DECISION
MODELING
USING
EXCEL

UPDATED FOR MICROSOFT OFFICE XP

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Detailed Contents

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Introduction to Decision Modeling

1.1 MODELS TO AID DECISION MAKING

Decision: irrevocable allocation of resources

Model: abstract representation of reality

What makes decision difficult?

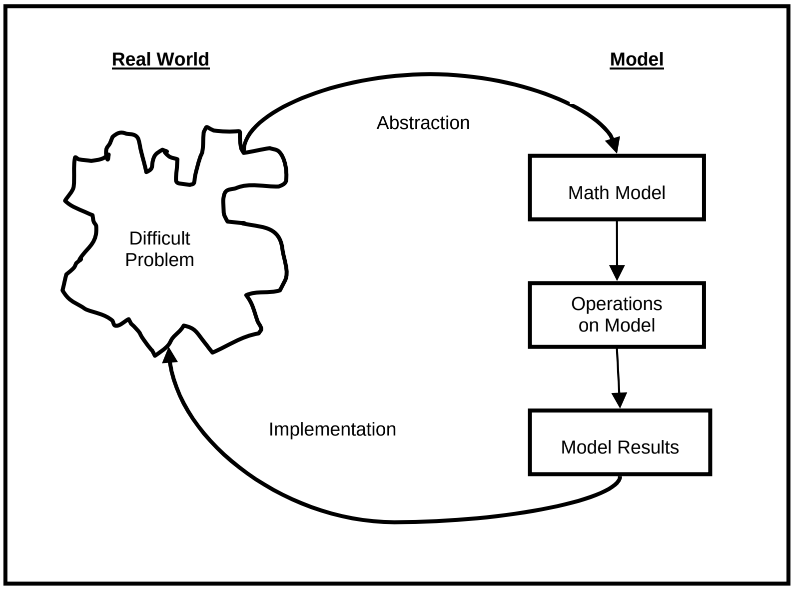
- Complexity
  - many factors to consider; relationships among factors
- Uncertainty
- Conflicting Objectives

How does modeling help?

Complexity	Model; consider each factor separately; consider relationships explicitly; avoid being overwhelmed
Uncertainty	Sensitivity Analysis and Probability
Conflicting Objectives	consider each objective; consider tradeoffs explicitly

Goals of modeling: recommended solution, insight, clarity of action

Figure 1.1 Overall Model-Building Flowchart



Components of a Decision Model

Controllable input variables

"What you can do," decision variables, alternatives

Uncontrollable input variables

"What you know and don't know," uncertainties, constraints

Relationships

how inputs are related to output, usually with intermediate variables, structure

Intermediate variables

useful for linking inputs to output

Output variable

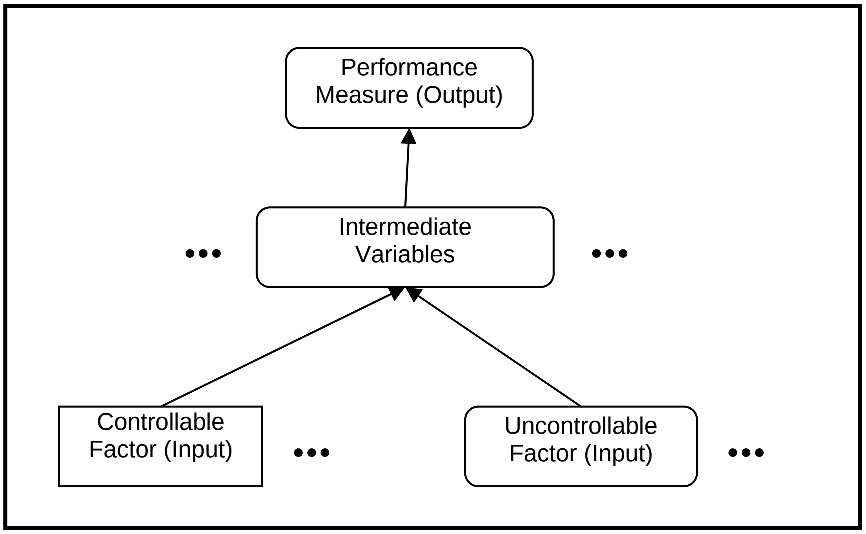
"What you want," performance measure, overall satisfaction

Influence chart

Rectangle for controllable inputs

Rounded rectangle or oval for other variables

Figure 1.2 Generic Influence Chart



1.2 BASIC WHAT-IF MODEL

Influence Diagram Representation

Figure 1.3 Typical Influence Diagram

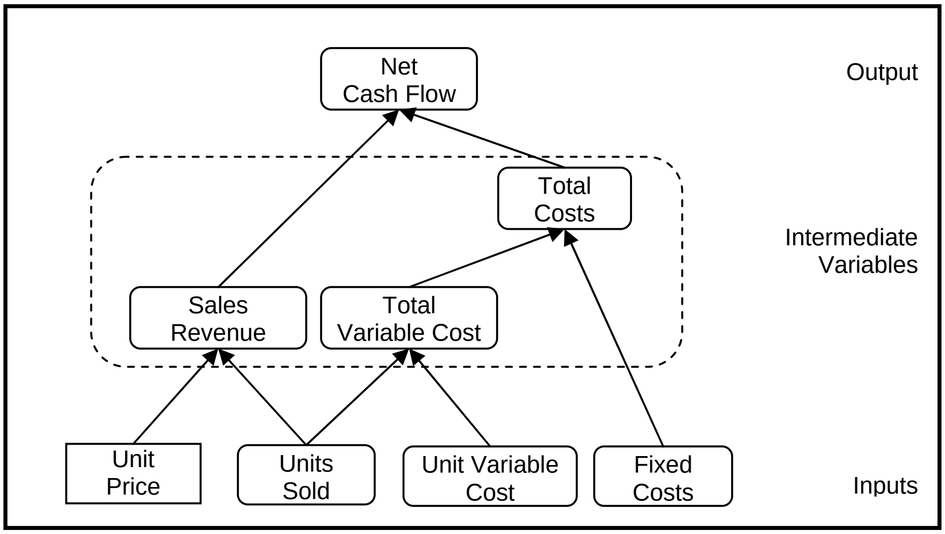
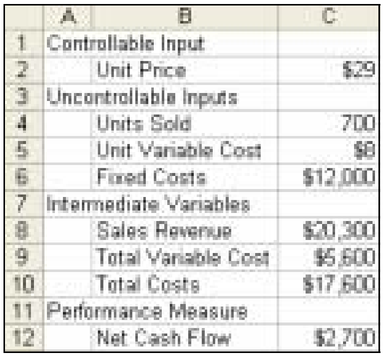


Figure 1.4 Typical Spreadsheet Model

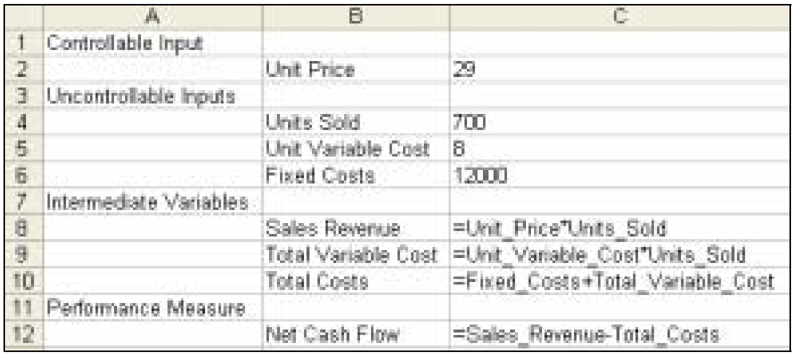


	A	B	C
1	Controllable Input		
2		Unit Price	$29
3	Uncontrollable Inputs		
4		Units Sold	700
5		Unit Variable Cost	$8
6		Fixed Costs	$12,000
7	Intermediate Variables		
8		Sales Revenue	$20,300
9		Total Variable Cost	$5,600
10		Total Costs	$17,600
11	Performance Measure		
12		Net Cash Flow	$2,700

Figure 1.5 Formulas for Typical Spreadsheet Model

	A	B	C
1	Controllable Input		
2		Unit Price	29
3	Uncontrollable Inputs		
4		Units Sold	700
5		Unit Variable Cost	8
6		Fixed Costs	12000
7	Intermediate Variables		
8		Sales Revenue	=C2*C4
9		Total Variable Cost	=C5*C4
10		Total Costs	=C6+C9
11	Performance Measure		
12		Net Cash Flow	=C8-C10

Figure 1.6 Defined Names for Typical Spreadsheet Model



	A	B	C
1	Controllable Input		
2		Unit Price	29
3	Uncontrollable Inputs		
4		Units Sold	700
5		Unit Variable Cost	8
6		Fixed Costs	12000
7	Intermediate Variables		
8		Sales Revenue	=Unit_Price*Units_Sold
9		Total Variable Cost	=Unit_Variable_Cost*Units_Sold
10		Total Costs	=Fixed_Costs+Total_Variable_Cost
11	Performance Measure		
12		Net Cash Flow	=Sales_Revenue-Total_Costs

Decision Tree Representation

Figure 1.7 Decision Fan and Event Fan

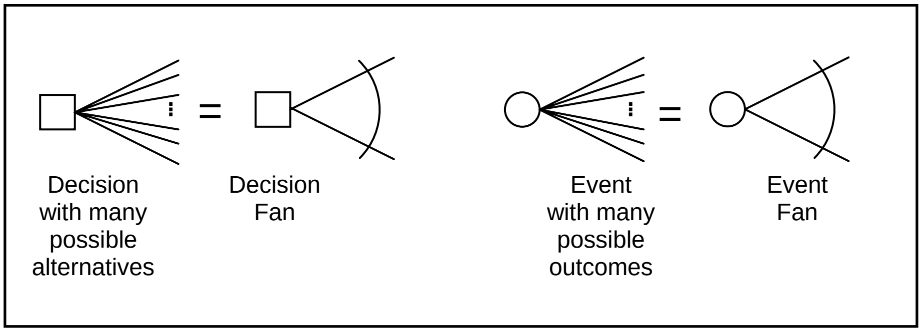
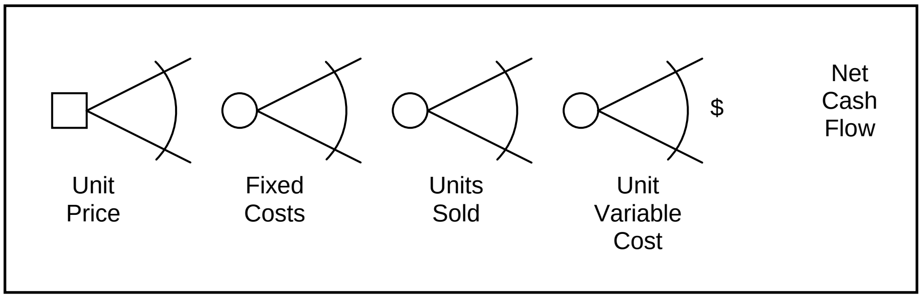


Figure 1.8 Conceptual Decision Tree



Consequence Table Representation

Figure 1.9 Professor's Summer Decision

	Conflicting Objectives			
Alternatives	Cash Flow	Hassle-Free	Happy Deans	Professional Fame
Develop Software	$2700	Yes	Maybe	Maybe
Teach MBAs	$4300	No	Yes	No
Vacation	$0	Yes	No	No

Sensitivity Analysis Using SensIt

2

SensIt is a sensitivity analysis add-in for Microsoft Excel 97 (and later versions of Excel) for Windows and Macintosh. It was written by Mike Middleton of the University of San Francisco and Jim Smith of Duke University.

2.1 HOW TO INSTALL SENSIT

Here are three ways to install SensIt:

(1) Start Excel, and use Excel's File | Open command to open the SensIt.xla file from floppy or hard drive.

(2) Copy the SensIt.xla file to the Excel | Library subdirectory of your hard drive. Start Excel, and use Excel's Tools | Add-Ins command to load and unload SensIt as needed.

(3) Copy the SensIt.xla file to the Excel | Startup subdirectory of your hard drive, in which case the file will be opened every time you start Excel.

All of SensIt's functionality, including its built-in help, is a part of the SensIt.xla file. There is no separate setup file or help file.

2.2 HOW TO UNINSTALL OR DELETE SENSIT

(A) First, use your file manager to locate SensIt.xla, and delete the file from your hard drive.

(B1) If SensIt is listed under Excel's add-in manager and the box is checked, when you start Excel you'll see "Cannot find ..." Click OK. Choose Tools | Add-Ins, uncheck the box for SensIt; you'll see "Cannot find ... Delete from list?" Click Yes.

(B2) If SensIt is listed under Excel's add-in manager and the box is not checked, start Excel and choose Tools | Add-Ins. Check the box for SensIt; you'll see "Cannot find ... Delete from list?" Click Yes.

2.3 SENSIT OVERVIEW

To run SensIt, start Excel and open the SensIt.xla file. Alternatively, install SensIt using one of the methods described above. SensIt adds a Sensitivity Analysis command to the Tools menu. The Sensitivity Analysis command has four subcommands: Plot, Spider, Tornado, and Help.

Before using the SensIt options, you must have a spreadsheet model with one or more inputs and an output. All three SensIt options make it easy for you to see how sensitive the output is to changes in the inputs.

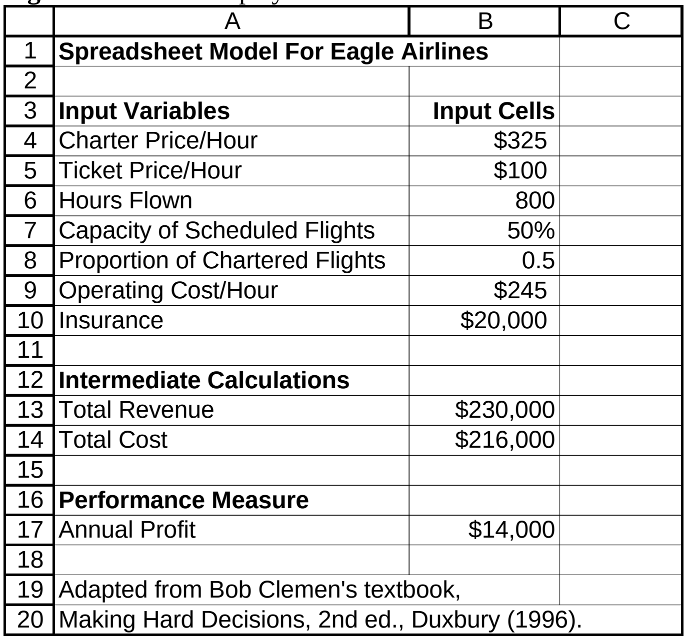
Use SensIt’s Plot option to see how your model’s output depends on changes in a single input variable.

Use SensIt’s Spider option to see how your model’s output depends on the same percentage changes for each of the model’s input variables.

Use SensIt’s Tornado option to see how your model’s output depends on ranges you specify for each of the model’s input variables.

2.4 EXAMPLE PROBLEM

Figure 2.1 Model Display



	A	B	C
1	**Spreadsheet Model For Eagle Airlines**		
2			
3	**Input Variables**	**Input Cells**	
4	Charter Price/Hour	$325	
5	Ticket Price/Hour	$100	
6	Hours Flown	800	
7	Capacity of Scheduled Flights	50%	
8	Proportion of Chartered Flights	0.5	
9	Operating Cost/Hour	$245	
10	Insurance	$20,000	
11			
12	**Intermediate Calculations**		
13	Total Revenue	$230,000	
14	Total Cost	$216,000	
15			
16	**Performance Measure**		
17	Annual Profit	$14,000	
18			
19	Adapted from Bob Clemen's textbook,		
20	Making Hard Decisions, 2nd ed., Duxbury (1996).		

Figure 2.2 Model Formulas

	A	B
11		
12	**Intermediate Calculations**	
13	Total Revenue	=(B8*B6*B4)+((1-B8)*B6*B5*B7*5)
14	Total Cost	=(B6*B9)+B10
15		
16	**Performance Measure**	
17	Annual Profit	=B13-B14
18		

2.5 PLOT

Use SensIt's Plot option to see how your model's output depends on changes in a single input variable.

Plot Input Variable

Plot Input Variable's Cells: Option: In the Label edit box, type a cell reference, or point to the cell containing a text label and click. Required: In the Cell edit box, type a cell reference, or point to the cell containing a numeric value that's an input to your model.

Plot Output Variable

Plot Output Variable's Cells: Option: In the Label edit box, type a cell reference, or point to the cell containing a text label and click. Required: In the Cell edit box, type a cell reference, or point to the cell containing a formula that's the output of your model.

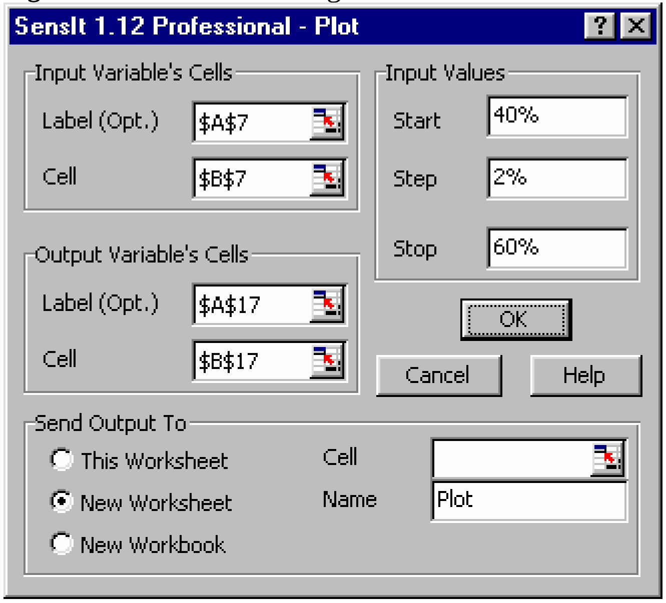
Plot Input Values

Plot Input Values: Type numbers in the Start, Step, and Stop edit boxes to specify values to be used in the input variable's cell. Cell references are not allowed.

Send Output To: Select the destination for the output table and chart. If you send output to This Worksheet, enter a Cell reference for the top left corner of the output. Output options are not available on the Macintosh; output is always sent to a new worksheet.

Click OK: SensIt Plot uses the Start, Step, and Stop values to prepare a table of values. Each value is copied to the input variable cell, the worksheet is recalculated, and the value of the output variable cell is copied to the table. (You could do this manually using the Edit | Fill | Series and Data | Table commands.) SensIt Plot uses the input and output values to prepare an XY (Scatter) chart; optionally, the text in the label cells you identified are used as the chart's axis labels. (You could do this manually using the ChartWizard.)

Figure 2.3 SensIt Plot Dialog Box



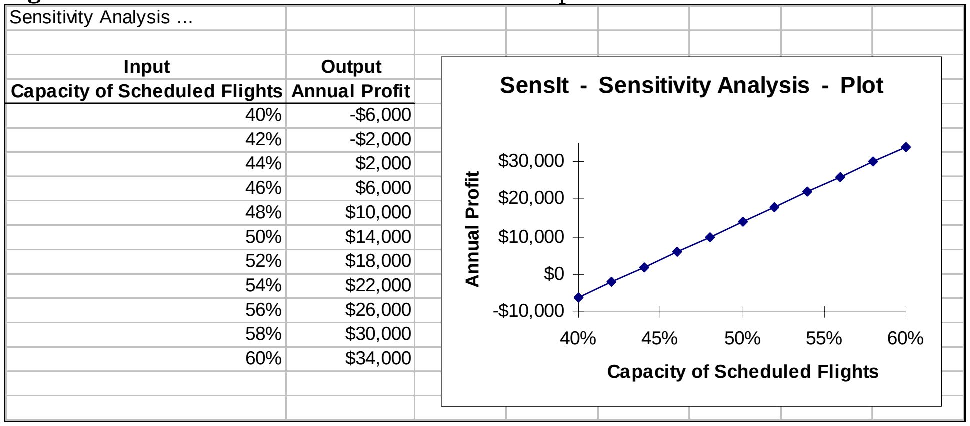
Input Variable's Cells
Label (Opt.) $A$7
Cell $B$7

Input Values
Start 40%
Step 2%
Stop 60%

Output Variable's Cells
Label (Opt.) $A$17
Cell $B$17

Send Output To
○ This Worksheet
● New Worksheet
○ New Workbook
Cell
Name Plot

Figure 2.4 SensIt Plot Numerical and Chart Output



Sensitivity Analysis ...

Input Capacity of Scheduled Flights	Output Annual Profit
40%	-$6,000
42%	-$2,000
44%	$2,000
46%	$6,000
48%	$10,000
50%	$14,000
52%	$18,000
54%	$22,000
56%	$26,000
58%	$30,000
60%	$34,000

2.6 SPIDER

Use SensIt's Spider option to see how your model's output depends on the same percentage changes for each of the model's input variables. Before using Spider, arrange your model input cells in adjacent cells in a single column, arrange corresponding labels in adjacent cells in a single column, and be sure your model's input cells contain base case values.

For example, if your model has five inputs, the names of the five inputs could be text in A1:A5. The input cells of your model could be numbers in B1:B5; when you change a number in one of these cells, the output of your model changes; enter base case values in the input cells B1:B5 before using Spider.

Spider Input Variables

Spider Input Variables' Ranges: Labels edit box: Type a range reference, or point to the range (click and drag) containing text labels. Cells edit box: Type a range reference, or point to the range containing numeric values that are inputs to your model. Each range must be adjacent cells in a single column.

Spider Output Variable

Spider Output Variable's Cells: Label edit box: Type a cell reference, or point to the cell containing a text label and click. Cell edit box: Type a cell reference, or point to the cell containing a formula that's the output of your model.

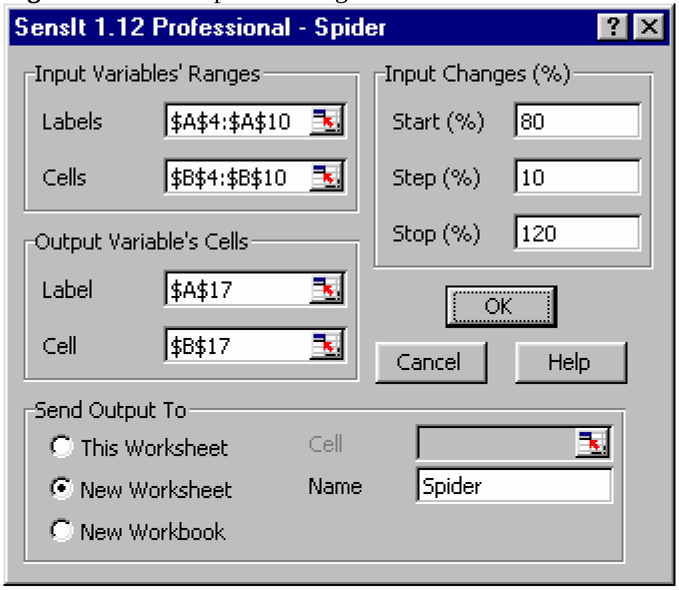
Spider Input Changes

Spider Input Changes (%): Type numbers in the Start (%), Step (%), and Stop (%) edit boxes to define the percents that will be multiplied times the current value in each input variable's cell. Cell references are not allowed.

Send Output To: Select the destination for the output table and chart. If you send output to This Worksheet, enter a Cell reference for the top left corner of the output. Output options are not available on the Macintosh; output is always sent to a new worksheet.

Click OK: SensIt Spider uses the Start (%), Step (%), and Stop (%) values and the original (base case) numeric value in each input variable cell to prepare a table of percentage change input values. For each input variable, all other input values are set at their base case values, each percentage change input value is copied to the input variable cell, the worksheet is recalculated, and the value of the output variable cell is copied to the table. The output variable values are also expressed as percentage change of the base case output value. SensIt Spider prepares two XY (Scatter) charts; the horizontal axis is percentage change of input variables; the vertical axis is model output value on one chart and percentage change of model output value on the other; the input variables' labels are used for chart legends.

Figure 2.5 SensIt Spider Dialog Box



Labels $A$4:$A$10

Cells $B$4:$B$10

Start (%) 80

Step (%) 10

Stop (%) 120

Label $A$17

Cell $B$17

○ This Worksheet

● New Worksheet

○ New Workbook

Cell

Name Spider

Figure 2.6 SensIt Spider Numerical Output

	A	B	C	D	E	F
1	**Input Variables Values**					
2		80%	90%	100%	110%	120%
3	Charter Price/Hour	$260	$293	$325	$358	$390
4	Ticket Price/Hour	$80	$90	$100	$110	$120
5	Hours Flown	640	720	800	880	960
6	Capacity of Scheduled Flights	40%	45%	50%	55%	60%
7	Proportion of Chartered Flights	0.4	0.45	0.5	0.55	0.6
8	Operating Cost/Hour	$196	$221	$245	$270	$294
9	Insurance	$16,000	$18,000	$20,000	$22,000	$24,000
10						
11						
12	**Output Variable Values (Annual Profit)**					
13		80%	90%	100%	110%	120%
14	Charter Price/Hour	-$12,000	$1,000	$14,000	$27,000	$40,000
15	Ticket Price/Hour	-$6,000	$4,000	$14,000	$24,000	$34,000
16	Hours Flown	$7,200	$10,600	$14,000	$17,400	$20,800
17	Scheduled Capacity	-$6,000	$4,000	$14,000	$24,000	$34,000
18	Chartered Proportion	$8,000	$11,000	$14,000	$17,000	$20,000
19	Operating Cost/Hour	$53,200	$33,600	$14,000	-$5,600	-$25,200
20	Insurance	$18,000	$16,000	$14,000	$12,000	$10,000

Figure 2.7 SensIt Spider Chart Output

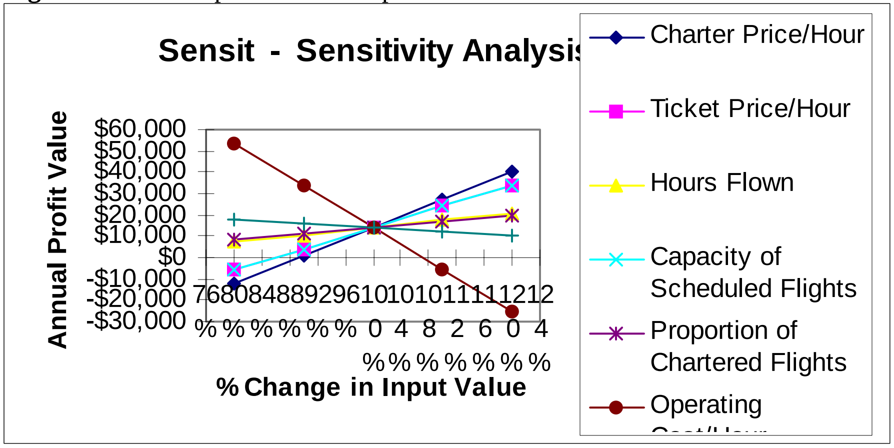
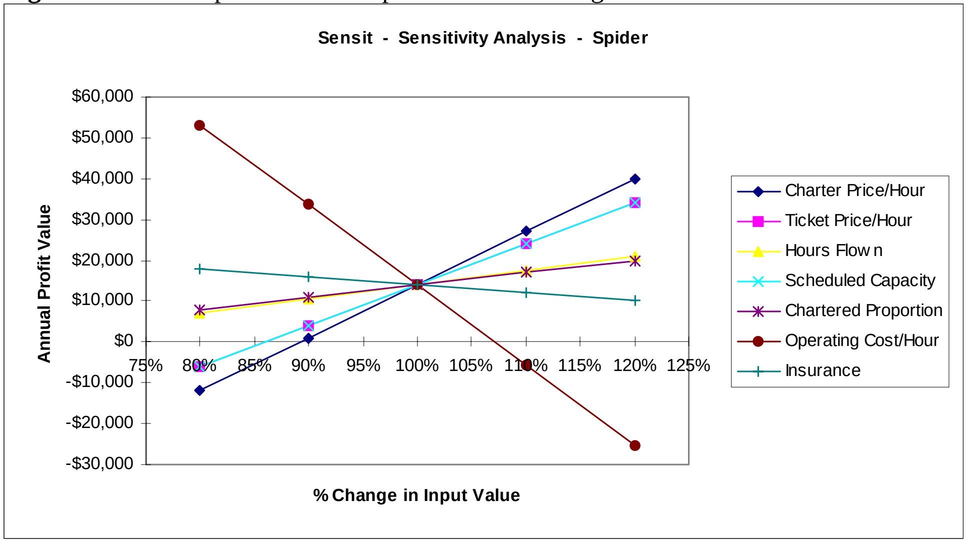


Figure 2.8 SensIt Spider Chart Output After Formatting



2.7 TORNADO

Use SensIt's Tornado option to see how your model's output depends on ranges you specify for each of the model's input variables. Before using Tornado, arrange your model input cells in adjacent cells in a single column, arrange corresponding labels in adjacent cells in a single column, and arrange Low, Base, and High input values for each input variable in three separate columns. Alternatively, the three columns containing input values can be worst case, likely case, and best case.

For example, if your model has five inputs, the names of the five inputs could be text in A1:A5. The input cells of your model could be numbers in B1:B5; when you change a number in one of these cells, the output of your model changes. The Low input values could be numbers in D1:D5, chosen as the minimum possible value you think each input variable could be. The Base input values could be numbers in E1:E5, chosen as the most likely value for each input; you might also have these same numbers in B1:B5 as current inputs to your model. The High input values could be numbers in F1:F5, chosen as the maximum possible value you think each input variable could be.

Tornado Input Variables

Tornado Input Variables' Ranges: Labels edit box: Type a range reference, or point to the range (click and drag) containing text labels. Cells edit box: Type a range reference, or point to the range containing numeric values that are inputs to your model. Each range must be adjacent cells in a single column.

Tornado Output Variable

Tornado Output Variable's Cells: Label edit box: Type a cell reference, or point to the cell containing a text label and click. Cell edit box: Type a cell reference, or point to the cell containing a formula that's the output of your model.

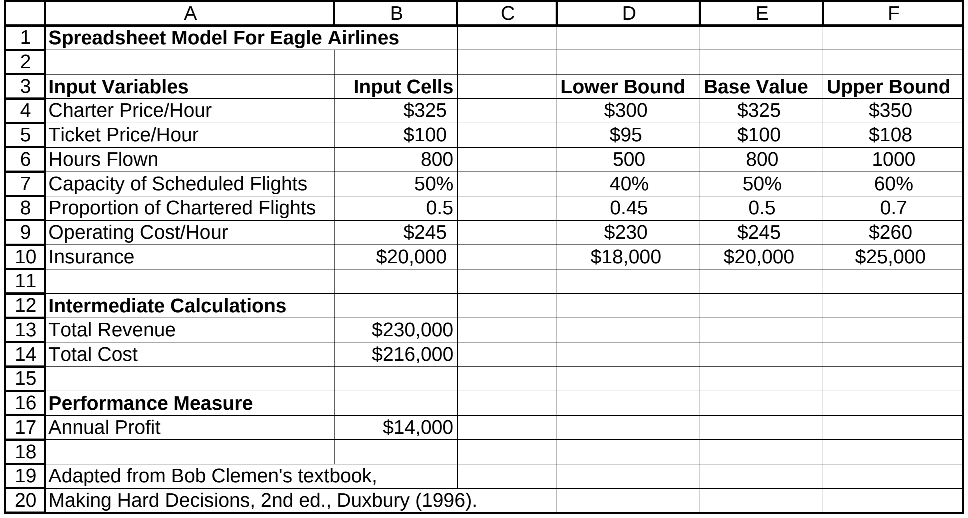
Tornado Input Values

Tornado Input Values' Ranges: In the Low, Base, and High edit boxes, type a range reference, or point to the range (click and drag) containing numeric values for each of your model's inputs.

Send Output To: Select the destination for the output table and chart. If you send output to This Worksheet, enter a Cell reference for the top left corner of the output. Output options are not available on the Macintosh; output is always sent to a new worksheet.

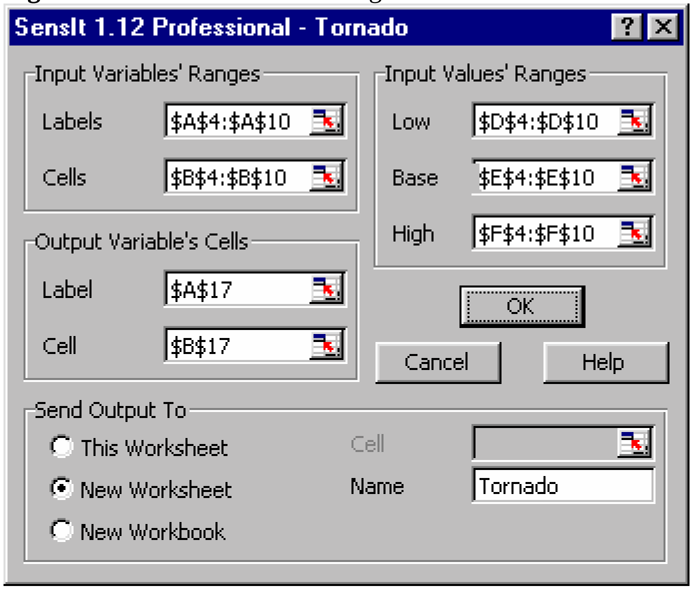
Click OK: For each input variable, SensIt Tornado sets all other input values at their Base case values, copies the Low input value to the input variable cell, recalculates the worksheet, and copies the value of the output variable cell to the table; the steps are repeated using each High input value. For each input variable, SensIt Tornado computes the range of the output variable values, sorts the table from largest range down to smallest range, and prepares a bar chart.

Figure 2.9 Example with Lower and Upper Bounds



	A	B	C	D	E	F
1	**Spreadsheet Model For Eagle Airlines**					
2						
3	**Input Variables**	**Input Cells**		**Lower Bound**	**Base Value**	**Upper Bound**
4	Charter Price/Hour	$325		$300	$325	$350
5	Ticket Price/Hour	$100		$95	$100	$108
6	Hours Flown	800		500	800	1000
7	Capacity of Scheduled Flights	50%		40%	50%	60%
8	Proportion of Chartered Flights	0.5		0.45	0.5	0.7
9	Operating Cost/Hour	$245		$230	$245	$260
10	Insurance	$20,000		$18,000	$20,000	$25,000
11						
12	**Intermediate Calculations**					
13	Total Revenue	$230,000				
14	Total Cost	$216,000				
15						
16	**Performance Measure**					
17	Annual Profit	$14,000				
18						
19	Adapted from Bob Clemen's textbook,					
20	Making Hard Decisions, 2nd ed., Duxbury (1996).					

Figure 2.10 SensIt Tornado Dialog Box



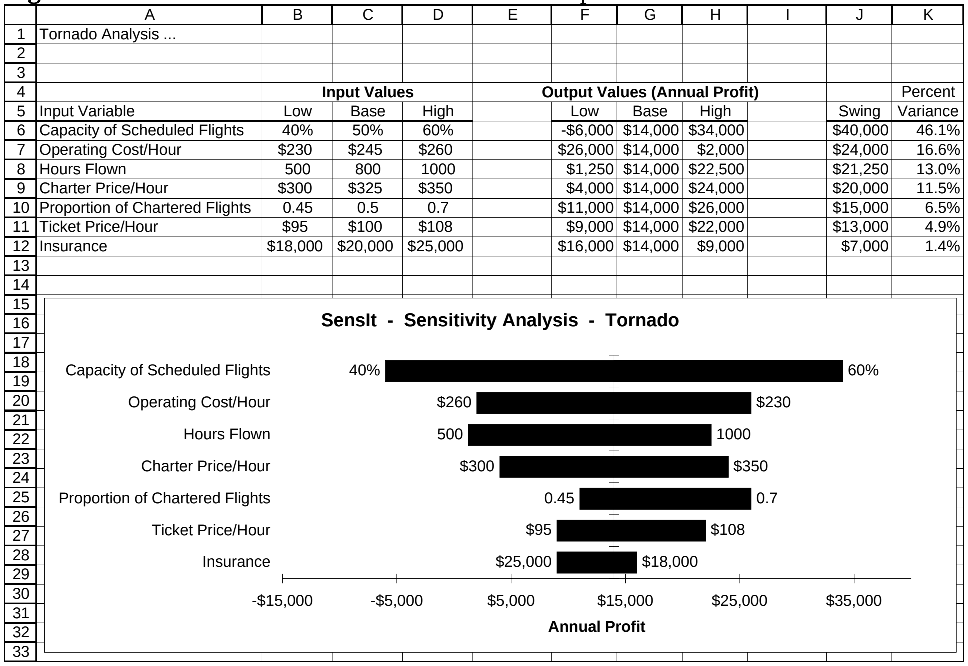
Input Variables' Ranges
Labels: $A$4:$A$10
Cells: $B$4:$B$10

Input Values' Ranges
Low: $D$4:$D$10
Base: $E$4:$E$10
High: $F$4:$F$10

Output Variable's Cells
Label: $A$17
Cell: $B$17

Send Output To
○ This Worksheet
● New Worksheet
○ New Workbook
Cell:
Name: Tornado

Figure 2.11 SensIt Tornado Numerical and Chart Output



	A	B	C	D	E	F	G	H	I	J	K
1	Tornado Analysis ...										
2											
3											
4			**Input Values**				**Output Values (Annual Profit)**				Percent
5	Input Variable	Low	Base	High		Low	Base	High		Swing	Variance
6	Capacity of Scheduled Flights	40%	50%	60%		-$6,000	$14,000	$34,000		$40,000	46.1%
7	Operating Cost/Hour	$230	$245	$260		$26,000	$14,000	$2,000		$24,000	16.6%
8	Hours Flown	500	800	1000		$1,250	$14,000	$22,500		$21,250	13.0%
9	Charter Price/Hour	$300	$325	$350		$4,000	$14,000	$24,000		$20,000	11.5%
10	Proportion of Chartered Flights	0.45	0.5	0.7		$11,000	$14,000	$26,000		$15,000	6.5%
11	Ticket Price/Hour	$95	$100	$108		$9,000	$14,000	$22,000		$13,000	4.9%
12	Insurance	$18,000	$20,000	$25,000		$16,000	$14,000	$9,000		$7,000	1.4%
13											
14											

2.8 TORNADO TIPS

When defining the high and low cases for each variable, it is important to be consistent so that the "high" cases are all equally high and the "low" cases are equally low. For example, you might take all of the base case values to be estimates of the mean of the input variable, take low cases to be values such there is a 1-in-10 chance of the variable being below this amount, and take the high cases to be values such that there is a 1-in-10 chance of the variable being above this amount. Alternatively, you may specify low and high values that are the absolute lowest and highest possible values.

When you click OK, SensIt sets all of the input variables to their base-case values and records the output value. Then SensIt goes through each of the input variables one at a time, plugs the low-case value into the input cell, and records the value in the output cell. It then repeats the process for the high case. For each substitution, all input values are kept at their base-case values except for the single input value that is setn at it low or high value. SensIt then produces a spreadsheet that lists the numerical results as shown in columns F, G, and H above.

In the worksheet, the variables are sorted by their "swing" -- the absolute value of the difference between the output values in the low and high cases. "Swing" serves as a rough measure of the impact of each input variable. The rows of numerical output are sorted from highest swing at the top down to lowest swing at the bottom. Then SensIt creates a bar chart of the sorted data.

"Percent variance" is a standardized measure of impact: it squares each swing, sums them up to get a "Total Variance", and reports the percentage of the "total variance" attributed to each input variable.

In general, you should focus your modeling efforts on those variables with the greatest impact on the value measure.

If your model has input variables that are discrete or categorical, you should create multiple tornado charts using different base case values of that input variable. For example, if your model has an input variable "Government Regulation" that has possible values 0 (zero) or 1, the low and high values will be 0 and 1, but you should run one tornado chart with base case = 0 and another tornado chart with base case = 1.

2.9 EAGLE AIRLINES PROBLEM

Figure 2.12 Eagle Model Display

	A	B	C	D	E	F
1	**Spreadsheet Model For Eagle Airlines**					
2						
3	**Variable**	**Input Cells**		**Lower Bound**	**Base Value**	**Upper Bound**
4	Hours Flown	800		500	800	1000
5	Charter Price/Hour	$325		$300	$325	$350
6	Ticket Price/Hour	$100		$95	$100	$108
7	Capacity of Scheduled Flights	50%		40%	50%	60%
8	Proportion Of Chartered Flights	0.5		0.45	0.5	0.7
9	Operating Cost/Hour	$245		$230	$245	$260
10	Insurance	$20,000		$18,000	$20,000	$25,000
11	Proportion Financed	0.4		0.3	0.4	0.5
12	Interest Rate	11.5%		10.5%	11.5%	13.0%
13	Purchase Price	$87,500		$85,000	$87,500	$90,000
14						
15	**Total Revenue**	$230,000				
16	**Total Cost**	$220,025				
17						
18	**Annual Profit**	$9,975				
19						
20	Adapted from Bob Clemen's textbook, Making Hard Decisions					

Figure 2.13 Eagle Model Formulas

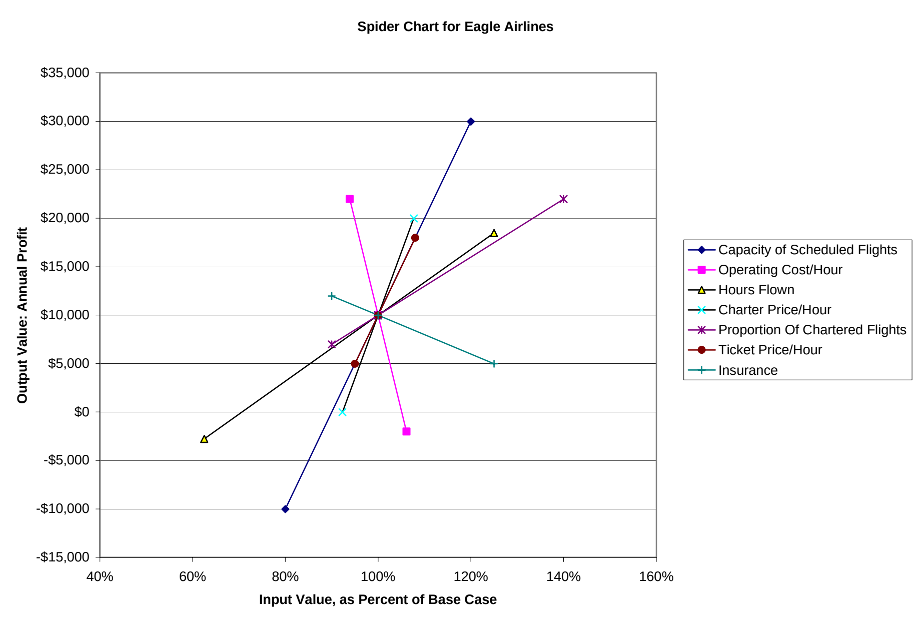
	A	B	C	D	E	F
14						
15	**Total Revenue**	=(B8*B4*B5)+((1-B8)*B4*B6*B7*5)				
16	**Total Cost**	=(B4*B9)+B10+(B13*B11*B12)				
17						
18	**Annual Profit**	=B15-B16				
19						

Figure 2.14 Worst Case and Best Case Inputs Determined by Solver

Variable	**Worst Case**	**Base Case**	**Best Case**
Hours Flown	1000	800	1000
Charter Price/Hour	$300	$325	$350
Ticket Price/Hour	$95	$100	$108
Capacity of Scheduled Flights	40%	50%	60%
Proportion Of Chartered Flights	0.45	0.5	0.7
Operating Cost/Hour	$260	$245	$230
Insurance	$25,000	$20,000	$18,000
Proportion Financed	0.5	0.4	0.3
Interest Rate	13.0%	11.5%	10.5%
Purchase Price	$90,000	$87,500	$85,000
Total Revenue	$239,500	$230,000	$342,200
Total Cost	$290,850	$220,025	$250,678
Annual Profit	-$51,350	$9,975	$91,523

2.10 SEVEN-VARIABLE NON-SENSIT SPIDER CHART

Figure 2.15 Extremes Associated With Tornado-Like Inputs



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Monte Carlo Simulation Using RiskSim

3

3.1 INTRODUCTION

Figure 3.1 Conceptual Simulation as a Sample of Tree Endpoints

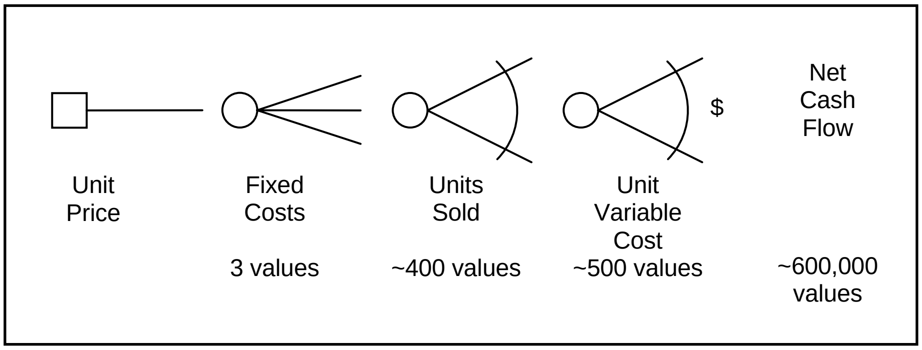


Figure 3.2 Probability Distributions for Sampling Tree Endpoints

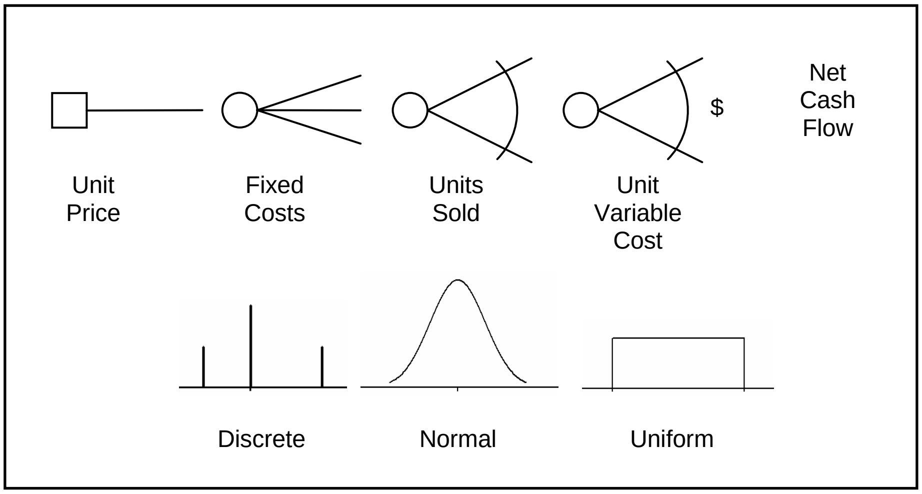
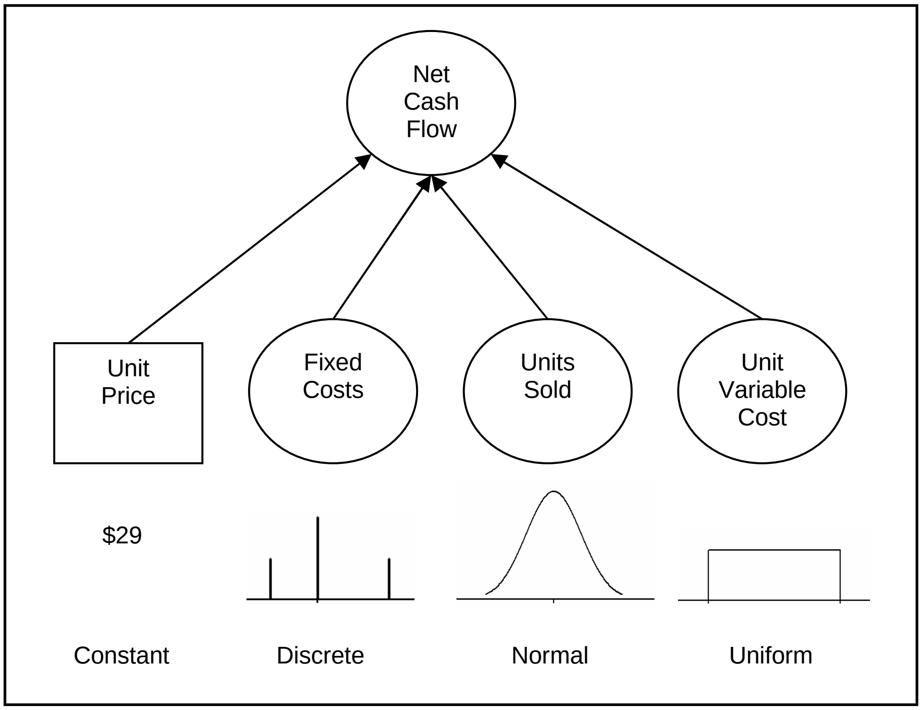


Figure 3.3 Conceptual Simulation as Influence Chart with Repeated What-Ifs



3.2 USING RISKSIM FUNCTIONS

RiskSim is a Monte Carlo Simulation add-in for Microsoft Excel 97 (and later versions of Excel) for Windows and Macintosh.

RiskSim provides random number generator functions as inputs for your model, automates Monte Carlo simulation, and creates charts. Your spreadsheet model may include various uncontrollable uncertainties as input assumptions (e.g., demand for a new product, uncertain variable cost of production, competitor reaction), and you can use simulation to determine the uncertainty associated with the model's output (e.g., annual profit). RiskSim automates the simulation by trying hundreds of what-ifs consistent with your assessment of the uncertainties.

To use RiskSim, you

(1) create a spreadsheet model

(2) optionally use SensIt to identify critical inputs

(3) enter one of RiskSim's nine random number generator functions in each input cell of your model

(4) choose Tools | Risk Simulation from Excel's menu

(5) specify the model output cell and the number of what-if trials

(6) interpret RiskSim's histogram and cumulative distribution charts.

RiskSim facilitates Monte Carlo simulation by providing:

Nine random number generator functions
Ability to set the seed for random number generation
Automatic repeated sampling for simulation

Frequency distribution of simulation results

Histogram and cumulative distribution charts

3.3 USING RISKSIM FUNCTIONS

RiskSim adds nine random number generator functions to Excel. You can use these functions as inputs to your model by typing in a worksheet cell or by using the Function Wizard. From the Insert menu choose Function, or click the Function Wizard button. RiskSim's functions are listed in a User Defined category. The nine functions are:

RANDBINOMIAL(trials,probability_s)

RANDCUMULATIVE(value_cumulative_table)

RANDDISCRETE(value_discrete_table)

RANDEXPONENTIAL(lambda)

RANDINTEGER(bottom,top)

RANDNORMAL(mean,standard_dev)

RANDPOISSON(mean)

RANDTRIANGULAR(minimum,most_likely,maximum)

RANDUNIFORM(minimum,maximum)

RiskSim's RAND... functions include extensive error checking of arguments. After verifying that the functions are working properly, you may want to substitute RiskSim's FAST... functions which have minimal error checking and therefore run faster. From the Edit menu choose Replace; in the Replace dialog box, type =RAND in the "Find What" edit box, type =FAST in the "Replace with" edit box, and click the Replace All button.

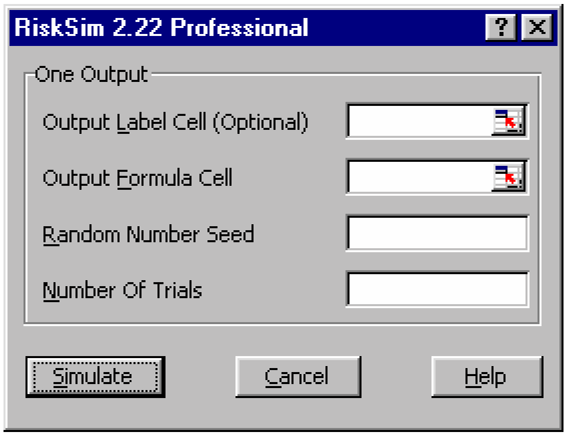
3.4 EXCEL ERROR MESSAGE

When you insert a RiskSim random number generator function in a worksheet cell, the function is linked to RiskSim.xla. When you save the workbook, Excel saves the complete path to the function in RiskSim.xla. When you open the workbook, Excel looks for RiskSim.xla using the saved path. If Excel cannot find RiskSim.xla at the saved path location (e.g., if you deleted RiskSim.xla or if you opened the workbook on another computer where RiskSim.xla isn't located at the same path), Excel displays a dialog box: "This document contains links. Re-establish links?" Click No. The workbook will be opened, but any cell containing a reference to a RiskSim function will display the #REF!, #NAME?, or other error code. To fix the links, be sure that RiskSim.xla is open (e.g., File | Open | RiskSim.xla), choose Edit | Links | Change Source, and locate the RiskSim.xla file that is open.

3.5 MONTE CARLO SIMULATION

After specifying random number generator functions as inputs to your model, from the Tools choose Risk Simulation | One Output.

Figure 3.4 RiskSim Dialog Box



Optionally, select the "Output Label Cell" edit box, and point or type a reference to a cell containing the name of the model output (for example, a cell whose contents is the text label "Net Profit").

Select the "Output Formula Cell" edit box, and point to a single cell on your worksheet or type a cell reference. The output cell of your model must contain a formula that depends, usually indirectly, on the model inputs determined by the random number generator functions.

Select the "Random Number Seed" edit box, and type a number between zero and one. (If you want to change the seed without performing a simulation, enter zero in the "Number of iterations" edit box.)

Select the "Number Of Trials" edit box, and type an integer value (for example, 100 or 500). This value, sometimes called the sample size or number of iterations, specifies the number of times the worksheet will be recalculated to determine output values of your model.

3.6 RANDOM NUMBER SEED

The "Random Number Seed" edit box on the RiskSim dialog box allows you to set the seed for RiskSim's random number generator functions. These functions depend on RiskSim's own uniform random number function that is completely independent of Excel's built-in RAND().

Random numbers generated by the computer are actually pseudo-random. The numbers appear to be random, and they pass various statistical tests for randomness. But they are actually calculated by an algorithm where each random number depends on the previous random number. Such an algorithm generates a repeatable sequence. The seed specifies where the algorithm starts in the sequence.

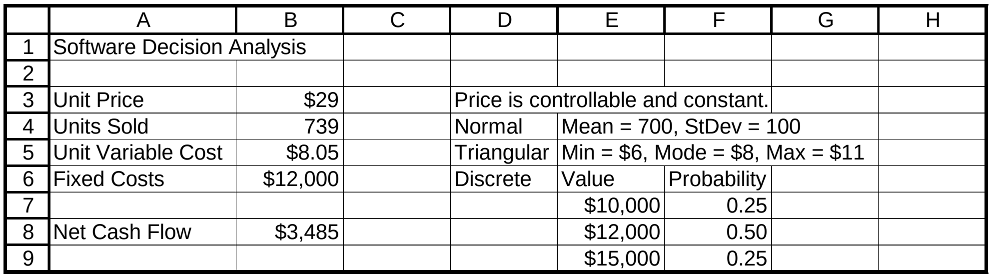
A Monte Carlo simulation model usually has uncontrollable inputs (uncertain quantities using random number generator functions), controllable inputs (decision variables that have fixed values for a particular set of simulation iterations), and an output variable (a performance measure or operating characteristic of the system).

For example, a simple queuing system model may have an uncertain arrival pattern, a controllable number of servers, and total cost (waiting time plus server cost) as output. To evaluate a different number of servers, you would specify the same seed before generating the uncertain arrivals. Then the variation in total cost should depend on the different number of servers, not on the particular sequence of random numbers that generates the arrivals.

3.7 ONE-OUTPUT EXAMPLE

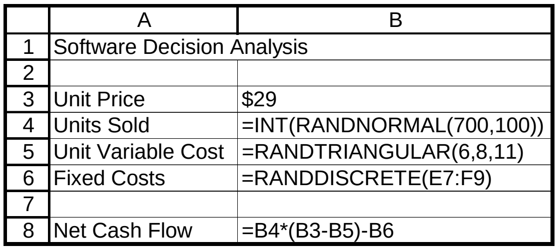
In this example the decision maker has described his subjective uncertainty using normal, triangular, and discrete probability distributions.

Figure 3.5 One-Output Example Model Display



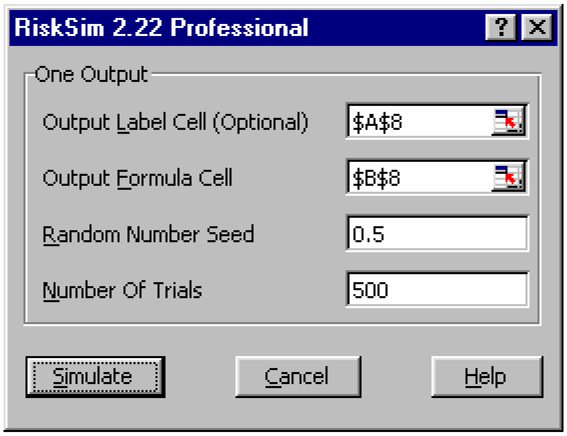
	A	B	C	D	E	F	G	H
1	Software Decision Analysis							
2								
3	Unit Price	$29		Price is controllable and constant.				
4	Units Sold	739		Normal	Mean = 700, StDev = 100			
5	Unit Variable Cost	$8.05		Triangular	Min = $6, Mode = $8, Max = $11			
6	Fixed Costs	$12,000		Discrete	Value	Probability		
7					$10,000	0.25		
8	Net Cash Flow	$3,485			$12,000	0.50		
9					$15,000	0.25		

Figure 3.6 One-Output Example Model Formulas



	A	B
1	Software Decision Analysis	
2		
3	Unit Price	$29
4	Units Sold	=INT(RANDNORMAL(700,100))
5	Unit Variable Cost	=RANDTRIANGULAR(6,8,11)
6	Fixed Costs	=RANDDISCRETE(E7:F9)
7		
8	Net Cash Flow	=B4*(B3-B5)-B6

Figure 3.7 RiskSim Dialog Box for One-Output Example



Output Label Cell (Optional) $A$8

Output Formula Cell $B$8

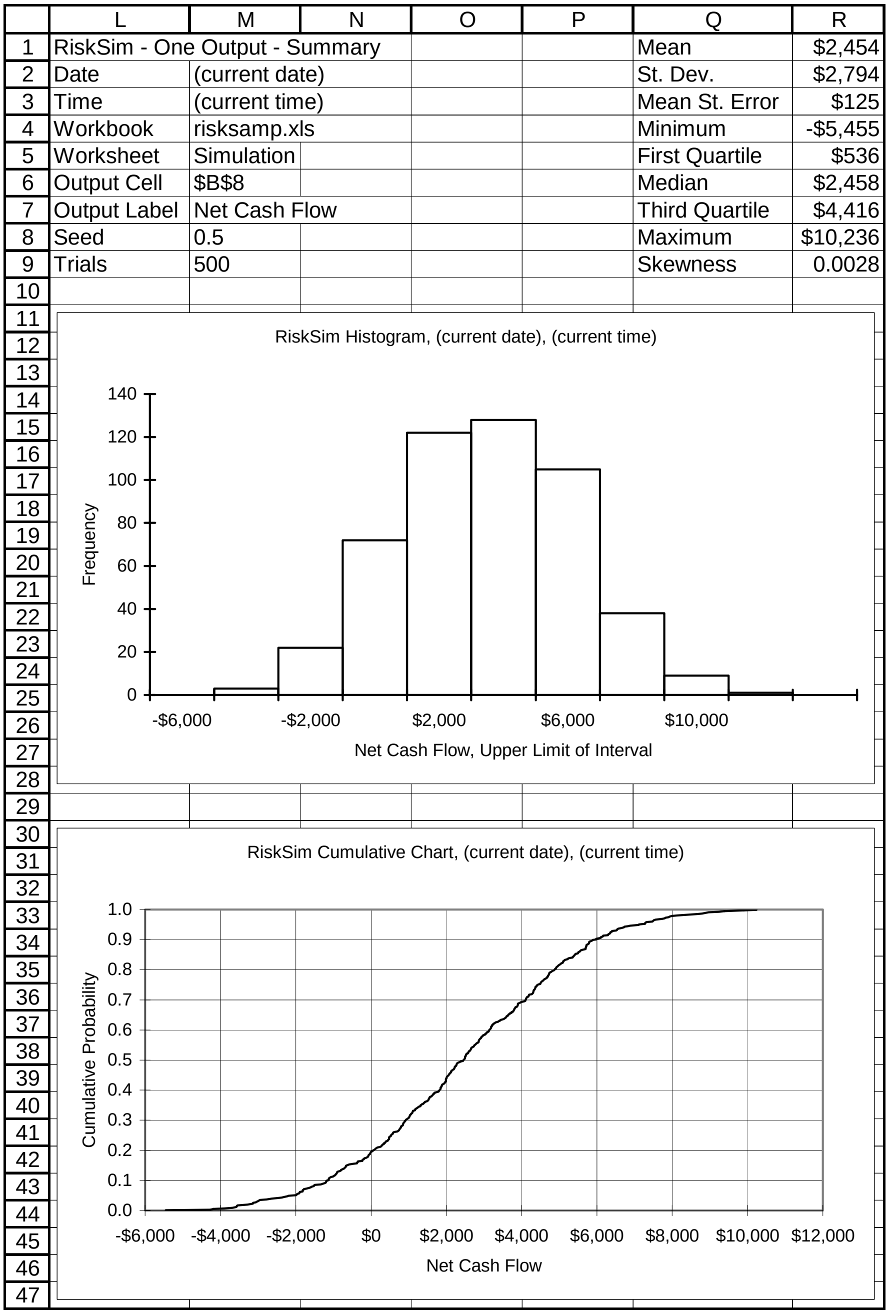
Random Number Seed 0.5

Number Of Trials 500

3.8 RISKSIM OUTPUT FOR ONE-OUTPUT EXAMPLE

When you click the Simulate button, RiskSim creates a new worksheet in your Excel workbook named "RiskSim Summary 1." A summary of your inputs and the output is shown in cells L1:R9 with the accompanying histogram and cumulative distribution charts.

Figure 3.8 RiskSim Summary Output for One-Output Example



	L	M	N	O	P	Q	R
1	RiskSim - One Output - Summary					Mean	$2,454
2	Date	(current date)				St. Dev.	$2,794
3	Time	(current time)				Mean St. Error	$125
4	Workbook	risksamp.xls				Minimum	-$5,455
5	Worksheet	Simulation				First Quartile	$536
6	Output Cell	$B$8				Median	$2,458
7	Output Label	Net Cash Flow				Third Quartile	$4,416
8	Seed	0.5				Maximum	$10,236
9	Trials	500				Skewness	0.0028
10							

The histogram is based on the frequency distribution in columns I:J. The cumulative distribution is based on the sorted output values in column C and the cumulative probabilities in column D.

Figure 3.9 RiskSim Numerical Output for One-Output Example

	A	B	C	D	E	F	G	H	I	J
1	Trial	Net Cash Flow	Sorted	Cumulative		Percent	Percentile		Upper Limit	Frequency
2	1	$1,653	-$5,455	0.0010		0%	-$5,455		-$6,000	0
3	2	$2,804	-$4,267	0.0030		5%	-$1,996		-$4,000	3
4	3	$2,280	-$4,185	0.0050		10%	-$1,132		-$2,000	22
5	4	$761	-$3,898	0.0070		15%	-$637		$0	72
6	5	-$1,817	-$3,675	0.0090		20%	$77		$2,000	122
7	6	-$692	-$3,582	0.0110		25%	$536		$4,000	128
8	7	$623	-$3,569	0.0130		30%	$923		$6,000	105
9	8	$5,575	-$3,562	0.0150		35%	$1,331		$8,000	38
10	9	$1,389	-$3,547	0.0170		40%	$1,823		$10,000	9
11	10	$445	-$3,275	0.0190		45%	$2,063		$12,000	1
12	11	$2,573	-$3,207	0.0210		50%	$2,458			0
13	12	$5,055	-$3,137	0.0230		55%	$2,756			
14	13	$1,430	-$3,135	0.0250		60%	$3,138			
15	14	$4,529	-$3,063	0.0270		65%	$3,644			
16	15	$701	-$3,036	0.0290		70%	$4,104			
17	16	-$903	-$3,008	0.0310		75%	$4,416			
18	17	$3,900	-$2,968	0.0330		80%	$4,867			
19	18	$7,282	-$2,950	0.0350		85%	$5,412			
20	19	$9,901	-$2,774	0.0370		90%	$5,897			
21	20	$285	-$2,649	0.0390		95%	$7,109			
22	21	$3,833	-$2,485	0.0410		100%	$10,236			
23	22	$4,369	-$2,370	0.0430						
24	23	$1,991	-$2,319	0.0450						
25	24	-$11	-$2,219	0.0470						
26	25	$1,100	-$2,195	0.0490						
27	26	-$1,100	-$1,986	0.0510						
28	27	-$5,455	-$1,969	0.0530						

The cumulative probabilities start at 1/(2*N), where N is the number of trials, and increase by 1/N. The rationale is that the lowest ranked output value of the sampled values is an estimate of the population's values in the range from 0 to 1/N, and the lowest ranked value is associated with the median of that range.

Column B contains the original sampled output values.

Columns F:G show percentiles based on Excel's PERCENTILE worksheet function. Refer to Excel's online help for the interpolation method used by the PERCENTILE function.

The summary measures in columns Q:R are also based on Excel worksheet functions: AVERAGE, STDEV, QUARTILE, and SKEW.

3.9 RANDOM NUMBER GENERATOR FUNCTIONS

RandBinomial

Returns a random value from a binomial distribution. The binomial distribution can model a process with a fixed number of trials where the outcome of each trial is a success or failure, the trials are independent, and the probability of success is constant. RANDBINOMIAL counts the total number of successes for the specified number of trials. If n is the number of trials, the possible values for RANDBINOMIAL are the non-negative integers 0,1,...,n.

RANDBINOMIAL Syntax: RANDBINOMIAL(trials,probability_s)

Trials (often denoted n) is the number of independent trials.

Probability_s (often denoted p) is the probability of success on each trial.

RANDBINOMIAL Remarks

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if an argument is text and the name is undefined.

Returns #NUM! if trials is non-integer or less than one, or probability_s is less than zero or more than one.

Returns #VALUE! if an argument is a defined name of a cell and the cell is blank or contains text.

RANDBINOMIAL Example

A salesperson makes ten unsolicited calls per day, where the probability of making a sale on each call is 30 percent. The uncertain total number of sales in one day is =RANDBINOMIAL(10,0.3)

RANDBINOMIAL Related Function

FASTBINOMIAL: Same as RANDBINOMIAL without any error checking of the arguments.

CRITBINOM(trials,probability_s,RAND()): Excel's inverse of the cumulative binomial, or CRITBINOM(trials,probability_s,RANDUNIFORM(0,1)) to use the RiskSim Seed feature.

RandCumulative

Returns a random value from a piecewise-linear cumulative distribution. This function can model a continuous-valued uncertain quantity, X, by specifying points on its cumulative distribution. Each point is specified by a possible value, x, and a corresponding left-tail cumulative probability, P(X<=x). Random values are based on linear interpolation between the specified points.

RANDCUMULATIVE Syntax: RANDCUMULATIVE(value_cumulative_table)

Value_cumulative_table must be a reference, or the defined name of a reference, for a two-column range, with values in the left column and corresponding cumulative probabilities in the right column.

RANDCUMULATIVE Remarks

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if the argument is text and the name is undefined.

Returns #NUM! if the first (top) cumulative probability is not zero, if the last (bottom) cumulative probability is not one, or if the values or cumulative probabilities are not in ascending order.

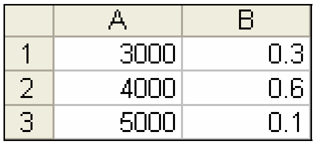
Returns #REF! if the number of columns in the table reference is not two.

Returns #VALUE! if the argument is not a reference, if the argument is a defined name but not for a reference, or if any cell of the table contains text or is blank.

RANDCUMULATIVE Example

A corporate planner thinks that minimum possible market demand is 1000 units, median is 5000, and maximum possible is 9000. Also, there is a ten percent chance that demand will be less than 4000 and a ten percent chance it will exceed 7000. The values, x, and cumulative probabilities, P(X<=x), are entered into spreadsheet cells A1:B5.

Figure 3.10 RandDiscrete Example Spreadsheet Data



	A	B
1	3000	0.3
2	4000	0.6
3	5000	0.1

The function is entered into another cell: =RANDCUMULATIVE(A1:B5)

RANDCUMULATIVE Related Function

FASTCUMULATIVE: Same as RANDCUMULATIVE without any error checking of the arguments.

Figure 3.11 RandCumulative Example Probability Density Function

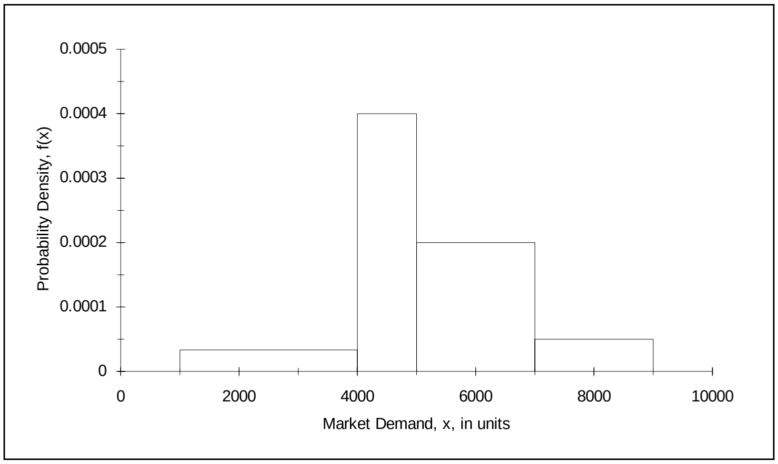
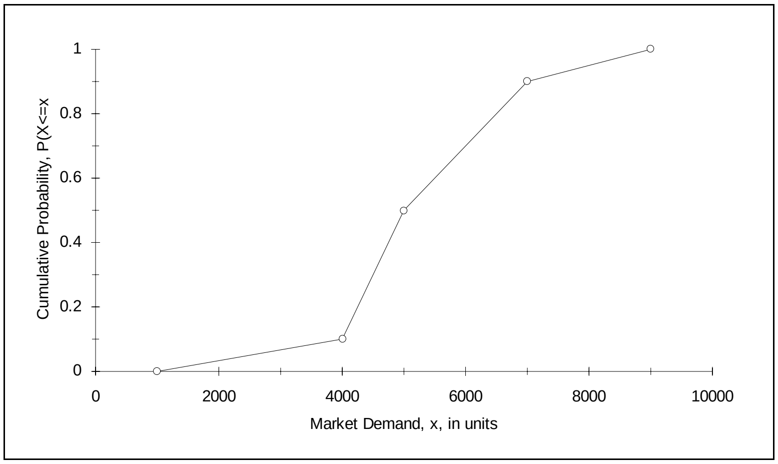


Figure 3.12 RandCumulative Example Cumulative Probability Function



RandDiscrete

Returns a random value from a discrete probability distribution. This function can model a discrete-valued uncertain quantity, X, by specifying its probability mass function. The function is specified by each possible discrete value, x, and its corresponding probability, P(X=x).

