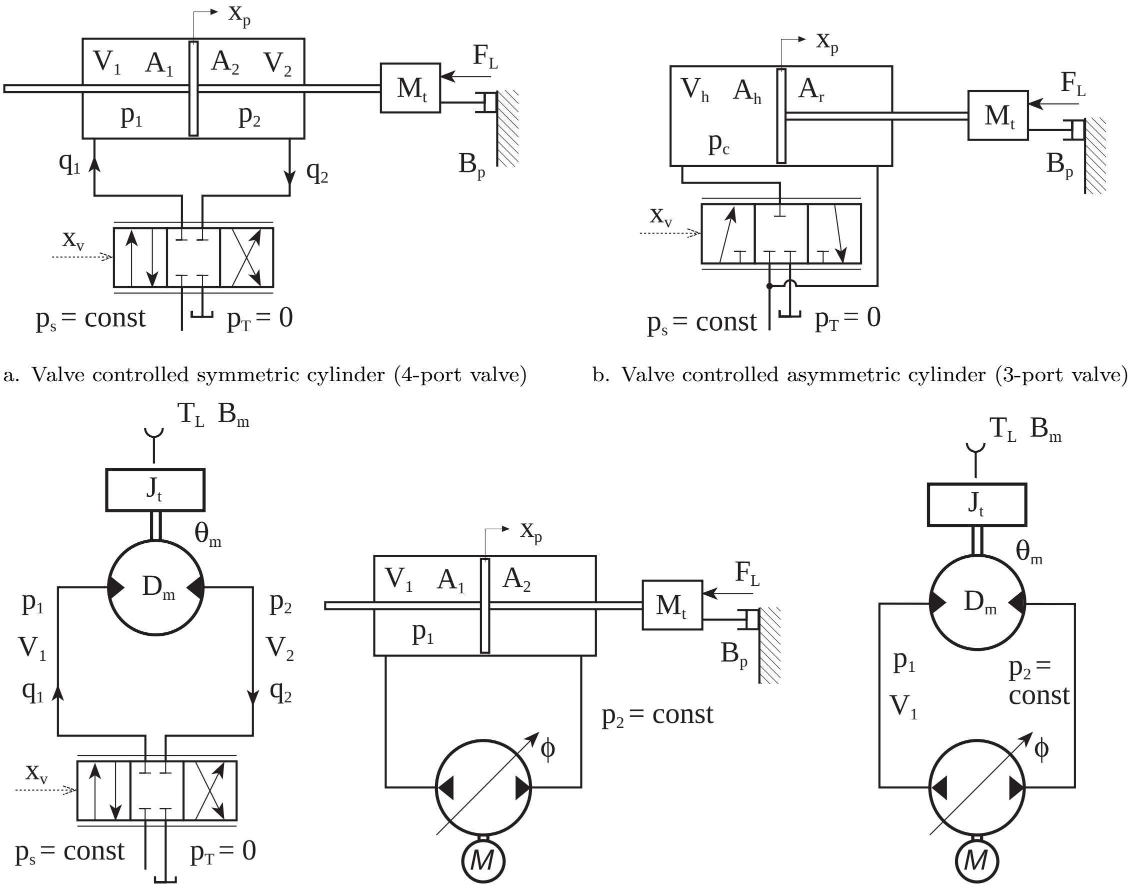
$$q_L = (q_{L1} + q_{L2})/2 \qquad p_L = p_1 - p_2$$

Pump controlled systems

The dynamic of the pump in the pump controlled systems is assumed to be negligible compared to the system. The pump's ideal flow is:

$$q_{p\,\text{ideal}} = \varepsilon_p D_p N_p = k_p \phi_p N_p$$

Linearised and Laplace transformed equations which describes the dynamic of the following hydraulic systems are presented in section 12.5.



a. Valve controlled symmetric cylinder (4-port valve) b. Valve controlled asymmetric cylinder (3-port valve)

c. Valve controlled motor d. Pump controlled cylinder e. pump controlled motor (transmission)

Figur 6: Different types of servo systems.

12.4 Servo valve

4-port zero lapped valve

Load flow: $q_L = C_q w x_v \sqrt{\frac{1}{\rho}\left(p_s - \frac{x_v}{|x_v|}p_L\right)}$

Centre flow: $q_c = 0$ ideal

Zero coefficients: $K_{q0} = C_q w \sqrt{\frac{p_s}{\rho}}$ $K_{c0} = 0$ ideal $K_{p0} = \infty$ ideal

$q_{L0} = 0$ $p_{L0} = 0$ $x_{v0} = 0$

4-port under lapped valve

Load flow: $$q_L = C_q w \left[(U + x_v) \sqrt{\frac{p_s - p_L}{\rho}} - (U - x_v) \sqrt{\frac{p_s + p_L}{\rho}} \right] \text{ for } |x_v| \leq U$$

Centre flow: $$q_c = 2 C_q w U \sqrt{\frac{p_s}{\rho}}$$

Zero coefficients: $$K_{q0} = 2 C_q w \sqrt{\frac{p_s}{\rho}} \qquad K_{c0} = C_q w U \sqrt{\frac{1}{p_s \rho}} \qquad K_{p0} = \frac{2 p_s}{U}$$

$$q_{L0} = 0 \qquad p_{L0} = 0 \qquad x_{v0} = 0$$

3-port zero-lapped valve

Load flow: $$q_L = \begin{cases} C_q w x_v \sqrt{\frac{2}{\rho} (p_s - p_c)} & \text{då } x_v \geq 0 \\ C_q w x_v \sqrt{\frac{2}{\rho} p_c} & \text{då } x_v \leq 0 \end{cases}$$

Centre flow: $q_c = 0 \quad \text{ideal}$

Zero coefficients: $$K_{q0} = C_q w \sqrt{\frac{p_s}{\rho}} \qquad K_{c0} = 0 \quad \text{ideal} \qquad K_{p0} = \infty \quad \text{ideal}$$

$$q_{L0} = 0 \qquad p_{c0} = \frac{p_s}{2} \qquad x_{v0} = 0$$

3-port under lapped valve

Load flow: $$q_L = C_q w \left[(U + x_v) \sqrt{\frac{2(p_s - p_c)}{\rho}} - (U - x_v) \sqrt{\frac{2 p_c}{\rho}} \right] \text{ for } |x_v| \leq U$$

Centre flow: $$q_c = C_q w U \sqrt{\frac{p_s}{\rho}}$$

Zero coefficients: $$K_{q0} = 2 C_q w \sqrt{\frac{p_s}{\rho}} \qquad K_{c0} = 2 C_q w U \sqrt{\frac{1}{p_s \rho}} \qquad K_{p0} = \frac{p_s}{U}$$

$$q_{L0} = 0 \qquad p_{L0} = \frac{p_s}{2} \qquad x_{v0} = 0$$

12.5 The hydraulic system's transfer function

Valve controlled symmetric cylinder with mass load (4-port valve)

$$\Delta X_p = \frac{\frac{K_q}{A_p} \Delta X_v - \frac{K_{ce}}{A_p^2} \left(1 + \frac{V_t}{4 \beta_e K_{ce}} s \right) \Delta F_L}{s \left(\frac{s^2}{\omega_h^2} + 2 \delta_h \frac{s}{\omega_h} + 1 \right)}$$

where $$\omega_h = \begin{cases} \sqrt{\frac{4 \beta_e A_p^2}{V_t M_t}} & \text{if } V_1 \approx V_2 \\ \sqrt{\frac{\beta_e A_p^2}{M_t} \left(\frac{1}{V_1} + \frac{1}{V_2} \right)} & \text{if } V_1 \neq V_2 \end{cases} \qquad \delta_h = \frac{K_{ce}}{A_p} \sqrt{\frac{\beta_e M_t}{V_t}} + \frac{B_p}{4 A_p} \sqrt{\frac{V_t}{\beta_e M_t}}$$

$$K_{ce} = K_c + C_{ip} + \frac{C_{ep}}{2} \qquad V_t = V_1 + V_2 \qquad A_p = A_1 = A_2$$

Valve controlled asymmetric cylinder with mass load (3-port valve)

$$\Delta X_p = \frac{\frac{K_q}{A_h}\Delta X_v - \frac{K_{ce}}{A_h^2}\left(1 + \frac{V_h}{\beta_e K_{ce}}s\right)\Delta F_L}{s\left(\frac{s^2}{\omega_h^2} + 2\delta_h\frac{s}{\omega_h} + 1\right)}$$

where $\omega_h = \sqrt{\frac{\beta_e A_h^2}{V_h M_t}}$ $\quad \delta_h = \frac{K_{ce}}{2A_h}\sqrt{\frac{\beta_e M_t}{V_h}} + \frac{B_p}{2A_h}\sqrt{\frac{V_h}{\beta_e M_t}}$ $\quad K_{ce} = K_c + C_{ip}$

Valve controlled motor with moment of inertia

$$\Delta\Theta_m = \frac{\frac{K_q}{D_m}\Delta X_v - \frac{K_{ce}}{D_m^2}\left(1 + \frac{V_t}{4\beta_e K_{ce}}s\right)\Delta T_L}{s\left(\frac{s^2}{\omega_h^2} + 2\delta_h\frac{s}{\omega_h} + 1\right)}$$

where $\omega_h = \sqrt{\frac{4\beta_e D_m^2}{V_t J_t}}$ $\quad \delta_h = \frac{K_{ce}}{D_m}\sqrt{\frac{\beta_e J_t}{V_t}} + \frac{B_m}{4D_m}\sqrt{\frac{V_t}{\beta_e J_t}}$

$K_{ce} = K_c + C_{im} + \frac{C_{em}}{2}$ $\quad V_t = V_1 + V_2, \quad V_1 = V_2$

Pump controlled cylinder with mass load

$$\Delta X_p = \frac{\frac{k_p N_p}{A_p}\Delta\phi_p - \frac{C_t}{A_p^2}\left(1 + \frac{V_0}{\beta_e C_t}s\right)\Delta F_L}{s\left(\frac{s^2}{\omega_h^2} + 2\delta_h\frac{s}{\omega_h} + 1\right)}$$

where $\omega_h = \sqrt{\frac{\beta_e A_p^2}{V_0 M_t}}$ $\quad \delta_h = \frac{C_t}{2A_p}\sqrt{\frac{\beta_e M_t}{V_0}} + \frac{B_p}{2A_p}\sqrt{\frac{V_0}{\beta_e M_t}}$

