TUTORIAL BOOK
FLUID MECHANIC
DDJ 3303

Universiti Teknologi Malaysia

Tutorial entry contains the following materials

1. **Flow Theory Disabilities Overview**
   Flow Tutorial Disabilities
   Final Examination Flow Disabilities compile

2. **Boundary Layer Flow Theory Overview**
   Boundary Layer Flow Tutorial
   Final Examination Boundary Layer Flow compile

3. **Simple Theory Turbo Machine**
   Turbo Machine Tutorial
   Final Examination Turbo Machine compile

Note to all students DDJ 3303

- **You are advised to solve all questions in this Tutorial Notes,**

- **According to our experience, in each semester, the percentage of students who pass excellently in this subjects of FLUID MECHANICS II is VERY LOW once**

- **We pray that hopefully with these notes you can help yourself to enhance your appreciation of the core subjects, and thus help us to pass your exams and tests of SMJ 3303.**

Prof amer n darus, c23 430 h/p:019 3239491

TURBO MACHINE OVERVIEW

UNIVERSITI TEKNOLOGI MALAYSIA

We can observe a few things about Turbo Machine:

a. Beat Turbine: Pelton Wheel

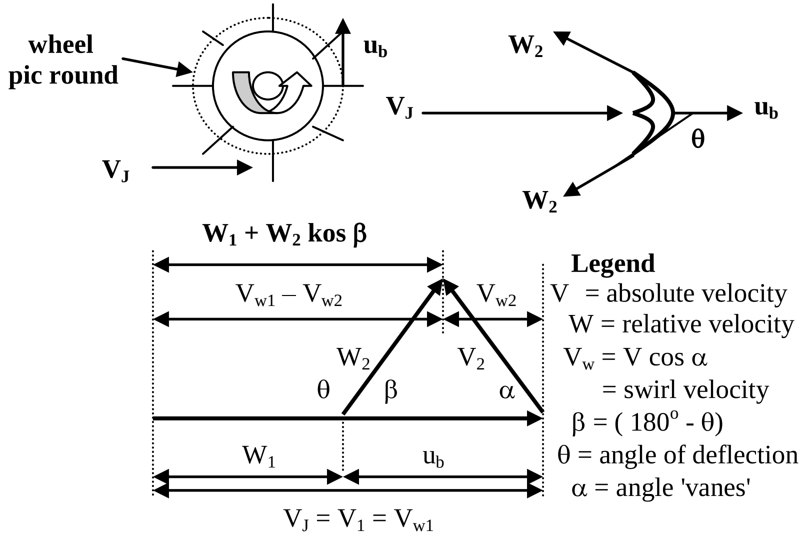


Diagram 3.1: The important thing of Pelton wheel

Jet velocity, $V_J = C_v \sqrt{2gH_B}$, head jet $h_{KE} = V_J^2/(C_v^2\ 2g)$

Effective head, H_B = Gross head - Penstok pipe friction head

$$= H_{rough} - (4fL/d)(u^2/2g)$$

with L and d respectively was the pipe length and diameter penstok; f and u also is the friction factor and flow velocity in the pipe.

Euler column = column generated by the engine turbo = [(UVW) 1 - (UVW) 2] / g

Special for Pelton turbine, U1 = U2 = ub = □ ND/60: D = pitch diameter wheels.

Euler column Pelton turbine,

$$h_E = (u_b/g)[V_{w1} - V_{w2}] = (u_b/g)[W_1 + k\ W_2 \cos \beta]$$

$$= (u_b/g)\ [(V_J - u_b)(1 + k \text{ kos } \beta)$$

with W1 = relative velocity = VJ - ub; k = 1 if there is no friction on the surface of the blade / scoop / dipper.

The power gained by Pelton turbine, Po

$= \rho g Q h_E = \rho g Q[(u_b/g)\ [(V_J - u_b)(1 + k \text{ kos } \beta)]$

Power provided by the nozzle, $P_{in} = \rho g Q h_{KE}$

$= \rho g Q\ [V_J^2/(C_v^2\ 2g)]$

Hydraulic efficiency, $\eta_h = h_E/h_{KE}$

$= 2C_v^2\ \phi\ (1-\phi)(1+ k \cos \beta)$; where $\beta = (180^o - \theta)$

Max efficiency. achieved when $d\eta_h/d\phi = 0$. This occurred in $\phi = ½$. In practice, $(\eta_h)_{maks}$ occur to $0{:}46 \leq \phi \leq 0.48$.

Reaction turbines: Francis turbine and Kaplan turbine.

Important feature: all stagnant water turbine system, from head to tail race race filled with water; mechanical energy derived from the transformation of energy by hydrodynamic pressure and also by the kinetic energy of water through the rotor blade. The choice of whether to install turbines Francis or Kaplan turbine based on the magnitude of the specific speed, $N_S = NP^{1/2}H^{-5/4}$. Typically for low H will produce large N_S, Kaplan turbine used. For high head H will produce small N_s, Pelton turbine installed.

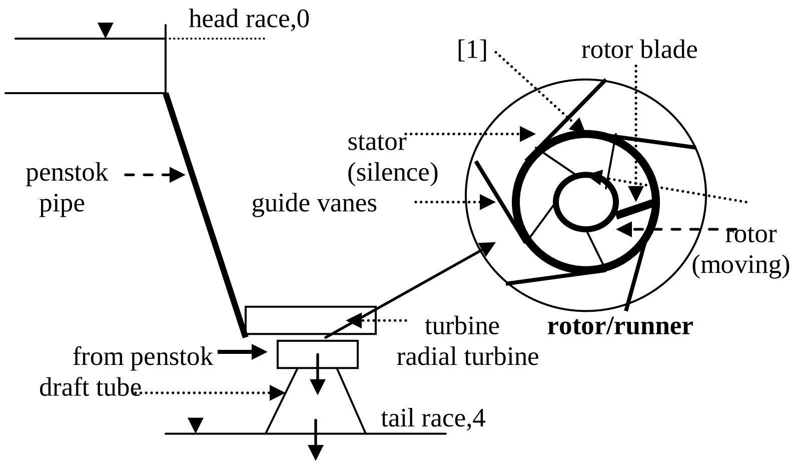


Figure 3.2: Enterence turbine floe

Apply Eq. energy from 0 → 4: $h_T = (H_{rough} - h_{Lpenstok}) - (h_{Lturbine})$; $h_{Lpenstok}$ is the pipe friction head losses $[(4fL/d)(u^2/2g)]$; $h_{Lturbine}$ the loss on the host, guide vanes, blade motion and the draft tube. So, $h_T = H_B - h_{Lturbine}$; where $\mathbf{H_B}$ **= effective head =** $\mathbf{H_{rough} - h_{Lpenstok}}$

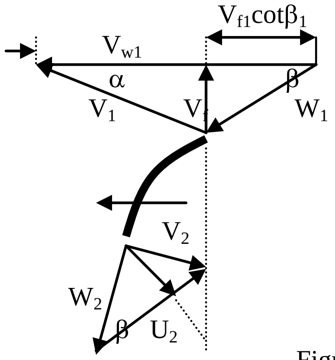
b. Francis turbine

Substitution of Eq. of energy [1]→[2] rotor, see Figure 3.2.

$$H_{01} = H_{02} + h_E + h_{Lroto} \Rightarrow h_E = H_{01} - H_{02} - h_{Lroto}$$

With H_0 = total head = $(p/\rho g + V^2/2g + z)$; h_E is the head Euler for reaction turbines, $\mathbf{h_E = [(UV_w)_1 - (UV_w)_2]/g}$

See the velocity triangles.



Legend

V = absolute velocity

U = velocity of rotation

W = relative velocity

V_w = whirl velocity = V cos α

V_f = flow velocity = V sin α

α = angle of guide vanes,

β = blade angle motion

Figure 3.3: Diagram velocity triangles reaction turbine

Typically $V_{w2} = 0$, , is radial oulflow, out without shock,

So, $\mathbf{h_E = U_1V_{w1}/g} = U_1[U_1 - V_{f1}\cot\beta]/g$;

With, $U_1 = V_{f1}[\cot\beta_1 + \cot\alpha_1]$

With $U = \pi DN/60$ = rotational speed,

Raised by the turbine power, $P_o = \rho gQh_E = \rho gQ[U_1V_{w1}/g]$

Available power, $P_{in} = \rho gQH_B$; $Q = \pi bDV_f$

With b = blade height, D = diameter rotor, V_f = radial velocity

Hydraulic efficiency,

$$\eta_h = U_1V_{w1}/gH_B$$

The overall efficiency,

$$\eta_0 = \eta_{hidraulik}\eta_{mekanikal}\,\eta_{volumatrik}$$

c. Turbin Kaplan

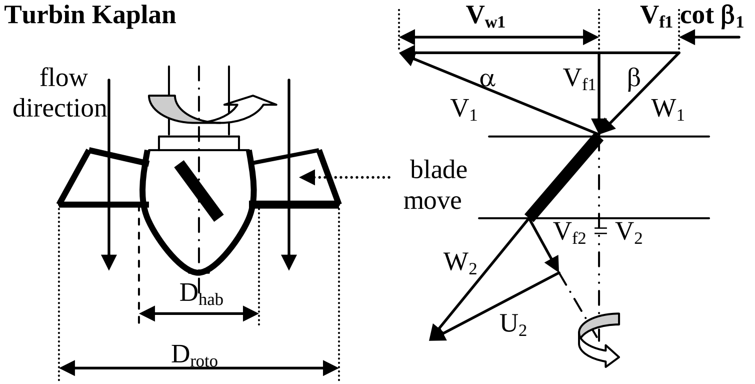


Figure 3.4: Kaplan Turbin

Here, the flow rate is determined from this eq, $Q = (\pi/4)(D^2 - D_{hab}^2)V_f$

Usually the Kaplan turbine hydrodynamic analysis identified,

Flow coefficient, $\psi = V_f/(\sqrt{2gH_B})$; V_f = radial velocity

Coefficient of velocity, $\phi = U/(\sqrt{2gH_B})$; U = velocity of rotation

Euler head with $V_{w2} = 0$, $\Rightarrow h_E = U_1V_{w1}/g$; net head = H_B

Turbine power, $P_o = \rho gQh_E = \rho gQ[U_1V_{w1}/g]$

Power $P_{in} = \rho gQH_B$; $Q = \pi bDV_f$

With b = blade height, D = diameter rotor, V_f = radial velocity

Hydraulic efficiency,

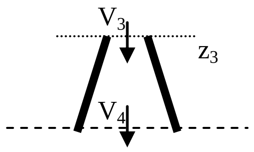
$$\eta_h = U_1 V_{w1}/gH_B$$

The overall efficiency,

$$\eta_0 = \eta_{hidraulik}\eta_{mekanikal}\ \eta_{volumatrik}$$

Draf tube

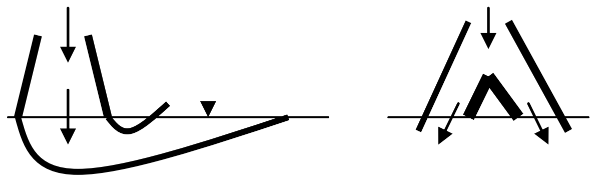
Draft tube is a reaction turbine. Draft tube should be installed because, 1. to capture a kinetic energy of water out of the turbine, $V_4^2/2g$ become smaller, 2. increase the effective head, 3. to facilitate the maintenance of plant and set repair work later.



$$p_3/\rho g + V_3^2/2g + z_3 = p_4/\rho g + V_4^2/2g + z_4 + h_{Ltd}$$

Figure 3.5: Draft tube

There are several types of draft tubes: 1. Types of cones, 2. Types of angles, 3. Spreading type. Draft tube in Figure 3.5 is of the conical type.



Angle type spreading type

Figure 3.6: The shape of draft tube

From the eq. energy that has been written above we see p_3 pressure can fall to a lower extent. But this should be controlled not to pass water vapor pressure.

$$(p_3 - p_4)/\rho g = (z_4 - z_3) + (V_4^2 - V_3^2)/2g + \Sigma h_{L34}$$

From the eq. continuity, $A_3V_3 = A_4V_4 \Rightarrow V_3/V_4 = A_4/A_3 = d_4^2/d_3^2$

Thus,

$$(p_3 - p_4)/\rho g = -L_{td} + (V_4^2/2g)[1 - (d_4^2/d_3^2)^2] + \Sigma h_{L34}$$

Look at the last equation $[1 - (d_4^2/d_3^2)^2]$ is negative. Then p_3 pressure [gauge] is always negative.

2. Hydrodynamic pump

If the turbine is a hydrodynamic device that embodies energy into mechanical energy hydro, the turn the pump is a device that transforms mechanical energy into hydro power. In principle, if the radial turbine rotation is reversed, the direction of rotation is reversed, we obtain a pump. Just as turbines, pumps can also be classified according to the operation and direction of rotation rotor work. For rotor pump known as the impeller.

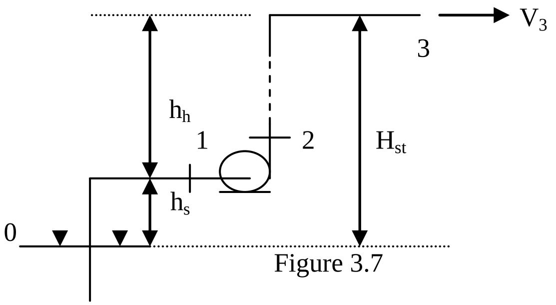


Figure 3.7

We see that happen head distribution systems, station [0] on the surface of water [3] the station end of the pipe form. We apply Eq. energy [PT] from one station to another station.

PT0-3: $p_o/\rho g+V_o^2/2g+z_o+h_p= p_3/\rho g+V_3^2/2g+z_3 + \Sigma h_{L03}$
PT0-1: $p_o/\rho g+V_o^2/2g+z_o = p_1/\rho g+V_1^2/2g+z_1+ \Sigma h_{L01}$
PT1-2: $p_1/\rho g+V_2^2/2g+z_2 + h_E = p_2/\rho g+V_2^2/2g+z_2+\Sigma h_{L12}$
PT2-3: $p_2/\rho g+V_2^2/2g+z_2 = p_3/\rho g+V_3^2/2g+z_3+\Sigma h_{L03}$

From Figure 3.7: $p_o = p_3 = 0$[gage], $V_o = 0$, $z_1-z_o=h_s, z_3-z_2=h_h$, $z_3-z_o=h_h$, h_L = head loss by a variety of reasons, h_p supply head and h_E = Euler head [see turbines]. Thus,

PT0-3: $z_o+h_p=V_3^2/2g+z_3 + \Sigma h_{L03} \Rightarrow h_p = H_{st} + \Sigma h_{L03}$
PT0-1: $z_o = p_1/\rho g+V_1^2/2g+z_1+ \Sigma h_{L01} \Rightarrow p_1/\rho g= -h_s-V_1^2/2g+h_{LP}$
PT1-2: $p_1/\rho g+V_2^2/2g+z_2 + h_E = p_2/\rho g+V_2^2/2g+z_2+\Sigma h_{L12}$, or
$(p_2/\rho g+V_2^2/2g)-(p_1/\rho g+V_1^2/2g) = h_E - h_{Lrotor+stator}$
PT2-3: $p_2/\rho g+V_2^2/2g+z_2 = V_3^2/2g+z_3+\Sigma h_{L03}$, or
$p_2/\rho g+V_2^2/2g+z_2 = V_3^2/2g+z_3+\Sigma h_{L03}$, or
$p_2/\rho g = h_h + (V_3^2/2g- V_2^2/2g) + h_{Lph}$

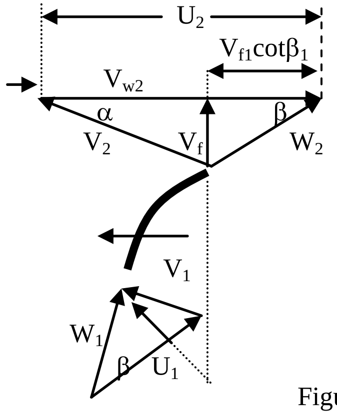
From this analysis, we get:

1. $h_p = H_{st} + \Sigma h_{L03}$ is the head system resistance,
2. $(p_2/\rho g+V_2^2/2g)-(p_1/\rho g+V_1^2/2g) = H_m$, manometric head
3. $h_E = H_M + h_{Lroto + stator}$, Euler head
4. $p_1/\rho g= -h_s-V_1^2/2g+h_{LP}$, suction head [usually negative]
5. $p_2/\rho g = h_h + (V_3^2/2g- V_2^2/2g) + h_{Lph}$, head post

So: h_p is the head supplied by the pump motor, used to overcome the static head and friction head pipe and pump system.
Head manometer, H_M is the head created by pump. We note as well if all forms of head losses in the pump are ignored then the Euler head = head manometer.
Euler Head, h_E is the head generated by the impeller rotation, as turbines, ie

$$h_E = [U_2V_{w2} - U_1V_{w1}]/g$$

See the velocity triangle of the pump.



Legend
V = absolute velocity
U = velocity of rotation
W = relative velocity
V_w = whirl velocity = V kos α
V_f = radial velocity = V sin α
α = angle of guide vanes,
β = blade angle motion

Figure 3.8: Diagram velocity triangles
Pump rotodynamic

Euler head may also be expressed as follows,

$$h_E = (V_2^2 - V_1^2)/2g + (U_2^2-U_1^2)/2g + (W_1^2-W_2^2)/2g$$

At the outset, V = 0, W = 0. So to start the pumping process, $h_E = (U_2^2-U_1^2)/2g$ with $U = \pi ND/60$ until

$$h_E = (\pi N/60)^2[1/D_2^2 - 1/D_1^2] = N^2\,[\pi^2(1/D_2^2 - 1/D_1^2)/3600]$$

Thus minimum rotation to start pumping :

$$N^2 = 3600\,h_E/[\pi^2(1/D_2^2 - 1/D_1^2)]$$

Pump efficiency : it is similar to the efficiency of a turbine.,

1. Manometer efficiency, $\eta_h = H_m/h_E = 1 - \Sigma h_L/h_E$
2. Mechanical efficiency, $\eta_m = (\rho g Q_{ung} h_E/P_{motor}$
   $\eta_{mechanical}$= power at pump shaft/power supplied by the motor
3. Volume efficiency, $\eta_v = Q_{actual}/Q_{unggul}$

Overall efficiency, $\eta_0 = \eta_h\,\eta_m\,\eta_v$

$$\eta_0 = \rho g Q H_M/P_{motor}$$
$$= \rho g\,(\eta_v Q_{unggul})\,(\eta_h h_E)/P_{motor} = \eta_h\eta_v(\rho g Q_{ung} h_E/P_{motor}) = \eta_h\eta_m\eta_v$$

Pump performance and the effect of the angle β_2:

Consider,

$$h_E = U_2 V_{w2}/g = U_2[U_2 - V_{f2}\mathrm{kot}\beta_2]/g$$

If we assume, Σh_L zero, thus $h_E = H_m$. Flow velocity $V_f = Q/A_{eff}$ thus,

$$h_E = U_2^2/g - Q\,[\mathrm{kot}\beta_2/gA_{eff}]$$

This last equation is a linear equation with respect to H and Q. We plot to see the importance in the theory of the pump!