RANDDISCRETE Syntax: RANDDISCRETE(value_discrete_table)

Value_discrete_table must be a reference, or the defined name of a reference, for a two-column range, with values in the left column and corresponding probability mass in the right column.

RANDDISCRETE Remarks

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if the argument is text and the name is undefined.

Returns #NUM! if a probability is negative or if the probabilities do not sum to one.

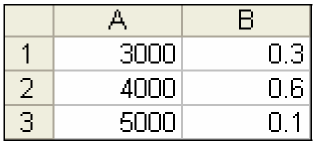
Returns #REF! if the number of columns in the table reference is not two.

Returns #VALUE! if the argument is not a reference, if the argument is a defined name but not for a reference, or if any cell of the table contains text or is blank.

RANDDISCRETE Example

A corporate planner thinks that uncertain market demand, X, can be approximated by three possible values and their associated probabilities: P(X=3000) = 0.3, P(X=4000) = 0.6, and P(X=5000) = 0.1. The values and probabilities are entered into spreadsheet cells A1:B3.

Figure 3.13 RandDiscrete Example Spreadsheet Data



	A	B
1	3000	0.3
2	4000	0.6
3	5000	0.1

The function is entered into another cell: =RANDDISCRETE(A1:B3)

RANDDISCRETE Related Function

FASTDISCRETE: Same as RANDDISCRETE without any error checking of the arguments.

RandDiscrete Example Probability Mass Function

Figure 3.14 RandDiscrete Example Probability Mass Function

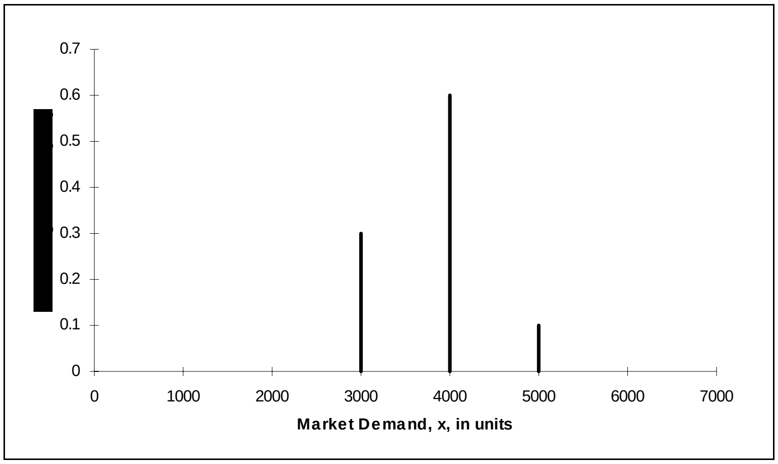
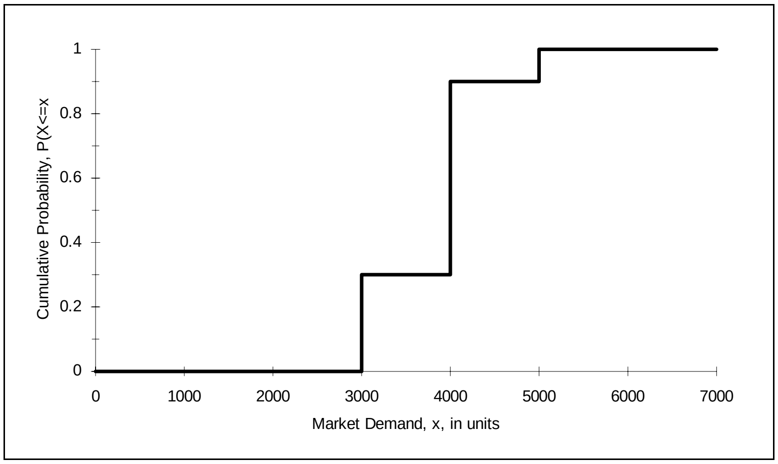


Figure 3.15 RandDiscrete Example Cumulative Probability Function



RandExponential

Returns a random value from an exponential distribution. This function can model the uncertain time interval between successive arrivals at a queuing system or the uncertain time required to serve a customer.

RANDEXPONENTIAL Syntax: RANDEXPONENTIAL(lambda)

Lambda is the mean number of occurrences per unit of time.

RANDEXPONENTIAL Remarks

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if the argument is text and the name is undefined.

Returns #NUM! if lambda is negative or zero.

Returns #VALUE! if the argument is a defined name of a cell and the cell is blank or contains text.

RANDEXPONENTIAL Examples

Cars arrive at a toll plaza with a mean rate of 3 cars per minute. The uncertain time between successive arrivals, measured in minutes, is =RANDEXPONENTIAL(3). The average value returned by repeated recalculation of RANDEXPONENTIAL(3) is 0.333.

A bank teller requires an average of two minutes to serve a customer. The uncertain customer service time, measured in minutes, is =RANDEXPONENTIAL(0.5). The average value returned by repeated recalculation of RANDEXPONENTIAL(0.5) is 2.

RANDEXPONENTIAL Related Functions

FASTEXPONENTIAL: Same as RANDEXPONENTIAL without any error checking of the arguments.

–LN(RAND())/lambda: Excel's inverse of the exponential, or
–LN(RANDUNIFORM(0,1))/lambda to use the RiskSim Seed feature.

RANDPOISSON: Counts number of occurrences for a Poisson process.

RandInteger

Returns a uniformly distributed random integer between two integers you specify.

RANDINTEGER Syntax: RANDINTEGER(bottom,top)

Bottom is the smallest integer RANDINTEGER will return.

Top is the largest integer RANDINTEGER will return.

RANDINTEGER Remarks

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if an argument is text and the name is undefined.

Returns #NUM! if top is less than or equal to bottom.

Returns #VALUE! if bottom or top is not an integer or if an argument is a defined name of a cell and the cell is blank or contains text.

RANDINTEGER Example

The number of orders a particular customer will place next year is between 7 and 11, with no number more likely than the others. The uncertain number of orders is =RANDINTEGER(7,11).

RANDINTEGER Related Function

FASTINTEGER: Same as RANDINTEGER without any error checking of the arguments.

RANDBETWEEN(bottom,top): Excel’s function for uniformly distributed integers, without RiskSim’s capability of setting the seed.

RandNormal

Returns a random value from a normal distribution. This function can model a variety of phenomena where the values follow the familiar bell-shaped curve, and it has wide application in statistical quality control and statistical sampling.

RANDNORMAL Syntax: RANDNORMAL(mean,standard_dev)

Mean is the arithmetic mean of the normal distribution.

Standard_dev is the standard deviation of the normal distribution.

RANDNORMAL Remarks

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if an argument is text and the name is undefined.

Returns #NUM! if standard_dev is negative.

Returns #VALUE! if an argument is a defined name of a cell and the cell is blank or contains text.

RANDNORMAL Example

The total market for a product is approximately normally distributed with mean 60,000 units and standard deviation 5,000 units. The uncertain total market is =RANDNORMAL(60000,5000).

RANDNORMAL Related Function

FASTNORMAL: Same as RANDNORMAL without any error checking of the arguments.

NORMINV(RAND(),mean,standard_dev): Excel's inverse of the normal, or NORMINV(RANDUNIFORM(0,1),mean,standard_dev) to use the RiskSim Seed feature.

RandPoisson

Returns a random value from a Poisson distribution. This function can model the uncertain number of occurrences during a specified time interval, for example, the number of arrivals at a service facility during an hour. The possible values of RANDPOISSON are the non-negative integers, 0, 1, 2, ...

RANDPOISSON Syntax: RANDPOISSON(mean)

Mean is the mean number of occurrences per unit of time.

RANDPOISSON Remarks

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if the argument is text and the name is undefined.

Returns #NUM! if mean is negative or zero.

Returns #VALUE! if mean is a defined name of a cell and the cell is blank or contains text.

RANDPOISSON Examples

Cars arrive at a toll plaza with a mean rate of 3 cars per minute. The uncertain number of arrivals in a minute is =RANDPOISSON(3). The average value returned by repeated recalculation of RANDPOISSON(3) is 3.

A bank teller requires an average of two minutes to serve a customer. The uncertain number of customers served in a minute is =RANDPOISSON(0.5). The average value returned by repeated recalculation of RANDPOISSON(0.5) is 0.5.

RANDPOISSON Related Functions

FASTPOISSON: Same as RANDPOISSON without any error checking of the arguments.

RANDEXPONENTIAL: Describes time between occurrences for a Poisson process.

RandTriangular

Returns a random value from a triangular probability density function. This function can model an uncertain quantity where the most likely value (mode) has the largest probability of occurrence, the minimum and maximum possible values have essentially zero probability of occurrence, and the probability density function is linear between the minimum and the mode and between the mode and the maximum. This function can also model a ramp density function where the minimum equals the mode or the mode equals the maximum.

RANDTRIANGULAR Syntax:
RANDTRIANGULAR(minimum,most_likely,maximum)

Minimum is the smallest value RANDTRIANGULAR will return.

Most_likely is the most likely value RANDTRIANGULAR will return.

Maximum is the largest value RANDTRIANGULAR will return.

RANDTRIANGULAR Remarks

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if an argument is text and the name is undefined.

Returns #NUM! if minimum is greater than or equal to maximum, if most_likely is less than minimum, or if most_likely is greater than maximum.

Returns #VALUE! if an argument is a defined name of a cell and the cell is blank or contains text.

RANDTRIANGULAR Example

The minimum time required to complete a particular task that is part of a large project is 4 hours, the most likely time required is 6 hours, and the maximum time required is 10 hours.

The function returning the uncertain time required for the task is entered into a cell: =RANDTRIANGULAR(4,6,10).

RANDTRIANGULAR Related Function

FASTTRIANGULAR: Same as RANDTRIANGULAR without any error checking of arguments.

Figure 3.16 RandTriangular Example Probability Density Function

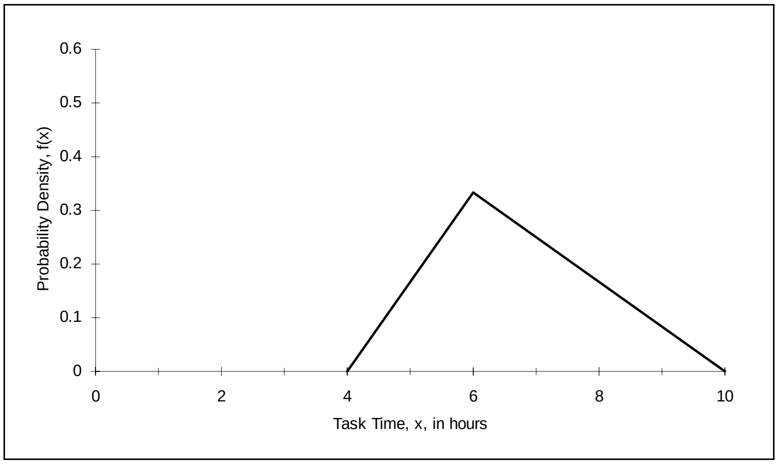
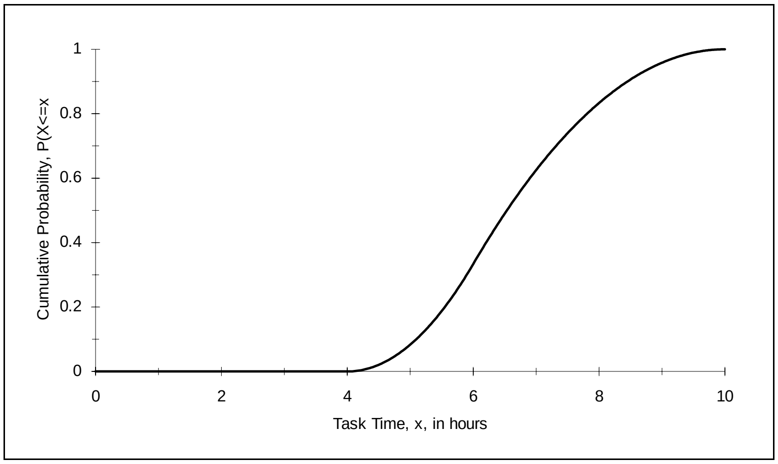


Figure 3.17 RandTriangular Example Cumulative Probability Function



RandUniform

Returns a uniformly distributed random value between two values you specify. As a special case, RANDUNIFORM(0,1) is the same as Excel's built-in RAND() function.

RANDUNIFORM Syntax: RANDUNIFORM(minimum,maximum)

Minimum is the smallest value RANDUNIFORM will return.

Maximum is the largest value RANDUNIFORM will return.

RANDUNIFORM Remarks

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if an argument is text and the name is undefined.

Returns #NUM! if minimum is greater than or equal to maximum.

Returns #VALUE! if an argument is a defined name of a cell and the cell is blank or contains text.

RANDUNIFORM Example

A corporate planner thinks that the company's product will garner between 10% and 15% of the total market, with all possible percentages equally likely in the specified range. The uncertain market proportion is =RANDUNIFORM(0.10,0.15).

RANDUNIFORM Related Function

FASTUNIFORM: Same as RANDUNIFORM without any error checking of the arguments.

3.10 RISKSIM TECHNICAL DETAILS

RiskSim's random number generator functions are based on a (0,1) uniformly distributed random number function called RandSeed which is not directly accessible by the user. Internally, decimal values for RandSeed are calculated by dividing a uniformly distributed random integer by 2147483647, which is RandSeed's period. Random integers are generated using the well-documented Park-Miller algorithm, where each random integer depends on the previous random integer.

When RiskSim starts, the initial integer seed depends on the system clock. Unlike Excel's RAND() function, you can use RiskSim at any time to specify an integer seed, which is used as the previous random integer for the sequence of random numbers generated by the RiskSim functions.

In the Risk Simulation dialog box, the "Random number seed" edit box changes the seed only for the RiskSim functions; it does not have any effect on Excel's built-in RAND() function.

Each of RiskSim's random number generator functions use RandSeed as a building block.

RANDBINOMIAL(trials,probability_s) uses RandSeed as the cumulative probability in Excel's built-in CRITBINOM function.

RANDCUMULATIVE(value_cumulative_table) uses the value of RandSeed, R, searches to find the adjacent cumulative probabilities that bracket R, and interpolates on the linear segment of the cumulative distribution to find the corresponding value.

RANDDISCRETE(value_discrete_table) compares RandSeed with summed probabilities of the input table until the sum exceeds the RandSeed value, and then returns the previous value from the input table.

RANDEXPONENTIAL(lambda) uses the value of RandSeed, R, as follows. If the exponential density function is f(t) = lambda*EXP(-lambda*t), the cumulative is P(T<=t) = 1 - EXP(-lambda*t). Associating R with P(T<=t), the inverse cumulative is t = -LN(1-R)/lambda. Since R and 1-R are both uniformly distributed between 0 and 1, RiskSim uses -LN(R)/lambda for the returned value.

RANDINTEGER(bottom,top) returns bottom + INT(RandSeed*(top-bottom+1)).

RANDNORMAL(mean,standard_dev) uses two RandSeed values in the well-documented Box-Muller method.7

RANDPOISSON(mean) compares RandSeed with cumulative probabilities of Excel's built-in POISSON function until the probability exceeds the RandSeed value, and then returns the previous value.

RANDTRIANGULAR(minimum,most_likely,maximum) uses RandSeed once. The triangular density function has two linear segments, so the cumulative distribution has two quadratic segments. The returned value is determined by interpolation on the appropriate quadratic segment.

RANDUNIFORM(minimum,maximum) returns minimum + RandSeed*(maximum-minimum). RANDUNIFORM(0,1) is equivalent to Excel's built-in RAND() function.

RiskSim includes a FAST... version of each of the nine functions, e.g., FASTBINOMIAL, FASTCUMULATIVE, etc. The FAST... functions are identical to the RAND... functions except there is no error checking of arguments.

3.11 MODELING UNCERTAIN RELATIONSHIPS

Base Model, Four Inputs

Price is fixed. The three uncontrollable inputs are independent.

Figure 3.18 Four Inputs Influence Chart

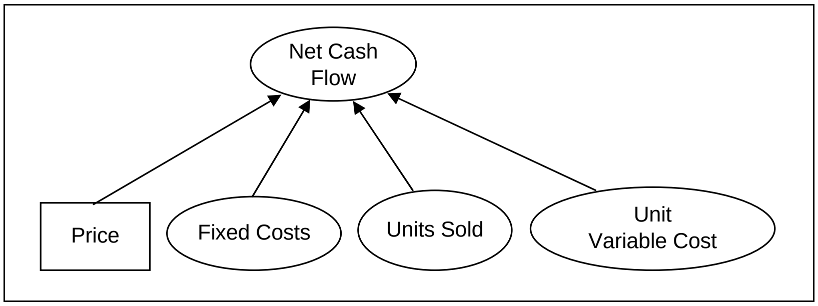


Figure 3.19 Four Inputs Display

	A	B
1	**Controllable Input**	
2	Price	$29
3	**Uncontrollable Inputs**	
4	Fixed Costs	$12,000
5	Units Sold	700
6	Unit Variable Cost	$8
7	**Output Variable**	
8	Net Cash Flow	$2,700

Figure 3.20 Four Inputs Formulas

	A	B
1	**Controllable Input**	
2	Price	29
3	**Uncontrollable Inputs**	
4	Fixed Costs	12000
5	Units Sold	700
6	Unit Variable Cost	8
7	**Output Variable**	
8	Net Cash Flow	=(B2-B6)*B5-B4

Three Inputs

Price is variable. Units sold depends on price. The two cost inputs are independent.

Figure 3.21 Three Inputs Influence Chart

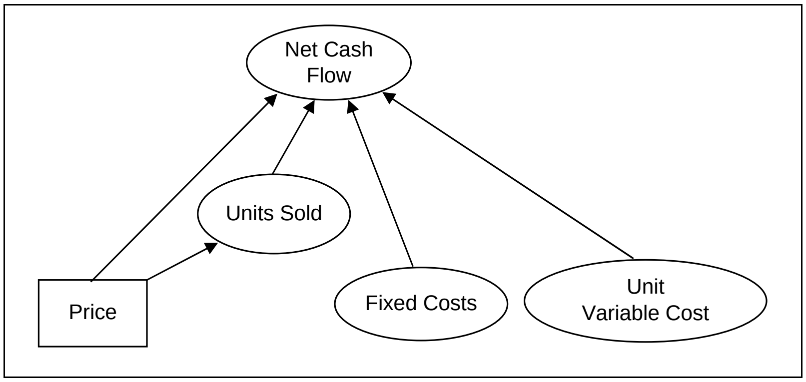


Figure 3.22 Three Inputs Display

	A	B	C	D	E
1	**Controllable Input**			Price	Units Sold
2	Price	$29		$29	700
3	**Uncontrollable Inputs**			$39	550
4	Fixed Costs	$12,000		$49	400
5	Unit Variable Cost	$8		$59	250
6	**Intermediate Variable**				
7	Units Sold	700		Slope	-15
8	**Output Variable**			Intercept	1135
9	Net Cash Flow	$2,700			

Figure 3.23 Three Inputs Formulas

	A	B	C	D	E
1	**Controllable Input**			Price	Units Sold
2	Price	29		29	700
3	**Uncontrollable Inputs**			39	550
4	Fixed Costs	12000		49	400
5	Unit Variable Cost	8		59	250
6	**Intermediate Variable**				
7	Units Sold	=E8+E7*B2		Slope	=SLOPE(E2:E5,D2:D5)
8	**Output Variable**			Intercept	=INTERCEPT(E2:E5,D2:D5)
9	Net Cash Flow	=(B2-B5)*B7-B4			

Two Inputs

Price is variable. Units sold depends on price. Unit variable cost depends on fixed costs.

Figure 3.24 Two Inputs Influence Chart

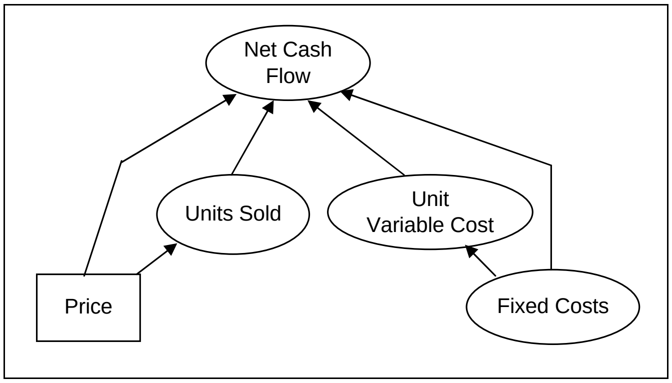
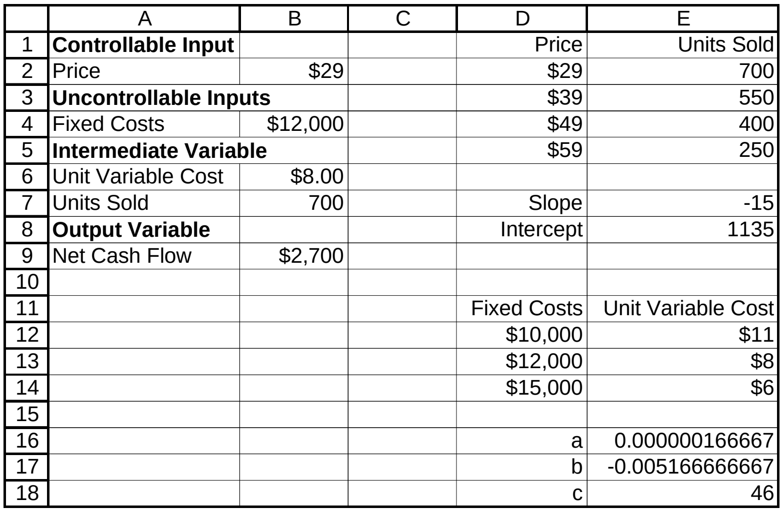
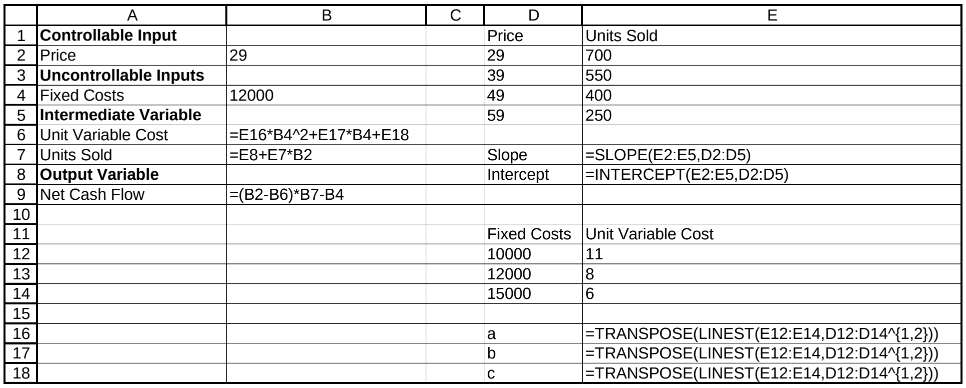


Figure 3.25 Two Inputs Display



	A	B	C	D	E
1	**Controllable Input**			Price	Units Sold
2	Price	$29		$29	700
3	**Uncontrollable Inputs**			$39	550
4	Fixed Costs	$12,000		$49	400
5	**Intermediate Variable**			$59	250
6	Unit Variable Cost	$8.00			
7	Units Sold	700		Slope	-15
8	**Output Variable**			Intercept	1135
9	Net Cash Flow	$2,700			
10					
11				Fixed Costs	Unit Variable Cost
12				$10,000	$11
13				$12,000	$8
14				$15,000	$6
15					
16				a	0.000000166667
17				b	-0.005166666667
18				c	46

Figure 3.26 Two Inputs Formulas



	A	B	C	D	E
1	**Controllable Input**			Price	Units Sold
2	Price	29		29	700
3	**Uncontrollable Inputs**			39	550
4	Fixed Costs	12000		49	400
5	**Intermediate Variable**			59	250
6	Unit Variable Cost	=E16*B4^2+E17*B4+E18			
7	Units Sold	=E8+E7*B2		Slope	=SLOPE(E2:E5,D2:D5)
8	**Output Variable**			Intercept	=INTERCEPT(E2:E5,D2:D5)
9	Net Cash Flow	=(B2-B6)*B7-B4			
10					
11				Fixed Costs	Unit Variable Cost
12				10000	11
13				12000	8
14				15000	6
15					
16				a	=TRANSPOSE(LINEST(E12:E14,D12:D14^{1,2}))
17				b	=TRANSPOSE(LINEST(E12:E14,D12:D14^{1,2}))
18				c	=TRANSPOSE(LINEST(E12:E14,D12:D14^{1,2}))

Four Inputs with Three Uncertainties

Price is variable. Units sold depends on price. Unit variable cost depends on fixed costs.

Fixed costs, units sold, and unit variable cost are uncertain.

Figure 3.27 Three Uncertainties Influence Chart

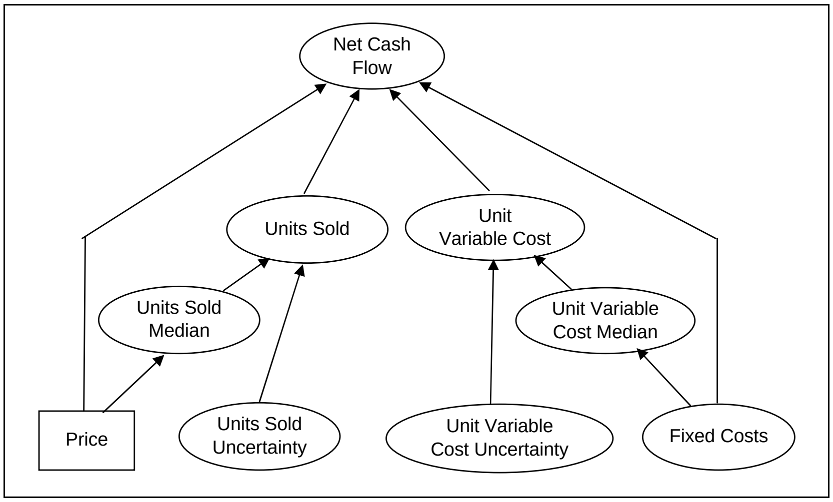
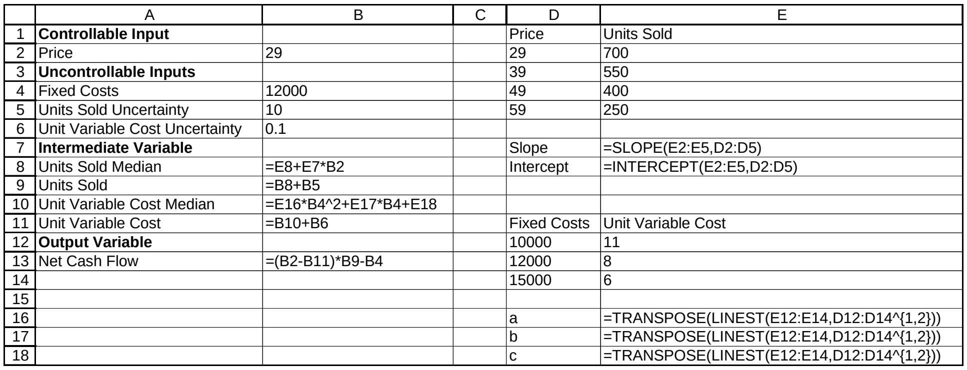


Figure 3.28 Three Uncertainties Display

	A	B	C	D	E
1	**Controllable Input**			Price	Units Sold
2	Price	$29		$29	700
3	**Uncontrollable Inputs**			$39	550
4	Fixed Costs	$12,000		$49	400
5	Units Sold Uncertainty	10		$59	250
6	Unit Variable Cost Uncertainty	$0.10			
7	**Intermediate Variable**			Slope	-15
8	Units Sold Median	700		Intercept	1135
9	Units Sold	710			
10	Unit Variable Cost Median	$8.00			
11	Unit Variable Cost	$8.10		Fixed Costs	Unit Variable Cost
12	**Output Variable**			$10,000	$11
13	Net Cash Flow	$2,839		$12,000	$8
14				$15,000	$6
15					
16				a	0.000000166667
17				b	-0.005166666667
18				c	46

Figure 3.29 Three Uncertainties Formulas



	A	B	C	D	E
1	**Controllable Input**			Price	Units Sold
2	Price	29		29	700
3	**Uncontrollable Inputs**			39	550
4	Fixed Costs	12000		49	400
5	Units Sold Uncertainty	10		59	250
6	Unit Variable Cost Uncertainty	0.1			
7	**Intermediate Variable**			Slope	=SLOPE(E2:E5,D2:D5)
8	Units Sold Median	=E8+E7*B2		Intercept	=INTERCEPT(E2:E5,D2:D5)
9	Units Sold	=B8+B5			
10	Unit Variable Cost Median	=E16*B4^2+E17*B4+E18			
11	Unit Variable Cost	=B10+B6		Fixed Costs	Unit Variable Cost
12	**Output Variable**			10000	11
13	Net Cash Flow	=(B2-B11)*B9-B4		12000	8
14				15000	6
15					
16				a	=TRANSPOSE(LINEST(E12:E14,D12:D14^{1,2}))
17				b	=TRANSPOSE(LINEST(E12:E14,D12:D14^{1,2}))
18				c	=TRANSPOSE(LINEST(E12:E14,D12:D14^{1,2}))

Intermediate Details

Price is variable. Units sold depends on price. Unit variable cost depends on fixed costs.

Fixed costs, units sold, and unit variable cost are uncertain.

Include revenue, total variable cost, and total costs as intermediate variables.

Figure 3.30 Intermediate Details Influence Chart

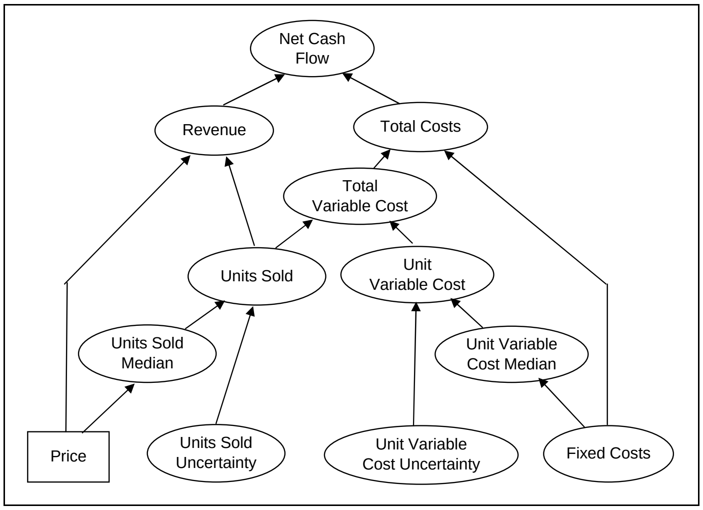


Figure 3.31 Intermediate Details Display

	A	B	C	D	E
1	**Controllable Input**			Price	Units Sold
2	Price	$29		$29	700
3	**Uncontrollable Inputs**			$39	550
4	Fixed Costs	$12,000		$49	400
5	Units Sold Uncertainty	10		$59	250
6	Unit Variable Cost Uncertainty	$0.10			
7	**Intermediate Variable**			Slope	-15
8	Units Sold Median	700		Intercept	1135
9	Units Sold	710			
10	Revenue	$20,590			
11	Unit Variable Cost Median	$8.00		Fixed Costs	Unit Variable Cost
12	Unit Variable Cost	$8.10		$10,000	$11
13	Total Variable Cost	$5,751		$12,000	$8
14	Total Costs	$17,751		$15,000	$6
15	**Output Variable**				
16	Net Cash Flow	$2,839		a	0.000000166667
17				b	-0.005166666667
18				c	46

Figure 3.32 Intermediate Details Formulas

	A	B	C	D	E
1	**Controllable Input**			Price	Units Sold
2	Price	29		29	700
3	**Uncontrollable Inputs**			39	550
4	Fixed Costs	12000		49	400
5	Units Sold Uncertainty	10		59	250
6	Unit Variable Cost Uncertainty	0.1			
7	**Intermediate Variable**			Slope	=SLOPE(E2:E5,D2:D5)
8	Units Sold Median	=E8+E7*B2		Intercept	=INTERCEPT(E2:E5,D2:D5)
9	Units Sold	=B8+B5			
10	Revenue	=B9*B2			
11	Unit Variable Cost Median	=E16*B4^2+E17*B4+E1		Fixed Costs	Unit Variable Cost
12	Unit Variable Cost	=B11+B6		10000	11
13	Total Variable Cost	=B12*B9		12000	8
14	Total Costs	=B4+B13		15000	6
15	**Output Variable**				
16	Net Cash Flow	=B10-B14		a	=TRANSPOSE(LINEST(E12:E14,D12:D14^{1,2}))
17				b	=TRANSPOSE(LINEST(E12:E14,D12:D14^{1,2}))
18				c	=TRANSPOSE(LINEST(E12:E14,D12:D14^{1,2}))

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Decision Trees Using TreePlan

4

TreePlan is a decision tree add-in for Microsoft Excel 97 (and later versions of Excel) for Windows and Macintosh. It was developed by Professor Michael R. Middleton at the University of San Francisco and modified for use at Fuqua (Duke) by Professor James E. Smith.

4.1 TREEPLAN INSTALLATION

All of TreePlan's functionality is in a single file, TreePlan.xla. Depending on your preference, there are three ways to install TreePlan. (These instructions also apply to the other Decision ToolPak add-ins: SensIt.xla and RiskSim.xla.)

Occasional Use

If you plan to use TreePlan on an irregular basis, simply use Excel's File | Open command to load TreePlan.xla each time you want to use it. You may keep the TreePlan.xla file on a floppy disk, your computer's hard drive, or a network server.

Selective Use

You can use Excel's Add-In Manager to install TreePlan. First, copy TreePlan.xla to a location on your computer's hard drive. Second, if you save TreePlan.xla in the Excel or Office Library subdirectory, go to the third step. Otherwise, run Excel, choose Tools | Add-Ins; in the Add-Ins dialog box, click the Browse button, use the Browse dialog box to specify the location of TreePlan.xla, and click OK. Third, in the Add-Ins dialog box, note that TreePlan is now listed with a check mark, indicating that its menu command will appear in Excel, and click OK.

If you plan to not use TreePlan and you want to free up main memory, uncheck the box for TreePlan in the Add-In Manager. When you do want to use TreePlan, choose Tools | Add-Ins and check TreePlan's box.

To remove TreePlan from the Add-In Manager, use Windows Explorer or another file manager to delete TreePlan.xla from the Library subdirectory or from the location you specified when you used the Add-In Manager’s Browse command. The next time you start Excel and choose Tools | Add-Ins, a dialog box will state “Cannot find add-in … treeplan.xla. Delete from list?” Click Yes.

Steady Use

If you want TreePlan’s options immediately available each time you run Excel, use Windows Explorer or another file manager to save TreePlan.xla in the Excel XLStart directory. Alternatively, in Excel you can use Tools | Options | General to specify an alternate startup file location and use a file manager to save TreePlan.xla there. When you start Excel, it tries to open all files in the XLStart directory and in the alternate startup file location.

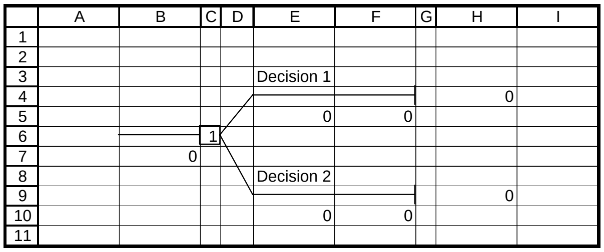
For additional information visit “TreePlan FAQ” at www.treeplan.com.

After opening TreePlan.xla in Excel, the command "Decision Tree" appears at the bottom of the Tools menu (or, if you have a customized main menu, at the bottom of the sixth main menu item).

4.2 BUILDING A DECISION TREE IN TREEPLAN

You can start TreePlan either by choosing **Tools | Decision Tree** from the menu bar or by pressing **Ctrl+t** (hold down the Ctrl key and press t). If the worksheet doesn't have a decision tree, TreePlan prompts you with a dialog box with three options; choose **New Tree** to begin a new tree. TreePlan draws a default initial decision tree with its upper left corner at the selected cell. For example, the figure below shows the initial tree when $B$2 is selected. (Note that TreePlan writes over existing values in the spreadsheet: begin your tree to the *right* of the area where your data is stored, and *do not subsequently add or delete rows or columns in the tree-diagram area*.) In Excel 5 and 95 a terminal node is represented by a triangle instead of a vertical bar.

Figure 4.1 TreePlan Initial Default Decision Tree



	A	B	C	D	E	F	G	H	I
1									
2									
3					Decision 1				
4								0	
5					0	0			
6			1						
7		0							
8					Decision 2				
9								0	
10					0	0			
11									

Build up a tree by adding or modifying branches or nodes in the default tree. To change the branch labels or probabilities, click on the cell containing the label or probability and type the new label or probability. To modify the structure of the tree (e.g., add or delete branches or nodes in the tree), select the node or the cell containing the node in the tree to modify, and choose **Tools | Decision Tree** or press **Ctrl**+**t**. TreePlan will then present a dialog box showing the available commands.

For example, to add an event node to the top branch of the tree shown above, select the square cell (cell G4) next to the vertical line at the end of a terminal branch and press **Ctrl**+**t**.. TreePlan then presents this dialog box.

Figure 4.2 TreePlan Terminal Dialog Box

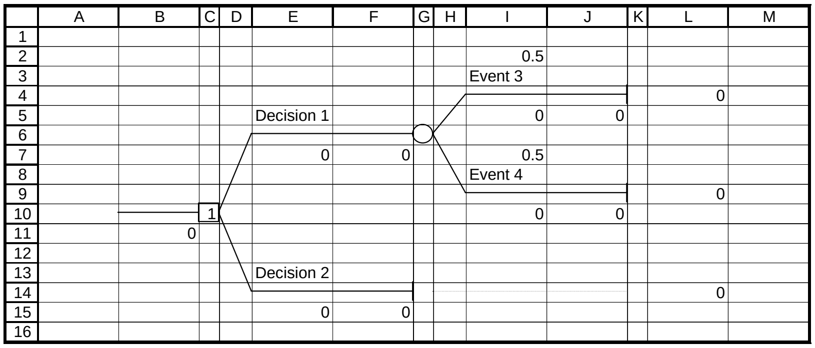
TreePlan...Terminal

● Change to decision node
○ Change to event node
○ Paste subtree
○ Remove previous branch

Branches
○ One
● Two
○ Three
○ Four
○ Five

To add an event node to the branch, we change the selected terminal node to an event node by selecting **Change to event node** in the dialog box, selecting the number of branches (here two), and pressing **OK**. TreePlan then redraws the tree with a chance node in place of the terminal node.

Figure 4.3



	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2									0.5				
3									Event 3				
4												0	
5					Decision 1				0	0			
6													
7					0	0			0.5				
8									Event 4				
9												0	
10			1						0	0			
11		0											
12													
13					Decision 2								
14												0	
15					0	0							
16													

The dialog boxes presented by TreePlan vary depending on what you have selected when you choose **Tools | Decision Tree** or press **Ctrl**+**t**. The dialog box shown below is presented when you press **Ctrl**+**t** with an event node selected; a similar dialog box is presented when you select a decision node. If you want to add a branch to the selected node, choose **Add branch** and press **OK**. If you want to insert a decision or event node before the selected node, choose **Insert decision** or **Insert event** and press **OK**. To get a description of the available commands, click on the **Help** button.

Figure 4.4

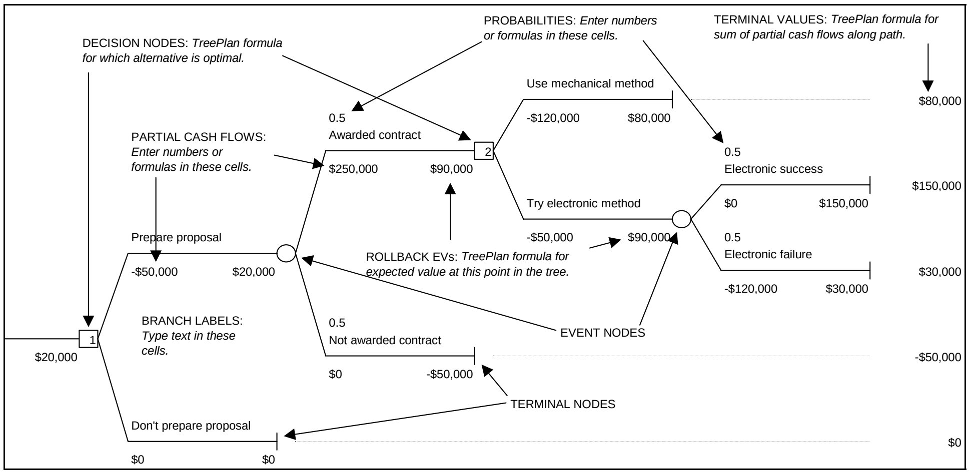
The **Copy subtree** command is particularly useful when building large trees. If two or more parts of the tree are similar, you can copy and paste "subtrees" rather than building up each part separately. To copy a subtree, select the node at the root of the subtree and choose **Copy subtree**. This tells TreePlan to copy the selected node and everything to the right of it in the tree. To paste this subtree, select a terminal node and choose **Paste subtree**. TreePlan then duplicates the specified subtree at the selected terminal node.

Since TreePlan decision trees are built directly in Excel, you can use Excel's commands to format your tree. For example, you can use bold or italic fonts for branch labels: select the cells you want to format and change them using Excel's formatting commands. To help you, TreePlan provides a **Select** dialog box that appears when you choose **Tools Decision Tree** or press **Ctrl+t** without a node selected. You can also bring up this dialog box by pressing the **Select** button on the **Node** dialog box. From here, you can select all items of a particular type in the tree. For example, if you choose **Probabilities** and press **OK**, TreePlan selects all cells containing probabilities in the tree. You can then format all of the probabilities simultaneously using Excel's formatting commands. (Because of limitations in Excel, the **Select** dialog box will not be available when working with very large trees.)

4.3 ANATOMY OF A TREEPLAN DECISION TREE

An example of a TreePlan decision tree is shown below. In the example, a firm must decide (1) whether to prepare a proposal for a possible contract and (2) which method to use to satisfy the contract. The tree consists of decision nodes, event nodes and terminal nodes connected by branches. Each branch is surrounded by cells containing formulas, cell references, or labels pertaining to that branch. You may edit the labels, probabilities, and partial cash flows associated with each branch. The partial cash flows are the amount the firm "gets paid" to go down that branch. Here, the firm pays $50,000 if it decides to prepare the proposal, receives $250,000 up front if awarded the contract, spends $50,000 to try the electronic method, and spends $120,000 on the mechanical method if the electronic method fails.

Figure 4.5



The trees are "solved" using formulas embedded in the spreadsheet. The *terminal values* sum all the partial cash flows along the path leading to that terminal node. The tree is then "rolled back" by computing expected values at event nodes and by maximizing at decision nodes; the *rollback EVs* appear next to each node and show the expected value at that point in the tree. The numbers in the decision nodes indicate which alternative is optimal for that decision. In the example, the "1" in the first decision node indicates that it is optimal to prepare the proposal, and the "2" in the second decision node indicates the firm should try the electronic method because that alternative leads to a higher expected value, $90,000, than the mechanical method, $80,000.

TreePlan has a few options that control the way calculations are done in the tree. To select these options, press the **Options** button in any of TreePlan's dialog boxes. The first choice is whether to **Use Expected Values** or **Use Exponential Utility Function** for computing certainty equivalents. The default is to rollback the tree using expected values. If you choose to use exponential utilities, TreePlan will compute utilities of endpoint cash flows at the terminal nodes and compute expected utilities instead of expected values at event nodes. Expected utilities are calculated in the cell below the certainty equivalents. You may also choose to **Maximize (profits)** or **Minimize (costs)** at decision nodes; the default is to maximize profits. If you choose to minimize costs instead, the cash flows are interpreted as costs, and decisions are made by choosing the minimum expected value or certainty equivalent rather than the maximum. See the Help file for details on these options.

4.4 STEP-BY-STEP TREEPLAN TUTORIAL

A decision tree can be used as a model for a sequential decision problems under uncertainty. A decision tree describes graphically the decisions to be made, the events that may occur, and the outcomes associated with combinations of decisions and events. Probabilities are assigned to the events, and values are determined for each outcome. A major goal of the analysis is to determine the best decisions.

Decision tree models include such concepts as nodes, branches, terminal values, strategy, payoff distribution, certainty equivalent, and the rollback method. The following problem illustrates the basic concepts.

DriveTek Problem

DriveTek Research Institute discovers that a computer company wants a new tape drive for a proposed new computer system. Since the computer company does not have research people available to develop the new drive, it will subcontract the development to an independent research firm. The computer company has offered a fee of $250,000 for the best proposal for developing the new tape drive. The contract will go to the firm with the best technical plan and the highest reputation for technical competence.

DriveTek Research Institute wants to enter the competition. Management estimates a cost of $50,000 to prepare a proposal with a fifty-fifty chance of winning the contract.

However, DriveTek's engineers are not sure about how they will develop the tape drive if they are awarded the contract. Three alternative approaches can be tried. The first approach is a mechanical method with a cost of $120,000, and the engineers are certain they can develop a successful model with this approach. A second approach involves electronic components. The engineers estimate that the electronic approach will cost only $50,000 to develop a model of the tape drive, but with only a 50 percent chance of satisfactory results. A third approach uses magnetic components; this costs $80,000, with a 70 percent chance of success.

DriveTek Research can work on only one approach at a time and has time to try only two approaches. If it tries either the magnetic or electronic method and the attempt fails, the second choice must be the mechanical method to guarantee a successful model.

The management of DriveTek Research needs help in incorporating this information into a decision to proceed or not.

[Source: The tape drive example is adapted from Spurr and Bonini, *Statistical Analysis for Business Decisions*, Irwin.]

Nodes and Branches

Decision trees have three kinds of nodes and two kinds of branches. A decision node is a point where a choice must be made; it is shown as a square. The branches extending from a decision node are decision branches, each branch representing one of the possible alternatives or courses of action available at that point. The set of alternatives must be mutually exclusive (if one is chosen, the others cannot be chosen) and collectively exhaustive (all possible alternatives must be included in the set).

There are two major decisions in the DriveTek problem. First, the company must decide whether or not to prepare a proposal. Second, if it prepares a proposal and is awarded the contract, it must decide which of the three approaches to try to satisfy the contract.

An event node is a point where uncertainty is resolved (a point where the decision maker learns about the occurrence of an event). An event node, sometimes called a "chance node," is shown as a circle. The event set consists of the event branches extending from an event node, each branch representing one of the possible events that may occur at that point. The set of events must be mutually exclusive (if one occurs, the others cannot occur) and collectively exhaustive (all possible events must be included in the set). Each event is assigned a subjective probability; the sum of probabilities for the events in a set must equal one.

The three sources of uncertainty in the DriveTek problem are: whether it is awarded the contract or not, whether the electronic approach succeeds or fails, and whether the magnetic approach succeeds or fails.

In general, decision nodes and branches represent the controllable factors in a decision problem; event nodes and branches represent uncontrollable factors.

Decision nodes and event nodes are arranged in order of subjective chronology. For example, the position of an event node corresponds to the time when the decision maker learns the outcome of the event (not necessarily when the event occurs).

The third kind of node is a terminal node, representing the final result of a combination of decisions and events. Terminal nodes are the endpoints of a decision tree, shown as the end of a branch on hand-drawn diagrams and as a triangle on computer-generated diagrams.

The following table shows the three kinds of nodes and two kinds of branches used to represent a decision tree.

Figure 4.6 Nodes and Symbols

Type of Node	Written Symbol	Computer Symbol	Node Successor
Decision	square	square	decision branches
Event	circle	circle	event branches
Terminal	endpoint	triangle or bar	terminal value

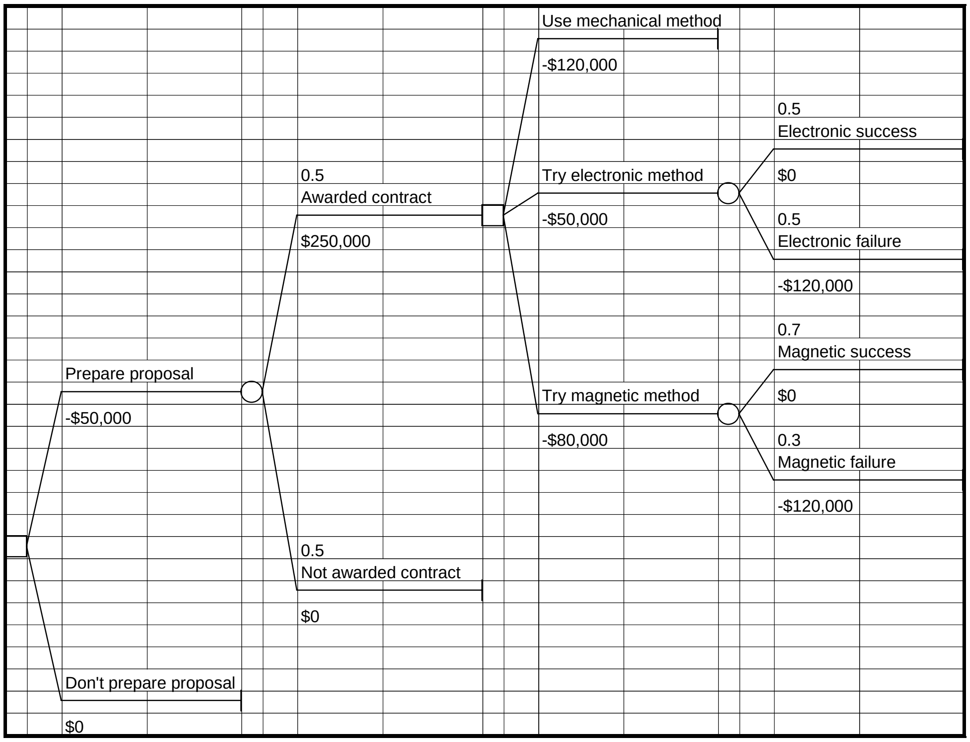
Terminal Values

Each terminal node has an associated terminal value, sometimes called a payoff value, outcome value, or endpoint value. Each terminal value measures the result of a *scenario*: the sequence of decisions and events on a unique path leading from the initial decision node to a specific terminal node.

To determine the terminal value, one approach assigns a cash flow value to each decision branch and event branch and then sum the cash flow values on the branches leading to a terminal node to determine the terminal value. In the DriveTek problem, there are distinct cash flows associated with many of the decision and event branches. Some problems require a more elaborate value model to determine the terminal values.

The following diagram shows the arrangement of branch names, probabilities, and cash flow values on an unsolved tree.

Figure 4.7



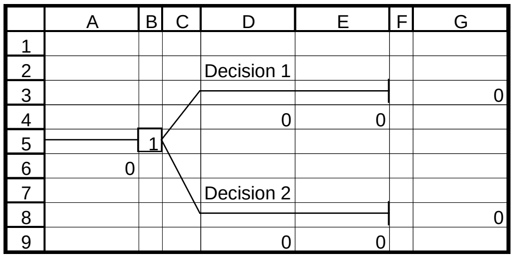
To build the decision tree, you use TreePlan's dialog boxes to develop the structure. You enter a branch name, branch cash flow, and branch probability (for an event) in the cells above and below the left side of each branch. As you build the tree diagram, TreePlan enters formulas in other cells.

Building the Tree Diagram

1. Start with a new worksheet. (If no workbook is open, choose File | New. If a workbook is open, choose Insert | Worksheet.)
2. Select cell A1. From the Tools menu, choose Decision Tree. In the TreePlan New dialog box, click the New Tree button. A decision node with two branches appears.

Figure 4.8

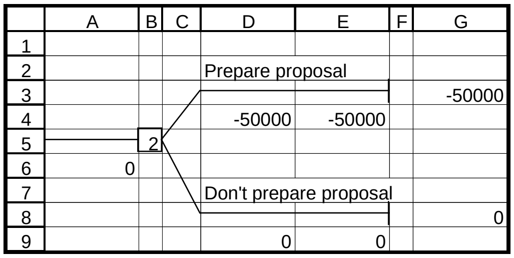
Figure 4.9



	A	B	C	D	E	F	G
1							
2				Decision 1			
3							0
4				0	0		
5		1					
6	0						
7				Decision 2			
8							0
9				0	0		

3. Do not type the quotation marks in the following instructions. Select cell D2, and enter **Prepare proposal**. Select cell D4, and enter **–50000**. Select cell D7, and enter **Don't prepare proposal**.

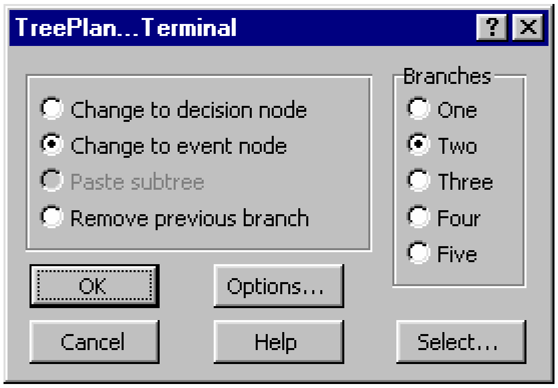
Figure 4.10



	A	B	C	D	E	F	G
1							
2				Prepare proposal			
3							-50000
4				-50000	-50000		
5		2					
6	0						
7				Don't prepare proposal			
8							0
9				0	0		

4. Select cell F3. From the Tools menu, choose Decision Tree. In the TreePlan Terminal dialog box, select Change To Event Node, select Two Branches, and click OK. The tree is redrawn.

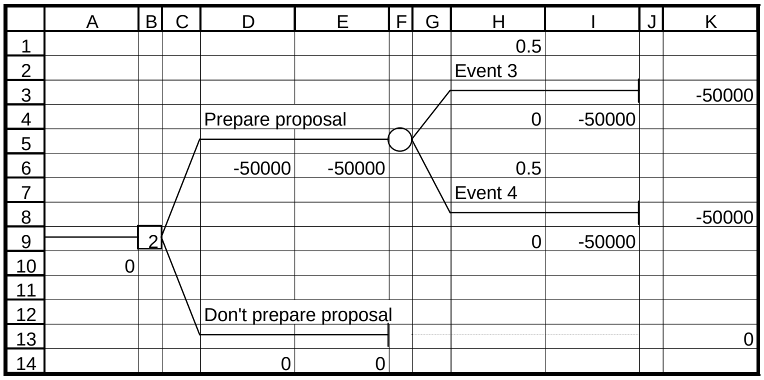
Figure 4.11


TreePlan...Terminal

○ Change to decision node
● Change to event node
○ Paste subtree
○ Remove previous branch

Branches
○ One
● Two
○ Three
○ Four
○ Five

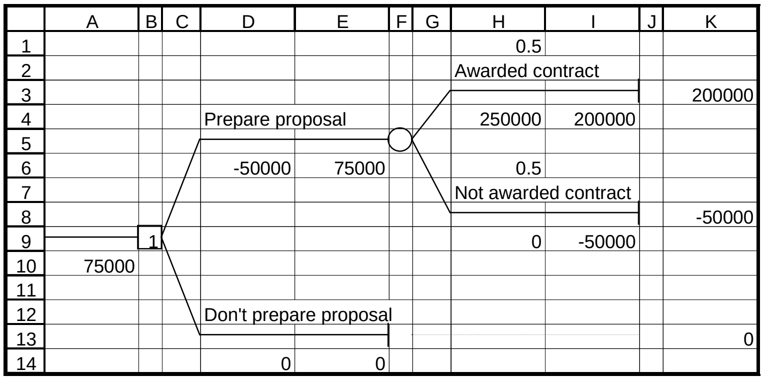
Figure 4.12



	A	B	C	D	E	F	G	H	I	J	K
1								0.5			
2								Event 3			
3											-50000
4				Prepare proposal				0	-50000		
5											
6				-50000	-50000			0.5			
7								Event 4			
8											-50000
9		2						0	-50000		
10	0										
11											
12				Don't prepare proposal							
13											0
14				0	0						

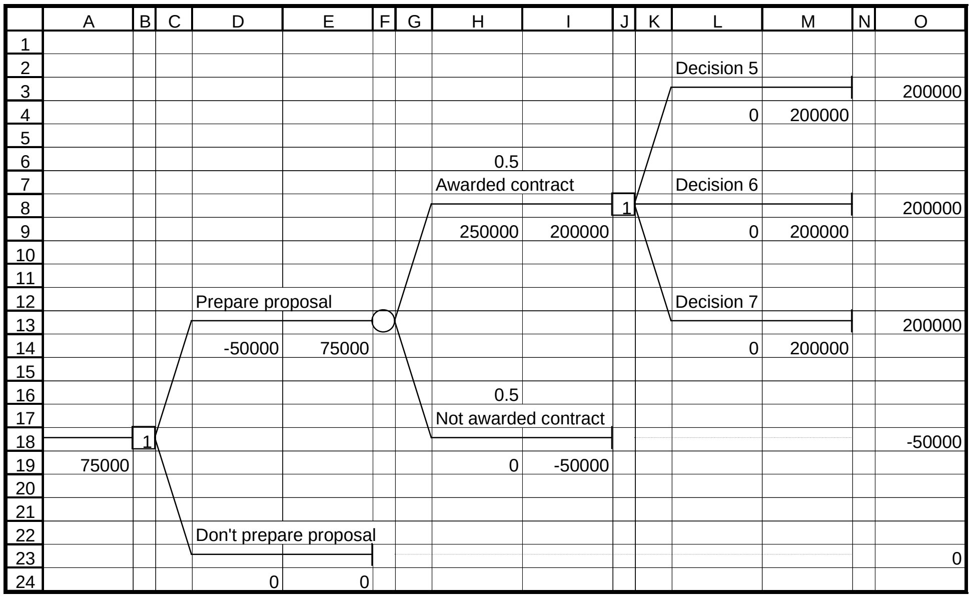
5. Select cell H2, and enter **Awarded contract**. Select cell H4, and enter **250000**. Select cell H7, and enter **Not awarded contract**.

Figure 4.13



6. Select cell J3. From the Tools menu, choose Decision Tree. In the TreePlan Terminal dialog box, select Change To Decision Node, select Three Branches, and click OK. The tree is redrawn.

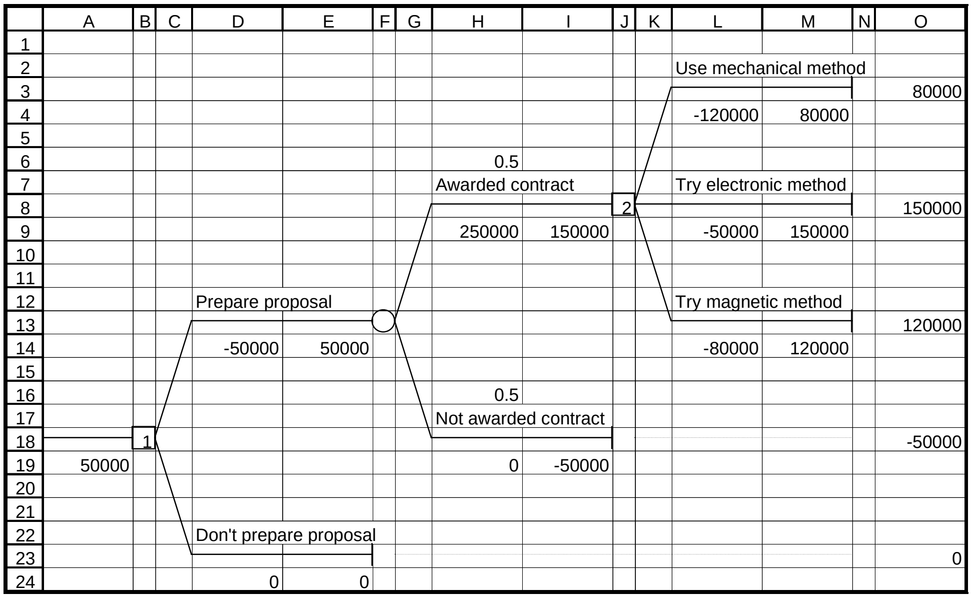
Figure 4.14



7. Select cell L2, and enter **Use mechanical method**. Select cell L4, and enter **–120000**. Select cell L7, and enter **Try electronic method**. Select cell L9, and

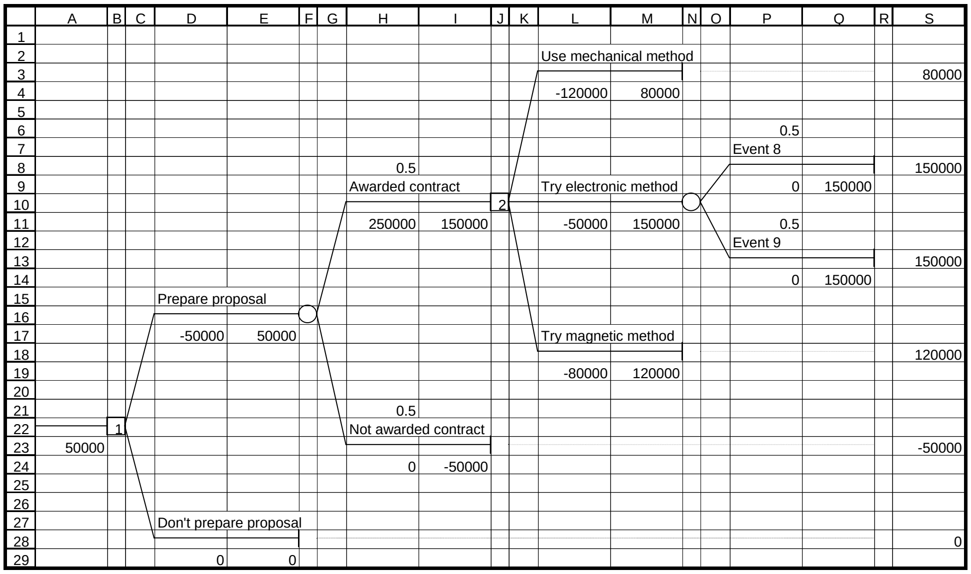
enter **–50000**. Select cell L12, and enter **Try magnetic method**. Select cell L14, and enter **–80000**.

Figure 4.15



8. Select cell N8. From the Tools menu, choose Decision Tree. In the TreePlan Terminal dialog box, select Change To Event Node, select Two Branches, and click OK. The tree is redrawn.

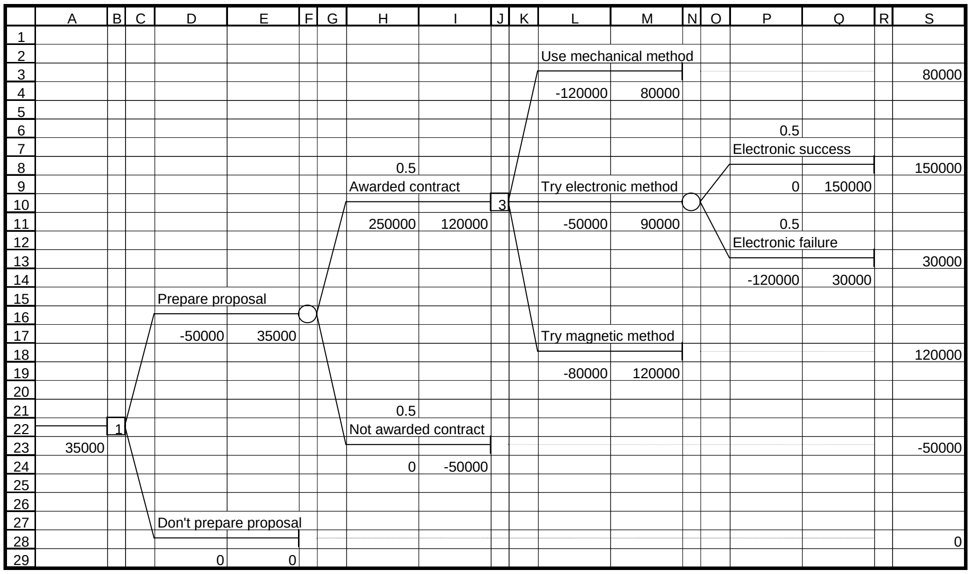
Figure 4.16



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1																			
2												Use mechanical method							
3																			80000
4												-120000	80000						
5																			
6																0.5			
7																Event 8			
8								0.5											150000
9								Awarded contract				Try electronic method				0	150000		
10										2									
11								250000	150000			-50000	150000			0.5			
12																Event 9			
13																			150000
14																0	150000		
15				Prepare proposal															
16																			
17				-50000	50000							Try magnetic method							
18																			120000
19												-80000	120000						
20																			
21								0.5											
22		1						Not awarded contract											
23	50000																		-50000
24								0	-50000										
25																			
26																			
27				Don't prepare proposal															
28																			0
29				0	0														

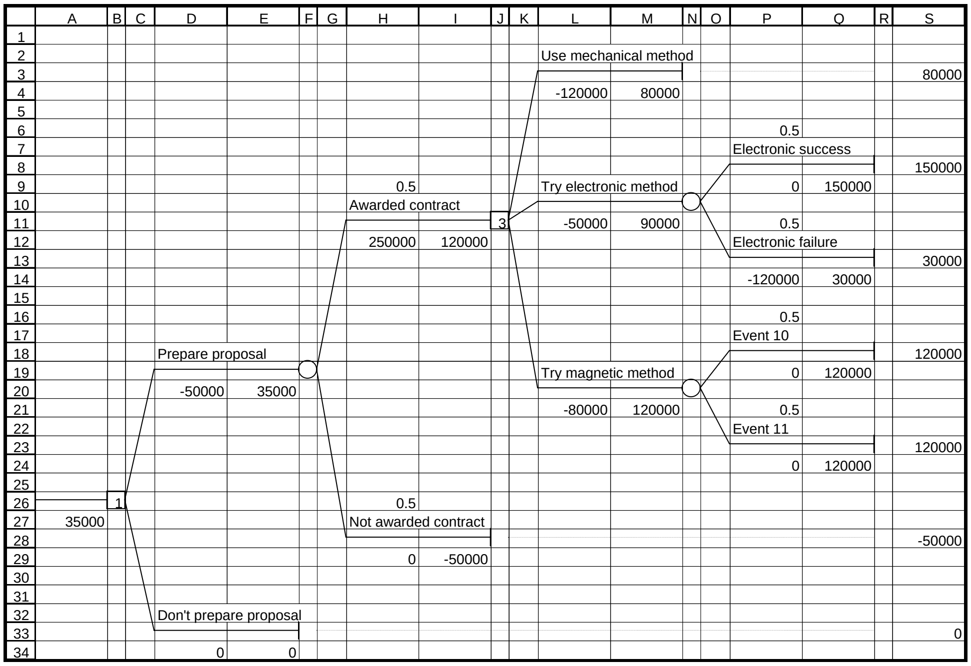
9. Select cell P7, and enter **Electronic success**. Select cell P12, and enter **Electronic failure**. Select cell P14, and enter **–120000**.

Figure 4.17



10. Select cell N18. From the Tools menu, choose Decision Tree. In the TreePlan Terminal dialog box, select Change To Event Node, select Two Branches, and click OK. The tree is redrawn.

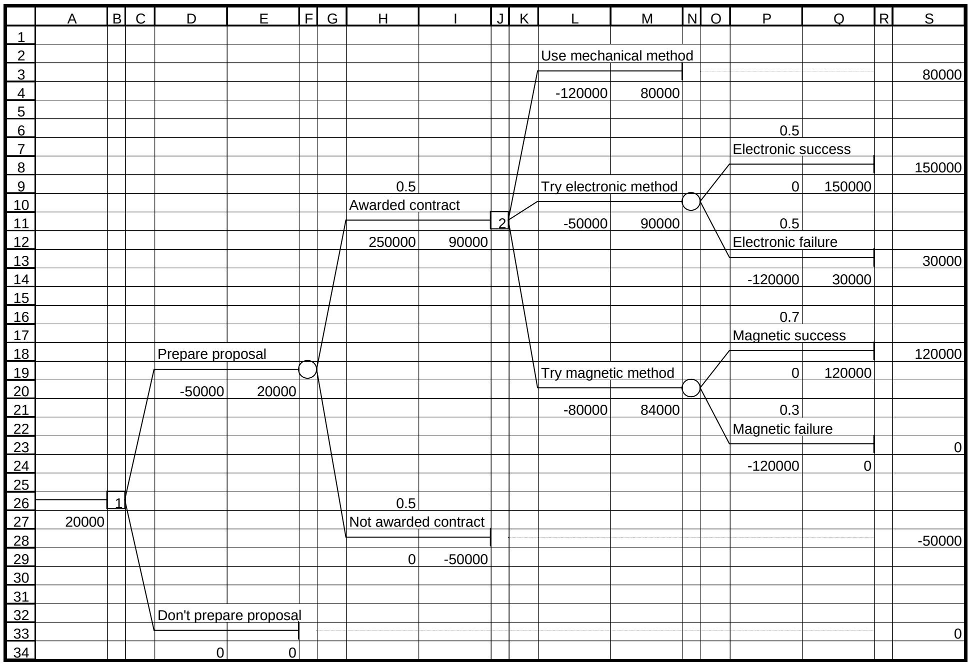
Figure 4.18



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1																			
2												Use mechanical method							
3																			80000
4												-120000	80000						
5																			
6																0.5			
7																Electronic success			
8																			150000
9								0.5				Try electronic method				0	150000		
10								Awarded contract											
11										3		-50000	90000			0.5			
12								250000	120000							Electronic failure			
13																			30000
14																-120000	30000		
15																			
16																0.5			
17																Event 10			
18				Prepare proposal															120000
19												Try magnetic method				0	120000		
20				-50000	35000														
21												-80000	120000			0.5			
22																Event 11			
23																			120000
24																0	120000		
25																			
26		1						0.5											
27	35000							Not awarded contract											
28																			-50000
29								0	-50000										
30																			
31																			
32				Don't prepare proposal															
33																			0
34				0	0														

11. Select cell P16, and enter **.7**. Select cell P17, and enter **Magnetic success**. Select cell P21, and enter **.3**. Select cell P22, and enter **Magnetic failure**. Select cell P24, and enter **–120000**.