$V_0 = V_1$ $\quad A_p = A_1 = A_2$

$C_t = C_{it} + C_{et} = C_{ip(iston)} + C_{ip(ump)} + C_{ep(iston)} + C_{ep(ump)}$

Pump controlled motor with moment of inertia (transmission)

$$\Delta\Theta_m = \frac{\frac{k_p N_p}{D_m}\Delta\phi_p - \frac{C_t}{D_m^2}\left(1 + \frac{V_0}{\beta_e C_t}s\right)\Delta T_L}{s\left(\frac{s^2}{\omega_h^2} + 2\delta_h\frac{s}{\omega_h} + 1\right)}$$

where $\omega_h = \sqrt{\frac{\beta_e D_m^2}{V_0 J_t}}$ $\quad \delta_h = \frac{C_t}{2D_m}\sqrt{\frac{\beta_e J_t}{V_0}} + \frac{B_m}{2D_m}\sqrt{\frac{V_0}{\beta_e J_t}}$

$V_0 = V_1$ $\quad C_t = C_{it} + C_{et} = C_{ip} + C_{im} + C_{ep} + C_{em}$

12.6 The servo stability

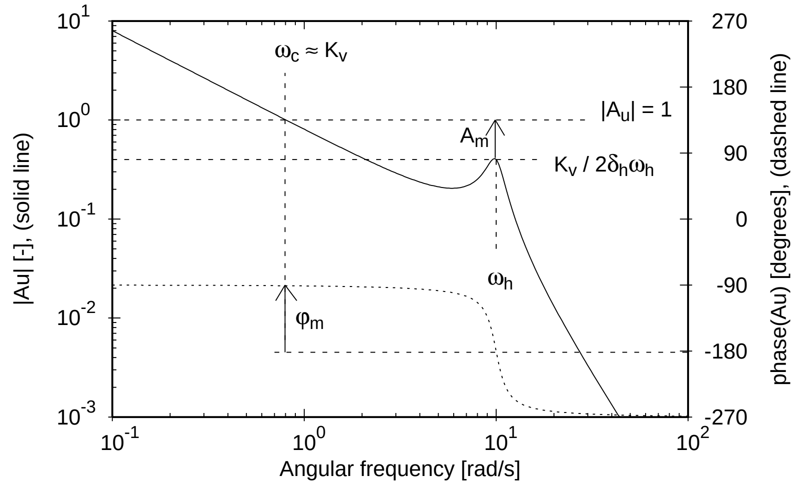
Feedback systems can become instable if the feedback is incorrect dimensioned. In this case we study a position servo with proportional feedback $G_{reg} = K_{reg}$. The open loop transfer function become:

$$A_u = G_{reg}G_o = \frac{K_v}{s\left(\frac{s^2}{\omega_h^2} + 2\delta_h\frac{s}{\omega_h} + 1\right)}$$

where G_o is the transfer function which describes the hydraulic system's output signal (cylinder position) as function of the hydraulic system's input signal (valve position) when the disturbance signal (ΔF_L) is zero. The steady state loop gain K_v (also called the velocity coefficient) is:

$K_v = \frac{K_q}{A_p} K_{reg}$	valve controlled symmetric cylinder (4-port valve)
$K_v = \frac{K_q}{A_h} K_{reg}$	valve controlled asymmetric cylinder (3-port valve)
$K_v = \frac{K_q}{D_m} K_{reg}$	valve controlled motor
$K_v = \frac{k_p N_p}{A_p} K_{reg}$	pump controlled cylinder
$K_v = \frac{k_p N_p}{D_m} K_{reg}$	pump controlled motor

Then, the open system's Bode-diagram for a position servo become as figure 7.



Figur 7: The open system's transfer function for a position servo. Amplitude, (solid line) and phase, (dashed).

Stability condition

A stable system is obtained when

- the amplitude margin $A_m > 0$ dB at $-180°$ phase shift. If the phase intersects $-180°$ more than one time the Nyquist diagram is needed.
- the phase margin $\varphi_m > 0°$ at 0 dB amplitude. If the amplitude curve intersects 0 dB more than one time the Nyquist diagram is needed.

For a proportional position servo with a feedback is, for the hydraulic, the amplitude margin the critical stability margin. It means that a stable system needs the open loop gain $|A_u|$ to be $<= 1$ (0 dB) when the phase shift is $<= -180°$.

Figure 7 shows that the stability condition becomes

$$\frac{K_v}{2\delta_h \omega_h} < 1$$

Stability margins

Position servo (with Bode-diagram according to figure 7) amplitude margin can be written as:

$$A_m = -20\ {}^{10}\log \left| \frac{K_v}{-2\delta_h \omega_h} \right| \qquad \text{[dB]}$$

The following margins should be used when the control parameters shall be dimensioned in a hydraulic system with a feedback.

amplitude margin: $A_m \approx 10$ dB

phase margin: $\varphi_m \geq 45°$

The system's critical working condition
Since hydraulic systems are non-linear systems, the stability margin will become different in different working condition. From the figure for the open system's transfer function A_u the stability margin become worst when both ω_h and δ_h are low and steady state loop gain K_v is big. This happened for a valve controlled symmetric cylinder when

- Cylinder piston is centered ($x_p = 0$), i.e. $V_1 = V_2 = \frac{V_t}{2}$. ω_h is minimised.
- The servo valve is closed ($x_v = q_L = 0$), i.e. when $K_c = K_{c0}$. K_{ce} and consequently δ_h is minimised.
- Cylinder piston is out balanced, i.e. when $p_L = 0$. K_q, which is proportional to $\sqrt{\Delta p_s - \Delta p_L}$, is maximised and consequently also K_v (proportional to K_q).

With similar discussion, the critical working condition can be decided for other systems.
In practical dimensioning, the hydraulic damping is often set to $\delta_h \approx 0,1$.

12.7 The servo's response – bandwidth

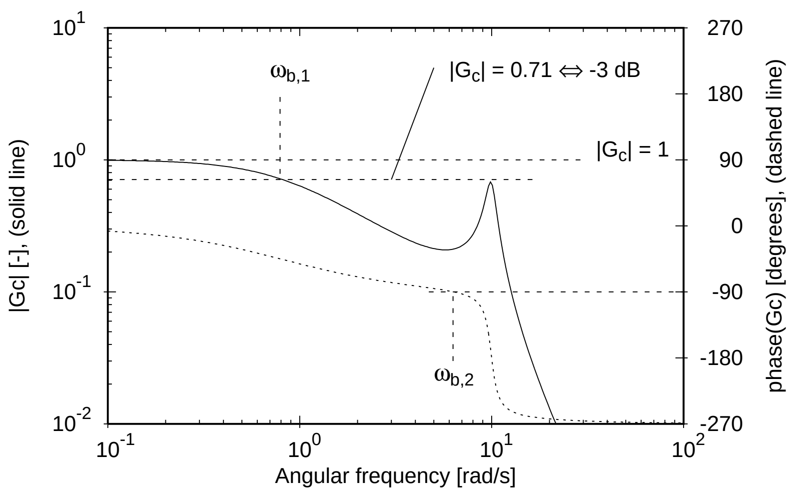
The bandwidth (ω_b) for the closed loop system specifies how high frequency the servo's output signal can follow a sinusoidal input signal, when the disturbance $\Delta F_L = 0$, without:

- the gain lowers more than 3 dB (29,3 %)
- or the phase shift become more than $-90°$

If the feedback is -1, the closed loop system's transfer function for a position servo with any of the earlier described hydraulic systems becomes:

$$G_c = \frac{A_u}{1 + A_u} = \frac{1}{\frac{1}{K_v \omega_h^2} s^3 + \frac{2\delta_h}{K_v \omega_h} s^2 + \frac{1}{K_v} s + 1}$$

where $G_c = \dfrac{\Delta X_p}{\Delta X_{p\,\mathrm{ref}}}$ for a position servo with a valve controlled cylinder.



Figur 8: The closed loop system's transfer function for a position servo. Amplitude (solid) and phase (dashed).

The closed loop system's transfer function can also be written as

$$G_c = \frac{1}{\left(\dfrac{s}{\omega_b}+1\right)\left(\dfrac{s^2}{\omega_{nc}^2}+2\delta_{nc}\dfrac{s}{\omega_{nc}}+1\right)}$$

If δ_h and K_v/ω_h is small

$\omega_{nc} \approx \omega_h$

$\omega_b \approx K_v$

$2\delta_{nc} \approx 2\delta_h - \dfrac{K_v}{\omega_h}$

12.8 The hydraulic system's and the servo's sensitivity to loading – stiffness

Cylinder - respective motor position sensitivity to disturbance force ΔF_L or a disturbance torque ΔT_L is described with its stiffness S. When the stiffness is studied is all other input signals assumed to be constant (ΔInsignal $= 0$). The stiffness is defined as

$$S = \frac{\Delta F_L}{\Delta X_p} \qquad \text{or} \qquad S = \frac{\Delta T_L}{\Delta \Theta_m}$$

The hydraulic system's transfer function can be found in section 12.5.
As example on a system without respective with a feedback, the stiffness for the valve controlled (4-port valve) symmetric cylinder is decided. The same approach is used when the stiffness is decided for the other hydraulic.