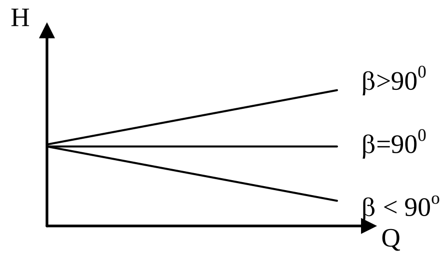
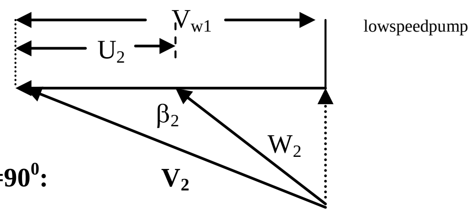
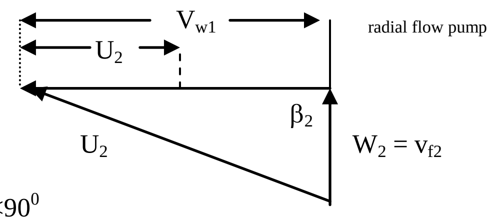


Diagram 3.9: Effect of β_2 to the curve H-Q pump

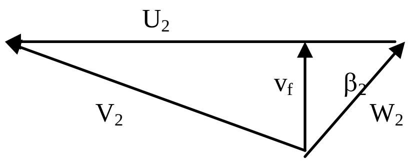
Case 1: $\beta_2 > 90^0$, triangular diagram at the exit become



Case 2: β=90^0:



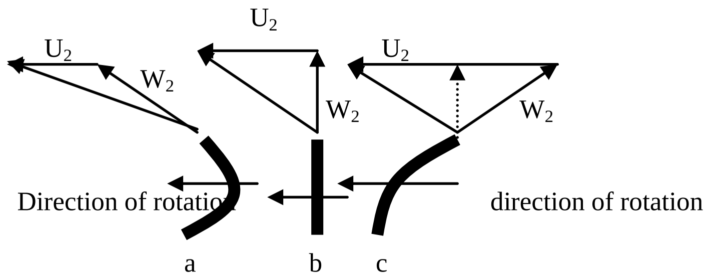
Case 3: β<90^0



Most widely used pump

Diagram 3:10: Three forms of the velocity triangle at the end.

Besides that, angle β_2 also determined the shape of the blade.



a. forward curved blade,
b. radial blade,
c. backward curved blade

Diagram 3.9 : pump impeller blade shape

Specific speed N_S : Magnitude N_S determine the shape of the pump impeller blades, and also the rotor blades of a turbine.

For pump: $N_S = NQ^{1/2}\ H^{-3/4}$

For turbine: $N_S = NP^{1/2}\ H^{-5/4}$

Expression N_S can be derived directly from the head coefficient K_H, power coefficient K_P and flow coefficient K_Q, which is:-

$$K_H = H/N^2D^2,\ K_P = P/\rho N^3D^5 \text{ dan } K_Q = Q/ND^3$$

For N_S turbine use K_H and K_P, remove D.
For N_S pump use K_H and K_Q, remove D

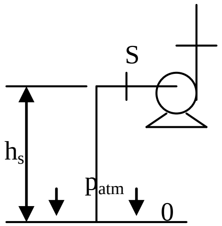
Pump type	radial flow	mixture flow	axial flow
Value N_S	10 until 80	80 until 160	above 160

Turbine type	Pelton wheel	radial flow	axial flow
Value N_S	12 until 70	70 until 400	above 400

Specific speed N_S a pump is a speed required by any similar pump to pump a total of 1 m3 / s of fluid of height 1 m. Specific speed of a turbine NS is the speed required by any similar turbine to produce 1 kW turbine work when the column was 1 m.

[Net positive Suction Head NPSH]

NPSH is the pressure difference at the inlet to the column corresponding to the vapor pressure of the pumped fluid. Refer to the diagram below.



Apply PT 0 - 1

$$p_o/\rho g+V_o^2/2g+z_o = p_1/\rho g+V_1^2/2g+z_1+ \Sigma h_{L01}$$

$$p_1/\rho g = p_o/\rho g - [V_1^2/2g+h_s+ \Sigma h_{L01}]$$

pump will operate properly when the pressure p1 is greater than pv, water vapor pressure. We mark the difference between $p_1/\rho g$ with this $p_v/\rho g$ with H_{sv}

Thus;

$$H_{sv} = p_1/\rho g- p_v/\rho g = p_{atm}/\rho g - [V_1^2/2g+h_s+ \Sigma h_{L01}]- p_v/\rho g$$

Or,

$$\mathbf{H_{sv} = H_{atm} - H_{static} - H_{vapor} \equiv NPSH}$$

[cavitation]

Cavitation occur when $p_1 < p_{vapor}$. At the pump or turbine parameters Thoma σ used as an indicator for the occurance of cavitation, this Thoma parameters is defined as

$$\boldsymbol{\sigma} = [H_{atm} - H_{static} - H_{vapor}]/H_{manometer} = \mathbf{H_{sv}/H_{manometer}}$$

$$= (N_S/[NQ^{1/2}\ (NPSH)^{-3/4}]\}^{4/3}$$

Note: H_{atm} = 10.29 m, H_{vapor} = 0.224 m

SUMMARY

FIELD THEORY
FLOW CAPACITY

Universiti Teknologi Malaysia

SUMMARY OF CAPACITY FLOW THEORY
Potential Flow Field Theory

We note some result of potential flow that we have learned in 3 weeks ago.

1. What is potential flow field?

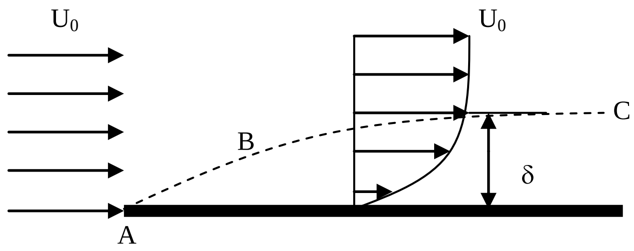


Diagram T1.0

Real fluid field on the plate can be divided into two parts, which is

1. $0 \leq y \leq \delta$ - in this region, the effect of viscous forces and inertial forces are comparable, or even identical, equally important, the velocity profile is sloping up in the fluid until shear stress exists, $\tau = \mu(du/dy)$, so with this one element of fluid in the region will rotate about its center point itself. This rotation raises the vorticity.
2. $y > \delta$ - in this region the effect of the inertia is higher than the viscous forces, we found that velocity profile is fixed to normal, which is $u = U_0$, one variable; until $(du/dy) = 0$, so there is no stress exist. A fluid element that is placed in the region will not spin. So there is no vorticity in the field.

A potential flow field is the flow field that in it does not exist any element of vorticity. Because of the vorticity is zero, thus flow velocity field can be represented by a continuous function called the potential function, $\phi = \phi(x,y,z)$ or $\phi = \phi(r, \theta,z)$.

2 Give the expression in terms of vorticity flow velocity components.
Vorticity is a vector quantity; vorticity is defined mathematically as follows,

$$\mathbf{\Omega} = \nabla \times \mathbf{V} \qquad [1.1$$

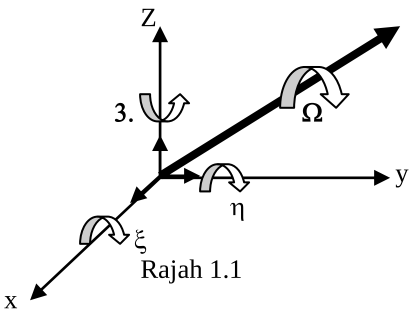
by $\mathbf{\Omega} = \mathbf{i}\xi + \mathbf{j}\eta + \mathbf{k}\varsigma$ and $\mathbf{V} = \mathbf{i}u + \mathbf{j}v + \mathbf{k}w$. so [1.1] can be further expanded as follows,

$$\nabla \times \mathbf{V} = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \mathbf{i}\xi + \mathbf{j}\eta + \mathbf{k}\varsigma \qquad [1.2$$

clear that,

$$\xi = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \ \eta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial y}, \varsigma = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \qquad [1.3$$

Please refer to the table below for the physical meaning of the components ξ , η and ς an earlier vorticity vector.



Rajah 1.1

3.0 Show the dimensions of the flow field - 2, only k components of Ω only exists.
Dimensional flow field - 2, the component of velocity in the direction of k is zero, ie w = 0. Now consider,

$$\xi = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \ \eta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial y}, \varsigma = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \qquad [1.3$$

Velocity components v and u may not be a function z. This means that,

$$u = u(x,y) \text{ or } v = v(x,y)$$

It is clear that, from [1.3] for two dimensional flow field, only the z component of **Ω** only the existence of,

$$\varsigma = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \qquad [1.4$$

4.0 Derive Eq. [1.4] from basic principles.
The rate of rotation of a fluid element in the field can be defined as,

$$\omega = \frac{\alpha + \beta}{2dt} \qquad [a$$

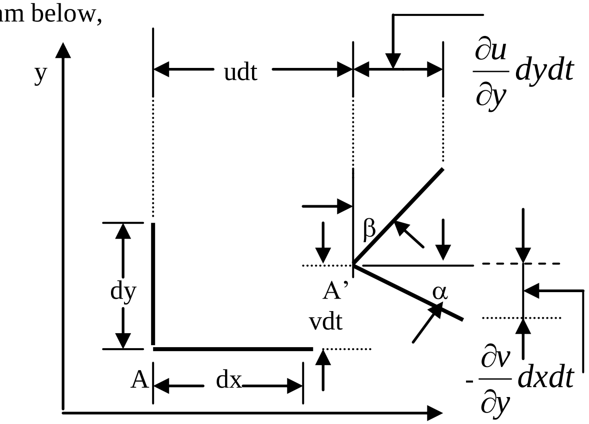
by α and β respectively - each is a corner that was built by the side of the element when creasing [deforms] during the cross-field flow.

With reference to the figure below, then

$$\alpha = \frac{\partial v}{\partial x} dt \ \beta \qquad [b$$

$$\beta = -\frac{\partial u}{\partial y} dt \qquad [c$$

Consider the diagram below,



Rajah 1.2

By replacing [b] and [c] into [a] we have,

$$\omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad [1.5$$

So, $$2\omega = \varsigma = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \qquad [1.6$$

Eqn. [1.7] shows,

$$\mathbf{2\omega = \varsigma} \qquad [1.7$$

Equation [1.7] once shows that when the fluid element does not rotate so, the vorticity is zero

5. Show, when the vorticity of a flow field is zero, then the velocity field can be represented by a function of the scale, ϕ(x,y).
This can be readily shown by vector. According to vector analysis,

$$\text{if } \nabla \times V = 0, \text{ so } V = \nabla\phi \qquad [1.8$$

Therefore,,

$$u = \frac{\partial \phi}{\partial x}, \ v = \frac{\partial \phi}{\partial y}, \text{ and } w = \frac{\partial \phi}{\partial z} \ [1.9$$

If we specify the field dimensions - 2 only so w = 0. Equation [1.9] in polar coordinates,

$$u_r = \frac{\partial \phi}{\partial r}, \text{ and } u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} \qquad [1.20$$

Now we consider the same in scale. Suppose there ϕ =ϕ(x,y) are continuous. Due ϕ(x,y) is continuous, then the sequence of differential or 'issue' is not important. This means that,

$$\frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial^2 \phi}{\partial y \partial x}$$

Or,

$$\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial x}\right) \qquad [1.21$$

Next see equation [4.11], which

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

Compare with equation [1.21]. Thus, it is clear that,

$$u = \frac{\partial \phi}{\partial x}, \text{ dan } v = \frac{\partial \phi}{\partial y}$$

Note that the last equation is consistent with equation [1.9]!

6. What is an easy flow fields, and expressed in terms of a simple stream function disabilities.

Simple flow fields, or 'simple' is the flow field has only one component of velocity in a certain direction. A simple example of the flow field

,

i. Uniform flow, $\mathbf{V} = U_0\mathbf{i}$, ie $u = U_0$,

ii. Source/sink flow, $\mathbf{V} = U_r\,\mathbf{e_r}$, $u_r = U_r$

iii. Vortecs flow, $V = U_\theta \mathbf{e_\theta}$, ie $u_\theta = U_\theta$

From eqn. [1.9], for uniform flow,,

$$\frac{d\phi}{dx} = U_0 \text{ to } \phi(x,y) = U_0 x + f(y)$$

From eqn.[1.20], to a source,

$$\frac{\partial \phi}{\partial r} = u_r = \frac{C}{r}, \text{ to } \phi(r,\theta) = C \ln r + f(\theta)$$

and, to a vortex, for a vortex flow,

$$U_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = \frac{C}{r}, \text{to } \phi(r,\theta) = C\theta + f(r)$$

7. Indicate whether the conditions have to be acquired by a flow field to the flow field may be regarded as a potential flow field?

Conditions is necessary is that, in the flow field should be free from any element of the vorticity. In more simple, we can say, functional disability $\phi(x,y)$ exist only if **∇ x V = Ω = 0.**

Note: *Note that the flow field first. do not have well-established, 2. no need to dimension 2,*

8. What is the current function?

Stream function $\psi(x,y)$ or $\psi(r,\theta)$ is a continuous function Looks exist in onedimensional flow field - 2, which satisfy the following definition,

$$u = \frac{\partial \psi}{\partial y}, \text{ and } v = -\frac{\partial \psi}{\partial x}$$

[1.22

or,

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \text{ and } u_\theta = -\frac{\partial \psi}{\partial r}$$

until dimensional continuity equation - 2 met identically, namely

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \text{ or } \frac{\partial (r u_r)}{\partial r} + \frac{\partial u_\theta}{\partial \theta} = 0 \qquad [1.23$$

9. What are the conditions of existence of the current function?

The flow field must be dimensioned - 2, and steady.

.

Note: *The free flow filed of vorticity is unnecessary. So, in all forms of flow field can be defined as the current function as long as the flow field is in two-dimensional.*

10. Write the Cauchy-Reimann relations?

Cauhy-Reimann relations,

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}, \text{ and } v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \qquad [1.24$$

In polar coordinate,

$$u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}, \text{ and } u_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad [1.25$$

12. Show that ϕ(x, y) and ψ(x, y) are harmonic.

A function ξ is harmonic if it is comply with the Laplace equation, $\nabla^2 \xi = 0$.

To prove that ϕ(x, y) is harmonic, substitute $u = \frac{\partial \phi}{\partial x}$ and $v = \frac{\partial \phi}{\partial y}$ into continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

Thus,

$$\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial y}\right) = 0, \text{ or } \nabla^2 \phi = 0.$$

To prove that ψ(x, y) is harmonic, substitute $u = \frac{\partial \psi}{\partial y}$, and $v = -\frac{\partial \psi}{\partial x}$ into vorticity equation, $\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$. We get,

$\nabla^2 \psi = 0$.

12. What is the importance of harmonic ϕ(x, y) and ψ(x, y) function?

A harmonic function is when it is satisfies the Laplace equation. Stream function ψ(x, y) and potential function ϕ(x, y) or polar coordinate are harmonic. Superposition method can be used because Laplace equation is harmonic. If a form of ψ or ϕ is complex, then the function of ψ or simple function ϕ can be used. The functions are combined together to form a complex, ψ and ϕ earlier.

For example, let say we want to study about the flow of a rotating cylinder. The flow field can be expressed by the combinations of the uniform flow, the doublet flow and the vortex flow.

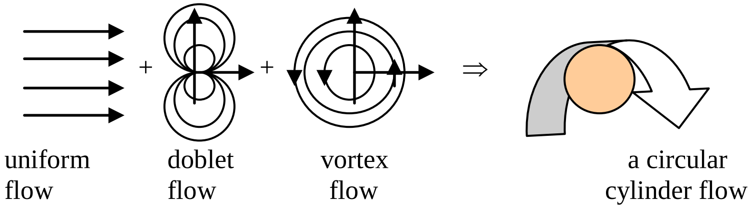


Figure C11.0

13. What is a simple flow field? Give some examples of simple flow fields.
A simple flow field is the basic flow field. Usually, the velocity field V only has one component.

Examples of the simple flow fields:

1. Uniform flow, $\mathbf{V} = U_0\mathbf{i}$
2. Sink flow, $\mathbf{V} = U_r\mathbf{e_r}$
3. Vortex flow, $\mathbf{V} = U_\theta\mathbf{e_\theta}$

Please refer 6.1 above!

14. Determine the current function ψ and potential function ϕ for the simple flow a. source, b. vortex and c. uniform flow.

Cauchy – Reimann relations, HCR is

$$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial\psi}{\partial r}, \text{ and } u_r = \frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

a. Flows resources only have a radial velocity component, U_r. The two-dimensional flow resource is a flow arising from a line. So, from the continuity equation, we get,

$$U_r\, 2\pi rt = Q \text{ so, } U_r = Q/2\pi rt, \text{ where } t = 1.$$

Refer diagram below,

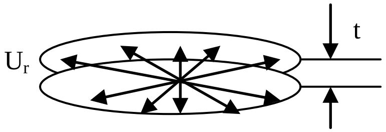


Diagram C13.0

Thus,

$$d\psi = [Q/2\pi r]\, dr \Rightarrow \psi(r,\theta) = (Q/2\pi) \ln r + f(\theta)$$

Note that, $U_\theta = 0$. So, $f(\theta) = C = 0$. Then,

$$\boldsymbol{\psi(r,\theta) = (Q/2\pi) \ln r}, \qquad [1.26$$

and,

$$\boldsymbol{\phi(r,\theta) = (Q/2r)\theta}, \qquad [1.27$$

for a flow resources. For the sink**,**

$$\boldsymbol{\psi(r,\theta) = (- Q/2\pi) \ln r.} \qquad [1.28$$

b. Vortex flow is a region within a fluid where the flow is mostly a swirling motion and only have a - θ component**,** $\mathbf{V} = U_\theta\, \mathbf{e_\theta}$. There are 2 types of vortex,
   1. Free vortex, $U_\theta = C/r$
   2. Forced vortex, $U_\theta = Cr$.

Which is the irrotational flow field?