Figure 4.19



12. Double-click the sheet tab (or right-click the sheet tab and choose Rename from the shortcut menu), and enter **Original**. Save the workbook.

Interpreting the Results

The $30,000 terminal value on the far right of the diagram in cell S13 is associated with the following scenario:

Figure 4.20

Branch Type	Branch Name	Cash Flow
Decision	Prepare proposal	–$50,000
Event	Awarded contract	$250,000
Decision	Try electronic method	–$50,000
Event	Electronic failure (Use mechanical method)	–$120,000
	Terminal value	$30,000

TreePlan put the formula =SUM(P14,L11,H12,D20) into cell S13 for determining the terminal value.

Other formulas, called rollback formulas, are in cells below and to the left of each node. These formulas are used to determine the optimal choice at each decision node.

In cell B26, a formula displays 1, indicating that the first branch is the optimal choice. Thus, the initial choice is to prepare the proposal. In cell J11, a formula displays 2, indicating that the second branch (numbered 1, 2, and 3, from top to bottom) is the optimal choice. If awarded the contract, DriveTek should try the electronic method. A subsequent chapter provides more details about interpretation.

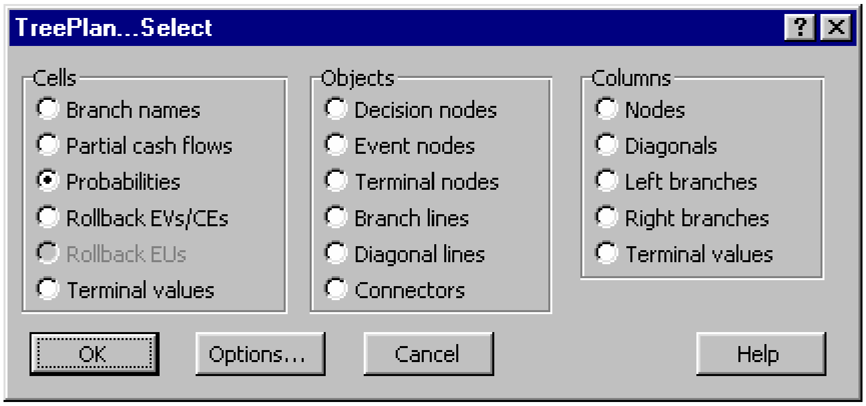
Formatting the Tree Diagram

The following steps show how to use TreePlan and Excel features to format the tree diagram. You may choose to use other formats for your own tree diagrams.

13. From the Edit menu, choose Move or Copy Sheet (or right-click the sheet tab and choose Move Or Copy from the shortcut menu). In the lower left corner of the Move Or Copy dialog box, check the Create A Copy box, and click OK.

14.On sheet Original (2), select cell H9. From the Tools menu, choose Decision Tree. In the TreePlan Select dialog box, verify that the option button for Cells with Probabilities is selected, and click OK. With all probability cells selected, click the Align Left button.

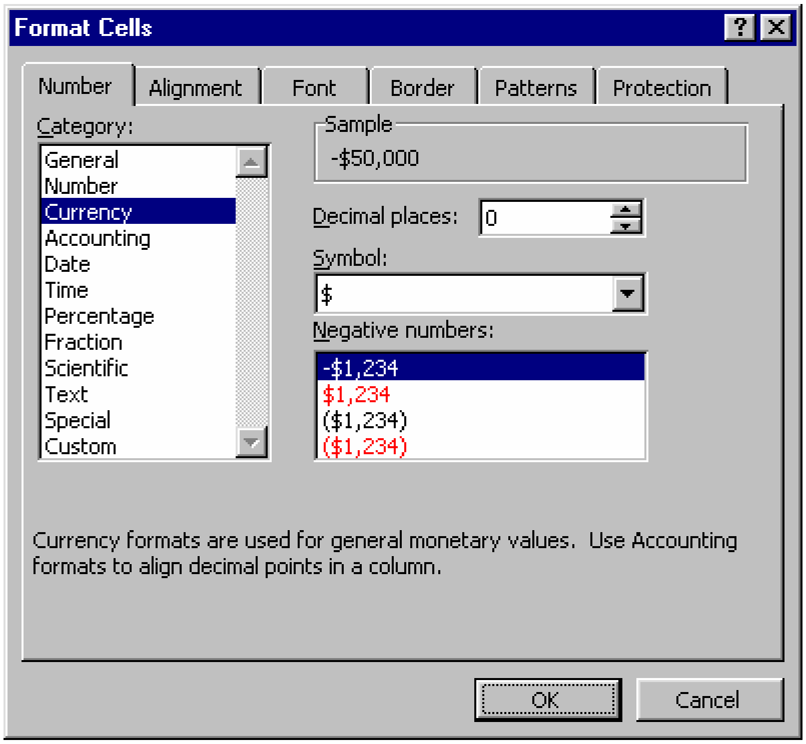
Figure 4.21



15. Select cell H12. From the Tools menu, choose Decision Tree. In the TreePlan Select dialog box, verify that the option button for Cells with Partial Cash Flows is selected, and click OK. With all partial cash flow cells selected, click the Align Left button. With those cells still selected, choose Format | Cells. In the Format Cells dialog box, click the Number tab. In the Category list box, choose

Currency; type **0** (zero) for Decimal Places; select $ in the Symbol list box; select -$1,234 for Negative Numbers. Click OK.

Figure 4.22



Sample: -$50,000

Decimal places: 0

Symbol: $

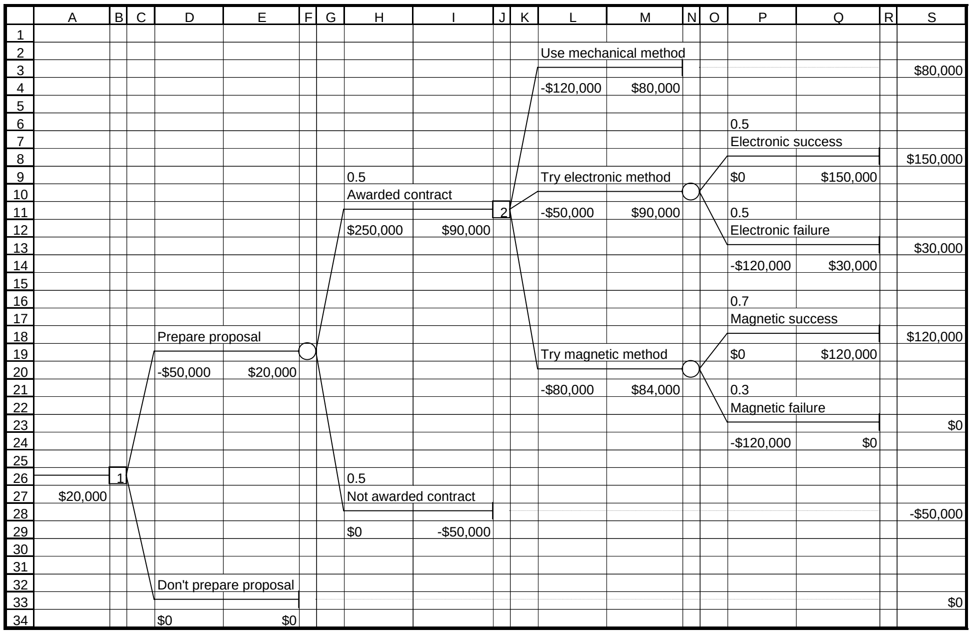
Negative numbers: -$1,234

Currency formats are used for general monetary values. Use Accounting formats to align decimal points in a column.

16. Select cell I12. From the Tools menu, choose Decision Tree. In the TreePlan Select dialog box, verify that the option button for Cells with Rollback EVs/CEs is selected, and click OK. With all rollback cells selected, choose Format | Cells. Repeat the Currency formatting of step 16 above.

17. Select cell S3. From the Tools menu, choose Decision Tree. In the TreePlan Select dialog box, verify that the option button for Cells with Terminal Values is selected, and click OK. With all terminal value cells selected, choose Format | Cells. Repeat the Currency formatting of step 16 above.

Figure 4.23



18. Double-click the Original (2) sheet tab (or right-click the sheet tab and choose Rename from the shortcut menu), and enter **Formatted**. Save the workbook.

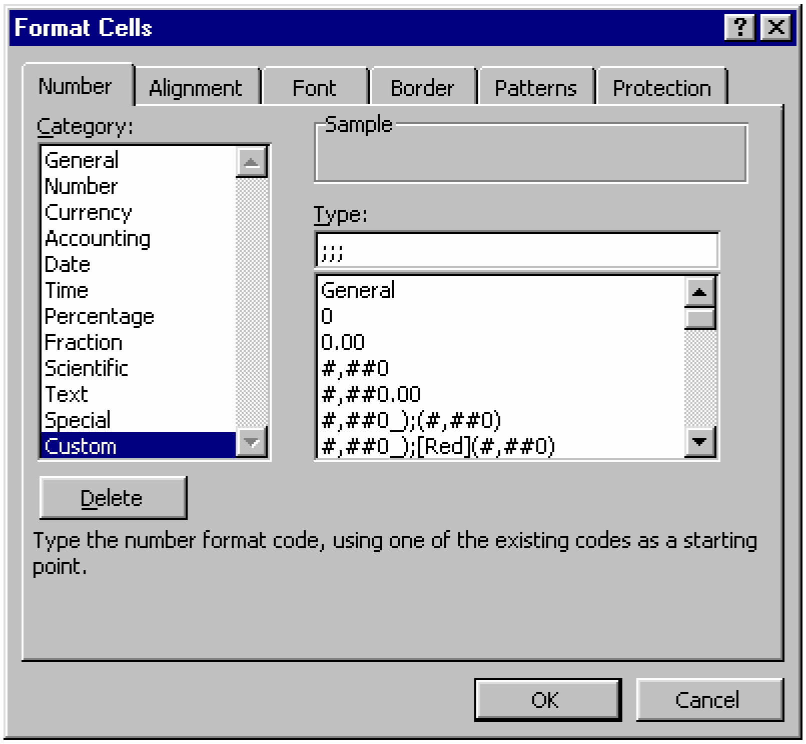
Displaying Model Inputs

When you build a decision tree model, you may want to discuss the model and its assumptions with co-workers or a client. For such communication it may be preferable to hide the results of formulas that show rollback values and decision node choices. The following steps show how to display only the model inputs.

19. From the Edit menu, choose Move or Copy Sheet (or right-click the sheet tab and choose Move Or Copy from the shortcut menu). In the lower left corner of the Move Or Copy dialog box, check the Create A Copy box, and click OK.

20. On sheet Formatted (2), select cell B1. From the Tools menu, choose Decision Tree. In the TreePlan Select dialog box, verify that the option button for Columns with Nodes is selected, and click OK. With all node columns selected, choose Format | Cells | Number. In the Category list box, select Custom. Select the entry in the Type edit box, and type **;;;** (three semicolons). Click OK.

Figure 4.24



Category: Custom

Type: ;;;

Type the number format code, using one of the existing codes as a starting point.

Explanation: A custom number format has four sections of format codes. The sections are separated by semicolons, and they define the formats for positive numbers, negative numbers, zero values, and text, in that order. When you specify three semicolons without format codes, Excel does not display positive numbers, negative numbers, zero values, or text. The formula remains in the cell, but its result is not displayed. Later, if you want to display the result, you can change the format without having to enter the formula again. Editing an existing format does not delete it. All formats are saved with the workbook unless you explicitly delete a format.

21. Select cell A27. From the Tools menu, choose Decision Tree. In the TreePlan Select dialog box, verify that the option button for Cells with Rollback EVs/CEs is selected, and click OK. With all rollback values selected, choose Format | Cells | Number. In the Category list box, select Custom. Scroll to the bottom of the Type list box, and select the three-semicolon entry. Click OK.

22. Double-click the Formatted (2) sheet tab (or right-click the sheet tab and choose Rename from the shortcut menu), and enter **Model Inputs**. Save the workbook.

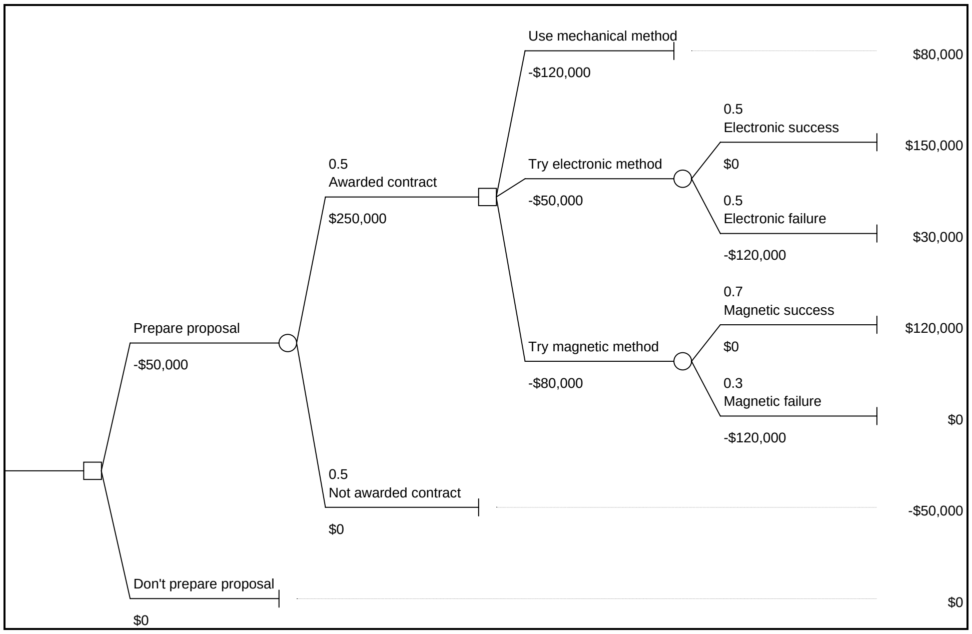
Printing the Tree Diagram

23. In the Name Box list box, select TreeDiagram (or select cells A1:S34).

24. To print the tree diagram from Excel, with the tree diagram range selected choose File | Print Area | Set Print Area. Choose File | Page Setup. In the Page Setup dialog box, click the Page tab; for Orientation click the option button for Landscape, and for Scaling click the option button for Fit To 1 Page Wide By 1 Page Tall. Click the Header/Footer tab; in the Header list box select None, and in the Footer list box select None (or select other appropriate headers and footers). Click the Sheet tab; clear the check box for Gridlines, and clear the check box for Row And Column Headings. Click OK. Choose File | Print and click OK.

25. To print the tree diagram from Word, clear the check boxes for Gridlines and for Row And Column Headings on Excel's Page Setup dialog box Sheet tab. Select the tree diagram range. Hold down the Shift key and from the Edit menu choose Copy Picture. In the Copy Picture dialog box, click the option button As Shown When Printed, and click OK. In Word select the location where you want to paste the tree diagram and choose Edit | Paste.

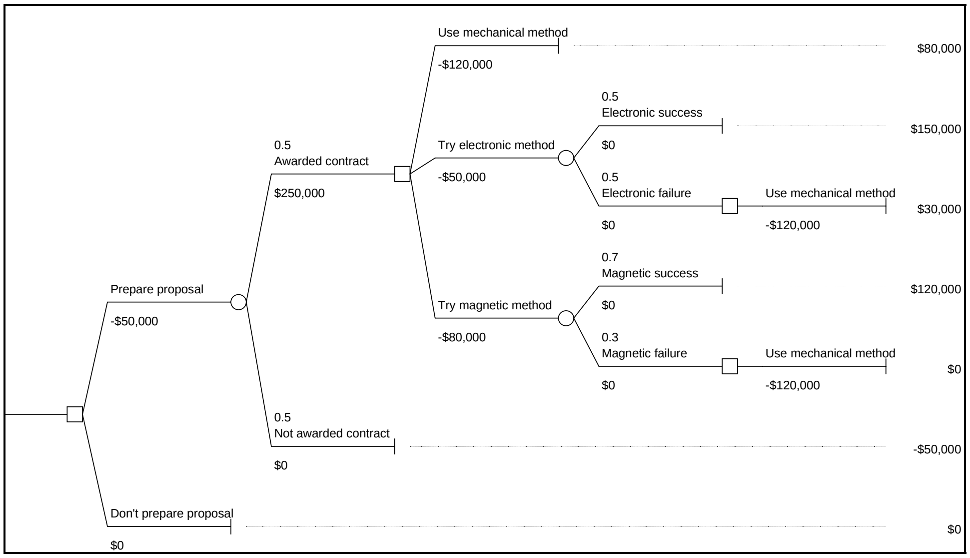
Figure 4.25



Alternative Model

If you want to emphasize that the time constraint forces DriveTek to use the mechanical approach if they try either of the uncertain approaches and experience a failure, you can change the terminal nodes in cells R13 and R23 to decision nodes, each with a single branch.

Figure 4.26



4.5 DECISION TREE SOLUTION

Strategy

A strategy specifies an initial choice and any subsequent choices to be made by the decision maker. The subsequent choices usually depend upon events. The specification of a strategy must be comprehensive; if the decision maker gives the strategy to a colleague, the colleague must know exactly which choice to make at each decision node.

Most decision problems have many possible strategies, and a goal of the analysis is to determine the optimal strategy, taking into account the decision maker's risk attitude. There are four strategies in the DriveTek problem. One of the strategies is: Prepare the proposal; if not awarded the contract, stop; if awarded the contract, try the magnetic method; if the magnetic method is successful, stop; if the magnetic method fails, use the mechanical method. The four strategies will be discussed in detail below.

Payoff Distribution

Each strategy has an associated payoff distribution, sometimes called a risk profile. The payoff distribution of a particular strategy is a probability distribution showing the probability of obtaining each terminal value associated with a particular strategy.

In decision tree models, the payoff distribution can be shown as a list of possible payoff values, x, and the discrete probability of obtaining each value, P(X=x), where X represents the uncertain terminal value associated with a strategy. Since a strategy specifies a choice at each decision node, the uncertainty about terminal values depends only on the occurrence of events. The probability of obtaining a specific terminal value equals the product of the probabilities on the event branches on the path leading to the terminal node.

DriveTek Strategies

In this section each strategy of the DriveTek problem is described by a shorthand statement and a more detailed statement. The possible branches following a specific strategy are shown in decision tree form, and the payoff distribution is shown in a table with an explanation of the probability calculations.

Strategy 1 (Mechanical): Prepare; if awarded, use mechanical.

Details: Prepare the proposal; if not awarded the contract, stop (payoff = -$50,000); if awarded the contract, use the mechanical method (payoff = $80,000).

Figure 4.27

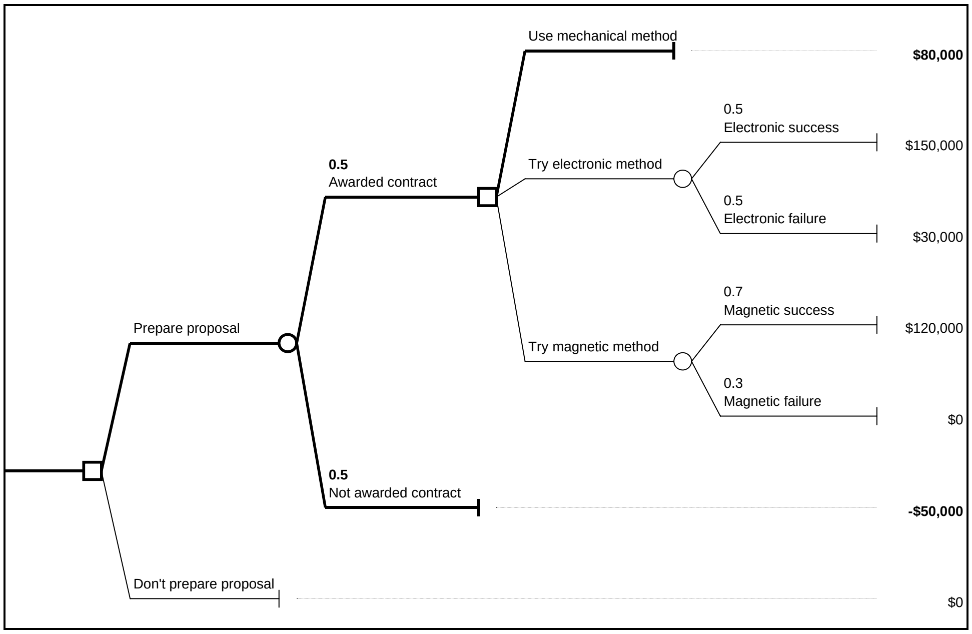


Figure 4.28

Value, x	Probability P(X=x)
$80,000	0.50
-$50,000	0.50
	1.00

Strategy 2 (Electronic): Prepare; if awarded, try electronic.

Details: Prepare the proposal; if not awarded the contract, stop (payoff = -$50,000); if awarded the contract, try the electronic method; if the electronic method is successful, stop (payoff = $150,000); if the electronic method fails, use the mechanical method (payoff = $30,000).

Figure 4.29

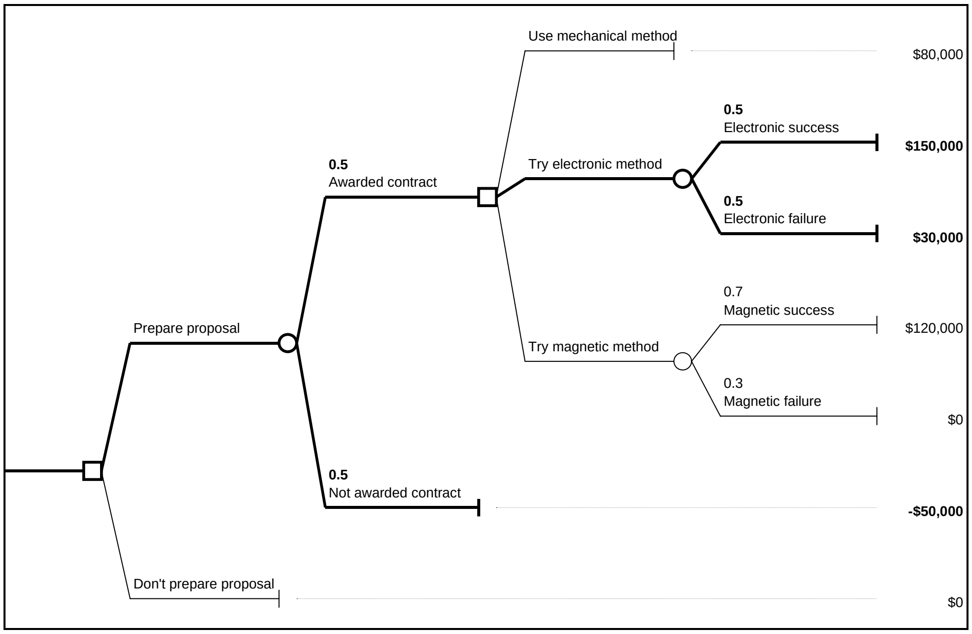


Figure 4.30

Value, x	Probability P(X=x)	
$150,000	0.25	= 0.5 * 0.5
$30,000	0.25	= 0.5 * 0.5
-$50,000	0.50	
	1.00	

Strategy 3 (Magnetic): Prepare; if awarded, try magnetic.

Details: Prepare the proposal; if not awarded the contract, stop (payoff = -$50,000); if awarded the contract, try the magnetic method; if the magnetic method is successful, stop (payoff = $120,000); if the magnetic method fails, use the mechanical method (payoff = $0).

Figure 4.31

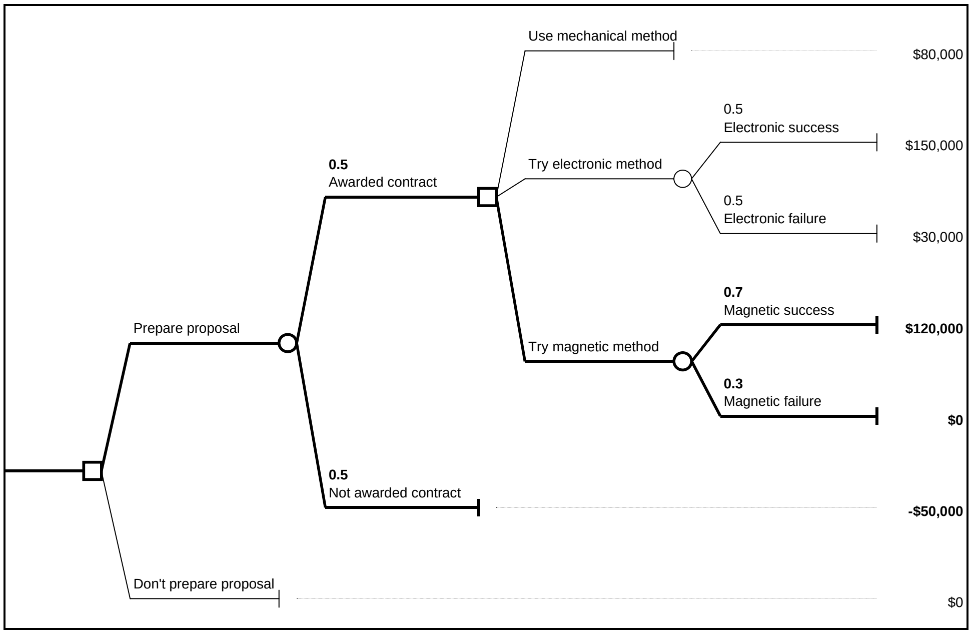


Figure 4.32

Value, x	Probability P(X=x)	
$120,000	0.35	= 0.5 * 0.7
$0	0.15	= 0.5 * 0.3
-$50,000	0.50	
	1.00	

Strategy 4 (Don't): Don't.

Details: Don't prepare the proposal (payoff = $0).

Figure 4.33

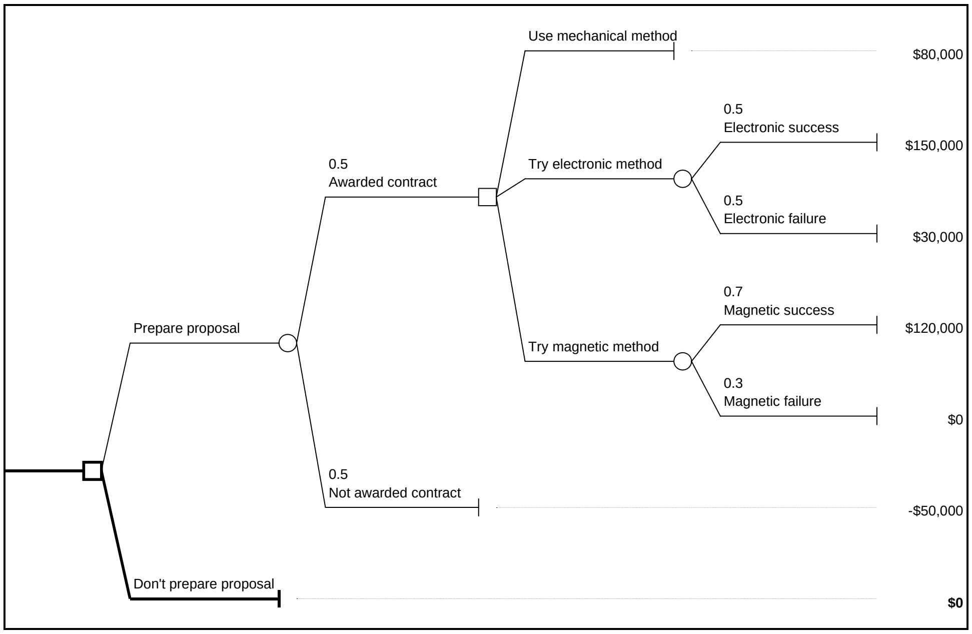


Figure 4.34

Value, x	Probability P(X=x)
$0	1.00
	1.00

Strategy Choice

Since each strategy can be characterized completely by its payoff distribution, selecting the best strategy becomes a problem of choosing the best payoff distribution.

One approach is to make a choice by direct comparison of the payoff distributions.

Figure 4.35

Strategy 1 (Mechanical)

Value, x	Probability P(X=x)
$80,000	0.50
-$50,000	0.50
	1.00

Strategy 2 (Electronic)

Value, x	Probability P(X=x)
$150,000	0.25
$30,000	0.25
-$50,000	0.50
	1.00

Strategy 3 (Magnetic)

Value, x	Probability P(X=x)
$120,000	0.35
$0	0.15
-$50,000	0.50
	1.00

Strategy 4 (Don't)

Value, x	Probability P(X=x)
$0	1.00
	1.00

Another approach for making choices involves certainty equivalents.

Certainty Equivalent

A certainty equivalent is a certain payoff value which is equivalent, for the decision maker, to a particular payoff distribution. If the decision maker can determine his or her certainty equivalent for the payoff distribution of each strategy, then the optimal strategy is the one with the highest certainty equivalent.

The certainty equivalent is the minimum selling price for a payoff distribution; it depends on the decision maker's personal attitude toward risk. A decision maker may be risk preferring, risk neutral, or risk avoiding.

If the terminal values are not regarded as extreme (relative to the decision maker's total assets), if the decision maker will encounter other decision problems with similar payoffs, and if the decision maker has the attitude that he or she will "win some and lose some," then the decision maker's attitude toward risk may be described as risk neutral.

If the decision maker is risk neutral, the expected value is the appropriate certainty equivalent for choosing among the strategies. Thus, for a risk neutral decision maker, the optimal strategy is the one with the highest expected value.

The expected value of a payoff distribution is calculated by multiplying each terminal value by its probability and summing the products. The expected value calculations for each of the four strategies of the DriveTek problem are shown below.

Figure 4.36

Strategy 1 (Mechanical)

Value, x	Probability P(X=x)	x * P(X=x)
$80,000	0.50	$40,000
-$50,000	0.50	-$25,000
		$15,000

Strategy 2 (Electronic)

Value, x	Probability P(X=x)	x * P(X=x)
$150,000	0.25	$37,500
$30,000	0.25	7,500
-$50,000	0.50	-$25,000
		$20,000

Strategy 3 (Magnetic)

Value, x	Probability P(X=x)	x * P(X=x)
$120,000	0.35	$42,000
$0	0.15	$0
-$50,000	0.50	-$25,000
		$17,000

Strategy 4 (Don't)

Value, x	Probability P(X=x)	x * P(X=x)
$0	1.00	$0
		$0

The four strategies of the DriveTek problem have expected values of $15,000, $20,000, $17,000, and $0. Strategy 2 (Electronic) is the optimal strategy with expected value $20,000.

A risk neutral decision maker's choice is based on the expected value. However, note that if strategy 2 (Electronic) is chosen, the decision maker does not receive $20,000. The actual payoff will be $150,000, $30,000, or -$50,000, with probabilities shown in the payoff distribution.

Rollback Method

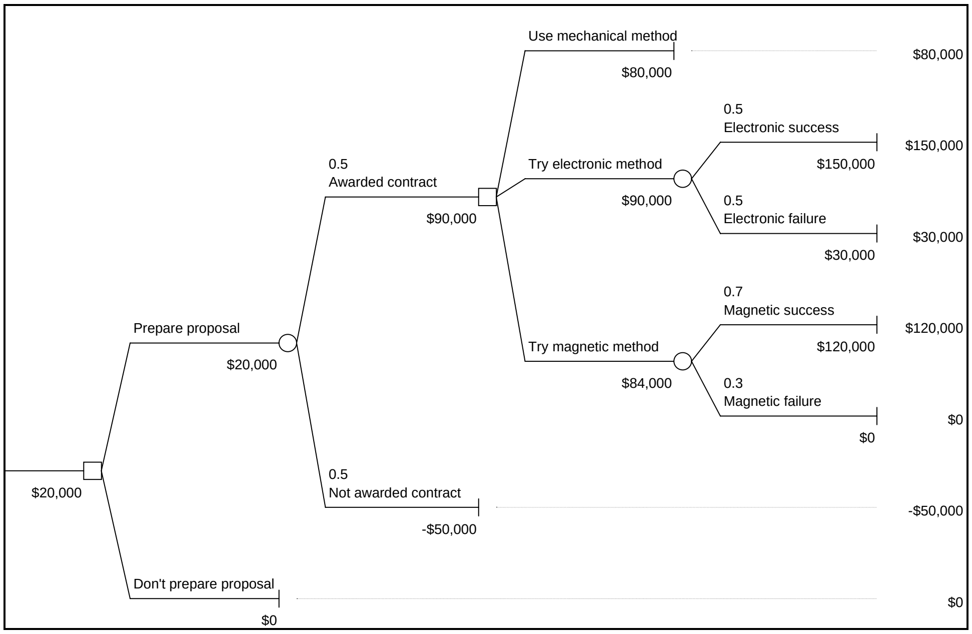
If we have a method for determining certainty equivalents (expected values for a risk neutral decision maker), we don't need to examine every possible strategy explicitly. Instead, the method known as rollback determines the single best strategy.

The rollback algorithm, sometimes called backward induction or "average out and fold back," starts at the terminal nodes of the tree and works backward to the initial decision node, determining the certainty equivalent rollback values for each node. Rollback values are determined as follows:

- At a terminal node, the rollback value equals the terminal value.
- At an event node, the rollback value for a risk neutral decision maker is determined using expected value; the branch probability is multiplied times the successor rollback value, and the products are summed.
- At a decision node, the rollback value is set equal to the highest rollback value on the immediate successor nodes.

In TreePlan tree diagrams the rollback values are located to the left and below each decision, event, and terminal node. Terminal values and rollback values for the DriveTek problem are shown below.

Figure 4.37



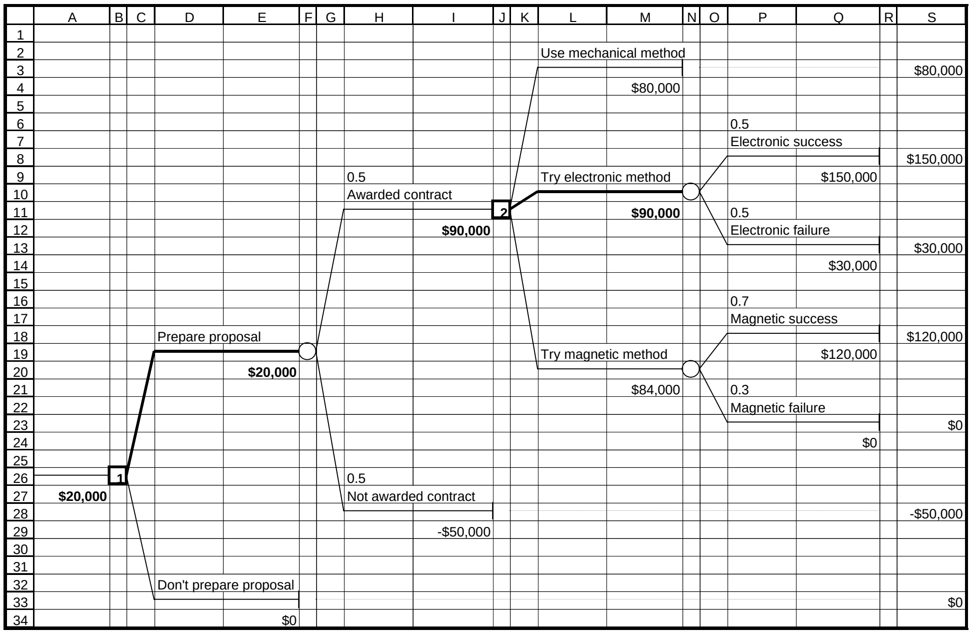
Optimal Strategy

After the rollback method has determined certainty equivalents for each node, the optimal strategy can be identified by working forward through the tree. At the initial decision node, the $20,000 rollback value equals the rollback value of the "Prepare proposal" branch, indicating the alternative that should be chosen. DriveTek will either be awarded the contract or not; there is a subsequent decision only if DriveTek obtains the contract. (In a more complicated decision tree, the optimal strategy must include decision choices for all decision nodes that might be encountered.) At the decision node following "Awarded contract," the $90,000 rollback value equals the rollback value of the "Try electronic method" branch, indicating the alternative that should be chosen. Subsequently, if the electronic method fails, DriveTek must use the mechanical method to satisfy the contract.

Cell B26 has the formula =IF(A27=E20,1,IF(A27=E34,2)) which displays 1, indicating that the first branch is the optimal choice. Thus, the initial choice is to prepare the proposal. Cell J11 has the formula =IF(I12=M4,1,IF(I12=M11,2,IF(I12=M21,3))) which displays 2, indicating that the second branch (numbered 1, 2, and 3, from top to bottom) is the optimal choice. If awarded the contract, DriveTek should try the electronic method.

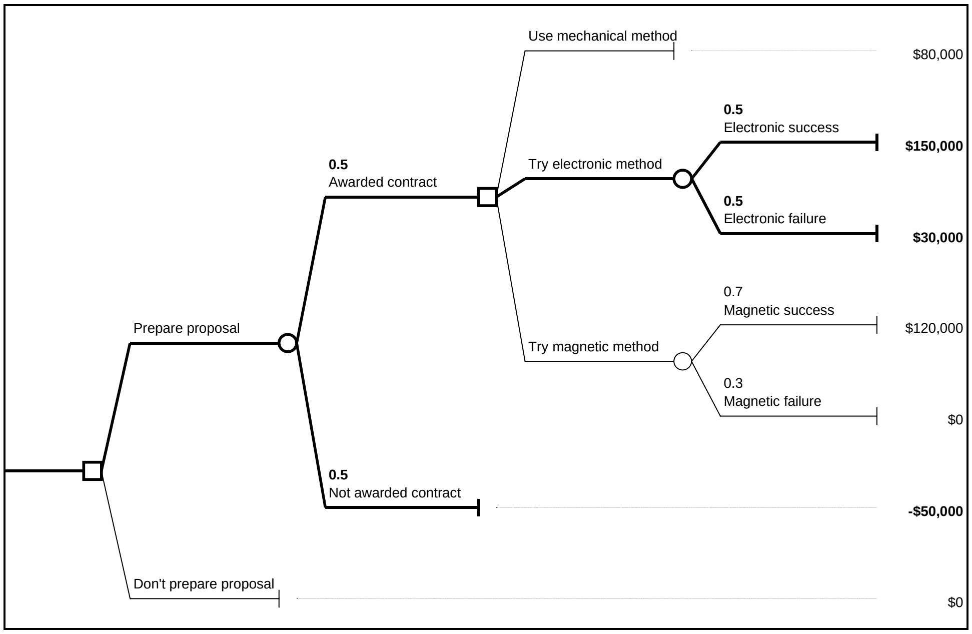
The pairs of rollback values at the relevant decision nodes ($20,000 and $90,000) and the preferred decision branches are shown below in bold.

Figure 4.38



Taking into account event branches with subsequent terminal nodes, all branches and terminal values associated with the optimal risk neutral strategy are shown below.

Figure 4.39



The rollback method has identified strategy 2 (Electronic) as optimal. The rollback value on the initial branch of the optimal strategy is $20,000, which must be the same as the expected value for the payoff distribution of strategy 2. Some of the intermediate calculations for the rollback method differ from the calculations for the payoff distributions, but both approaches identify the same optimal strategy with the same initial expected value. For decision trees with a large number of strategies, the rollback method is more efficient.

4.6 NEWOX DECISION TREE PROBLEM

The Newox Company is considering whether or not to drill for natural gas on its own land. If they drill, their initial expenditure will be $40,000 for drilling costs. If they strike gas, they must spend an additional $30,000 to cap the well and provide the necessary hardware and control equipment. (This $30,000 cost is not a decision; it is associated with the event "strike gas.") If they decide to drill but no gas is found, there are no other subsequent alternatives, so their outcome value is $-40,000.

If they drill and find gas, there are two alternatives. Newox could sell to West Gas, which has made a standing offer of $200,000 to purchase all rights to the gas well's production (assuming that Newox has actually found gas). Alternatively, if gas is found, Newox can decide to keep the well instead of selling to West Gas; in this case Newox manages the gas production and takes its chances by selling the gas on the open market.

At the current price of natural gas, if gas is found it would have a value of $150,000 on the open market. However, there is a possibility that the price of gas will rise to double its current value, in which case a successful well will be worth $300,000.

The company's engineers feel that the chance of finding gas is 30 percent; their staff economist thinks there is a 60 percent chance that the price of gas will double.

4.7 BRANDON DECISION TREE PROBLEM

Brandon Appliance Corporation, a predominant producer of microwave ovens, is considering the introduction of a new product. The new product is a microwave oven that will defrost, cook, brown, and boil food as well as sense when the food is done.

Brandon must decide on a course of action for implementing this new product line. An initial decision must be made to (1) nationally distribute the product from the start, (2) conduct a marketing test first, or (3) not market the product at all. If a marketing test is conducted, Brandon will consider the result and then decide whether to abandon the product line or make it available for national distribution.

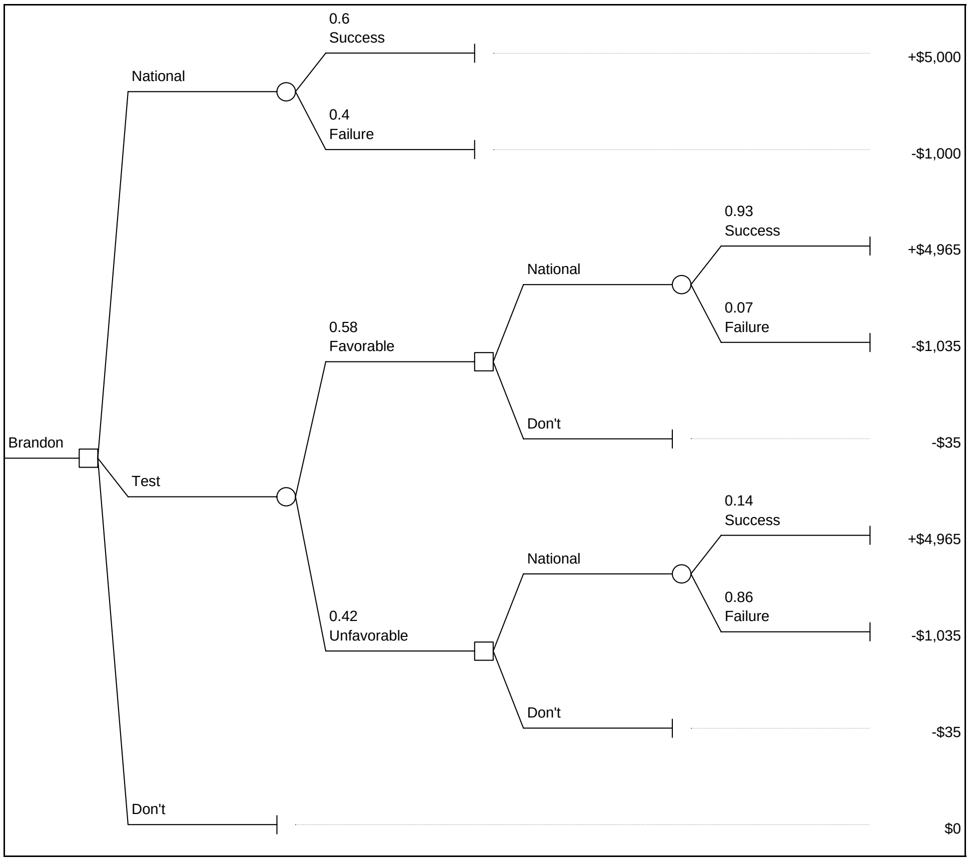
The finance department has provided some cost information and probability assignments relating to this decision. The preliminary costs for research and development have already been incurred and are considered irrelevant to the marketing decision. A success nationally will increase profits by $5,000,000, and failure will reduce them by $1,000,000, while abandoning the product will not affect profits. The test market analysis will cost Brandon an additional $35,000.

If a market test is not performed, the probability of success in a national campaign is 60 percent. If the market test is performed, the probability of a favorable test result is 58 percent. With favorable test results, the probability for national success is approximately 93 percent. However, if the test results are unfavorable, the national success probability is approximately 14 percent.

Decision Tree Strategies

Brandon Appliance Corporation must decide on a course of action for implementing this new microwave oven. An initial decision must be made to (1) nationally distribute the product from the start, (2) conduct a marketing test first, or (3) not market the product at all. If a marketing test is conducted, Brandon will consider the result and then decide whether to abandon the product line or make it available for national distribution. The following decision tree is based on information about cash flows and probability assignments.

Figure 4.40



In a decision tree model, a strategy is a specification of an initial choice and any subsequent choices that must be made by the decision maker.

How many strategies are there in the Brandon problem?

Describe each strategy.

Figure 4.41 Strategy 1: National

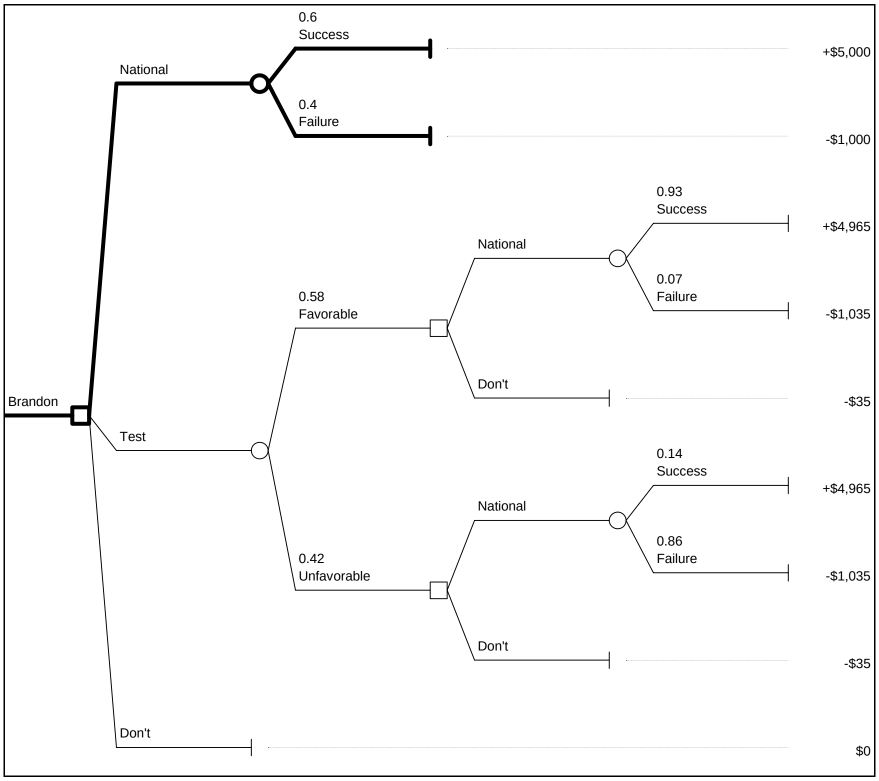


Figure 4.42 Strategy 2: Test; if Favorable, National; if Unfavorable, National

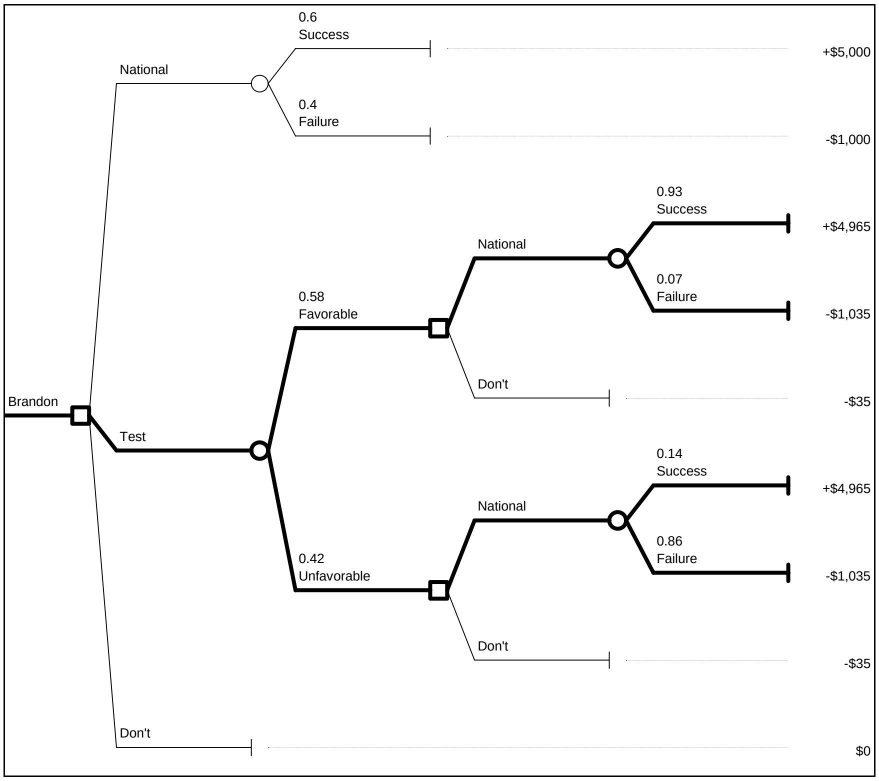


Figure 4.43 Strategy 3: Test; if Favorable, National; if Unfavorable, Don't

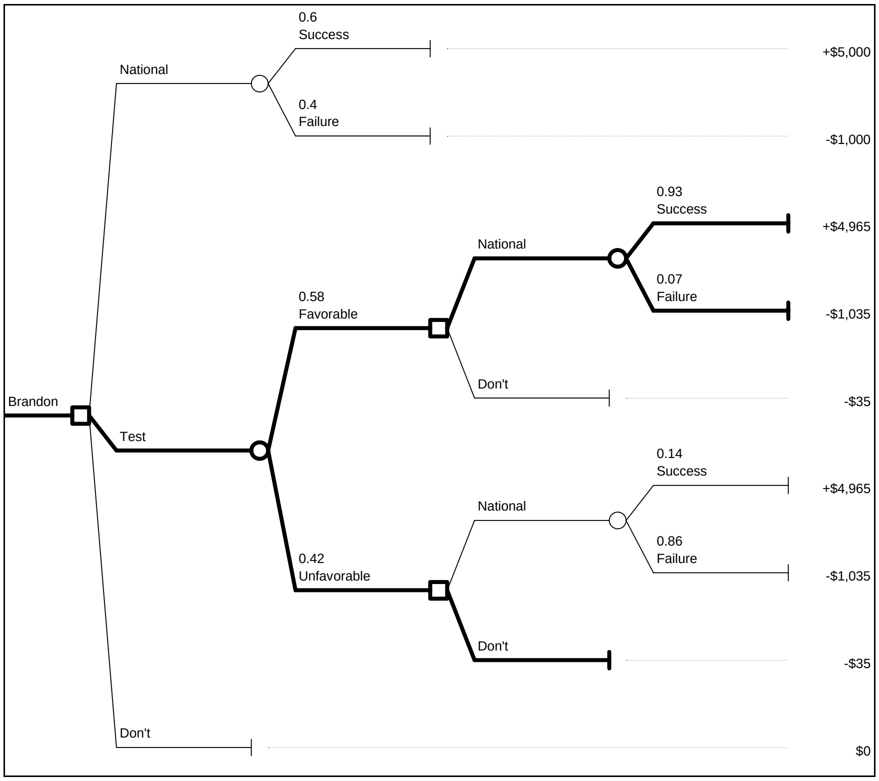


Figure 4.44 Strategy 4: Test; if Favorable, Don't; if Unfavorable, National

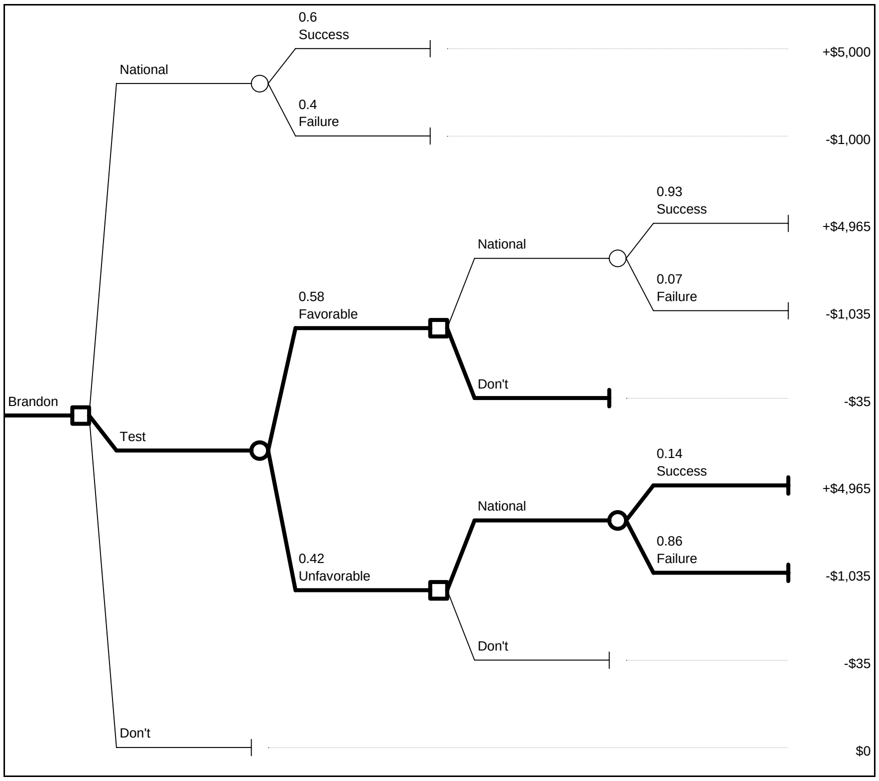


Figure 4.45 Strategy 5: Test; if Favorable, Don't; if Unfavorable, Don't

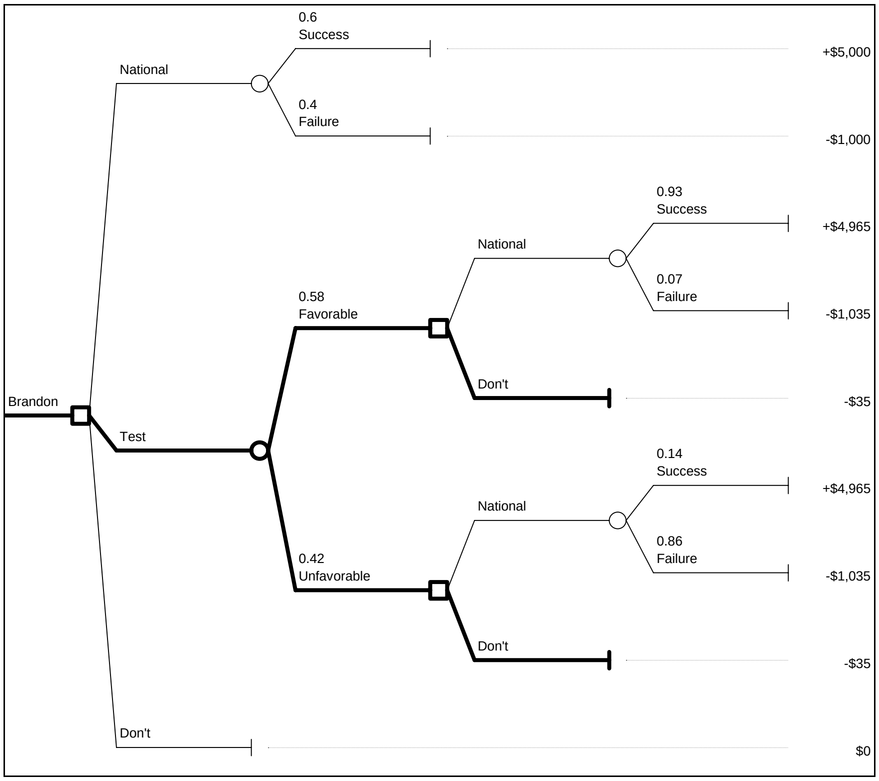
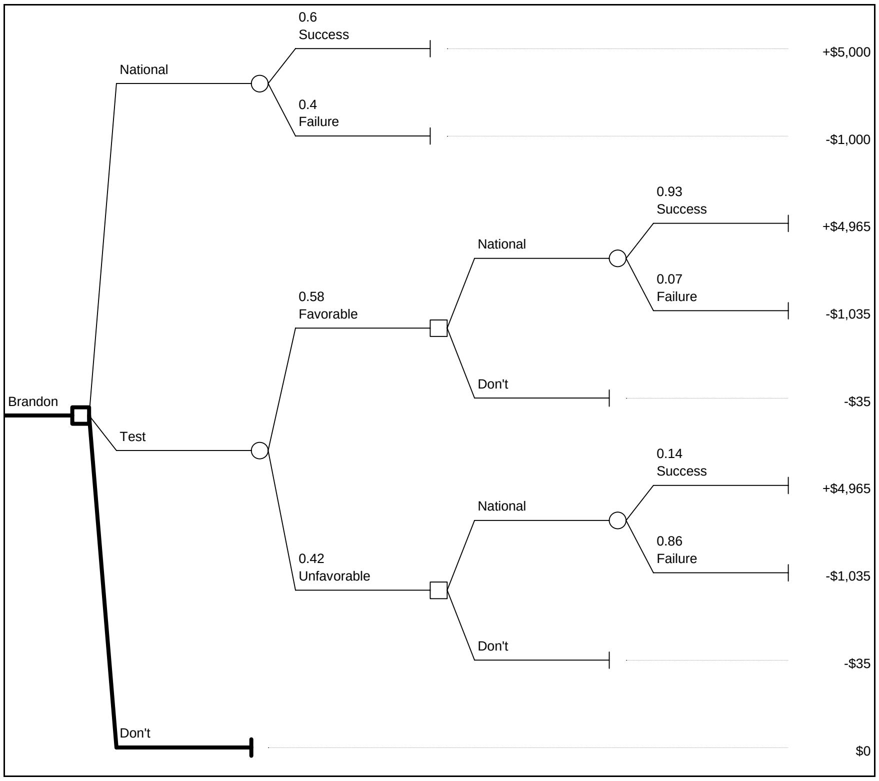


Figure 4.46 Strategy 6: Don't



Multiattribute Utility

5

5.1 APPLICATIONS OF MULTI-ATTRIBUTE UTILITY

- Strategy for Dealing with Microcomputer Networking
    - Impact on microcomputer users
        - Productivity enhancement
        - User satisfaction
    - Impact on mainframe capacity
    - Costs
    - Upward compatibility of the network
    - Impacts on organizational structure
    - Risks
- Purchase of manufacturing machinery
    - Price
    - Technical features
    - Service
- Choosing a manager candidate
    - Education
    - Management skills
    - Technical skills
    - Personal skills
- Choosing a beverage container (soft drink industry)

- Energy to produce
- Cost
- Environmental waste
- Customer service

Selecting a best job

- Monetary compensation
- Geographical location
- Travel requirements
- Nature of work

5.2 MULTIATTRIBUTE UTILITY SWING WEIGHTS

Excel Workbook Clemen15.xls

Conflicting Objectives: Fundamental Objectives versus Means Objectives

Clemen, Making Hard Decisions, Ch. 15

Multiattribute Utility

Set of Objectives should be

1) complete
2) as small as possible
3) not redundant
4) decomposable ("independent" or unrelated)

Additive Utility Function

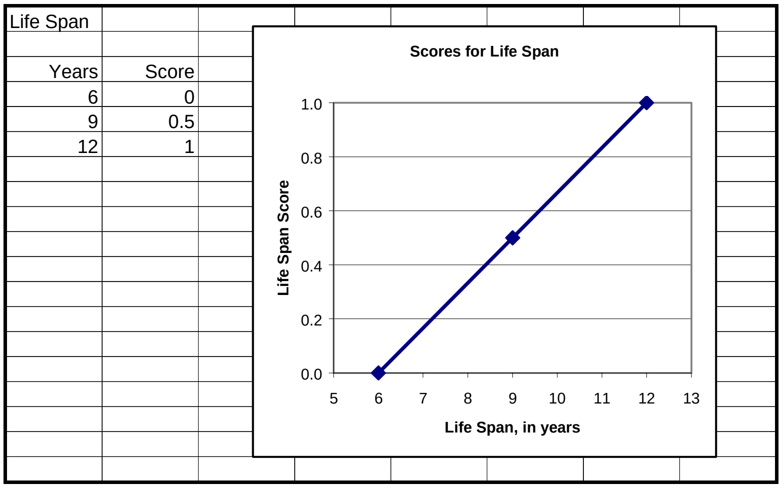
Overall Score of Alternative = Sum [Weight times Attribute Score of Alternative]

Figure 5.1 Data for Example

Attribute	Red Portalo	Blue Norushi	Yellow Standard
Life span, in years	12	9	6
Price	$17,000	$10,000	$8,000
Color	Red	Blue	Yellow

Attribute Scores

Figure 5.2 Individual Utility for Life Span



Life Span

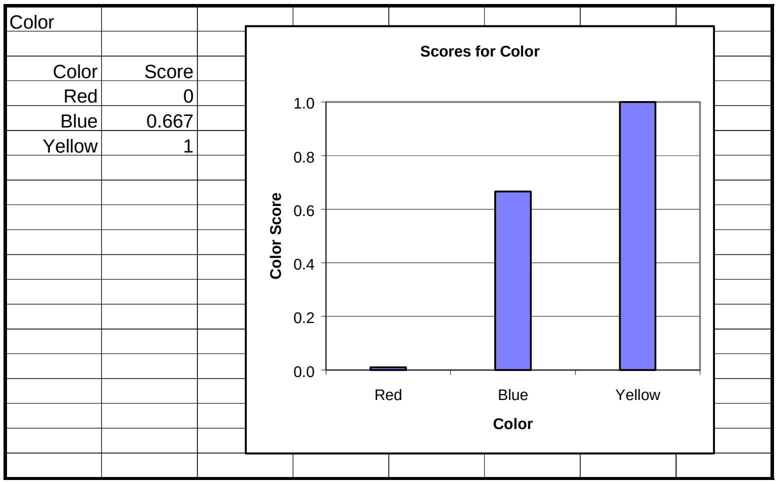
Years	Score
6	0
9	0.5
12	1

Figure 5.3 Individual Utility for Price

Price

Price	Score
$17,000	0
$10,000	0.78
$8,000	1

Figure 5.4 Individual Utility for Color



Color

Color	Score
Red	0
Blue	0.667
Yellow	1

Swing Weights

Figure 5.5 Swing Weight Assessment Display

	A	B	C	D	E	F	G
1	Swing Weights						
2							
3		Consequence to Compare					
4	Attribute Swung from						
5	Worst to Best	Life span	Price	Color	Rank	Rate	Weight
6	(Benchmark)	6 years	$17,000	red	4	0	0.000
7	Life span	12 years	$17,000	red	2	75	0.405
8	Price	6 years	$8,000	red	1	100	0.541
9	Color	6 years	$17,000	yellow	3	10	0.054
10						185	

1) Hypothetical alternatives (number of attributes plus one)

Benchmark alternative is worst for all attributes

Each other hypothetical alternative has one attribute at best, all others at worst

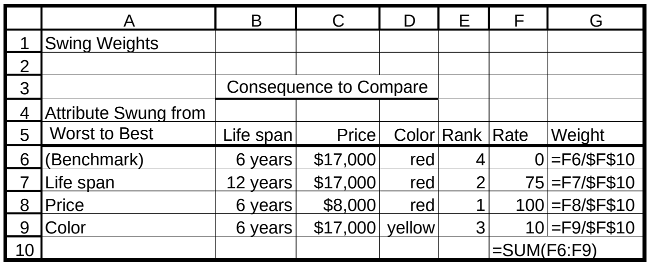
2) Rank the hypothetical alternatives

3) Benchmark has rating zero, first ranked alternative has rating 100

Assign level-of-satisfaction ratings to the intermediate alternatives

4) Weight equals rating divided by sum of ratings

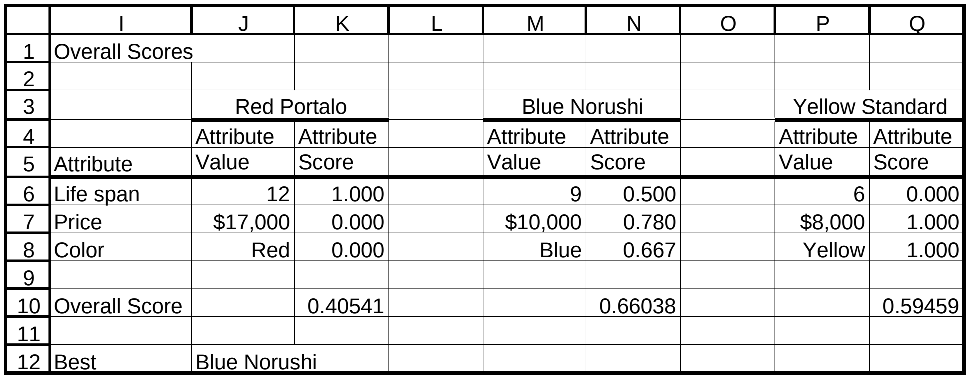
Figure 5.6 Swing Weight Assessment Formulas



	A	B	C	D	E	F	G
1	Swing Weights						
2							
3		Consequence to Compare					
4	Attribute Swung from						
5	Worst to Best	Life span	Price	Color	Rank	Rate	Weight
6	(Benchmark)	6 years	$17,000	red	4	0	=F6/$F$10
7	Life span	12 years	$17,000	red	2	75	=F7/$F$10
8	Price	6 years	$8,000	red	1	100	=F8/$F$10
9	Color	6 years	$17,000	yellow	3	10	=F9/$F$10
10						=SUM(F6:F9)	

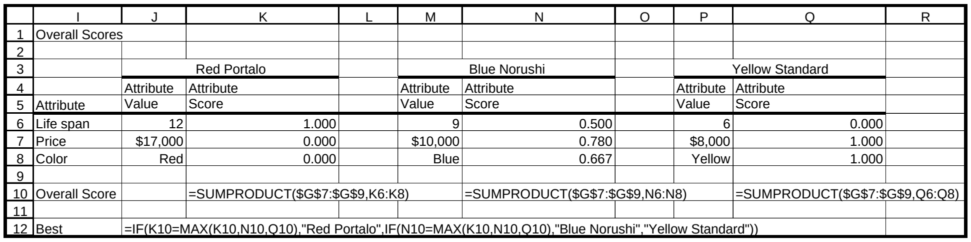
Overall Scores

Figure 5.7 Swing Weight Overall Scores Display



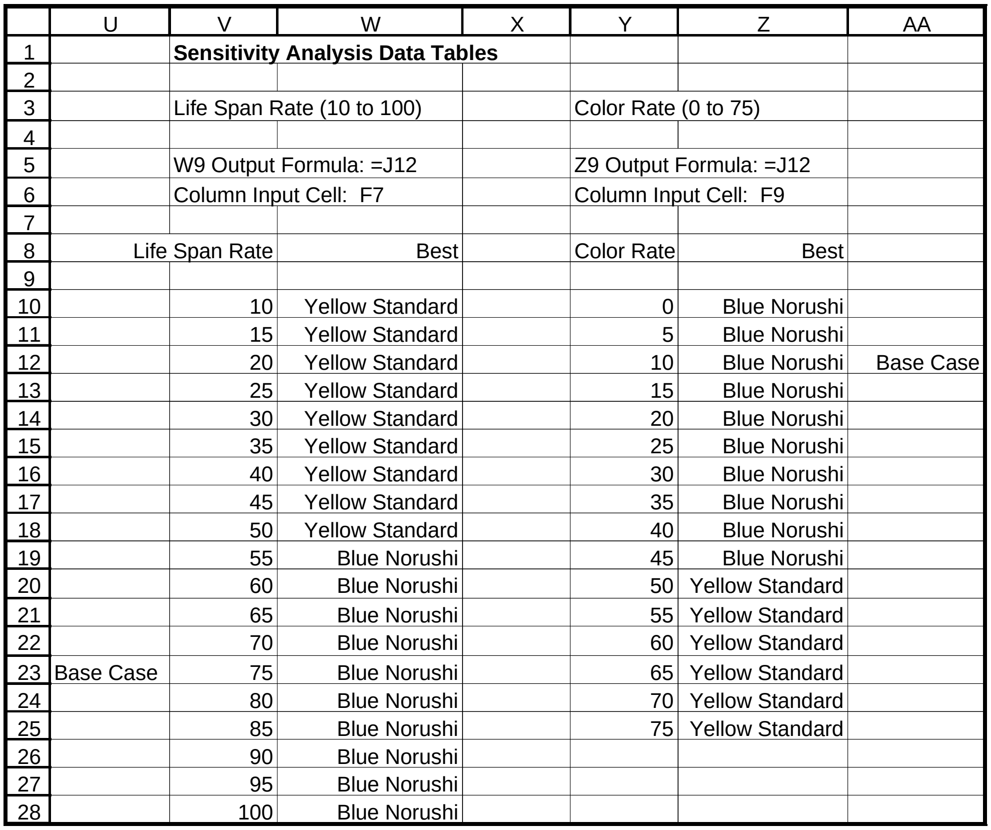
	I	J	K	L	M	N	O	P	Q
1	Overall Scores								
2									
3		Red Portalo			Blue Norushi			Yellow Standard	
4		Attribute	Attribute		Attribute	Attribute		Attribute	Attribute
5	Attribute	Value	Score		Value	Score		Value	Score
6	Life span	12	1.000		9	0.500		6	0.000
7	Price	$17,000	0.000		$10,000	0.780		$8,000	1.000
8	Color	Red	0.000		Blue	0.667		Yellow	1.000
9									
10	Overall Score		0.40541			0.66038			0.59459
11									
12	Best	Blue Norushi							

Figure 5.8 Swing Weight Overall Scores Formulas



	I	J	K	L	M	N	O	P	Q	R
1	Overall Scores									
2										
3		Red Portalo			Blue Norushi			Yellow Standard		
4		Attribute	Attribute		Attribute	Attribute		Attribute	Attribute	
5	Attribute	Value	Score		Value	Score		Value	Score	
6	Life span	12	1.000		9	0.500		6	0.000	
7	Price	$17,000	0.000		$10,000	0.780		$8,000	1.000	
8	Color	Red	0.000		Blue	0.667		Yellow	1.000	
9										
10	Overall Score		=SUMPRODUCT($G$7:$G$9,K6:K8)			=SUMPRODUCT($G$7:$G$9,N6:N8)			=SUMPRODUCT($G$7:$G$9,Q6:Q8)	
11										
12	Best	=IF(K10=MAX(K10,N10,Q10),"Red Portalo",IF(N10=MAX(K10,N10,Q10),"Blue Norushi","Yellow Standard"))								

Figure 5.9 Sensitivity Analysis



	U	V	W	X	Y	Z	AA
1		**Sensitivity Analysis Data Tables**					
2							
3		Life Span Rate (10 to 100)			Color Rate (0 to 75)		
4							
5		W9 Output Formula: =J12			Z9 Output Formula: =J12		
6		Column Input Cell: F7			Column Input Cell: F9		
7							
8		Life Span Rate	Best		Color Rate	Best	
9							
10		10	Yellow Standard		0	Blue Norushi	
11		15	Yellow Standard		5	Blue Norushi	
12		20	Yellow Standard		10	Blue Norushi	Base Case
13		25	Yellow Standard		15	Blue Norushi	
14		30	Yellow Standard		20	Blue Norushi	
15		35	Yellow Standard		25	Blue Norushi	
16		40	Yellow Standard		30	Blue Norushi	
17		45	Yellow Standard		35	Blue Norushi	
18		50	Yellow Standard		40	Blue Norushi	
19		55	Blue Norushi		45	Blue Norushi	
20		60	Blue Norushi		50	Yellow Standard	
21		65	Blue Norushi		55	Yellow Standard	
22		70	Blue Norushi		60	Yellow Standard	
23	Base Case	75	Blue Norushi		65	Yellow Standard	
24		80	Blue Norushi		70	Yellow Standard	
25		85	Blue Norushi		75	Yellow Standard	
26		90	Blue Norushi				
27		95	Blue Norushi				
28		100	Blue Norushi				

5.3 SENSITIVITY ANALYSIS METHODS

SENSITIVITY ANALYSIS FOR MULTI-ATTRIBUTE UTILITY USING EXCEL

This paper describes several standard methods for analyzing decisions where the outcomes have multiple attributes. The example problem concerns a large company that is planning to purchase several hundred cars for use by the sales force. The company wants a car that is inexpensive, safe, and lasts a long time. Figure 1 shows data for seven cars that are being considered.