System without feedback

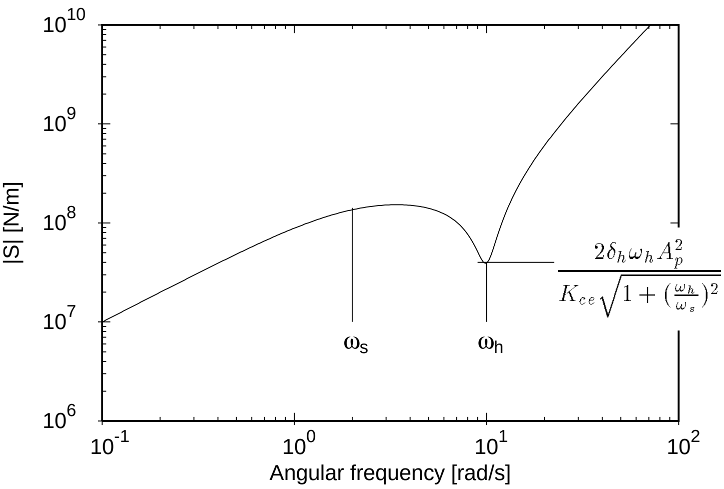
For a hydraulic system without feedback, the stiffness for the valve controlled cylinder is calculated as

$$S = \frac{\Delta F_L}{\Delta X_p} = \frac{1}{\dfrac{\Delta X_p}{\Delta F_L}} = -\frac{s\left(\dfrac{s^2}{\omega_h^2}+2\delta_h\dfrac{s}{\omega_h}+1\right)}{\dfrac{K_{ce}}{A_p^2}\left(1+\dfrac{s}{\omega_s}\right)}$$

where

$$\omega_s = \frac{4\beta_e K_{ce}}{V_t} = \{\text{if } B_p \text{ isneglected}\} = 2\delta_h\omega_h$$

The transfer function for the stiffness is shown in figure 9.



Figur 9: The transfer function for the stiffness for the non-feedback valve controlled cylinder.

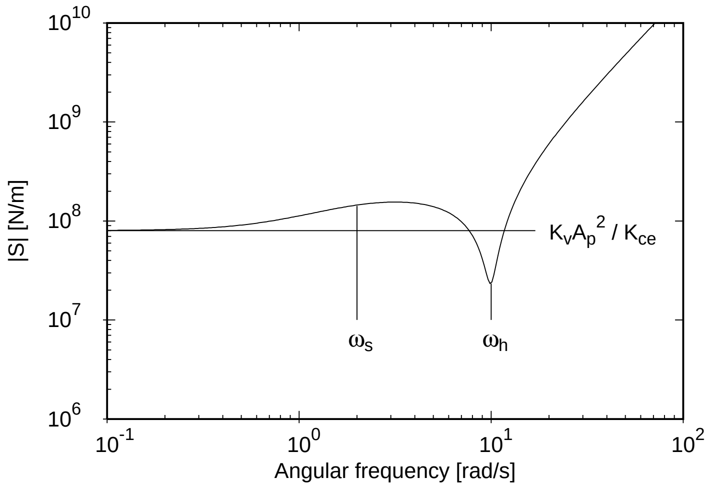
System with feedback
For a system with feedback with the feedback -1 and proportional gain of the control error is following stiffness obtained for a valve controlled cylinder($\Delta X_{p\,\mathrm{ref}}$ are set to 0).

$$S = \frac{\Delta F_L}{\Delta X_p} = \frac{1}{\dfrac{\Delta X_p}{\Delta F_L}} = -K_v \frac{\dfrac{1}{K_v\omega_h^2}s^3 + \dfrac{2\delta_h}{K_v\omega_h}s^2 + \dfrac{1}{K_v}s + 1}{\dfrac{K_{ce}}{A_p^2}\left(1 + \dfrac{s}{\omega_s}\right)}$$

where

$$\omega_s = \frac{4\beta_e K_{ce}}{V_t} = \{\text{if } B_p \text{ isneglected}\} = 2\delta_h\omega_h$$

The transfer function for the stiffness in the position servo is shown in figure 10.



Figur 10: The transfer function for the stiffness for the valve controlled cylinder with proportional feedback.

12.9 The servo's steady state error

The control error $e(t)$ in a servo system is defined as the difference between the output value and the input value when $\Delta\text{Disturbance signal} = 0$.
According to end value theorem, the steady state control error will be:

$$\varepsilon_0 = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\Delta E(s)$$

where the error gets the following expression if the feedback is -1:

$$\Delta E(s) = \Delta X_{\text{pref}} - \Delta X_p = \Delta X_{\text{pref}} \frac{1}{1+A_u}$$

The end value theorem is only usable on an asymptotic stable system, i.e. if the output signal has a finite limit value. For all systems can the transient be studied in the time domain (inverse transformation, see section 12.10).

The input signal is a step
If the input signal ΔX_{pref} respective $\Delta\Theta_{m\text{ref}}$ is a step with the amplitude A becomes the input signal A/s in frequency domain. The end value theorem becomes

$$\varepsilon_0 = \lim_{s\to 0} s\,\frac{A}{s}\,\frac{1}{1+A_u} = \frac{A}{1+\lim\limits_{s\to 0} A_u} \to \frac{A}{\infty} = 0$$

Practical, the steady state error does never become 0, because of the components which are included in the control loop do not have ideal characteristic.

The input signal is a ramp
If the input signal ΔX_{pref} respective $\Delta\Theta_{m\text{ref}}$ is a ramp $A \cdot t$ (i.e. the speed A is desired), becomes the input signal A/s^2. The steady state position error becomes

$$\varepsilon_0 = \lim_{s\to 0} s\,\frac{A}{s^2}\,\frac{1}{1+A_u} = \to \frac{A}{K_v}$$

12.10 Control technical resources

Linearize

Non-linear differential equations are linearized around a working point with the first term of the Taylor series according to

$$\Delta f\Big|_0 = \Delta f(x_{10}, x_{20}, \ldots, x_{n0}) = \sum_{j=1}^{n} \Delta x_j \frac{\partial f(x_1, x_2, \ldots, x_n)}{\partial x_j}\Big|_{(x_{10}, x_{20}, \ldots, x_{n0})}$$

where the working point is assumed to have stationary working conditions.

Laplace transformation

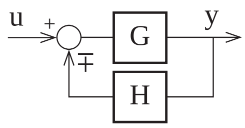
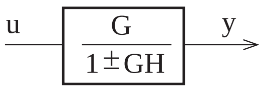
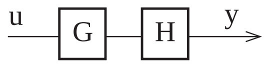
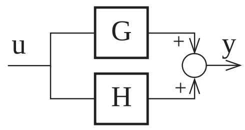
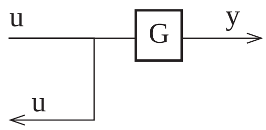
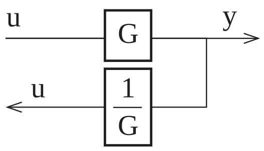
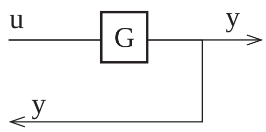
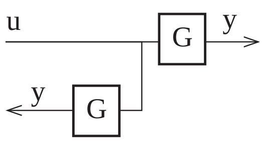
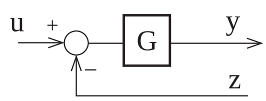
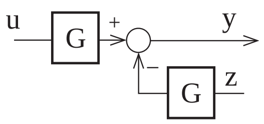
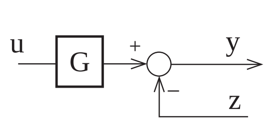
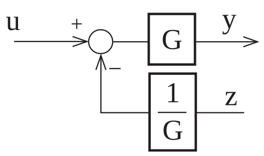
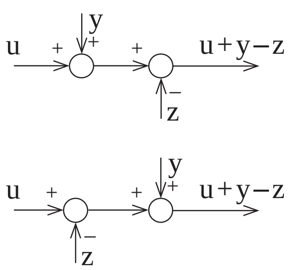
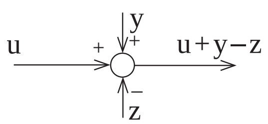
The following Laplace transformation table translates equations in the time domain to equations in frequency domain (rad/s) and vice versa.

Definition $\quad \mathcal{L}\{f(t)\} = F(s) = \int_0^\infty \mathrm{e}^{-st} f(t)dt$

Function in time domain		Transform to frequency domain
$f(t)$		$F(s)$
$af(t) + bg(t)$	a, b constants	$aF(s) + bG(s)$
$f(t-a)$	$a \geq 0$	$e^{-as}F(s)$
$f(at)$	$a > 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
$t^n f(t)$	$n = 0, 1, 2, \ldots$	$(-1)^n F^{(n)(s)}$
$\frac{f(t)}{t}$	$\lim_{t\to 0^+} \frac{f(t)}{t}$ exists	$\int_s^\infty F(u)du$
$\mathrm{e}^{-at}f(t)$	a constant	$F(s+a)$
$f(t-a)H(t-a)$	$a > 0, H(t) \begin{cases} 0 & \text{then } t < 0 \\ 1 & \text{otherwise} \end{cases}$	$\mathrm{e}^{-as}F(s)$
$f'(t)$		$sF(s) - f(0)$
$f^{(n)}(t)$	$n = 0, 1, 2, \ldots$	$s^n F(s) - \sum_{k=1}^{n} s^{n-k} f^{(k-1)}(0)$
$\int_0^t f(u)du$		$\frac{F(s)}{s}$
$f(t) * g(t) = \int_0^t f(u)g(t-u)du = \int_0^t g(u)f(t-u)du$		$F(s)G(s)$

Function in time domain		Transform to frequency domain	
t^n	$n = 0, 1, 2, \ldots$	$\frac{n!}{s^{n+1}}$	$\mathrm{Re}(s) > 0$
e^{at}	a constant	$\frac{1}{s-a}$	$\mathrm{Re}(s) > \mathrm{Re}(a)$
$t^n \mathrm{e}^{at}$	$n = 0, 1, 2, \ldots$	$\frac{n!}{(s-a)^{n+1}}$	$\mathrm{Re}(s) > \mathrm{Re}(a)$
$\cos at$		$\frac{s}{s^2+a^2}$	$\mathrm{Re}(s) > \lvert\mathrm{Im}(a)\rvert$
$\sin at$		$\frac{a}{s^2+a^2}$	$\mathrm{Re}(s) > \lvert\mathrm{Im}(a)\rvert$
$e^{at}\cos bt$		$\frac{s-a}{(s-a)^2+b^2}$	$\mathrm{Re}(s) > \lvert\mathrm{Im}(a)\rvert$
$e^{at}\sin bt$		$\frac{b}{(s-a)^2+b^2}$	$\mathrm{Re}(s) > \lvert\mathrm{Im}(a)\rvert$
$t\cos at$		$\frac{s^2-a^2}{(s^2+a^2)^2}$	$\mathrm{Re}(s) > \lvert\mathrm{Im}(a)\rvert$
$2\cos at - at\sin at$		$\frac{2s^3}{(s^2+a^2)^2}$	$\mathrm{Re}(s) > \lvert\mathrm{Im}(a)\rvert$
$\sin at + at\cos at$		$\frac{2as^2}{(s^2+a^2)^2}$	$\mathrm{Re}(s) > \lvert\mathrm{Im}(a)\rvert$
$\sin at - at\cos at$		$\frac{2a^3}{(s^2+a^2)^2}$	$\mathrm{Re}(s) > \lvert\mathrm{Im}(a)\rvert$