If **∇ x V = Ω = 0,** the velocity field V is not rotate. In polar coordinates, **∇ x V = Ω = 0,** for two–dimensional field,

$$\varsigma = \frac{1}{r}\frac{\partial(rU_\theta)}{\partial r} - \frac{1}{r}\frac{\partial U_r}{\partial\theta} = 0$$

For the irrotational flow, $\varsigma = 0$. Because of $U_\theta \neq f(\theta)$ for the both types of vortex, the differential component of the velocity V against θ is equal to zero. But for forced vortex, $U_\theta = Cr$ whereas for free vortex, $U_\theta = C/r$. Therefore, $\varsigma=0$ is only for free vortex.

Both types of vortex have stream function but only free vortex has potential function.

From CR relations, a free vortex,

$$U_\theta = -\partial\psi/\partial r = \partial\phi/r\partial\theta = C/r$$

Thus,

$$\boldsymbol{\psi(r,\theta) = -C \ln r, \text{ and } \phi(r,\theta) = C\theta} \qquad \textbf{[1.29}$$

Compare the expression of a stream function and potential function of the free vortex flow with the flow of resources. You will find that the expression for the free vortex flow function is same as the stream source potential function. Besides, the vortex flow potential functions also same as the resource flow stream function.

This is clearly shown in the following diagram.

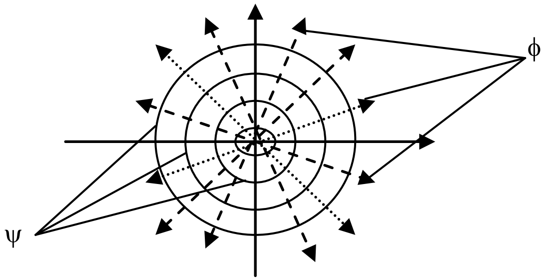


Figure C13.b: Free vortex flow

If the function of ψ and ϕ for a source was drawn, we get

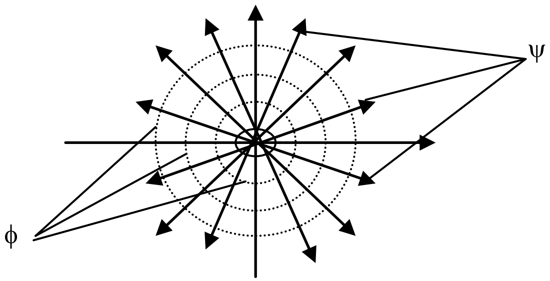


Figure C13.c: Flow resources

How to determine the constant, C?

Constant, C in a free vortex is determined directly from the definition of circulation, Γ. The distribution of Γ is the strength/ability of the vortex. It is similar to Q in a single source, or sink, Q is a source of strength, or sink.

14. Show that the circulation that includes the origin is finite, while the circulation which not includes the origin of vortex is zero.

Consider the diagram below.

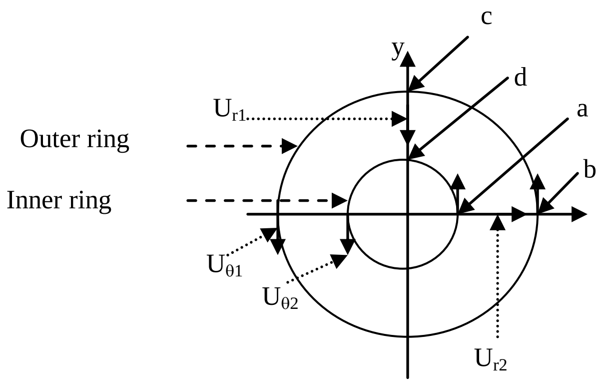


Figure C14.0

From the definition of circulation,

$$\Gamma = \oint_C \mathbf{V.\ ds} \qquad \textbf{[1.30}$$

Using closed integration.

First we consider, an integral value taken following the path on the outer shell or inner circle in Figure C14.0 above,

$$\mathbf{V} = U_\theta\, \mathbf{e_\theta} \text{ and } \mathbf{ds} = r d\theta\, \mathbf{e_\theta}$$

Then,

$$\Gamma = \oint_C U_\theta\, \mathbf{e_\theta} \,\mathbf{.}\, r d\theta\, \mathbf{e_\theta} = \oint_C (C/r)\, r d\theta = 2\pi C$$

That,

$$C = \Gamma/2\pi \qquad [1.31$$

Next, we consider a path that does not include the origin 0, ABCDA crossings, see Figure C14.0,

$$\Gamma = \int \mathbf{V\ .\ ds} = \int_a^b \mathbf{V\ .\ ds} + \int_b^c \mathbf{V\ .\ ds} + \int_c^d \mathbf{V\ .\ ds} + \int_d^a \mathbf{V\ .\ ds}$$

Or,

$$\Gamma_{abcda} = U_r\, dr + U_{\theta 1}\, r_1 d\theta - U_r dr - U_{\theta 2}\, r d\theta_2$$

For one vortex, $U_\theta = (\Gamma/2\pi)/r$. so,

$$\Gamma_{abcda} = U_r\, dr + [(\Gamma/2\pi)/r_1]\, r_1 d\theta_1 - U_r dr - [(\Gamma/2\pi)/r_2\,]\, r_2\, d\theta_2$$

On Figure C14.0, $d\theta_1 = d\theta_2 = \pi/4$, and for one vortex $U_r = 0$. ***Thus, Γ_{abcda} is zero.***

Now we look at the definition of circulation, ie, $\Gamma = \oint_C$ **V. ds.** As we all know, in a non-rotating flow fields, $\mathbf{V} = \nabla\phi$. This show,

$$\Gamma = \oint_C \mathbf{V\ .\ ds} = \oint_C \nabla\phi\ \mathbf{.\ ds} \qquad \textbf{[1.32}$$

Can be shown that, $\nabla\phi\mathbf{.\ ds} = d\phi$. With this,

$$\Gamma = \phi_a - \phi_a = \phi_b - \phi_b = 0$$

Not taking the integral on the outer shell or inner shell.

As occurred in a potential flow field, there is no circulation element that exists

This can be seen more clearly from the relationship between the distributions of vorticity.

15. Shown that in one potential flow field, flow is zero.
Considering definition of circulation,

$$\Gamma = \oint_C \mathbf{V\ .\ ds}$$

Using Green Theorem,

$$\Gamma = \oint_C \mathbf{V\ .\ ds} = \oint_C \nabla \times \mathbf{V.\ dA} = \oint_C \mathbf{\Omega.\ dA} = \Omega_n A \qquad [1.33$$

dA is the area with closed integral,∫.

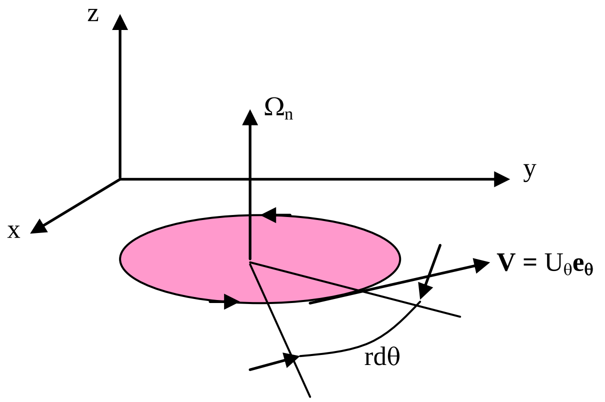


Figure C15.0

When, $\Omega_n = 0$, then $\Gamma = 0$.

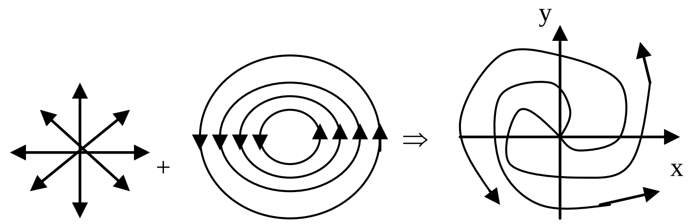
For field 2 - dim we have seen that the vorticity components that exist only, ς. From ***Subject 4.0***, we had shown that $2\omega = \varsigma$. so if $\varsigma = 0$, then $\omega = 0$. With $\omega = 0$, so $\Gamma = 0$.

16. Find the function of combine flow,

a. Between a source and a vortex
b. Between a source [-a,0] with a sink [a,0],
c. Between a vortex [0,-a] with vortexes [0,a],
d. Between doublet with the uniform flow,
e. Between doublet, uniform flow and vortexes,

a. combination of one source + one vortex

$$\psi = (Q/2\pi)\theta + (\Gamma/2\pi)\ln r \qquad [1.34$$



Rajah C16.0

To get a generated body, we set $\psi = 0$. Thus,

$$\psi = (Q/2\pi)\theta + (\Gamma/2\pi)\ln r = 0$$

That,

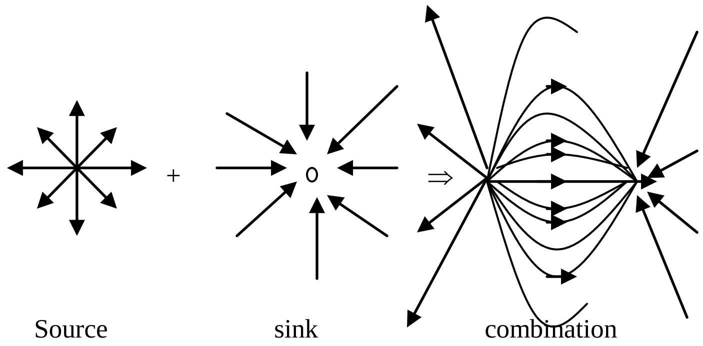
$$\ln r = -(Q/\Gamma)\theta$$

Or,

$$r = \exp[-(Q/\Gamma)\theta] \qquad [1.35$$

b. combination of source [-a.0] + sink [+a.0]

$$\psi = (Q/2\pi)[\theta_{source} - \theta_{sink}] \qquad [1.36$$



Rajah C16.a

An interesting flow occur when the source and sink overwritten, when a $\rightarrow$ 0. Observe the diagram below

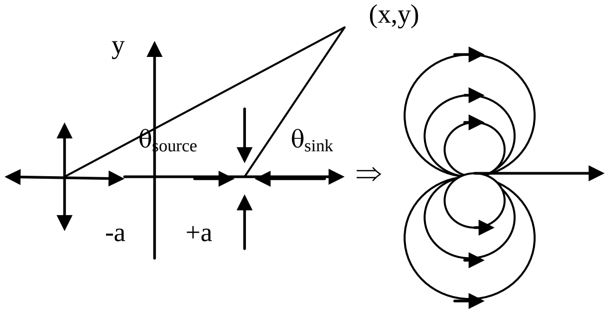


Figure C16.b

$$\theta_{Source} = \tan^{-1} [\, y/(x+a)],\ \theta_{sink} = \tan^{-1} [y/(x-a)]$$

So,

$$\theta_{Source} - \theta_{sink} = \tan^{-1}[y/(x+a)] - \tan^{-1}[y/(x-a)]$$

Which,

$$\tan^{-1} A - \tan^{-1} B = \tan^{-1} [(A-B)/(1 + AB)] \qquad [1.37$$

with that,

$$\psi_g = (Q/2\pi) \tan^{-1} \{[(y/(x+a) - y/(x-a)]/[1 + y^2/(x+a)(x-a)]\}$$

When simplify the relationship became, m = $Q/2\pi$

$$\psi_g = - m \tan^{-1} 2ay/(x^2+y^2-a^2) \qquad [1.38$$

In limit, when a $\rightarrow$0, 2am $\rightarrow$ μ, is a strenght doublet. Then we have,

$$\boldsymbol{\psi_g = - \mu y/(x^2 + y^2) = - \mu/(r \sin \theta)} \qquad [1.39$$

How does the expression of function disable ϕ for doublet flow?
How does the expression U_r and U_θ in a doublet flow?

A very interesting form of flow occurs when the uniform flow superposed doublets,

c. Overlapping between uniform flow with the flow of doublets.

Considering a uniform flow which cross the field of doublet flow.

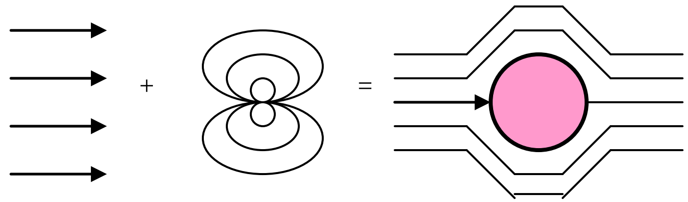


Figure C16.c

Combined flow field current function can be written as follows,

$$\boldsymbol{\psi_g = U\, r \sin \theta - (\mu \sin \theta/r)} \qquad \textbf{[1.40}$$

The body, $\boldsymbol{\psi_g = 0}$.

$$0 = Ur \sin\theta\, [\,1 - a^2/r^2] \text{ with } a^2 = \mu/U$$

Here , [Ur sinθ] $\neq$ 0. That, r = a, is a circle.

d. Combine vortex [0,-a] + vortex [0, +a]

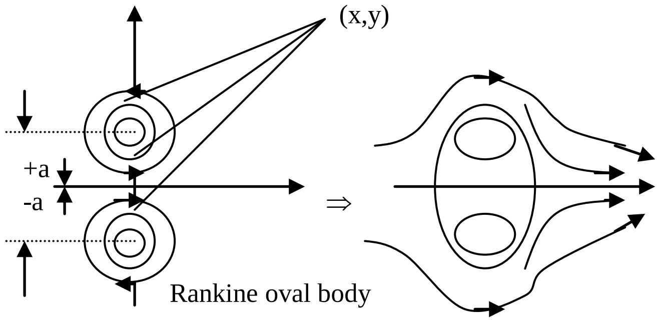


figure C16.d

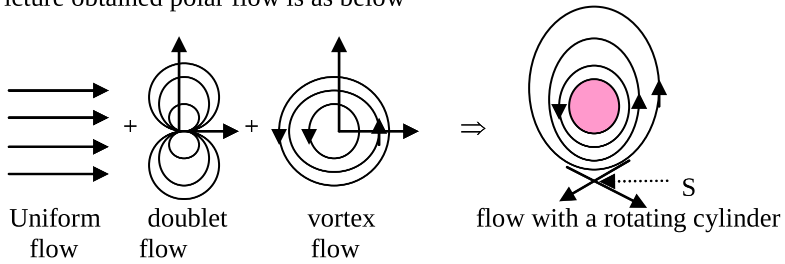
Combination of 2 vortexes,

$$\psi = (\Gamma/2\pi) \ln (r_1/r_2) \qquad [1.40$$

Try to remember, a vortex current function is disabled Similar to the functions of the sources, and functional capacity of the vortex is similar to the current function of the sources. So?

e. combination of uniform flow doublet + vortex

Picture obtained polar flow is as below



Rajah C11.0

Stagnation point position depends on magnitude Γ and U_0.

Combined flow stream function is:

$$\boldsymbol{\psi = U_0\, r \sin\theta\, [\, 1 - a^2/r^2] - (\Gamma/2\pi) \ln (\, r/a\,)} \qquad [1.41$$

This is a cylinder with a radius of rotating equipment.

Try to set r = a, what happen to ψ?

e. combination of uniform flow + source (-a,0) + sink (a,0)

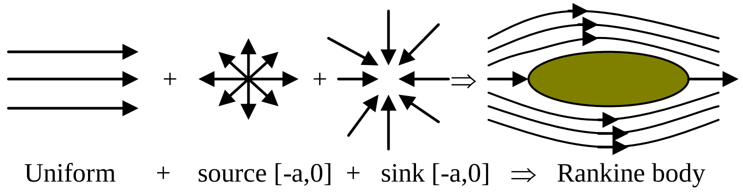


figure C16.e

Combined flow stream function,

$$\boldsymbol{U_0\, r \sin\theta + (Q/2\pi)[\theta_{source} - \theta_{sink}] = \psi_g} \qquad \textbf{[1.42}$$

Body's pattern, $\psi_g = 0$.

$$r = \frac{Q}{2\pi}\frac{[\theta_{\sin ki} - \theta_{source}]}{U_0 \sin\theta} \qquad [1.43$$

Body's length:

$$U_r = \frac{\partial \psi}{r \partial \theta} = U_0 \cos\theta + \frac{Q}{2\pi}[\frac{1}{r_1} - \frac{1}{r_2}]$$

And,

$$U_\theta = -\frac{\partial \psi}{\partial r} = U \sin\theta$$

Note:

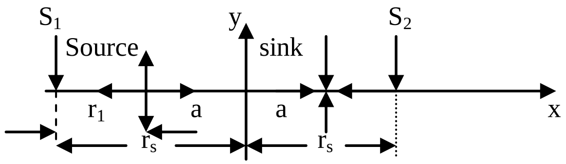


Figure C16.e

Stagnation point is a point in the flow field with the local velocity is zero, i.e. at the stagnation point, V = 0. Therefore, Ur = 0 and $U_\theta = 0$.

Thus,

$$U_\theta = -\frac{\partial \psi}{\partial r} = U \sin\theta = 0 \quad \Rightarrow \boldsymbol{\theta = 0, \pi,}$$

And,

$$U_r = \frac{\partial \psi}{r\partial \theta} = U_0 \cos\theta + \frac{Q}{2\pi}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 0$$

In S_2, θ = 0 and S_1, θ = π. Now from figure C16.e, in S_1, $r_1 = r_s - a$, and at S_2, $r_2 = r_S + a$. Therefore,

$$U_0 - \frac{Q}{2\pi}\left[\frac{1}{r_s - a} - \frac{1}{r_s + a}\right] = 0$$

Thus,

$$r_s = a\sqrt{\left(\frac{Q}{a\pi U_0}\right) + 1} \qquad [1.44$$

See the next to see the width and length of the body. Body length is,

$$L = 2r_s \qquad [1.45$$

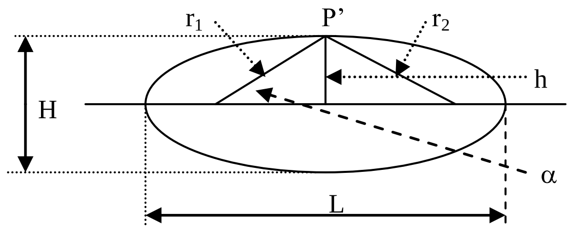


Figure C16.f

Body's width:
Referred from figure C16.f, $\theta_{source} = \alpha$, $\theta_{sink} = \pi - \alpha$, and θ = π/2. Thus by r = h in eq. [1.43] we produce,

$$h = \frac{m}{U_0}[\pi - 2\alpha]$$

Or,

$$\alpha = \frac{\pi}{2} - \frac{U_0 h}{2m}$$

From figure C16.f,

$$h = a \tan\alpha = a \cot\left(\frac{U_0 h}{2m}\right) \qquad [1.45$$

With m = Q/2π.

Height or width of the Rankine body,

$$H = 2h \qquad [1.46$$

Note that Eq. [1:45] well. Eqn. This is not linear. Try to find the value of h, when the value of U0, a and m = Q/2π given!

17. Show the drag force on an arbitrary object that is placed in an ideal flow field is zero. You can use a cylindrical body to facilitate analysis. Consider the diagram below.