Figure 1 Attribute Data for Seven Alternatives

	A	B	C	D	E	F	G	H
1		Alternatives						
2	Attribute	Alta	Bulldog	Cruiser	Delta	Egret	Fleet	Garnett
3	Cost	$20	$18	$16	$14	$12	$10	$15
4	Lifetime	10	10	8	8	6	6	8
5	Safety	High	Medium	High	Medium	Medium	Low	Low
6								
7	Cost	thousands of dollars						
8	Lifetime	expected years						
9	Safety	third-party rating						

Other attributes might be important, e.g., comfort and prestige. The cost attribute should include operating costs, insurance, and salvage value, in addition to purchase price. It might be appropriate to combine the cost and lifetime attributes into a single attribute, e.g., cost per year. Clemen [1] suggests that a set of attributes should be complete (so that all important objectives are included), as small as possible (to facilitate analysis), not redundant (to avoid double-counting a common underlying characteristic), and decomposable (so that the decision maker can think about each attribute separately).

Dominance

An alternative can be eliminated if another alternative is better on some objectives and no worse on the others. The Garnett is more expensive than the Delta, has the same lifetime, and has a lower safety rating. So the Garnett can be eliminated from further consideration.

Monetary Equivalents Assessment

One method for comparing multi-attribute alternatives is to subjectively assign monetary values to the non-monetary attributes. For example, the decision maker may determine that each additional year of expected lifetime is worth $500, medium safety is $4,000 better than low safety, and high safety is $6,000 better than low safety. Arbitrarily using Fleet as the base case with total equivalent cost of $10,000, Figure 2 shows costs and equivalent costs, in thousands of dollars, in rows 9:11. The negative entries for Lifetime and Safety correspond to positive benefits relative to the Fleet car's base case values.

Based on this method, the Egret is chosen. Sensitivity analysis, not shown here, would involve seeing how the choice depends on subjective equivalents different from the $500 per year lifetime and the $4,000 and $6,000 safety assessments.

Hammond et al. [3] describe another method involving even swaps that could be used to select the best alternative.

Figure 2 Monetary Equivalents for Non-Dominated Alternatives

	A	B	C	D	E	F	G
1		Non-Dominated Alternatives					
2	Attribute	Alta	Bulldog	Cruiser	Delta	Egret	Fleet
3	Cost	$20	$18	$16	$14	$12	$10
4	Lifetime, years	10	10	8	8	6	6
5	Safety rating	High	Medium	High	Medium	Medium	Low
6							
7		Non-Dominated Alternatives					
8	Attribute	Alta	Bulldog	Cruiser	Delta	Egret	Fleet
9	Cost	$20	$18	$16	$14	$12	$10
10	Lifetime, $	-$2	-$2	-$1	-$1	$0	$0
11	Safety, $	-$6	-$4	-$6	-$4	-$4	$0
12							
13	Equiv. Cost	$12	$12	$9	$9	$8	$10

Additive Utility Function

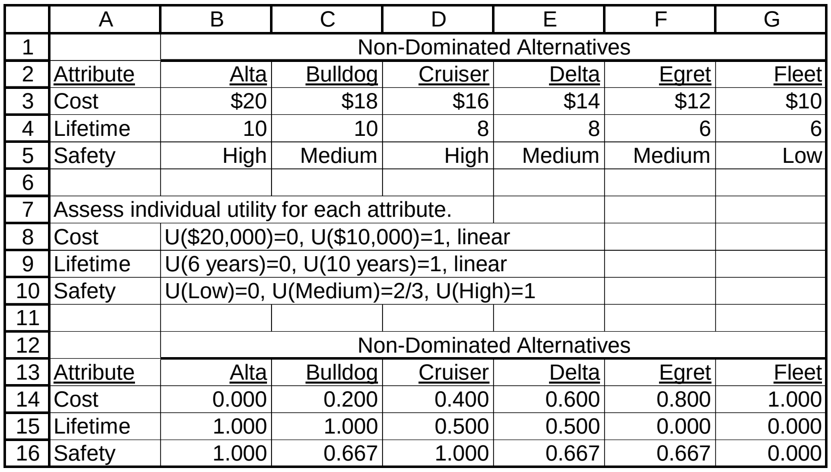
The additive multi-attribute utility function U includes individual utility functions U_i for each attribute x_i, usually scaled from 0 to 1, and weights w_i that reflect the decision maker's tradeoffs among the attributes.

$$U(x_1,x_2,x_3) = w_1 \cdot U_1(x_1) + w_2 \cdot U_2(x_2) + w_3 \cdot U_3(x_3), \text{ where } w_1 + w_2 + w_3 = 1 \qquad (1)$$

Weights may be specified directly, as ratios, or using a swing weight procedure. Individual utility functions are assessed using the range of attribute values for the alternatives being considered.

The individual utility values for Cost and Lifetime shown in Figure 3 are based on proportional scores, corresponding to linear utility functions. For example, each thousand dollar difference in cost is associated with a 0.1 difference in utility. The utility values for Safety are subjective judgments. For example, the decision maker thinks that a change in Safety from Low to Medium achieves only two-thirds of the satisfaction associated with a change from Low to High.

Figure 3 Individual Utilities



	A	B	C	D	E	F	G
1		Non-Dominated Alternatives					
2	Attribute	Alta	Bulldog	Cruiser	Delta	Egret	Fleet
3	Cost	$20	$18	$16	$14	$12	$10
4	Lifetime	10	10	8	8	6	6
5	Safety	High	Medium	High	Medium	Medium	Low
6							
7	Assess individual utility for each attribute.						
8	Cost	U($20,000)=0, U($10,000)=1, linear					
9	Lifetime	U(6 years)=0, U(10 years)=1, linear					
10	Safety	U(Low)=0, U(Medium)=2/3, U(High)=1					
11							
12		Non-Dominated Alternatives					
13	Attribute	Alta	Bulldog	Cruiser	Delta	Egret	Fleet
14	Cost	0.000	0.200	0.400	0.600	0.800	1.000
15	Lifetime	1.000	1.000	0.500	0.500	0.000	0.000
16	Safety	1.000	0.667	1.000	0.667	0.667	0.000

Compared to the assessments for individual utility, the assessments for tradeoffs are usually much more difficult to make. The following sections focus on assessments of tradeoff weights and sensitivity analysis.

Weight Ratio Assessment

One method for measuring trade-offs among the conflicting objectives is to assess weight ratios. For example, the decision maker may judge that cost is five times as important as lifetime, which may be interpreted to mean that the change in overall satisfaction corresponding to a change in cost from $20,000 to $10,000 is five times the change in overall satisfaction corresponding to a change in lifetime from 6 years to 10 years. Similarly, the decision maker may judge that a $10,000 decrease in cost is one and a half times as satisfying as a change from a low to a high safety rating. The assessments are shown in cells J4:J5 in Figure 4.

Figure 4 Weight Ratio Assessment and Choice

	A	B	C	D	E	F	G	H	I	J
1		Non-Dominated Alternatives							Assess weight ratios.	
2	Attribute	Alta	Bulldog	Cruiser	Delta	Egret	Fleet			
3	Cost	0.000	0.200	0.400	0.600	0.800	1.000		Weight Ratio	Input
4	Lifetime	1.000	1.000	0.500	0.500	0.000	0.000		Cost/Lifetime	5.0
5	Safety	1.000	0.667	1.000	0.667	0.667	0.000		Cost/Safety	1.5
6										
7	Overall	0.464	0.452	0.625	0.613	0.667	0.536		Weights	
8									Cost	0.536
9	Max Value	0.667							Lifetime	0.107
10	Location	5							Safety	0.357
11	Choice	Egret								
12										
13	Choice	Egret								

With three attributes, the two assessed weight ratios determine two equations and the requirement that the weights sum to one determines a third equation. Using algebra, a solution for the three unknown weights is shown in cells J8:J10 in Figure 5.

The formula for overall utility in cell B7, with a relative reference to the attribute utilities in B3:B5 and an absolute reference to the weights in J8:J10, is copied to cells C7:G7.

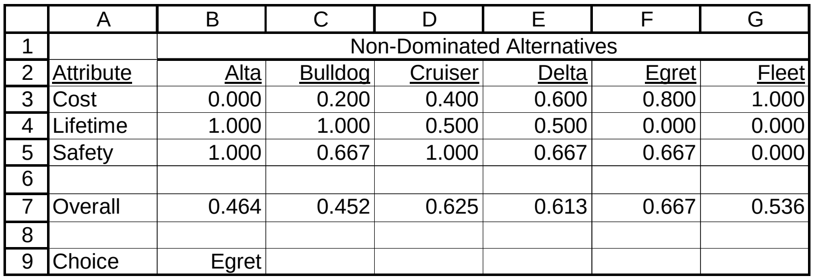
The MAX worksheet function determines the maximum overall utility in B7:G7, the MATCH function determines the location of that maximum in B7:G7, and the INDEX function returns the alternative name located in B2:G2. The zero argument in the MATCH function is needed to specify that an exact match is required; the zero argument in the INDEX function is used as a placeholder and could be omitted in this application without affecting the results. Cell B13 combines these functions into a single formula.

Figure 5 Formulas for Weight Ratio Assessment and Choice

	A	B	H	I	J
1		Non-Dominated Alternatives		Assess weight ratios.	
2	Attribute	Alta			
3	Cost	0		Weight Ratio	Input
4	Lifetime	1		Cost/Lifetime	5
5	Safety	1		Cost/Safety	1.5
6					
7	Overall	=SUMPRODUCT(B3:B5,$J$8:$J$10)		Weights	
8				Cost	=1/(1/J4+1/J5+1)
9	Max Value	=MAX(B7:G7)		Lifetime	=J8/J4
10	Location	=MATCH(B9,B7:G7,0)		Safety	=J8/J5
11	Choice	=INDEX(B2:G2,0,B10)			
12					
13	Choice	=INDEX(B2:G2,0,MATCH(MAX(B7:G7),B7:G7,0))			

After deleting cells A9:B12, the single formula is in cell B9. The arrangement shown in Figure 6 is used for the remaining analyses.

Figure 6 Weight Ratio Choice for Sensitivity Analysis

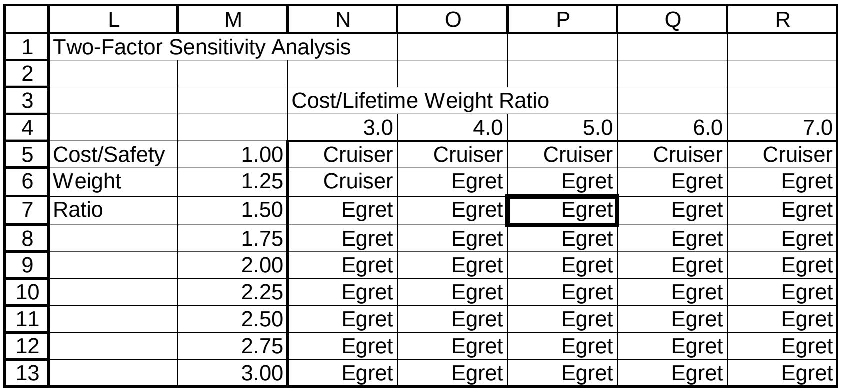


	A	B	C	D	E	F	G
1		Non-Dominated Alternatives					
2	Attribute	Alta	Bulldog	Cruiser	Delta	Egret	Fleet
3	Cost	0.000	0.200	0.400	0.600	0.800	1.000
4	Lifetime	1.000	1.000	0.500	0.500	0.000	0.000
5	Safety	1.000	0.667	1.000	0.667	0.667	0.000
6							
7	Overall	0.464	0.452	0.625	0.613	0.667	0.536
8							
9	Choice	Egret					

Weight Ratio Sensitivity Analysis

The decision maker specified tradeoffs using weight ratios, so it is appropriate to see whether the choice is sensitive to changes in those assessed values. To construct a two-way data table for sensitivity analysis of the weight ratios as shown in Figures 7 and 8, enter a set of values in a row, N4:R4, and another set of values in a column, M5:M13. In the top left cell of the data table, M4, enter a formula for determining the data table's output values, =B9. (To improve the appearance of the table, cell M4 is formatted with a custom three-semicolon format so that the formula result is not displayed.) Select M4:R13. Choose Data | Table. In the Data Table dialog box, specify J4 as the Row Input Cell and J5 as the Column Input Cell. Click OK.

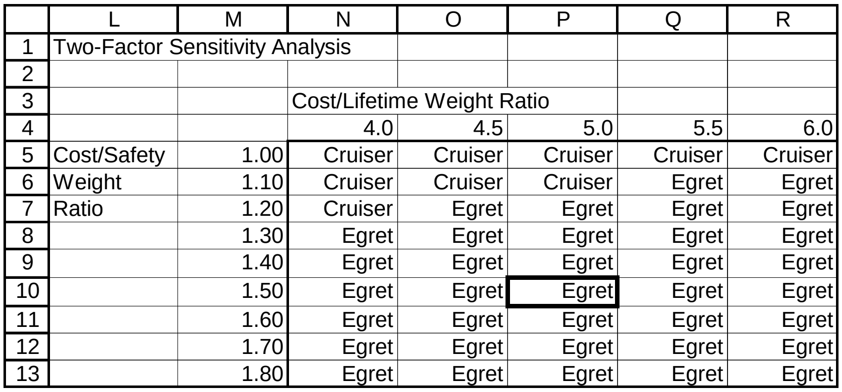
Figure 7 Coarse Two-Factor Sensitivity Analysis of Weight Ratios



	L	M	N	O	P	Q	R
1	Two-Factor Sensitivity Analysis						
2							
3			Cost/Lifetime Weight Ratio				
4			3.0	4.0	5.0	6.0	7.0
5	Cost/Safety	1.00	Cruiser	Cruiser	Cruiser	Cruiser	Cruiser
6	Weight	1.25	Cruiser	Egret	Egret	Egret	Egret
7	Ratio	1.50	Egret	Egret	Egret	Egret	Egret
8		1.75	Egret	Egret	Egret	Egret	Egret
9		2.00	Egret	Egret	Egret	Egret	Egret
10		2.25	Egret	Egret	Egret	Egret	Egret
11		2.50	Egret	Egret	Egret	Egret	Egret
12		2.75	Egret	Egret	Egret	Egret	Egret
13		3.00	Egret	Egret	Egret	Egret	Egret

Cell P7, corresponding to the original assessments, has a border. The data table is dynamic, so the macro view may be refined near the base-case assessments by specifying different input values.

Figure 8 Fine Two-Factor Sensitivity Analysis of Weight Ratios



	L	M	N	O	P	Q	R
1	Two-Factor Sensitivity Analysis						
2							
3			Cost/Lifetime Weight Ratio				
4			4.0	4.5	5.0	5.5	6.0
5	Cost/Safety	1.00	Cruiser	Cruiser	Cruiser	Cruiser	Cruiser
6	Weight	1.10	Cruiser	Cruiser	Cruiser	Egret	Egret
7	Ratio	1.20	Cruiser	Egret	Egret	Egret	Egret
8		1.30	Egret	Egret	Egret	Egret	Egret
9		1.40	Egret	Egret	Egret	Egret	Egret
10		1.50	Egret	Egret	Egret	Egret	Egret
11		1.60	Egret	Egret	Egret	Egret	Egret
12		1.70	Egret	Egret	Egret	Egret	Egret
13		1.80	Egret	Egret	Egret	Egret	Egret

Figure 8 shows that the Cost/Safety weight ratio must be less than 1.2 to affect the choice. If the decision maker regards 1.2 as "far away" from 1.5, then the Egret choice is appropriate. Otherwise, the decision maker should think more carefully about the original assessments before making a choice based on this analysis. The assessment of the Cost/Lifetime weight ratio is not as critical, because any value between 4 and 6 yields the same choice.

Swing Weight Assessment

Compared to weight ratio assessment, the swing weight method requires assessments that are similar to directly assigning an overall utility to an alternative. However, the hypothetical alternatives requiring assessment in this method are constructed so that it should be easier for the decision maker to assign overall utilities to them instead of to the actual alternatives.

The swing weight method involves four steps as shown in Figure 9.

1) Develop the hypothetical alternatives. The number of hypothetical alternatives equals the number of attributes plus one. The benchmark alternative in column J is worst for all attributes. Each other hypothetical alternative, shown in columns K, L, and M, has one attribute at best and all others at worst.

2) Rank the hypothetical alternatives, as shown in row 7. This is an intermediate step that facilitates assigning overall utilities.

3) Assign overall utility scores reflecting overall satisfaction for the hypothetical alternatives. The benchmark worst case has score zero, and the first-ranked alternative has score 100. Then assign level-of-satisfaction scores to the intermediate alternatives, as shown in cells L9 and M9.

4) Sum the scores, as shown in cell N9. In the additive utility function, the weight for each attribute equals the score divided by sum of the scores. (The algebra solution, not shown here, is based on the special zero and one individual utility values of the hypothetical alternatives.) Formulas are shown in Figure 10.

Figure 9 Hypothetical Alternatives and Weights for Swing Weight Assessment

	I	J	K	L	M	N
1		Hypothetical Alternatives				
2	Attribute	Worst	Best Cost	Best Lifetime	Best Safety	
3	Cost	$20	$10	$20	$20	
4	Lifetime	6	6	10	6	
5	Safety	Low	Low	Low	High	
6						
7	Rank	4	1	3	2	
8						Total
9	Overall Score	0	100	20	70	190
10						
11	Weight	0.000	0.526	0.105	0.368	
12						
13	Decision Maker's Inputs Underlined					

Figure 10 Formulas for Swing Weight Assessment

	I	J	K	L	M	N
1		Hypothetical Alternatives				
2	Attribute	Worst	Best Cost	Best Lifetime	Best Safety	
3	Cost	20	10	20	20	
4	Lifetime	6	6	10	6	
5	Safety	Low	Low	Low	High	
6						
7	Rank	4	1	3	2	
8						Total
9	Overall Score	0	100	20	70	=SUM(J9:M9)
10						
11	Weight	=J9/$N$9	=K9/$N$9	=L9/$N$9	=M9/$N$9	
12						
13	Decision Maker's Inputs Underlined					

The individual utility values are in a column, and the weights are in a row. The SUMPRODUCT function requires that the two arrays for its arguments have the same orientation, so the TRANSPOSE function converts the weights into a column format, as shown in Figure 11. The function in B7 must be array-entered; after typing the function, hold down Control and Shift while you press Enter.

Figure 11 Formulas for Swing Weight Choice

	A	B
1		Non-Dominated Alternatives
2	Attribute	Alta
3	Cost	0
4	Lifetime	1
5	Safety	1
6		
7	Overall	=SUMPRODUCT(B3:B5,TRANSPOSE($K$11:$M$11))
8		
9	Choice	=INDEX(B2:G2,0,MATCH(MAX(B7:G7),B7:G7,0))

Figure 12 Swing Weight Choice

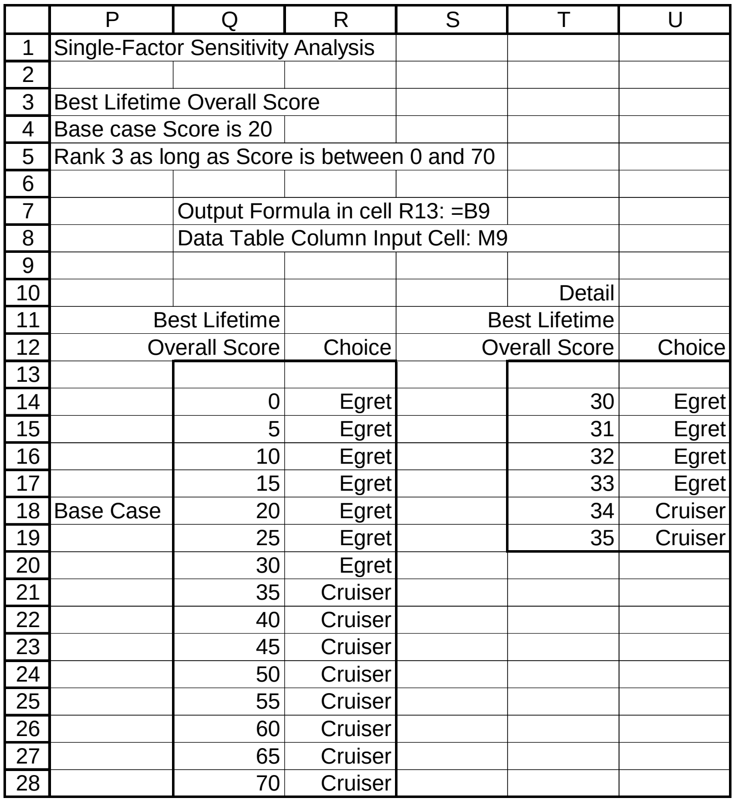
	A	B	C	D	E	F	G
1		Non-Dominated Alternatives					
2	Attribute	Alta	Bulldog	Cruiser	Delta	Egret	Fleet
3	Cost	0.000	0.200	0.400	0.600	0.800	1.000
4	Lifetime	1.000	1.000	0.500	0.500	0.000	0.000
5	Safety	1.000	0.667	1.000	0.667	0.667	0.000
6							
7	Overall	0.474	0.456	0.632	0.614	0.667	0.526
8							
9	Choice	Egret					

Swing Weight Sensitivity Analysis

The decision maker specified tradeoffs using overall scores for the hypothetical alternatives, so it is appropriate to see whether the choice is sensitive to changes in those assessed values. Figure 13 shows the sensitivity for the Best-Lifetime score that was specified as 20 relative to the worst-case benchmark and the highest-ranked Best-Cost hypothetical alternative. The Best-Lifetime alternative is still ranked 3 as long as its score is between 0 and 70.

To improve the appearance of the sensitivity analysis tables in Figure 13, the output formula cells, R13 and T13, have a three-semicolon custom format.

Figure 13 Sensitivity Analysis of Swing Weight Best-Lifetime Score



	P	Q	R	S	T	U
1	Single-Factor Sensitivity Analysis					
2						
3	Best Lifetime Overall Score					
4	Base case Score is 20					
5	Rank 3 as long as Score is between 0 and 70					
6						
7		Output Formula in cell R13: =B9				
8		Data Table Column Input Cell: M9				
9						
10					Detail	
11		Best Lifetime			Best Lifetime	
12		Overall Score	Choice		Overall Score	Choice
13						
14		0	Egret		30	Egret
15		5	Egret		31	Egret
16		10	Egret		32	Egret
17		15	Egret		33	Egret
18	Base Case	20	Egret		34	Cruiser
19		25	Egret		35	Cruiser
20		30	Egret			
21		35	Cruiser			
22		40	Cruiser			
23		45	Cruiser			
24		50	Cruiser			
25		55	Cruiser			
26		60	Cruiser			
27		65	Cruiser			
28		70	Cruiser			

The results in the left table Figure 13, cells Q13:R28, indicate that the Best-Lifetime score must be greater than 30 to affect the choice. A refined data table in cells T13:U19 shows that the score must be greater than 33 before the choice changes from Egret to Cruiser. If the decision maker regards 33 as "far away" from 20, then the Egret choice is appropriate.

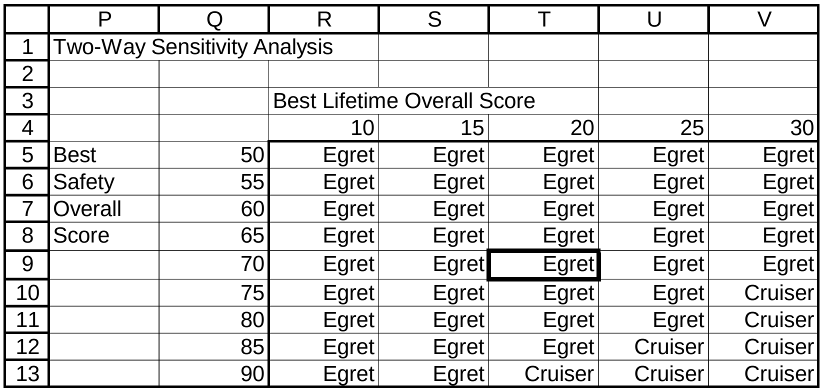
Figure 14 shows a similar sensitivity analysis for the Best-Safety score. The assessed score of 70 must be greater than 89 to affect the choice.

Figure 14 Sensitivity Analysis of Swing Weight Best-Safety Score

	W	X	Y	Z	AA	AB
1	Single-Factor Sensitivity Analysis					
2						
3	Best Safety Overall Score					
4	Base case Score is 70					
5	Rank 2 as long as Score is between 20 and 100					
6						
7		Output Formula in cell Y13 and cell AB13: =B9				
8		Data Table Column Input Cell: N9				
9						
10					Detail	
11		Best Safety			Best Safety	
12		Overall Score	Choice		Overall Score	Choice
13						
14		20	Fleet		85	Egret
15		25	Fleet		86	Egret
16		30	Fleet		87	Egret
17		35	Egret		88	Egret
18		40	Egret		89	Egret
19		45	Egret		90	Cruiser
20		50	Egret			
21		55	Egret			
22		60	Egret			
23		65	Egret			
24	Base Case	70	Egret			
25		75	Egret			
26		80	Egret			
27		85	Egret			
28		90	Cruiser			
29		95	Cruiser			
30		100	Cruiser			

To construct a two-way data table for sensitivity analysis of the swing weight assessments as shown in Figure 15, enter a set of values in a row, R4:V4, and another set of values in a column, Q5:Q13. In the top left cell of the data table, Q4, enter a formula for determining the data table's output values, =B9. (To improve the appearance of the table, cell Q4 is formatted with a custom three-semicolon format so that the formula result is not displayed.) Select Q4:V13. Choose Data | Table. In the Data Table dialog box, specify L9 as the Row Input Cell and M9 as the Column Input Cell. Click OK.

Figure 15 Sensitivity Analysis of Both Swing Weight Scores



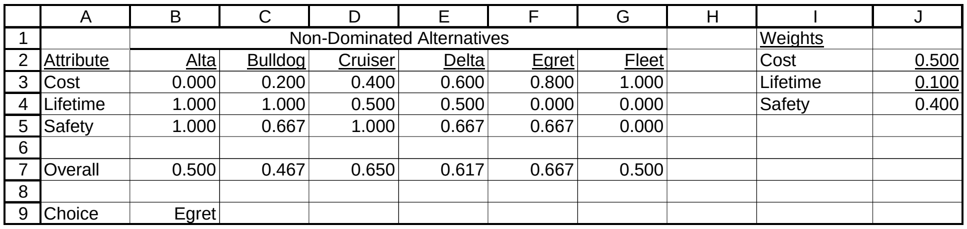
	P	Q	R	S	T	U	V
1	Two-Way Sensitivity Analysis						
2							
3			Best Lifetime Overall Score				
4			10	15	20	25	30
5	Best	50	Egret	Egret	Egret	Egret	Egret
6	Safety	55	Egret	Egret	Egret	Egret	Egret
7	Overall	60	Egret	Egret	Egret	Egret	Egret
8	Score	65	Egret	Egret	Egret	Egret	Egret
9		70	Egret	Egret	Egret	Egret	Egret
10		75	Egret	Egret	Egret	Egret	Cruiser
11		80	Egret	Egret	Egret	Egret	Cruiser
12		85	Egret	Egret	Egret	Cruiser	Cruiser
13		90	Egret	Egret	Cruiser	Cruiser	Cruiser

The table shows that the choice changes from Egret to Cruiser if the combination of assessments is changed from 20 & 70 to 30 & 75. This table could be refined to examine the exact threshold values.

Direct Weight Assessment and Sensitivity Analysis

In some situations the decision maker may be able to assign tradeoff weights directly. Figure 16 shows results using the formulas shown in Figure 17.

Figure 16 Direct Weight Assessment



	A	B	C	D	E	F	G	H	I	J
1		Non-Dominated Alternatives							Weights	
2	Attribute	Alta	Bulldog	Cruiser	Delta	Egret	Fleet		Cost	0.500
3	Cost	0.000	0.200	0.400	0.600	0.800	1.000		Lifetime	0.100
4	Lifetime	1.000	1.000	0.500	0.500	0.000	0.000		Safety	0.400
5	Safety	1.000	0.667	1.000	0.667	0.667	0.000			
6										
7	Overall	0.500	0.467	0.650	0.617	0.667	0.500			
8										
9	Choice	Egret								

The formula in cell B9 includes an IF function to verify that each weight is between 0 and 1, inclusive, and that the sum of the weights equals one. If not, the formula returns empty text. This formula must be array-entered; after typing the function, hold down Control and Shift while you press Enter.

Figure 17 Formulas for Direct Weight Assessment

	A	B	H	I	J
1		Non-Dominated Alternatives		Weights	
2	Attribute	Alta		Cost	0.5
3	Cost	0		Lifetime	0.1
4	Lifetime	1		Safety	=1-J3-J2
5	Safety	1			
6					
7	Overall	=SUMPRODUCT(B3:B5,$J$2:$J$4)			
8					
9	Choice	=IF(AND(SUM(J2:J4)<=1,J2:J4>=0),INDEX(B2:G2,0,MATCH(MAX(B7:G7),B7:G7,0)),"")			

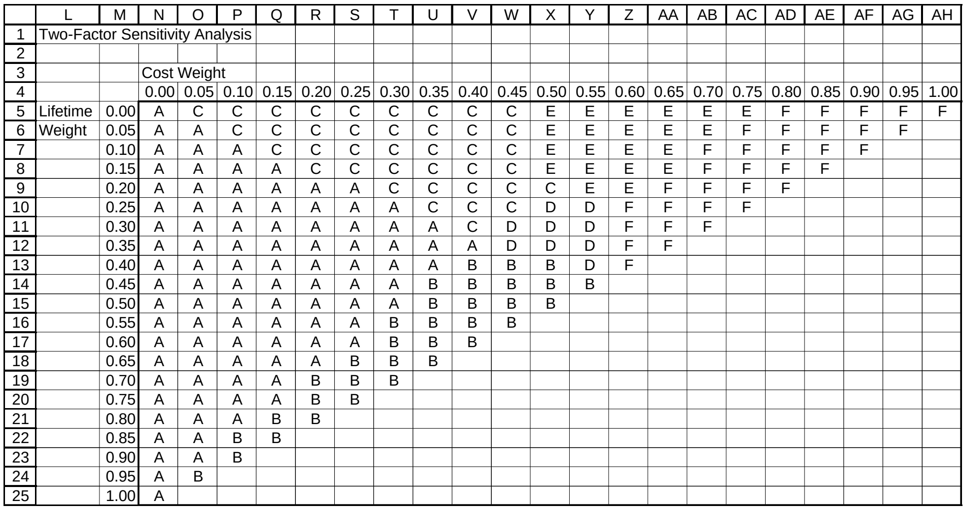
Figure 18 shows a two-way table for sensitivity analysis of the weights. Cell R5 corresponds to the approximate base case assessments in the weight ratio and swing weight methods.

Figure 18 Sensitivity Analysis of Direct Weight Assessment

	L	M	N	O	P	Q	R	S	T	U	V
1	Two-Factor Sensitivity Analysis										
2											
3			Cost Weight								
4			0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
5	Lifetime	0.1	Alta	Cruiser	Cruiser	Cruiser	Egret	Egret	Fleet	Fleet	Fleet
6	Weight	0.2	Alta	Alta	Cruiser	Cruiser	Cruiser	Egret	Fleet	Fleet	
7		0.3	Alta	Alta	Alta	Cruiser	Delta	Fleet	Fleet		
8		0.4	Alta	Alta	Alta	Bulldog	Bulldog	Fleet			
9		0.5	Alta	Alta	Alta	Bulldog	Bulldog				
10		0.6	Alta	Alta	Bulldog	Bulldog					
11		0.7	Alta	Bulldog	Bulldog						
12		0.8	Alta	Bulldog							
13		0.9	Bulldog								

Figure 19 is a more detailed view. The choice formula in cell B9 is modified by placing the INDEX function inside the LEFT function so that only the first letter of the alternative's name is returned.

Figure 19 Detailed Sensitivity Analysis of Direct Weight Assessment



	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH
1	Two-Factor Sensitivity Analysis																						
2																							
3			Cost Weight																				
4			0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
5	Lifetime	0.00	A	C	C	C	C	C	C	C	C	C	E	E	E	E	E	E	F	F	F	F	F
6	Weight	0.05	A	A	C	C	C	C	C	C	C	C	E	E	E	E	E	F	F	F	F	F	
7		0.10	A	A	A	C	C	C	C	C	C	C	E	E	E	E	F	F	F	F	F		
8		0.15	A	A	A	A	C	C	C	C	C	C	E	E	E	E	F	F	F	F			
9		0.20	A	A	A	A	A	A	C	C	C	C	C	E	E	F	F	F	F				
10		0.25	A	A	A	A	A	A	A	C	C	C	D	D	F	F	F	F					
11		0.30	A	A	A	A	A	A	A	A	C	D	D	D	F	F	F						
12		0.35	A	A	A	A	A	A	A	A	A	D	D	D	F	F							
13		0.40	A	A	A	A	A	A	A	A	B	B	B	D	F								
14		0.45	A	A	A	A	A	A	A	B	B	B	B	B									
15		0.50	A	A	A	A	A	A	A	B	B	B	B										
16		0.55	A	A	A	A	A	A	B	B	B	B											
17		0.60	A	A	A	A	A	A	B	B	B												
18		0.65	A	A	A	A	A	B	B	B													
19		0.70	A	A	A	A	B	B	B														
20		0.75	A	A	A	A	B	B															
21		0.80	A	A	A	B	B																
22		0.85	A	A	B	B																	
23		0.90	A	A	B																		
24		0.95	A	B																			
25		1.00	A																				

The results in Figure 19 show that all alternatives in this data set are candidates depending on the tradeoffs specified by the decision maker. In general, moving left to right, if more weight is given to cost, a less expensive alternative is chosen.

Summary

This paper considered three methods for assessing tradeoffs in the additive utility function. For each method sensitivity analysis is useful for gaining insight into which tradeoff assumptions are critical. Kirkwood [2] includes Excel VBA methods for sensitivity analysis of individual utility functions in addition to weights.

Sensitivity Analysis Examples References

[1] Clemen, R.T. Making Hard Decisions: An Introduction to Decision Analysis, 2nd Edition. Duxbury Press, 1996.

[2] Kirkwood, C.W. Strategic Decision Making: Multiobjective Decision Analysis with Spreadsheets. Duxbury Press, 1997.

[3] Hammond, J.S., Keeney, R.L., and Raiffa, H. Smart Choices: A Practical Guide to Making Better Decisions. Harvard Business School Press, 1999.

Screenshots from Excel to Word

To copy Excel displays for the figures in this paper, choose File | Page Setup | Sheet | Gridlines and File | Page Setup | Sheet | Row And Column Headings. Select the cell range, hold down the Shift key, and in Excel's main menu choose Edit | Copy Picture | As Shown When Printed. In Word, position the pointer in an empty paragraph and choose Edit | Paste.

Product Mix Optimization

6

6.1 LINEAR PROGRAMMING CONCEPTS

Formulation

Decision variables (Excel Solver "Changing Cells")

Objective function ("Target Cell")

Constraints and right-hand-side values ("Constraints")

Non-negativity constraints ("Constraints")

Graphical Solution

Constraints

Feasible region

Corner points (extreme points)

Objective function value at each corner point

Total enumeration vs. simplex algorithm (search)

Optimal solution

Sensitivity Analysis

Post-optimality analysis and interpretation of computer print-outs

Shadow price (a marginal value)

(Excel Solver Sensitivity Report, Constraints section, "Shadow Price")

The shadow price for a particular constraint is the amount of change in the value of the objective function corresponding to a unit change in the right-hand-side value of the constraint.

Range on a right-hand-side (RHS) value

> (Excel Solver Sensitivity Report, Constraints section, “Allowable Increase/Decrease”)
>
> Range over which the shadow price applies. The optimal values of the decision variables would change depending on the exact RHS value, but the current mix of decision variables remains optimal over the specified range of RHS values.

Range on an objective function coefficient

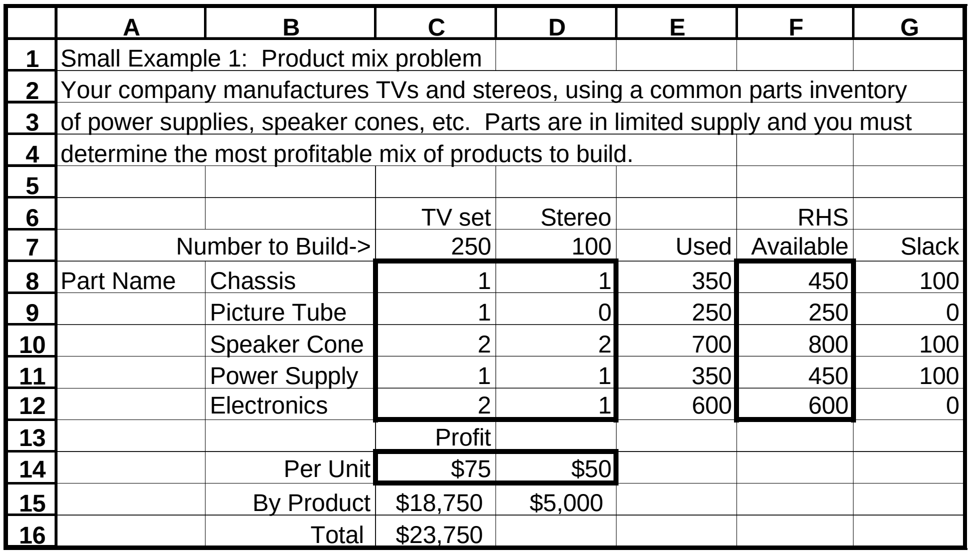
> (Excel Solver Sensitivity Report, Changing Cells section, “Allowable Increase/Decrease”)
>
> Range over which an objective function coefficient could change with the current optimal solution remaining optimal (same mix and values of decision variables). The value of the objective function would change depending on the exact value of the objective function coefficient.

Simplex algorithm terminology

> Slack, surplus, and artificial variables
>
> Basic variables (variables "in the solution," typically with non-zero values)
>
> Non-basic variables (value equal to zero)
>
> Complementary slackness

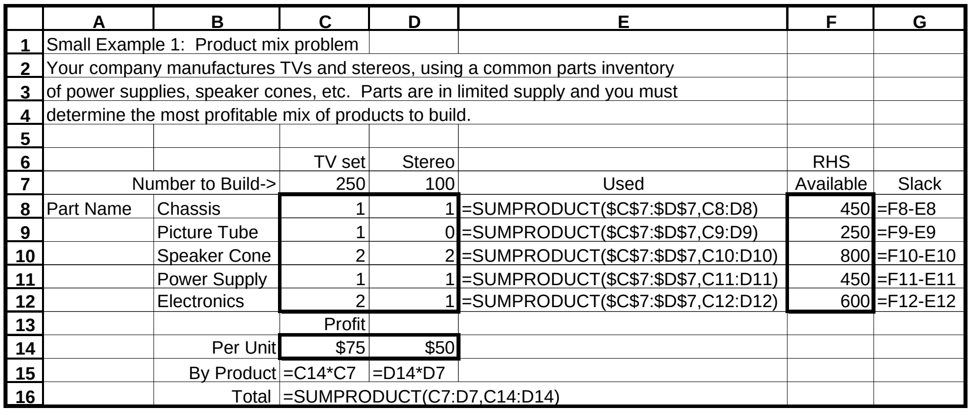
6.2 BASIC PRODUCT MIX PROBLEM

Figure 6.1 Display



	A	B	C	D	E	F	G
1	Small Example 1: Product mix problem						
2	Your company manufactures TVs and stereos, using a common parts inventory						
3	of power supplies, speaker cones, etc. Parts are in limited supply and you must						
4	determine the most profitable mix of products to build.						
5							
6			TV set	Stereo		RHS	
7		Number to Build->	250	100	Used	Available	Slack
8	Part Name	Chassis	1	1	350	450	100
9		Picture Tube	1	0	250	250	0
10		Speaker Cone	2	2	700	800	100
11		Power Supply	1	1	350	450	100
12		Electronics	2	1	600	600	0
13			Profit				
14		Per Unit	$75	$50			
15		By Product	$18,750	$5,000			
16		Total	$23,750				

Figure 6.2 Formulas



	A	B	C	D	E	F	G
1	Small Example 1: Product mix problem						
2	Your company manufactures TVs and stereos, using a common parts inventory						
3	of power supplies, speaker cones, etc. Parts are in limited supply and you must						
4	determine the most profitable mix of products to build.						
5							
6			TV set	Stereo		RHS	
7		Number to Build->	250	100	Used	Available	Slack
8	Part Name	Chassis	1	1	=SUMPRODUCT($C$7:$D$7,C8:D8)	450	=F8-E8
9		Picture Tube	1	0	=SUMPRODUCT($C$7:$D$7,C9:D9)	250	=F9-E9
10		Speaker Cone	2	2	=SUMPRODUCT($C$7:$D$7,C10:D10)	800	=F10-E10
11		Power Supply	1	1	=SUMPRODUCT($C$7:$D$7,C11:D11)	450	=F11-E11
12		Electronics	2	1	=SUMPRODUCT($C$7:$D$7,C12:D12)	600	=F12-E12
13			Profit				
14		Per Unit	$75	$50			
15		By Product	=C14*C7	=D14*D7			
16		Total	=SUMPRODUCT(C7:D7,C14:D14)				

Figure 6.3 Graphical Solution

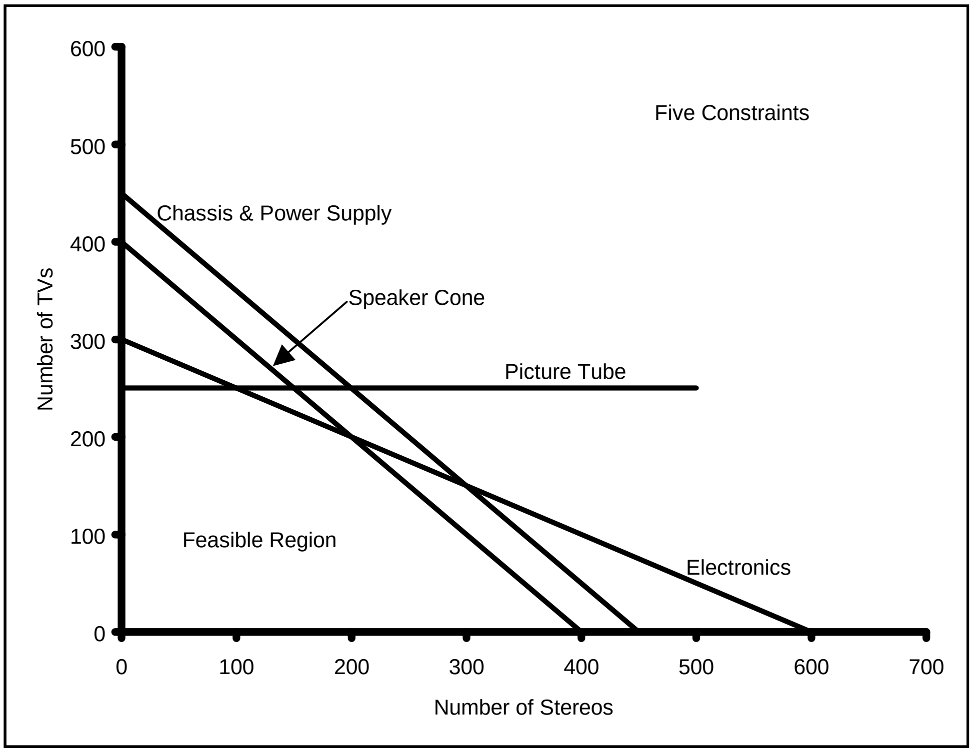
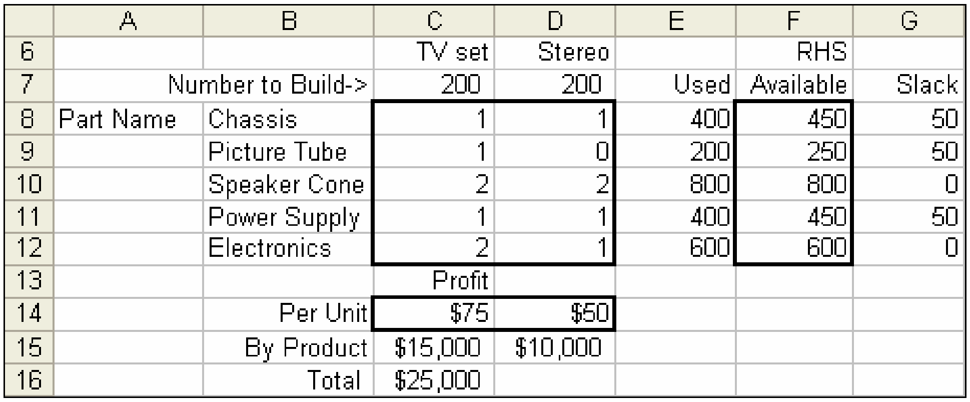
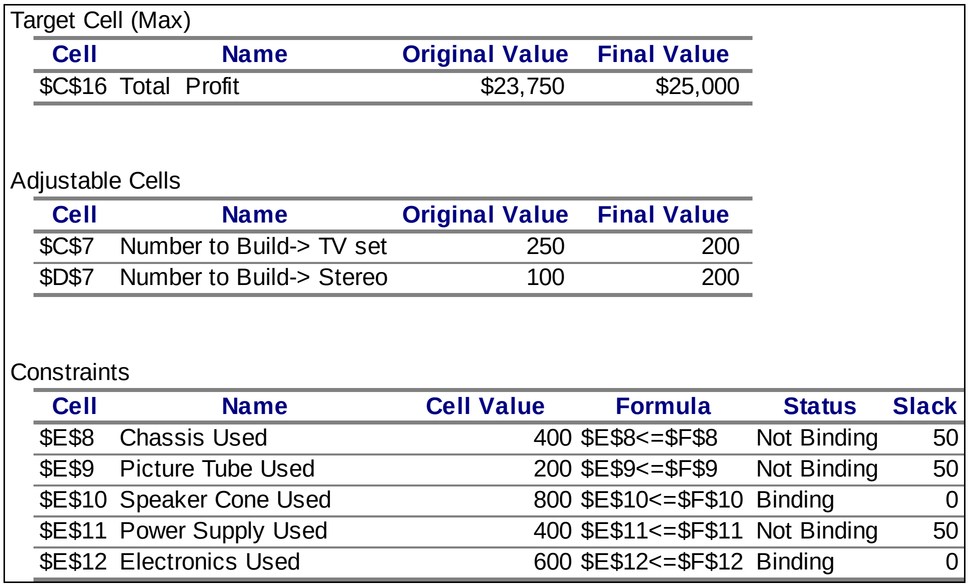


Figure 6.4 Solver Solution



	A	B	C	D	E	F	G
6			TV set	Stereo		RHS	
7		Number to Build->	200	200	Used	Available	Slack
8	Part Name	Chassis	1	1	400	450	50
9		Picture Tube	1	0	200	250	50
10		Speaker Cone	2	2	800	800	0
11		Power Supply	1	1	400	450	50
12		Electronics	2	1	600	600	0
13			Profit				
14		Per Unit	$75	$50			
15		By Product	$15,000	$10,000			
16		Total	$25,000				

Figure 6.5 Solver Answer Report



Target Cell (Max)

Cell	Name	Original Value	Final Value
$C$16	Total Profit	$23,750	$25,000

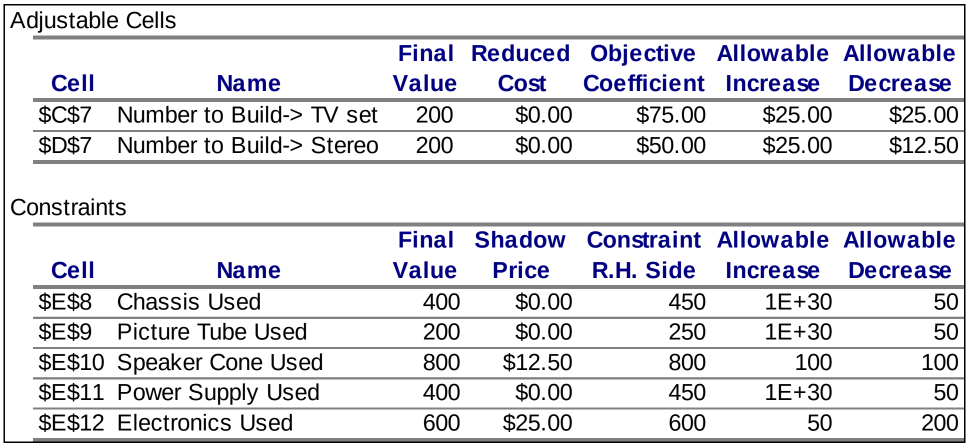
Adjustable Cells

Cell	Name	Original Value	Final Value
$C$7	Number to Build-> TV set	250	200
$D$7	Number to Build-> Stereo	100	200

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$8	Chassis Used	400	$E$8<=$F$8	Not Binding	50
$E$9	Picture Tube Used	200	$E$9<=$F$9	Not Binding	50
$E$10	Speaker Cone Used	800	$E$10<=$F$10	Binding	0
$E$11	Power Supply Used	400	$E$11<=$F$11	Not Binding	50
$E$12	Electronics Used	600	$E$12<=$F$12	Binding	0

Figure 6.6 Solver Sensitivity Report



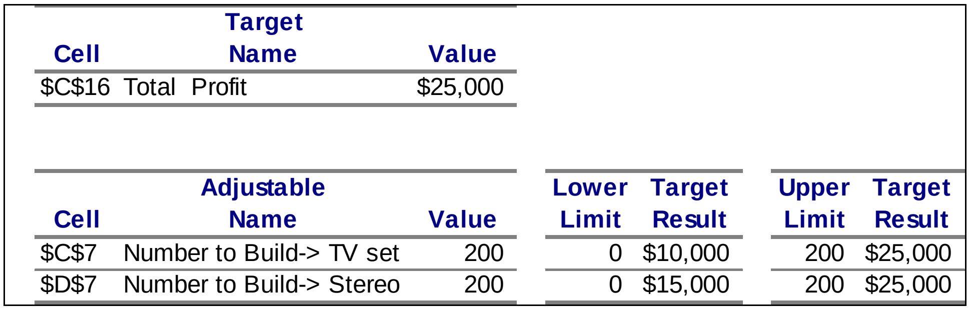
Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$7	Number to Build-> TV set	200	$0.00	$75.00	$25.00	$25.00
$D$7	Number to Build-> Stereo	200	$0.00	$50.00	$25.00	$12.50

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$8	Chassis Used	400	$0.00	450	1E+30	50
$E$9	Picture Tube Used	200	$0.00	250	1E+30	50
$E$10	Speaker Cone Used	800	$12.50	800	100	100
$E$11	Power Supply Used	400	$0.00	450	1E+30	50
$E$12	Electronics Used	600	$25.00	600	50	200

Figure 6.7 Solver Limits Report



Cell	Target Name	Value
$C$16	Total Profit	$25,000

Cell	Adjustable Name	Value	Lower Limit	Target Result	Upper Limit	Target Result
$C$7	Number to Build-> TV set	200	0	$10,000	200	$25,000
$D$7	Number to Build-> Stereo	200	0	$15,000	200	$25,000

6.3 OUTDOORS PROBLEM

Outdoors, Inc., has lawn furniture as one of its product lines. They currently have three items in that line: a lawn chair, a standard bench, and a table. These products are produced in a two-step manufacturing process involving the tube bending department and the welding department. The hours required by each item in each department is as follows:

	Product			
Department	Chair	Bench	Table	Present Capacity
Bending	1.2	1.7	1.2	1,000 hours
Welding	0.8	0.0	2.3	1,200 hours

The profit contribution that Outdoors receives from manufacture and sale of one unit of each product is $3 for a chair, $3 for a bench, and $5 for a table.

The company is trying to plan its production mix for the current selling season. They feel that they can sell any number they produce, but unfortunately production is further limited by available material because of a prolonged strike. The company currently has on hand 2,000 pounds of tubing. The three products require the following amounts of this tubing: 2 pounds per chair, 3 pounds per bench, and 4.5 pounds per table.

In order to determine the optimal product mix, the production manager has formulated the linear programming problem as shown below.

	Product				
	Chair	Bench	Table		
Contribution	$3	$3	$5		
Constraint				Relation	Limit
Bending	1.2	1.7	1.2	<=	1,000
Welding	0.8	0.0	2.3	<=	1,200
Tubing	2.0	3.0	4.5	<=	2,000

A. The inventory manager suggests that the company produce 200 units of each product. Is the plan to produce 200 units of each product a feasible plan, i.e., does it satisfy all contraints? If not, which constraints are not satisfied?

B. If the company produces 200 chairs, 200 benches, and 200 tables, how much tubing, if any, will be left over?

Each of the following questions refer to the solution of the original linear programming problem.

C. A local manufacturing firm has excess capacity in its welding department and has offered to sell 100 hours of welding time to Outdoors for $3 per hour. This arrangement would cost $300 and would increase welding capacity from 1,200 hours to 1,300 hours. Should Outdoors purchase the additional welding capacity? Why or why not?

D. The marketing manager thinks that the original estimate of $3 profit contribution per chair should be changed to $2.50 per chair. Should the production manager solve the linear programming problem again using the $2.50 value, or should Outdoors go ahead with the plan to produce 700 chairs, zero benches, and 133 tables? Why or why not?

E. A local metal products distributor has offered to sell Outdoors some additional metal tubing for 60 cents per pound. Should Outdoors buy additional tubing at this price? If so, how much would their contribution increase if they bought 500 pounds and used it in an optimal fashion?

F. The R&D department has been redesigning the bench to make it more profitable. The new design will require 1.1 hours of tube bending time, 2 hours of welding time, and 2.0 pounds of metal tubing. If they can sell one unit of this bench with a unit contribution of $3, what effect will it have on overall contribution?

G. Marketing has suggested a new patio awning that would require 1.8 hours of tube bending time, 0.5 hours of welding time, and 1.3 pounds of metal tubing.

What contribution must this new product have to make it attractive to produce this season?

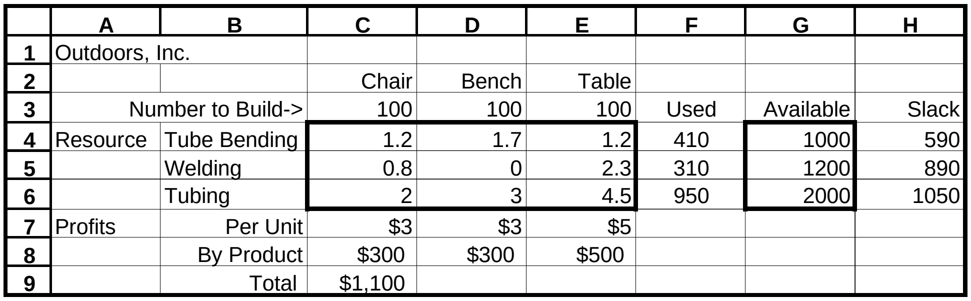
H. Outdoors, Inc., has a chance to sell some of its capacity in tube bending at a price of $1.50 per hour. If it sells 200 hours at that price, how will this affect contribution?

I. If Outdoors, Inc., feels that it must produce benches to round out its production line, what effect will production of benches have on overall contribution?

Adapted from Vatter et al., *Quantitative Methods in Management*, Irwin, 1978.

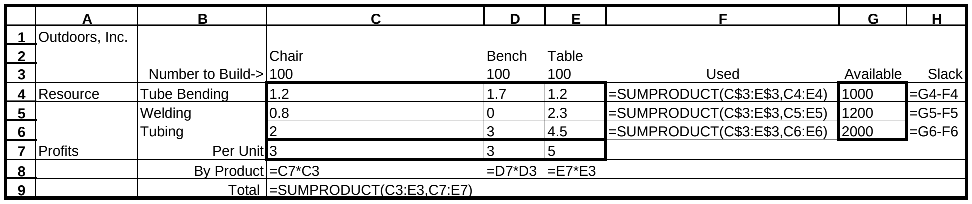
Spreadsheet Model

Figure 6.8 Model



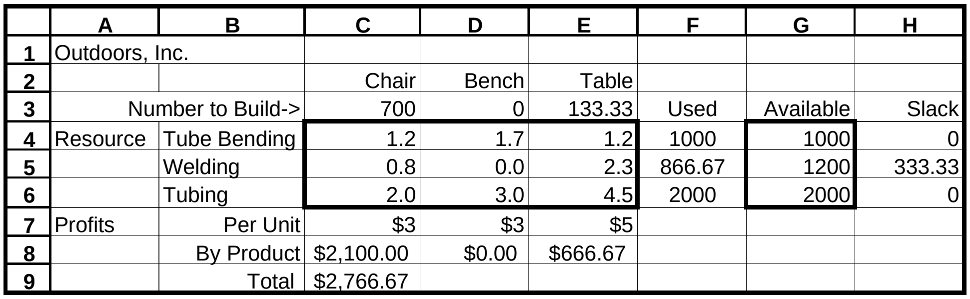
	A	B	C	D	E	F	G	H
1	Outdoors, Inc.							
2			Chair	Bench	Table			
3		Number to Build->	100	100	100	Used	Available	Slack
4	Resource	Tube Bending	1.2	1.7	1.2	410	1000	590
5		Welding	0.8	0	2.3	310	1200	890
6		Tubing	2	3	4.5	950	2000	1050
7	Profits	Per Unit	$3	$3	$5			
8		By Product	$300	$300	$500			
9		Total	$1,100					

Figure 6.9 Formulas



	A	B	C	D	E	F	G	H
1	Outdoors, Inc.							
2			Chair	Bench	Table			
3		Number to Build->	100	100	100	Used	Available	Slack
4	Resource	Tube Bending	1.2	1.7	1.2	=SUMPRODUCT(C$3:E$3,C4:E4)	1000	=G4-F4
5		Welding	0.8	0	2.3	=SUMPRODUCT(C$3:E$3,C5:E5)	1200	=G5-F5
6		Tubing	2	3	4.5	=SUMPRODUCT(C$3:E$3,C6:E6)	2000	=G6-F6
7	Profits	Per Unit	3	3	5			
8		By Product	=C7*C3	=D7*D3	=E7*E3			
9		Total	=SUMPRODUCT(C3:E3,C7:E7)					

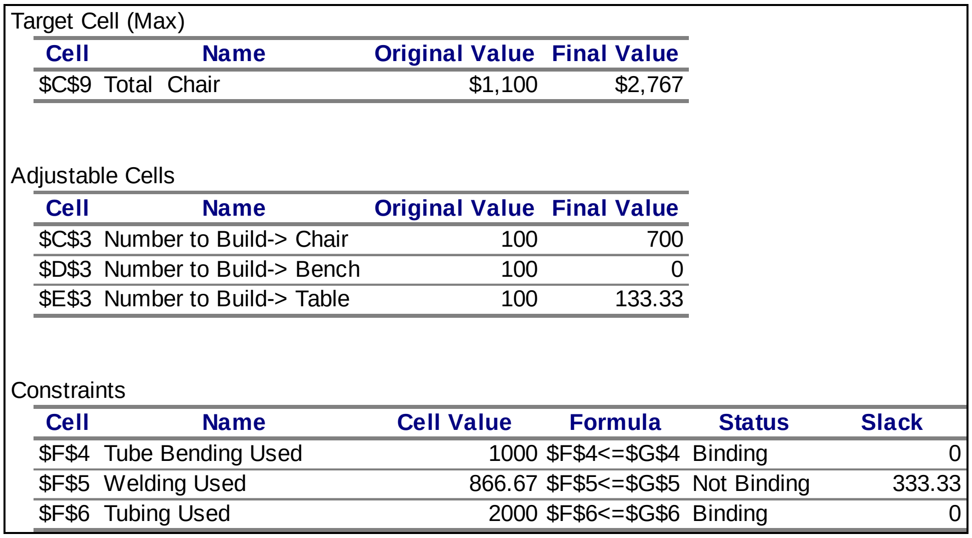
Figure 6.10 Solution



	A	B	C	D	E	F	G	H
1	Outdoors, Inc.							
2			Chair	Bench	Table			
3		Number to Build->	700	0	133.33	Used	Available	Slack
4	Resource	Tube Bending	1.2	1.7	1.2	1000	1000	0
5		Welding	0.8	0.0	2.3	866.67	1200	333.33
6		Tubing	2.0	3.0	4.5	2000	2000	0
7	Profits	Per Unit	$3	$3	$5			
8		By Product	$2,100.00	$0.00	$666.67			
9		Total	$2,766.67					

Solver Reports

Figure 6.11 Answer Report



Target Cell (Max)

Cell	Name	Original Value	Final Value
$C$9	Total Chair	$1,100	$2,767

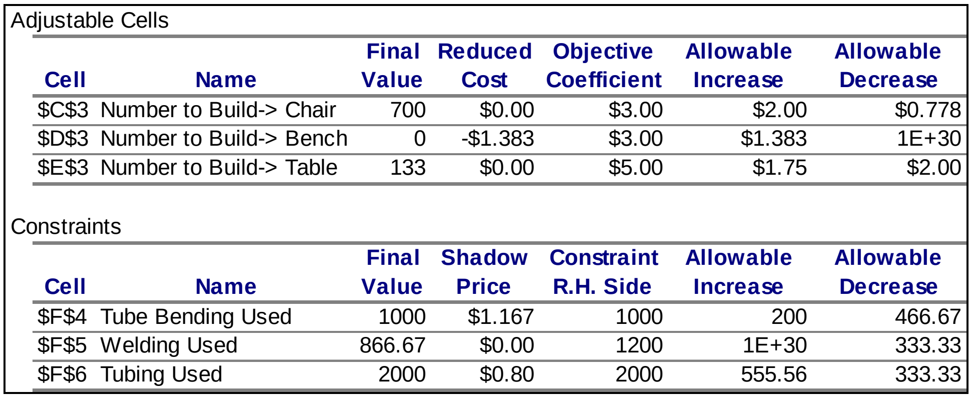
Adjustable Cells

Cell	Name	Original Value	Final Value
$C$3	Number to Build-> Chair	100	700
$D$3	Number to Build-> Bench	100	0
$E$3	Number to Build-> Table	100	133.33

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$F$4	Tube Bending Used	1000	$F$4<=$G$4	Binding	0
$F$5	Welding Used	866.67	$F$5<=$G$5	Not Binding	333.33
$F$6	Tubing Used	2000	$F$6<=$G$6	Binding	0

Figure 6.12 Sensitivity Report



Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$3	Number to Build-> Chair	700	$0.00	$3.00	$2.00	$0.778
$D$3	Number to Build-> Bench	0	-$1.383	$3.00	$1.383	1E+30
$E$3	Number to Build-> Table	133	$0.00	$5.00	$1.75	$2.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$4	Tube Bending Used	1000	$1.167	1000	200	466.67
$F$5	Welding Used	866.67	$0.00	1200	1E+30	333.33
$F$6	Tubing Used	2000	$0.80	2000	555.56	333.33

Uncertain Quantities

7

7.1 DISCRETE UNCERTAIN QUANTITIES

Discrete UQ: a few, distinct values

Assign probability mass to each value (probability mass function).

Contrast discrete UQs with continuous UQs. Continuous UQs have an infinite number of values or so many distinct values that it is difficult to assign probability to each value. Instead, for a continuous UQ we assign probability only to ranges of values.

7.2 CONTINUOUS UNCERTAIN QUANTITIES

Probability Density Functions and Cumulative Probability for Continuous Uncertain Quantities

The total area under a probability density function equals one.

A portion of the area under a density function is a probability.

The height of a density function is not a probability.

The simplest probability density function is the uniform density function.

Case A: Uniform Density

The number of units of a new product that will be sold is an uncertain quantity.

What is the minimum quantity?	“1000 units”
What is the maximum quantity?	“5000 units”

Are any values in the range between 1000 and 5000 more likely than others? “No”

Represent the uncertainty using a uniform density function.

Technical point: For a continuous UQ, P(X=x) = 0.

For a continuous UQ, probability is non-zero only for a range of values.

For convenience in computation and assessment, we may use a continuous UQ to approximate a discrete UQ, and vice versa.

In Figure 1, the range of values is 5000 – 1000 = 4000, which is the width of the total area under the uniform (rectangular) density function. The area of a rectangle is Width * Height = Area, and the area under the uniform density function in Figure 1 must equal 1. So, Height = Area / Base. Here the Base is 5000 – 1000 = 4000 units. Therefore, Height = 1/4000 = 0.00025.

Figure 7.1 Uniform Density Function

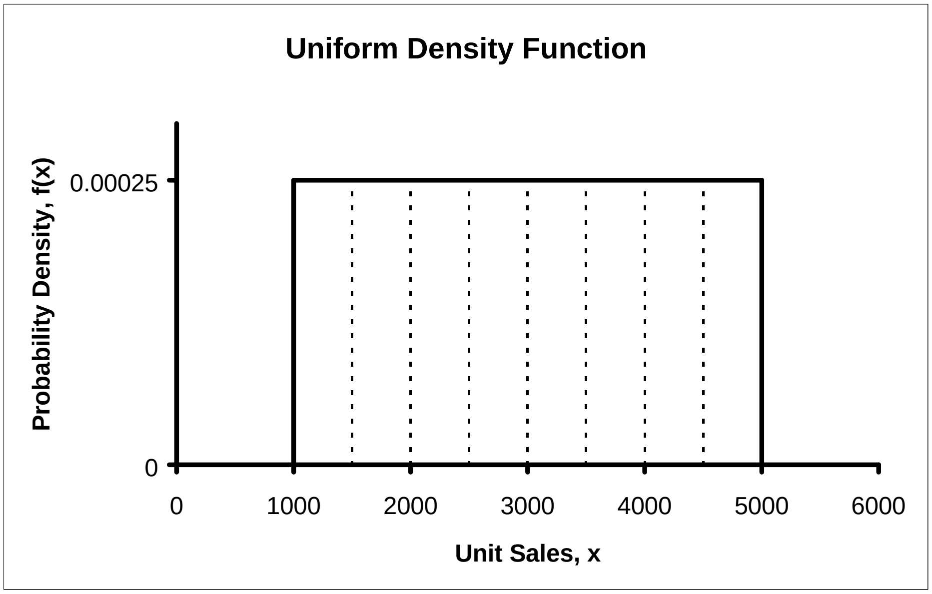
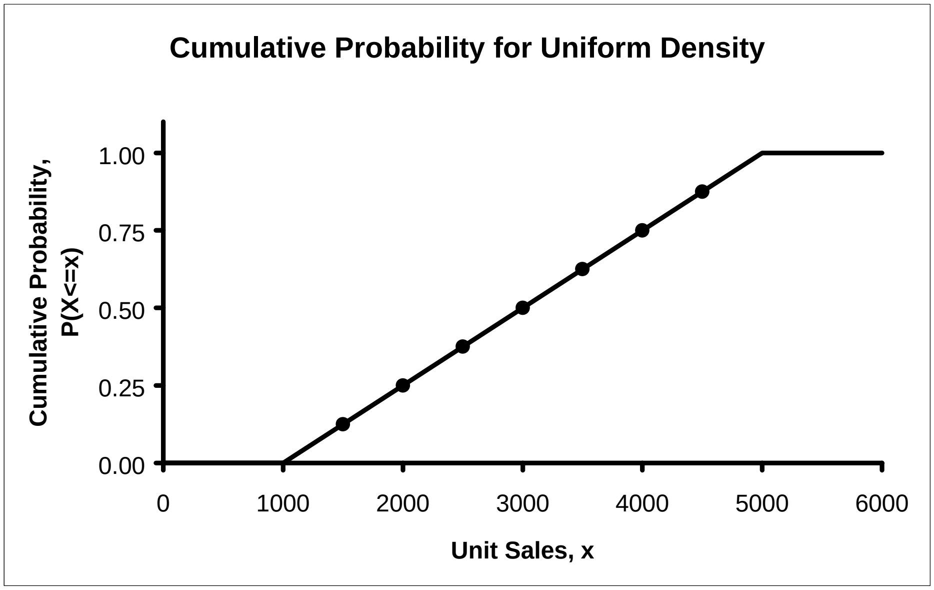


Figure 7.2 Figure 2



Both probability mass functions (for discrete UQs) and probability density functions (for continuous UQs) have corresponding cumulative probability functions.

It is important to understand the relationship between a density function and its cumulative probability function.