Block diagram reduction

Reduction	System	Equivalent system
1. Feedback		
2. Series connection		
3. Parallel connection		
4. Move branch point after a block		
5. Move branch point in front of a block		
6. Move summation point after a block		
7. Move summation point in front of a block		
8. Shift summation order		

Bode diagram

For the transfer functions

1. $$G(s) = \frac{K}{s^p}\frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\cdots\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\cdots\left(1+\frac{s}{p_n}\right)} \qquad \text{(real numbers)}$$

2. $$G(s) = \frac{1}{\frac{s^2}{\omega_0^2} + 2\delta_0\frac{s}{\omega_0} + 1} \qquad \delta_0 < 1 \text{ (complex numbers)}$$

the amplitude curve becomes

1. $$\begin{aligned}\log|G(i\omega)| = \; & \log K - p\log|\omega| + \log\left|1+\frac{i\omega}{z_1}\right| + \log\left|1+\frac{i\omega}{z_2}\right| + \cdots \\ & \cdots + \log\left|1+\frac{i\omega}{z_m}\right| - \log\left|1+\frac{i\omega}{p_1}\right| - \log\left|1+\frac{i\omega}{p_2}\right| - \cdots - \log\left|1+\frac{i\omega}{p_n}\right|\end{aligned}$$

2. $$|G(i\omega)| = \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_0^2}\right)^2 + \left(2\delta_0\frac{\omega}{\omega_0}\right)^2}}$$

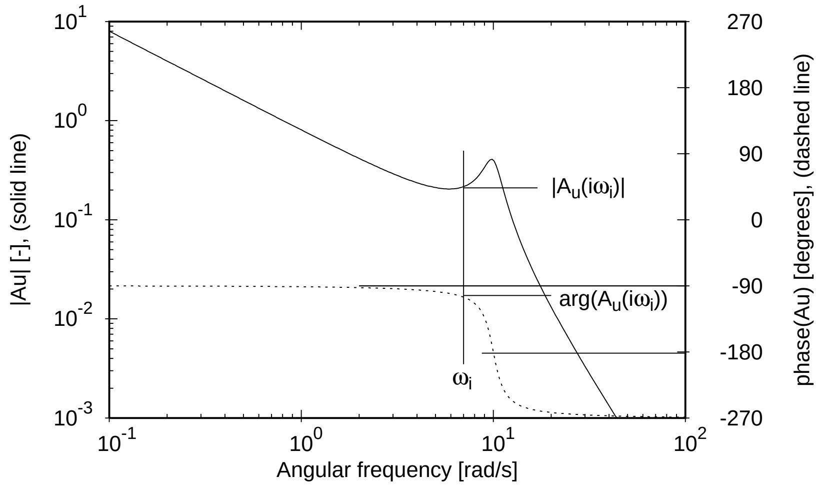
and the phase becomes

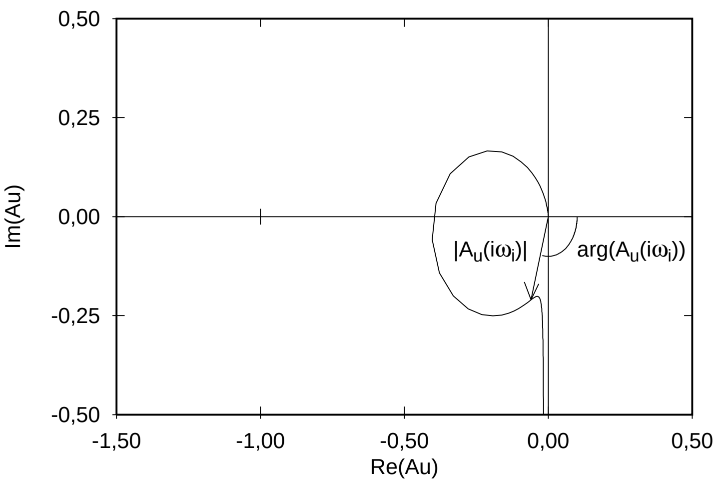
1. $$\begin{aligned}\arg G(i\omega) = \; & -p90^\circ + \arctan\left(\frac{\omega}{z_1}\right) + \arctan\left(\frac{\omega}{z_2}\right) + \cdots + \arctan\left(\frac{\omega}{z_m}\right) - \\ & \arctan\left(\frac{\omega}{p_1}\right) - \arctan\left(\frac{\omega}{p_2}\right) - \cdots - \arctan\left(\frac{\omega}{p_n}\right)\end{aligned}$$

2. $$\arg G(i\omega) = \begin{cases} -\arctan\left(\dfrac{2\delta_0\frac{\omega}{\omega_0}}{1-\frac{\omega^2}{\omega_0^2}}\right) & \text{då } 0 \le \omega \le \omega_0 \\ -90^\circ & \text{då } \omega = \omega_0 \\ -180^\circ + \arctan\left(\dfrac{2\delta_0\frac{\omega}{\omega_0}}{\frac{\omega^2}{\omega_0^2}-1}\right) & \text{då } \omega > \omega_0 \end{cases}$$

Nyquist diagram

If the transfer function is plotted direct in the complex domain, the Nyquist diagram is obtained which is more usable than the Bode diagram. From the Bode diagram can Nyquist diagram be constructed in following way:





Linear systems - linearized system
Under presumption that A_u does not have poles in the right side are following valid:

- For a feedback system shall be stable must not the open loop system's transfer function enclose $\mathrm{Re}(A_u) = -1$ in the Nyquist diagram.
- Mnemonic rule: Pull the "rope" downward. If $\mathrm{Re}(A_u) = -1$ follows - the system is instable!

Non-linear systems
With non-linear systems can a host of phenomenon occur due to occurrence of play, hysteresis in the system etc despite the system is seemingly stable. A analyse method for investigation of the stability is *the descriptive functions* where the open loop system's transfer function is divided in a linear part and a non-linear part according to

$$A_u = G_{\text{linear}} G_{\text{non-linear}}$$

which results in the stability condition

$$G_{\text{linear}} G_{\text{non-linear}} = -1$$

By plotting G_{linear} and $(-1/G_{\text{non-linear}})$ in the Nyquist diagram can the stability be investigated. The intersection point gives in many cases the frequency and the amplitude for the self oscillation. See other literature for determination of the non-linear transfer function.

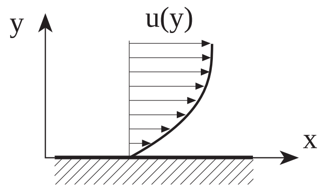
13 Hydraulic fluids

13.1 Nomenclature

V	:volume	[m^3]	x_0	:amount of air in the oil (gas volume/total volume at normal state, NTP)	[-]
V_1	:start volume (secant)	[m^3]	ν	:kinematic viscosity	[m^2/s]
V_2	:end volume (secant)	[m^3]	ρ	:density	[kg/m^3]
V_c	:volume (reservoir)	[m^3]	β_e	:effective bulk modulus	[Pa]
V_g	:volume (gas)	[m^3]	β_c	:bulk modulus (reservoir)	[Pa]
V_ℓ	:volume (fluid)	[m^3]	β_g	:bulk modulus (gas)	[Pa]
V_t	:total volume	[m^3]	β_ℓ	:bulk modulus (fluid)	[Pa]
p	:absolute pressure	[Pa]	β_t	:bulk modulus with no air in the oil (tangent)	[Pa]
p_0	:absolute pressure at NTP (= 0,1 MPa)	[Pa]	β_s	:bulk modulus with no air in the olja (secant)	[Pa]
p_1	:start pressure (secant)	[Pa]	β_{bt}	:bulk modulus with air in the oil (tangent)	[Pa]
p_2	:end pressure (secant)	[Pa]	β_{bs}	:bulk modulus with air in the oil (secant)	[Pa]
u	:flow velocity in x-led	[m/s]	τ	:skjuvspänning	[N/m^2]
n	:polytrophic exponent	[-]	η	:dynamic viscosity	[Ns/m^2]
y_s	:correction coefficient (secant)	[-]			
y_t	:correction coefficient (tangent)	[-]			

Definition, η
dynamic viscosity
for Newton fluid

$$\tau = \eta \frac{du}{dy}$$



Kinematic viscosity

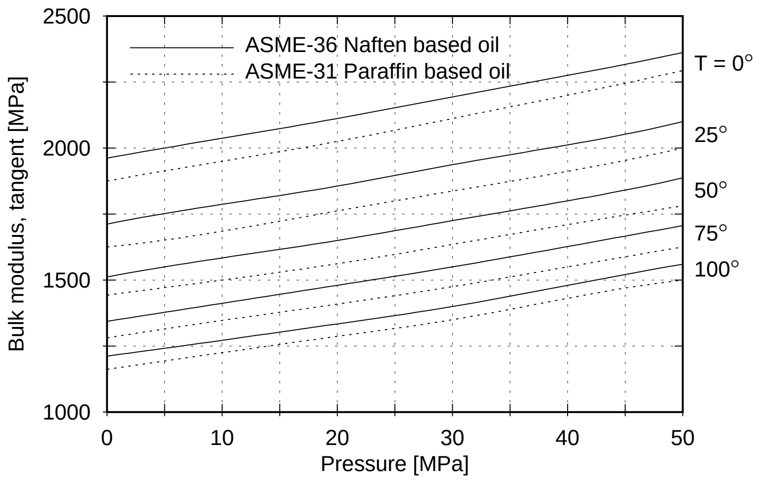
$$\nu = \frac{\eta}{\rho}$$

Bulk modulus for oil with no air

Tangent value, is used at small changes

$$\beta_t = -V \frac{dp}{dV}$$

β_t is shown in figure 11. Most of the normal hydraulic oils have bulk modulus between the Naften based oil and Paraffin based oil.