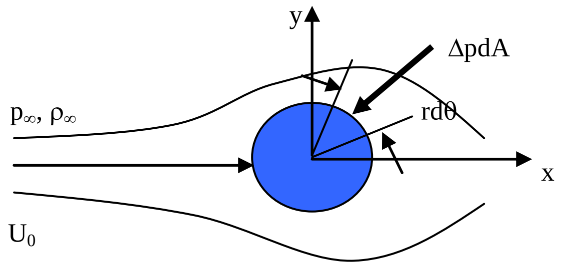


figure C17.0

Current function, cover flow cylinder of radius and we have produced, namely

$$\boldsymbol{\psi_g = U_0\ r \sin\theta - (\mu \sin\theta / r)}$$

With this,

$$U_r = \frac{\partial \psi}{r\partial\theta} = U_0 \cos\theta\ [1 - \frac{a^2}{r^2}],\ \text{and}$$

$$U_\theta = -\ \frac{\partial \psi}{\partial r} = U_0 \sin\theta\ [1 + \frac{a^2}{r^2}],$$

On the cylindrical surface, r = a. Then Ur = 0, and $\boldsymbol{U_\theta = 2U_0 \sin\theta}$.

Now apply the eq. Bernoulli on the second point. A distant point in front cylinder p_∞, ρ_∞ and the velocity U_0 and the other is located on the cylindrical surface with p_s, ρ_∞ and the velocity U_θ cause $U_r = 0$.

According to Bernoulli,

$$p_\infty + \frac{1}{2}\rho_\infty U_0^2 = p_s + \frac{1}{2}\rho_\infty U_\theta^2$$

That is,

$$\Delta p = p_s - p_\infty = \frac{1}{2}\rho_\infty U_0^2\ [1 - \left(\frac{U_\theta}{U_0}\right)^2]$$

$$= \frac{1}{2}\rho_\infty U_0^2\ [1 - 4\sin^2\theta] \qquad [1.47$$

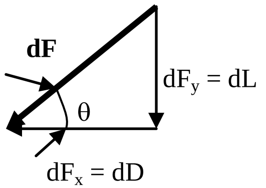
With reference to Figure C17.0, elemental forces acting on the area element dA is

$$dF = \Delta p\ dA = \Delta p\ rd\theta$$

Elements of this force can be resolved in the x and y, as

$$\mathbf{dF} = \mathbf{i}\ dD + \mathbf{j}\ dL \qquad [1.48$$

Refer to the diagram below.



With, $dF_x = \Delta p \cos\theta\ rd\theta$, and $dF_y = \Delta p \sin\theta\ rd\theta$

Figure C17.a

So,

$$dD = \frac{1}{2}\rho_\infty U_0^2 [1\text{- } 4 \sin^2 \theta] \cos \theta \, rd\theta, \qquad [1.48$$

And,

$$dL = \frac{1}{2}\rho_\infty U_0^2 [1\text{- } 4 \sin^2 \theta] \sin \theta \, rd\theta, \qquad [1.49$$

If Equation [1.48] and Equation [1.49] is integrated from θ = 0 to θ = 2π, we will find that the drag force, **D = 0 and the lift force, L = 0.**

Haulage condition is called d' Alembert Paradox.

For the condition where D = 0 and L = 0 can actually be seen in Equation [1.47]. If Equation [1.47] is plotted we will obtain the picture as shown below, which is the pressure distribution on the cylinder surface.

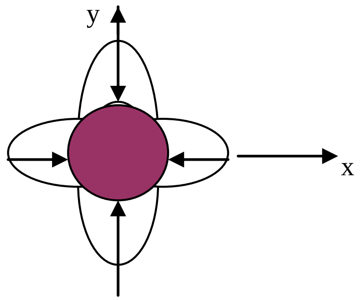


Diagram C17.b

Take note, the pressure distribution on the cylinder surface is symmetrical to the y axis and x axis. No excess force, in the positive and negative x or y. As evident, D = 0 and L = 0.

18. Proving the Kutta-Jouskousky Theorem, namely the lift force L = ρΓU.

From the previous matter 17.0, we can see that when a cylinder is placed in a potential flow field, then drag force D and lift force L, are both equaled to zero.

Next, we will see the pressure distribution on a rotating cylinder which is placed in a potential flow field. We will then determine D and L.

Consider the diagram below.

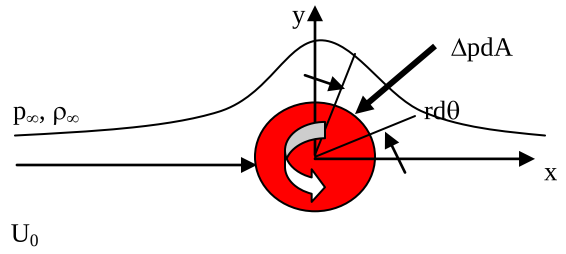


Diagram C18.0

This condition can be modeled by,

$$\boldsymbol{\psi = U_0 \, r \sin \theta \, [\, 1 - a^2/r^2] - (\Gamma/2\pi) \ln (\, r/a \,)}$$

Velocity component U_r and U_θ are,

$$U_r = U_0 \cos \theta \, [1 - a^2/r^2] \qquad [1.50$$

And,

$$U_\theta = - U_0 \sin \theta \, [1 + a^2/r^2] + (\Gamma/2\pi r) \qquad [1.51$$

Obviously, on the cylinder surface, r = a, U_r = 0. However,

$$U_\theta = - 2 \, U_0 \sin \theta + (\Gamma/2\pi a) \qquad [1.52$$

Now apply the Bernoulli Equation to two specific points. One point is placed distant and in front of the cylinder with p_∞, ρ_∞ and velocity U_0 and the other point is placed on the surface of the cylinder with p_s, ρ_∞ and velocity U_θ because $U_r = 0$. According to Bernoulli,

$$p_\infty + \frac{1}{2}\rho_\infty U_0^2 = p_s + \frac{1}{2}\rho_\infty U_\theta^2$$

Namely,

$$\Delta p = p_s - p_\infty = \frac{1}{2}\rho_\infty U_0^2 [1 - \left(\frac{U_\theta}{U_0}\right)^2]$$

or,

$$\mathbf{\Delta p} = \frac{1}{2}\rho_\infty U_0^2 \mathbf{[1- (-2\sin\theta + (\Gamma/2U_0\pi a)^2]} \qquad [1.53$$

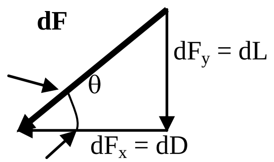
With reference to the diagram C 18.0, elements of the force acting on the area element dA is,

$$dF = \Delta p\, dA = \Delta p\, rd\theta$$

Elements of this force can be resolved in the x and y, as follows

$$\mathbf{dF} = \mathbf{i}\, dD + \mathbf{j}\, dL$$

Refer to the diagram below.



with, $dF_x = \Delta p \cos\theta\, ad\theta$, and $dF_y = \Delta p \sin\theta\, ad\theta$

Diagram C18.a

So,

$$dD = \frac{1}{2}\rho_\infty U_0^2 [\mathbf{1- (-2\sin\theta + (\Gamma/2U_0\pi a)^2}] \cos\theta\, rd\theta, \quad [1.54$$

And,

$$dL = \frac{1}{2}\rho_\infty U_0^2 [\mathbf{1- (-2\sin\theta + (\Gamma/2U_0\pi a)^2}] \sin\theta\, rd\theta, \quad [1.55$$

Next if we integrate dD and dL from $\theta = 0$ to $\theta = 2\pi$, we will find that,

$$D = 0$$

And,

$$L = - \rho\Gamma U_0 \qquad [1.56$$

Note:

1. We find out that in the potential flow field, despite the drag circulation remains zero. This condition is called d' Alembert's Paradox.

2. Equation [1.56] is known as Kutta-Jouskousky Theorem.

@prof amer/21/jun/2001

FLUID OF MECHANICS II: SMJ/SMM 3303
TAHUN 98/99 MAC 1999

Question 1.0

a. Define the vorticity, ξ and derive the vorticity equation in terms of terms of the components U_r and U_θ. Show that,

$$\xi = -\left(\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta} \right)$$

With ψ is the function of current.

[5 mark]

b. Polar velocity component of a current 2 – dimension given by,

$U_r = U \sin\theta \left(1 - \frac{a^2}{r^2}\right)$, and $U_\theta = U \cos\theta \left(1 + \frac{a^2}{r^2}\right) + \frac{k}{2\pi r}$

Show the function equation ψ and show the trend is irrotational

[14 marks]

c. Draw the diagram of the flow field 1(b) complete for moderate values of k distribution. Mark the stagnation point.

[5 marks]

Question 2

a. What is meant by velocity potential function ϕ. Obtain the relationship between the velocity potential ϕ with velocity components u and v in a flow field.

[5 marks]

b. The ability velocity of a flow field is given by,

$$\phi(x,y) = 10x + 5 \ln (x^2 + y^2)$$

Find the current function ψ (x,y) is the flow field.

[14 marks]

c. Determine the pressure at the point (2.0) if the pressure is not exceed 100kPa.

[6 marks]

FLUID OF MECHANICS II: SMJ/SMM 3303
TAHUN 1998/1999 Mac 1999

Question 1.0

a. What is the potential flow field.

[5 marks]

b. Publish the potential function of the flow velocity,

i. Source and sink, respectively with strength Q m^3/s

ii. Dublet flow formed when the flow of resources in (-a, 0) and sink at (a,0) is brought near to $a \rightarrow 0$

[8 marks]

c. Bridge pole with diameter at 3.0 m buried in the water flowing at a velocity of 10 m / s. If the water is considered to be superior, determine the force per unit length [N/m] which exists at the front bridge pole, which is to $\pi / 2 \le \theta \le 3\pi / 2$.

[12 marks]

QUESTION 2.0

a. Describe briefly,

i. Free vortex,

ii. Forced vortex

[5 marks]

b. based on basic principles, show that the vorticity ξ of a forced vortex is expressed as,

$$\xi = 2\omega$$

with ω is the angular velocity. Use the sketch to support your answer.

a. A cylinder diameter 2.0 m is rotated with a speed of 100 rounds per minute in a clockwise, in an air flow field. The air velocity is 10 m/s in the direction perpendicular to the cylinder axis in the positive direction.

Show, in principle, the lift(elevator) can be expressed as,

$$L = -\rho U \Gamma$$

With Γ ialah is the lift, ρ air density and U free flow velocity. Specify the lift generated if the density of air is 1.2 kg/m3. Air can be considered superior.

Fluid Mechanics II: SMJ/SMM 3303
Year 1999/2000 October, 1999

Question 1.0

a. Starting from basic principles, show that the current function for uniform flow field across a rotating cylinder can be expressed as,

$$\psi = U_0 \sin\theta\left(r - \frac{a^2}{r}\right) - \frac{\Gamma}{2\pi}\ln\left(\frac{r}{a}\right)$$

with a = outer radius of the cylinder, Γ = cycle and U_0 = uniform flow velocity.

[6 marks]

b. Show that the stagnation point can be determined by the relationship

$$\sin\theta = \frac{\Gamma}{4\pi a U_0}$$

[6 marks]

c. In a long cylinder axis perpendicular to the direction of the uniform flow there are two points on the corner $\theta = 60^0$ and $\theta = 120^0$. Show that the coefficient of lift,

$$C_L = 2\pi\sqrt{3}$$

Given: $U_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$, and $U_\theta = -\frac{\partial\psi}{\partial r}$

Question 2.
A flow field is expressed as,

$$\phi = 10x + 5\ln(x^2 + y^2)$$

a. Determine the current function, $\psi(x,y)$,
[5 marks]

b. Specify the nature of the flow,
[3 marks]

c. Determine the pressure along the x-axis if pressure p = 100 kPa at $x \to \infty$
[7 marks]

d. Determine the location of the stagnation point
[5 marks]

Given relationship Cauchy – Reimann:

Given: $U_r = \dfrac{1}{r}\dfrac{\partial \psi}{\partial \theta} = \dfrac{\partial \phi}{\partial r}$ and $U_\theta = -\dfrac{\partial \psi}{\partial r} = \dfrac{1}{r}\dfrac{\partial \phi}{\partial \theta}$

Fluid Mechanics II: SMJ/SMM 3303
Year 1999/2000 March/April, 2000

Question 1.0

a. Explain briefly the distribution, Γ, how the relationship between the distribution Γ with Ω and function disabilities, ϕ in a potential flow field.
[5 marks]

b. Show that the function of a free vortex flow is

$$\psi = \frac{\Gamma}{2\pi}\ln r$$

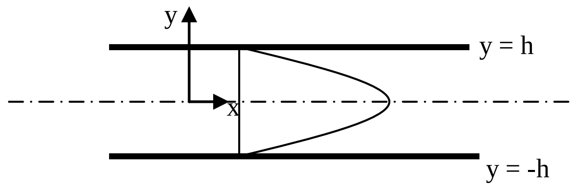
Sketch the current function is. [5 marks]

c. In the next experiment with a free vortex Γ= 850 m^2/s will be overlaid with a sink flow. Sink strength is not known. But at a distance measurement r = 4 m from the center of a free vortex shows p = 220 kPa less than the pressure at $r \to \infty$. With this, determine the magnitude of sink strength, ie m[m^2/s] when the air density is 1.2 kg/m^3.
[10 marks]

QUESTION 2.0

a. Indicate if ψa function of current, then the current line of is ψ = C, with C is a constant. Show that also $d\psi = \psi_2 - \psi_1$ = volumetric flow rate Q [m^3/s] ideal fluid past between ψ_2 with ψ_1.
[10 marks]

b. Refer to the diagram below.



Rajah S2.0

The velocity profile of viscous fluid flow between the two plates can be recorded using the following equation,

$$u(y) = U_0\left[1 - \left(\frac{y}{h}\right)^2\right]$$

With U_0 is the maximum velocity occurs at y = 0. Determine,

i. current function ψ and functional disability ϕ given flow field,
ii. vorticity field of the flow velocity field,
iii. stream lines at y = 0 and y = h, respectively ψ_0 and ψ_1
iv. value of the difference between ψ_1 and ψ_0,
v. volumetric flow rate through the slit $0 \le y \le h$.

[15 marks]

Fluid Mechanics II: SMJ/SMM 3303
Year 2000/2001 October 2000

Question 1.0

Give explanation on the stream function, ψ and the potential function, ϕ.

Next, elaborate on the relationship between the particle velocity u, which is parallel to x-axis and the particle velocity v, which is parallel to y-axis towards the stream function ψ and potential function ϕ. State the assumption used.

[6 marks]

a. If the stream function of the flow field is given by,

$$\psi = -Vy\left[1 - \frac{a^2}{x^2 + y^2}\right]$$

with V as the free velocity component parallel to y-axis, a radius of cylinder and x, y as the center coordinate at the mid-point of the circle cylinder,

i. show that the stream function on a non-rotating stream function equation,

ii. Find the potential function, ϕ,

iii. Velocity distribution on the surface of the cylindrical boundary.

[14 marks]

Question 2.0

a. Prove that the stream function, ψ to a combination of three types of doublet flow, vortex and uniform flow through a horizontal rotating cylinder can be expressed as follows,

$$\psi = -U\sin\theta\left(r - \frac{R^2}{r}\right) - \frac{\Gamma}{2\pi}\ln\left(\frac{r}{R}\right)$$

With U is the uniform flow velocity horizontally to the left, R as the radius of cylinder, T as the period, r and θ was the polar coordinate with the origin placed at the middle of the cylinder.

[10 marks]

b. Based on Question 2a, sketch the stream field for the following situations,

i. When $\dfrac{\Gamma}{2\pi RU} < 1$

ii. When $\dfrac{\Gamma}{2\pi RU} = 1$

iii. When $\dfrac{\Gamma}{2\pi RU} > 1$

Show the direction of the flow and the stagnation point position for each condition.

[10 marks]

Fluid Mechanics II: SMJ/SMM 3303
Year 2000/2001, April 2001

Question 1.0

a. What is the meaning of vorticity? Explain the relationship between vorticity and rotational flow.

[3 marks]

b. Prove that the distribution Γ can be written as,

$$\Gamma = 2\pi\omega r^2$$

With ω as the angular rotation [rad/s] and r as the radius [m].

[5 marks]

c. A cylinder with a diameter of 20 cm rotates with a speed of 10000 rpm, the airflow blows in the direction perpendicular to the cylinder axis. If the air velocity of 5 m / s. Define,

i. Force per unit length,
ii. Position of the stationary point, and
iii. Minimum pressure on the cylinder surface,

Density of air is 1.2 kg/m^3.

[12 marks]

Question 2.0

a. What is source or cause of the flow? State the stream function ψ to the source. **[5 marks]**

b. A Rankine body formed by the combination of an outflow of resources is at the origin with a uniform flow from right to left parallel to the x-axis. If the width of the Rankine body is 100 mm, uniform flow velocity U= 15m/s, air density 1.2 kg/m^3, find

i. Power source,
ii. Position of stationary point,
iii. Maximum pressure on the body – Rankine.

[15 marks]

Fluid Mechanics II SMJ/SMM 3303
Year 2000/2001 May 2001

Question 1.0

a. Show that the combination of uniform flow with a doublet flow will produce a flow field which consists of a cylinder with a radius a = $\sqrt{(\mu/U_0)}$ with μ and U_0 as the strength and magnitude of the uniform flow. **[10 marks]**

b. A cylinder was formed by bolting the inside, two semi-cylinders as shown in the following figure.

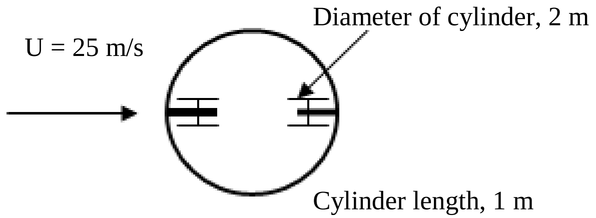


Figure S1

There are 10 bolts in each meter length as shown in the Figure S1. If the pressure in the cylinder is 50 kPa (gauge pressure), calculate the traction on the bolts when the fluid outside is air with density ρ = 1.2 kg/m^3.

[15 marks]

QUESTION 2.0

a. A tornado of wind putting a tornado may be modeled by flowing as shown in the following figure.