Cumulative probability can be expressed in four ways:

P(X<=x)	probability that UQ X is less than or equal to x	inclusive left -tail
P(X<x)	probability that UQ X is strictly less than x	exclusive left -tail
P(X>=x)	probability that UQ X is greater than or equal to x	inclusive right -tail
P(X>x)	probability that UQ X is strictly greater than x	exclusive right -tail

For continuous UQs the cumulative probability is the same for inclusive and exclusive.

P(X<=x) is the most common type.

Figure 2 is the cumulative probability function corresponding to the uniform density function shown in Figure 1.

What is the probability that sales will be between 3,500 and 4,000 units?

P(3500<=X<=4000) = 0.125

P(3500<=X<=4000) = P(X<=4000) – P(X<=3500) = 0.750 – 0.625 = 0.125

Mathematical observation: The uniform density function is a constant; the corresponding cumulative function (the integral of the constant function) is linear.

Case B: Ramp Density

The number of units of a new product that will be sold is an uncertain quantity.

What is the minimum quantity?	"1000 units"
What is the maximum quantity?	"5000 units"

Are any values in the range between 1000 and 5000 more likely than others?

"Yes, values close to 5000 are much more likely than values close to 1000."

Represent the uncertainty using a ramp density function.

The area of a triangle is Base * Height / 2, and the area under the ramp density function in Figure 3 must equal 1. So, Height = 2 / Base. Here, the Base is 5000 – 1000 = 4000 units. Therefore, Height = 2 / 4000 = 0.0005.

Figure 7.3 Figure 3

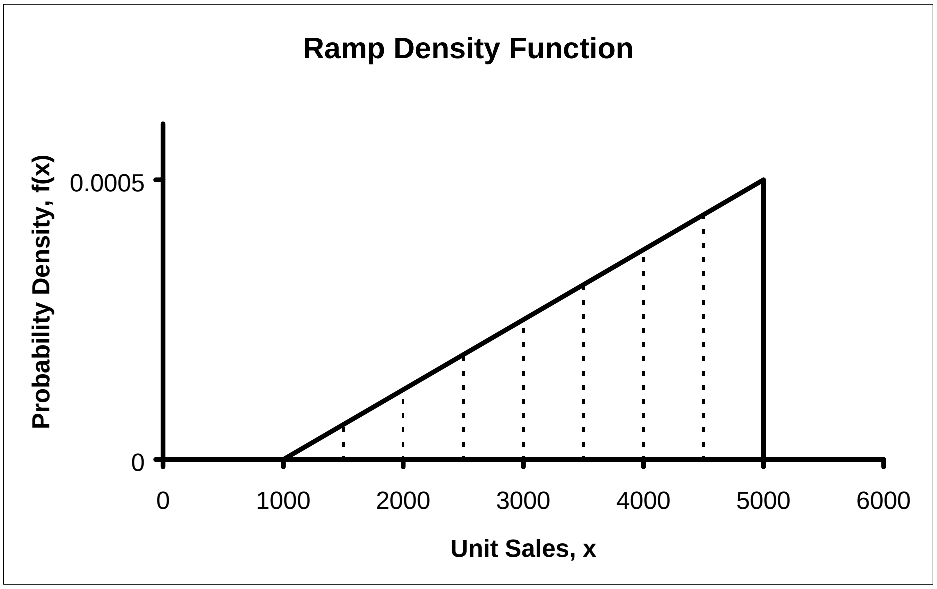
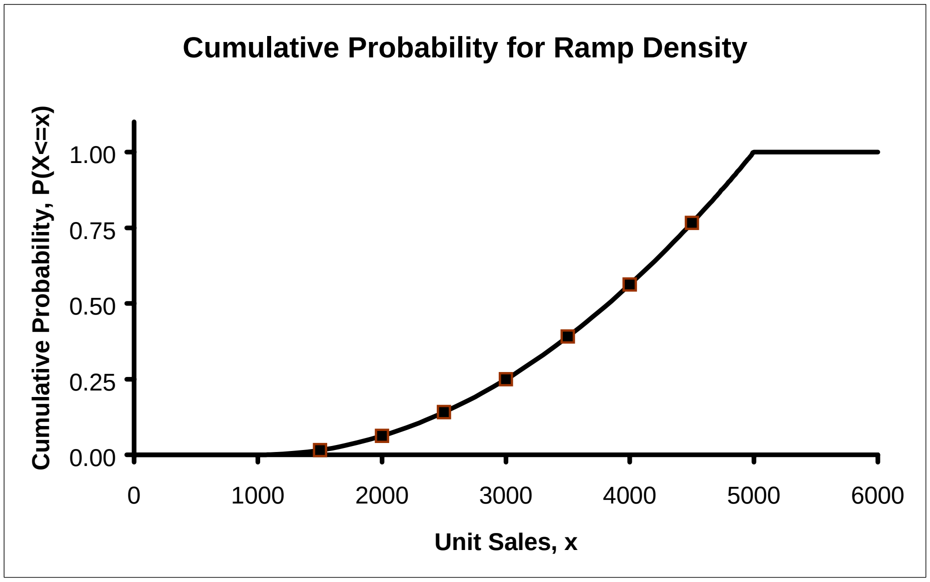


Figure 7.4 Figure 4



An important observation is that flatter portions of a cumulative probability function correspond to ranges with low probability. Steeper portions of a cumulative probability function correspond to ranges with high probability.

What is the probability that sales will be between 3,500 and 4,000 units?

P(3500<=X<=4000) = 0.171875

P(3500<=X<=4000) = P(X<=4000) – P(X<=3500) = 0.562500 – 0.390625 = 0.171875

The ramp density may not be appropriate for describing uncertainty in many situations, but it is an important building block for the extremely useful triangular density function.

Mathematical observation: The ramp density function is linear; the corresponding cumulative function (the integral of the linear function) is quadratic.

Case C: Triangular Density

The number of units of a new product that will be sold is an uncertain quantity.

What is the minimum quantity?	"1000 units"
What is the maximum quantity?	"5000 units"

Are any values in the range between 1000 and 5000 more likely than others?

"Yes, values close to 4000 are more likely."

Represent the uncertainty using a triangular density function.

The area of a triangle is Base * Height / 2, and the area under the triangular density function in Figure 5 must equal 1. So, Height = 2 / Base. Here, the Base is 5000 – 1000 = 4000 units. Thus, Height = 2 / 4000 = 0.0005.

Figure 7.5 Figure 5

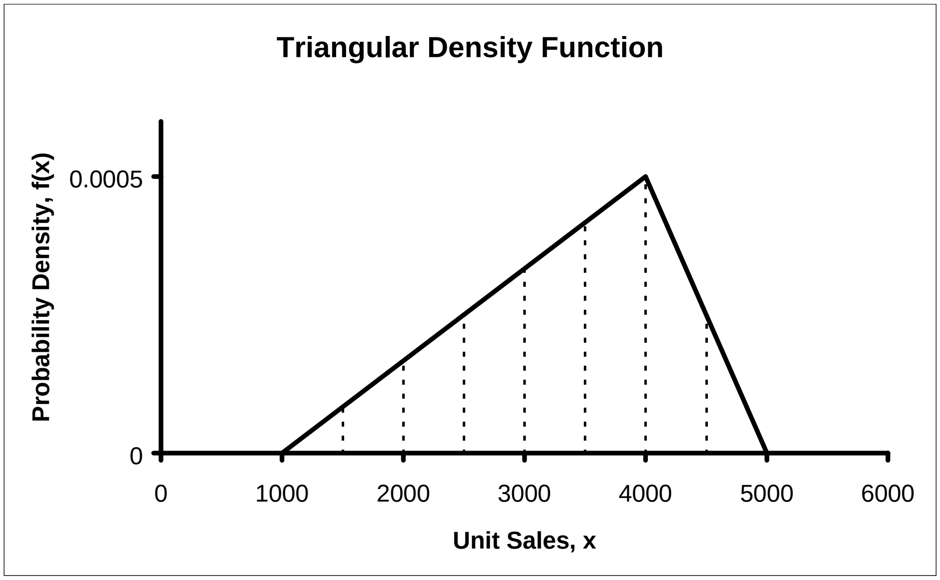
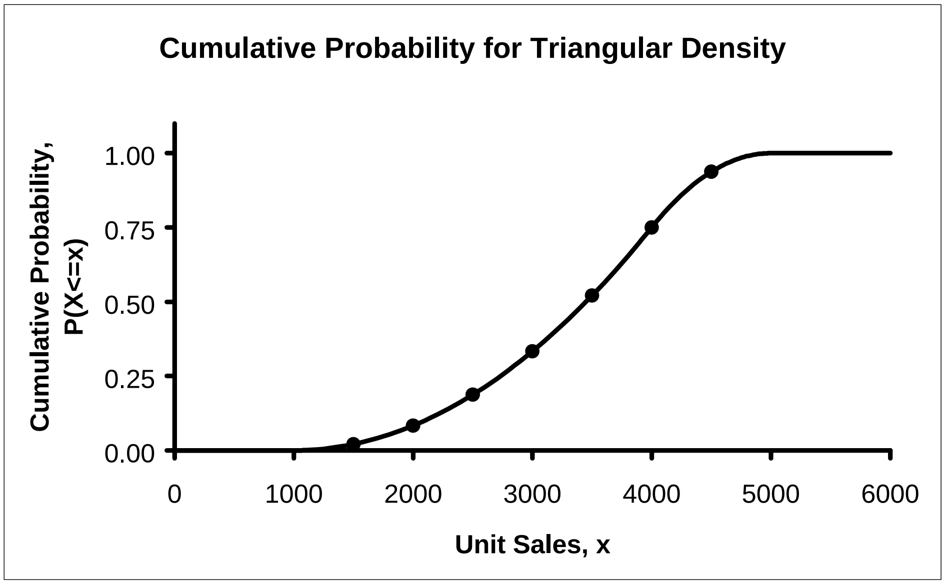


Figure 7.6 Figure 6



Again, an important observation is that flatter portions of a cumulative probability function correspond to ranges with low probability (the range close to 1000 and the range close to 5000 in Figure 6). Steeper portions of a cumulative probability function correspond to ranges with high probability (the range close to 4000).

What is the probability that sales will be between 3,500 and 4,000 units?

P(3500<=X<=4000) = 0.229167

P(3500<=X<=4000) = P(X<=4000) – P(X<=3500) = 0.750000 – 0.520833 = 0.229167

The triangular density function is extremely useful for describing uncertainty in many situations. It requires only three inputs: minimum, mode (most likely value), and maximum.

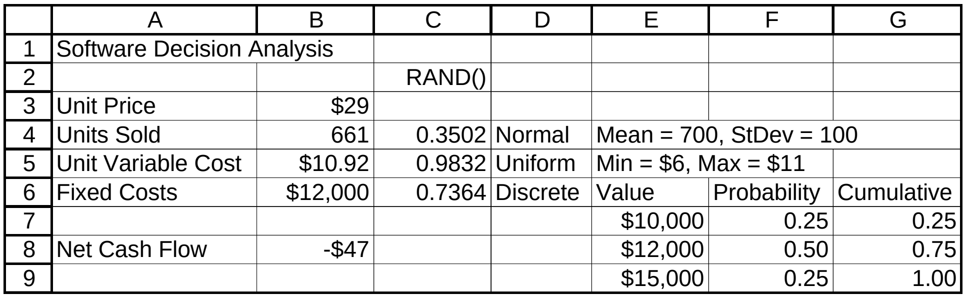
Mathematical observation: The triangular density function has two linear segments, i.e., piecewise linear; the corresponding cumulative function (the integral of each linear function) is two quadratic segments, i.e., piecewise quadratic.

Simulation Without Add-Ins

8

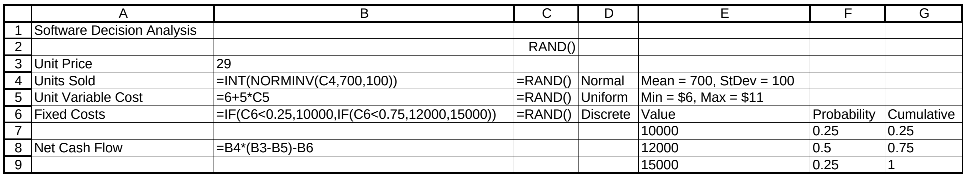
8.1 SIMULATION USING EXCEL FUNCTIONS

Figure 8.1 Display



	A	B	C	D	E	F	G
1	Software Decision Analysis						
2			RAND()				
3	Unit Price	$29					
4	Units Sold	661	0.3502	Normal	Mean = 700, StDev = 100		
5	Unit Variable Cost	$10.92	0.9832	Uniform	Min = $6, Max = $11		
6	Fixed Costs	$12,000	0.7364	Discrete	Value	Probability	Cumulative
7					$10,000	0.25	0.25
8	Net Cash Flow	-$47			$12,000	0.50	0.75
9					$15,000	0.25	1.00

Figure 8.2 Formulas



	A	B	C	D	E	F	G
1	Software Decision Analysis						
2			RAND()				
3	Unit Price	29					
4	Units Sold	=INT(NORMINV(C4,700,100))	=RAND()	Normal	Mean = 700, StDev = 100		
5	Unit Variable Cost	=6+5*C5	=RAND()	Uniform	Min = $6, Max = $11		
6	Fixed Costs	=IF(C6<0.25,10000,IF(C6<0.75,12000,15000))	=RAND()	Discrete	Value	Probability	Cumulative
7					10000	0.25	0.25
8	Net Cash Flow	=B4*(B3-B5)-B6			12000	0.5	0.75
9					15000	0.25	1

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Multiperiod What-If Modeling

9

9.1 APARTMENT BUILDING PURCHASE PROBLEM

You are considering the purchase of an apartment building in northern California. The building contains 25 units and is listed for $2,000,000. You plan to keep the building for three years and then sell it.

You know that the annual taxes on the property are currently $20,000 and will increase to $25,000 after closing. You estimate that these taxes will grow at a rate of 2 percent per year. You estimate that it will cost about $1,000 per unit per year to maintain the apartments, and these maintenance costs are expected to grow at a 15 percent per year rate.

You have not decided on the rent to charge. Currently, the rent is $875 per unit per month, but there is substantial turnover, and the occupancy is only 75 percent. That is, on average, 75 percent of the units are rented at any time. You estimate that if you lowered the rent to $675 per unit per month, you would have 100 percent occupancy. You think that intermediate rental charges would produce intermediate occupancy percentages; for example, a $775 rental charge would have 87.5 percent occupancy.

You will decide on the monthly rental charge for the first year, and you think the rental market is such that you will be able to increase it 7 percent per year for the second and third years. Furthermore, whatever occupancy percentage occurs in the first year will hold for the second and third years. For example, if you decide on the $675 monthly rental charge for the first year, the occupancy will be 100 percent all three years.

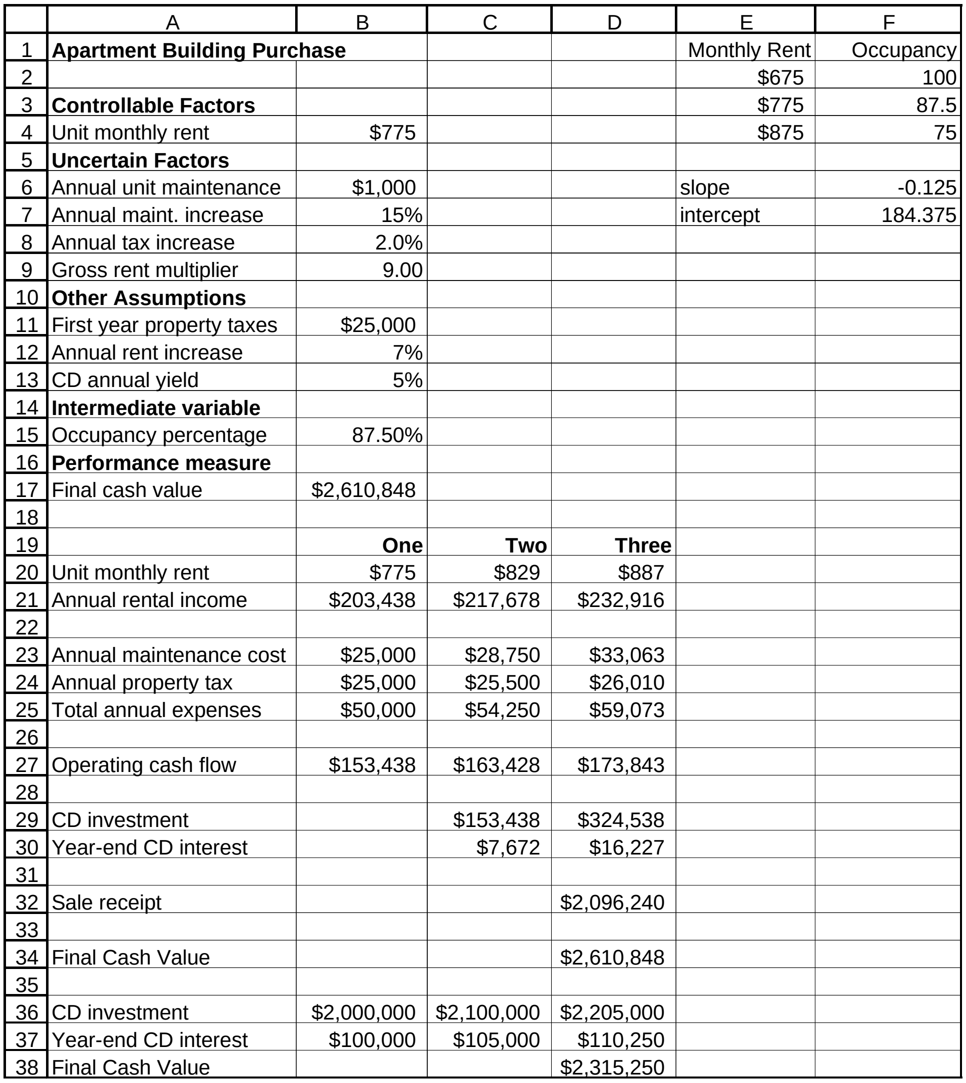
At the end of three years, you will sell the apartment building. The realtors in your area usually estimate the selling price of a rental property as a multiple of its annual rental income (before expenses). You estimate that this multiple will be 9. That is, if the rental income in the third year is $200,000, then the sale price will be $1,800,000.

Your objective is to achieve the highest total accumulated cash at the end of the three year period. If rental income exceeds expenses in the first or second years, you will invest the excess in one-year certificates of deposits (CDs) yielding 5 percent. Thus, total accumulated cash will include net cash flow (income minus expense) in each of the three years, interest from CDs received at the end of the second and third years, and cash from the sale of the property at the end of the third year.

In your initial analysis you have decided to ignore depreciation and other issues related to income taxes.

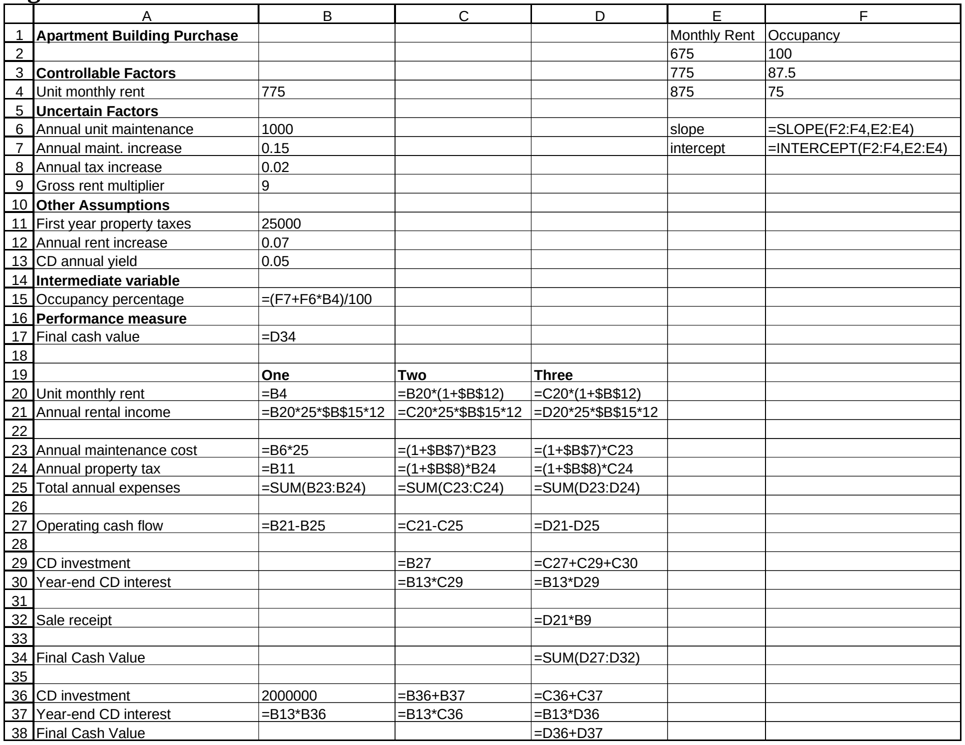
Instead of purchasing the apartment building, you could invest the entire $2,000,000 in certificates of deposits yielding 5 percent per year.

Figure 9.1 Base Case Model Display



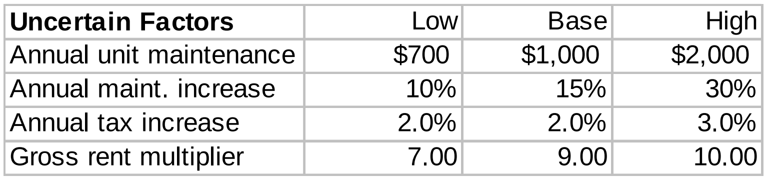
	A	B	C	D	E	F
1	**Apartment Building Purchase**				Monthly Rent	Occupancy
2					$675	100
3	**Controllable Factors**				$775	87.5
4	Unit monthly rent	$775			$875	75
5	**Uncertain Factors**					
6	Annual unit maintenance	$1,000			slope	-0.125
7	Annual maint. increase	15%			intercept	184.375
8	Annual tax increase	2.0%				
9	Gross rent multiplier	9.00				
10	**Other Assumptions**					
11	First year property taxes	$25,000				
12	Annual rent increase	7%				
13	CD annual yield	5%				
14	**Intermediate variable**					
15	Occupancy percentage	87.50%				
16	**Performance measure**					
17	Final cash value	$2,610,848				
18						
19		**One**	**Two**	**Three**		
20	Unit monthly rent	$775	$829	$887		
21	Annual rental income	$203,438	$217,678	$232,916		
22						
23	Annual maintenance cost	$25,000	$28,750	$33,063		
24	Annual property tax	$25,000	$25,500	$26,010		
25	Total annual expenses	$50,000	$54,250	$59,073		
26						
27	Operating cash flow	$153,438	$163,428	$173,843		
28						
29	CD investment		$153,438	$324,538		
30	Year-end CD interest		$7,672	$16,227		
31						
32	Sale receipt			$2,096,240		
33						
34	Final Cash Value			$2,610,848		
35						
36	CD investment	$2,000,000	$2,100,000	$2,205,000		
37	Year-end CD interest	$100,000	$105,000	$110,250		
38	Final Cash Value			$2,315,250		

Figure 9.2 Base Case Model Formulas



	A	B	C	D	E	F
1	**Apartment Building Purchase**				Monthly Rent	Occupancy
2					675	100
3	**Controllable Factors**				775	87.5
4	Unit monthly rent	775			875	75
5	**Uncertain Factors**					
6	Annual unit maintenance	1000			slope	=SLOPE(F2:F4,E2:E4)
7	Annual maint. increase	0.15			intercept	=INTERCEPT(F2:F4,E2:E4)
8	Annual tax increase	0.02				
9	Gross rent multiplier	9				
10	**Other Assumptions**					
11	First year property taxes	25000				
12	Annual rent increase	0.07				
13	CD annual yield	0.05				
14	**Intermediate variable**					
15	Occupancy percentage	=(F7+F6*B4)/100				
16	**Performance measure**					
17	Final cash value	=D34				
18						
19		**One**	**Two**	**Three**		
20	Unit monthly rent	=B4	=B20*(1+$B$12)	=C20*(1+$B$12)		
21	Annual rental income	=B20*25*$B$15*12	=C20*25*$B$15*12	=D20*25*$B$15*12		
22						
23	Annual maintenance cost	=B6*25	=(1+$B$7)*B23	=(1+$B$7)*C23		
24	Annual property tax	=B11	=(1+$B$8)*B24	=(1+$B$8)*C24		
25	Total annual expenses	=SUM(B23:B24)	=SUM(C23:C24)	=SUM(D23:D24)		
26						
27	Operating cash flow	=B21-B25	=C21-C25	=D21-D25		
28						
29	CD investment		=B27	=C27+C29+C30		
30	Year-end CD interest		=B13*C29	=B13*D29		
31						
32	Sale receipt			=D21*B9		
33						
34	Final Cash Value			=SUM(D27:D32)		
35						
36	CD investment	2000000	=B36+B37	=C36+C37		
37	Year-end CD interest	=B13*B36	=B13*C36	=B13*D36		
38	Final Cash Value			=D36+D37		

Figure 9.3 Ranges based on decision maker's or expert's judgment



Uncertain Factors	Low	Base	High
Annual unit maintenance	$700	$1,000	$2,000
Annual maint. increase	10%	15%	30%
Annual tax increase	2.0%	2.0%	3.0%
Gross rent multiplier	7.00	9.00	10.00

Apartment Building Analysis Notes

Influence Diagram (for single period)

Modeling effect of rent on occupancy rate

Linear fit:	algebra (slope and intercept)
	XY Scatter chart; Insert Trendline
Quadratic fit:	if $775 yields 82.5% occupancy instead of 87.5%

Base Case model

Use Solver to find optimum rent to maximize final cash value

Use Sensit.xla Plot of final cash value depending on rent; relatively insensitive

Use Sensit.xla Spider

Sensitivity Cases

Ranges based on decision maker's or expert's judgment

Sensit.xla Tornado chart: identify critical variables

Monte Carlo simulation

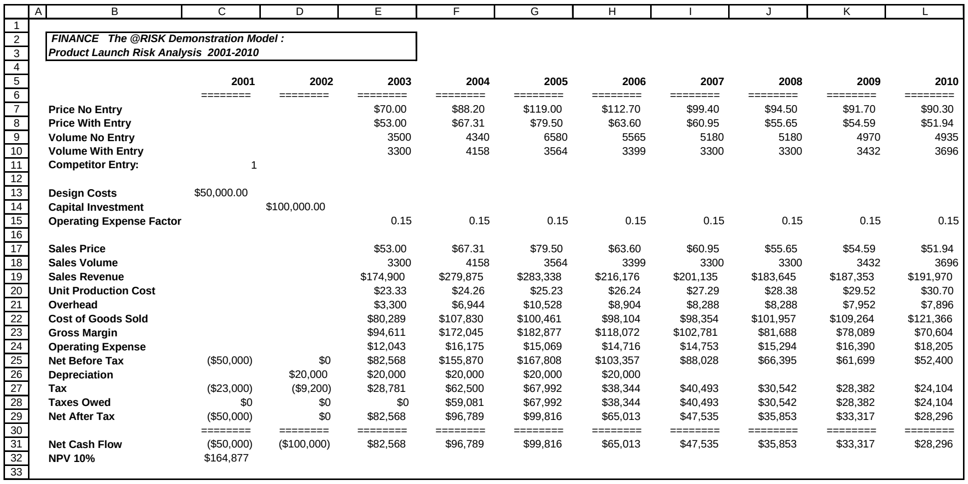
RiskSim.xla

Triangular distributions for critical variables

What is probability that final cash will be less than $2,315,250?

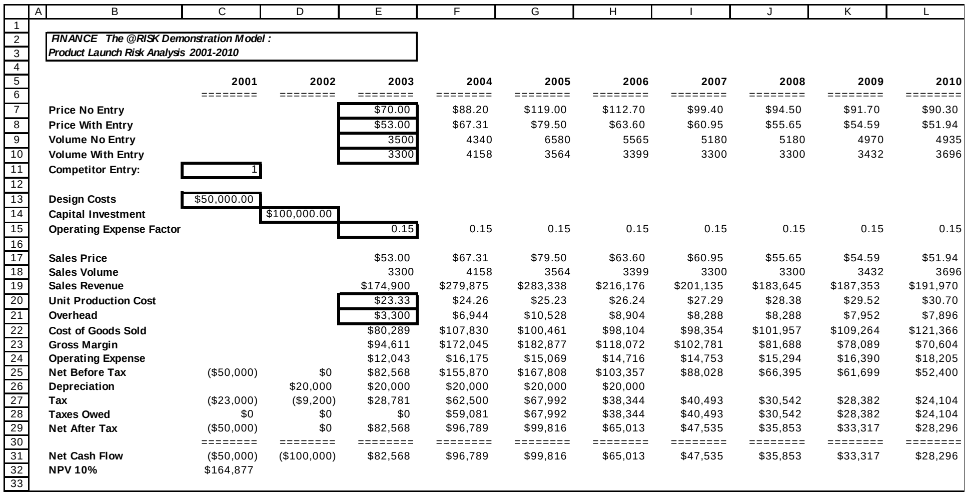
9.2 PRODUCT LAUNCH FINANCIAL MODEL

Figure 9.4 Original Model Display



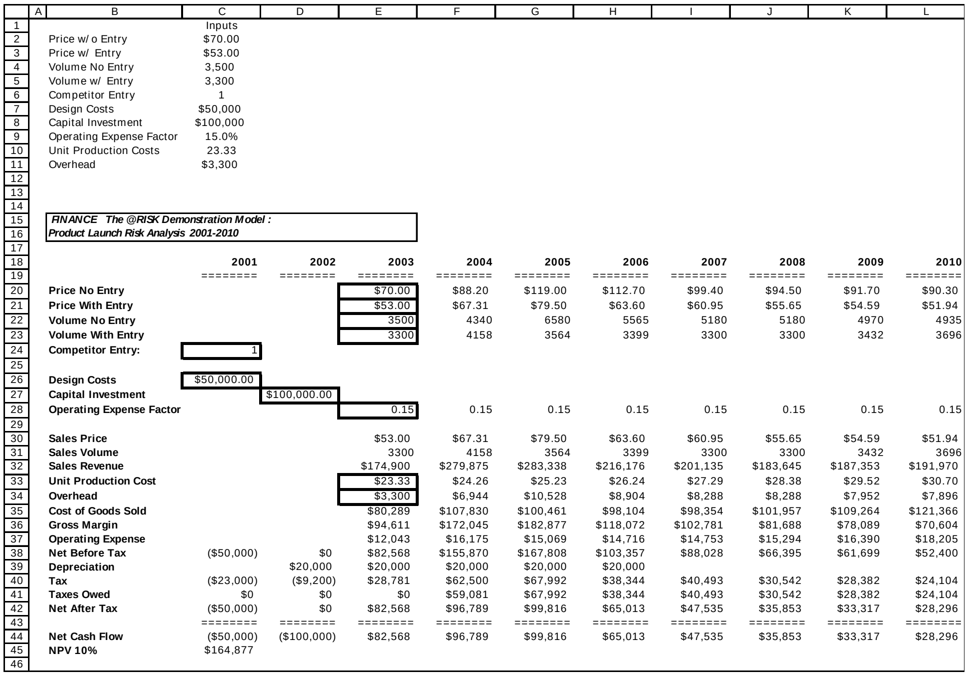
	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		***FINANCE The @RISK Demonstration Model :***										
3		***Product Launch Risk Analysis 2001-2010***										
4												
5			**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**
6			========	========	========	========	========	========	========	========	========	========
7		**Price No Entry**			$70.00	$88.20	$119.00	$112.70	$99.40	$94.50	$91.70	$90.30
8		**Price With Entry**			$53.00	$67.31	$79.50	$63.60	$60.95	$55.65	$54.59	$51.94
9		**Volume No Entry**			3500	4340	6580	5565	5180	5180	4970	4935
10		**Volume With Entry**			3300	4158	3564	3399	3300	3300	3432	3696
11		**Competitor Entry:**	1									
12												
13		**Design Costs**	$50,000.00									
14		**Capital Investment**		$100,000.00								
15		**Operating Expense Factor**			0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
16												
17		**Sales Price**			$53.00	$67.31	$79.50	$63.60	$60.95	$55.65	$54.59	$51.94
18		**Sales Volume**			3300	4158	3564	3399	3300	3300	3432	3696
19		**Sales Revenue**			$174,900	$279,875	$283,338	$216,176	$201,135	$183,645	$187,353	$191,970
20		**Unit Production Cost**			$23.33	$24.26	$25.23	$26.24	$27.29	$28.38	$29.52	$30.70
21		**Overhead**			$3,300	$6,944	$10,528	$8,904	$8,288	$8,288	$7,952	$7,896
22		**Cost of Goods Sold**			$80,289	$107,830	$100,461	$98,104	$98,354	$101,957	$109,264	$121,366
23		**Gross Margin**			$94,611	$172,045	$182,877	$118,072	$102,781	$81,688	$78,089	$70,604
24		**Operating Expense**			$12,043	$16,175	$15,069	$14,716	$14,753	$15,294	$16,390	$18,205
25		**Net Before Tax**	($50,000)	$0	$82,568	$155,870	$167,808	$103,357	$88,028	$66,395	$61,699	$52,400
26		**Depreciation**		$20,000	$20,000	$20,000	$20,000	$20,000				
27		**Tax**	($23,000)	($9,200)	$28,781	$62,500	$67,992	$38,344	$40,493	$30,542	$28,382	$24,104
28		**Taxes Owed**	$0	$0	$0	$59,081	$67,992	$38,344	$40,493	$30,542	$28,382	$24,104
29		**Net After Tax**	($50,000)	$0	$82,568	$96,789	$99,816	$65,013	$47,535	$35,853	$33,317	$28,296
30			========	========	========	========	========	========	========	========	========	========
31		**Net Cash Flow**	($50,000)	($100,000)	$82,568	$96,789	$99,816	$65,013	$47,535	$35,853	$33,317	$28,296
32		**NPV 10%**	$164,877									
33												

Figure 9.5 Input Assumptions



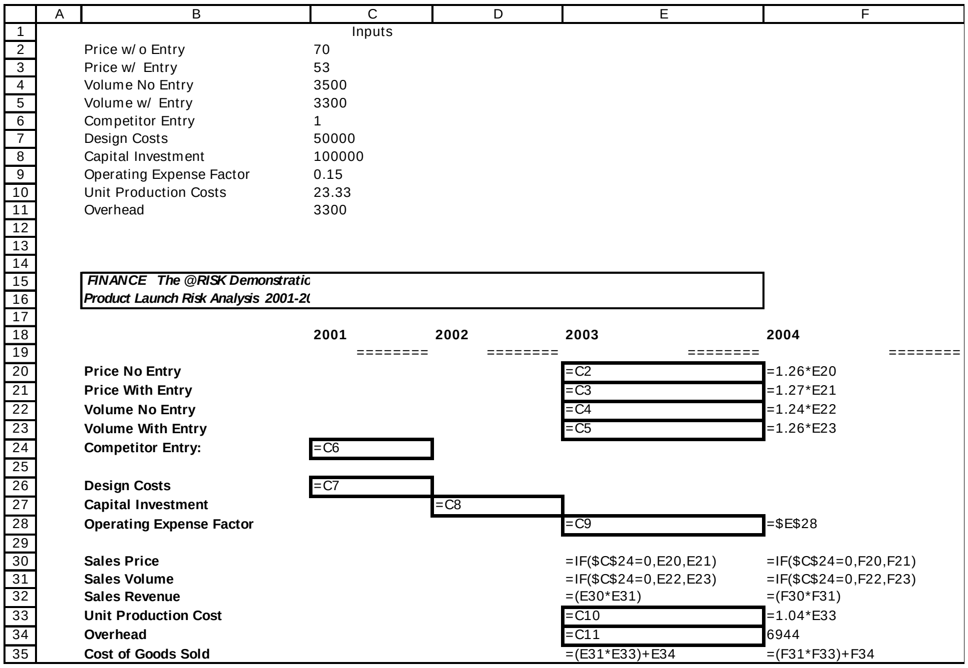
	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		*FINANCE The @RISK Demonstration Model :*										
3		*Product Launch Risk Analysis 2001-2010*										
4												
5			**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**
6			========	========	========	========	========	========	========	========	========	========
7		**Price No Entry**			$70.00	$88.20	$119.00	$112.70	$99.40	$94.50	$91.70	$90.30
8		**Price With Entry**			$53.00	$67.31	$79.50	$63.60	$60.95	$55.65	$54.59	$51.94
9		**Volume No Entry**			3500	4340	6580	5565	5180	5180	4970	4935
10		**Volume With Entry**			3300	4158	3564	3399	3300	3300	3432	3696
11		**Competitor Entry:**	1									
12												
13		**Design Costs**	$50,000.00									
14		**Capital Investment**		$100,000.00								
15		**Operating Expense Factor**			0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
16												
17		**Sales Price**			$53.00	$67.31	$79.50	$63.60	$60.95	$55.65	$54.59	$51.94
18		**Sales Volume**			3300	4158	3564	3399	3300	3300	3432	3696
19		**Sales Revenue**			$174,900	$279,875	$283,338	$216,176	$201,135	$183,645	$187,353	$191,970
20		**Unit Production Cost**			$23.33	$24.26	$25.23	$26.24	$27.29	$28.38	$29.52	$30.70
21		**Overhead**			$3,300	$6,944	$10,528	$8,904	$8,288	$8,288	$7,952	$7,896
22		**Cost of Goods Sold**			$80,289	$107,830	$100,461	$98,104	$98,354	$101,957	$109,264	$121,366
23		**Gross Margin**			$94,611	$172,045	$182,877	$118,072	$102,781	$81,688	$78,089	$70,604
24		**Operating Expense**			$12,043	$16,175	$15,069	$14,716	$14,753	$15,294	$16,390	$18,205
25		**Net Before Tax**	($50,000)	$0	$82,568	$155,870	$167,808	$103,357	$88,028	$66,395	$61,699	$52,400
26		**Depreciation**		$20,000	$20,000	$20,000	$20,000	$20,000				
27		**Tax**	($23,000)	($9,200)	$28,781	$62,500	$67,992	$38,344	$40,493	$30,542	$28,382	$24,104
28		**Taxes Owed**	$0	$0	$0	$59,081	$67,992	$38,344	$40,493	$30,542	$28,382	$24,104
29		**Net After Tax**	($50,000)	$0	$82,568	$96,789	$99,816	$65,013	$47,535	$35,853	$33,317	$28,296
30			========	========	========	========	========	========	========	========	========	========
31		**Net Cash Flow**	($50,000)	($100,000)	$82,568	$96,789	$99,816	$65,013	$47,535	$35,853	$33,317	$28,296
32		**NPV 10%**	$164,877									
33												

Figure 9.6 Modifications for SensIt Display



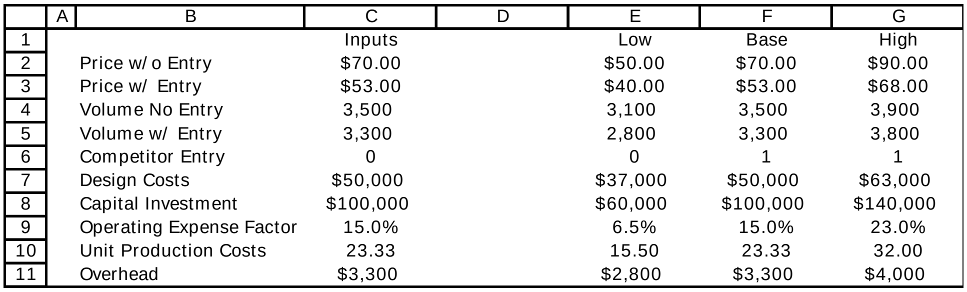
	A	B	C	D	E	F	G	H	I	J	K	L
1			Inputs									
2		Price w/ o Entry	$70.00									
3		Price w/ Entry	$53.00									
4		Volume No Entry	3,500									
5		Volume w/ Entry	3,300									
6		Competitor Entry	1									
7		Design Costs	$50,000									
8		Capital Investment	$100,000									
9		Operating Expense Factor	15.0%									
10		Unit Production Costs	23.33									
11		Overhead	$3,300									
12												
13												
14												
15		*FINANCE The @RISK Demonstration Model :*										
16		*Product Launch Risk Analysis 2001-2010*										
17												
18			**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**
19			========	========	========	========	========	========	========	========	========	========
20		**Price No Entry**			$70.00	$88.20	$119.00	$112.70	$99.40	$94.50	$91.70	$90.30
21		**Price With Entry**			$53.00	$67.31	$79.50	$63.60	$60.95	$55.65	$54.59	$51.94
22		**Volume No Entry**			3500	4340	6580	5565	5180	5180	4970	4935
23		**Volume With Entry**			3300	4158	3564	3399	3300	3300	3432	3696
24		**Competitor Entry:**	1									
25												
26		**Design Costs**	$50,000.00									
27		**Capital Investment**		$100,000.00								
28		**Operating Expense Factor**			0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
29												
30		**Sales Price**			$53.00	$67.31	$79.50	$63.60	$60.95	$55.65	$54.59	$51.94
31		**Sales Volume**			3300	4158	3564	3399	3300	3300	3432	3696
32		**Sales Revenue**			$174,900	$279,875	$283,338	$216,176	$201,135	$183,645	$187,353	$191,970
33		**Unit Production Cost**			$23.33	$24.26	$25.23	$26.24	$27.29	$28.38	$29.52	$30.70
34		**Overhead**			$3,300	$6,944	$10,528	$8,904	$8,288	$8,288	$7,952	$7,896
35		**Cost of Goods Sold**			$80,289	$107,830	$100,461	$98,104	$98,354	$101,957	$109,264	$121,366
36		**Gross Margin**			$94,611	$172,045	$182,877	$118,072	$102,781	$81,688	$78,089	$70,604
37		**Operating Expense**			$12,043	$16,175	$15,069	$14,716	$14,753	$15,294	$16,390	$18,205
38		**Net Before Tax**	($50,000)	$0	$82,568	$155,870	$167,808	$103,357	$88,028	$66,395	$61,699	$52,400
39		**Depreciation**		$20,000	$20,000	$20,000	$20,000	$20,000				
40		**Tax**	($23,000)	($9,200)	$28,781	$62,500	$67,992	$38,344	$40,493	$30,542	$28,382	$24,104
41		**Taxes Owed**	$0	$0	$0	$59,081	$67,992	$38,344	$40,493	$30,542	$28,382	$24,104
42		**Net After Tax**	($50,000)	$0	$82,568	$96,789	$99,816	$65,013	$47,535	$35,853	$33,317	$28,296
43			========	========	========	========	========	========	========	========	========	========
44		**Net Cash Flow**	($50,000)	($100,000)	$82,568	$96,789	$99,816	$65,013	$47,535	$35,853	$33,317	$28,296
45		**NPV 10%**	$164,877									
46												

Figure 9.7 Modifications for SensIt Formulas



	A	B	C	D	E	F
1			Inputs			
2		Price w/ o Entry	70			
3		Price w/ Entry	53			
4		Volume No Entry	3500			
5		Volume w/ Entry	3300			
6		Competitor Entry	1			
7		Design Costs	50000			
8		Capital Investment	100000			
9		Operating Expense Factor	0.15			
10		Unit Production Costs	23.33			
11		Overhead	3300			
12						
13						
14						
15		*FINANCE The @RISK Demonstratic*				
16		*Product Launch Risk Analysis 2001-2(*				
17						
18			**2001**	**2002**	**2003**	**2004**
19			========	========	========	========
20		**Price No Entry**			=C2	=1.26*E20
21		**Price With Entry**			=C3	=1.27*E21
22		**Volume No Entry**			=C4	=1.24*E22
23		**Volume With Entry**			=C5	=1.26*E23
24		**Competitor Entry:**	=C6			
25						
26		**Design Costs**	=C7			
27		**Capital Investment**		=C8		
28		**Operating Expense Factor**			=C9	=$E$28
29						
30		**Sales Price**			=IF($C$24=0,E20,E21)	=IF($C$24=0,F20,F21)
31		**Sales Volume**			=IF($C$24=0,E22,E23)	=IF($C$24=0,F22,F23)
32		**Sales Revenue**			=(E30*E31)	=(F30*F31)
33		**Unit Production Cost**			=C10	=1.04*E33
34		**Overhead**			=C11	6944
35		**Cost of Goods Sold**			=(E31*E33)+E34	=(F31*F33)+F34

Figure 9.8 Data for Competitor Entry as Base Case



	A	B	C	D	E	F	G
1			Inputs		Low	Base	High
2		Price w/ o Entry	$70.00		$50.00	$70.00	$90.00
3		Price w/ Entry	$53.00		$40.00	$53.00	$68.00
4		Volume No Entry	3,500		3,100	3,500	3,900
5		Volume w/ Entry	3,300		2,800	3,300	3,800
6		Competitor Entry	0		0	1	1
7		Design Costs	$50,000		$37,000	$50,000	$63,000
8		Capital Investment	$100,000		$60,000	$100,000	$140,000
9		Operating Expense Factor	15.0%		6.5%	15.0%	23.0%
10		Unit Production Costs	23.33		15.50	23.33	32.00
11		Overhead	$3,300		$2,800	$3,300	$4,000

Figure 9.9 Tornado Chart for Competitor Entry as Base Case

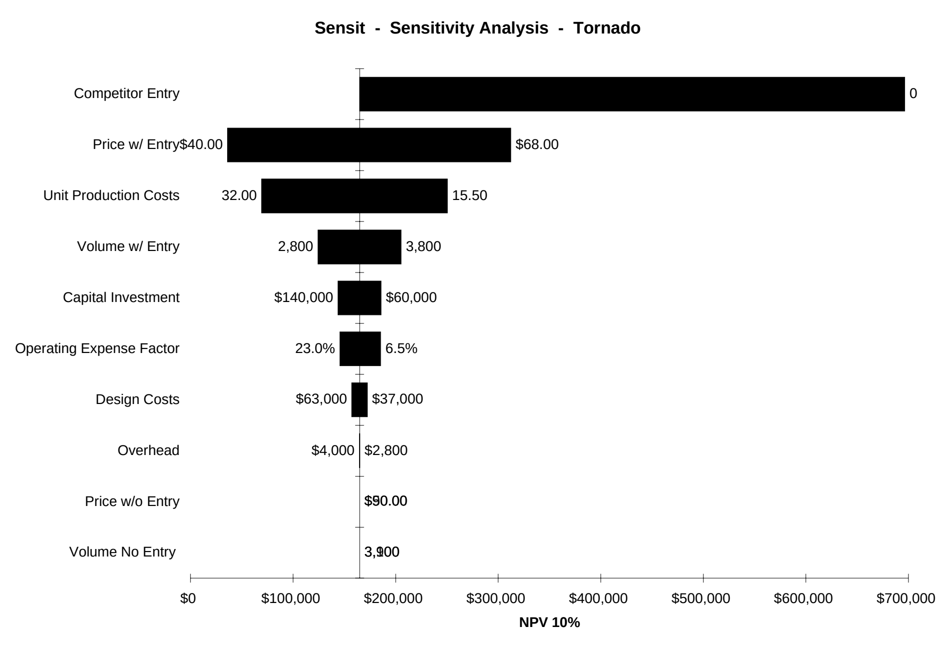
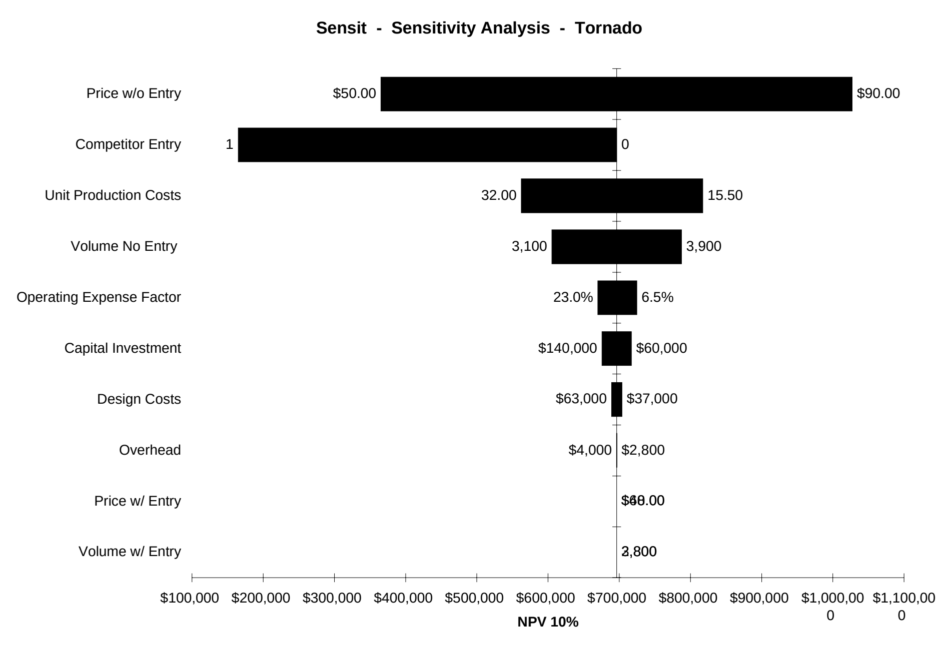


Figure 9.10 Data for No Competitor Entry as Base Case

	A	B	C	D	E	F	G
1			Inputs		Low	Base	High
2		Price w/ o Entry	$70.00		$50.00	$70.00	$90.00
3		Price w/ Entry	$53.00		$40.00	$53.00	$68.00
4		Volume No Entry	3,500		3,100	3,500	3,900
5		Volume w/ Entry	3,300		2,800	3,300	3,800
6		Competitor Entry	0		0	0	1
7		Design Costs	$50,000		$37,000	$50,000	$63,000
8		Capital Investment	$100,000		$60,000	$100,000	$140,000
9		Operating Expense Factor	15.0%		6.5%	15.0%	23.0%
10		Unit Production Costs	23.33		15.50	23.33	32.00
11		Overhead	$3,300		$2,800	$3,300	$4,000

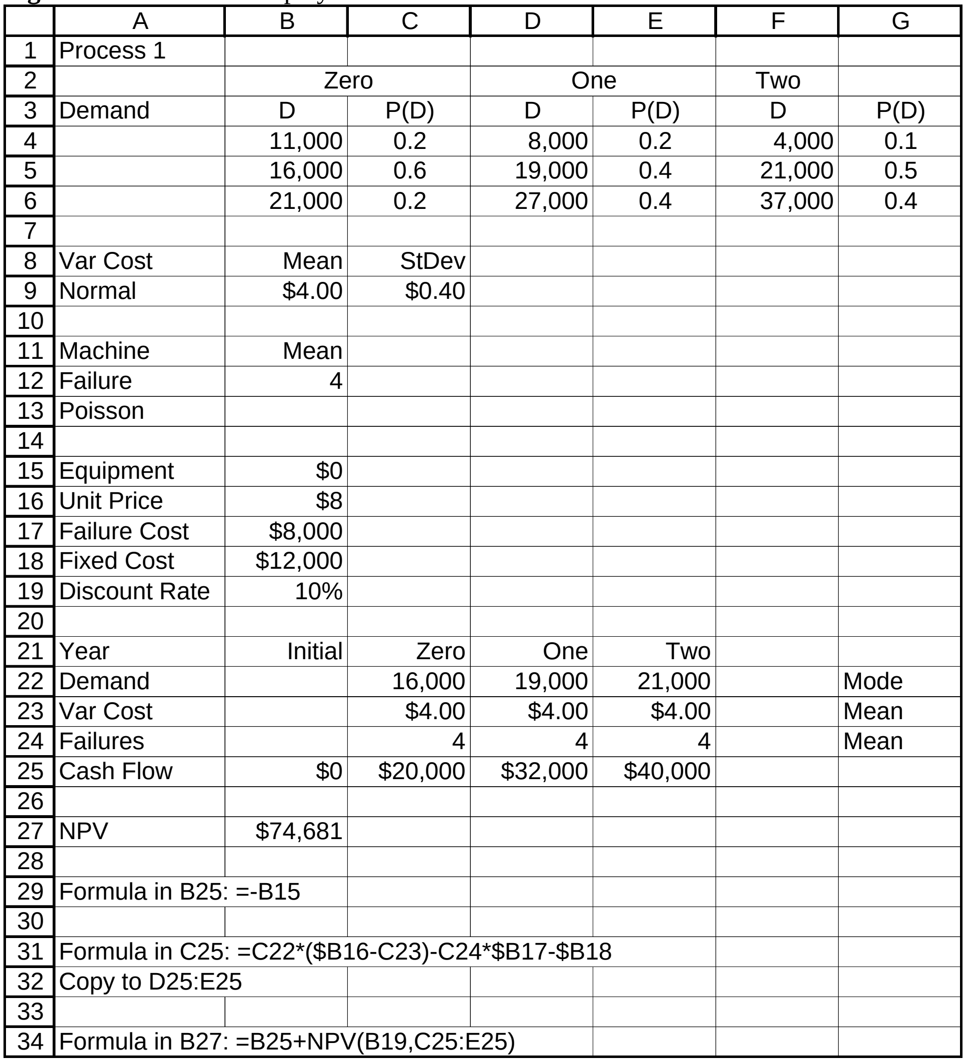
Figure 9.11 Tornado Chart for No Competitor Entry as Base Case



9.3 MACHINE SIMULATION MODEL

Clemen's AJS Problem

Figure 9.12 Process 1 Display and Formulas



	A	B	C	D	E	F	G
1	Process 1						
2		Zero		One		Two	
3	Demand	D	P(D)	D	P(D)	D	P(D)
4		11,000	0.2	8,000	0.2	4,000	0.1
5		16,000	0.6	19,000	0.4	21,000	0.5
6		21,000	0.2	27,000	0.4	37,000	0.4
7							
8	Var Cost	Mean	StDev				
9	Normal	$4.00	$0.40				
10							
11	Machine	Mean					
12	Failure	4					
13	Poisson						
14							
15	Equipment	$0					
16	Unit Price	$8					
17	Failure Cost	$8,000					
18	Fixed Cost	$12,000					
19	Discount Rate	10%					
20							
21	Year	Initial	Zero	One	Two		
22	Demand		16,000	19,000	21,000		Mode
23	Var Cost		$4.00	$4.00	$4.00		Mean
24	Failures		4	4	4		Mean
25	Cash Flow	$0	$20,000	$32,000	$40,000		
26							
27	NPV	$74,681					
28							
29	Formula in B25: =-B15						
30							
31	Formula in C25: =C22*($B16-C23)-C24*$B17-$B18						
32	Copy to D25:E25						
33							
34	Formula in B27: =B25+NPV(B19,C25:E25)						

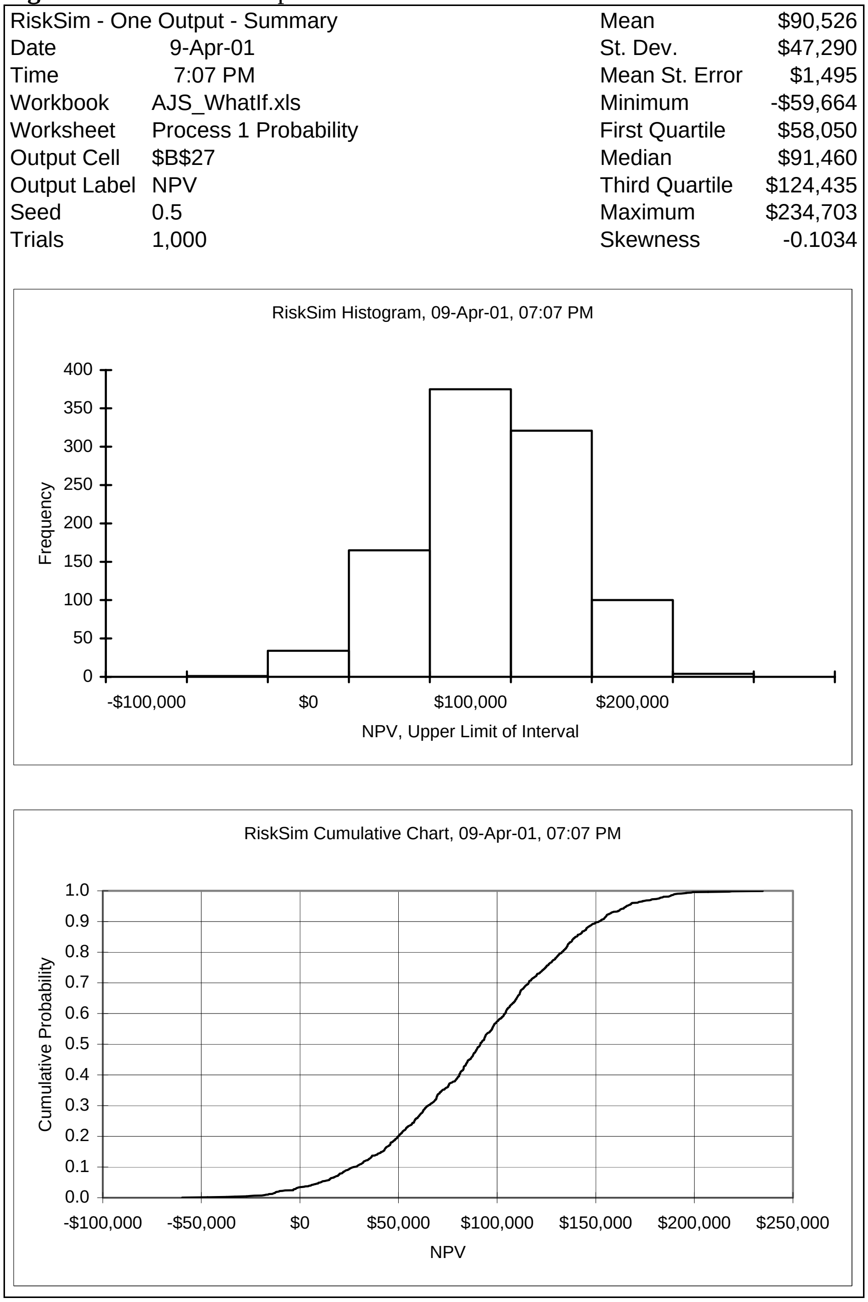
Figure 9.13 Process 2 Display

	A	B	C	D	E	F	G
1	Process 2						
2		Zero		One		Two	
3	Demand	D	P(D)	D	P(D)	D	P(D)
4		14,000	0.3	12,000	0.36	9,000	0.4
5		19,000	0.4	23,000	0.36	26,000	0.1
6		24,000	0.3	31,000	0.28	42,000	0.5
7							
8	Var Cost	Mean	StDev				
9	Normal	$3.50	$1.00				
10							
11	Machine	Mean					
12	Failure	3					
13	Poisson						
14							
15	Equipment	$60,000					
16	Unit Price	$8					
17	Failure Cost	$6,000					
18	Fixed Cost	$12,000					
19	Discount Rate	10%					
20							
21	Year	Initial	Zero	One	Two		
22	Demand		19,000	23,000	26,000		Mode
23	Var Cost		$3.50	$3.50	$3.50		Mean
24	Failures		3	3	3		Mean
25	Cash Flow	-$60,000	$55,500	$73,500	$87,000		
26							
27	NPV	$116,563					

Figure 9.14 RiskSim Functions for Process 1 and Process 2

	A	B	C	D	E
20					
21	Year	Initial	Zero	One	Two
22	Demand		=randdiscrete(B4:C6)	=randdiscrete(D4:E6)	=randdiscrete(F4:G6)
23	Var Cost		=randnormal($B$9,$C$9)	=randnormal($B$9,$C$9)	=randnormal($B$9,$C$9)
24	Failures		=randpoisson($B$12)	=randpoisson($B$12)	=randpoisson($B$12)
25	Cash Flow	=-B15	=C22*($B16-C23)-C24*$B17-$B18	=D22*($B16-D23)-D24*$B17-$B18	=E22*($B16-E23)-E24*$B17-$B18
26					
27	NPV	=B25+NPV(B19,C25:E25)			

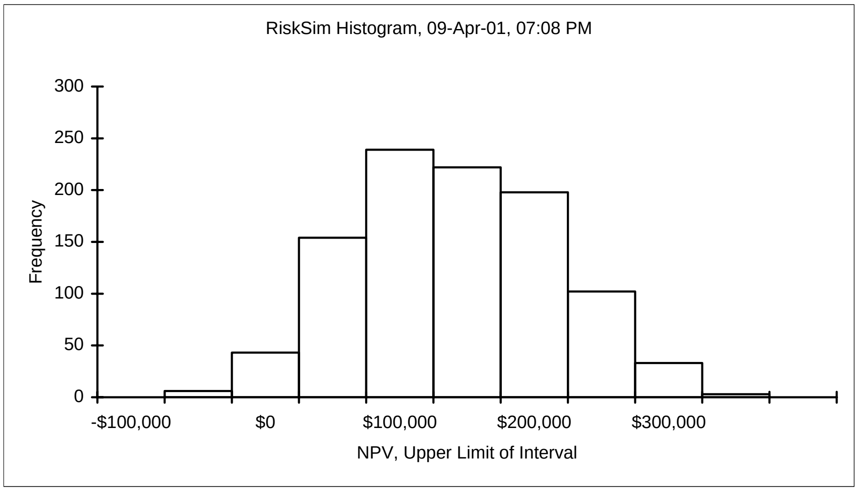
Figure 9.15 RiskSim Output for Process 1

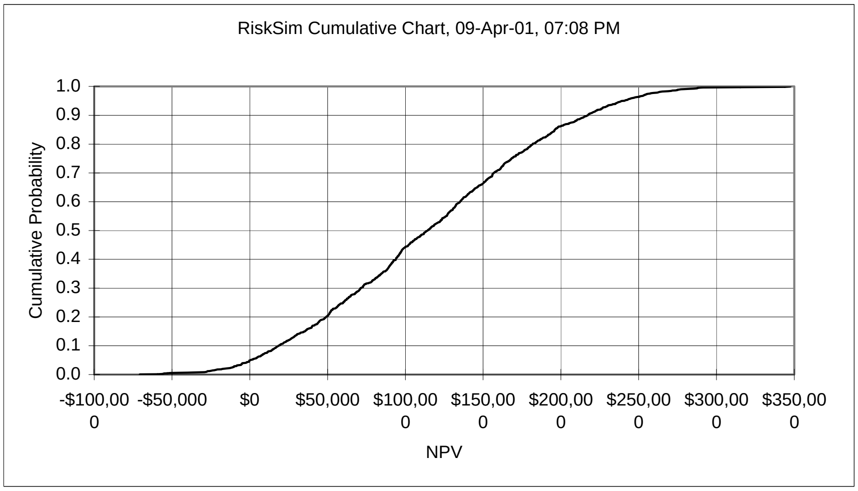


RiskSim - One Output - Summary		Mean	$90,526
Date	9-Apr-01	St. Dev.	$47,290
Time	7:07 PM	Mean St. Error	$1,495
Workbook	AJS_WhatIf.xls	Minimum	-$59,664
Worksheet	Process 1 Probability	First Quartile	$58,050
Output Cell	$B$27	Median	$91,460
Output Label	NPV	Third Quartile	$124,435
Seed	0.5	Maximum	$234,703
Trials	1,000	Skewness	-0.1034

Figure 9.16 RiskSim Output for Process 2

```
RiskSim - One Output - Summary                          Mean            $116,159
Date            9-Apr-01                                St. Dev.         $73,675
Time             7:08 PM                                Mean St. Error    $2,330
Workbook        AJS_WhatIf.xls                          Minimum         -$70,685
Worksheet       Process 2 Probability                   First Quartile   $60,199
Output Cell     $B$27                                   Median          $114,335
Output Label    NPV                                     Third Quartile  $168,191
Seed            0.5                                     Maximum         $347,514
Trials          1,000                                   Skewness          0.1390
```





Follow these instructions to show two or more risk profiles on the same chart.

Use RiskSim to obtain the sorted values, cumulative probabilities, and XY charts for strategy A and strategy B.

To add the data for strategy B to the existing plot for strategy A, select the sorted values and cumulative probabilities for strategy B (without including the text labels in row 1), and choose Edit | Copy.

Click just inside the outer border of the strategy A chart to select it. From the main menu, choose Edit | Paste Special. In the Paste Special dialog box, select "Add cells as New series," select "Values (Y) in Columns," check the box for "Categories (X Values) in First Column," and click OK.

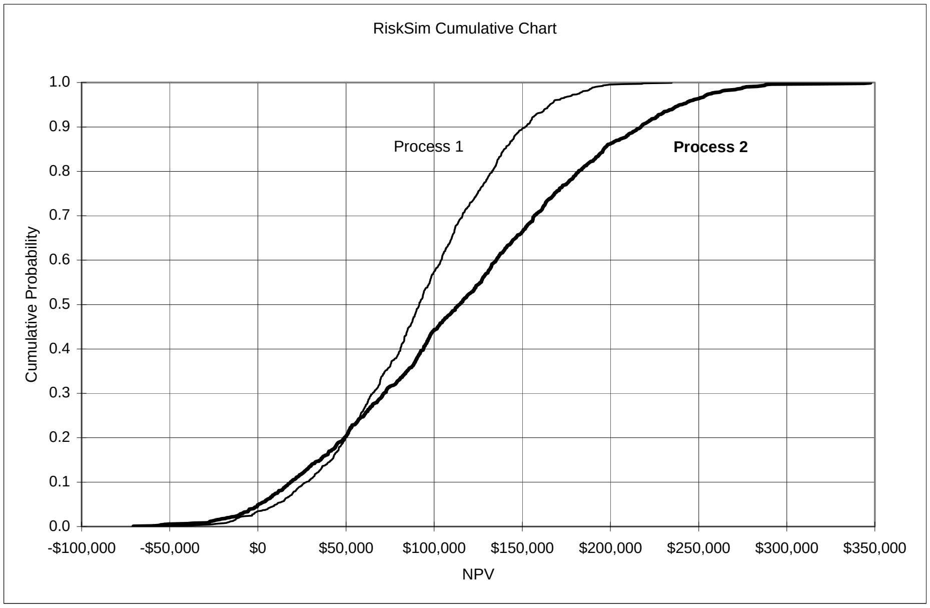
Use the same method to add data for other strategies to the strategy A chart.

To change the lines and markers of a data series, click a data point on the chart to select the data series, and choose Format | Selected Data Series | Patterns.

If the X values are quite different for the various strategies, it may be necessary to adjust the minimum and maximum values on the Scale tab of the Format Axis dialog box.

Figure 9.17 Comparison of Process1 and Process 2

Process 1		Process 2	
Mean	$90,526	Mean	$116,159
St. Dev.	$47,290	St. Dev.	$73,675
Mean St. Error	$1,495	Mean St. Error	$2,330
Minimum	-$59,664	Minimum	-$70,685
First Quartile	$58,050	First Quartile	$60,199
Median	$91,460	Median	$114,335
Third Quartile	$124,435	Third Quartile	$168,191
Maximum	$234,703	Maximum	$347,514
Skewness	-0.1034	Skewness	0.1390



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Modeling Waiting Lines

10

10.1 QUEUE SIMULATION

A warehouse has one dock used to unload railroad freight cars. Incoming freight cars are delivered to the warehouse during the night. It takes exactly half a day to unload a car. If more than two cars are waiting to be unloaded on a given day, the unloading of some of the cars is postponed until the following day. The cost is $100 per day for each car delayed.

Past experience has indicated that the number of cars arriving during the night have the frequencies shown in the table below. Furthermore, there is no apparent pattern, so that the number arriving on any night is independent of the number arriving on any other night.