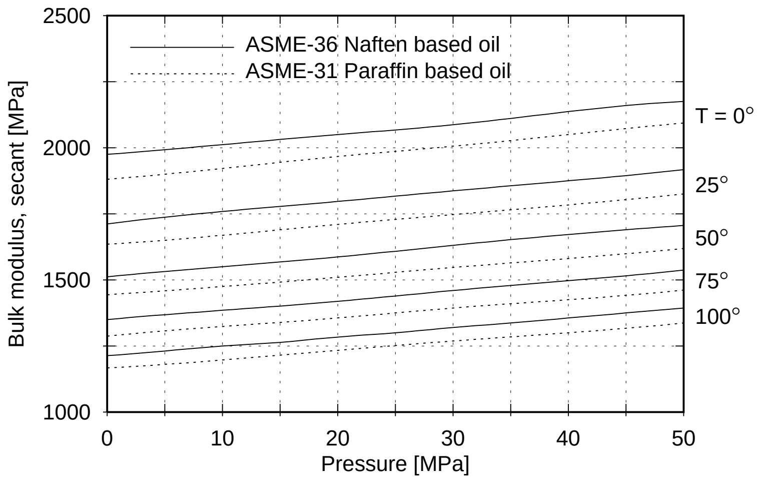


Figur 11: Tangent value of the bulk modulus for oil with no air.

Secant value, is used at big changes from normal pressure
$(p_1 = 0, V_1)$ till (p_2, V_2)

$$\beta_s = -V_1 \frac{p_2}{V_2 - V_1}$$

β_s is shown in figure 12. Most of the normal hydraulic oils have bulk modulus between the Naften base oil and Paraffin base oil.



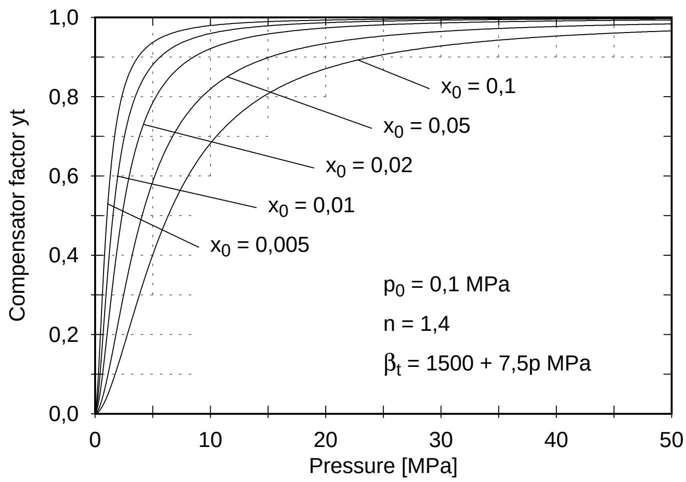
Figur 12: Secant value of bulk modulus for oil with no air.

Bulk modulus for oil with air

Tangent value
Simplified model and can be used when $x_0 \leq 0{,}1$.

$$\beta_{bt} = y_t \beta_t \qquad \text{där} \qquad y_t = \frac{1}{1 + \dfrac{x_0}{np} \dfrac{\beta_t}{(1 - x_0)} \left(\dfrac{p_0}{p}\right)^{\frac{1}{n}}}$$

y_t is shown in figure 13 for different amount of air in the oil, x_0, at the special case:
polytrophic exponent $n = 1{,}4$
$\beta_t = 1500 + 7{,}5\Delta p$ [MPa]

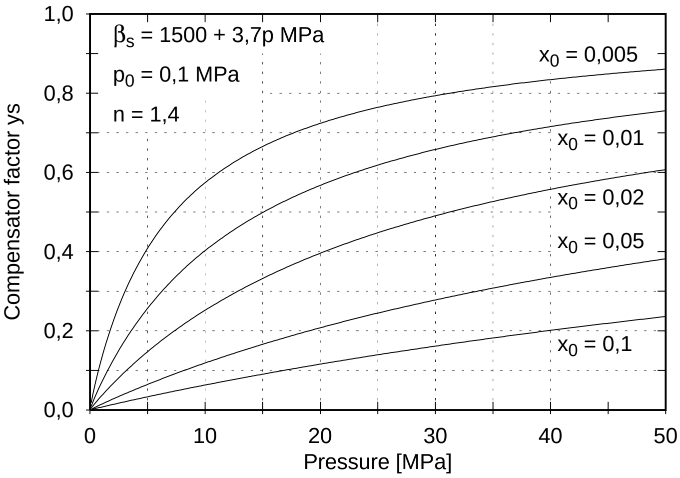


Figur 13: Correction for the air included in oil, tangent value.

Secant value
Simplified model and can be used when $x_0 \leq 0{,}1$.

$$\beta_{bs} = y_s \beta_s \qquad \text{där} \qquad y_s = \frac{1}{1 - x_0 + \dfrac{\beta_s x_0}{(p - p_0)} \left[1 - \left(\dfrac{p_0}{p}\right)^{\frac{1}{n}}\right]}$$

y_s is shown in figure 14 for different amount of air in the oil, x_0, at the special case:
polytropexponent $n = 1{,}4$
$\beta_s = 1500 + 3{,}7\Delta p$ [MPa]



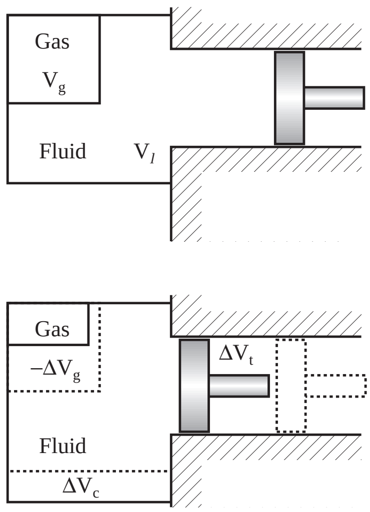
Figur 14: Correction for the air included in oil, secant value.

Effective bulk modulus, β_e

The effective bulk modulus, β_e, is defined as

$$\frac{1}{\beta_e} = \frac{\Delta V_t}{V_t \Delta p}$$

Total initial volume: $V_t = V_\ell + V_g$
At compression: $\Delta V_t = -\Delta V_\ell - \Delta V_g + \Delta V_c$



where ℓ, g och c refer to fluid, gas respective reservoir.

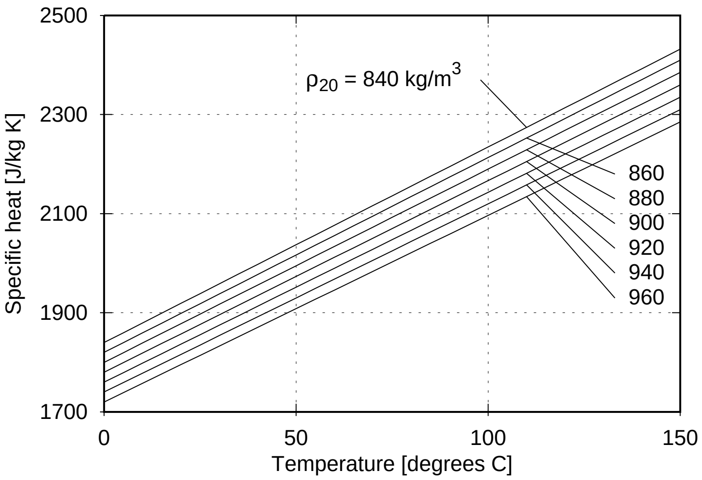
In general, the bulk modulus is calculated as (secant value)

$$\frac{1}{\beta_e} = \frac{V_g}{V_t}\left(\frac{1}{\beta_g}\right) + \frac{V_\ell}{V_t}\left(\frac{1}{\beta_\ell}\right) + \frac{1}{\beta_c}$$

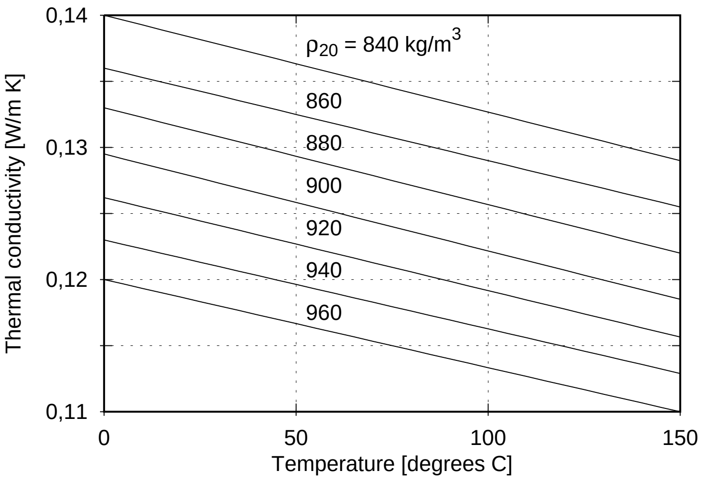
For oil with no air:

$$\frac{1}{\beta_e} = \frac{1}{\beta_\ell} + \frac{1}{\beta_c}$$

Specific heat capacity



Thermal conductivity ability



14 Pneumatic

14.1 Nomenclature

A_0	:min. cross-section area of the orifice	[m^2]	b	:critical pressure ratio	[-]
A_{12}	:effective entrance area	[m^2]	b_i	:b-value for component i	[-]
A_{23}	:effective exit area	[m^2]	b_s	:b-value for system	[-]
A_e	:effective orifice area ($C_d A_0$)	[m^2]	$\dot{m}$	:mass flow	[kg/s]
C_d	:flow coefficient	[-]	p_1	:upstream total absolute pressure (= static + dynamic pressure)	[Pa]
C_i	:C-value for component i	[-]			
C_s	:C-value for system	[-]	p_2	:downstream static absolute pressure	[Pa]
K	:constant	[$\sqrt{\text{kgK/J}}$]	p_3	:atmospheric pressure (0,1 MPa)	[Pa]
K_t	:temperature correction ($\sqrt{T_0/T_1}$)	[-]	p_v	:atmospheric pressure in volume	[Pa]
N	:parameter	[-]	q	:volume flow	[m^3/s]
R	:gas constant (287 for air)	[J/kg K]	t	:time	[s]
T_0	:reference temperature (NTP)	[K]	α	:parameter	[-]
T_1	:upstream total temperature	[K]	κ	:isentropic exponent	[-]
T_3	:downstream total temperature	[K]	ω	:parameter	[-]
T_v	:total temperature i volume	[K]	τ	:dimension free time ($= \frac{\sqrt{RT}At}{V}$)	[-]