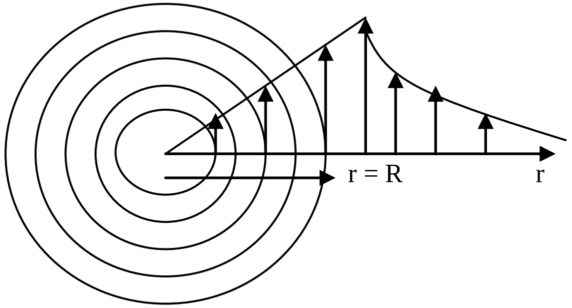


Figure S2

With,

$$U_\theta = \begin{cases} r\omega, \text{ for } r < R \\ R^2\omega/r, \text{ for } r > R \end{cases}$$

Determine:

i. potential flow area putting a tornado wind field,

ii. pressure distribution p (r) in putting a tornado,

iii. position and magnitude of the lowest pressure in this field,

[15 marks]

b. By using Answer Question 2a, above, and the following data,

Tornado Radius, R = 100 m,

Velocity $U_{\theta maks}$ = 65 m/s,

Plot pressure distribution p (r) from r = 0 to r = 400 m.

[10 marks]

@prof amer/mei30/2001

FINAL EXAM QUESTION PAST YEAR
PART 1: FLOW BOUNDARY LAYER

Fluid Mechanics II SMJ / SMM 3303
Year 98/99 October 1998

Question 3.0

a. Explain why and how the boundary layer flow can occur on the surface. [8 marks]

b. A piece of flat plate located in a uniform flow U = 0.5 m / s as shown in Figure S3.

i. derive the equation for the boundary layer thickness, δ

[10 marks]

ii. determine the boundary layer thickness and the shear stress at the position x = 3 m and x = 6 m.

[7 marks]

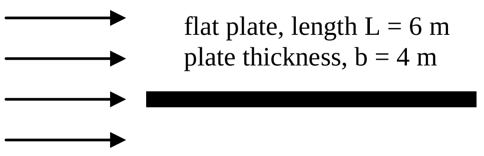


Figure S3

Question 4.0

a. With suitable sketches, give a physical meaning of the thick layer shift, δ* [10 marks]

b. With reference to Figure S4, derive an expression for the drag force acting on a wall F_F a channel length L in terms of layer thickness, δ*and layer thicknesses momentum shift θ.

[15 marks]

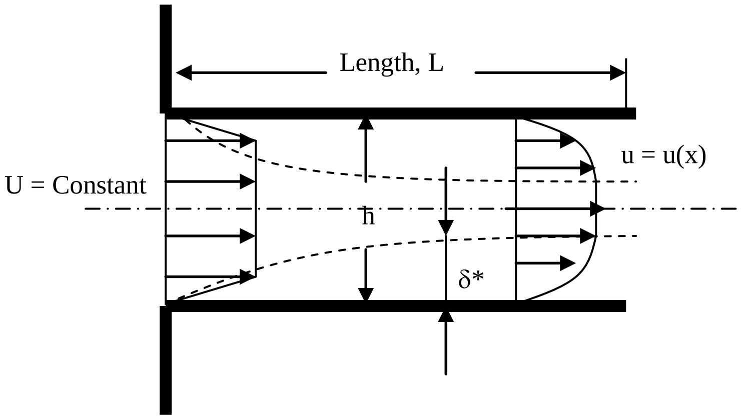


Figure S4

Guide:

i. $\tau_0 = \rho \dfrac{d}{dx}(U^2\theta) + \rho\delta^* U \dfrac{dU}{dx}$

ii. obtain the velocity profile shape u = u(x)

Fluid Mechanics II SMJ / SMM 3303
Year 98/99 Mac 1999

Question 3

a. Turbulent flow velocity distribution on the surface of a flat plate can be expressed as,

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$$

The pressure gradient is zero and the shear stress on surface is given as,

$$\tau_0 = 0.0225 U_\infty^2 \left(\frac{\upsilon}{U_\infty \delta}\right)^{1/4}$$

Derive the following relationship:

i. $\dfrac{\delta}{x} = 0.370 \mathrm{Re}_x^{-1/5}$

ii. $C_f = 0.0577\ \mathrm{Re}_x^{-1/5}$

iii. $C_F = 0.074\ \mathrm{Re}_L^{-1/5}$

Where L is the plate length.

[12 marks]

b. Calculate the force acted deceitfully [drag] on the plate surface measuring 10 m length and 2.0 m wide, if the velocity of the free flow of [air] U_∞ = 1.5 m / s. Given the density and absolute viscosity of air respectively was 1.2 kg/m^3 and 1.8 x 10^{-5} N s/m^2.

[13 marks]

Question 4.0

a. Describe briefly the formation of the boundary layer on the surface of the flat plate for laminar and turbulent flow.

[5 marks]

b. A flow of incompressible flow along a flat plate with constant free stream velocity, U_0. The velocity profile can be expressed as,

$$\frac{u}{U_\infty} = a\eta + b\eta^2 \text{ with } \eta = \frac{y}{\delta}$$

i. By using the integral momentum, proved expression of boundary layer thickness δ in terms Re_x as,

$$\delta = \frac{4.614}{\sqrt{\mathrm{Re}_x}}\,x$$

[6 marks]

ii. Derive, fiction coefficient

$$C_F = \frac{0.646}{\sqrt{\mathrm{Re}_L}}$$

[6 marks]

iii. Determine the friction on the surface of the plate if the plate is 2 m long and 1.0 m wide. Free stream velocity given, U_∞ = 3 m/s, kinematic viscosity and density, respectively 1.5 x 10^{-5} m^2/s and 1.2 kg/m^3.

[8 marks]

Fluid Mechanic II: SMJ/SMM 3303
Year 1999/2000 October 1999

Question 3.0

a. By using the Von Karman integral equation, derive an expression for the displacement thickness δ* and friction F_F on a piece of flat plate where the velocity profile is,

$$\frac{u}{U_0} = \sin\left(\frac{\pi}{2}\eta\right)$$

With U_∞ = free stream velocity, and $\eta = \frac{y}{\delta}$

[12 marks]

b. Air creating a uniform flow field in a water tunnel, some straightening plates is installed horizontally Plate length is 1.0 m. Water at a velocity of 1.3 m/s is used in this experiment. Determine,

i. displacement boundary layer thickness at the ends of the plates,

ii. expel water at x = 3.0 m between two piece of the plates,

iii. friction that occur in the entire length of the plates,

iv. the power required to overcome friction between two piece the plates,

[8 marks]

Question 4.0

a. Drag coefficient, C_F for laminar and turbulent flow is as follows

$$C_F = \frac{1.328}{Re_L^{1/2}} \qquad \text{for laminar flow,}$$

$$C_F = \frac{0.074}{Re_L^{1/5}} \qquad \text{for } Re_L < 10^7$$

$$C_F = \frac{0.455}{\left(\log_{10} Re_L\right)^{2.58}} \qquad \text{for} 10^7 < Re < 10^9$$

Assuming a critical Reynolds number occurs at Re = 5 x 105, and on the surface of a flat plate boundary layer formed laminar and turbulent flow, show that the CF for Re> 107 is,

$$C_F = \frac{0.455}{\left(\log_{10} Re_L\right)^{2.58}} - \frac{1740}{Re_L}$$

[10 marks]

b. The calculation of the friction on the surface of the moving train with a velocity of 160 km / h can be considered as a condition that occurs on the surface of a flat plate sized 110 m long and 8.52 m wide. Calculate,

i. power required to overcome the friction force on the surface,
ii. distance of laminar boundary layer formed on the surface,

Air data: $\rho = 1.2$ kg/m^3, $\mu = 1.8 \times 10^{-5}$ Ns/m^2

[10 marks]

Fluid Mechanic II: SMJ/SMM 3303
Year 1999/2000 April 2000

Question 3.0

a. Explain briefly what is meant by displacement thickness δ^* and momentum thickness, θ. Draw a suitable diagram.

[6marks]

b. The velocity profile of a laminar boundary layer flow without pressure gradient can be expressed as,

$$\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

With

U = free stream velocity, and δ = boundary layer thickness.

By using the momentum integral equation, show that,

i. $$\frac{\delta}{x} = \frac{4.80}{\sqrt{Re_x}}$$

ii. coefficient of friction, $C_D = \dfrac{1.316}{\sqrt{Re_x}}$

Question 4.0

a. The velocity profile of a turbulent boundary layer without pressure gradient can be expressed as a function of the power profile of 1/8, which

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/8}$$

U = velocity independent, and δ = boundary layer thickness.

By using the Von Karman momentum integral equation and the surface stress,

$$\tau_W = 0.0225\rho U^2 \left(\frac{\nu}{U\delta}\right)^{1/4}$$

Show that,

i.

$$\frac{\delta}{x} = \frac{0.3982}{\mathrm{Re}_x^{1/5}}$$

ii. friction coefficient, $CD = \dfrac{1.316}{\mathrm{Re}_L^{1/5}}$

b. The length of a ship is 360 m wide and 70 m high side setting is 25 m. Estimate the force and the power required to overcome friction surface at a speed of 6.5 m / s. Sea water temperature is 10 ^{0}C. Sea water data at 10^{0}C.

ν = 14 x 10-6 m2 / s.

Fluid Mechanics II: SMJ / SMM 3303
Year 2000/2001 October 2000

Question 3.0

a. Explain 3 importance of boundary-layer theory in practical engineering.

b. A piece of flat plate of length, L and width, b m embedded in a stream. Laminar boundary layer occurs in the second - two sides of the plate surface. If the boundary layer flow velocity profile is, and can be ignored, Show that,

$$\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) \text{ and } \frac{dp}{dx} \text{ can be neglected,}$$

Show that,

$$\tau_w = 0.328\ \rho\ U^2 \left(\frac{\nu}{Ux}\right)^{1/2}$$

Next prove that,

$$F = 1.31\ b\ (\rho\mu U^3 L)^{1/2}$$

Question 4.0

a. Explain the formation of the boundary layer when a fluid flows over a flat plate. Next explain how the transfer occurs from a surface boundary layer.

b. A laminar boundary layer velocity profiles,

$$\frac{u}{U} = a\left(\frac{y}{\delta}\right)^3 + b\left(\frac{y}{\delta}\right)$$

Where a and b are constants.

Determine a and b and the boundary layer thickness at the point δ transitions. Take the transition Reynolds number as the 4 x 105. Stress on the surface is,

$$\tau_w = \frac{\rho U^2}{7.2}\frac{\partial \delta}{\partial x}$$

with U = 10.0 m/s and $\nu = 1 \times 10^{-6}$ m^2/s.

Fluid Mechanics II: SMJ/SMM 3303
Year 2000/2001 April 2001

Question 3.0

a. What is the boundary layer. Why the boundary layer thickness increases along the flow across a flat plate from the front fringe.

b. Distribution of the laminar boundary layer flow velocity is given by the following equation,

$$\frac{u}{U} = \frac{3}{2}\eta - \frac{1}{2}\eta^3$$

with $\eta = \left(\frac{y}{\delta}\right)$, u = velocity range $0 \le y \le \delta$,

U = free stream velocity, and δ = boundary layer thickness.
Find the equations of the boundary layer thickness and the coefficient friction average C_F in terms of Reynolds number.
Oil with a velocity of 1.2 m / s across the surface of the flat plate with 3 m long and 2 m wide. If the density of oil is 860 kg/m3 and the kinematic viscosity is 1.2 x 10-5 m2 / s, based on the velocity distribution, determine;

i. thick layers and the shear stress in the middle of the plate, and
ii. total drag.

Take a critical Reynolds number as 5 x 105.

Question 4.0

a. Draw a boundary layer flow across the horizontal flat plate. Indicate the flow parts and explain how the boundary layer is formed.

b. An equilateral triangle traffic signboard shown in Figure S4.0. The wind was blowing at a velocity of 5 m / s parallel and horizontally through the surface of the signboard. If the density of air is 1.2 kg/m3 and kinematic viscosity 1.5 x 10-5 m2 / s, determine the wind drag.

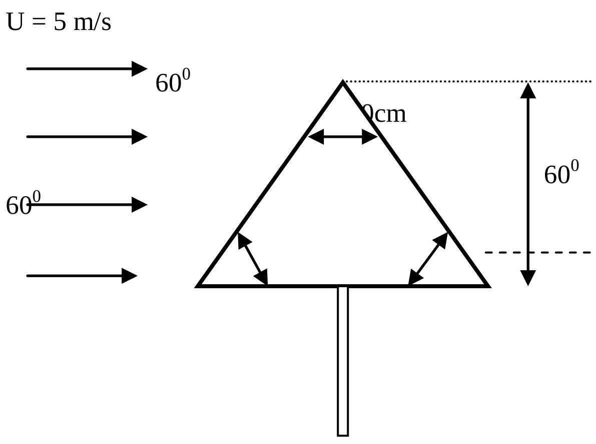


Figure S4.0

Fluid Mechanics II: SMJ/SMM 3303
Year 2000/2001 April 2001

Question 3.0

a. Explain why the field of boundary layer should be known and studied in fluid mechanics.

b. Sketch the flow field in the boundary layer on a piece of flat plate placed at an angle of attack, $\alpha = 0$.

c. Describe what happened when the plate boundary layer flow on Question 3b is now positioned at the angle of attack, $\alpha \neq 0$.

d. Water flowing in a closed channel is square side, with the length L. Derive an expression that can determine the drag force on the inner surface of the channel. You can assume that the flow field in the channel is laminar, because L is short enough.

e. By using the Answers above 3d, if a = 4 cm and L = 25 cm, and the flow velocity is U = 12 m / s, determine the drag caused by the boundary layer flow field in it.

Data for water:
Density, $\rho = 1000$ kg/m^3
Absolute viscosity, $\mu = 1.78 \times 10^{-5}$ kg/m.s

Question 4.0

a. State three advantages Von Karman integral method compared with the methods in the analysis of differential Blasius boundary layer flow.

b. In the determination of boundary layer parameters such as δ, δ^*, θ and C_F with Von Karman method, the shape of the velocity profile used should be selected very well. Mentioned three conditions that must be owned by an equation of the velocity profile until the equation is able to capture a flow velocity profiles in the range of $0 \le y \le \delta$.

By using a parabolic velocity profile, then determine the drag force experienced by a wide rectangular channel, a x a = 4 x 4 cm and length L = 25 cm, when the velocity of flow in this channel is U = 12 m/s.

Data for water:

Density, $\rho = 1000\ kg/m^3$

Absolute viscosity, $\mu = 1.78 \times 10^{-5}\ kg/m.s$

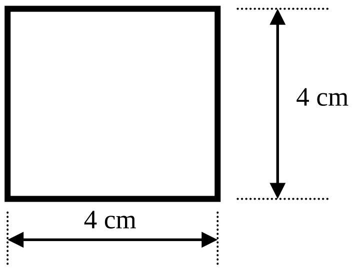


Figure S4.0

PART 2: TURBO MACHINE

Fluid Mechanics II SMJ/SMM 3303
Year of 98/99 October 1998

Question 5

a. In a radial flow turbine [radial flow turbine], angled vanes on rotor tangent. Water enters the rotor with velocity V_f at an angle of 90^0. The water leaves the rotor at kV_f. If the head is H, show that for maximum efficiency, velocity U_r the rotor should,

$$U_r = \sqrt{\frac{2gH}{2 + k^2 \tan^2 \alpha}}$$

[10 marks]

b. In a radial flow machine, rotor the outside diameter is 0.6 m and the diameter of the inside hand is 0.3 m. Water enters the rotor with velocity 1.8 m / s, and rotor rotates at 75 rpm. Water left in the radial rotor. If k = 1 and the flow rate is $1.4\ m^3/s$, determine the rotor blade angle at the exit.

[15 marks]

Question 6

a. A centrifugal pump has an impeller outer diameter D and rotates at N rpm. If the static head is H [m], show that in order to start pumping,

$$N = \frac{84.5\sqrt{H}}{D}$$

b. A centrifugal pump delivers water through a static column of 36 m. The losses due to pipe friction are 9 m. Manometer pump efficiency is 80%. Suction pipe and send their pipes - each the same diameter, 38 cm. Exterior diameter of the impeller is 38 cm and the width is 2.5 cm. Exit blade angle is 250 and the impeller rotation is 1320 rpm. Determine the pump.

Question 7

a. From formula of flow coefficient K_Q and head coefficient K_H , we can get

$$K_Q = \frac{Q}{ND^3}\text{, and } K_H = \frac{gH}{N^2D^2}$$

Hence,

D = diameter impellers,
G = gravitational acceleration,
H = head,
N = rotation speed,
Q = flow rate,

Derive an expression for the dimensionless specific speed N_S which is independent of the size of rotor dynamic pump. State all assumptions made and list all SI units for each dimension that should be used..

b. A centrifugal pump with an impeller outer diameter 300 mm rotating at a speed of 1750 rpm, and pump water at a rate of 4000 liters / min. Pump net head is 25 m, while hydraulic efficiency is 70%. Get the number of identical pumps in order to lift a row of 5000 liters / min to a height of 175 m at a speed of 1500 rpm.

Question 8

A variable speed pump [variable speed pump] that have 1450 r pm at a speed in the table attached to the handle of the pump station flow rates - changing. Static lift is 15 m, 250 mm pipe diameter, pipe length 2 km, pipe roughness k = 0:06 mm. Small loss can be expressed as $10c^2/2g$, where c is the mean velocity of the fluid in the pipe.

Determine the pump head and discharge pipes to send up to 1000 rpm and 500 rpm.

Q [L/s]	0	10	20	30	40	50	60	70
H [m]	45.0	44.0	42.5	39.5	35.5	29.0	20.0	6.0

Fluid Mechanic II SMJ/SMM 3303
Year 98/99 Mac 1999

Question 5

a) If the work done by the turbine per unit mass can be expressed as,

$$W.D/m = U_1 V_{w1} - U_2 V_{w2},$$

Prove that the efficiency of a Pelton wheel hydraulic η h can be expressed as follows

$$\eta_h = \frac{2u(V_J - u)(1 + k kos(180 - \beta)}{V_J^2}$$

With positive Vw2 with β are scoop angle of deflection and k is the ratio of the relative velocity between the exit and entry.

b) An electric power plant using three four-jet Pelton wheel for each wheel. Wheel speed is 250 rpm with a head at the nozzle are 200 m. Dimensionless specific speed of each jet are 0.022 and also the nozzle coefficient of C_v = 0.97. If the hydraulic efficiency are 93%, angle of deflection for scoop are 165^0 and the exit velocity relative are reduced by 10% of the relative velocity of the inlet, then calculate,

i) Pelton wheel diameter that are suitable to used,
ii) total power output of this power plants

Question 6

a. Guide vanes are typically installed around the impeller blades [rotor] in hydraulic turbines. State three important function of these guide vanes.

b. A Francis turbine has vanes angle α =18^0. The angle of the impeller blades at the inlet are 115^0. Outside diameters are 1.28 m and the actuator rotates at 150 rpm speed on the effective head of 10 m. due to Impeller blades friction, the exit relative velocity reduced by 5% of incoming relative velocity. Determine,
   a) Hydraulic efficiency
   b) The angle of the impeller blades at the exit
   c) Impeller diameter at the exit.