Figure 10.1 Arrival Frequency

Number of cars arriving	Relative frequency
0	0.23
1	0.30
2	0.30
3	0.10
4	0.05
5	0.02
6 or more	0.00
	1.00

<u>Concepts for Queuing (waiting-line) Models</u>

Arrival pattern

Service time

Number of servers

Queue discipline

Performance measures

Equilibrium

Average waiting time

Average number of customers in line

System utilization, rho = mean arrival rate / mean service rate

Stable system: rho < 1

Figure 10.2 Influence Chart for Simulation Model

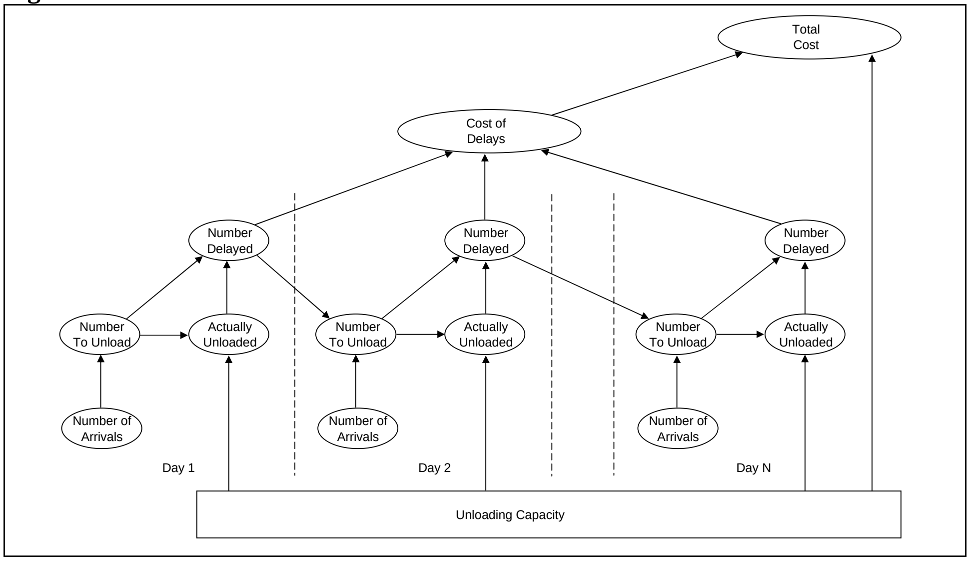


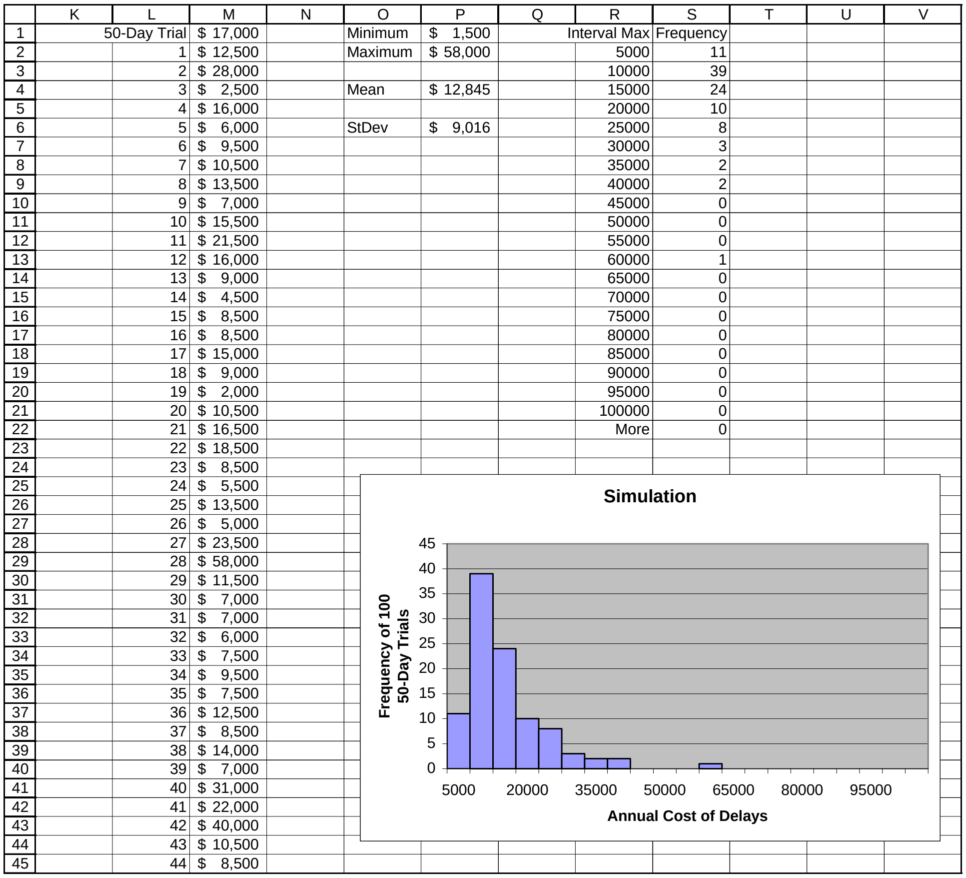
Figure 10.3 Simulation Model Spreadsheet Model Display

	A	B	C	D	E	F	G	H	I
1	Unloading Capacity		2					Daily Delay Cost	$ 100
2									
3		Random	Number of	Number	Actually	Number		Annual Delay Cost	$ 16,500
4	Day	Number	Arrivals	To Unload	Unloaded	Delayed			
5	1	0.812	3	3	2	1			
6	2	0.524	2	3	2	1			
7	3	0.671	2	3	2	1			
8	4	0.250	1	2	2	0			
9	5	0.940	3	3	2	1			
10	6	0.771	2	3	2	1			
11	7	0.026	0	1	1	0			
12	8	0.178	0	0	0	0			
13	9	0.683	2	2	2	0			
14	10	0.727	2	2	2	0			
44	40	0.082	0	0	0	0			
45	41	0.425	1	1	1	0			
46	42	0.826	3	3	2	1			
47	43	0.855	3	4	2	2			
48	44	0.971	3	5	2	3			
49	45	0.429	1	4	2	2			
50	46	0.592	2	4	2	2			
51	47	0.085	0	2	2	0			
52	48	0.018	0	0	0	0			
53	49	0.678	2	2	2	0			
54	50	0.510	2	2	2	0			
55									
56	Total		86			33			
57									
58	Daily Average		1.72			0.66			

Figure 10.4 Simulation Model Spreadsheet Model Formulas

	A	B	C	D	E	F	G	H	I
1	Unloading Capacity		2					Daily Delay Cost	100
2									
3		Random	Number of	Number	Actually	Number		Annual Delay Cost	=250*F58*I1
4	Day	Number	Arrivals	To Unload	Unloaded	Delayed			
5	1	=RAND()	=IF(B5<0.2,0,IF(B5<0.5,1,IF(B5<0.8,2,3)))	=C5	=MIN(D5,$C$1)	=D5-E5			
6	2	=RAND()	=IF(B6<0.2,0,IF(B6<0.5,1,IF(B6<0.8,2,3)))	=F5+C6	=MIN(D6,$C$1)	=D6-E6			
7	3	=RAND()	=IF(B7<0.2,0,IF(B7<0.5,1,IF(B7<0.8,2,3)))	=F6+C7	=MIN(D7,$C$1)	=D7-E7			
8	4	=RAND()	=IF(B8<0.2,0,IF(B8<0.5,1,IF(B8<0.8,2,3)))	=F7+C8	=MIN(D8,$C$1)	=D8-E8			
9	5	=RAND()	=IF(B9<0.2,0,IF(B9<0.5,1,IF(B9<0.8,2,3)))	=F8+C9	=MIN(D9,$C$1)	=D9-E9			
10	6	=RAND()	=IF(B10<0.2,0,IF(B10<0.5,1,IF(B10<0.8,2,3)))	=F9+C10	=MIN(D10,$C$1)	=D10-E10			
11	7	=RAND()	=IF(B11<0.2,0,IF(B11<0.5,1,IF(B11<0.8,2,3)))	=F10+C11	=MIN(D11,$C$1)	=D11-E11			
12	8	=RAND()	=IF(B12<0.2,0,IF(B12<0.5,1,IF(B12<0.8,2,3)))	=F11+C12	=MIN(D12,$C$1)	=D12-E12			
13	9	=RAND()	=IF(B13<0.2,0,IF(B13<0.5,1,IF(B13<0.8,2,3)))	=F12+C13	=MIN(D13,$C$1)	=D13-E13			
14	10	=RAND()	=IF(B14<0.2,0,IF(B14<0.5,1,IF(B14<0.8,2,3)))	=F13+C14	=MIN(D14,$C$1)	=D14-E14			
44	40	=RAND()	=IF(B44<0.2,0,IF(B44<0.5,1,IF(B44<0.8,2,3)))	=F43+C44	=MIN(D44,$C$1)	=D44-E44			
45	41	=RAND()	=IF(B45<0.2,0,IF(B45<0.5,1,IF(B45<0.8,2,3)))	=F44+C45	=MIN(D45,$C$1)	=D45-E45			
46	42	=RAND()	=IF(B46<0.2,0,IF(B46<0.5,1,IF(B46<0.8,2,3)))	=F45+C46	=MIN(D46,$C$1)	=D46-E46			
47	43	=RAND()	=IF(B47<0.2,0,IF(B47<0.5,1,IF(B47<0.8,2,3)))	=F46+C47	=MIN(D47,$C$1)	=D47-E47			
48	44	=RAND()	=IF(B48<0.2,0,IF(B48<0.5,1,IF(B48<0.8,2,3)))	=F47+C48	=MIN(D48,$C$1)	=D48-E48			
49	45	=RAND()	=IF(B49<0.2,0,IF(B49<0.5,1,IF(B49<0.8,2,3)))	=F48+C49	=MIN(D49,$C$1)	=D49-E49			
50	46	=RAND()	=IF(B50<0.2,0,IF(B50<0.5,1,IF(B50<0.8,2,3)))	=F49+C50	=MIN(D50,$C$1)	=D50-E50			
51	47	=RAND()	=IF(B51<0.2,0,IF(B51<0.5,1,IF(B51<0.8,2,3)))	=F50+C51	=MIN(D51,$C$1)	=D51-E51			
52	48	=RAND()	=IF(B52<0.2,0,IF(B52<0.5,1,IF(B52<0.8,2,3)))	=F51+C52	=MIN(D52,$C$1)	=D52-E52			
53	49	=RAND()	=IF(B53<0.2,0,IF(B53<0.5,1,IF(B53<0.8,2,3)))	=F52+C53	=MIN(D53,$C$1)	=D53-E53			
54	50	=RAND()	=IF(B54<0.2,0,IF(B54<0.5,1,IF(B54<0.8,2,3)))	=F53+C54	=MIN(D54,$C$1)	=D54-E54			
55									
56	Total		=SUM(C5:C54)			=SUM(F5:F54)			
57									
58	Daily Average		=C56/50			=F56/50			

Figure 10.5 Simulation Model Dynamic Histogram Display



	K	L	M	N	O	P	Q	R	S	T	U	V
1		50-Day Trial	$ 17,000		Minimum	$ 1,500		Interval Max	Frequency			
2		1	$ 12,500		Maximum	$ 58,000		5000	11			
3		2	$ 28,000					10000	39			
4		3	$ 2,500		Mean	$ 12,845		15000	24			
5		4	$ 16,000					20000	10			
6		5	$ 6,000		StDev	$ 9,016		25000	8			
7		6	$ 9,500					30000	3			
8		7	$ 10,500					35000	2			
9		8	$ 13,500					40000	2			
10		9	$ 7,000					45000	0			
11		10	$ 15,500					50000	0			
12		11	$ 21,500					55000	0			
13		12	$ 16,000					60000	1			
14		13	$ 9,000					65000	0			
15		14	$ 4,500					70000	0			
16		15	$ 8,500					75000	0			
17		16	$ 8,500					80000	0			
18		17	$ 15,000					85000	0			
19		18	$ 9,000					90000	0			
20		19	$ 2,000					95000	0			
21		20	$ 10,500					100000	0			
22		21	$ 16,500					More	0			
23		22	$ 18,500									
24		23	$ 8,500									
25		24	$ 5,500									
26		25	$ 13,500									
27		26	$ 5,000									
28		27	$ 23,500									
29		28	$ 58,000									
30		29	$ 11,500									
31		30	$ 7,000									
32		31	$ 7,000									
33		32	$ 6,000									
34		33	$ 7,500									
35		34	$ 9,500									
36		35	$ 7,500									
37		36	$ 12,500									
38		37	$ 8,500									
39		38	$ 14,000									
40		39	$ 7,000									
41		40	$ 31,000									
42		41	$ 22,000									
43		42	$ 40,000									
44		43	$ 10,500									
45		44	$ 8,500									

Figure 10.6 Simulation Model Dynamic Histogram Formulas

	L	M	N	O	P	Q	R	S
1	50-Day Trial	=I3		Minimum	=MIN(M2:M101)		Interval Max	Frequency
2	1	=TABLE(,K1)		Maximum	=MAX(M2:M101)		5000	=FREQUENCY(M2:M101,R2:R21)
3	2	=TABLE(,K1)					10000	=FREQUENCY(M2:M101,R2:R21)
4	3	=TABLE(,K1)		Mean	=AVERAGE(M2:M101)		15000	=FREQUENCY(M2:M101,R2:R21)
5	4	=TABLE(,K1)					20000	=FREQUENCY(M2:M101,R2:R21)
6	5	=TABLE(,K1)		StDev	=STDEV(M2:M101)		25000	=FREQUENCY(M2:M101,R2:R21)
7	6	=TABLE(,K1)					30000	=FREQUENCY(M2:M101,R2:R21)
8	7	=TABLE(,K1)					35000	=FREQUENCY(M2:M101,R2:R21)
9	8	=TABLE(,K1)					40000	=FREQUENCY(M2:M101,R2:R21)
10	9	=TABLE(,K1)					45000	=FREQUENCY(M2:M101,R2:R21)
11	10	=TABLE(,K1)					50000	=FREQUENCY(M2:M101,R2:R21)
12	11	=TABLE(,K1)					55000	=FREQUENCY(M2:M101,R2:R21)
13	12	=TABLE(,K1)					60000	=FREQUENCY(M2:M101,R2:R21)
14	13	=TABLE(,K1)					65000	=FREQUENCY(M2:M101,R2:R21)
15	14	=TABLE(,K1)					70000	=FREQUENCY(M2:M101,R2:R21)
16	15	=TABLE(,K1)					75000	=FREQUENCY(M2:M101,R2:R21)
17	16	=TABLE(,K1)					80000	=FREQUENCY(M2:M101,R2:R21)
18	17	=TABLE(,K1)					85000	=FREQUENCY(M2:M101,R2:R21)
19	18	=TABLE(,K1)					90000	=FREQUENCY(M2:M101,R2:R21)
20	19	=TABLE(,K1)					95000	=FREQUENCY(M2:M101,R2:R21)
21	20	=TABLE(,K1)					100000	=FREQUENCY(M2:M101,R2:R21)
22	21	=TABLE(,K1)					More	=FREQUENCY(M2:M101,R2:R21)
23	22	=TABLE(,K1)						
24	23	=TABLE(,K1)						
25	24	=TABLE(,K1)						
26	25	=TABLE(,K1)						

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Introduction to Data Analysis

11

object of analysis: person, thing, business entity, etc.

characteristic of interest: weight, hair color, diameter, sales, etc.

measurement of the characteristic: pounds, blond/brunette/red/etc., inches, dollars, etc.

11.1 LEVELS OF MEASUREMENT

called measurement scales by some authors

important distinctions because analysis and summary methods are very different

two general levels of measurement, each with two specific levels

Categorical Measure

also called qualitative measure

assign a category level to each object of analysis

Nominal Measure: simple classification, "assign a name"

Ordinal Measure: ranked categories, "assign an ordered classification"

Numerical Measure

also called quantitative measure

assign a numerical value to each object of analysis

Interval Measure:, rankings and numerical differences are meaningful

Ratio Measure: natural zero and numerical ratios are meaningful

11.2 DESCRIBING CATEGORICAL DATA

List each categorical level with frequencies (counts) or relative frequencies (percentages).

Use an Excel pivot table to obtain frequencies.

Use an Excel bar chart, column chart, or pie chart.

To display the relationship between two categorical measures, use a two-way classification table.

For nominal data, the appropriate summary measure is the mode (most frequently occurring level)

For ordinal data, the appropriate summary measures are the mode and median (the middle-ranked category level with approximately 50% of the counts below and approximately 50% above).

Do not assign meaningless numerical values to the categorical levels.

Do not use the mean and standard deviation.

11.3 DESCRIBING NUMERICAL DATA

Frequency Distribution and Histogram

Determine the range (maximum minus minimum), generally use between 5 and 15 equally-spaced intervals, and pick "nice" numbers for the upper limit of each interval (Excel "bins").

Use Excel's Histogram analysis tool, or use Excel's FREQUENCY array-entered worksheet function with an Excel Column chart (vertical bars).

Numerical Summary Measures

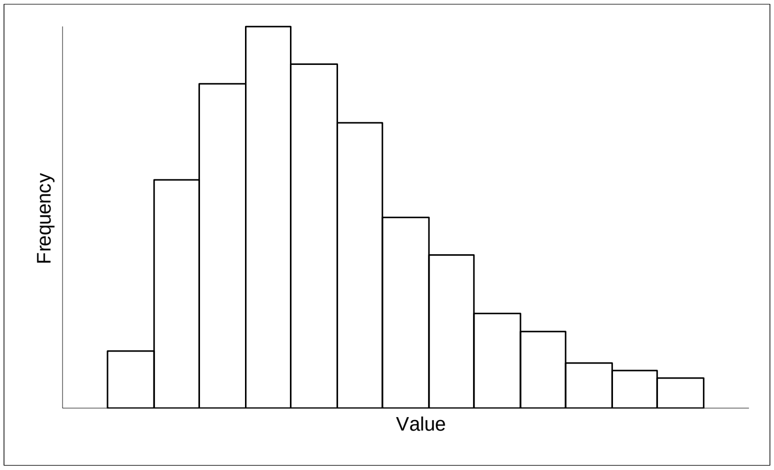
Appropriate summary measures for central tendency ("What's a typical value?") include mean (average, most appropriate for mound-shaped data), median, and mode.

Appropriate summary measures for dispersion ("How typical is the typical value?") include range, standard deviation (most appropriate for mound-shaped distributions), and fractiles (first quartile, or 25th percentile, is a value with approximately 25% of the values below it and approximately 75% of the values above).

Appropriate summary measures for shape are Excel's SKEW worksheet function and Pearson's coefficient of skewness.

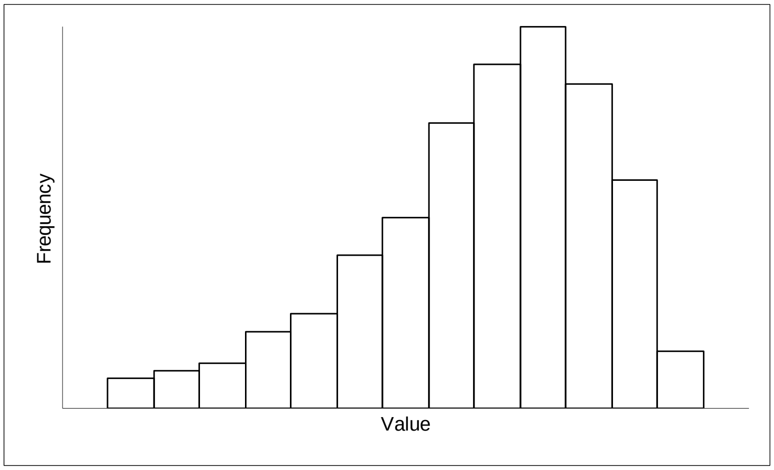
Distribution Shapes

Figure 11.1 Positively Skewed Distribution (Skewed to the Right)



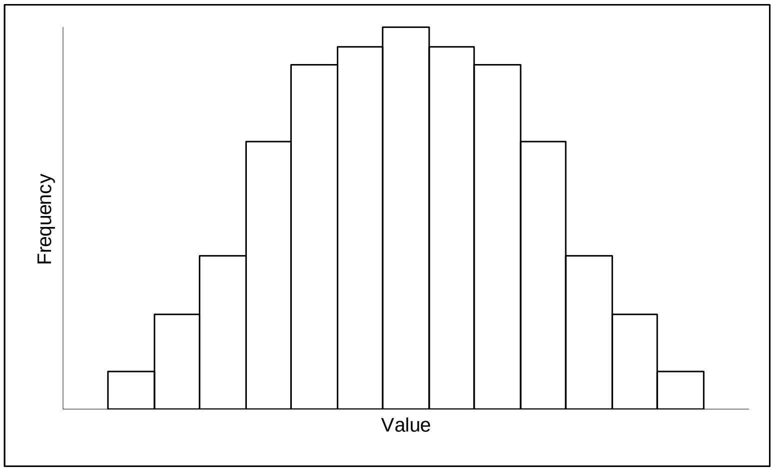
In a distribution with positive skew, the mean is greater than the median.

Figure 11.2 Negatively Skewed Distribution (Skewed to the Left)



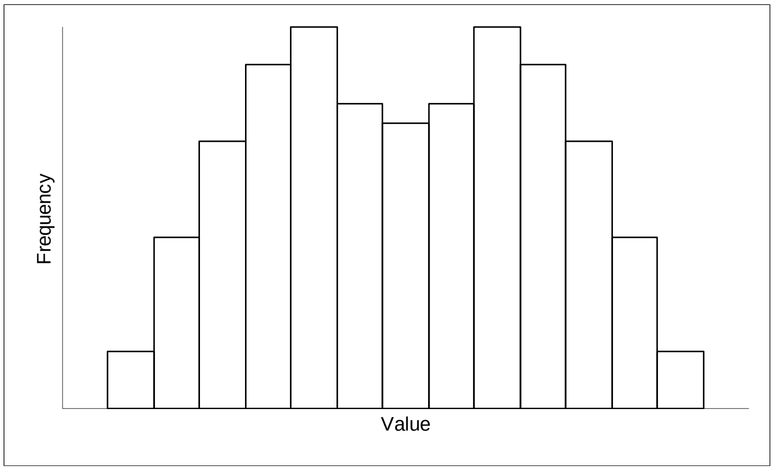
In a distribution with negative skew, the mean is less than the median.

Figure 11.3 Mound-Shaped Distribution (Symmetric)



In a symmetric distribution, the mean and median are equal.

Figure 11.4 Bimodal Distribution



In a bimodal distribution, there is often a distinguishing characteristic for the two groups of data that have been combined into a single distribution.

Regression Models for Cross-Sectional Data

12

12.1 CROSS-SECTIONAL REGRESSION CHECKLIST

Plot Y versus each X

1 Verify that the relationship agrees with your prior judgment, e.g., positive vs negative relationship, linear vs nonlinear, strong vs weak

2 Identify outliers or unusual observations and decide whether to exclude

3 Determine whether the relationship is linear; if not, consider using a nonlinear form, e.g., quadratic (include X and X^2 in the model)

Examine the correlation matrix

4 Identify potential multicollinearity problems, i.e., high correlation between a pair of X variables; if so, consider using only one X of the pair in the model

Calculate the regression model with diagnostics

5 Verify that the sign of each regression coefficient agrees with your prior judgment, i.e., positive vs negative relationship; otherwise, consider excluding that X and rerun the regression

6 Examine each plot of residuals vs X; if there is a non-random pattern (e.g., U-shape or upside-down-U-shape), use a nonlinear form for that X in a new model

7 Identify key X variables by comparing standardized regression coefficients, usually computed by multiplying an X coefficient by the standard deviation of that X and dividing by the standard deviation of Y. This dimensionless standardized regression coefficient measures how much Y (in standard deviation units) is affected by a change in X (in standard deviation units).

8 If a goal is to find a model with small standard error of estimate (approx. standard deviation of residuals), use the t-stat screening method. Disregard the t-stat for the intercept. If there are X variables with a t-stat between -1 and +1, remove the single X variable whose t-stat is closest to zero, and rerun the regression. Remove only one X variable at a time.

9 Before using the final model, examine each plot of residuals vs X to verify that the random scatter is the same for all values of X. If there is more scatter for higher values of X, consider using a log transformation of X in the model (instead of using X itself). If the scatter is not uniform with respect to X, the standard error of estimate may not be a useful measure of uncertainty because it overstates the uncertainty for some values of X and understates the uncertainty for other values of X.

Use the model

10 If the purpose is to identify unusual observations, examine the residuals directly for large negative or large positive values, or examine the standardized residuals (each residual divided by the standard deviation of residuals) for values more extreme than +2 or -2 or for values more extreme than +3 or -3.

11 If the purpose is to make predictions, use the X values for a new observation to compute a predicted Y. Use the standard error of estimate to provide an interval estimate, e.g., an approximate 95% prediction interval that ranges from two standard errors below to two standard errors above the predicted Y. Avoid extrapolation, i.e., do not make predictions using X values outside the range of the original data.

Time Series Data and Forecasts

13

13.1 TIME SERIES PATTERNS

Meandering time series pattern: Small changes from period to period, possible larger changes over a longer period of time

Use an autoregressive model

Figure 13.1 Typical Meandering Time Series Pattern

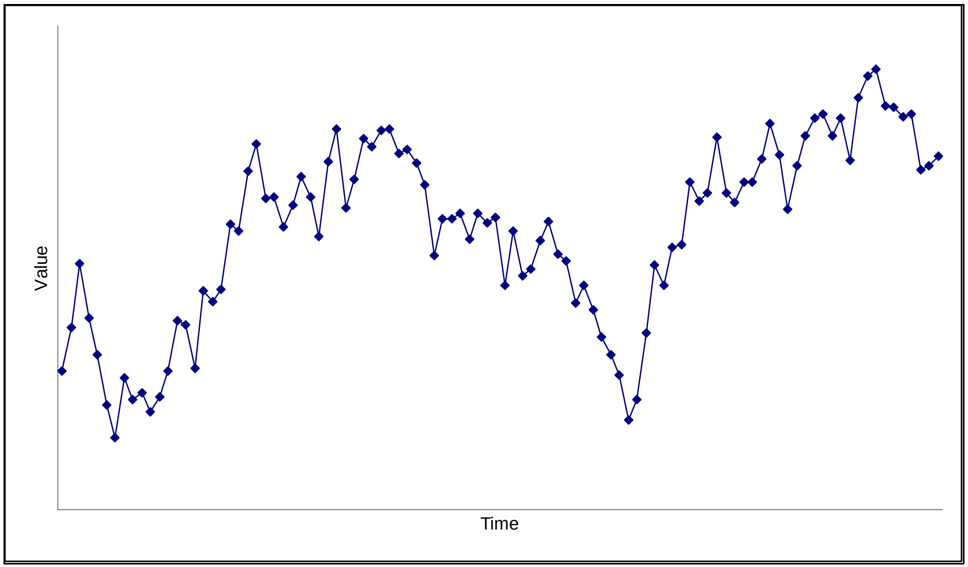


Figure 13.2 Typical Long-Term Trend Time Series Patterns

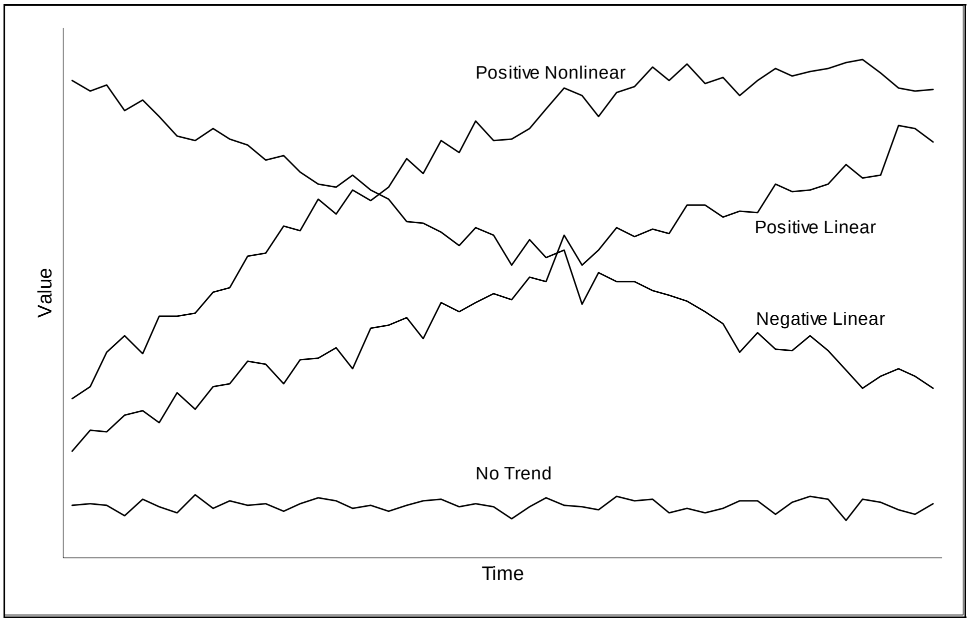


Figure 13.3 Typical Quarterly Seasonal Time Series with Linear Trend

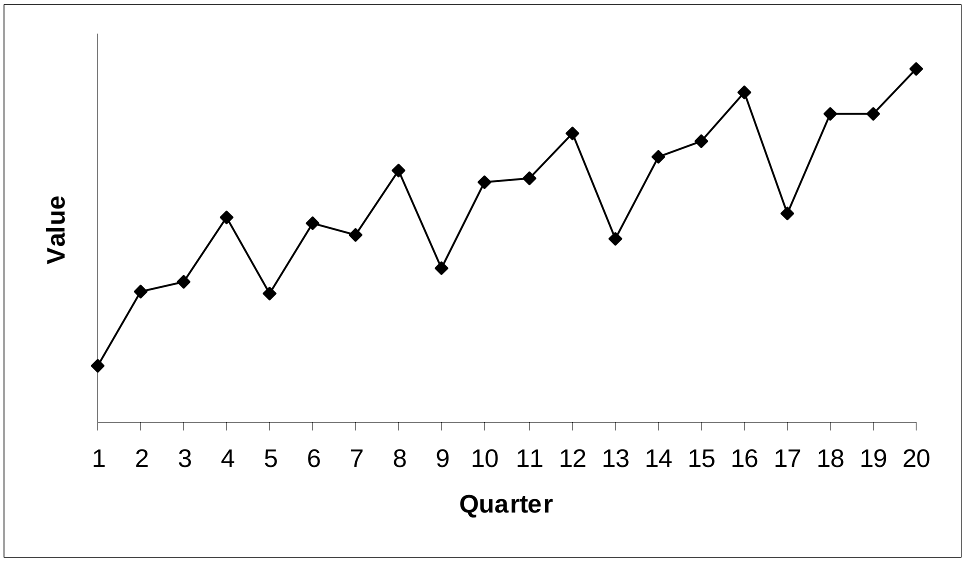
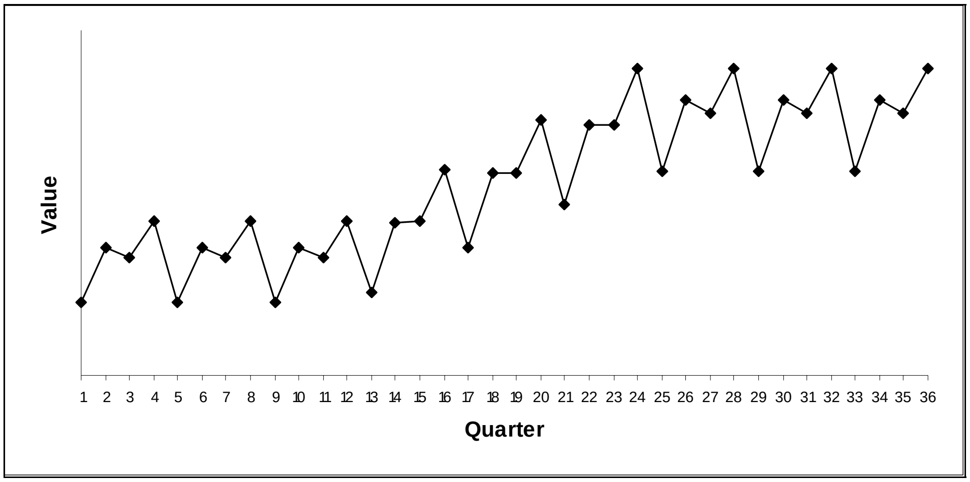


Figure 13.4 Quarterly Seasonal Pattern with Nonlinear Trend



Strong seasonal pattern, no trend during first 12 quarters, positive trend during middle 12 quarters, no trend during last 12 quarters

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Regression Models for Time Series Data

14

14.1 TIME SERIES REGRESSION CHECKLIST

Relevant explanatory variables (X) for time series data related to business activity (Y), e.g., sales over time, include several general types:

a Internal business activity, like advertising, promotion, research and development

b Competitor business activity, like competitor sales and competitor advertising

c Industry activity, like number of competitors and market size

d General economic activity, like personal disposable income

Plot Y versus time

1 Identify any systematic pattern to help determine an appropriate model

Plot Y versus each X

2 Verify that the relationship agrees with your prior judgment, e.g., positive vs negative relationship, linear vs nonlinear, strong vs weak

3 Identify outliers or unusual observations and decide whether to exclude

4 Determine whether the relationship is linear; if not, consider using a nonlinear form, e.g., quadratic (include X and X^2 in the model)

Examine the correlation matrix

5 Include a time period variable in the correlation matrix. For example, if there are n equally-spaced time periods, include a variable in your data set with values 1,2,...,n.

6 Identify potential multicollinearity problems, i.e., high correlation between a pair of X variables; if so, consider using only one X of the pair in the model

Calculate the regression model with diagnostics

7 Verify that the sign of each regression coefficient agrees with your prior judgment, i.e., positive vs negative relationship; otherwise, consider excluding that X and rerun the regression

8 Examine each plot of residuals vs X; if there is a non-random pattern (e.g., U-shape or upside-down-U-shape), use a nonlinear form for that X in a new model

9 In addition to the residual plots generated automatically by Excel's Regression tool, prepare and examine a plot of residuals vs time. If there is a snake-like pattern of residuals, consider adding lag Y as an explanatory variable. Optionally, compute the Durbin-Watson statistic to detect autocorrelation of residuals.

10 Identify key X variables by comparing standardized regression coefficients, usually computed by multiplying an X coefficient by the standard deviation of that X and dividing by the standard deviation of Y. This dimensionless standardized regression coefficient measures how much Y (in standard deviation units) is affected by a change in X (in standard deviation units).

11 If a goal is to find a model with small standard error of estimate (approx. standard deviation of residuals), use the t-stat screening method. Disregard the t-stat for the intercept. If there are X variables with a t-stat between -1 and +1, remove the single X variable whose t-stat is closest to zero, and rerun the regression. Remove only one X variable at a time.

12 Before using the final model, examine each plot of residuals vs X to verify that the random scatter is the same for all values of X. If there is more scatter for higher values of X, consider using a log transformation of X in the model (instead of using X itself). If the scatter is not uniform with respect to X, the standard error of estimate may not be a useful measure of uncertainty because it overstates the uncertainty for some values of X and understates the uncertainty for other values of X.

Use the model

13 If the purpose is to identify unusual observations, examine the residuals directly for large negative or large positive values, or examine the standardized residuals (each residual divided by the standard deviation of residuals) for values more extreme than +2 or -2 or for values more extreme than +3 or -3.

14 If the purpose is to make predictions, use the X values for a new observation to compute a predicted Y. Use the standard error of estimate to provide an interval estimate, e.g., an approximate 95% prediction interval that ranges from two standard errors below to two standard errors above the predicted Y. Note that a

time series forecast usually extrapolates beyond the original range of data, so the standard error of estimate is a minimum indication of the uncertainty surrounding a forecast.

14.2 AUTOCORRELATION OF RESIDUALS

Figure 14.1 Undesirable Extreme Negative Autocorrelation

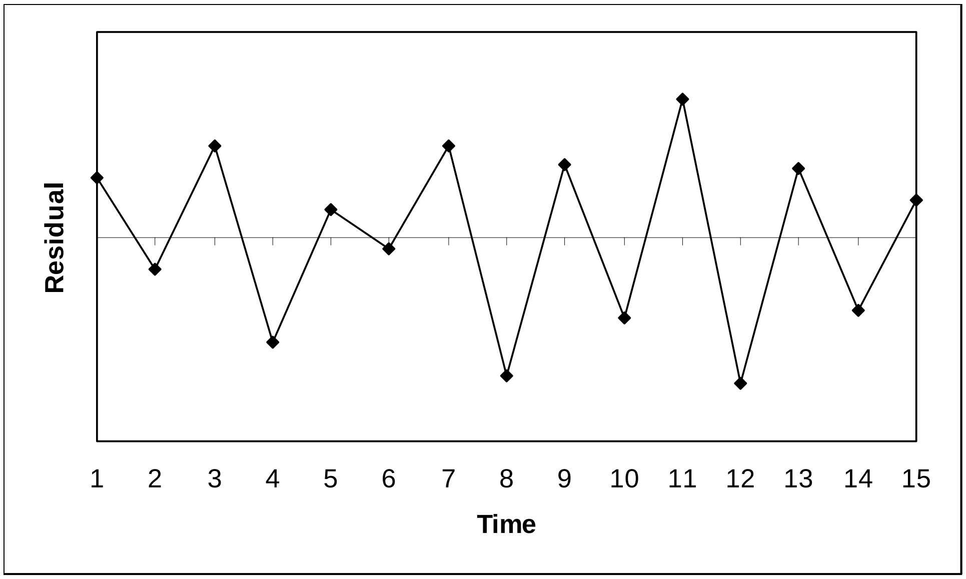


Figure 14.2 Undesirable Extreme Positive Autocorrelation

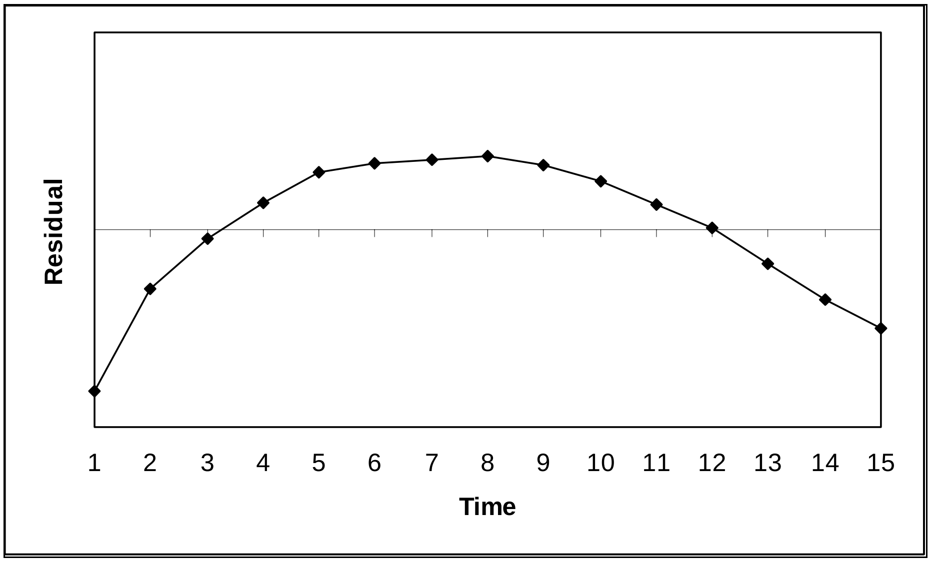
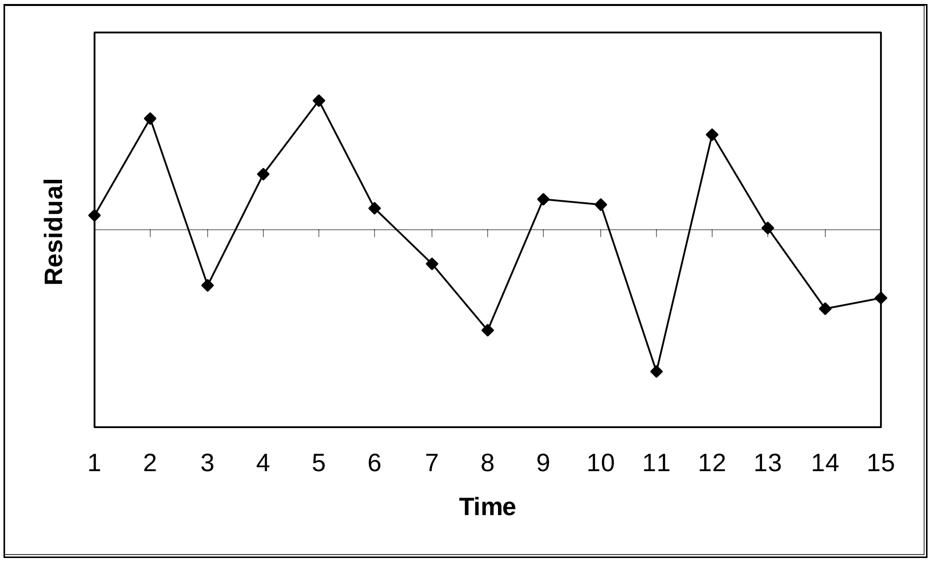


Figure 14.3 Desirable Zero Autocorrelation



Sensitivity Analysis for Decision Trees

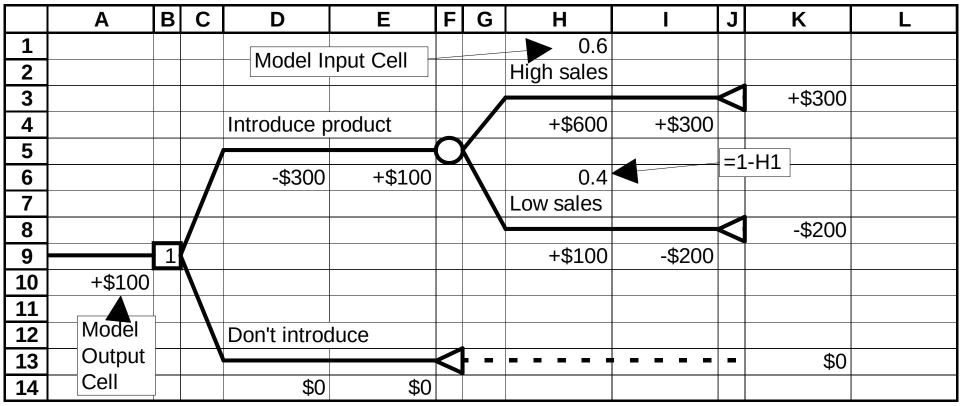
15

15.1 ONE-VARIABLE SENSITIVITY ANALYSIS

One-Variable Sensitivity Analysis using an Excel data table

1. Construct a decision tree model or financial planning model.
2. Identify the model input cell (H1) and model output cell (A10).
3. Modify the model so that probabilities will always sum to one. (That is, enter the formula =1-H1 in cell H6.)

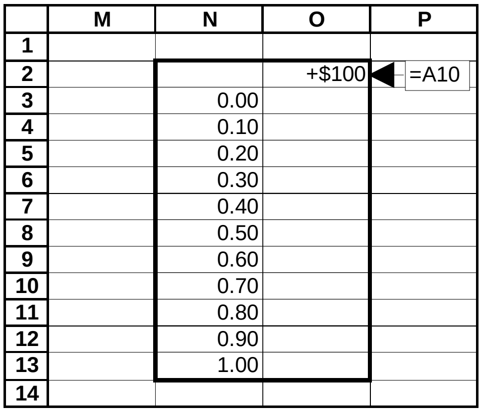
Figure 15.1 Display for One-Variable Sensitivity Analysis



4. Enter a list of input values in a column (N3:N13).
5. Enter a formula for determining output values at the top of an empty column on the right of the input values (=A10 in cell O2).
6. Select the data table range (N2:O13).

7. From the Data menu choose the Table command.

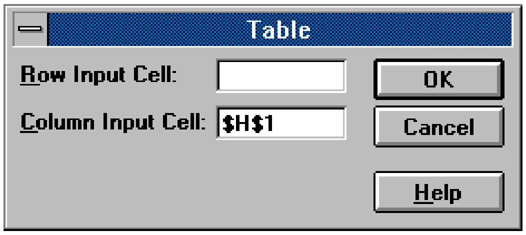
Figure 15.2



	M	N	O	P
1				
2			+$100	=A10
3		0.00		
4		0.10		
5		0.20		
6		0.30		
7		0.40		
8		0.50		
9		0.60		
10		0.70		
11		0.80		
12		0.90		
13		1.00		
14				

8. In the Data Table dialog box, select the Column Input Cell edit box. Type the model input cell (H1), or point to the model input cell (in which case the edit box displays $H$1). Click OK.

Figure 15.3



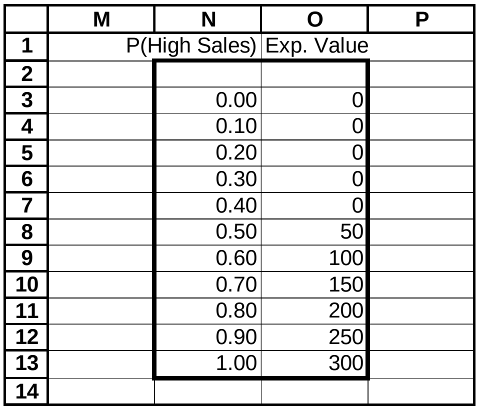
Row Input Cell:

Column Input Cell: $H$1

9. The Data Table command substitutes each input value into the model input cell, recalculates the worksheet, and displays the corresponding model output value in the table.

10. Optional: Change the formula in cell O2 to =CHOOSE(B9,"Introduce","Don't").

Figure 15.4

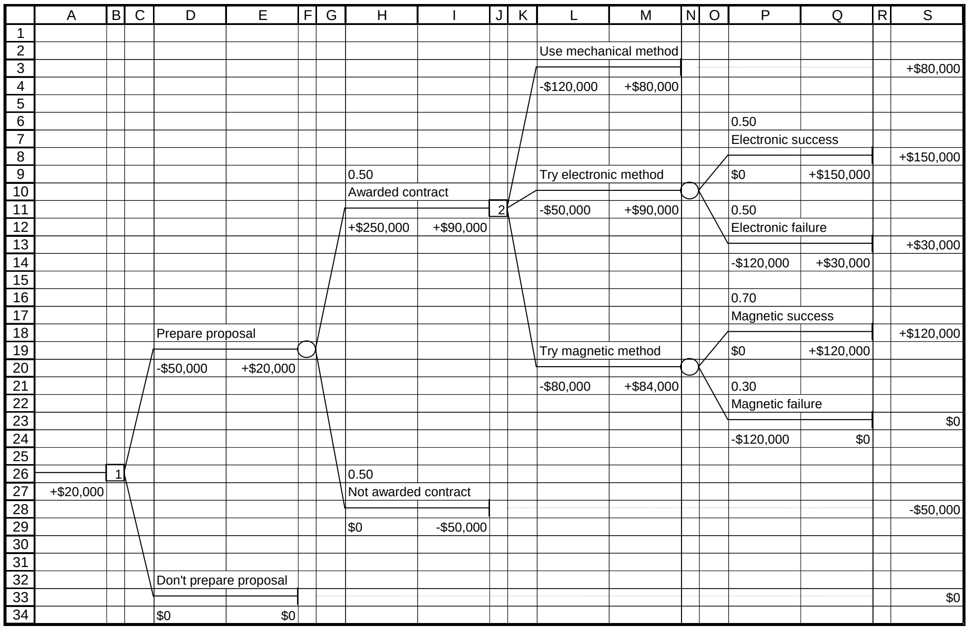


	M	N	O	P
1		P(High Sales)	Exp. Value	
2				
3		0.00	0	
4		0.10	0	
5		0.20	0	
6		0.30	0	
7		0.40	0	
8		0.50	50	
9		0.60	100	
10		0.70	150	
11		0.80	200	
12		0.90	250	
13		1.00	300	
14				

15.2 TWO-VARIABLE SENSITIVITY ANALYSIS

Two-Variable Sensitivity Analysis using an Excel data table

Figure 15.5 Decision Tree for Strategy Region Table



Optional: Activate the Base Case worksheet. From the Edit menu, choose Move Or Copy Sheet. In the Move Or Copy dialog box, check the box for Create A Copy, and click OK. Double-click the new worksheet tab and enter Strategy Region Table.

Setup for Data Table

Select cell P11, and enter the formula =1–P6. Select cell P21, and enter the formula =1–P16.

In cell U3 enter P(Elec OK). In cell V3 enter 1, and in cell V4 enter 0.9. Select cells V3:V4. In the lower right corner of cell V4, click the fill handle and drag down to cell V13. With cells V3:V13 still selected, click the Increase Decimal button once so that all values are displayed with one decimal place.

Select columns V:AG. (Select column V. Click and drag the horizontal scroll bar until column AG is visible. Hold down the Shift key and click column AG.) From the Format menu choose Column | Width. In the Column Width edit box type 5 and click OK.

In cell W1 enter P(Mag OK). In cell W2 enter 0 (zero), and in cell X2 enter 0.1. Select cells W2:X2. In the lower right corner of cell X2, click the fill handle and drag right to cell AG2. With cells W2: AG2 still selected, click the Increase Decimal button once so that all values are displayed with one decimal place.

Select cell V2 and enter the formula =CHOOSE(J11,"Mech","Elec","Mag"). With the base case assumptions the formula shows Elec.

Figure 15.6

	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG
1			P(Mag OK)										
2		Elec	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
3	P(Elec OK)	1.0											
4		0.9											
5		0.8											
6		0.7											
7		0.6											
8		0.5											
9		0.4											
10		0.3											
11		0.2											
12		0.1											
13		0.0											

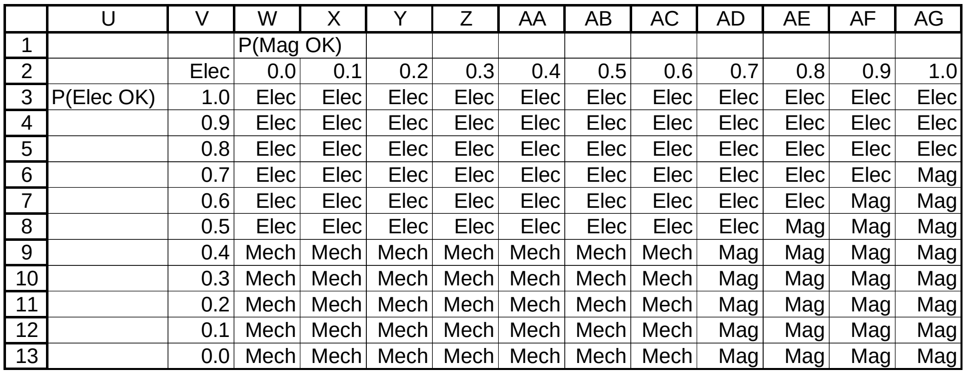
Obtaining Results Using Data Table Command

Select the entire data table, cells V2:AG13.

From the Data menu, choose Table. In the Table dialog box, type P16 in the Row Input Cell edit box, type P6 in the Column Input Cell edit box, and click OK.

With cells V2:AG13 still selected, click the Align Right button.

Figure 15.7



	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	AF	AG
1			P(Mag OK)										
2		Elec	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
3	P(Elec OK)	1.0	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
4		0.9	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
5		0.8	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
6		0.7	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag
7		0.6	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag	Mag
8		0.5	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag	Mag	Mag
9		0.4	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
10		0.3	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
11		0.2	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
12		0.1	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
13		0.0	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag

Embellishments

Select cells U1:AG13, and click the Copy button. Select cell AI1, right-click, and from the shortcut menu choose Paste Special. In the Paste Special dialog box, click the Values option button, and click OK. Right-click again, choose Paste Special, click the Formats option button, and click OK.

Select columns AJ:AU. Choose Format | Cells | Width, type 5, and click OK.

Select cell AJ2, right-click, and from the shortcut menu choose Clear Contents. Select cells AK2:AU2, move the cursor near the border of the selection until it becomes an arrow, click and drag the selection down to cells AK14:AU14. Similarly, select cell AK1 and move its contents down to cell AP15. Also, move the contents of cell AI3 to cell AI8. Select cell AN1, and enter Strategy Region Table.

Figure 15.8

	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR	AS	AT	AU
1						Strategy Region Table							
2													
3		1.0	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
4		0.9	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
5		0.8	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
6		0.7	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag
7		0.6	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag	Mag
8	P(Elec OK)	0.5	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag	Mag	Mag
9		0.4	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
10		0.3	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
11		0.2	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
12		0.1	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
13		0.0	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
14			0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
15								P(Mag OK)					

Apply borders to appropriate ranges and cells to show the strategy regions. Apply shading to cell AR8 to show the base case strategy.

Figure 15.9

	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR	AS	AT	AU
1						**Strategy Region Table**							
2													
3		1.0	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
4		0.9	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
5		0.8	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec
6		0.7	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag
7		0.6	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag	Mag
8	P(Elec OK)	0.5	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Elec	Mag	Mag	Mag
9		0.4	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
10		0.3	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
11		0.2	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
12		0.1	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
13		0.0	Mech	Mech	Mech	Mech	Mech	Mech	Mech	Mag	Mag	Mag	Mag
14			0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
15								P(Mag OK)					

15.3 MULTIPLE-OUTCOME SENSITIVITY ANALYSIS

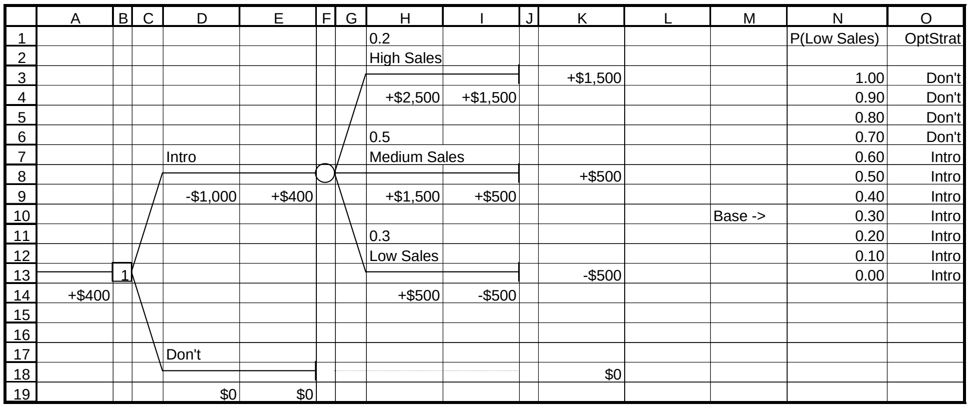
Sensitivity Analysis for Multiple-Outcome Event Probabilities

Choose one of the outcome probabilities that will be explicitly changed.

For example, focus on P(Low Sales).

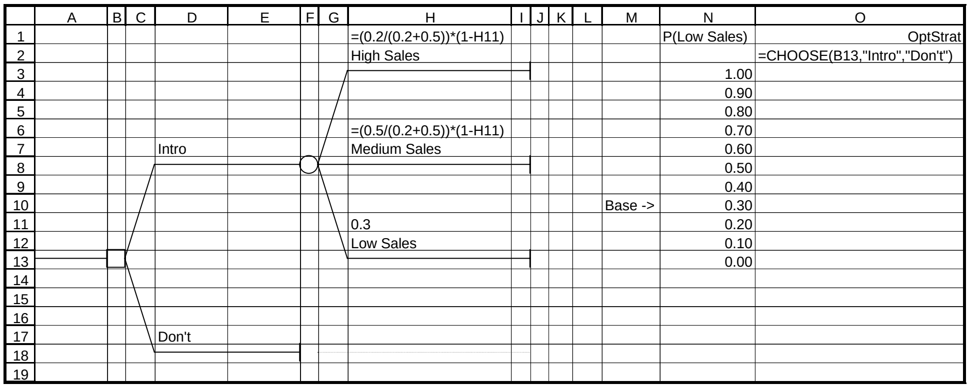
Keep same relative likelihood (base case) for the other probabilities.

Figure 15.10



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1								0.2						P(Low Sales)	OptStrat
2								High Sales							
3											+$1,500			1.00	Don't
4								+$2,500	+$1,500					0.90	Don't
5														0.80	Don't
6								0.5						0.70	Don't
7				Intro				Medium Sales						0.60	Intro
8											+$500			0.50	Intro
9				-$1,000	+$400			+$1,500	+$500					0.40	Intro
10													Base ->	0.30	Intro
11								0.3						0.20	Intro
12								Low Sales						0.10	Intro
13		1									-$500			0.00	Intro
14	+$400							+$500	-$500						
15															
16															
17				Don't											
18											$0				
19				$0	$0										

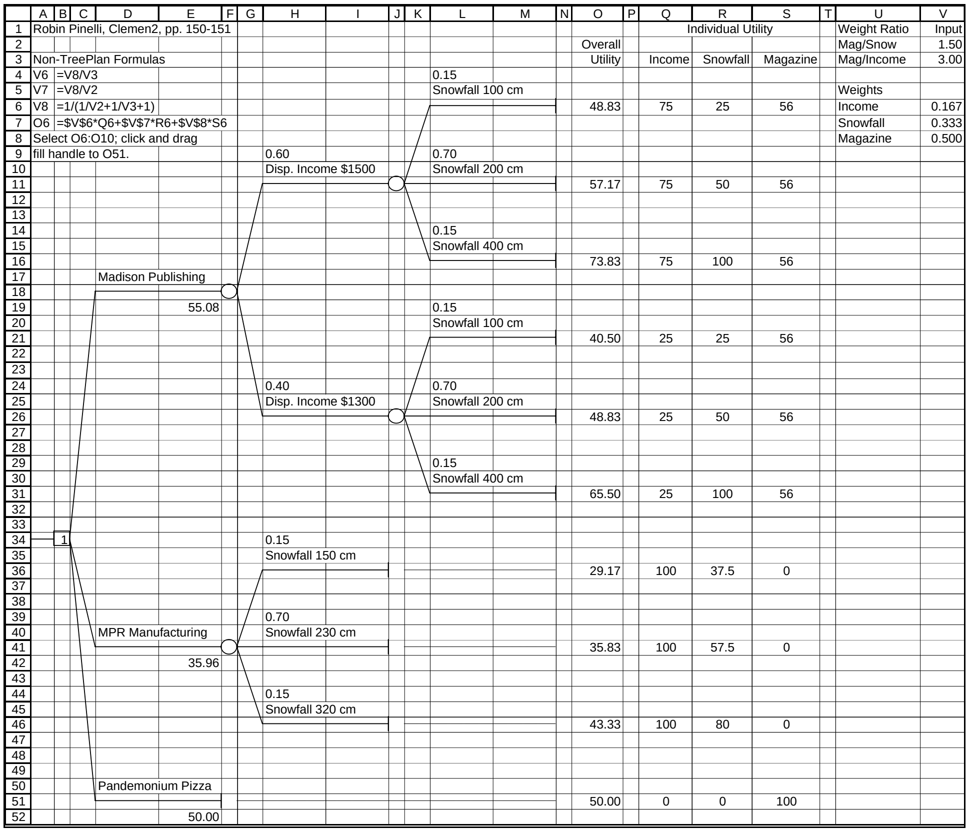
Figure 15.11



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1								=(0.2/(0.2+0.5))*(1-H11)						P(Low Sales)	OptStrat
2								High Sales							=CHOOSE(B13,"Intro","Don't")
3														1.00	
4														0.90	
5														0.80	
6								=(0.5/(0.2+0.5))*(1-H11)						0.70	
7				Intro				Medium Sales						0.60	
8														0.50	
9														0.40	
10													Base ->	0.30	
11								0.3						0.20	
12								Low Sales						0.10	
13														0.00	
14															
15															
16															
17				Don't											
18															
19															

15.4 ROBIN PINELLI'S SENSITIVITY ANALYSIS

Figure 15.12 Decision Tree and Multi-Attribute Utility (Robin Pinelli)



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1	Robin Pinelli, Clemen2, pp. 150-151																Individual Utility				Weight Ratio	Input
2															Overall						Mag/Snow	1.50
3	Non-TreePlan Formulas														Utility		Income	Snowfall	Magazine		Mag/Income	3.00
4	V6	=V8/V3										0.15										
5	V7	=V8/V2										Snowfall 100 cm									Weights	
6	V8	=1/(1/V2+1/V3+1)													48.83		75	25	56		Income	0.167
7	O6	=$V$6*Q6+$V$7*R6+$V$8*S6																			Snowfall	0.333
8	Select O6:O10; click and drag																				Magazine	0.500
9	fill handle to O51.							0.60				0.70										
10								Disp. Income $1500				Snowfall 200 cm										
11															57.17		75	50	56			
12																						
13																						
14												0.15										
15												Snowfall 400 cm										
16															73.83		75	100	56			
17				Madison Publishing																		
18																						
19					55.08							0.15										
20												Snowfall 100 cm										
21															40.50		25	25	56			
22																						
23																						
24								0.40				0.70										
25								Disp. Income $1300				Snowfall 200 cm										
26															48.83		25	50	56			
27																						
28																						
29												0.15										
30												Snowfall 400 cm										
31															65.50		25	100	56			
32																						
33																						
34		1						0.15														
35								Snowfall 150 cm														
36															29.17		100	37.5	0			
37																						
38																						
39								0.70														
40				MPR Manufacturing				Snowfall 230 cm														
41															35.83		100	57.5	0			
42					35.96																	
43																						
44								0.15														
45								Snowfall 320 cm														
46															43.33		100	80	0			
47																						
48																						
49																						
50				Pandemonium Pizza																		
51															50.00		0	0	100			
52					50.00																	

Figure 15.13 Sensitivity Analysis of Weight-Ratio Input Assumptions

	X	Y	Z	AA	AB	AC	AD	AE	AF	AG	AH
1	Sensitivity Analysis										
2			Mag/Income Weight Ratio								
3			1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
4	Mag/Snow	1.00	Madison	Madison	Madison	Madison	Madison	Madison	Madison	Madison	Madison
5	Weight	1.25	Madison	Madison	Madison	Madison	Madison	Madison	Madison	Madison	Madison
6	Ratio	1.50	Madison	Madison	Madison	Madison	Madison	Madison	Madison	Madison	Madison
7		1.75	Madison	Madison	Madison	Madison	Madison	Madison	Madison	Pizza	Pizza
8		2.00	Madison	Madison	Madison	Madison	Madison	Pizza	Pizza	Pizza	Pizza
9		2.25	Madison	Madison	Madison	Madison	Pizza	Pizza	Pizza	Pizza	Pizza
10		2.50	Madison	Madison	Madison	Pizza	Pizza	Pizza	Pizza	Pizza	Pizza
11		2.75	Madison	Madison	Madison	Pizza	Pizza	Pizza	Pizza	Pizza	Pizza
12		3.00	Madison	Madison	Madison	Pizza	Pizza	Pizza	Pizza	Pizza	Pizza
13											
14											
15											
16			Mag/Income Weight Ratio								
17			0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00	3.50
18	Mag/Snow	0.25	MPR	MPR	MPR	MPR	Madison	Madison	Madison	Madison	Madison
19	Weight	0.50	MPR	MPR	MPR	Madison	Madison	Madison	Madison	Madison	Madison
20	Ratio	0.75	MPR	MPR	MPR	Madison	Madison	Madison	Madison	Madison	Madison
21		1.00	MPR	MPR	MPR	Madison	Madison	Madison	Madison	Madison	Madison
22		1.25	MPR	MPR	MPR	Madison	Madison	Madison	Madison	Madison	Madison
23		1.50	MPR	MPR	MPR	Madison	Madison	Madison	Madison	Madison	Madison
24		1.75	MPR	MPR	MPR	Madison	Madison	Madison	Madison	Madison	Madison
25		2.00	MPR	MPR	MPR	Madison	Madison	Madison	Madison	Madison	Pizza
26		2.25	MPR	MPR	MPR	Madison	Madison	Madison	Madison	Pizza	Pizza
27											
28											
29	Formulas										
30	Y3	=CHOOSE(B34,"Madison","MPR", "Pizza")									
31	Y17	=CHOOSE(B34,"Madison","MPR", "Pizza")									
32											
33	Data Tables Y3:AH11 and Y17:AH26										
34	V3	Row Input Cell									
35	V2	Column Input Cell									

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Value of Information in Decision Trees

16

16.1 VALUE OF INFORMATION

Useful concept for

- Evaluation potential information-gathering activities
- Comparing importance of multiple uncertainties

16.2 EXPECTED VALUE OF PERFECT INFORMATION

Several computational methods

- Flipping tree, moving an event set of branches, appropriate for any decision tree
- Payoff table, most appropriate only for single-stage tree (one set of uncertain outcomes with no subsequent decisions)
- Expected improvement

All three methods start by determining Expected Value Under Uncertainty, EVUU, which is the expected value of the optimal strategy without any additional information.

Figure 16.1 Basic Probability Decision Tree

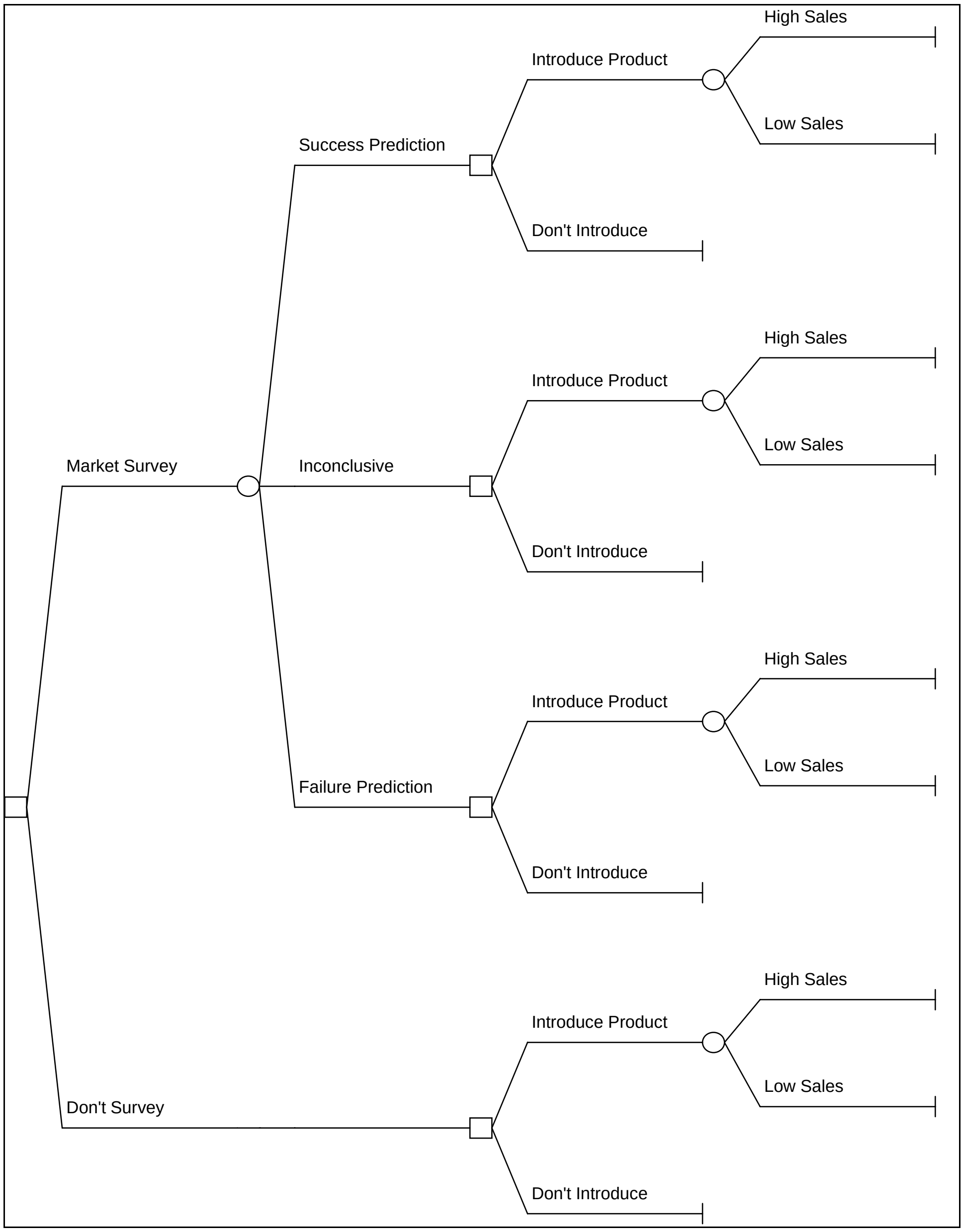
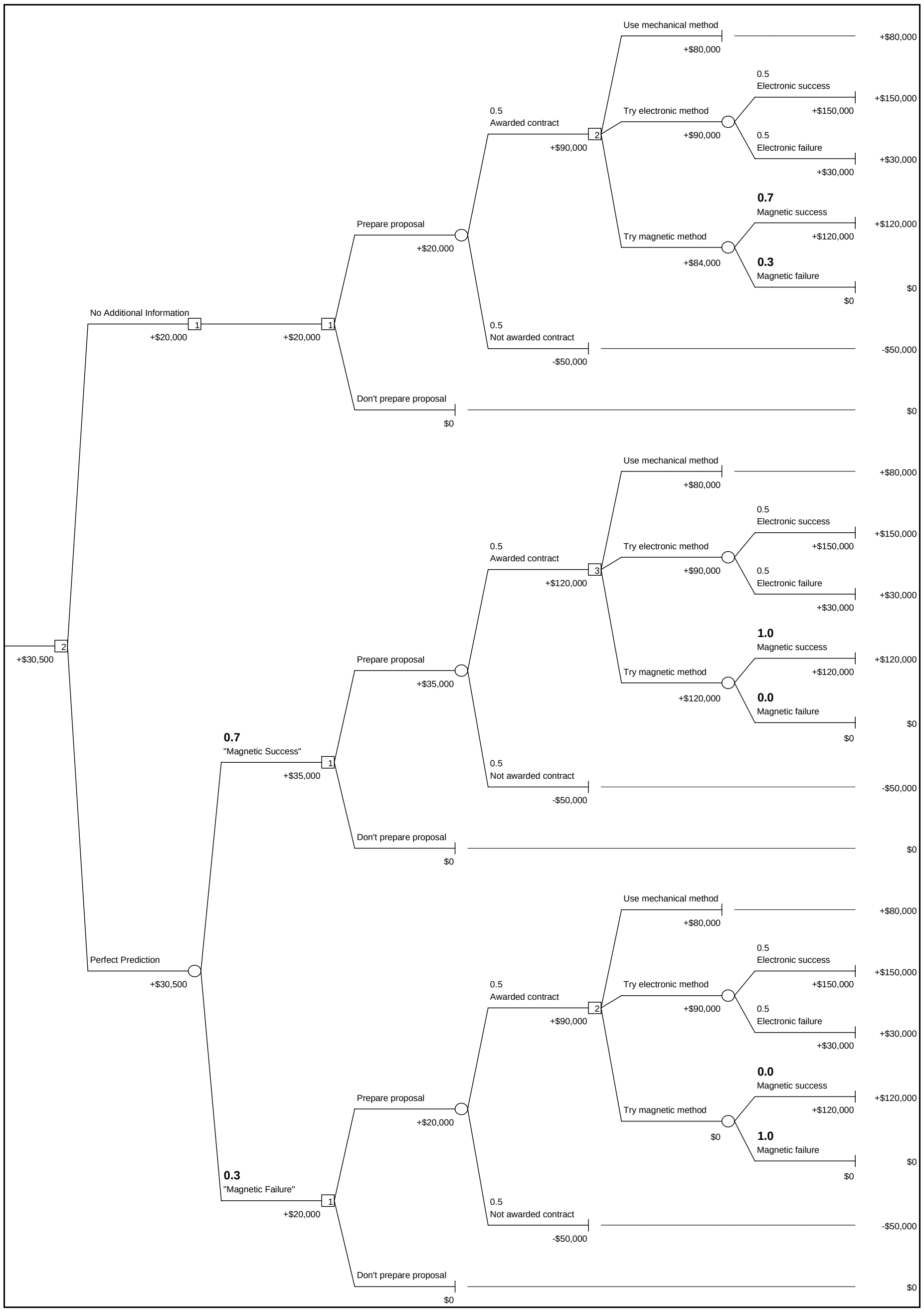


Figure 16.2 DriveTek EVPI Magnetic Success/Failure



16.3 DRIVETEK POST-CONTRACT-AWARD PROBLEM

DriveTek decided to prepare the proposal, and it turned out that they were awarded the contract. The $50,000 cost and $250,000 up-front payment are in the past. The current decision is to determine which method to use to satisfy the contract.

The following decision trees show costs as negative cash flows, so the decision criterion is to maximize expected cash flow. An alternative formulation (not shown here) would show all costs as positive values and would minimize expected cost.