14.2 Stream through nozzle

According to the thermodynamic, the mass flow $\dot{m}$ through a nozzle can be written as

$$\dot{m} = \frac{p_1 C_d A_0 K N}{\sqrt{T_1}} \qquad \text{where} \qquad K = \sqrt{\frac{\kappa}{R}\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}}$$

$$N = \begin{cases} 1 & \text{for } \frac{p_2}{p_1} \leq \left(\frac{p_2}{p_1}\right)^* \\ \sqrt{\dfrac{\left(\frac{p_2}{p_1}\right)^{\frac{2}{\kappa}} - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa+1}{\kappa}}}{\frac{\kappa-1}{2}\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}}} & \text{for } \frac{p_2}{p_1} > \left(\frac{p_2}{p_1}\right)^* \end{cases}$$

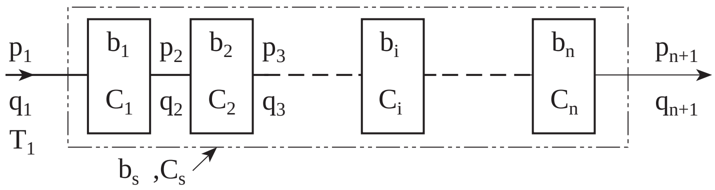
critical pressure ratio:
$$b = \left(\frac{p_2}{p_1}\right)^* = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}$$

With b- and C-value hold for volume flow q following expression at NTP

$$q = p_1 K_t C \omega$$

$$\omega = \begin{cases} 1 & \text{for } \frac{p_2}{p_1} \leq b \\ \sqrt{1 - \left(\dfrac{\frac{p_2}{p_1} - b}{1-b}\right)^2} & \text{for } \frac{p_2}{p_1} > b \end{cases}$$

14.3 Series connection of pneumatic components



Figur 15: Series connection of pneumatic components with b- and C-values

On condition that

- every component can be described with $q = p_1 K_t C \omega$
- b- and C-value is known for every component
- absolute static pressure after one component is equal to the absolute total pressure before the next component
- the entrance temperature holds for all system
- every component have the same mass flow ($q_1 = q_2 = \cdots = q_n$)

$$q_s = p_1 K_t C_s \omega \qquad \text{med} \quad K_t = \sqrt{\frac{T_0}{T_1}}$$

$$\omega = \begin{cases} 1 & \text{för } \dfrac{p_{n+1}}{p_1} \le b_s \\ \sqrt{1 - \left(\dfrac{\frac{p_{n+1}}{p_1} - b_s}{1 - b_s} \right)^2} & \text{för } \dfrac{p_{n+1}}{p_1} > b_s \end{cases}$$

Case A If b- and C-value is about the same

$$\frac{1}{C_s^3} = \sum_{i=1}^{n} \frac{1}{C_i^3} \qquad b_s = 1 - C_s^2 \sum_{i=1}^{n} \frac{1 - b_i}{C_i^2}$$

Case B In cases of, the components' characteristics show large divergence, the components sequence have to be considered (gradual reduction)

Calculation sequence:

$$\alpha_{12} = \frac{C_1}{C_2 b_1}$$

$\alpha_{12} < 1$ Critical pressure drop first over component 1, and because of the decrease of pressure ratio as well in component 2.

$\alpha_{12} = 1$ Both components is critical at the same time

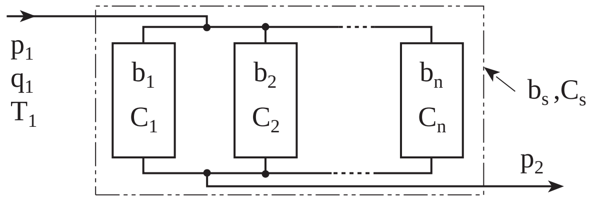
$\alpha_{12} > 1$ Critical pressure drop only in component 2

$$C_{12} = \begin{cases} C_1 & \text{for } \alpha_{12} \le 1 \\ C_2 \alpha_{12} \dfrac{\alpha_{12} b_1 + (1 - b_1)\sqrt{\alpha_{12}^2 + \left(\frac{1 - b_1}{b_1}\right)^2 - 1}}{\alpha_{12}^2 + \left(\frac{1 - b_1}{b_1}\right)^2} & \text{for } \alpha_{12} > 1 \end{cases}$$

$$b_{12} = 1 - C_{12}^2 \left(\frac{1-b_1}{C_1^2} + \frac{1-b_2}{C_2^2} \right)$$

$$\left\{ \alpha_{13} = \frac{C_{12}}{C_3 b_{12}} \quad \text{osv} \ \ldots \right\}$$

14.4 Parallel connected pneumatic components



Figur 16: Parallel connected pneumatic components with b- and C-values

$$q_s = p_1 K_t C_s \omega \qquad \text{med} \quad K_t = \sqrt{\frac{T_0}{T_1}}$$

$$\omega = \begin{cases} 1 & \text{forr } \dfrac{p_2}{p_1} \leq b_s \\ \sqrt{1 - \left(\dfrac{\frac{p_2}{p_1} - b_s}{1 - b_s} \right)^2} & \text{for } \dfrac{p_2}{p_1} > b_s \end{cases}$$

For systems with the same further line (see figure 16) relates to

$$C_s = \sum_{i=1}^{n} C_i \qquad \frac{C_s}{\sqrt{1-b_s}} = \sum_{i=1}^{n} \frac{C_i}{\sqrt{1-b_i}}$$

14.5 Parallel- and series connected pneumatic components

For system with separate further lines are dealt as series links, after which the total flow is obtained as the sum of all the partial flows in every series links.

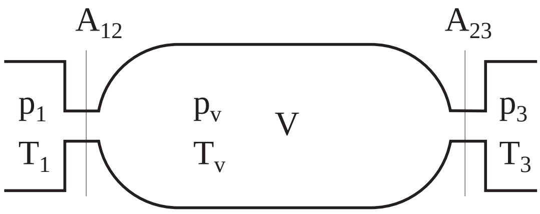
14.6 Filling and emptying of volumes

Assumptions

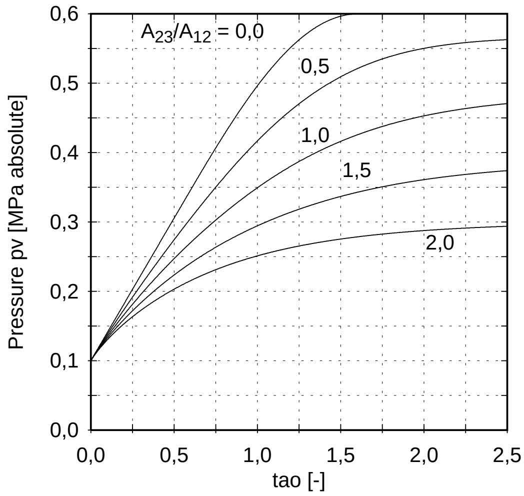
- isotherm process ($T = T_1 = T_v = T_3$)
- stationary conditions $\left(\frac{dT}{dt} = 0,\ \frac{dA_e}{dt} = 0,\ \frac{dV}{dt} = 0,\ \frac{dp_1}{dt} = 0,\ \frac{dp_3}{dt} = 0 \right)$
- p_3 = atmospheric pressure
- p_1 and p_v are absolute pressure
- A_{12} and A_{23} are effective areas

Charging a volume

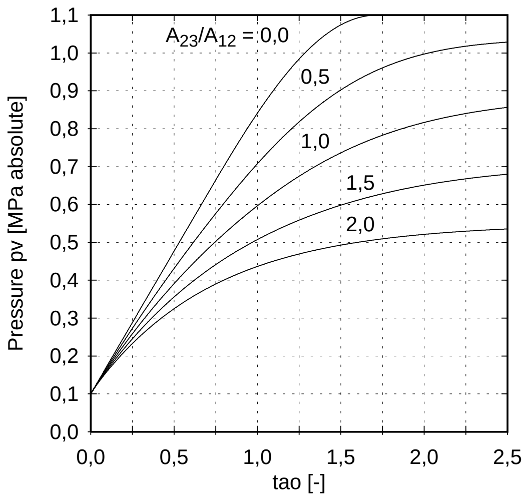
The diagrams below shows p_v as a function of dimensionless time $\tau = \dfrac{\sqrt{RT}A_{12}t}{V}$ with the area relation of $\dfrac{A_{23}}{A_{12}}$ as parameter.



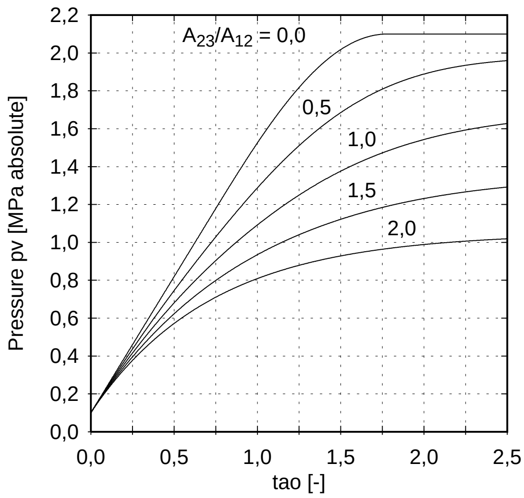
(a) The downstream pressure $p_3 = 0{,}1$ MPa absolute.