Question 7

a. Fluid enters the impeller of a centrifugal pump without vortex. Assuming the inlet and outlet pressure points are on the same level, show that the increase in pressure head pump impeller can be written as follows,

$$= \frac{1}{2g}\left(V_{f1} + U_0 - V_{f0}^2 ko\sec 2\beta_0\right)$$

With Vf1 and Vf0 – both was flow velocity at the inlet and outlet, U0 is the linear velocity [tangent] at the output and β0 is the angle of the impeller blades curved backwards. Ignore friction and other secondary loss.

b. Impeller of a centrifugal pump has a diameter of 20 cm and the outside diameter of 40 cm. Width of impeller 1.5 cm at the input and at the output is 0.8 cm. If the pump rotates at a speed of 1500 rpm and flow rate of water is $0.015 m^3/s$, determine the pressure head increase in the impeller. Neglect all losses and use the 35^0 blade angle.

Question 8

Two centrifugal pumps P1 and P2, are connected to a piping system as shown in Figure S2. Both pumps are used to pump water into a reservoir, T3 which is higher. P1 pump that delivers water at the rate of 140 lit/s, need to overcome static lift 30 m, while the pump P2 will have to lift static force 20 m. Pipes system resistance can be expressed as,

$$E = 40 + 0.10Q - 0.0012Q^2$$

Q is the flow rate of the pump in lit/s and E in meters.
AB pipe in length is 100m, and with diameter of 300mm. While BC pipe in lengths is 200m and with diameter of 450mm.
Coefficient of friction for pipe is 0.0075. Determine

i. Mass flow rate for pump P2
ii. flow rates which enter reservoirs T3
iii. Head loss on valve I

Neglect all kind of loss

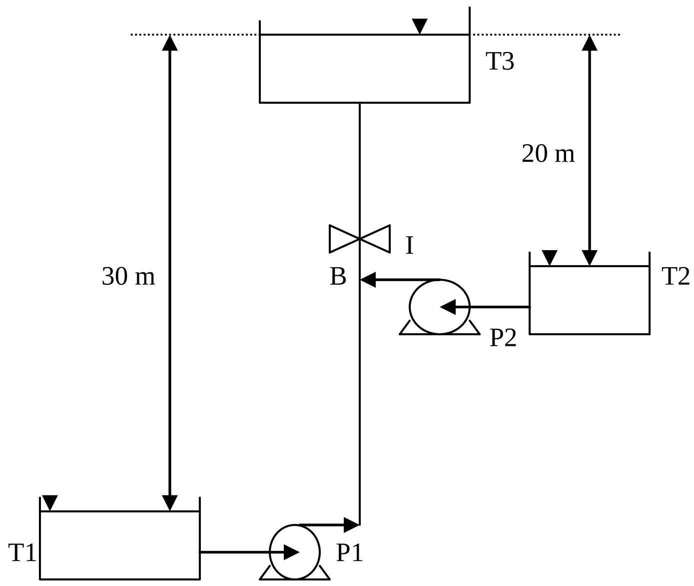


Diagram S8

Fluid Mechanics: II SMJ/SMM 3303
Year 1999/2000: October 1999

Question 5

a. What is centrifugal pump?

b. Describe the purpose or function of the former spiral [volute casing] of a centrifugal pump.

c. A centrifugal pump has an impeller diameter of 400 mm at the exit and 200 mm at the entrance. Pump operates at a speed of 1000 revolutions per minute [ppm] and be able to produce 32 m of water column. Flow velocity in the radial direction is constant, 2.5 m / s. If backward curved impeller blades 300 at the exit pump, determine the :
   i. Blade angle at the inlet
   ii. magnitude and direction of the absolute velocity of the exit,
   iii. hydraulic efficiency.

Question 6

a. The theoretical maximum hydraulic efficiency of a Pelton wheel can be achieved by using a reverse angle intake [bucket] 1800. Prove this statement by using the energy equation description of the flow through the Pelton wheel.

b. A Pelton wheel working under a head of 36 m of water. Water jet flow rate is 20 liters / s. Average velocity intake [bucket] is 15 m / s. Sauk deflect the jet stream through the corner 1600. If the nozzle velocity coefficient, CV = 0.0, then determine the,

   i. Nozzle diameter,
   ii. Output power of pelton wheel,
   iii. Hydraulic efficiency.

Question 7

A mixed flow reaction turbine [Francis] working under a head of 46 m and generate effective shaft power of 3700 kW and overall efficiency of 80%. The shaft rotates at a speed of 280 rpm, and the hydraulic efficiency is 90%. The entrance to the impeller is 1.5 m above the tail race with pressure gauge 250 kN/m2. The exit is 1.2 m above the tail race with vacuum pressure 14 kN/m2. No vortex in the draft tube [draft tube] and water out with a velocity of 3 m / s. Outside diameter of 1:55 m and driving here is the flow velocity of 6 m / s. Define,

i. angle of the impeller blades at inlet,
i. diameter at the tip of draft tube exit,
ii. head loss in the inlet guide vanes, driver and draft tube.

Fluid Mechanics II SMJ/SMM 3303
Year 1999/2000: Mac/April 2000

Question 5

a. Centrifugal pumps are widely used in industry. Explain briefly the working principle of centrifugal pump. Illustrate your answer with sketches of centrifugal pumps, and show the main components of this pump.

b. A centrifugal pump is capable of pumping water at the head of 14.5 m with up to 1000 ppm. Pump impeller blades curved backwards at an angle 300 to the tangent. Outside diameter of 300 mm and thicker pressure at the exit of the impeller is 50 mm. Determine the pump flow rate and shaft power if manometric efficiency is 95% and an overall efficiency of 80%. Ignore the blade thickness and also assume no vortex at pump inlet.

Question 6

a. In theory, the maximum hydraulic efficiency of a Pelton wheel can be achieved by using a scoop with reverse angle 1800. Prove this statement by using Pelton wheel energy equation.

b. A Pelton wheel working under a head of 35 m of water with water jet flow rate 20.5 lit / s. Wheel rotational speed is 450 rpm. If the Pelton wheel diameter is 650 mm and the intake deflect the jet stream at an angle of 1600, determine

   i. diameter of the nozzle,
   ii. output power,
   iii. hydraulic power.

   Given speed coefficient of the nozzle, $C_V = 0.9$

Question 7

a. Francis turbine and Kaplan turbine usually connected together with a draft tube [draft tube] at the turbine exit, where the flow of the spillway earthed [tail race]. Draw a diagram of the turbine draft tube and spillway. Name the two main purposes of the draft tube is used.

b. A Kaplan turbine can produce 20 MW of output power with a head of 50 m. Dh hub diameter is 0.35 times the diameter of the blade tip rotor, D1. Assume that the velocity ratio $\phi = 2.0$, and the ratio of the = 0.65, and the overall efficiency is 90%, calculate the

   i. diameter at the end of the blade, D_1
   ii. rotation speed of the rotor in ‘rpm’.
   iii. specific rate, NS in 'radians'.

Fluid Mechanics II SMJ/SMM 3303

Year 1999/2000: October 2000

Question 5

a. What is an impulse turbine?

b. Pelton wheel is an example of impulse turbine. Sketch the turbine and specify three main components of Pelton Turbine.

c. A Pelton wheel capable of producing 1,700 kW under a head of 558 m gross. Flow in the pipeline before the nozzle experiencing friction head losses of 5.0 m. The minimum diameter of the Pelton wheel intake is 90 cm. Water jet flow entering the intake reversed through the angle and velocity of 1700 decreased 10% relative flow between the inlet and the outlet intake. If velocity ratio is $\phi = 0.47$, CV = 0.98 coefficient of velocity and efficiency of the Pelton wheel is 90%, determine the :

   i. hydraulic efficiency,
   ii. speed of the wheel,
   iii. diameter of the nozzle.

Question 6

a. What is the effect of inlet swirl on the amount of energy transferred per unit weight of the fluid pressure? Illustrate with relevant formulas.

b. To send water at a rate of 50 lit / s, a diesel engine with a capacity of 18.65 kW power is used to drive a centrifugal pump at a speed of 1500 rpm. Pump impeller has a diameter of 150 mm input and output diameter 250 mm. Flow velocity in the impeller is constant at a value of 2.5 m / s. Impeller blades are curved at the back at an angle 300 to the tangent to the circumference of the impeller. Suction pipe diameter to be 150 mm while the diameter of the delivery pipe is about 100 mm.

c. If the head pressure at the suction of the pump is 4 m below and the pressure in the pipe did send is 18 m above the atmospheric value, determine
   i. blade angle at the impeller inlet,,
   ii. overall efficiency, and
   iii. hydraulic efficiency,

Question 7

a. For Francis turbine, draw the velocity triangles difference for the low-speed rotor and the rotor high velocity. Also show its impact on the blade profile and rotor design.

b. A hydroelectric generating stations using Francis turbine with high speed rotor, at 300 rpm to produce 2600 kW under a head of 30 m. Linear velocity of the rotor blade inlet flow velocity was 0.95 $\sqrt{2gH}$ while the 0.30 $\sqrt{2gH}$. If the percentage is 15% loss of hydraulic, get
   i. volume flow rate through the rotor,
   ii. guide vanes angle,
   iii. input angle rotor blades, and
   iv. the diameter of the rotor inlet.

   Assume no swirl in the rotor exit.

Fluid Mechanics II SMJ/SMM 3303
Year 2000/2001: April 2001

Question 6

a. Francis turbine has a non-dimensional specific speed in the range of 0,064 [low velocity runners] to 0,318 [high velocity runners]. For the runner on second - the two ends of this range draw the velocity triangles at inlet runners complete. Describe all notation and symbols used in your sketch.

b. A Francis turbine flow to operate under a head of 45 m. Diameter runner inlet is 90 cm and a diameter of 60 cm at the output. This runner has a constant width of 12 cm. The outlet blade angle is 15^0 [measured on the tangent to the artificial runners]. The flow velocity is constant at a value of 3.5 m / s and water out, leaving runners without swirl. If the hydraulic efficiency is 90%, calculate the
   i. turbine's velocity,
   ii. discharge in m^3/s,
   iii. guide vanes angle,
   iv. runner blade angle at inlet, and
   v. Power produced.

Question 7

a. Starting with the definition of the head coefficient, K_H coefficient of power, K_P, derives the formula for the non-dimensional specific speed for hydraulic turbines. Explain the meaning of all notation and symbols used, and state the SI units for each parameter involved to produce the non-dimensional specific speed.

b. A Pelton wheel dual - jet needed to generate 5510 kW at 0.0165 dimensionless specific speeds. Water is supplied through pipes empis air throughout 1000 m, the reservoir level is 350 m above the position of the nozzle spout. Nozzle velocity coefficient is 0.97, the velocity ratio is 0.46, the overall efficiency is 85%, and Darcy friction coefficient f = 0024. If the friction loss in the pipe water empis totaling 5% of the gross head, determine the
   i. turbine speed in rpm,
   ii. water jet diameter,
   iii. mean radius circle intakes, and
   iv. empis diameter water pipe.

Fluid Mechanics II SMJ/SMM 3303
Year 2000/2001: May 2001

Question 5

a. Point out the difference superior speed velocity triangles [ideal] and triangular actual velocity of a pump or fan, centrifugal on a thumbnail.

b. The diameter of the inlet and outlet of a centrifugal fan that spins at 1450 rpm respectively - each, is 475 mm wide and 700 mm while the corresponding pressure is about 190 mm and 145 mm. Performance of the fan is controlled by a series of vanes for producing swirl component in the direction of rotation so that the velocity of the inlet air relative worth of 31 m / s at an angle of 15^0 to the tangential inlet blade ring. Shock loss caused by this vortex component of 0.6 $V_1^2/2g$ with V1 is the absolute velocity of the air at the inlet. *Delengkungkan* impeller blades back and corners each input and output - are 12^0 and 38^0 respectively, measured from the tangent to the ring input and output.

c. By assuming that, because *gelinsiran*, actual swirl component at the output is 0.78 of the theoretical value and the loss in the impeller is 0.43 of the velocity head at the exit pressure, calculate the total fan flow through the head developed by the impeller.

Question 6

a. Velocity triangles show the differences for the low-speed Francis turbine, Francis turbine and high velocity. In a low-speed Francis turbine, the total head pressure and kinetic energy input to the container or containers *berlinkar* spiral is 120 m and the distance of this section above the tail race is 3 m. Francis turbine is the type of water flow into and leaves the rotor without friction. The linear velocity of the rotor blade at inlet is 30 m / s, while the flow velocity is constant at a value of 10 m / s. Loss - the loss of others, including hydraulic,
   - Between the inlet guide vanes input rotor of 4.5 m, and
   - The rotor, 8 m

   Define,

   i. corner vanes,
   ii. angle of the rotor blades in the input,
   iii. pressure head at the inlet rotor, and
   iv. pressure head at the outlet of the rotor.

@prof amer/o6/06/02

Question and Answer for Final Examination

Question 1

a. Publish stream function ψ and ϕ potential function of two-dimensional sink flow.

[5 marks]

b. A uniform flow with constant velocity U_o on a plate. A hole is punched in the length, t of this plate, see figure below:

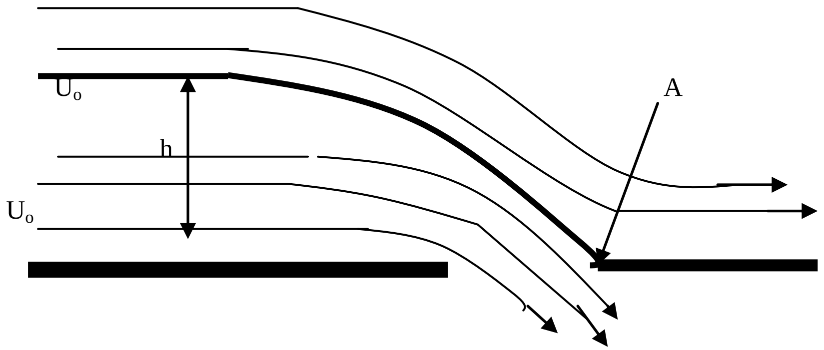


Figure Q1

If the water sucked into the pores is Q = 0.1 m^3/s per m length of the pores, and the velocity U_o = 5 m/s, then determine,

i. Stagnation point position, A

ii. Stagnation streamlines equation,

iii. Height, h above h will not be inhaled water in the pores,

[15 marks]

Answer:

b. Combined flow stream function:

$$\psi = Ur \sin\theta - (Q/2\pi)\,\theta$$

Velocity components,

$$U_r = (1/r)\partial\psi/\partial\theta = U \cos\theta - (Q/2\pi r)$$

And,

$$U_\theta = -\,\partial\psi/\partial r = -\,U \sin\theta$$

i. This is the position of the stagnation point A, which is

$$\mathbf{r_s = Q/2\pi U}$$

With Q* = (2 x 0.1) m^3/s, U = 5 m/s. [Note: a power source of 0.2 m3 / s is required to produce half of the resources at pores on the board].

So,

$$\mathbf{r_s = 0.00637\ m}$$

ii. Stagnation streamline obtained by setting ψ = 0.

Therefore,

$$\psi = Ur \sin\theta - (Q/2\pi)\,\theta = 0$$

Then,

$$r \sin\theta = (Q/2\pi U)\,\theta$$

However,

$$r \sin\theta = y,$$

Thus, the stream line is requested inundation,

$$\mathbf{y = (Q/2\pi U)\,\theta}$$

iii. The maximum distance the water is not sucked into the pores occurs when,

$$\theta \rightarrow \pi$$

The equation,

$$\mathbf{y = (Q/2\pi U)\ \theta}$$

With this, when, $\theta \rightarrow \pi$ then $y \rightarrow h$, until

$$\mathbf{h = Q/2U}$$

So with Q = 2 x 0.1 m^3/s and U = 5 m/s, we obtain

$$\mathbf{h = 0.0200\ m}$$

Question 2

a. Determine the magnitude or scale circulation Γ in a cylinder with a diameter, D spinning at the speed of N in the air. Ignore the viscosity of the air.

[5 marks]

b. In an experiment, a student would like to generate a lift force of 10 N/m on a cylinder with a diameter, D = 500 mm in the uniform flow, U_o = 5 m/s. Temperature and pressure were measured which is 300 K and 100 kPa respectively. Assuming the air is an ideal gas, incompressible fluid, no viscosity, and then determines the rotation, N [rpm] cylinder needed to be producing the desired lift.

Note: ideal gas equation p = ρRT, with R = 0,287 kJ / kg.K

[15 marks]

Answer:

c. From the answers [a],

$$\Gamma = \oint \mathbf{V.ds} = \int_0^{2\pi} a\omega a d\theta = \mathbf{2\pi a^2\omega = 2\pi a^2\ [2\pi N/60) = 4\pi^2 a^2 N/60}$$

or,

$\Gamma = \pi^2 a^2 N/15$ [a

According to the theorem Kutta-Jouskousky, to a rotating cylinder in a uniform flow field of an ideal fluid, lift force is,

$L = \rho U\Gamma$ [b

With Γ given by the press. [a].

Thus, the lift force per unit length of the cylinder is,

$L = \rho U\,[\pi^2 a^2 N/15]$ c]

With the air density is determined from the equation of ideal gas, ie,

$p = \rho RT$ [d

Therefore, the

$L = pU\pi^2 a^2 N/15RT$ [e

Thus, the

$N = 15\ RTL/[p\mathbf{U\pi^2 a^2}]$

Question 5

a. Prove from first base that the maximum hydraulic efficiency of a Pelton turbine is,

$$\eta_{maks} = \frac{1}{2}\left(1 + k kos\beta\right)$$

with k = W2/W1 ratio of the relative velocity at the exit [2] of its value at the inlet [1]; $\beta = 180^o - \theta$, θ is the angle of deflection of the water jet after hitting the blade on a Pelton wheel.

[5 marks]

b. A Pelton wheel is designed to produce 6000 kW of power. Available net head is 300 m. Spun the wheel as fast as 550 rounds per minute. The ratio between the diameter of the jet to the diameter of the wheel is d / D = 1/10. Overall hydraulic efficiency is 85%. With regard,

$V_J/\sqrt{(2gH)} = 0.98$, and $u/\sqrt{(2gH)} = 0.46$

Determine,

i. required water flow rate,

ii. Pelton wheel diameter,

iii. used jet diameter,

iv. number of jets mounted on wheels.

[15 marks]

Answer:

c. With $V_J = 0.98\sqrt{(2gH)}$, the velocity of the jet,

$V_J = 0.98(2x9.81x300)^{1/2} = 75.19$ m/s.