Figure 16.3 EVUU

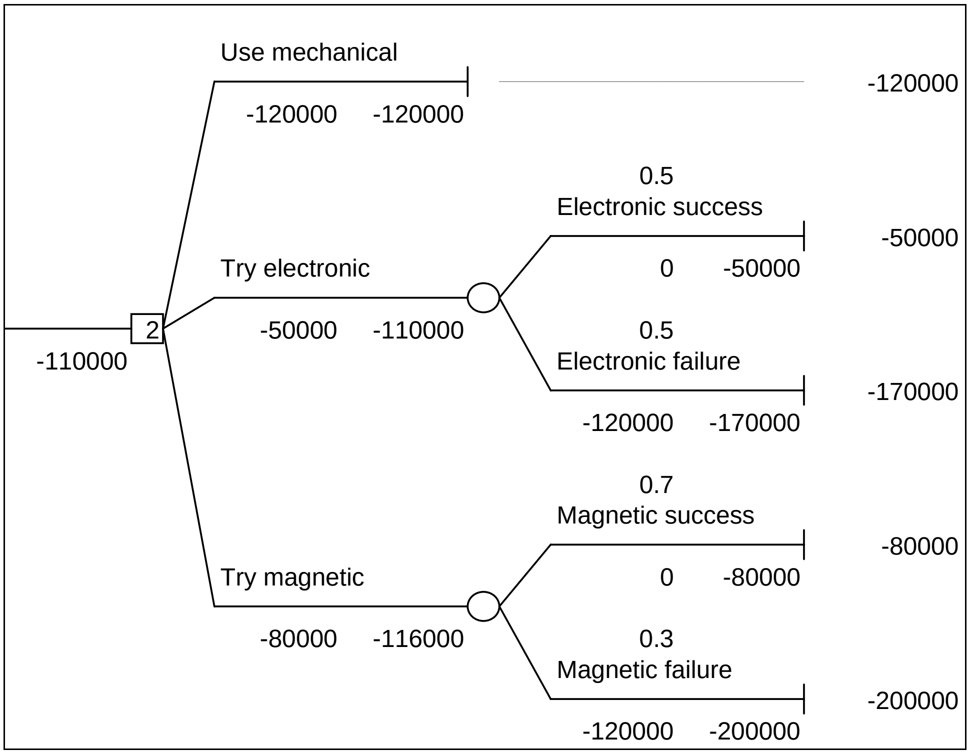


Figure 16.4 EVPP Elec

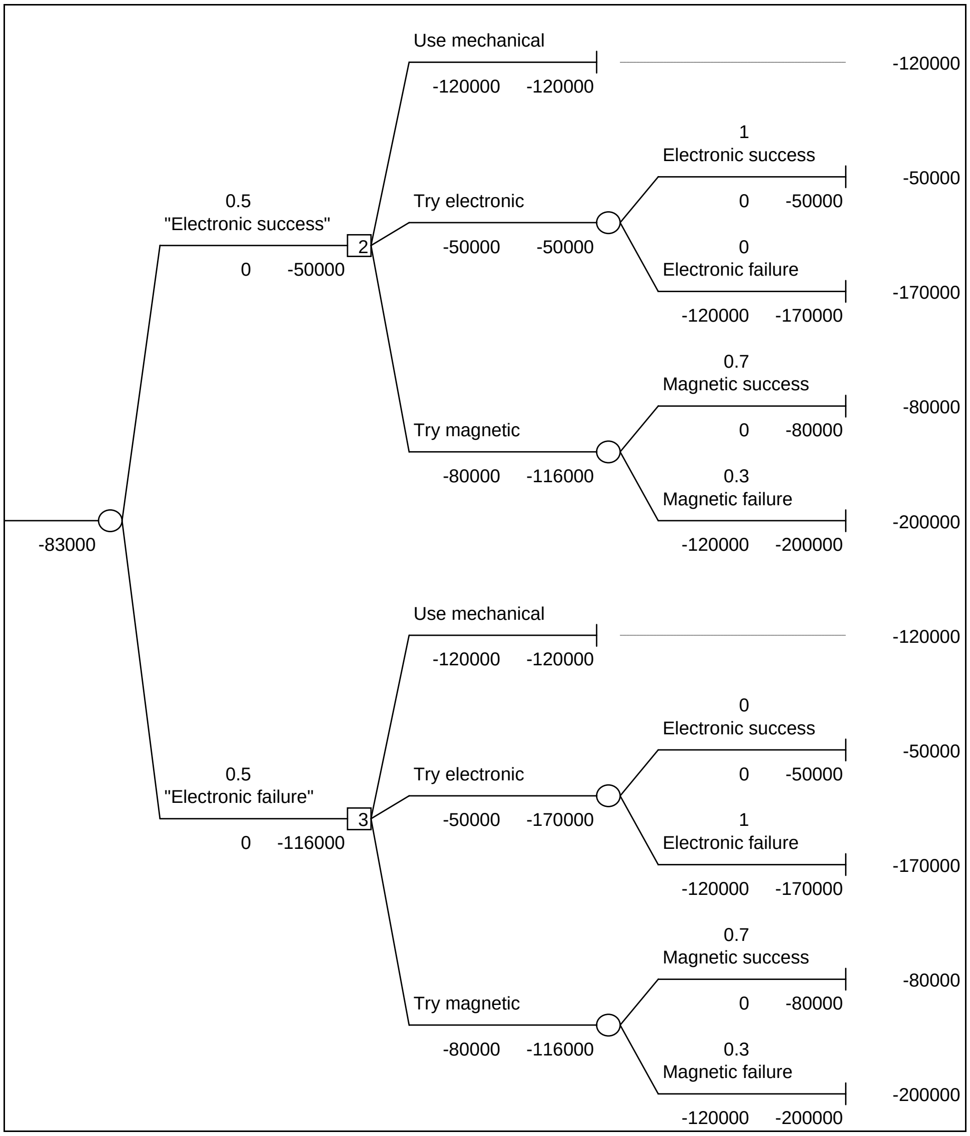


Figure 16.5 EVPP Mag

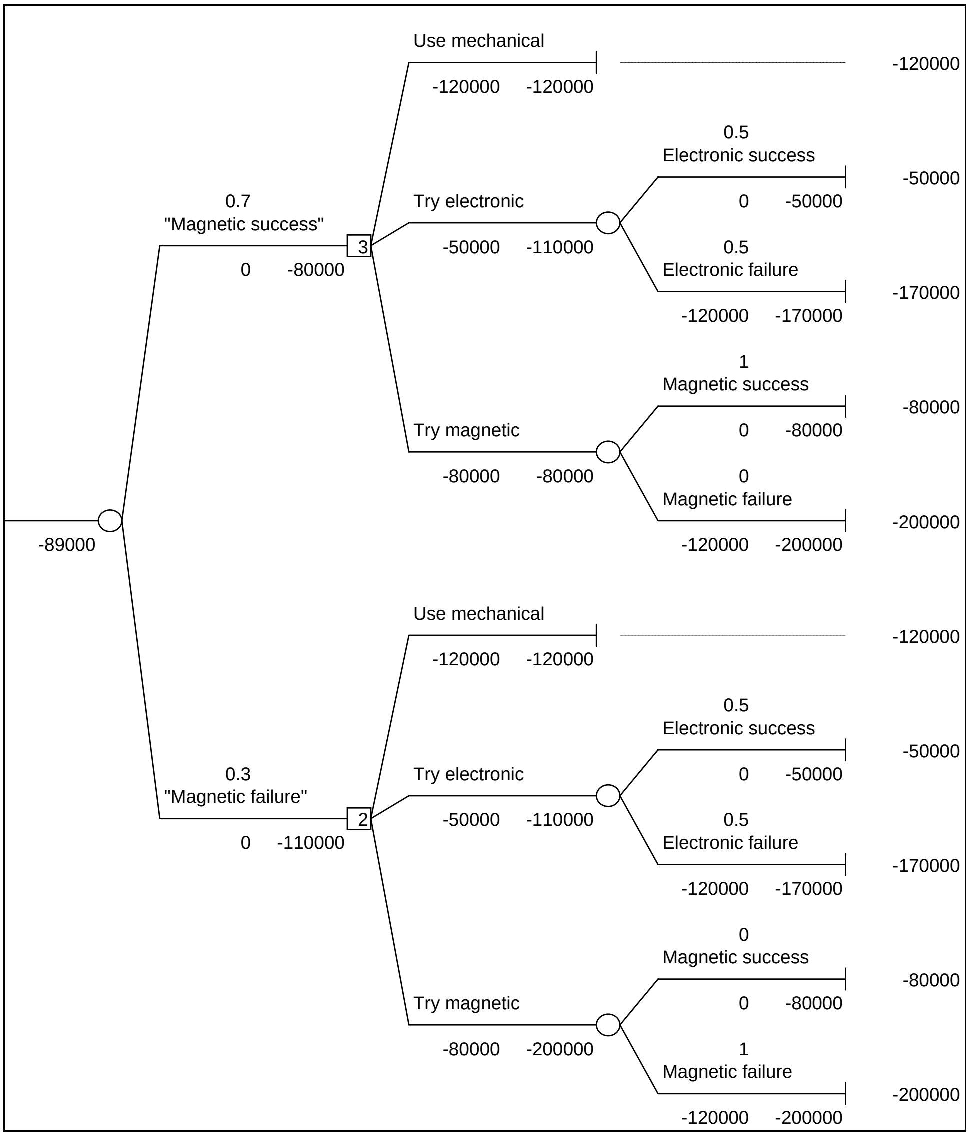
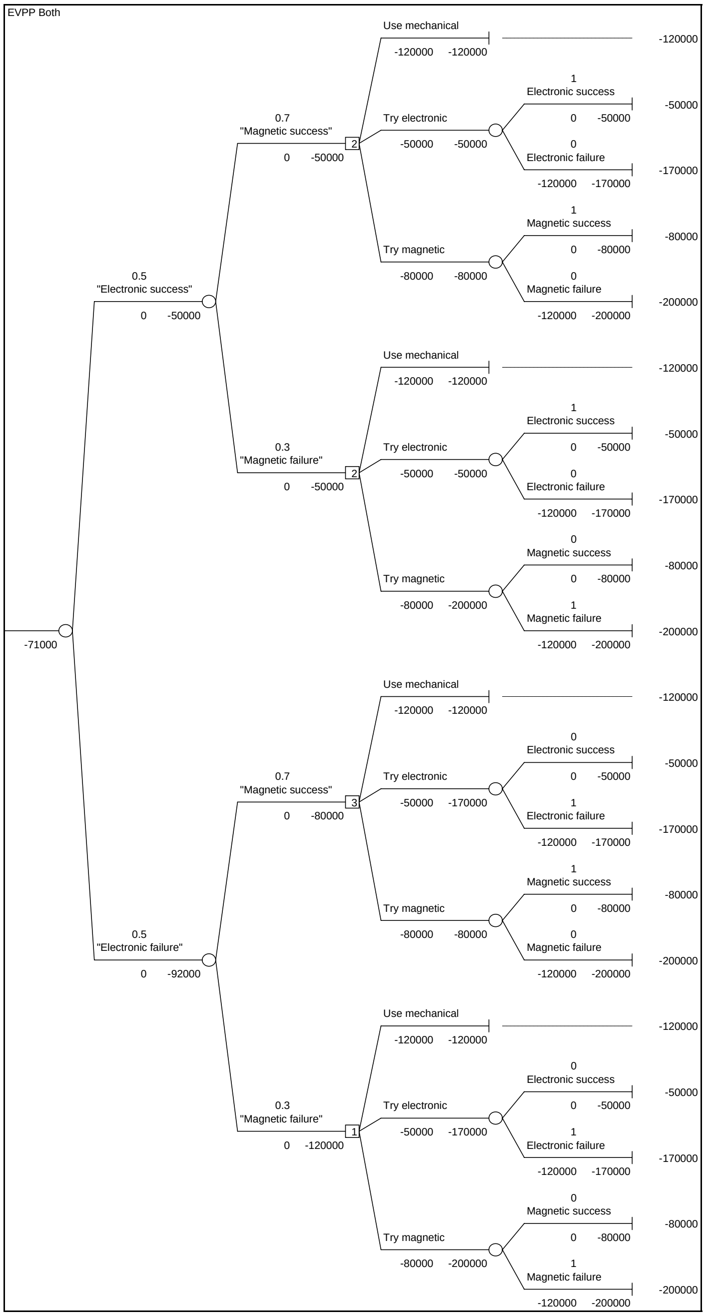


Figure 16.6 EVPP Both



16.4 SENSITIVITY ANALYSIS VS EVPI

Working Paper Title: Do Sensitivity Analyses Really Capture Problem Sensitivity? An Empirical Analysis Based on Information Value

Authors: James C. Felli, Naval Postgraduate School and Gordon B. Hazen, Northwestern University

Date: March 1998

The most common methods of sensitivity analysis (SA) in decision-analytic modeling are based either on proximity in parameter-space to decision thresholds or on the range of payoffs that accompany parameter variation. As an alternative, we propose the use of the expected value of perfect information (EVPI) as a sensitivity measure and argue from first principles that it is the proper measure of decision sensitivity. EVPI has significant advantages over conventional SA, especially in the multiparametric case, where graphical SA breaks down. In realistically sized problems, simple one- and two-way SAs may not fully capture parameter interactions, raising the disturbing possibility that many published decision analyses might be overconfident in their policy recommendations. To investigate the extent of this potential problem, we re-examined 25 decision analyses drawn from the published literature and calculated EVPI values for parameters on which sensitivity analyses had been performed, as well as the entire set of problem parameters. While we expected EVPI values to indicate greater problem sensitivity than conventional SA due to revealed parameter interaction, we in fact found the opposite: compared to EVPI, the one- and two-parameter SAs accompanying these problems dramatically overestimated problem sensitivity to input parameters. This phenomenon can be explained by invoking the flat maxima principle enunciated by von Winterfeldt and Edwards.

http://www.mccombs.utexas.edu/faculty/jim.dyer/DA_WP/WP980019.pdf

Value of Imperfect Information

17

17.1 TECHNOMETRICS PROBLEM

Prior Problem

Technometrics, Inc., a large producer of electronic components, is having some problems with the manufacturing process for a particular component. Under its current production process, 25 percent of the units are defective. The profit contribution of this component is $40 per unit. Under the contract the company has with its customers, Technometrics refunds $60 for each component that the customer finds to be defective; the customers then repair the component to make it usable in their applications. Before shipping the components to customers, Technometrics could spend an additional $30 per component to rework any components thought to be defective (regardless of whether the part is really defective). The reworked components can be sold at the regular price and will definitely not be defective in the customers' applications. Unfortunately, Technometrics cannot tell ahead of time which components will fail to work in their customers' applications. The following payoff table shows Technometrics' net cash flow per component.

Figure 17.1 Payoff Table

Component	Technometrics' Choice	
Condition	Ship as is	Rework first
Good	+$40	+$10
Defective	-$20	+$10

What should Technometrics do?

How much should Technometrics be willing to pay for a test that could evaluate the condition of the component before making the decision to ship as is or rework first?

Imperfect Information

An engineer at Technometrics has developed a simple test device to evaluate the component before shipping. For each component, the test device registers positive, inconclusive, or negative. The test is not perfect, but it is consistent for a particular component; that is, the test yields the same result for a given component regardless of how many times it is tested. To calibrate the test device, it was run on a batch of known good components and on a batch of know defective components. The results in the table below, based on relative frequencies, show the probability of a test device result, conditional on the true condition of the component.

Figure 17.2 Likelihoods

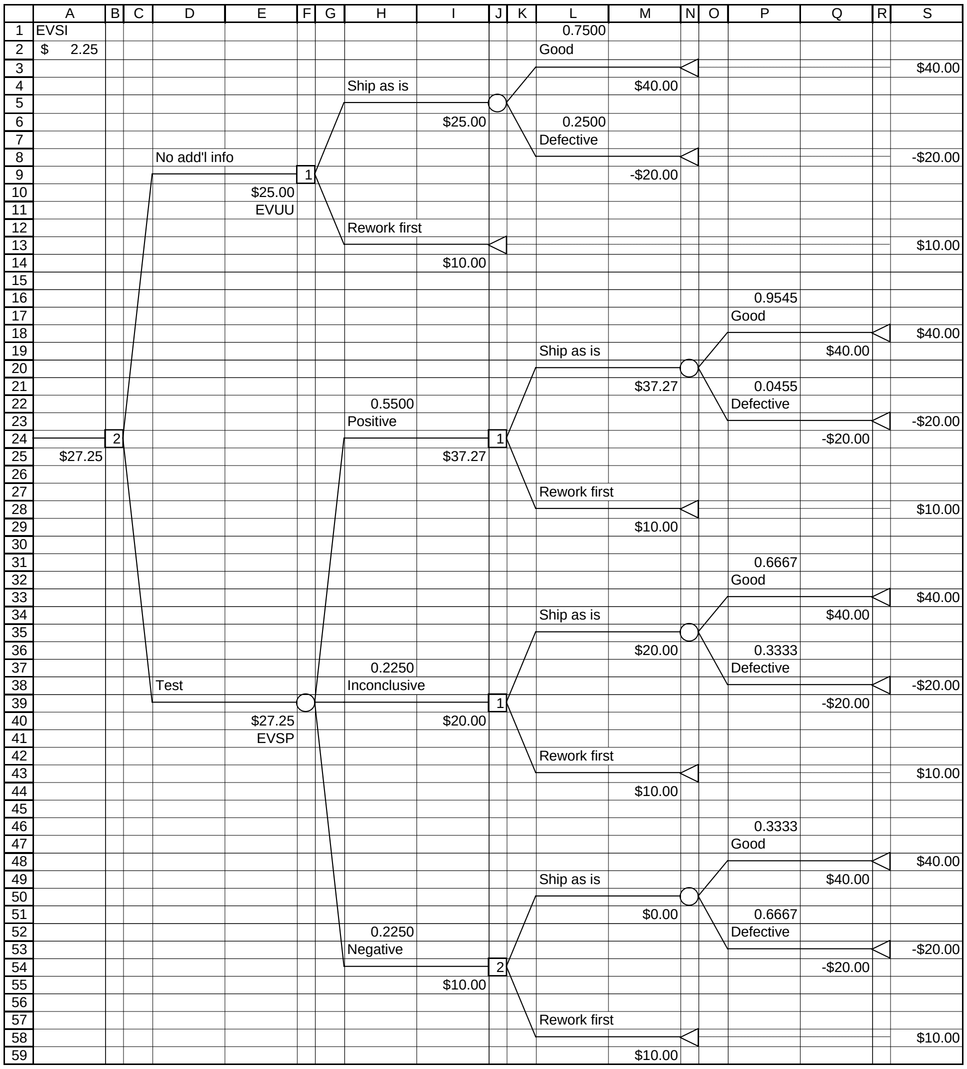
	Component Condition	
Test Result	Good	Defective
Positive	0.70	0.10
Inconclusive	0.20	0.30
Negative	0.10	0.60

For example, of the known defective components tested, sixty percent had a negative test result.

An analyst at Technometrics suggested using Bayesian revision of probabilities to combine the assessments about the reliability of the test device (shown above) with the original assessment of the components' condition (25 percent defectives).

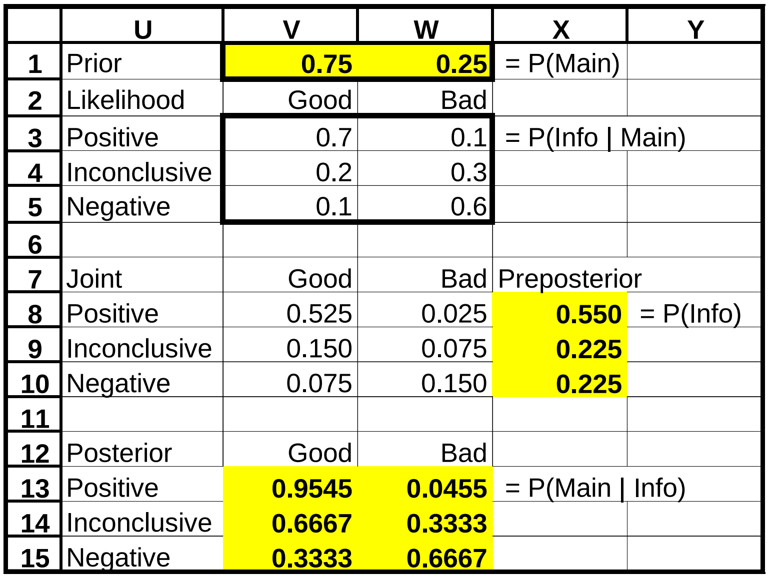
Technometrics uses expected monetary value for making decisions under uncertainty. What is the maximum (per component) the company should be willing to pay for using the test device?

Figure 17.3 Decision Tree Model



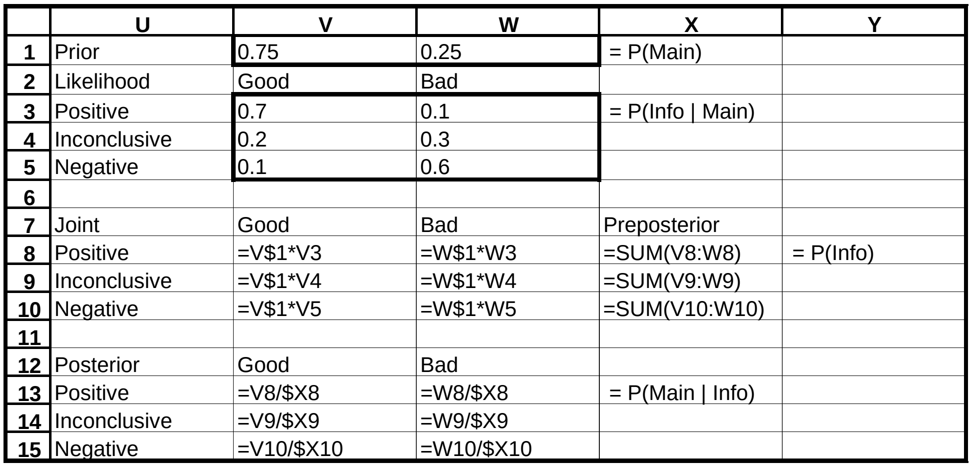
Revision of Probability

Figure 17.4 Display



	U	V	W	X	Y
1	Prior	**0.75**	**0.25**	= P(Main)	
2	Likelihood	Good	Bad		
3	Positive	0.7	0.1	= P(Info | Main)	
4	Inconclusive	0.2	0.3		
5	Negative	0.1	0.6		
6					
7	Joint	Good	Bad	Preposterior	
8	Positive	0.525	0.025	**0.550**	= P(Info)
9	Inconclusive	0.150	0.075	**0.225**	
10	Negative	0.075	0.150	**0.225**	
11					
12	Posterior	Good	Bad		
13	Positive	**0.9545**	**0.0455**	= P(Main | Info)	
14	Inconclusive	**0.6667**	**0.3333**		
15	Negative	**0.3333**	**0.6667**		

Figure 17.5 Formulas



	U	V	W	X	Y
1	Prior	0.75	0.25	= P(Main)	
2	Likelihood	Good	Bad		
3	Positive	0.7	0.1	= P(Info | Main)	
4	Inconclusive	0.2	0.3		
5	Negative	0.1	0.6		
6					
7	Joint	Good	Bad	Preposterior	
8	Positive	=V$1*V3	=W$1*W3	=SUM(V8:W8)	= P(Info)
9	Inconclusive	=V$1*V4	=W$1*W4	=SUM(V9:W9)	
10	Negative	=V$1*V5	=W$1*W5	=SUM(V10:W10)	
11					
12	Posterior	Good	Bad		
13	Positive	=V8/$X8	=W8/$X8	= P(Main | Info)	
14	Inconclusive	=V9/$X9	=W9/$X9		
15	Negative	=V10/$X10	=W10/$X10		

Modeling Attitude Toward Risk

18

18.1 RISK UTILITY FUNCTION

A certainty equivalent is a certain payoff value which is equivalent, for the decision maker, to a particular payoff distribution. If the decision maker can determine his or her certainty equivalent for the payoff distribution of each strategy in a decision problem, then the optimal strategy is the one with the highest certainty equivalent.

The certainty equivalent, i.e., the minimum selling price for a payoff distribution, depends on the decision maker's personal attitude toward risk. A decision maker may be risk preferring, risk neutral, or risk avoiding.

If the terminal values are not regarded as extreme relative to the decision maker's total assets, if the decision maker will encounter other decision problems with similar payoffs, and if the decision maker has the attitude that he or she will "win some and lose some," then the decision maker's attitude toward risk may be described as risk neutral.

If the decision maker is risk neutral, the certainty equivalent of a payoff distribution is equal to its expected value. The expected value of a payoff distribution is calculated by multiplying each terminal value by its probability and summing the products.

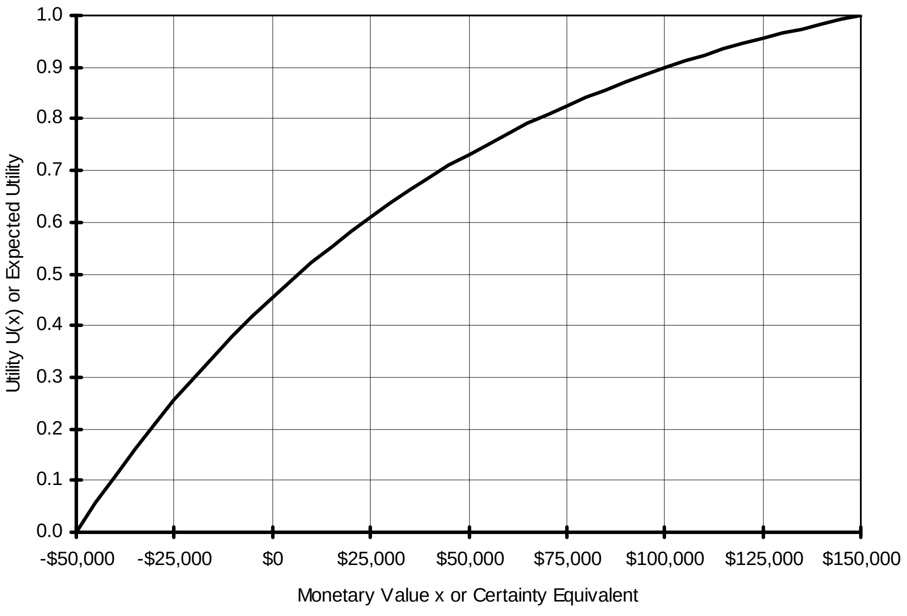
If the terminal values in a decision situation are extreme or if the situation is "one-of-a-kind" so that the outcome has major implications for the decision maker, an expected value analysis may not be appropriate. Such situations may require explicit consideration of risk.

Unfortunately, it can be difficult to determine one's certainty equivalent for a complex payoff distribution. We can aid the decision maker by first determining his or her certainty equivalent for a simple payoff distribution and then using that information to infer the certainty equivalent for more complex payoff distributions.

A utility function, U(x), can be used to represent a decision maker's attitude toward risk. The values or certainty equivalents, x, are plotted on the horizontal axis; utilities or expected utilities, u or U(x), are on the vertical axis. You can use the plot of the function by finding a value on the horizontal axis, scanning up to the plotted curve, and looking left to the vertical axis to determine the utility.

A typical risk utility function might have the general shape shown below if you draw a smooth curve approximately through the points.

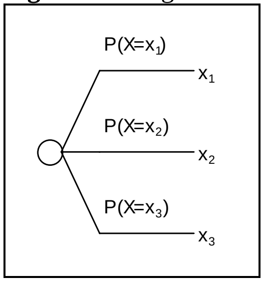
Figure 18.1 Typical Risk Utility Function



Since more value generally means more utility, the utility function is monotonically non-decreasing, and its inverse is well-defined. On the plot of the utility function, you locate a utility on the vertical axis, scan right to the plotted curve, and look down to read the corresponding value.

The concept of a payoff distribution, risk profile, gamble, or lottery is important for discussing utility functions. A payoff distribution is a set of payoffs, e.g., x_1, x_2, and x_3, with corresponding probabilities, $P(X=x_1)$, $P(X=x_2)$, and $P(X=x_3)$. For example, a payoff distribution may be represented in decision tree form as shown below.

Figure 18.2 Figure 2 Payoff Distribution Probability Tree



The fundamental property of a utility function is that the utility of the certainty equivalent CE of a payoff distribution is equal to the expected utility of the payoffs, i.e,

$$U(CE) = P(X=x_1)*U(x_1) + P(X=x_2)*U(x_2) + P(X=x_3)*U(x_3).$$

It follows that if you compute the expected utility (EU) of a lottery,

$$EU = P(X=x_1)*U(x_1) + P(X=x_2)*U(x_2) + P(X=x_3)*U(x_3),$$

the certainty equivalent of the payoff distribution can be determined using the inverse of the utility function. That is, you locate the expected utility on the vertical axis, scan right to the plotted curve, and look down to read the corresponding certainty equivalent.

If a utility function has been determined, you can use this fundamental property to determine the certainty equivalent of any payoff distribution. Calculations for the Magnetic strategy in the DriveTek problem are shown below. First, using a plot of the utility function, locate each payoff x on the horizontal axis and determine the corresponding utility U(x) on the vertical axis. Second, compute the expected utility EU of the lottery by multiplying each utility by its probability and summing the products. Third, locate the expected utility on the vertical axis and determine the corresponding certainty equivalent CE on the horizontal axis.

Figure 18.3 Calculations Using Risk Utility Function

P(X=x)	x	U(x)	P(X=x)*U(x)	
0.50	-$50,000	0.00	0.0000	
0.15	$0	0.45	0.0675	
0.35	$120,000	0.95	0.3325	
			0.4000	EU
			-$8,000	CE

18.2 EXPONENTIAL RISK UTILITY

Instead of using a plot of a utility function, an exponential function may be used to represent risk attitude. The general form of the exponential utility function is

$$U(x) = A - B*EXP(-x/RT).$$

The risk tolerance parameter RT determines the curvature of the utility function reflecting the decision maker’s attitude toward risk. Subsequent sections cover three methods for determining RT.

EXP is Excel's standard exponential function, i.e., EXP(z) represents the value e raised to the power of z, where e is the base of the natural logarithms.

The parameters A and B determine scaling. After RT is determined, if you want to plot a utility function so that U(High) = 1.0 and U(Low) = 0.0, you can use the following formulas to determine the scaling parameters A and B.

$$A = EXP\ (-Low/RT) / [EXP\ (-Low/RT) - EXP\ (-High/RT)]$$

$$B = 1 / [EXP\ (-Low/RT) - EXP\ (-High/RT)]$$

The inverse function for finding the certainty equivalent CE corresponding to an expected utility EU is

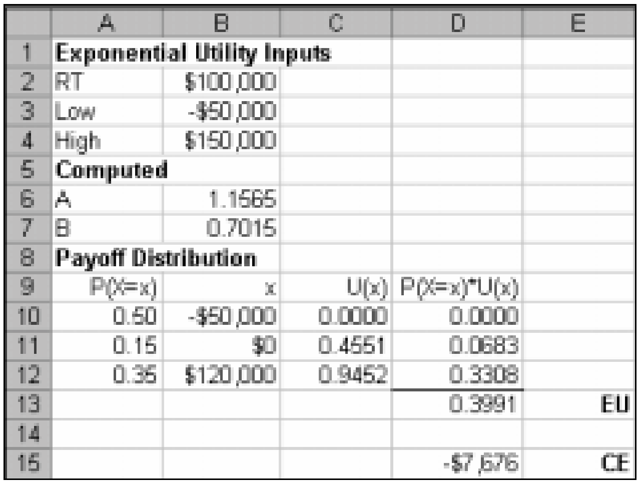
$$CE = -RT*LN[(A-EU)/B],$$

where LN(y) represents the natural logarithm of y.

After the parameters A, B, and RT have been determined, the exponential utility function and its inverse can be used to determine the certainty equivalent for any lottery.

Calculations for the Magnetic strategy in the DriveTek problem are shown in Figure 4.

Figure 18.4 Exponential Risk Utility Results

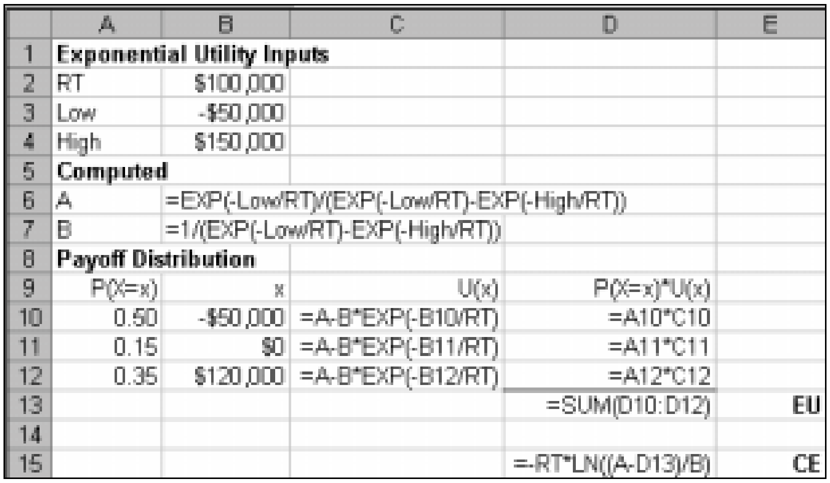


	A	B	C	D	E
1	**Exponential Utility Inputs**				
2	RT	$100,000			
3	Low	-$50,000			
4	High	$150,000			
5	**Computed**				
6	A	1.1565			
7	B	0.7015			
8	**Payoff Distribution**				
9	P(X=x)	x	U(x)	P(X=x)*U(x)	
10	0.50	-$50,000	0.0000	0.0000	
11	0.15	$0	0.4551	0.0683	
12	0.35	$120,000	0.9452	0.3308	
13				0.3991	EU
14					
15				-$7,676	CE

Computed values are displayed with four decimal places, but Excel's 15-digit precision is used in all calculations. For a decision maker with a risk tolerance parameter of $100,000, the payoff distribution for the Magnetic strategy has a certainty equivalent of -$7,676. That is, if the decision maker is facing the payoff distribution shown in A9:B12 in Figure 4, he or she would be willing to pay $7,676 to be relieved of the obligation.

Formulas are shown in Figure 5. To construct the worksheet, enter the text in column A and the monetary values in column B. To define names, select A2:B4, and choose Insert | Name | Create. Similarly, select A6:B7, and choose Insert | Name | Create. Then enter the formulas in B6:B7. Enter formulas in C10 and D10, and copy down. Finally, enter the EU formula in D13 and the CE formula in D15. The defined names are absolute references by default.

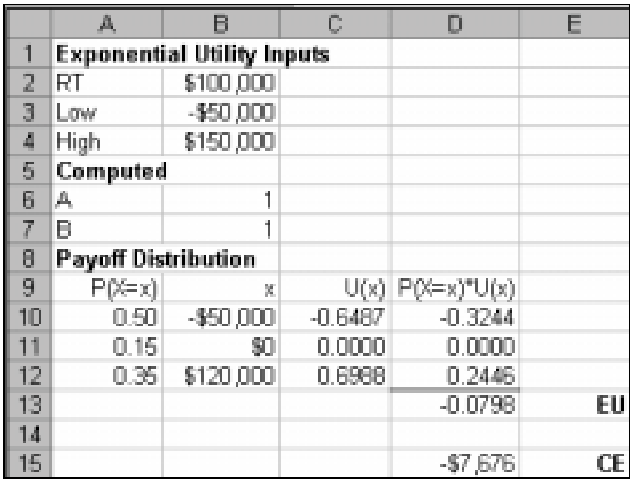
Figure 18.5 Exponential Risk Utility Formulas



	A	B	C	D	E
1	**Exponential Utility Inputs**				
2	RT	$100,000			
3	Low	-$50,000			
4	High	$150,000			
5	**Computed**				
6	A	=EXP(-Low/RT)/(EXP(-Low/RT)-EXP(-High/RT))			
7	B	=1/(EXP(-Low/RT)-EXP(-High/RT))			
8	**Payoff Distribution**				
9	P(X=x)	x	U(x)	P(X=x)*U(x)	
10	0.50	-$50,000	=A-B*EXP(-B10/RT)	=A10*C10	
11	0.15	$0	=A-B*EXP(-B11/RT)	=A11*C11	
12	0.35	$120,000	=A-B*EXP(-B12/RT)	=A12*C12	
13				=SUM(D10:D12)	**EU**
14					
15				=-RT*LN((A-D13)/B)	**CE**

Figure 6 shows results for the same payoff distribution using a simplified form of the exponential risk utility function with A = 1 and B = 1. This function could be represented as U(x) = 1–EXP(–x/RT) with inverse CE = –RT*LN(1–EU). The utility and expected utility calculations are different, but the certainty equivalent is the same.

Figure 18.6 Simplified Exponential Risk Utility Results

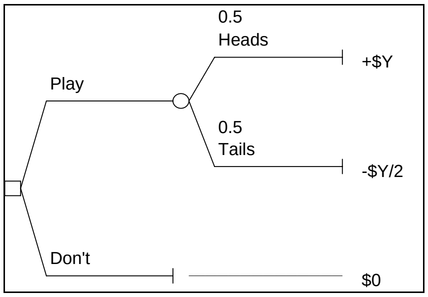


	A	B	C	D	E
1	**Exponential Utility Inputs**				
2	RT	$100,000			
3	Low	-$50,000			
4	High	$150,000			
5	**Computed**				
6	A	1			
7	B	1			
8	**Payoff Distribution**				
9	P(X=x)	x	U(x)	P(X=x)*U(x)	
10	0.50	-$50,000	-0.6487	-0.3244	
11	0.15	$0	0.0000	0.0000	
12	0.35	$120,000	0.6988	0.2446	
13				-0.0798	**EU**
14					
15				-$7,676	**CE**

18.3 APPROXIMATE RISK TOLERANCE

The value of the risk tolerance parameter RT is approximately equal to the maximum value of Y for which the decision maker is willing to accept a payoff distribution with equally-likely payoffs of $Y and −$Y/2 instead of accepting $0 for certain.

Figure 18.7 Approximate Risk Tolerance



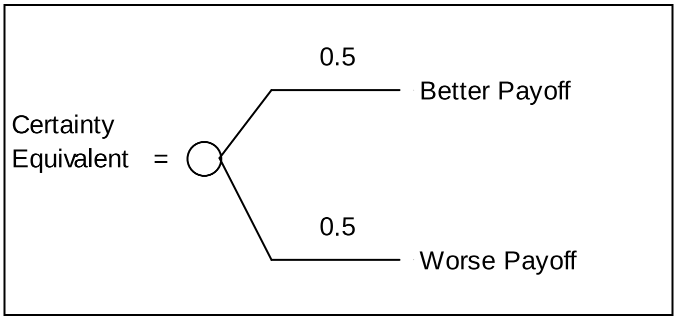
For example, in a personal decision, you may be willing to play the game shown in Figure 7 with equally-likely payoffs of $100 and –$50, but you might not play with payoffs of $100,000 and –$50,000. As the better payoff increases from $100 to $100,000 (and the corresponding worse payoff increases from –$50 to –$50,000), you reach a value where you are indifferent between playing the game and receiving $0 for certain. At that point, the value of the better payoff is an approximation of RT for an exponential risk utility function describing your risk attitude.

In a business decision for a small company, the company may be willing to play the game with payoffs of $200,000 and –$100,000 but not with payoffs of $20,000,000 and -$10,000,000. Somewhere between a better payoff of $200,000 and $20,000,000, the company would be indifferent between playing the game and not playing, thereby determining the approximate RT for their business decision.

18.4 EXACT RISK TOLERANCE USING EXCEL

A simple payoff distribution, called a risk attitude assessment lottery, may be used to determine the decision maker's attitude toward risk. This lottery has equal probability of obtaining each of the two payoffs. It is good practice to use a better payoff at least as large as the highest payoff in the decision problem and a worse payoff as small as or smaller than the lowest payoff. In any case, the payoffs should be far enough apart that the decision maker perceives a definite difference in the two outcomes. Three values must be specified for the fifty-fifty lottery: the Better payoff, the Worse payoff, and the Certainty Equivalent, as shown in Figure 8.

Figure 18.8 Risk Attitude Assessment Lottery



According to the fundamental property of a risk utility function, the utility of the certainty equivalent equals the expected utility of the lottery, so the three values are related as follows.

U(CertEquiv) = 0.5∗U(BetterPayoff) + 0.5∗U(WorsePayoff)

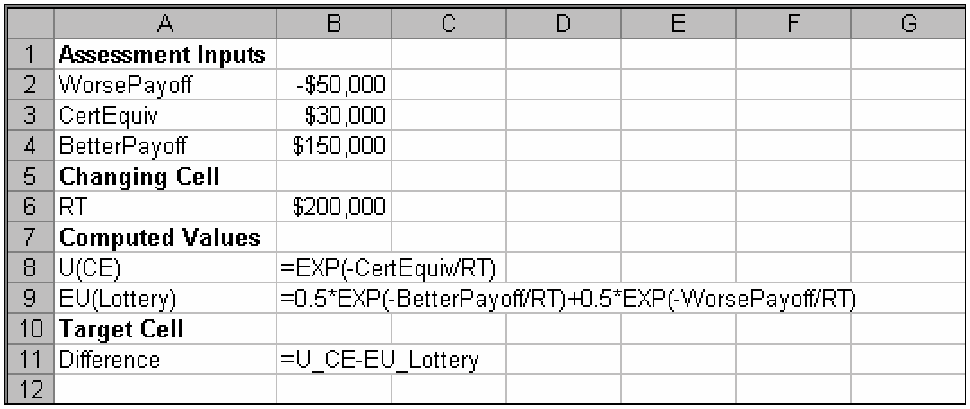
If you use the general form for an exponential utility function with parameters A, B, and RT, and if you simplify terms, it follows that RT must satisfy the following equation.

Exp(–CertEquiv/RT) = 0.5∗Exp(–BetterPayoff/RT) + 0.5∗Exp(–WorsePayoff/RT)

Given the values for CE, Better, and Worse, you could use trial-and-error to find the value of RT that exactly satisfies the equation. In Excel you can use Goal Seek or Solver by creating a worksheet like Figure 9.

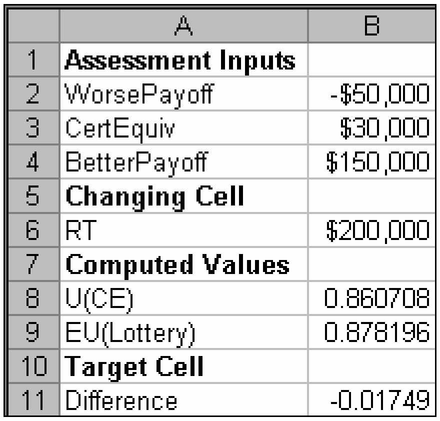
Enter the text in column A. Enter the assessment lottery values in B2:B4. Enter a tentative RT value in B6. Select A2:B4, and use Insert | Name | Create; repeat for A6:B6 and A8:B9. Note that the parentheses symbol is not allowed in a defined name, so Excel changes U(CE) to U_CE and EU(Lottery) to EU_Lottery.

Figure 18.9 Formulas for Risk Tolerance Search



	A	B	C	D	E	F	G
1	**Assessment Inputs**						
2	WorsePayoff	-$50,000					
3	CertEquiv	$30,000					
4	BetterPayoff	$150,000					
5	**Changing Cell**						
6	RT	$200,000					
7	**Computed Values**						
8	U(CE)	=EXP(-CertEquiv/RT)					
9	EU(Lottery)	=0.5*EXP(-BetterPayoff/RT)+0.5*EXP(-WorsePayoff/RT)					
10	**Target Cell**						
11	Difference	=U_CE-EU_Lottery					
12							

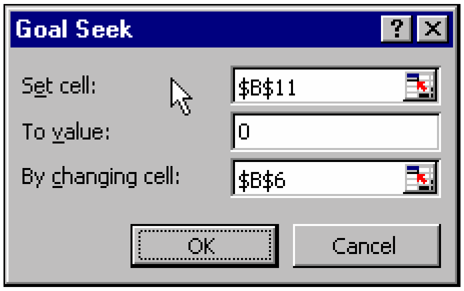
Figure 18.10 Tentative Values for Risk Tolerance Search



	A	B
1	**Assessment Inputs**	
2	WorsePayoff	-$50,000
3	CertEquiv	$30,000
4	BetterPayoff	$150,000
5	**Changing Cell**	
6	RT	$200,000
7	**Computed Values**	
8	U(CE)	0.860708
9	EU(Lottery)	0.878196
10	**Target Cell**	
11	Difference	-0.01749

Figure 10 shows tentative values for the search. From the Tools menu, choose Goal Seek. In the Goal Seek dialog box, enter B11, 0, and B6. If you point to cells, the reference appears in the edit box as an absolute reference, as shown in Figure 11. Click OK.

Figure 18.11 Goal Seek Dialog Box



Goal Seek

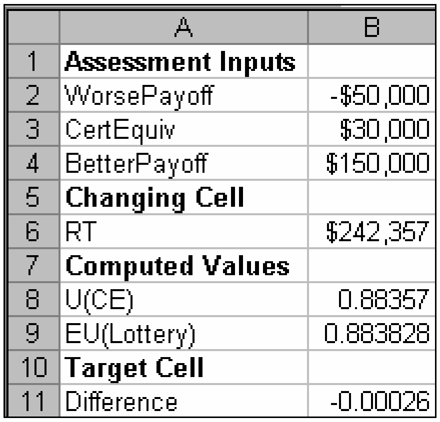
Set cell: $B$11

To value: 0

By changing cell: $B$6

The Goal Seek Status dialog box shows that a solution has been found. Click OK. The worksheet appears as shown in Figure 12.

Figure 18.12 Results of Goal Seek Search



	A	B
1	**Assessment Inputs**	
2	WorsePayoff	-$50,000
3	CertEquiv	$30,000
4	BetterPayoff	$150,000
5	**Changing Cell**	
6	RT	$242,357
7	**Computed Values**	
8	U(CE)	0.88357
9	EU(Lottery)	0.883828
10	**Target Cell**	
11	Difference	-0.00026

The difference between U(CE) and EU(Lottery) is not exactly zero. If you start at $250,000, the Goal Seek converges to a difference of –6.2E–05 or 0.000062, which is closer to zero, resulting in a RT of $243,041.

If extra precision is needed, use Solver. With Solver's default settings, the difference is 2.39E–08 with RT equal to $243,261. If you change the precision from 0.000001 to 0.00000001 or an even smaller value in Solver's Options, the difference will be even closer to zero.

18.5 EXACT RISK TOLERANCE USING RISKTOL.XLA

The Goal Seek and Solver methods for determining the risk tolerance parameter RT yield static results. For a dynamic result, use the risktol.xla add-in function. A major advantage of risktol.xla is that it facilitates sensitivity analysis. Whenever an input to the function changes, the result is recalculated. The function syntax is

RISKTOL(*WorsePayoff*,*CertEquiv*,*BetterPayoff*,*BetterProb*).

When you open the risktol.xla file, the function is added to the Math & Trig function category list.

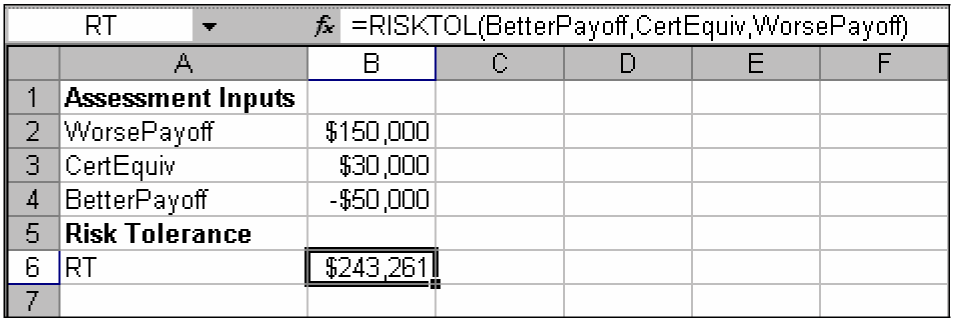
The function returns a very precise value of the risk tolerance parameter for an exponential utility function. The result is consistent with CertEquiv as the decision maker's certainty equivalent for a two-payoff assessment lottery with payoffs WorsePayoff and BetterPayoff, with probability BetterProb of obtaining BetterPayoff and probability 1 – BetterProb of obtaining WorsePayoff.

In case of an error, the RISKTOL function returns:

> #N/A if there are too few or too many arguments. The first three arguments (WorsePayoff, CertEquiv, and BetterPayoff) are required; the fourth argument (BetterProb) is optional, with default value 0.5.
>
> #VALUE! if WorsePayoff >= CertEquiv, or CertEquiv >= Better Payoff, or BetterProb (if specified) <= 0 or >= 1.
>
> #NUM! if the search procedure fails to converge.

In Figure 13, the text in cells A2:A4 has been used as defined names for cells B2:B4, and the text in cell A6 is the defined name for cell B6, as shown in the name box. After opening the risktol.xla file, enter the function name and arguments, as shown in the formula bar. If one of the three inputs change, the result in cell B6 is recalculated.

Figure 18.13 Exact Risk Tolerance Using RiskTol.xla



=RISKTOL(BetterPayoff,CertEquiv,WorsePayoff)

	A	B	C	D	E	F
1	**Assessment Inputs**					
2	WorsePayoff	$150,000				
3	CertEquiv	$30,000				
4	BetterPayoff	-$50,000				
5	**Risk Tolerance**					
6	RT	$243,261				
7						

18.6 EXPONENTIAL UTILITY AND TREEPLAN

TreePlan's default is to rollback the tree using expected values. If you choose to use exponential utilities in TreePlan's Options dialog box, TreePlan will redraw the decision tree diagram with formulas for computing the utility and certainty equivalent at each node. For the Maximize option, the rollback formulas are U = A–B*EXP(–X/RT) and CE = -LN((A-EU)/B)*RT, where X and EU are cell references. For the Minimize option, the formulas are U = A-B*EXP(X/RT) and CE = LN((A-EU)/B)*RT.

TreePlan uses the name RT to represent the risk tolerance parameter of the exponential utility function. The names A and B determine scaling. If the names A, B, and RT don't exist on the worksheet when you choose to use exponential utility, they are initially defined as A=1, B=1, and RT=999999999999. You can redefine the names using the Insert | Name | Define or Insert | Name | Create commands.

To plot the utility curve, enter a list of X values in a column on the left, and enter the formula =A−B*EXP(−X/RT) in a column on the right, where X is a reference to the corresponding cell on the left. Select the values in both columns, and use the ChartWizard to develop an XY (Scatter) chart.

If RT is specified using approximate risk tolerance values, you can perform sensitivity analysis by (1) using the defined name RT for a cell, (2) constructing a data table with a list of possible RT values and an appropriate output formula (usually a choice indicator at a decision node or a certainty equivalent), and (3) specifying the RT cell as the input cell in the Data Table dialog box.

18.7 EXPONENTIAL UTILITY AND RISKSIM

After using RiskSim to obtain model output results, select the column containing the Sorted Data, copy to the clipboard, select a new sheet, and paste. Alternatively, you can use the unsorted values, and you can also do the following calculations on the original sheet containing the model results. This example uses only ten iterations; 500 or 1,000 iterations are more appropriate.

Use one of the methods described previously to specify values of RT, A, and B. Since the model output values shown in Figures 14 and 15 range from approximately $14,000 to $176,000, the utility function is defined for a range from worse payoff $0 to better payoff $200,000. RT was determined using risktol.xla with a risk-seeking certainty equivalent of $110,000.

To obtain the utility of each model output value in cells A2:A11, select cell B2, and enter the formula =A−B*EXP(−A2/RT). Select cell B2, click the fill handle in the lower right corner of the cell and drag down to cell B11. Enter the formulas in cells A13:C13 and the labels in row 14.

Figure 18.14 Risk Utility Formulas for RiskSim

	A	B	C
1	Sorted Data	Utility	
2	14229.56	=A-B*EXP(-A2/RT)	
3	32091.92	=A-B*EXP(-A3/RT)	
4	51091.48	=A-B*EXP(-A4/RT)	
5	66383.79	=A-B*EXP(-A5/RT)	
6	69433.32	=A-B*EXP(-A6/RT)	
7	87322.23	=A-B*EXP(-A7/RT)	
8	95920.93	=A-B*EXP(-A8/RT)	
9	135730.71	=A-B*EXP(-A9/RT)	
10	154089.36	=A-B*EXP(-A10/RT)	
11	175708.87	=A-B*EXP(-A11/RT)	
12			
13	=AVERAGE(A2:A11)	=AVERAGE(B2:B11)	=-LN((A-B13)/B)*RT
14	Exp. Value	Exp.Util.	CE

Figure 18.15 Risk Utility Results for RiskSim

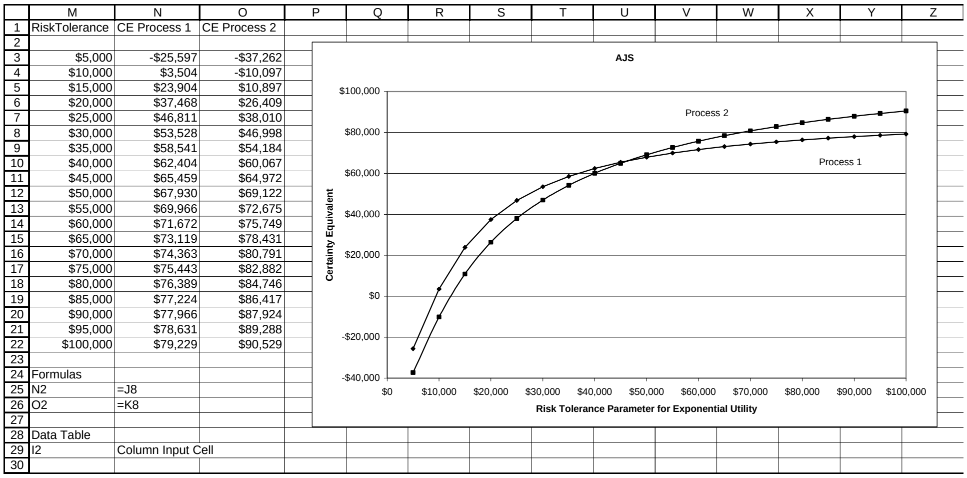
	A	B	C
1	Sorted Data	Utility	
2	$ 14,230	0.05862	
3	$ 32,092	0.13462	
4	$ 51,091	0.21851	
5	$ 66,384	0.28841	
6	$ 69,433	0.30260	
7	$ 87,322	0.38767	
8	$ 95,921	0.42966	
9	$ 135,731	0.63382	
10	$ 154,089	0.73363	
11	$ 175,709	0.85600	
12			
13	$ 88,200	0.40435	$ 90,757
14	Exp. Value	Exp.Util.	CE

18.8 RISK SENSITIVITY FOR MACHINE PROBLEM

Figure 18.16

	A	B	C	D	E	F	G	H	I	J	K	L
1	Process 1	NPV	Utility		Process 2	NPV	Utility		RT		AJS, Clemen2	
2		$107,733	0.102133			$86,161	0.082554		$1,000,000		pp. 428-430	
3		$39,389	0.038623			$58,417	0.056744					
4		$125,210	0.117689			$171,058	0.157228			Process 1	Process 2	
5		$66,032	0.063899			$263,843	0.231906					
6		$32,504	0.031982			$37,180	0.036498		ExpUtility	0.085527	0.107258	
7		$138,132	0.129016			$254,027	0.224329					
8		$83,000	0.079649			$118,988	0.112181		CertEquiv	$89,407	$113,458	
9		$48,178	0.047036			$133,862	0.125289					
10		$20,130	0.019928			$26,597	0.026247		ExpValue	$90,526	$116,159	
11		$31,445	0.030956			$187,063	0.170608					
12		$19,739	0.019546			$88,060	0.084294					
13		$4,641	0.00463			$114,837	0.108489					
14		$92,368	0.08823			$130,638	0.122465		Goal Seek			
15		$102,585	0.097498			$138,882	0.12967					
16		$106,411	0.100945			$226,909	0.203006		CE2 - CE1	$24,050		
17		$110,528	0.104639			$156,102	0.144528					
18		$171,524	0.15762			$193,209	0.17569					
19		$87,698	0.083963			$92,004	0.087898					
20		$123,907	0.116538			$163,780	0.151071		NPV values from RiskSim Summary			
21		$69,783	0.067404			$22,176	0.021932		Cell I2 has defined name RT			
22		$144,052	0.134157			$135,190	0.12645		Formulas			
23		$131,461	0.123187			$61,013	0.059189		C2	=1-EXP(-B2/RT)		
24		$34,938	0.034335			$184,907	0.168819		Copy down to C1001			
25		$75,551	0.072768			$70,967	0.068507		G2	=1-EXP(-F2/RT)		
26		$32,144	0.031633			-$10,251	-0.010304		Copy down to G1001			
27		$61,719	0.059853			$89,645	0.085744		J6	=AVERAGE(C2:C1001)		
28		$139,568	0.130266			$119,405	0.112551		K6	=AVERAGE(G2:G1001)		
29		$89,107	0.085252			$96,670	0.092144		J8	=-RT*LN(1-J6)		
30		$94,158	0.089861			$114,124	0.107853		K8	=-RT*LN(1-K6)		
31		$81,459	0.07823			$208,778	0.188425		J10	=AVERAGE(B2:B1001)		
32		$139,258	0.129997			$24,580	0.02428		K10	=AVERAGE(F2:F1001)		
33		$58,190	0.056529			$155,958	0.144405		J16	=K8-J8		
34		-$13,104	-0.01319			$198,519	0.180056					
35		$36,529	0.035869			$167,568	0.154281					
36		$91,239	0.0872			$36,676	0.036011					
37		$147,155	0.13684			$225,777	0.202104					
38		$154,168	0.142872			$195,738	0.177773					
39		$180,770	0.165372			$53,467	0.052063					
40		$112,313	0.106235			$213,920	0.192587					

Figure 18.17



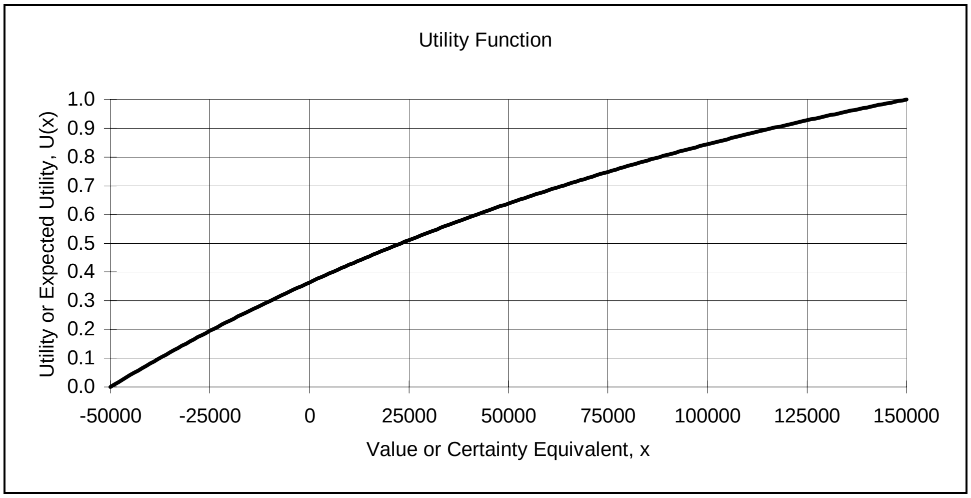
	M	N	O
1	RiskTolerance	CE Process 1	CE Process 2
2			
3	$5,000	-$25,597	-$37,262
4	$10,000	$3,504	-$10,097
5	$15,000	$23,904	$10,897
6	$20,000	$37,468	$26,409
7	$25,000	$46,811	$38,010
8	$30,000	$53,528	$46,998
9	$35,000	$58,541	$54,184
10	$40,000	$62,404	$60,067
11	$45,000	$65,459	$64,972
12	$50,000	$67,930	$69,122
13	$55,000	$69,966	$72,675
14	$60,000	$71,672	$75,749
15	$65,000	$73,119	$78,431
16	$70,000	$74,363	$80,791
17	$75,000	$75,443	$82,882
18	$80,000	$76,389	$84,746
19	$85,000	$77,224	$86,417
20	$90,000	$77,966	$87,924
21	$95,000	$78,631	$89,288
22	$100,000	$79,229	$90,529
23			
24	Formulas		
25	N2	=J8	
26	O2	=K8	
27			
28	Data Table		
29	I2	Column Input Cell	
30			

18.9 RISK UTILITY SUMMARY

Concepts

Strategy, Payoff Distribution, Certainty Equivalent

Figure 18.18 Utility Function



Fundamental Property of Utility Function

The utility of the CE of a lottery equals the expected utility of the lottery's payoffs.

U(CE) = EU = p1*U(x1) + p2*U(x2) + p3*U(x3)

Using a Utility Function To Find the CE of a Lottery

1. U(x): Locate each payoff on the horizontal axis and determine the corresponding utility on the vertical axis.
2. EU: Compute the expected utility of the lottery by multiplying each utility by its probability and summing the products.
3. CE: Locate the expected utility on the vertical axis and determine the corresponding certainty equivalent on the horizontal axis.

Exponential Utility Function

General form: U(x) = A – B*EXP(–x/RT)

Parameters A and B affect scaling.

Parameter RT (RiskTolerance) depends on risk attitude and affects curvature.

Inverse: CE = –RT*LN[(A–EU)/B]

TreePlan's Simple Form of Exponential Utility

Set A and B equal to 1.

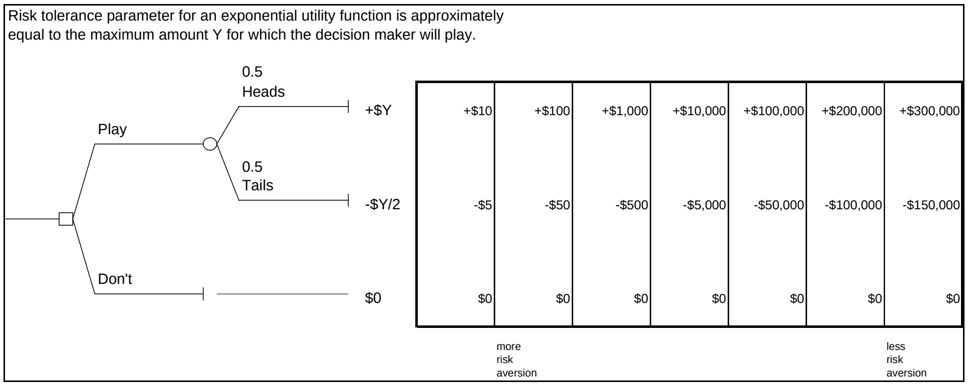
U(x) = 1 – EXP(–x/RT)

CE = –RT*LN(1–EU)

Approximate Assessment of RiskTolerance

Refer to the Clemen textbook, Figure 13.12, on page 478.

Figure 18.19 Assessing ApproximateRisk Tolerance



Risk tolerance parameter for an exponential utility function is approximately equal to the maximum amount Y for which the decision maker will play.

+$Y	+$10	+$100	+$1,000	+$10,000	+$100,000	+$200,000	+$300,000
-$Y/2	-$5	-$50	-$500	-$5,000	-$50,000	-$100,000	-$150,000
$0	$0	$0	$0	$0	$0	$0	$0

Exact Assessment of RiskTolerance

The RISKTOL.XLA Excel add-in file adds the following function to the Math & Trig function category list:

RISKTOL(WorsePayoff,CertEquiv,BetterPayoff,BetterProb)

The first three arguments are required, and the last argument is optional with default value 0.5. WorsePayoff and BetterPayoff are payoffs of an assessment lottery, and CertEquiv is the decision maker's certainty equivalent for the lottery.

RISKTOL returns #N/A if there are too few or too many arguments, #VALUE! if WorsePayoff >= CertEquiv, or CertEquiv >= Better Payoff, or BetterProb (if specified) <= 0 or >= 1, and #NUM! if the search procedure fails to converge.

For example, consider a 50-50 lottery with payoffs of $100,000 and $0. A decision maker has decided that the certainty equivalent is $43,000. If you open the RISKTOL.XLA file and type =RISKTOL(0,43000,100000) in a cell, the result is 176226. Thus, the value of the RiskTolerance parameter in an exponential utility function for this decision maker should be 176226.

Using Exponential Utility for TreePlan Rollback Values

1. Select a cell, and enter a value for the RiskTolerance parameter.
2. With the cell selected, choose Insert Name | Define, and enter RT.
3. From TreePlan's Options dialog box, select Use Exponential Utility. The new decision tree diagram includes the EXP and LN functions for determining U(x) and the inverse.

Using Exponential Utility for a Payoff Distribution

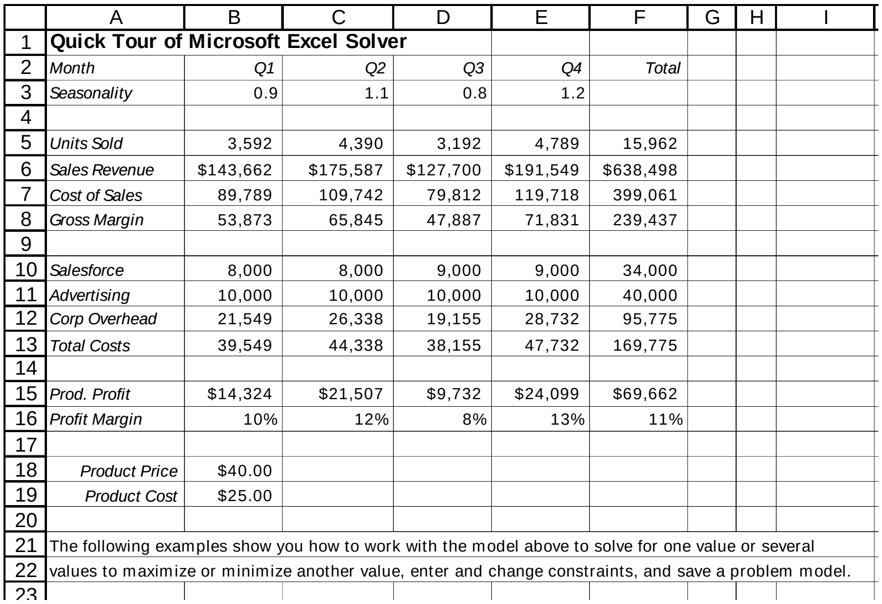
Enter the exponential utility function directly, using the appropriate value for RiskTolerance. If the payoff values are equally-likely, use the AVERAGE function to determine the expected utility; otherwise, use SUMPRODUCT. Enter the inverse function directly to obtain the certainty equivalent.

19

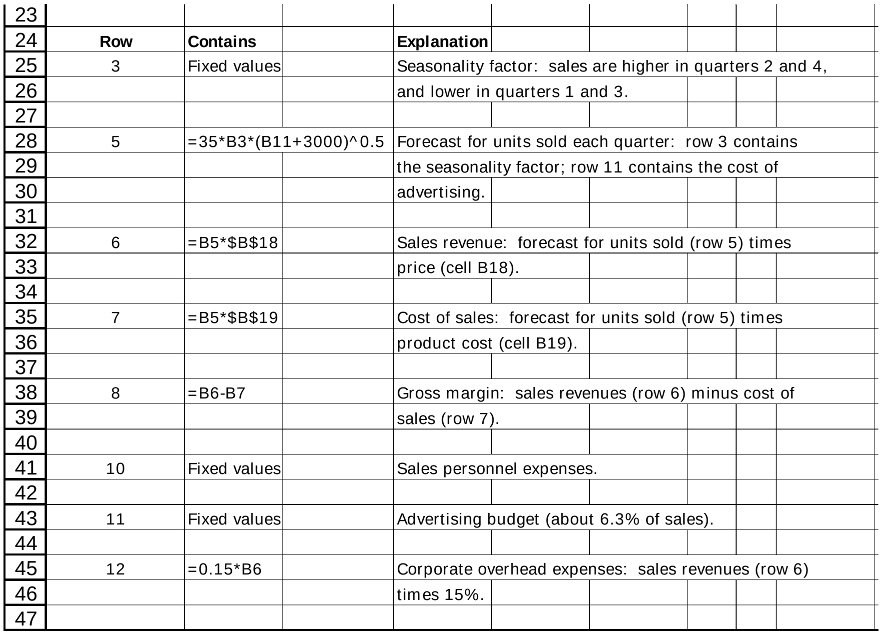
Modeling Marketing Decisions

19.1 ALLOCATING ADVERTISING EXPENDITURES

Figure 19.1 Quick Tour



	A	B	C	D	E	F	G	H	I
1	**Quick Tour of Microsoft Excel Solver**								
2	*Month*	*Q1*	*Q2*	*Q3*	*Q4*	*Total*			
3	*Seasonality*	0.9	1.1	0.8	1.2				
4									
5	*Units Sold*	3,592	4,390	3,192	4,789	15,962			
6	*Sales Revenue*	$143,662	$175,587	$127,700	$191,549	$638,498			
7	*Cost of Sales*	89,789	109,742	79,812	119,718	399,061			
8	*Gross Margin*	53,873	65,845	47,887	71,831	239,437			
9									
10	*Salesforce*	8,000	8,000	9,000	9,000	34,000			
11	*Advertising*	10,000	10,000	10,000	10,000	40,000			
12	*Corp Overhead*	21,549	26,338	19,155	28,732	95,775			
13	*Total Costs*	39,549	44,338	38,155	47,732	169,775			
14									
15	*Prod. Profit*	$14,324	$21,507	$9,732	$24,099	$69,662			
16	*Profit Margin*	10%	12%	8%	13%	11%			
17									
18	*Product Price*	$40.00							
19	*Product Cost*	$25.00							
20									
21	The following examples show you how to work with the model above to solve for one value or several								
22	values to maximize or minimize another value, enter and change constraints, and save a problem model.								
23									



23			
24	**Row**	**Contains**	**Explanation**
25	3	Fixed values	Seasonality factor: sales are higher in quarters 2 and 4,
26			and lower in quarters 1 and 3.
27			
28	5	=35*B3*(B11+3000)^0.5	Forecast for units sold each quarter: row 3 contains
29			the seasonality factor; row 11 contains the cost of
30			advertising.
31			
32	6	=B5*$B$18	Sales revenue: forecast for units sold (row 5) times
33			price (cell B18).
34			
35	7	=B5*$B$19	Cost of sales: forecast for units sold (row 5) times
36			product cost (cell B19).
37			
38	8	=B6-B7	Gross margin: sales revenues (row 6) minus cost of
39			sales (row 7).
40			
41	10	Fixed values	Sales personnel expenses.
42			
43	11	Fixed values	Advertising budget (about 6.3% of sales).
44			
45	12	=0.15*B6	Corporate overhead expenses: sales revenues (row 6)
46			times 15%.
47			

	A	B	C	D	E	F	G	H	I
48	13	=SUM(B10:B12)		Total costs: sales personnel expenses (row 10) plus					
49				advertising (row 11) plus overhead (row 12).					
50									
51	15	=B8-B13		Product profit: gross margin (row 8) minus total costs					
52				(row 13).					
53									
54	16	=B15/B6		Profit margin: profit (row 15) divided by sales revenue					
55				(row 6).					
56									
57	18	Fixed values		Product price.					
58									
59	19	Fixed values		Product cost.					
60									
61	This is a typical marketing model that shows sales rising from a base figure (perhaps due to the sales								
62	personnel) along with increases in advertising, but with diminishing returns. For example, the first								
63	$5,000 of advertising in Q1 yields about 1,092 incremental units sold, but the next $5,000 yields only								
64	about 775 units more.								
65									
66	You can use Solver to find out whether the advertising budget is too low, and whether advertising								
67	should be allocated differently over time to take advantage of the changing seasonality factor.								
68									
69	**Solving for a Value to Maximize Another Value**								
70	One way you can use Solver is to determine the maximum value of a cell by changing another cell. The								
71	two cells must be related through the formulas on the worksheet. If they are not, changing the value in								
72	one cell will not change the value in the other cell.								
73									
74	For example, in the sample worksheet, you want to know how much you need to spend on advertising								
75	to generate the maximum profit for the first quarter. You are interested in maximizing profit by changing								
76	advertising expenditures.								
77									
78	▪	On the **Tools** menu, click **Solver**. In the **Set target cell** box, type **b15** or							
79		select cell B15 (first-quarter profits) on the worksheet. Select the **Max** option.							
80		In the **By changing cells** box, type **b11** or select cell B11 (first-quarter advertising)							
81		on the worksheet. Click **Solve**.							
82									
83	You will see messages in the status bar as the problem is set up and Solver starts working. After a								
84	moment, you'll see a message that Solver has found a solution. Solver finds that Q1 advertising of								
85	$17,093 yields the maximum profit $15,093.								
86									
87	▪	After you examine the results, select **Restore original values** and click **OK** to							
88		discard the results and return cell B11 to its former value.							
89									
90	**Resetting the Solver Options**								
91									
92	If you want to return the options in the **Solver Parameters** dialog box to their original settings so that								
93	you can start a new problem, you can click **Reset All**.								
94									

Solving for a Value by Changing Several Values

You can also use Solver to solve for several values at once to maximize or minimize another value. For example, you can solve for the advertising budget for each quarter that will result in the best profits for the entire year. Because the seasonality factor in row 3 enters into the calculation of unit sales in row 5 as a multiplier, it seems logical that you should spend more of your advertising budget in Q4 when the sales response is highest, and less in Q3 when the sales response is lowest. Use Solver to determine the best quarterly allocation.