(b) The upstream pressure $p_1 = 0{,}6$ MPa (absolute)



(c) The upstream pressure $p_1 = 1{,}1$ MPa (absolute)

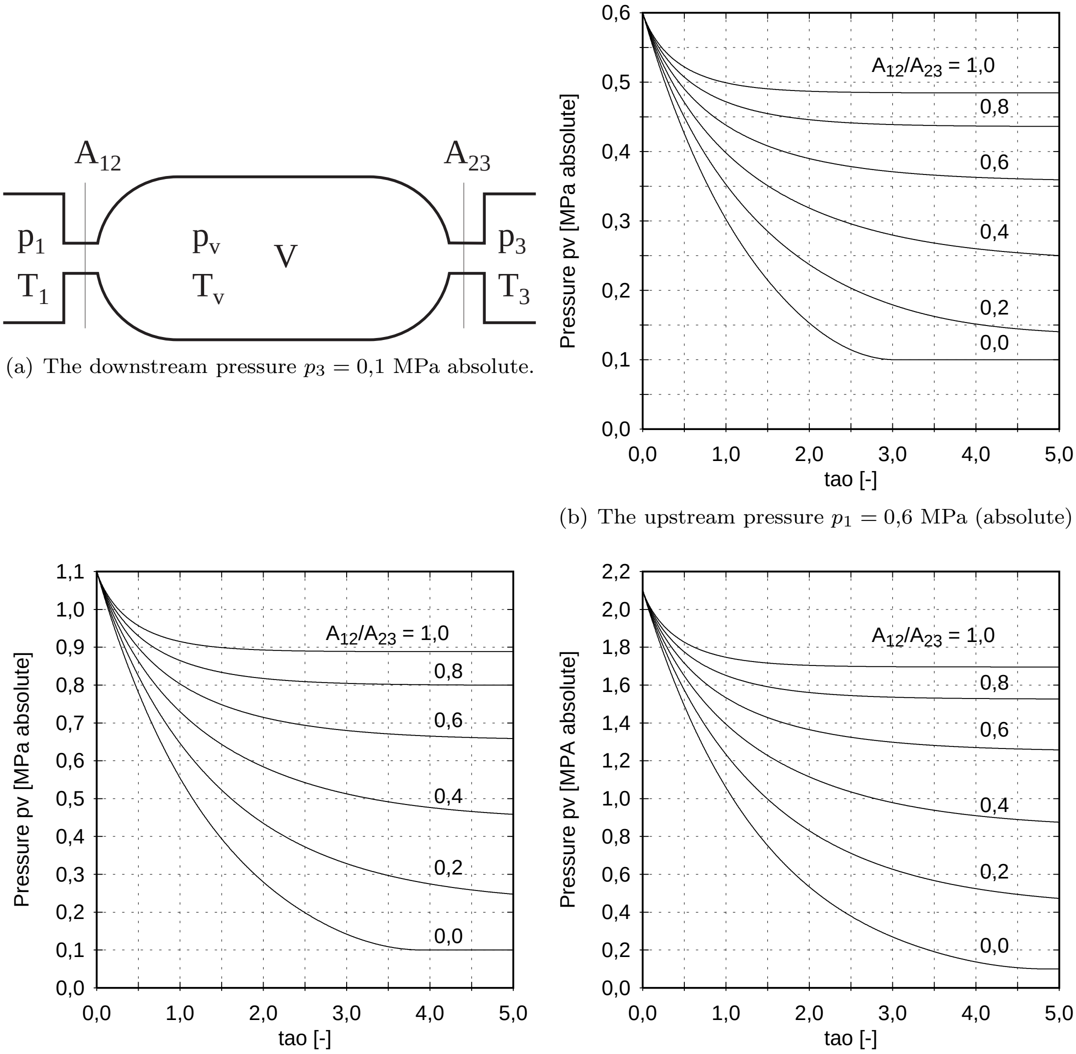


(d) The upstream pressure $p_1 = 2{,}1$ MPa (absolute)

Figur 17: Charging of the volume V.

Discharging a volume

The diagrams below shows p_v as a function of dimensionless time $\tau = \dfrac{\sqrt{RT}A_{23}t}{V}$ with the area relation of $\dfrac{A_{12}}{A_{23}}$ as parameter.



(a) The downstream pressure $p_3 = 0{,}1$ MPa absolute.

(b) The upstream pressure $p_1 = 0{,}6$ MPa (absolute)

(c) The upstream pressure $p_1 = 1{,}1$ MPa (absolute)

(d) The upstream pressure $p_1 = 2{,}1$ MPa (absolute)

Figur 18: Discharging the volume V.

For both charging and discharging a volume

If p_1 is between the assumed levels in the diagrams can the time be calculated with linear interpolation of the two diagram according to the equation below ((p_1 in MPa)).

$$t = \begin{cases} t_5 + \dfrac{p_1 - 0{,}6}{0{,}5}(t_{10} - t_5) & \text{for } 0{,}6 \le p_1 \le 1{,}1 \text{ MPa} \\ t_{10} + (p_1 - 1{,}1)(t_{20} - t_{10}) & \text{for } 1{,}1 \le p_1 \le 2{,}1 \text{ MPa} \end{cases}$$

Appendix A

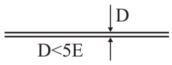
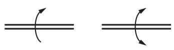
Symbols for hydraulic diagrams

Correspond to the international standard CETOP RP3 and the Swedish SMS 712. It is specified when the two standards differ.

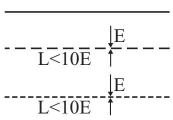
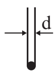
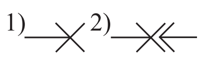
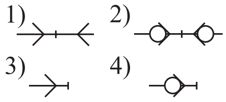
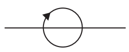
General symbols

	Flow direction	1)Hydraulic 2)Pneumatic
	Variability	
	Joined components	Components belonging to one assembly or functional group

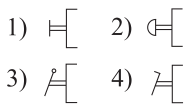
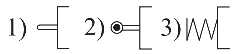
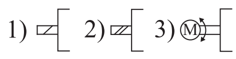
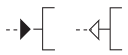
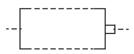
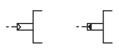
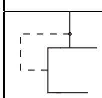
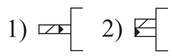
Mechanical elements

	Shaft, lever arm, bar, rod piston	Rod piston can be drawn with a single line
	Rotational shaft	One respective two rotational directions
	Spring	

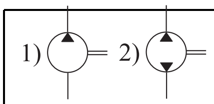
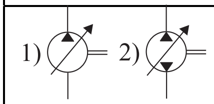
Pipes and connections

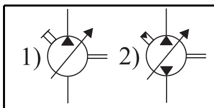
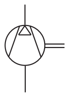
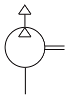
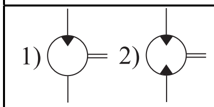
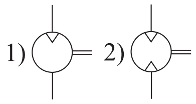
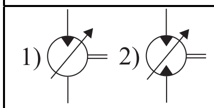
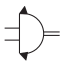
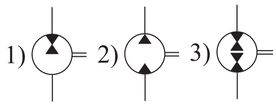
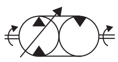
	Line	Solid line = main pipe Dotted line with L>10E = control pipe Dotted line with l<5E = drain pipe (E = line width)
	Flexible pipe, hose	The symbol is used mainly for movable parts
	Pipe connection	d = 5E (E = line width)
	Air-drain and air outlet	1) Air-drain for hydraulic pipe 2) Air outlet without possibility of connection 3) Air outlet with possibility of connection
	Connection	1) Plugged connection 2) Connection with connected pipe. (Normally is the connection plugged.)
	Quick connect fittings	Enable connection of pipes without tools. 1) Quick connect fitting without valve (interconnected) 2) Quick connect fitting with valve (interconnected) 3) Half of a quick connect fitting without valve 4) Half of a quick connect fitting with valve
	Rotatable connection	Can rotate during operation, with one line

Control systems

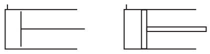
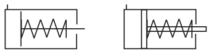
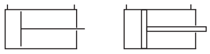
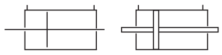
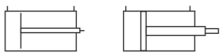
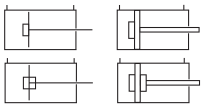
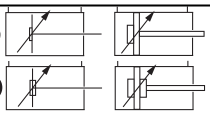
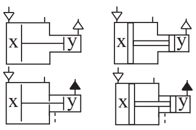
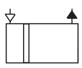
	Manual controls	1) General symbol 2) Control with push button 3) Control with lever arm 4) Control with pedal
	Mechanical controls	1) Plunge 2) Roll 3) Spring
	Electric controls	1) Electromagnetic with one winding 2) Electromagnetic with two winding (active in two directions) 3) Control with electric motor
	Pressure control	Pressure control through pressure rise respective pressure reduction
	Pressure control	Direct control through differential pressure
	Indirect pressure control	Simplified symbol for pre-controlled valve. Control at pressure rise respective pressure reduction.
	Internal control line	Control line is inside the valve.
	Combined controls	1) Control with electromagnetic controlled pre-control valve. 2) Control with electromagnetic or pre-control valve.

Pumps and Motors

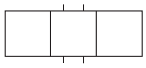
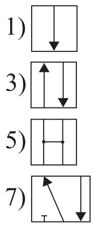
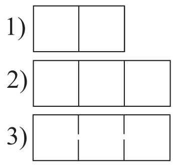
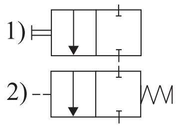
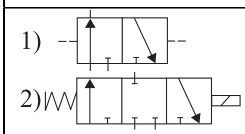
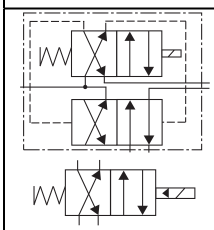
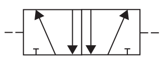
	Pump with constant displacement	1) One flow direction 2) Two flow directions
	Pump with variable displacement	1) One flow direction 2) Two flow directions

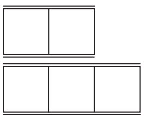
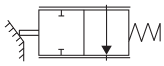
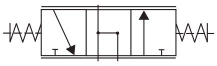
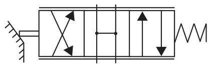
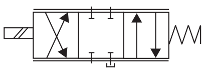
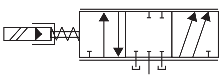
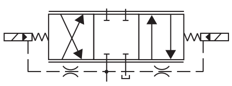
	Example of controls for variable pump	1) Manual control 2) Pressure control via control valve
	Compressor with one flow direction	The two distorted lines is not included in SMS
	Vacuum pump	
	Motor with constant displacement	1) One flow direction 2) Two flow directions
	Pneumatic motor with constant displacement	1) One flow direction 2) Two flow directions
	Motor with variable displacement	1) One flow direction 2) Two flow directions
	Motor with limited angle of twist	
	Pump/motor with constant displacement	Component working as pump/motor at: 1) Shifted flow direction, maintained pressure side. 2) Maintained flow direction, shifted pressure side. 3) Shifted flow direction and pressure side.
	Hydrostatic gear	Pump and motor together as one unit without external pipe system.