The velocity of the blades is,

$= 0.46(2x9.80x300)^{1/2} = 35.29$ m/s

From a given hydraulic efficiency, $\eta_o = P/\rho gQH$, then,

$Q = P/\ \eta_o\rho gH = 6000 \times 10^3/[0.85x10^3x9.81x300] = 2.399\ m^3/s$

Wheel diameter is determined from the value of the velocity, u = 35.29 m / s, which is

$u = \pi DN/60$

up,

$D = 60u/\pi N = 60 \times 35.29/[550\pi] = 1.225$ m

From, the ratio d / D = 1/10, then

d = 1.225/10 = 0.1225 m = 122.5 mm

Wide stream to produce P = 6000 kW,

is A = Q / VJ = 2.399/75.19 = 0.0319 m2

Broad stream of the jet, $a = \pi d^2/4 = \pi(0.1225)^2/4 = 0.0118\ m^2$

So the total jet is mounted on wheels,

n = A/a = 0.0319/0.0118 = 2.7 = 3

Question 6

a. Show from first base that the overall efficiency of a reaction turbine is,

$$\eta_o = \eta_h\ \eta_m\ \eta_q$$

with η_h, η_m, and η_q respectively the hydraulic efficiency, mechanical and flow rate,

[5 marks]

b. A reaction turbine, flow into, the overall efficiency as high as 80% is required to raise 150 kW. Net head available is 8 m. Rotational speed of the wheel is $0.96\sqrt{(2gH)}$, and also the radius is $0.36\sqrt{(2gH)}$. The rotational speed of the wheel is 150 rounds per minute. The losses in hydraulic turbine is 22% of the total energy available. Charged water radial. Then determine,

i. angle at the inlet guide vanes, α_1

ii. blade angle at inlet, β_1

iii. wheel diameter,

iv. wide wheels at the entrance, b_1

[15 marks]

Answer:

Than 22% loss of hydraulic Therefore we have high hydraulic efficiency,

$$\eta_h = 100 - 22 = 78\%$$

By the definition of hydraulic efficiency,

$$\eta_h = V_{W1}\, U_1/gH$$

we get

$$V_{W1} = \eta_h gH/U_1$$

with $U_2 = 0.96\sqrt{(2gH)} = 0.96\sqrt{(2x9.81x8)} = 12.03$ m/s.

So swirl velocity,

$$V_{W1} = 0.78x9.81x8/12/03 = 5.09 \text{ m/s}$$

Next refer the velocity triangles figure below.

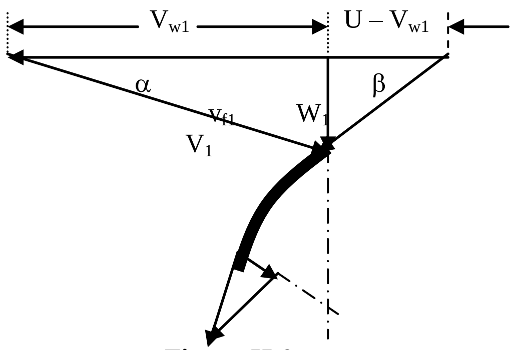


Figure J5.0

From figure J5.0,

$$\tan \alpha_1 = v_{f1}/V_{w1}$$

with $v_{f1} = 0.36\sqrt{(2gH)} = 0.36\sqrt{(2x9.81x8)} = 4.51$ m/s.

therefore, $\boldsymbol{\alpha_1 = 41^o\ 33'}$

From velocity triangle diagram above, we also find that,

$$\tan \beta_1 = 4.51/[12.03 - 5.09]$$

up,

$$\boldsymbol{\beta_1 = 33^o}$$

Of the value of the rotational speed of the wheels,

$$U = \pi ND/60$$

We have,

$$D = UD/\pi N = 12.03 \text{ x } 60/[150\pi] = \mathbf{1.532\ m}$$

The overall efficiency is defined as,

$$\eta_o = P/\rho gQH$$

then,

$$Q = P/\rho g\eta_o H$$

so,

$$Q = 150 \text{ x } 10^3/[0.80x10^3x9.81x8] = \mathbf{2.389\ m^3/s}$$

From the press. continuity,

$$Q = \pi bD\, v_f$$

where b is the width of the blade. so,

$$b = Q/\pi Dv_f$$
$$= 2.389/[\pi \text{ x } 1.532 \text{ x } 4.51]$$
$$\mathbf{= 0.110\ m = 110\ mm.}$$

Question 7.0

a. Show that the increase of the cross-sectional head pressure of an impeller from a centrifugal pump is,

$$h = \frac{1}{2g}[v_f^2 + U_2^2 - v_{f2}^2 \text{ kosec}^2 \beta_2] - h_{L12}$$

with v_f = flow velocity at the inlet of the rotating blade, U_2 and v_{f2} are the rotational and flow velocity at the outlet of the blade, and h_{L12} is the decrease in head between the inlet and outlet of the blade.

[5 marks]

b. A centrifugal pump sends 1.5 liters of water per second. The inlet and outlet diameters of the impellers are 200mm and 400mm respectively. The blade width at the inlet and outlet of the impeller are 16mm and 8mm.The pump is rotated at 1200 rounds per minute. The water enters the pump radially. The outer blade angle is 30^o towards the tangent. If the cross-sectional head loss is 1.5m tall, determine the pressure increase Δp of the cross-sectional impeller, of this centrifugal pump.

[15 marks]

Answer:

a. Refer to the diagram below the velocity triangles.

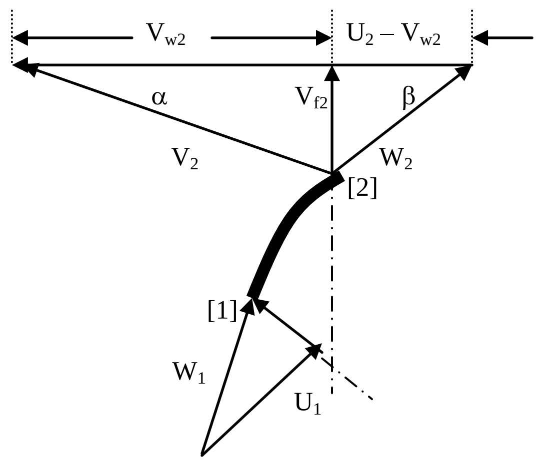


Figure J6.0

Apply the energy equation from the station [1] to the station [2]. Thus, the

$$p_1/\rho g + V_1^2/2g + z_1 = p_2/\rho g + V_2^2/2g + z_2 - h_E + h_{L12}$$

So,

$$(p_2 - p_1)/\rho g = h = V_1^2/2g + h_E - V_2^2/2g - h_{L12}$$

[a

Now $V_1 = v_{f1}$ and $h_E = V_{w2}U_2/g$. From Figure J6.0, we find that,

$$V_{w2} = U_2 - v_{f2} \cot \beta_2$$

and,

$$V_2^2 = v_{f2}^2 + V_{w2}^2 = v_{f2}^2 + (U_2 - v_{f2} \text{ kot } \beta_2)^2 = v_{f2}^2 + U_2^2 + v_{f2}^2 \text{ kot}^2\beta - 2U_2v_{f2}\text{kot } \beta_2$$

or,

$$V_2^2 = U_2^2 + v_{f2}^2 [1 + \cot^2 \beta] - 2U_2 v_{f2} \cot \beta_2 = U_2^2 + v_{f2}^2 \operatorname{cosec}^2\beta - 2U_2 v_{f2} \cot \beta_2$$

With this,

$(p_2 - p_1)/\rho g = h = v_{f1}^2/2g + U_2 (U_2 - v_{f2} \cot \beta_2)g - [U_2^2 + v_{f2}^2 \operatorname{cosec}^2\beta - 2U_2 v_{f2} \cot \beta_2]/2g - h_{L12}$

$$= v_{f1}^2/2g + [2U_2^2 - 2U_2 v_{f2} \cot \beta_2 - U_2^2 - v_{f2}^2 \operatorname{cosec}^2\beta - 2U_2 v_{f2} \cot \beta_2]/2g - h_{L12}$$

That is,

$$\mathbf{h = [v_{f1}^2 + [U_2^2 - v_{f2}^2 \cot \beta_2]/2g - h_{L12}}$$

a. From the continuity equation,

$$Q = \pi\, bDv_f,$$

Thus, the flow velocity at the inlet is,

$$v_{f1} = Q/\pi b_1 D_1 = 15 \times 10^{-3}/[\pi x0.016x0.2) = 1.49 \text{ m/s}$$

Flow velocity at the exit,

$$V_{f2} = Q/\pi b_2 D_2 = 15 \times 10^{-3}/[\pi x0.008x0.4) = 1.49 \text{ m/s}$$

Rotational speed of the impeller also is, $U = \pi ND/60$

Rotation velocity at the inlet,

$$U_1 = \pi N_1 D_1/60 = \pi x1200x0.2/60 = 12.57 \text{ m/s}$$

Rotation velocity at the exit,

$$U_2 = \pi N_2 D_2/60 = \pi x1200x0.4/60 = 25.13 \text{ m/s}$$

Answer from 6a. pressure head rise impeller (same plane) of a centrifugal pump is,

$$h = \frac{1}{2g} [v_{f1}^2 + U_2^2 - v_{f2}^2 \operatorname{cosec}^2 \beta] - h_{L12}$$

with v_f = velocity of flow in the inlet blade motion, U_2 and their vf_2 - each turn is the rotation velocity and the flow velocity at the exit of the blade, and h_{L12} is the head loss in head between the inlet and outlet blade.

In this case, $v_{f1} = v_{f2}$. so,

$$h = \frac{1}{2g} [U_2^2 + v_{f1}^2 (1 - \operatorname{kosec}^2 \beta_2)] - h_{L12}$$

That is,

$$h = [25.13^2 + 1.49^2 (1 - \operatorname{kosec}^2 30^o)]/(2x9.81) - 1.15 = \mathbf{30.7\ m}$$

@profamer/0102

FINAL EXAMINATION

FLUID MECHANICS II
SMJ 3303
OCTOBER 2001/2002

Instructions:

1. Answer five [5] questions only

2 questions from Part A
2 questions from Part B
1 question from Part A or part B

2. The time allocated is three [3] hour only.

Part A: Flow Disability and Boundary Layer Flow

Question 1.0

a. Give three [3] important feature of the potential flow field, briefly explain the method of analysis of this potential flow field methods.

[5 marks]

b. Consider of the flow field is formed when a uniform flow coming from the negative x, coincides with a source of strength q [m3 / s], located at the original point of the axis x - y familiar. Define,

i. Point x, the position of the stagnation point of the flow field is

ii. Pressure coefficient distribution, $C_p = \dfrac{p - p_\infty}{\frac{1}{2}\rho U_\infty^2}$, in terms (x_s/x).

iii. Physically describe what happens to the C_p when $x \rightarrow \pm\infty$ and $x \rightarrow x_s$

[15 marks]

Refer to the diagram below.

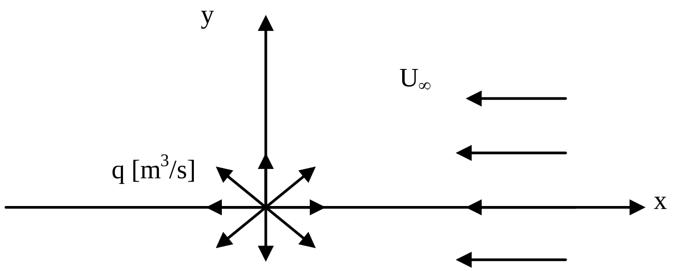


Diagram S1.0

Question 2.0

a. Show that the streamlines and lines of constant effort, is orthogonal,, at any point in a potential flow field..

[5 marks

b. A long cylinder is used to cover a hole found on a piece of flat wall placed horizontally, refer to the figure below.

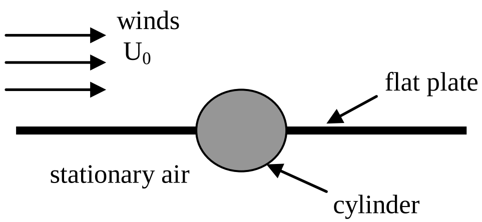


Diagram S2.0

This cylinder of diameter D = 7.5 cm, and made of a material with relative density s = 0.98. Determine the magnitude of the wind that can lift cylinder from the position shown. Ignore all frictional effects between the cylinder and the pores of the plates. Wind pressure and density respectively - each is 1.01 x 102 kN/m2 and 1.22 kg/m3.

The formula used should be issued with clear **[15 marks]**

Question 3.0

a. Studied boundary layer flow theory in fluid mechanics is simply - points to determine the drag force on the body that *digenang* by fluid motion. Usually observed is the parameter C_F and δ*.

Mention the importance of values C_f and δ* this? Why not accurately count Von Karman Blasius count in determining the value C_F and δ* this. Give reasons two reasons.

[5 marks]

b. A flow straightener consists of a long box L = 30 cm, sectional area A = 4 x 4 cm^2 installed in a wind tunnel. If 20 similar boxes used, determine the drag caused by the flow straightener. Flow velocity entering the box is uniform on 10 m/s.

[15 marks]

Note:

1. You can use the following formula without prove the answer, if necessary.

$$C_F = \frac{1.328}{\sqrt{(\rho UL/\mu)}}, \quad \text{for laminar,}$$

$$C_F = \frac{0.072}{(\rho UL/\mu)^{1/5}}, \quad \text{to the turbulence}$$

2. properties of the working fluid, is

density of air, $\rho = 1.22\ kg/m^3$,

Absolute viscosity of air, $\mu = 18.46 \times 10^{-6}\ N.s/m^2$

Question 4.0

a.

i. Give three [3] key conditions that must be fulfilled by any function u = u (y) to be suitable in the final equation Von Karman integral method, for a flat plate with dp / dx = 0, ie

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx} \qquad [4.1$$

[3 marks]

ii. Special to turbulent boundary layer flow field, can value τ_0, shear stress on the plate surface, obtained from u = u (y)? If not, give reasons and how recoverable value τ_0 this?

b. A child using a piece of flat plate water slider length 1 m, width 0.5 m drawn by his father with a speedboat. The child's weight is 40 kN. The boat was pulling at 60 km / h. The relative density of sea water can be estimated to be about s = 1.02. Absolute viscosity is approximately, μ = 0.001 kg / ms Determine the tension in the draw rope of the child.

Note: 1. If the situation is turbulent you are advised to use the profile $u/U_0 = \eta^{1/8}$ with $\tau_0 = 0.0142\rho U_0^2(\nu/U_0\delta)^{1/5}$. If the condition is laminar, use cubic physique, ie $u = a + by^2 + cy^3$ only.

[15 marks]

Part B: Turbo Machine

Question 5

b. From hydrodynamic analysis of a Pelton wheel, it can be shown that the hydraulic efficiency is a function of the ratio $\phi = u_b/V_J$ and jet deflection angle $\beta = (180^o - \theta)$, for ideal conditions,

$$\eta_h = 2\phi(1 - \phi)(1 + \cos\beta)$$

i. Show η_h is maximum at = 0.5,

ii. Actual operating conditions, the maximum occurs for η_h, $0.46 \le \phi \le 0.48$. Give three [3] the reason.

iii. If $W_2 = kW_1$, the relative exit velocity [2] decreased k times the relative velocity in [1], the maximum efficiency has also occurred at $\phi = 0.5$.

[10 marks]

A Pelton turbine with 2 jets produces 2 MW of power at 400 revolutions per minute rotation [rpm]. Wheel diameter is 1.5 m. Measured gross head at the nose nozzle to the surface water in the reservoir is 200 m. Efficiency of power transmission through pipes and nozzles stocker is 90%. Relative velocities out of the intake [or blade] were reduced by 10%. Blade deflecting water jet at an angle $\theta = 165^0$.

i. Sketch velocity triangle Pelton wheel, the
ii. Determine the net head, H_B
iii. Determine the Euler column, H_E
iv. Determine the hydraulic efficiency, η_h, and
v. Determine the diameter of the jet,

[10 marks]

Question 6

a. From the hydrodynamic analysis, the hydraulic efficiency of a reaction turbine can be expressed as follows,

$$\eta_h = 1 - \frac{\sum h_L}{H_B} \qquad [6.1$$

with Σh_L and H_B respectively - were all head losses in the turbine and the net head. If Σh_L minimum H_L, obviously η_h is the maximum.

Obtain an expression for V_{w2}, the swirl velocity at the exit rotor, if the total head losses in the turbine can be expressed as follows,

$$\sum h_L = k_1 \frac{W_2^2}{2g} + k_2 \frac{V_2^2}{2g} \qquad [6.2$$

that can maximize the value of η_h. In Eq. [6.2], W_2 and V_2 respectively - each is the relative velocity and absolute velocity at exit rotor, k_1 and k_2 are constants.

[10 marks

b. In a reaction turbine, the rotor head loss is 5% of the kinetic head out rotor relative velocity, and the head loss in the draft tube is 20% of the absolute velocity kinetic head out rotor. Same plane flow velocity is constant turbine, which is 7.5 m / s. Around at the exit velocity is given high rotor, U_2 = 15 m / s. Sketch the velocity triangle problem. Next at maximum efficiency; determine

i. swirl velocity, V_{w2}, ii. Guide vanes angle, α_2, iii. Blade angle, □ 1 iv. The value of such a turbine hydraulic efficiency when the net head is 90 m.

[10 marks

Question 7

a. By using sketches, obtain an expression for a column manometer H_M rotor dynamic pump that pumps water across a pressure difference $(p_h - p_s) / \rho g$, with a p_h and p_s respectively - each is the pressure head in the pipe to send and suction pipe.

[5 marks

b. A centrifugal pump is driven by an electric motor at a speed of 1450 revolutions per minute, rpm, delivering a total of 0.3 m^3/s of water. The diameter of the impeller blade width and angle at the exit of each is 600 mm, 400 mm and 300. The diameter of the impeller blade width and angle at inlet respectively is 300 mm, 80 mm and 200. Level-term pressure mounted, on tap send and suction pipe, respectively - showed a positive reading of negative 13.5 bar and 0.5 bar.

With regard diameter suction pipe and send the same size, determine the

i. column manometer, H_M

ii. manometric efficiency, η_h

iii. power must be supplied by the motor if the mechanical efficiency is 98%.

[15 marks

Answer Questions Final Exam

Question 1.0

a. Give three [3] features of the potential flow field, briefly explain the method of analysis of this potential flow field methods.

[5 marks]

c. Consider a flow field is formed when a uniform flow coming from the negative x, coincides with a source of strength q [m3 / s], located at the original point of the axis x - y. Define,

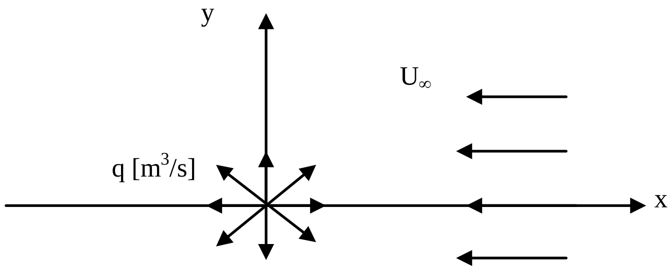
i. point x_s, the position of the stagnation point of the flow field, the

ii. distribute pressure coefficient, C_p =, in terms of (x_s / x).

iii. physically describe what happens to the Cp when x $\rightarrow \pm\infty$ and x$\rightarrow$ x_s

[15 marks]

Refer to the diagram below.