- On the **Tools** menu, click **Solver**. In the **Set target cell** box, type **f15** or select cell F15 (total profits for the year) on the worksheet. Make sure the **Max** option is selected. In the **By changing cells** box, type **b11:e11** or select cells B11:E11 (the advertising budget for each of the four quarters) on the worksheet. Click **Solve**.

- After you examine the results, click **Restore original values** and click **OK** to discard the results and return all cells to their former values.

You've just asked Solver to solve a moderately complex nonlinear optimization problem; that is, to find values for the four unknowns in cells B11 through E11 that will maximize profits. (This is a nonlinear problem because of the exponentiation that occurs in the formulas in row 5). The results of this unconstrained optimization show that you can increase profits for the year to $79,706 if you spend $89,706 in advertising for the full year.

However, most realistic modeling problems have limiting factors that you will want to apply to certain values. These constraints may be applied to the target cell, the changing cells, or any other value that is related to the formulas in these cells.

Adding a Constraint

So far, the budget recovers the advertising cost and generates additional profit, but you're reaching a point of diminishing returns. Because you can never be sure that your model of sales response to advertising will be valid next year (especially at greatly increased spending levels), it doesn't seem prudent to allow unrestricted spending on advertising.

Suppose you want to maintain your original advertising budget of $40,000. Add the constraint to the problem that limits the sum of advertising during the four quarters to $40,000.

- On the **Tools** menu, click **Solver**, and then click **Add**. The **Add Constraint** dialog box appears. In the **Cell reference** box, type **f11** or select cell F11 (advertising total) on the worksheet. Cell F11 must be less than or equal to $40,000. The relationship in the **Constraint** box is **<=** (less than or equal to) by default, so you don't have to change it. In the box next to the relationship, type **40000**. Click **OK**, and then click **Solve**.

- After you examine the results, click **Restore original values** and then click **OK** to discard the results and return the cells to their former values.

The solution found by Solver allocates amounts ranging from $5,117 in Q3 to $15,263 in Q4. Total Profit has increased from $69,662 in the original budget to $71,447, without any increase in the advertising budget.

Changing a Constraint

When you use Microsoft Excel Solver, you can experiment with slightly different parameters to decide the best solution to a problem. For example, you can change a constraint to see whether the results are better or worse than before. In the sample worksheet, try changing the constraint on advertising dollars to $50,000 to see what that does to total profits.

- On the **Tools** menu, click **Solver**. The constraint, **$F$11<=40000**, should already be selected in the **Subject to the constraints** box. Click **Change**. In the **Constraint** box, change **40000** to **50000**. Click **OK**, and then click **Solve**. Click **Keep solver solution** and then click **OK** to keep the results that are displayed on the worksheet.

Solver finds an optimal solution that yields a total profit of $74,817. That's an improvement of $3,370 over the last figure of $71,447. In most firms, it's not too difficult to justify an incremental investment of $10,000 that yields an additional $3,370 in profit, or a 33.7% return on investment. This solution also results in profits of $4,889 less than the unconstrained result, but you spend $39,706 less to get there.

Saving a Problem Model

When you click **Save** on the **File** menu, the last selections you made in the **Solver Parameters** dialog box are attached to the worksheet and retained when you save the workbook. However, you can define more than one problem for a worksheet by saving them individually using **Save Model** in the **Solver Options** dialog box. Each problem model consists of cells and constraints that you entered in the **Solver Parameters** dialog box.

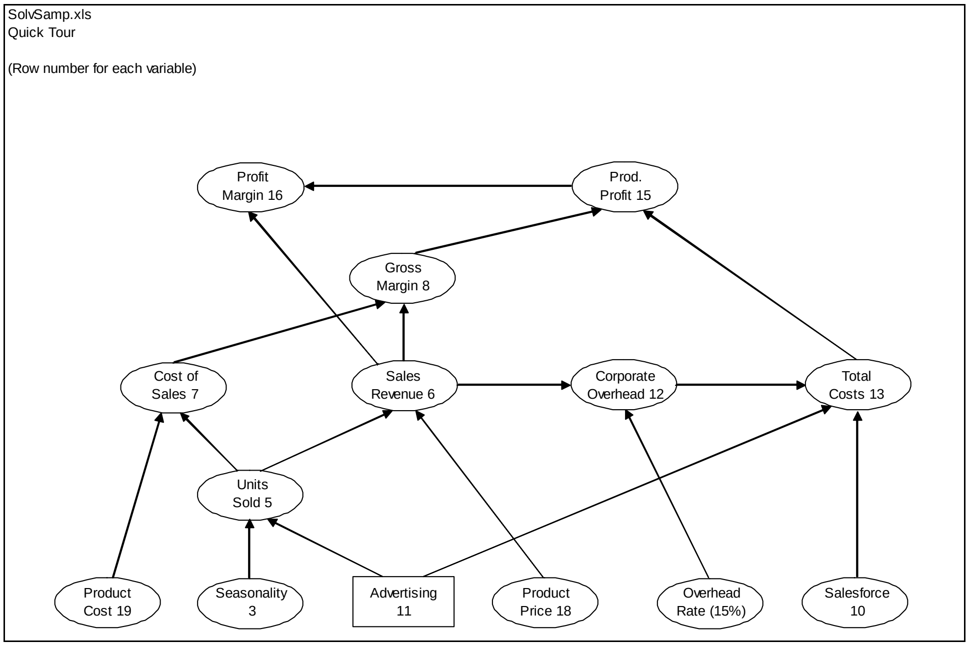
When you click **Save Model**, the **Save Model** dialog box appears with a default selection, based on the active cell, as the area for saving the model. The suggested range includes a cell for each constraint plus three additional cells. Make sure that this cell range is an empty range on the worksheet.

- On the **Tools** menu, click **Solver**, and then click **Options**. Click **Save Model**. In the **Select model area** box, type **h15:h18** or select cells H15:H18 on the worksheet. Click **OK**.

Note You can also enter a reference to a single cell in the **Select model area** box. Solver will use this reference as the upper-left corner of the range into which it will copy the problem specifications.

To load these problem specifications later, click **Load Model** on the **Solver Options** dialog box, type **h15:h18** in the **Model area** box or select cells H15:H18 on the sample worksheet, and then click **OK**. Solver displays a message asking if you want to reset the current Solver option settings with the settings for the model you are loading. Click **OK** to proceed.

Figure 19.2 Quick Tour Influence Chart

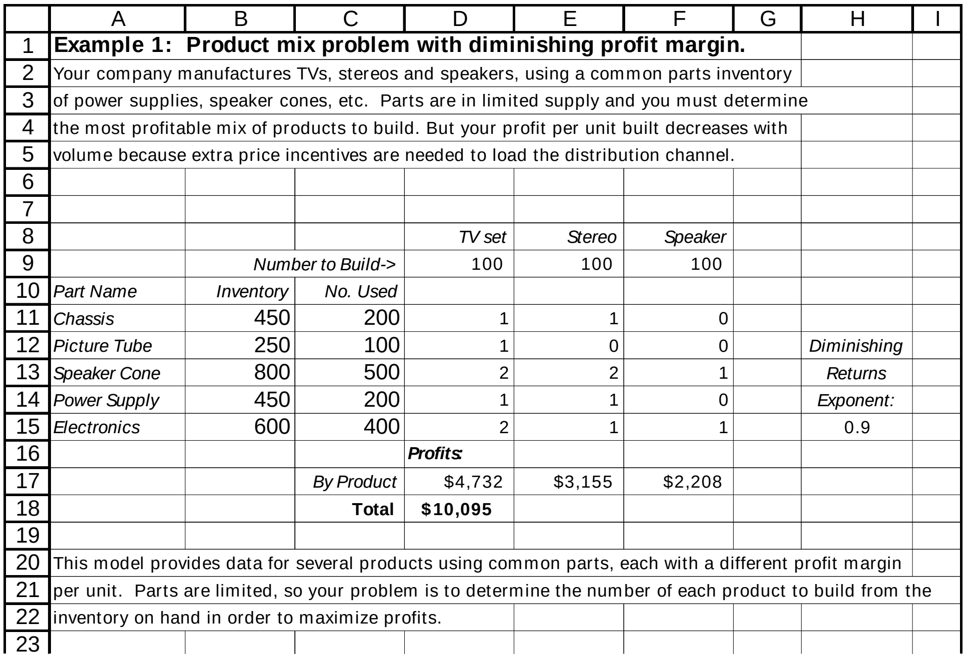


Nonlinear Product Mix Optimization

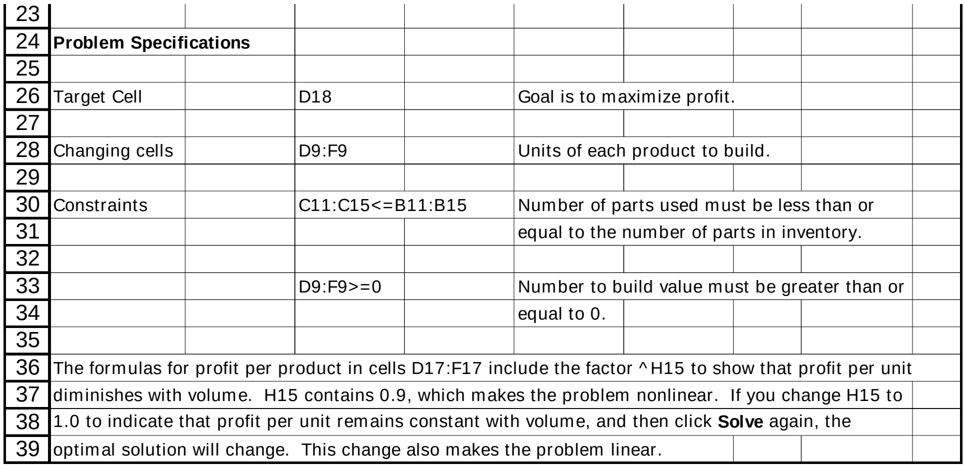
20

20.1 DIMINISHING PROFIT MARGIN

Figure 20.1 Product Mix Problem



	A	B	C	D	E	F	G	H	I
1	**Example 1: Product mix problem with diminishing profit margin.**								
2	Your company manufactures TVs, stereos and speakers, using a common parts inventory								
3	of power supplies, speaker cones, etc. Parts are in limited supply and you must determine								
4	the most profitable mix of products to build. But your profit per unit built decreases with								
5	volume because extra price incentives are needed to load the distribution channel.								
6									
7									
8				*TV set*	*Stereo*	*Speaker*			
9			*Number to Build->*	100	100	100			
10	*Part Name*	*Inventory*	*No. Used*						
11	*Chassis*	450	200	1	1	0			
12	*Picture Tube*	250	100	1	0	0		*Diminishing*	
13	*Speaker Cone*	800	500	2	2	1		*Returns*	
14	*Power Supply*	450	200	1	1	0		*Exponent:*	
15	*Electronics*	600	400	2	1	1		0.9	
16				***Profits:***					
17			*By Product*	$4,732	$3,155	$2,208			
18			**Total**	**$10,095**					
19									
20	This model provides data for several products using common parts, each with a different profit margin								
21	per unit. Parts are limited, so your problem is to determine the number of each product to build from the								
22	inventory on hand in order to maximize profits.								
23									



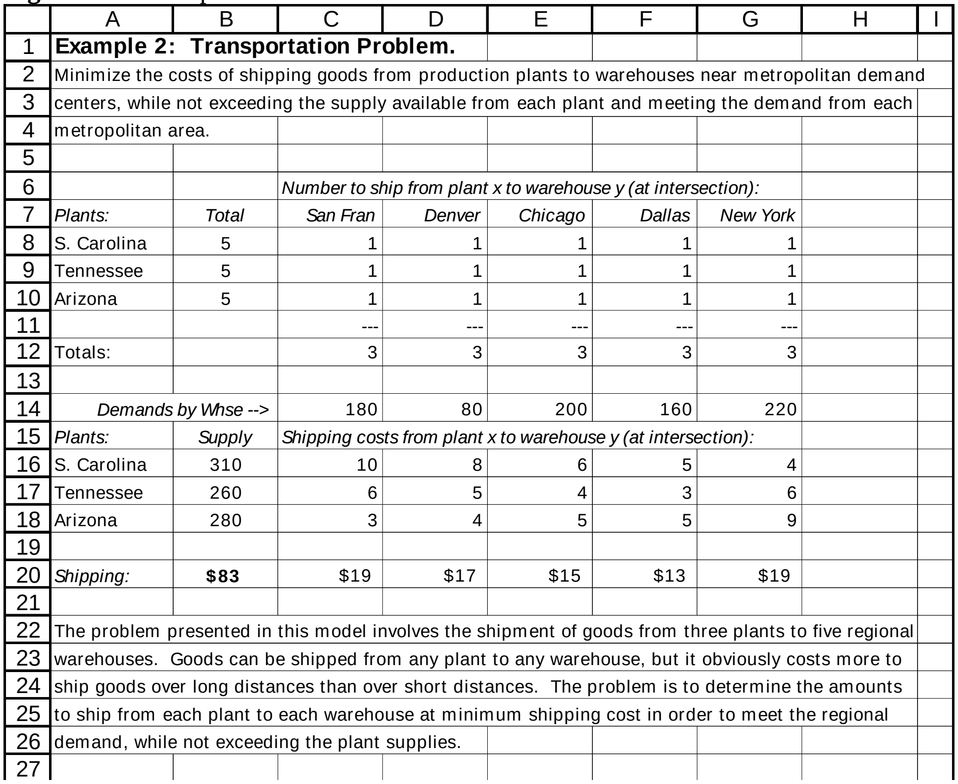
23			
24	**Problem Specifications**		
25			
26	Target Cell	D18	Goal is to maximize profit.
27			
28	Changing cells	D9:F9	Units of each product to build.
29			
30	Constraints	C11:C15<=B11:B15	Number of parts used must be less than or
31			equal to the number of parts in inventory.
32			
33		D9:F9>=0	Number to build value must be greater than or
34			equal to 0.
35			
36	The formulas for profit per product in cells D17:F17 include the factor ^H15 to show that profit per unit		
37	diminishes with volume. H15 contains 0.9, which makes the problem nonlinear. If you change H15 to		
38	1.0 to indicate that profit per unit remains constant with volume, and then click **Solve** again, the		
39	optimal solution will change. This change also makes the problem linear.		

Integer-Valued Optimization Models

21

21.1 TRANSPORTATION PROBLEM

Figure 21.1 Transportation Problem



	A	B	C	D	E	F	G	H	I
1	**Example 2: Transportation Problem.**								
2	Minimize the costs of shipping goods from production plants to warehouses near metropolitan demand								
3	centers, while not exceeding the supply available from each plant and meeting the demand from each								
4	metropolitan area.								
5									
6			*Number to ship from plant x to warehouse y (at intersection):*						
7	*Plants:*	*Total*	*San Fran*	*Denver*	*Chicago*	*Dallas*	*New York*		
8	S. Carolina	5	1	1	1	1	1		
9	Tennessee	5	1	1	1	1	1		
10	Arizona	5	1	1	1	1	1		
11			---	---	---	---	---		
12	Totals:		3	3	3	3	3		
13									
14	*Demands by Whse -->*		180	80	200	160	220		
15	*Plants:*	*Supply*	*Shipping costs from plant x to warehouse y (at intersection):*						
16	S. Carolina	310	10	8	6	5	4		
17	Tennessee	260	6	5	4	3	6		
18	Arizona	280	3	4	5	5	9		
19									
20	*Shipping:*	**$83**	$19	$17	$15	$13	$19		
21									
22	The problem presented in this model involves the shipment of goods from three plants to five regional								
23	warehouses. Goods can be shipped from any plant to any warehouse, but it obviously costs more to								
24	ship goods over long distances than over short distances. The problem is to determine the amounts								
25	to ship from each plant to each warehouse at minimum shipping cost in order to meet the regional								
26	demand, while not exceeding the plant supplies.								
27									

27					
28	**Problem Specifications**				
29					
30	Target cell		B20		Goal is to minimize total shipping cost.
31					
32	Changing cells		C8:G10		Amount to ship from each plant to each
33					warehouse.
34					
35	Constraints		B8:B10<=B16:B18		Total shipped must be less than or equal to
36					supply at plant.
37					
38			C12:G12>=C14:G14		Totals shipped to warehouses must be greater
39					than or equal to demand at warehouses.
40					
41			C8:G10>=0		Number to ship must be greater than or equal
42					to 0.
43					
44	You can solve this problem faster by selecting the **Assume linear model** check box in the **Solver**				
45	**Options** dialog box before clicking **Solve**. A problem of this type has an optimum solution at which				
46	amounts to ship are integers, if all of the supply and demand constraints are integers.				

21.2 MODIFIED TRANSPORTATION PROBLEM

Figure 21.2 Display

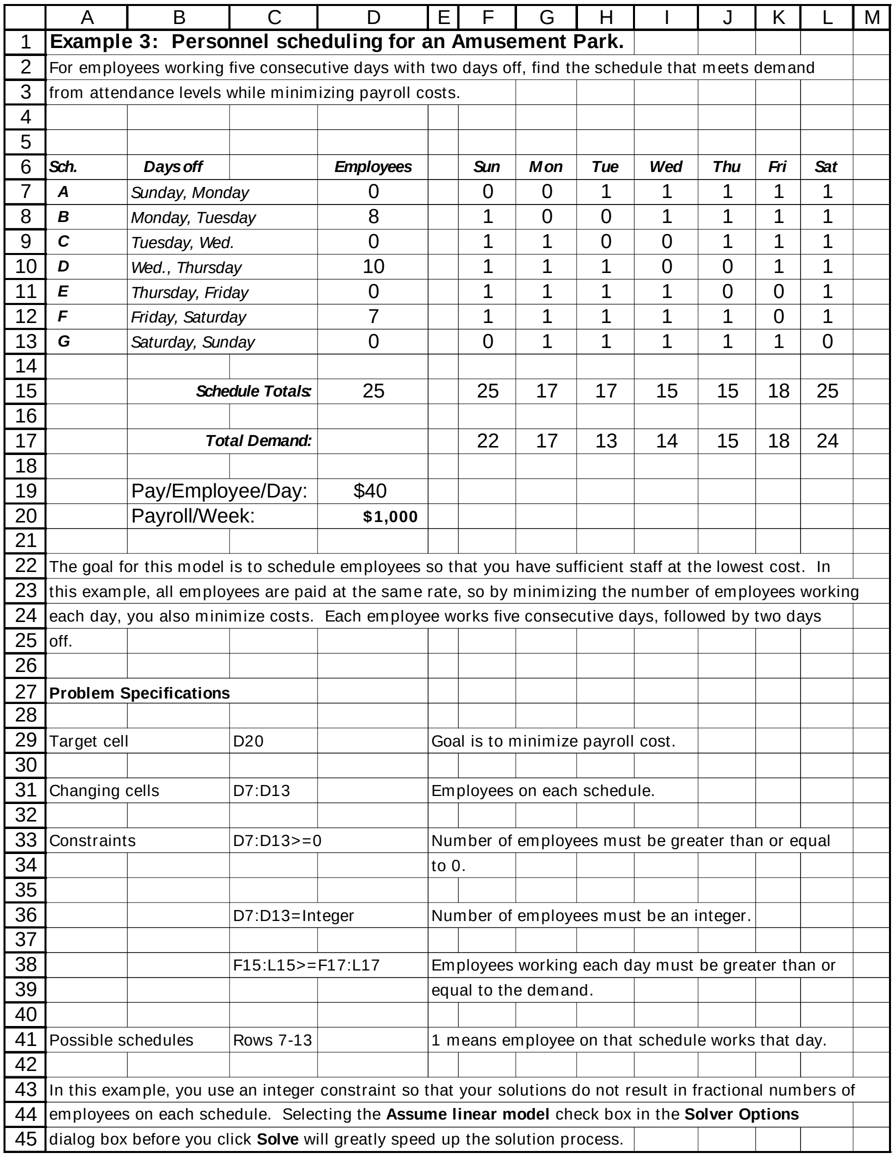
	A	B	C	D	E	F	G	H	I
1	Modified Example 2: Transportation Problem.								
2									
3	Minimize the costs of shipping goods from production plants to warehouses near metropolitan demand								
4	centers, while not exceeding the supply available from each plant and meeting the demand from each								
5	metropolitan area.								
6									
7			Number to ship from plant to warehouse						
8					Warehouse			Shipped	Plant
9	Plant		San Fran	Denver	Chicago	Dallas	New York	from plant	supply
10	S. Carolina		1	1	1	1	1	5	310
11	Tennessee		1	1	1	1	1	5	260
12	Arizona		1	1	1	1	1	5	280
13	Shipped to warehouse		3	3	3	3	3		
14	Warehouse demand		180	80	200	160	220		
15									
16			Shipping cost from plant to warehouse						
17					Warehouse				
18	Plant		San Fran	Denver	Chicago	Dallas	New York		
19	S. Carolina		$10	$8	$6	$5	$4		
20	Tennessee		$6	$5	$4	$3	$6		
21	Arizona		$3	$4	$5	$5	$9		
22									
23	Total shipping cost		$83						

Figure 21.3 Formulas

	A	B	C	D	E	F	G	H	I
1	Modified Example 2: Transportation Problem.								
2									
3	Minimize the costs of shipping goods from production plants to warehouses near metropolitan demand								
4	centers, while not exceeding the supply available from each plant and meeting the demand from each								
5	metropolitan area.								
6									
7			Number to ship from plant to warehouse						
8					Warehouse			Shipped	Plant
9	Plant		San Fran	Denver	Chicago	Dallas	New York	from plant	supply
10	S. Carolina		1	1	1	1	1	=SUM(C10:G10)	310
11	Tennessee		1	1	1	1	1	=SUM(C11:G11)	260
12	Arizona		1	1	1	1	1	=SUM(C12:G12)	280
13	Shipped to warehouse		=SUM(C10:C12)	=SUM(D10:D12)	=SUM(E10:E12)	=SUM(F10:F12)	=SUM(G10:G12)		
14	Warehouse demand		180	80	200	160	220		
15									
16			Shipping cost from plant to warehouse						
17					Warehouse				
18	Plant		San Fran	Denver	Chicago	Dallas	New York		
19	S. Carolina		$10	$8	$6	$5	$4		
20	Tennessee		$6	$5	$4	$3	$6		
21	Arizona		$3	$4	$5	$5	$9		
22									
23	Total shipping cost		=SUMPRODUCT(C10:G12,C19:G21)						

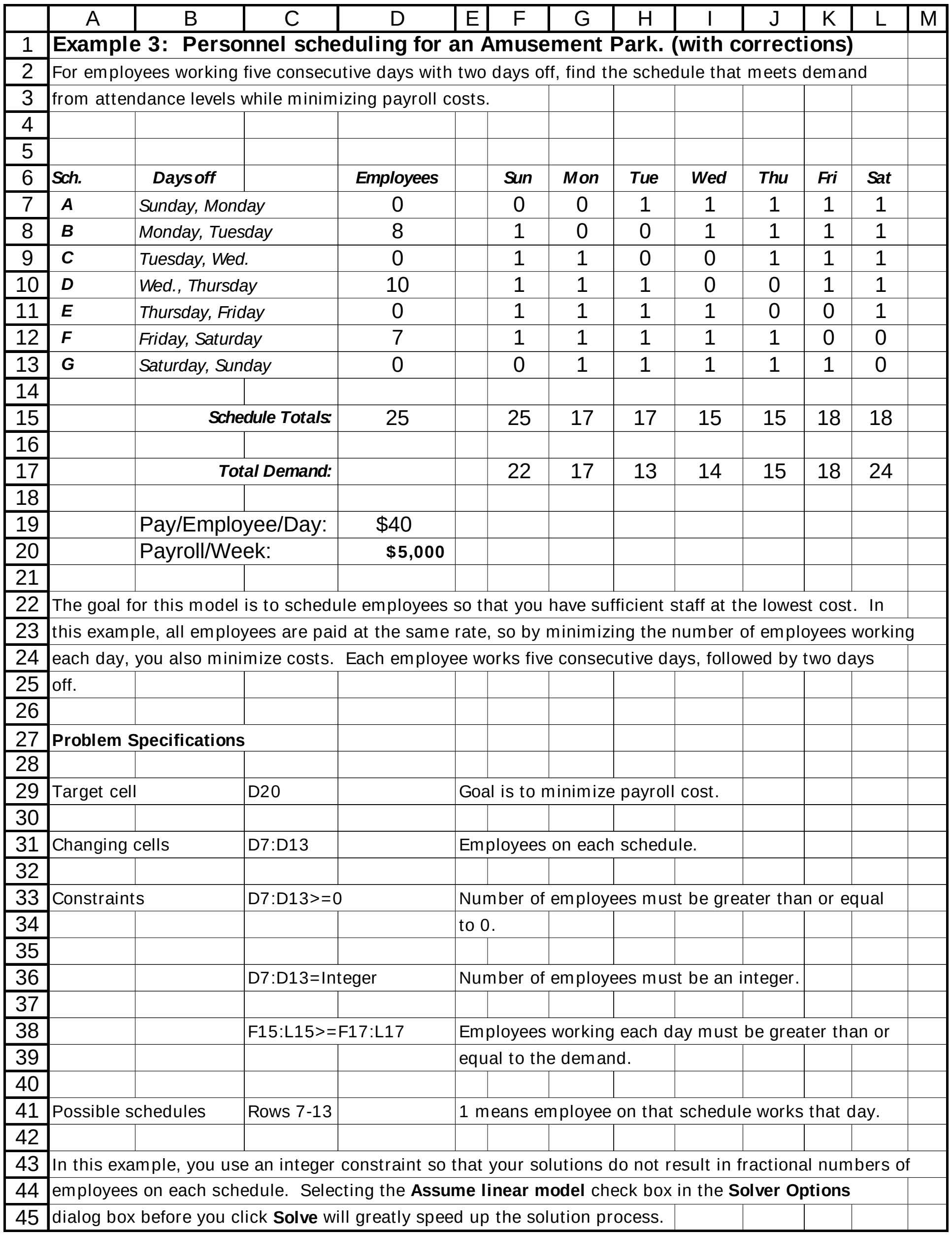
21.3 SCHEDULING PROBLEM

Figure 21.4 Personnel Scheduling



	A	B	C	D	E	F	G	H	I	J	K	L	M
1	**Example 3: Personnel scheduling for an Amusement Park.**												
2	For employees working five consecutive days with two days off, find the schedule that meets demand												
3	from attendance levels while minimizing payroll costs.												
4													
5													
6	***Sch.***	***Days off***		***Employees***		***Sun***	***Mon***	***Tue***	***Wed***	***Thu***	***Fri***	***Sat***	
7	***A***	*Sunday, Monday*		0		0	0	1	1	1	1	1	
8	***B***	*Monday, Tuesday*		8		1	0	0	1	1	1	1	
9	***C***	*Tuesday, Wed.*		0		1	1	0	0	1	1	1	
10	***D***	*Wed., Thursday*		10		1	1	1	0	0	1	1	
11	***E***	*Thursday, Friday*		0		1	1	1	1	0	0	1	
12	***F***	*Friday, Saturday*		7		1	1	1	1	1	0	1	
13	***G***	*Saturday, Sunday*		0		0	1	1	1	1	1	0	
14													
15			***Schedule Totals:***	25		25	17	17	15	15	18	25	
16													
17			***Total Demand:***			22	17	13	14	15	18	24	
18													
19		Pay/Employee/Day:		$40									
20		Payroll/Week:		**$1,000**									
21													
22	The goal for this model is to schedule employees so that you have sufficient staff at the lowest cost. In												
23	this example, all employees are paid at the same rate, so by minimizing the number of employees working												
24	each day, you also minimize costs. Each employee works five consecutive days, followed by two days												
25	off.												
26													
27	**Problem Specifications**												
28													
29	Target cell		D20		Goal is to minimize payroll cost.								
30													
31	Changing cells		D7:D13		Employees on each schedule.								
32													
33	Constraints		D7:D13>=0		Number of employees must be greater than or equal								
34					to 0.								
35													
36			D7:D13=Integer		Number of employees must be an integer.								
37													
38			F15:L15>=F17:L17		Employees working each day must be greater than or								
39					equal to the demand.								
40													
41	Possible schedules		Rows 7-13		1 means employee on that schedule works that day.								
42													
43	In this example, you use an integer constraint so that your solutions do not result in fractional numbers of												
44	employees on each schedule. Selecting the **Assume linear model** check box in the **Solver Options**												
45	dialog box before you click **Solve** will greatly speed up the solution process.												

Figure 21.5 Personnel Scheduling with Corrections



	A	B	C	D	E	F	G	H	I	J	K	L	M
1	**Example 3: Personnel scheduling for an Amusement Park. (with corrections)**												
2	For employees working five consecutive days with two days off, find the schedule that meets demand												
3	from attendance levels while minimizing payroll costs.												
4													
5													
6	***Sch.***	***Days off***		***Employees***		***Sun***	***Mon***	***Tue***	***Wed***	***Thu***	***Fri***	***Sat***	
7	***A***	*Sunday, Monday*		0		0	0	1	1	1	1	1	
8	***B***	*Monday, Tuesday*		8		1	0	0	1	1	1	1	
9	***C***	*Tuesday, Wed.*		0		1	1	0	0	1	1	1	
10	***D***	*Wed., Thursday*		10		1	1	1	0	0	1	1	
11	***E***	*Thursday, Friday*		0		1	1	1	1	0	0	1	
12	***F***	*Friday, Saturday*		7		1	1	1	1	1	0	0	
13	***G***	*Saturday, Sunday*		0		0	1	1	1	1	1	0	
14													
15		***Schedule Totals:***		25		25	17	17	15	15	18	18	
16													
17		***Total Demand:***				22	17	13	14	15	18	24	
18													
19		Pay/Employee/Day:		$40									
20		Payroll/Week:		**$5,000**									
21													
22	The goal for this model is to schedule employees so that you have sufficient staff at the lowest cost. In												
23	this example, all employees are paid at the same rate, so by minimizing the number of employees working												
24	each day, you also minimize costs. Each employee works five consecutive days, followed by two days												
25	off.												
26													
27	**Problem Specifications**												
28													
29	Target cell		D20		Goal is to minimize payroll cost.								
30													
31	Changing cells		D7:D13		Employees on each schedule.								
32													
33	Constraints		D7:D13>=0		Number of employees must be greater than or equal								
34					to 0.								
35													
36			D7:D13=Integer		Number of employees must be an integer.								
37													
38			F15:L15>=F17:L17		Employees working each day must be greater than or								
39					equal to the demand.								
40													
41	Possible schedules		Rows 7-13		1 means employee on that schedule works that day.								
42													
43	In this example, you use an integer constraint so that your solutions do not result in fractional numbers of												
44	employees on each schedule. Selecting the **Assume linear model** check box in the **Solver Options**												
45	dialog box before you click **Solve** will greatly speed up the solution process.												

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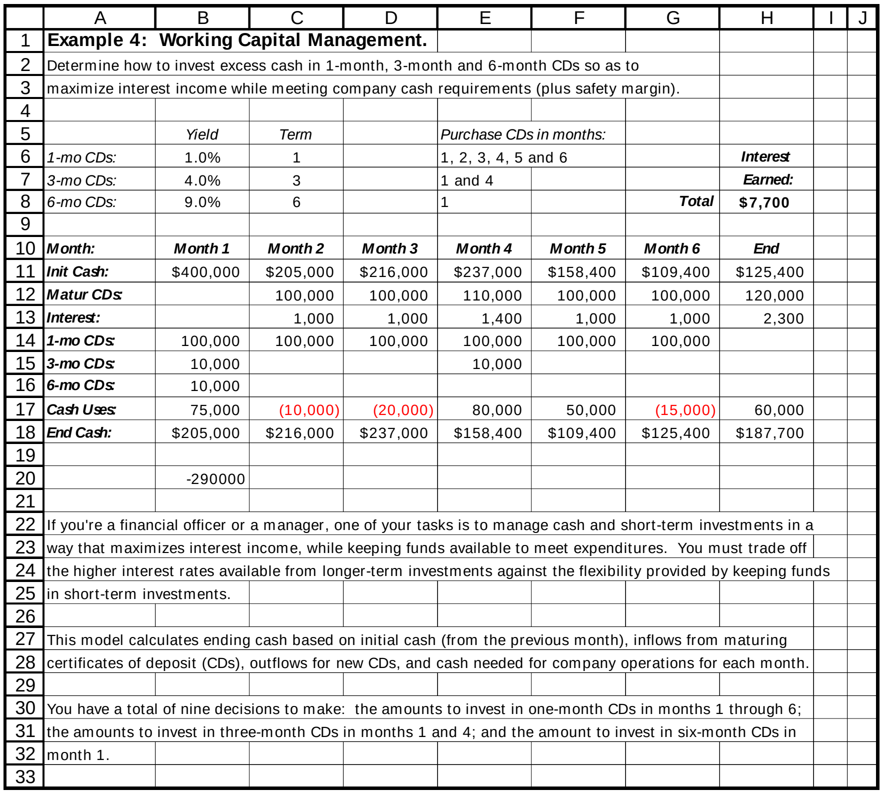
22

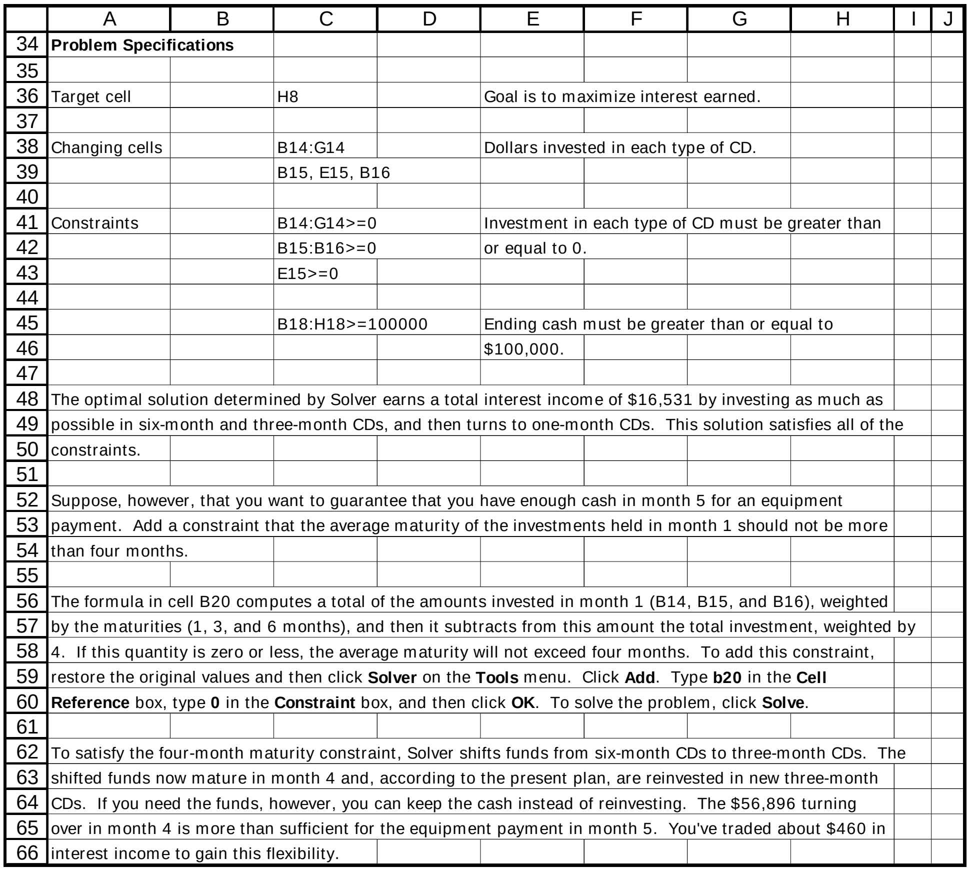
Optimization Models for Finance Decisions

22.1 WORKING CAPITAL MANAGEMENT PROBLEM

Figure 22.1 Working Capital Management



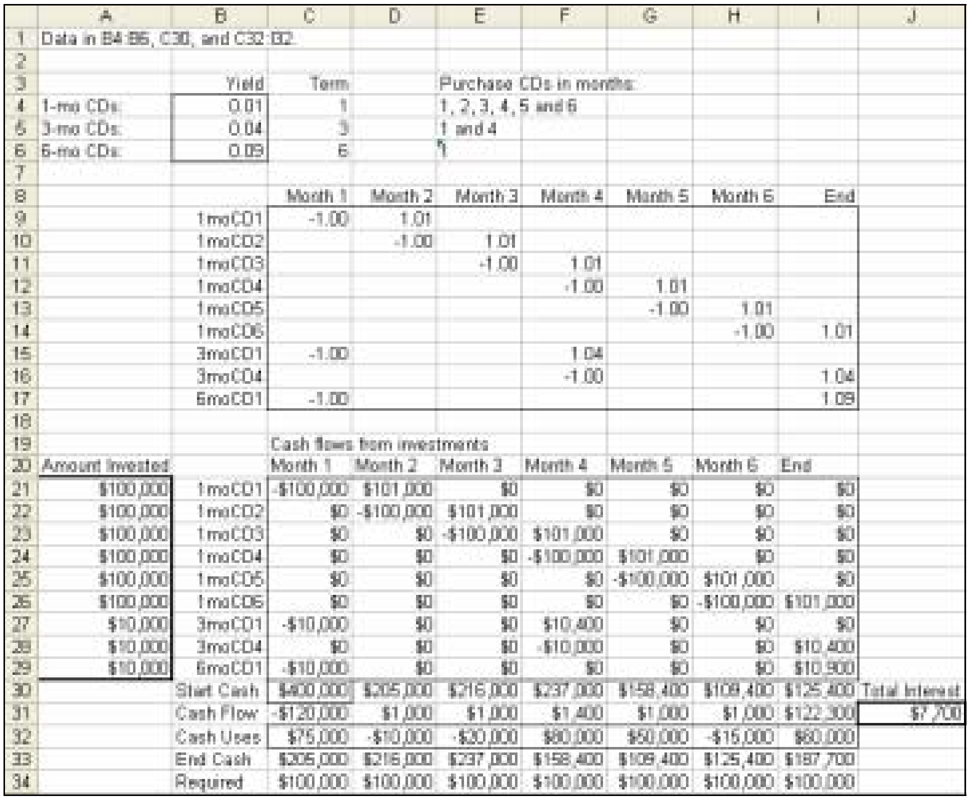
	A	B	C	D	E	F	G	H	I	J
1	**Example 4: Working Capital Management.**									
2	Determine how to invest excess cash in 1-month, 3-month and 6-month CDs so as to									
3	maximize interest income while meeting company cash requirements (plus safety margin).									
4										
5		*Yield*	*Term*		*Purchase CDs in months:*					
6	*1-mo CDs:*	1.0%	1		1, 2, 3, 4, 5 and 6			***Interest***		
7	*3-mo CDs:*	4.0%	3		1 and 4			***Earned:***		
8	*6-mo CDs:*	9.0%	6		1		***Total***	**$7,700**		
9										
10	***Month:***	***Month 1***	***Month 2***	***Month 3***	***Month 4***	***Month 5***	***Month 6***	***End***		
11	***Init Cash:***	$400,000	$205,000	$216,000	$237,000	$158,400	$109,400	$125,400		
12	***Matur CDs:***		100,000	100,000	110,000	100,000	100,000	120,000		
13	***Interest:***		1,000	1,000	1,400	1,000	1,000	2,300		
14	***1-mo CDs:***	100,000	100,000	100,000	100,000	100,000	100,000			
15	***3-mo CDs:***	10,000			10,000					
16	***6-mo CDs:***	10,000								
17	***Cash Uses:***	75,000	(10,000)	(20,000)	80,000	50,000	(15,000)	60,000		
18	***End Cash:***	$205,000	$216,000	$237,000	$158,400	$109,400	$125,400	$187,700		
19										
20		-290000								
21										
22	If you're a financial officer or a manager, one of your tasks is to manage cash and short-term investments in a									
23	way that maximizes interest income, while keeping funds available to meet expenditures. You must trade off									
24	the higher interest rates available from longer-term investments against the flexibility provided by keeping funds									
25	in short-term investments.									
26										
27	This model calculates ending cash based on initial cash (from the previous month), inflows from maturing									
28	certificates of deposit (CDs), outflows for new CDs, and cash needed for company operations for each month.									
29										
30	You have a total of nine decisions to make: the amounts to invest in one-month CDs in months 1 through 6;									
31	the amounts to invest in three-month CDs in months 1 and 4; and the amount to invest in six-month CDs in									
32	month 1.									
33										



	A	B	C	D	E	F	G	H	I	J
34	**Problem Specifications**									
35										
36	Target cell		H8		Goal is to maximize interest earned.					
37										
38	Changing cells		B14:G14		Dollars invested in each type of CD.					
39			B15, E15, B16							
40										
41	Constraints		B14:G14>=0		Investment in each type of CD must be greater than					
42			B15:B16>=0		or equal to 0.					
43			E15>=0							
44										
45			B18:H18>=100000		Ending cash must be greater than or equal to					
46					$100,000.					
47										
48	The optimal solution determined by Solver earns a total interest income of $16,531 by investing as much as									
49	possible in six-month and three-month CDs, and then turns to one-month CDs. This solution satisfies all of the									
50	constraints.									
51										
52	Suppose, however, that you want to guarantee that you have enough cash in month 5 for an equipment									
53	payment. Add a constraint that the average maturity of the investments held in month 1 should not be more									
54	than four months.									
55										
56	The formula in cell B20 computes a total of the amounts invested in month 1 (B14, B15, and B16), weighted									
57	by the maturities (1, 3, and 6 months), and then it subtracts from this amount the total investment, weighted by									
58	4. If this quantity is zero or less, the average maturity will not exceed four months. To add this constraint,									
59	restore the original values and then click **Solver** on the **Tools** menu. Click **Add**. Type **b20** in the **Cell**									
60	**Reference** box, type **0** in the **Constraint** box, and then click **OK**. To solve the problem, click **Solve**.									
61										
62	To satisfy the four-month maturity constraint, Solver shifts funds from six-month CDs to three-month CDs. The									
63	shifted funds now mature in month 4 and, according to the present plan, are reinvested in new three-month									
64	CDs. If you need the funds, however, you can keep the cash instead of reinvesting. The $56,896 turning									
65	over in month 4 is more than sufficient for the equipment payment in month 5. You've traded about $460 in									
66	interest income to gain this flexibility.									

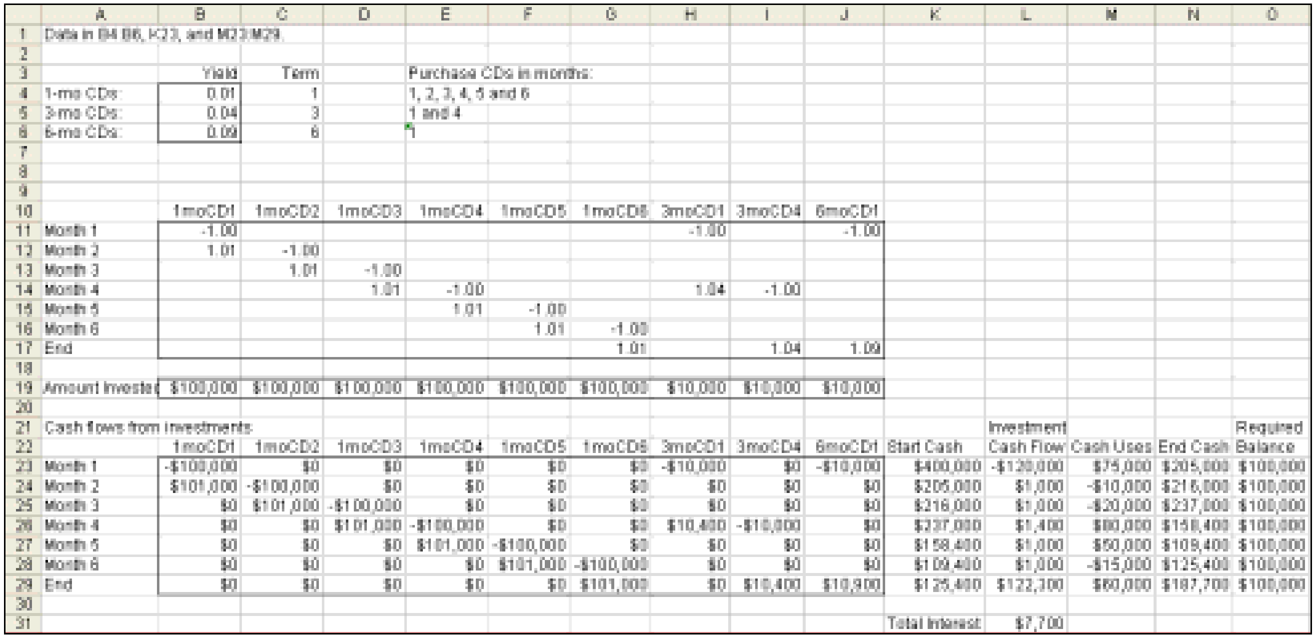
22.2 WORK CAP ALTERNATE FORMULATIONS

Figure 22.2 Working Capital Management Horizontal Time



	A	B	C	D	E	F	G	H	I	J
1	Data in B4:B6, C30, and C32:I32.									
2										
3		Yield	Term		Purchase CDs in months:					
4	1-mo CDs:	0.01	1		1, 2, 3, 4, 5 and 6					
5	3-mo CDs:	0.04	3		1 and 4					
6	6-mo CDs:	0.09	6		1					
7										
8			Month 1	Month 2	Month 3	Month 4	Month 5	Month 6	End	
9		1moCD1	-1.00	1.01						
10		1moCD2		-1.00	1.01					
11		1moCD3			-1.00	1.01				
12		1moCD4				-1.00	1.01			
13		1moCD5					-1.00	1.01		
14		1moCD6						-1.00	1.01	
15		3moCD1	-1.00			1.04				
16		3moCD4				-1.00			1.04	
17		6moCD1	-1.00						1.09	
18										
19			Cash flows from investments							
20	Amount Invested		Month 1	Month 2	Month 3	Month 4	Month 5	Month 6	End	
21	$100,000	1moCD1	-$100,000	$101,000	$0	$0	$0	$0	$0	
22	$100,000	1moCD2	$0	-$100,000	$101,000	$0	$0	$0	$0	
23	$100,000	1moCD3	$0	$0	-$100,000	$101,000	$0	$0	$0	
24	$100,000	1moCD4	$0	$0	$0	-$100,000	$101,000	$0	$0	
25	$100,000	1moCD5	$0	$0	$0	$0	-$100,000	$101,000	$0	
26	$100,000	1moCD6	$0	$0	$0	$0	$0	-$100,000	$101,000	
27	$10,000	3moCD1	-$10,000	$0	$0	$10,400	$0	$0	$0	
28	$10,000	3moCD4	$0	$0	$0	-$10,000	$0	$0	$10,400	
29	$10,000	6moCD1	-$10,000	$0	$0	$0	$0	$0	$10,900	
30		Start Cash	$400,000	$205,000	$216,000	$237,000	$158,400	$109,400	$125,400	Total Interest
31		Cash Flow	-$120,000	$1,000	$1,000	$1,400	$1,000	$1,000	$122,300	$7,700
32		Cash Uses	$75,000	-$10,000	-$20,000	$80,000	$50,000	-$15,000	$60,000	
33		End Cash	$205,000	$216,000	$237,000	$158,400	$109,400	$125,400	$187,700	
34		Required	$100,000	$100,000	$100,000	$100,000	$100,000	$100,000	$100,000	

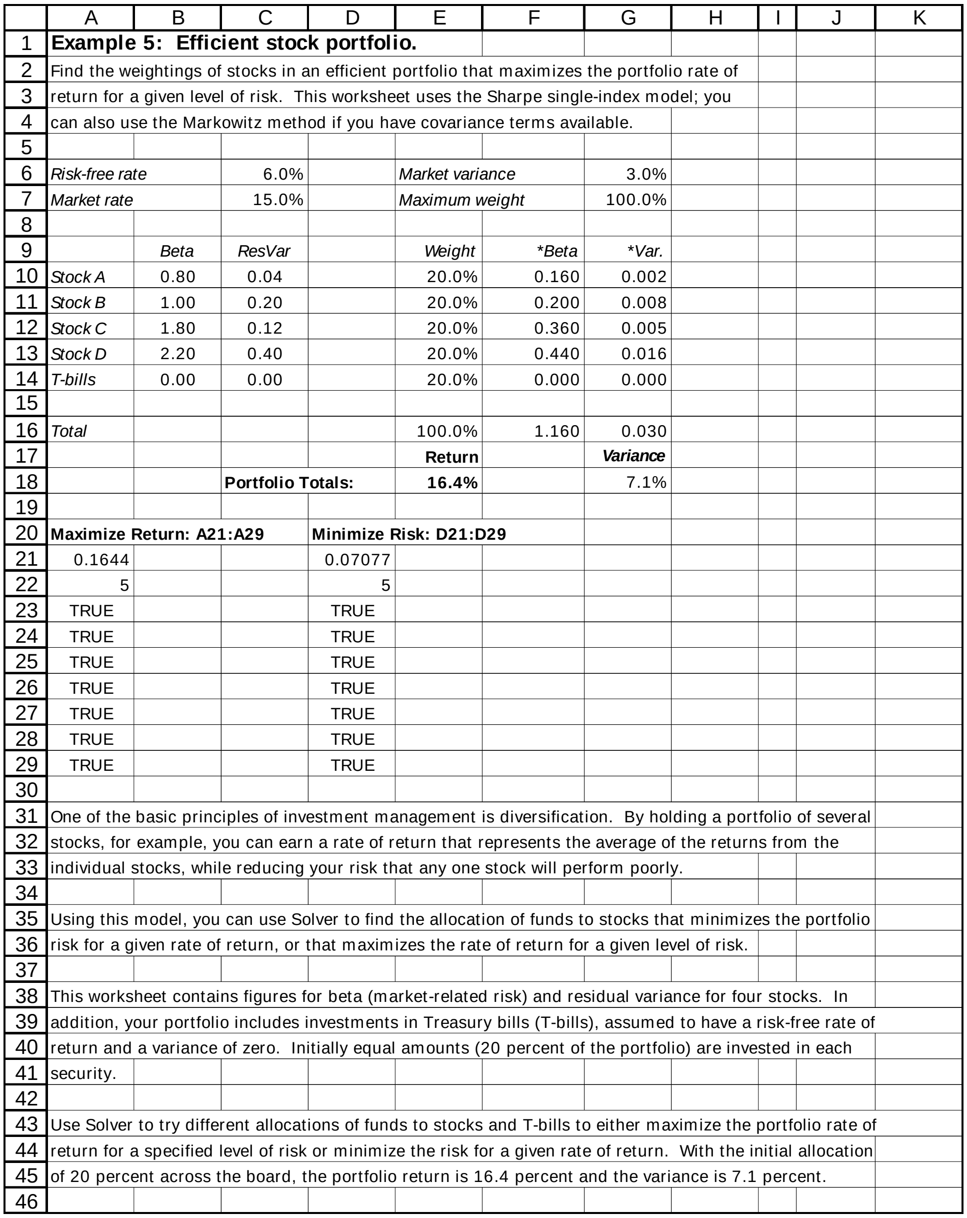
Figure 22.3 Working Capital Management Vertical Time



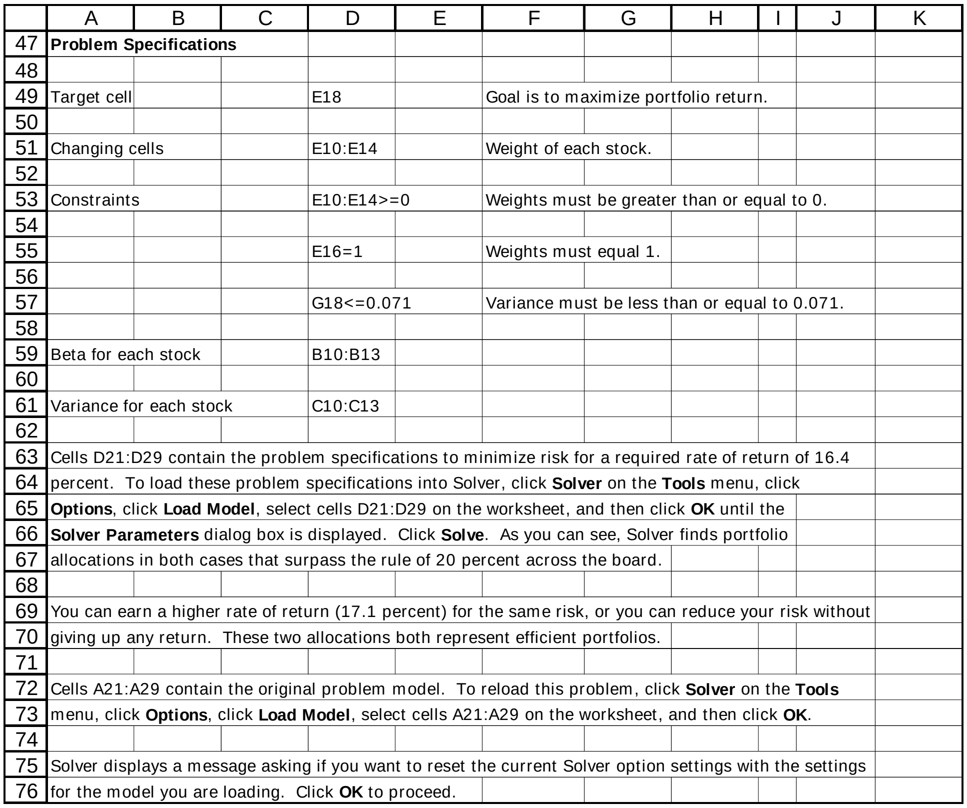
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Data in B4:B6, K23, and M23:M29.														
2															
3		Yield	Term		Purchase CDs in months:										
4	1-mo CDs:	0.01	1		1, 2, 3, 4, 5 and 6										
5	3-mo CDs:	0.04	3		1 and 4										
6	6-mo CDs:	0.09	6		1										
7															
8															
9															
10		1moCD1	1moCD2	1moCD3	1moCD4	1moCD5	1moCD6	3moCD1	3moCD4	6moCD1					
11	Month 1	-1.00						-1.00		-1.00					
12	Month 2	1.01	-1.00												
13	Month 3		1.01	-1.00											
14	Month 4			1.01	-1.00			1.04	-1.00						
15	Month 5				1.01	-1.00									
16	Month 6					1.01	-1.00								
17	End						1.01		1.04	1.09					
18															
19	Amount Investe	$100,000	$100,000	$100,000	$100,000	$100,000	$100,000	$10,000	$10,000	$10,000					
20															
21	Cash flows from investments:											Investment			Required
22		1moCD1	1moCD2	1moCD3	1moCD4	1moCD5	1moCD6	3moCD1	3moCD4	6moCD1	Start Cash	Cash Flow	Cash Uses	End Cash	Balance
23	Month 1	-$100,000	$0	$0	$0	$0	$0	-$10,000	$0	-$10,000	$400,000	-$120,000	$75,000	$205,000	$100,000
24	Month 2	$101,000	-$100,000	$0	$0	$0	$0	$0	$0	$0	$205,000	$1,000	-$10,000	$216,000	$100,000
25	Month 3	$0	$101,000	-$100,000	$0	$0	$0	$0	$0	$0	$216,000	$1,000	-$20,000	$237,000	$100,000
26	Month 4	$0	$0	$101,000	-$100,000	$0	$0	$10,400	-$10,000	$0	$237,000	$1,400	$80,000	$158,400	$100,000
27	Month 5	$0	$0	$0	$101,000	-$100,000	$0	$0	$0	$0	$158,400	$1,000	$50,000	$109,400	$100,000
28	Month 6	$0	$0	$0	$0	$101,000	-$100,000	$0	$0	$0	$109,400	$1,000	-$15,000	$125,400	$100,000
29	End	$0	$0	$0	$0	$0	$101,000	$0	$10,400	$10,900	$125,400	$122,300	$60,000	$187,700	$100,000
30															
31											Total Interest	$7,700			

22.3 STOCK PORTFOLIO PROBLEM

Figure 22.4 Efficient Stock Portfolio



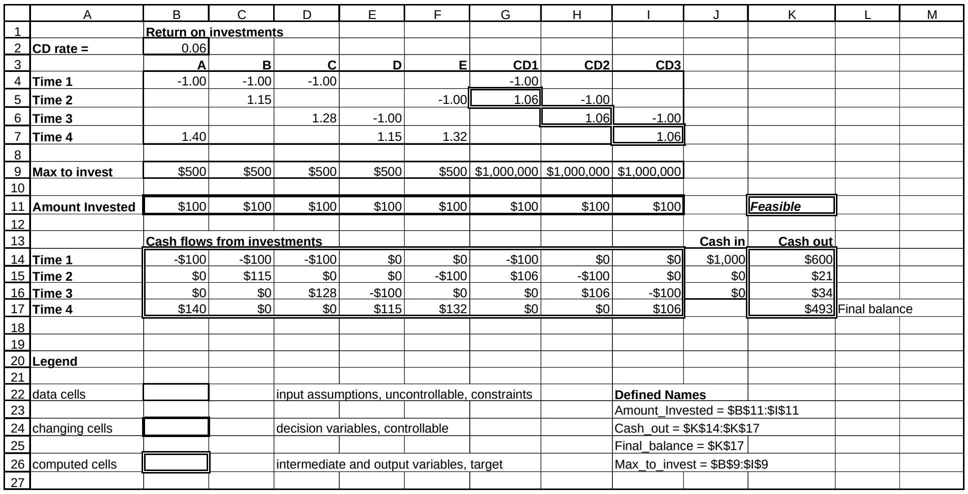
	A	B	C	D	E	F	G	H	I	J	K
1	**Example 5: Efficient stock portfolio.**										
2	Find the weightings of stocks in an efficient portfolio that maximizes the portfolio rate of										
3	return for a given level of risk. This worksheet uses the Sharpe single-index model; you										
4	can also use the Markowitz method if you have covariance terms available.										
5											
6	*Risk-free rate*		6.0%		*Market variance*		3.0%				
7	*Market rate*		15.0%		*Maximum weight*		100.0%				
8											
9		*Beta*	*ResVar*		*Weight*	**Beta*	**Var.*				
10	*Stock A*	0.80	0.04		20.0%	0.160	0.002				
11	*Stock B*	1.00	0.20		20.0%	0.200	0.008				
12	*Stock C*	1.80	0.12		20.0%	0.360	0.005				
13	*Stock D*	2.20	0.40		20.0%	0.440	0.016				
14	*T-bills*	0.00	0.00		20.0%	0.000	0.000				
15											
16	*Total*				100.0%	1.160	0.030				
17					**Return**		***Variance***				
18			**Portfolio Totals:**		**16.4%**		7.1%				
19											
20	**Maximize Return: A21:A29**			**Minimize Risk: D21:D29**							
21	0.1644			0.07077							
22	5			5							
23	TRUE			TRUE							
24	TRUE			TRUE							
25	TRUE			TRUE							
26	TRUE			TRUE							
27	TRUE			TRUE							
28	TRUE			TRUE							
29	TRUE			TRUE							
30											
31	One of the basic principles of investment management is diversification. By holding a portfolio of several										
32	stocks, for example, you can earn a rate of return that represents the average of the returns from the										
33	individual stocks, while reducing your risk that any one stock will perform poorly.										
34											
35	Using this model, you can use Solver to find the allocation of funds to stocks that minimizes the portfolio										
36	risk for a given rate of return, or that maximizes the rate of return for a given level of risk.										
37											
38	This worksheet contains figures for beta (market-related risk) and residual variance for four stocks. In										
39	addition, your portfolio includes investments in Treasury bills (T-bills), assumed to have a risk-free rate of										
40	return and a variance of zero. Initially equal amounts (20 percent of the portfolio) are invested in each										
41	security.										
42											
43	Use Solver to try different allocations of funds to stocks and T-bills to either maximize the portfolio rate of										
44	return for a specified level of risk or minimize the risk for a given rate of return. With the initial allocation										
45	of 20 percent across the board, the portfolio return is 16.4 percent and the variance is 7.1 percent.										
46											



	A	B	C	D	E	F	G	H	I	J	K
47	**Problem Specifications**										
48											
49	Target cell			E18		Goal is to maximize portfolio return.					
50											
51	Changing cells			E10:E14		Weight of each stock.					
52											
53	Constraints			E10:E14>=0		Weights must be greater than or equal to 0.					
54											
55				E16=1		Weights must equal 1.					
56											
57				G18<=0.071		Variance must be less than or equal to 0.071.					
58											
59	Beta for each stock			B10:B13							
60											
61	Variance for each stock			C10:C13							
62											
63	Cells D21:D29 contain the problem specifications to minimize risk for a required rate of return of 16.4										
64	percent. To load these problem specifications into Solver, click **Solver** on the **Tools** menu, click										
65	**Options**, click **Load Model**, select cells D21:D29 on the worksheet, and then click **OK** until the										
66	**Solver Parameters** dialog box is displayed. Click **Solve**. As you can see, Solver finds portfolio										
67	allocations in both cases that surpass the rule of 20 percent across the board.										
68											
69	You can earn a higher rate of return (17.1 percent) for the same risk, or you can reduce your risk without										
70	giving up any return. These two allocations both represent efficient portfolios.										
71											
72	Cells A21:A29 contain the original problem model. To reload this problem, click **Solver** on the **Tools**										
73	menu, click **Options**, click **Load Model**, select cells A21:A29 on the worksheet, and then click **OK**.										
74											
75	Solver displays a message asking if you want to reset the current Solver option settings with the settings										
76	for the model you are loading. Click **OK** to proceed.										

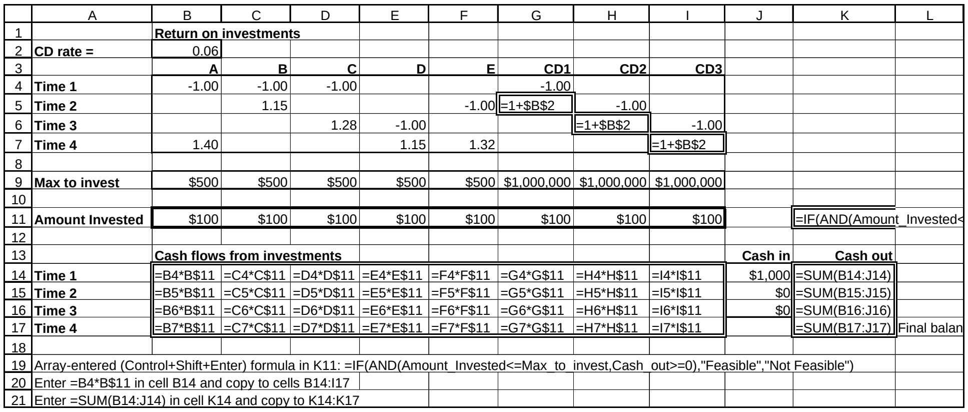
22.4 MONEYCO PROBLEM

Figure 22.5 Display



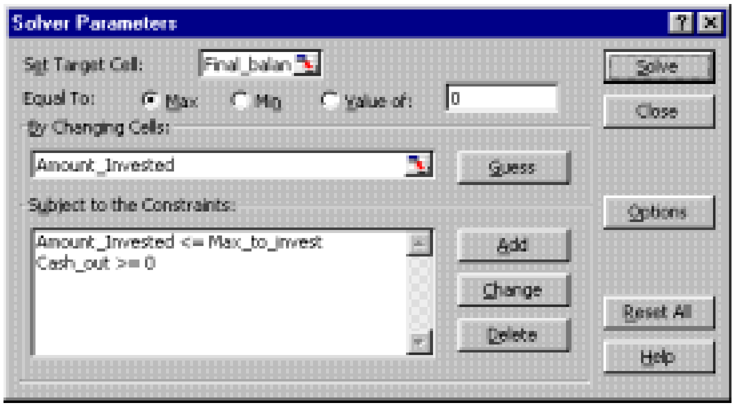
	A	B	C	D	E	F	G	H	I	J	K	L	M
1		**Return on investments**											
2	**CD rate =**	0.06											
3		**A**	**B**	**C**	**D**	**E**	**CD1**	**CD2**	**CD3**				
4	**Time 1**	-1.00	-1.00	-1.00			-1.00						
5	**Time 2**		1.15			-1.00	1.06	-1.00					
6	**Time 3**			1.28	-1.00			1.06	-1.00				
7	**Time 4**	1.40			1.15	1.32			1.06				
8													
9	**Max to invest**	$500	$500	$500	$500	$500	$1,000,000	$1,000,000	$1,000,000				
10													
11	**Amount Invested**	$100	$100	$100	$100	$100	$100	$100	$100		***Feasible***		
12													
13		**Cash flows from investments**								**Cash in**	**Cash out**		
14	**Time 1**	-$100	-$100	-$100	$0	$0	-$100	$0	$0	$1,000	$600		
15	**Time 2**	$0	$115	$0	$0	-$100	$106	-$100	$0	$0	$21		
16	**Time 3**	$0	$0	$128	-$100	$0	$0	$106	-$100	$0	$34		
17	**Time 4**	$140	$0	$0	$115	$132	$0	$0	$106		$493	Final balance	
18													
19													
20	**Legend**												
21													
22	data cells			input assumptions, uncontrollable, constraints					**Defined Names**				
23									Amount_Invested = $B$11:$I$11				
24	changing cells			decision variables, controllable					Cash_out = $K$14:$K$17				
25									Final_balance = $K$17				
26	computed cells			intermediate and output variables, target					Max_to_invest = $B$9:$I$9				
27													

Figure 22.6 Formulas



	A	B	C	D	E	F	G	H	I	J	K	L
1		**Return on investments**										
2	**CD rate =**	0.06										
3		**A**	**B**	**C**	**D**	**E**	**CD1**	**CD2**	**CD3**			
4	**Time 1**	-1.00	-1.00	-1.00			-1.00					
5	**Time 2**		1.15			-1.00	=1+$B$2	-1.00				
6	**Time 3**			1.28	-1.00			=1+$B$2	-1.00			
7	**Time 4**	1.40			1.15	1.32			=1+$B$2			
8												
9	**Max to invest**	$500	$500	$500	$500	$500	$1,000,000	$1,000,000	$1,000,000			
10												
11	**Amount Invested**	$100	$100	$100	$100	$100	$100	$100	$100		=IF(AND(Amount_Invested<	
12												
13		**Cash flows from investments**								**Cash in**	**Cash out**	
14	**Time 1**	=B4*B$11	=C4*C$11	=D4*D$11	=E4*E$11	=F4*F$11	=G4*G$11	=H4*H$11	=I4*I$11	$1,000	=SUM(B14:J14)	
15	**Time 2**	=B5*B$11	=C5*C$11	=D5*D$11	=E5*E$11	=F5*F$11	=G5*G$11	=H5*H$11	=I5*I$11	$0	=SUM(B15:J15)	
16	**Time 3**	=B6*B$11	=C6*C$11	=D6*D$11	=E6*E$11	=F6*F$11	=G6*G$11	=H6*H$11	=I6*I$11	$0	=SUM(B16:J16)	
17	**Time 4**	=B7*B$11	=C7*C$11	=D7*D$11	=E7*E$11	=F7*F$11	=G7*G$11	=H7*H$11	=I7*I$11		=SUM(B17:J17)	Final balan
18												
19	Array-entered (Control+Shift+Enter) formula in K11: =IF(AND(Amount_Invested<=Max_to_invest,Cash_out>=0),"Feasible","Not Feasible")											
20	Enter =B4*B$11 in cell B14 and copy to cells B14:I17											
21	Enter =SUM(B14:J14) in cell K14 and copy to K14:K17											

Figure 22.7 Solver Dialog Box



Set Target Cell: Final_balan

Equal To: ● Max ○ Min ○ Value of: 0

By Changing Cells:

Amount_Invested

Subject to the Constraints:

Amount_Invested <= Max_to_invest
Cash_out >= 0