Cylinders

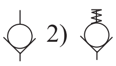
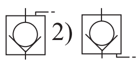
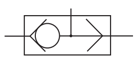
	Single-acting cylinder	The fluid pressure exercises a force in one direction only.
	Single-acting cylinder	The fluid pressure exercises a force in one direction only and the return stroke by return spring.
	Double-acting cylinder	The fluid operates alternatively in both directions.
	Double-acting symmetric cylinder	
	Differential cylinder	Double-acting cylinder where the area different between the both sides are essential for the function.
1) 2) 	Cylinder with cushion	1) Single acting cushion. 2) Double acting cushion.
1) 2) 	Cylinder with variable cushion	1) Single acting cushion. 2) Double acting cushion.
1) 2) 	Pressure intensifiers	Unit converting a pressure X into a higher pressure Y. 1) For one medium (air). 2) For two mediums (air and oil).
	Air-oil actuators	Unit converting a pneumatic pressure into an equal hydraulic pressure.

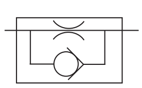
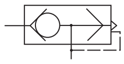
Directional control valves

	Opening and closing of one or more flow paths. Symbols with several squares. The external flow lines are normally situated at the square which indicates the neutral or normal position. Other positions can be shown by displacement of the squares until the external flow lines are situated at the corresponding square.	
	Example of squares with different flow paths.	1) Two pipe connections and free throughflow. 2) Two connections that are closed. 3) 4) Four connections and free throughflow. 5) Four connections where all are tied to each other. 6) Four connections were two are closed and two are tied together. 7) Five connections were one is closed.
	Non-throttling directional control valve	Several service positions each shown by a square; 1) Valve with two distinct positions. 2) Valve with three distinct positions. 3) Valve with two distinct positions (outer squares), but between the distinct positions the valve passes a central position with essential function.
	2/2 directional control valve	First number in the description denotes the number of ports, the second the number of positions. Pilot ports are not included. Control e.g. 1) manual 2)by pressure against return spring (unloading valve).
	3/2 directional control valve	1) Control by pressure from both ends. 2) Control by solenoid against return spring, with transient intermediate position.
	Electro hydraulic servo valve with pilot and mechanical feedback. 4/2 directional control valve	Combined with solenoid operated pilot valve with return spring. Representation 1) detailed 2) simplified.
	5/2 directional valve	Control by pressure in both directions.

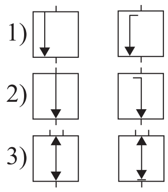
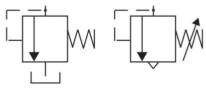
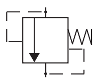
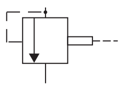
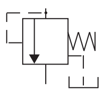
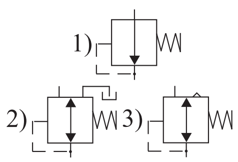
1) 2) 	Throttling directional control valve	Two end positions and intermediate throttling positions. 1) Shows only the end positions. 2) Shows the end positions and the centre (neutral) position. All valve symbols have parallel lines outside the envelope.
	One throttling orifice (2 ports)	Tracer valve; plunger operated against return spring.
	Two throttling orifices (3 ports)	Pressure controlled against return spring in two directions.
	Four throttling orifices (4 ports)	Tracer valve; plunger operated against return spring.
	Single stage electro-hydraulic servo valve	Amplification of infinitely variable electrical input signals transformed onto hydraulic output; without pilot operation.
	Two stage electro-hydraulic servo valve with mechanical feedback	
	Two stage electro-hydraulic servo valve with hydraulic feedback	

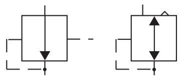
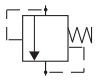
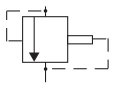
Check valves or non-return valves

1) 2) 	Check valve	1) without, 2) with back pressure. Opening if inlet pressure is 1) higher than the outlet pressure, 2) higher than the outlet pressure plus spring pressure.
1) 2) 	Pilot controlled check valve	Pilot controlled 1) Opening can be prevented 2) Closing can be prevented
	Shuttle valve	The inlet under pressure is automatically connected to the common outlet and the other inlet is closed.

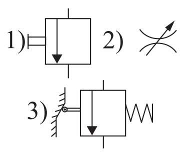
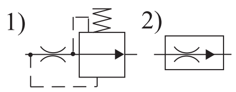
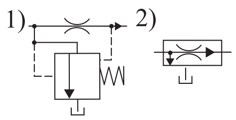
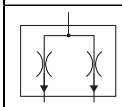
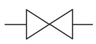
	One way restrictor	Valve which allows free flow in one direction and restricted flow in the other direction.
	Quick exhaust valve	When pressure falls at the inlet connection, the outlet is automatically opened to exhaust.

Pressure control valves

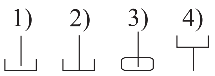
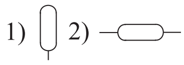
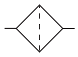
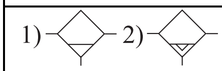
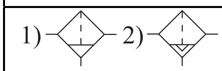
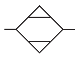
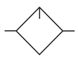
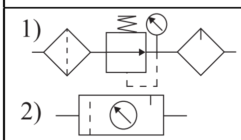
	Pressure control valves	Automatic control of pressure; 1) One throttling orifice, normally closed. 2) One throttling orifice, normally open. 3) Two throttling orifices. Arrows with or without tails. Pictures in the right column are not according to SMS standard.
	Pressure relief valve	Inlet pressure is controlled by opening the exhaust port to the reservoir or the atmosphere against an opposing force. The left valve has fixed preloaded spring force, the right has variable.
	Pilot controlled pressure relief valve	Inlet pressure is controlled by spring force or by a value determined by the pressure in a outer pilot port. In the left figure pilot pressure acts against the spring force. In the right figure it acts together with the spring force.
	Proportioning pressure relief valve	Inlet pressure is limited to a value proportional to the pilot pressure.
	Sequence valve	When the inlet pressure exceeds the opposing force of the spring, the valve opens permitting flow through the outlet port.
	Pressure regulator	With varying inlet pressure the outlet pressure remains substantially constant. Inlet pressure must however remain higher than the selected outlet pressure. 1) without, 2) with unloading device.

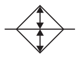
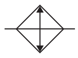
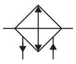
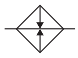
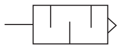
	Pilot controlled pressure regulator	Outlet port pressure proportional to pilot pressure.
	Differential pressure regulator	The outlet pressure is reduced by a fixed amount with regard to the inlet pressure.
	Proportioning pressure regulator	The outlet pressure is reduced by a fixed ratio with regard to the inlet pressure.

Flow control valves

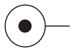
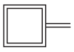
	Variable flow control valve	Manual controlled throttle valve. 1) Detailed 2) Simplified Mechanical controlled throttle valve. 3) Mechanical controlled against spring (braking valve).
	Series flow control valve	Regulator with fixed setting, without exceeding oil bleed off. 1) Detailed 2) Simplified
	By-pass flow control valve	Regulator with fixed setting, with exceeding oil bleed off. 1) Detailed 2) Simplified
	Flow divider	The flow is divided into two fixed flows substantially independent of pressure variations.
	Shut-off valve	Simplified symbol.

Components for cooling, filtering, energy storage etc.

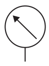
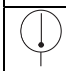
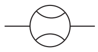
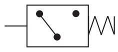
	Orifices	Acc. CETOP: 1) Viscosity dependent (pipe orifice) 2) 1) Viscosity dependent (sharp edged) Acc. SMS: 1) General symbol 3) Viscosity dependent (pipe orifice)
	Reservoirs	1),2), and 4) Reservoirs with atmospheric pressure. 3) Pressured reservoir. 1),3) The flow line flow into the reservoir above the the fluid level. 2) The flow line flow into the reservoir below the the fluid level. 4) The flow line flow connected under the reservoir.
	Accumulators	1) Hydraulic, the fluid is subjected to pressure from a spring, weight or gas (air, nitrogen, etc) 2) pneumatic (receiver)
	Filter; strainer	
	Water trap	1) Manual control of draining, 2) automatic draining.
	Filter with water trap	The apparatus is a combination of filter and water trap..
	Desiccator	Air drying by chemicals.
	Lubricator	For lubrication of apparatus small quantities of oil are added to the air which is flowing through the lubricator.
	Maintenance unit	Apparatus comprising filter, pressure regulator and lubricator assembled as a unit, 1) detailed 2) simplified.

	Temperature controller	The fluid temperature is controlled between two predetermined values. The arrows indicate both heat introduction and dissipation.
1)  2) 	Cooler	1) Without 2) with indication of the flow lines of the coolant. The arrows in the square indicate the heat dissipation.
	Heater	The arrows indicate introduction of heat.
	Silencer	

Energy sources

	Pressure source	
	Electric motor	
1)  2) 	Combustion engine	1) According to CETOP 2) According to SMS

Measurement equipments

	Manometer (Pressure transducer)	
	Thermometer	The symbol can be placed arbitrarily.
	Flow rate meter	Measure flow (volume per time unit)
	Integrated flow meter	Measure the total volume which is passed.
	Pressure switch	The symbol shows a switching contact.