Rajah S1.0

Answer:

a. Three important features of the flow field potentials:

i. there is no vorticity, which $\nabla \times V = \Omega = 0$

ii. due $\Omega = 0$, then there exists ϕ, ie $V = \nabla\phi$,

iii. due to field must fulfill the continuity equation namely $\nabla . V$, then the potential flow field we have the situation $\nabla^2 \phi = 0$.

[3 marks]

The method of analyzing the potential flow field: -

In line with the three characteristic flow field was cleared above, the method of analysis of this field can be summarized as follows,

i. due to the flow field can be represented by the function disabled, □, which meet the press. Laplace, namely

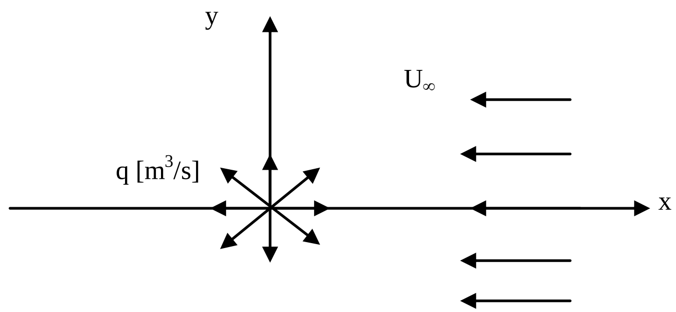
$$\nabla^2\phi = 0,$$

ii. Laplace equation can be solved analytically, or numerically, depending on the geometry of the studied field and boundary conditions involved. The method of separation of variables and superposition method is commonly used analytical techniques. Computational methods can be finite difference or finite elements

iii. step ii produce velocity field distribution in the potential flow field. Knowingly velocity field then we can determine the pressure distribution that covers the body. From the pressure field we determine the force. However, the potential flow field would not exist drag.

[2 marks]

b. Consider the diagram below.



Rajah J1.0

Uniform flow potential function / constant,

$$\phi_{constant} = - U_\infty r \cos\theta \qquad [1.1]$$

Potential function for a stream of the resource is,

$$\phi_{sumber} = [q/2\pi] \ln r. \qquad [1.2]$$

Field which is formed,

$$\phi_{gabungan} = - U_\infty r \cos\theta + [q/2\pi] \ln r.[1.3$$

Then the velocity component in the direction of r and q respectively,

$$u_r = \partial\phi/\partial r = - U_\infty \cos\theta + [q/2\pi]. \qquad [1.4]$$

and,

$$u_\theta = \partial\phi/r\partial\theta = + U_\infty \sin\theta \qquad [1.5]$$

[3 marks]

Along x - axis, $\theta = 0$ or, $\theta = \pi$, it is clear that $u_\theta = 0$. It also shows that the plane y = 0, x - axis, is a surface current. Then y = 0 is a streamline.

Stagnation point is one or a few points in the flow field where the velocity slowed isentropic isentropically deceleration], Until it reaches zero. With this, due to u_θ = 0.

$$u_r = 0: \quad - U_\infty \cos\theta + [q/2\pi] = 0$$

That is,

$$r = x_o = q/2\pi U_\infty \qquad [1.6]$$

[2 marks]

ii. Since we only consider the flow over the plate, the plane y = 0, we can interchange r and q in the coordinate (x, y). so,

$$u = - U_\infty + (q/2\pi x)$$

From equation. [1.6],

$$q = 2\pi x_o U_\infty$$

So,

$$u = U_\infty [x_o/x - 1] \qquad [1.7]$$

[3 marks]

Equa. [1.7] Gives the velocity distribution at the plate, in which the plane y = 0. From Bernoulli equation,

$$p_\infty + \rho_\infty U_\infty^2/2 = p_x + \rho_\infty u^2/2$$

or,

$$C_p = (p_x - p_\infty)/\rho_\infty U_\infty^2/2 = 1 - u^2/U_\infty^2$$

[3 marks]

With equa. [1.7],

$$C_p = 1 - [x_o/x - 1]^2 = 2(x_o/x) - (x_o/x)^2 \qquad [1.8]$$

[2 marks]

c. it is clear from equal. [1.8],

$$C_p = 0, \text{ when } x \to \infty$$

and

$$C_p = 1, \text{ when } x \to x_o.$$

[2 marks]

Question 2.0

c. Show that the streamlines and lines of constant effort, is orthogonal, at any point in a potential flow field.

[5 marks]

d. A long cylinder is used to cover a hole available on a piece of flat wall that placed horizontally, refer to the figure below.

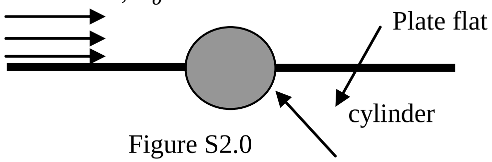


Figure S2.0

This cylinder of diameter D = 7.5 cm, and made of a material with relative density s = 0.98. Determine the magnitude of the wind flow that can lift cylinder from the position shown. Ignore all frictional effects between the cylinder and the pores of the plates. Wind pressure and density respectively - each is 1.01 x 102 kN/m2 and 1.22 kg/m3.

The formula used must be clearly derived.

[15 marks]

Answer:

Ortogonality on the line between ψ with ϕ can be shown as follows. Consider the diagram below.

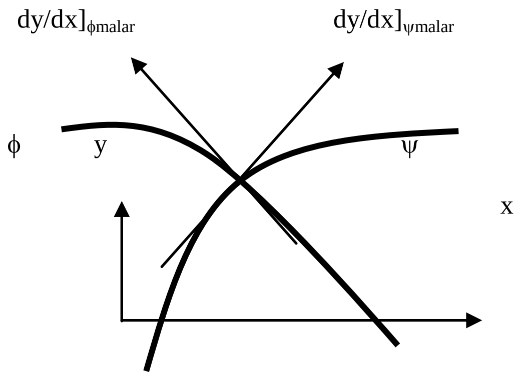


Figure 7.5 : Ortogonality between ψ with ϕ

For ϕ = constant,

$$d\phi = (\partial\phi/\partial x)dx + (\partial\phi/\partial y)dy = 0 \qquad [a]$$

And for ψ = constant,

$$d\psi = (\partial\psi/\partial x)dx + (\partial\psi/\partial y)dy = 0 \qquad [b]$$

From equation. [a],

$$[dy/dx]_{\phi = malar} = - (\partial\phi/\partial x)/(\partial\phi/\partial y) = -u/v \qquad [c]$$

And from equation. [b],

$$[dy/dx]_{\psi = malar} = - (\partial\psi/\partial x)/(\partial\psi/\partial y) = -(-v)/u = v/u \qquad [d]$$

Now,

$$[dy/dx]_{\phi = malar} \,.\, [dy/dx]_{\psi = malar} = - (u/v).(v/u) = - 1 \qquad [7.14]$$

[5 marks]

Equation. [7.14] prove that the constant line ϕ and ψ is perpendicular, ie orthogonal.

b. Consider the diagram below.

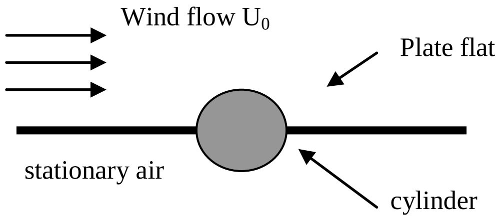


Figure J2.2

A cylinder in a potential flow field can be expressed by superposing doublet flow with uniform flow, ie

$$\psi = U_o r\sin\theta \, [\, 1 - a^2/r^2 \,], \text{ with } a^2 = \mu/U_o, \text{ radius of the cylinder.} \qquad [2.1$$

[2 marks]

From equation. [2.1], we get the velocity components u_r and u_θ

$$u_r = \partial\psi/r\partial\theta = U_o \, r \text{ kos } \theta \, [\, 1 - a^2/r] \qquad [2.2$$

And,

$$u_\theta = -\,\partial\psi/\partial r = -\,U_o \sin\theta \, [\, 1 + a^2/r^2] \qquad [2.3$$

[3 marks]

The obvious one on the cylindrical surface, r = a, then $u_r = 0$ and,

$$u_\theta = -\,2U_o\sin\theta \qquad [2.4$$

From Bernoulli equation,

$$p + \rho U_o^2/2 = p_s + \rho U_s^2/2 \qquad [2.5$$

Then the coefficient of the covering surface of the cylinder pressure is,

$$C_p = [p - p_s]/\rho U_o^2/2 = 1 - 4\sin^2\theta \qquad [2.6$$

Then lift, or lift per meter length of the cylinder is, refer to Figure J2.2,

$$L = -\,(\rho U_o^2/2)a \int_0^{\pi} (1 - 4\sin^2\theta)\sin\theta \, d\theta \qquad [2.7$$

By integrating Equation. [2.7] directly, we have,

$$L = \frac{10}{3}\frac{1}{2}\rho U_0^2 a \qquad [2.8$$

Where a is the radius of the cylinder. Equation. [2.8] is the lift per unit meter length of the cylinder.

This force must be balanced by the weight per unit length of the cylinder, Ws. The weight per unit length of the cylinder is,

$$W_s = s\rho_{air} A = s\rho_{air} \pi a^2$$

Then the velocity of the air can lift cylinder is when,

3 $L = W_s$.

That is,

$$\frac{10}{3}\frac{1}{2}\rho U_0^2 a = s\rho_{air} \pi a^2$$

Then,

$$\mathbf{U_0} = \sqrt{\frac{3}{5}\frac{\rho_{air}}{\rho_{udara}} s\pi\ \mathbf{a}}$$

[2 marks]

By substituting the parameters involved in the given numerical values, we obtain

$$U_0 = 7.5 \text{ m/s}$$

[1 marks]

Question 3.0

a. The boundary layer flow theory is studied in fluid mechanics solely to determine the drag force on the body that *digenang* by fluid motion. Usually observed are the parameter s C_F and δ^*.

   State what are the importance of these values, C_F and δ^*. Why does Von Karman's calculation inaccurate as Blasius' calculation in determining the values of C_F and δ^*. Give 2 reasonable reasons.

[5 marks]

b. A flow straightener consists of a box of length, L=30 cm, with cross-sectional area A = 4 x 4 cm^2 mounted in a wind tunnel. If 20 similar boxes used, determine the drag caused by the flow straightener. Flow velocity entering the box is uniform at 10 m/s.

[15 marks]

Note: 1.You could use the following formula without proving it in the answer, if necessary.

$$C_F = \frac{1.328}{\sqrt{(\rho UL/\mu)}} \text{, for lamina condition.}$$

$$C_F = \frac{0.072}{(\rho UL/\mu)^{1/5}} \text{, for turbulent condition}$$

2. Properties of working fluids are,

Density of Air, $\rho = 1.22$ kg/m^3,

Absolute viscosity of air, $\mu = 18.46 \times 10^{-6}$ N.s/m^2

Answers:

a. The boundary layer flow parameter C_f dan δ^* have important roles in the viscous flow theory.

C_f is the coefficient of friction that exists when the moving fluid covering the body. Contact between the body and fluid will create shear force, τdA. The entire force acting on the body surface is,

$$dF = \frac{1}{2}\rho U_o^2 c_f\, dA \qquad [3.1]$$

C_f value will be determined by whether a field that covers the body is laminar or turbulent.

[3 marks]

δ^* parameter is thickness of displacement layer. Physically this thickness is a thickness on the surface of the body. In this thickness, no flow occurs. Therefore, with the presence of δ^*, the flow will be displaced with a height of δ^* from the surface of the body.

If the flow occurs in a channel, refer to the figure below, the area of the entrance and exit area will be different. So the velocity along the channel will change.

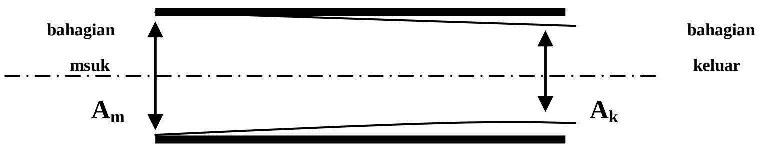


Figure J2.1

AlS is a 2 - dimensional field. However, the flow in a given channel is a 3 – dimensional field. Therefore, we should only review the $0 < y < \delta$ region. By this, the area of the outlet mentioned is,

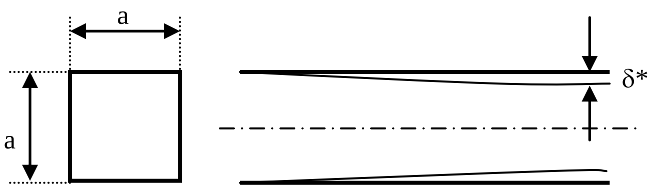
$$A_2 = A_1 - A^*$$

With $A^* = h - \delta^*$

So in practical, the wall at the test section will be tilted by δ^* so that the flow velocity at section 1 and 2are the same size.

[2 marks]

b. Consider the figure below.



[2 marks]

Figure J2.2

The channel length L = 30 cm = 0.3 m, flow velocity U = 10 m/s, working fluid is air with, Density of air, $\rho = 1.22$ kg/m^3, and

Absolute viscosity of air, $\mu = 18.46 \times 10^{-6}$ N.s/m^2

Therefore, the magnitude of the Reynolds no. flow is,

$$Re = \rho UL/\mu = 1.2 \times 10 \times 0.3/(18.46 \times 10^{-6}) = 1.98 \times 10^5$$

[2 marks]

Due to Re <5 x 105, the flow field in the channel is laminar. Thus, the drag on one of the walls is,

$$F_1 = \frac{1}{2}\rho U^2 C_F A \qquad [3.1]$$

With,

$$C_F = \frac{1.328}{\sqrt{(\rho UL/\mu)}}, \text{ for laminar condition,}$$

Therefore,

$$F_1 = \frac{1}{2}\rho U^2 \left[\frac{1.328}{\sqrt{(\rho UL/\mu)}}\right] A \qquad [3.2]$$

[3 marks]

Forces on the four surfaces is the surface N^2,

$$F = 4F_1 = 2\rho U^2 \left[\frac{1.328}{\sqrt{(\rho UL/\mu)}}\right] aLN^2 \qquad [3.3]$$

[5 marks]

Or,

$$\mathbf{F = 2.656\, N^2\, (\rho\mu L)^{1/2}\, U^{3/2} a} \qquad [3.4]$$

By substituting the parameters involved with the data given, we could determine the force, F.

Question 4.0

a. i. Give 3 key conditions that must be fulfilled by any function $u = u(y)$ to be suitable in the end equation of Von Karman integral method, for a flat plate with $dp/dx = 0$, ie

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx} \qquad [4.1]$$

[3 marks]

ii. Special to turbulent boundary layer flow field, can the τ_0, shear stress on the plate surface, that is obtained from $u = u(y)$? If not, give reasons and how to get the value τ_0 ?

[2 marks]

b. A child used a long flat plate surfers with length1 m, width 0.5 m which is drawn by his father with a speedboat. The weight of the child is 40 kN. The boat drawn him at 60 km/h. The relative density of sea water can be estimated, s = 1.02. The absolute viscosity is, μ = 0.001 kg/m.s. Determine the tension in the rope that drawn the child.

Note: 1. If the situation is turbulent, you are advised to use $u/U_0 = \eta^{1/8}$ and $\tau_0 = 0.0142\rho U_0^2(\nu/U_0\delta)$. If it is laminar, you must use $u = a + by^2 + cy^3$.
2. Mention all the assumptions, and show all the publishing formulas that is used in force analysis.

[15 marks]

Answer:

a. Three important requirements that must be fulfill by a form of velocity, $u = u(y)$ is as follows,

i. at y = 0, velocity u = 0, which u(0) = 0

ii. at y = δ, velocity u = U_o, which $u(\delta) = U_o$,

iii. at y = δ, velocity gradient, du/dy = 0,

Refer to the diagram below. **[2 marks]**

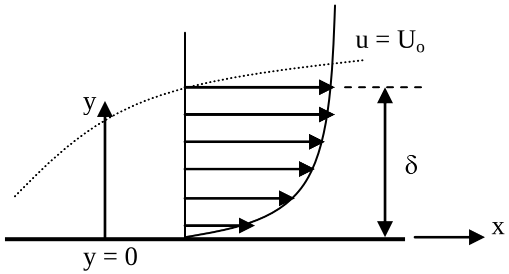


Diagram J4.1

[1 marks]

ii. In application of the intergral equation Von Karman, which is

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx} \qquad [4.1$$

the stresses τ_o on the plate surface cannot be determined directly from the equation u = u(y). Eligible physique form equation used in the turbulent boundary layer flow,

$$u = U_o\,(y/\delta)^{1/n}$$

to,

$$du/dy = (1/n)\,U_o\,(y/\delta)^{1/n-1} = (1/n)U_o(y/\delta)^{1/n}\,[\delta/y]$$

Obviously when y = 0, du/dy $\rightarrow\infty$.Then the velocity is not physically reasonable. Therefore, in the analysis of turbulent boundary layer flow, stress values is τ_o on the surface of the plate, y = 0, will be taken directly from the experiment data separately.

[2 marks]

b. Consider the diagram below,

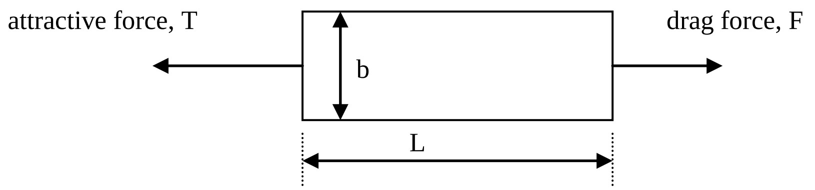


Diagram J4.2

From diagram J4.2,

The attractive force, T = drag force, F by fluid friction

Velocity of the boat, U = velocity of the drawn plate = 60 km/h = 16.6 m/s
Length of the plate, L = 1 m, the working fluid is sea water, μ = 0.001 kg/m.s, and density $\rho = 1.02 \times 10^3$ kg/m^3.

So the magnitude of the flow Reynolds number

$$Re = \rho UL/\mu = 1.02 \times 10^3 \times 16.6 \times 1/(0.001) = 1.7 \times 10^7$$

[2 marks]

This clearly shows that the boundary layer flow fields exist under the plate. Thus, the drag force that arrises is,

$$F_1 = \frac{1}{2}\rho U^2 C_F A \qquad [4.2$$

The coefficient of friction will be determined by using the Von Karman intergral equation,

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx} \qquad [4.1$$

By using $u/U_0 = \eta^{1/8}$ with $\tau_0 = 0.0142\rho U_0^2(\nu/U_0\delta)^{1/5}$.

The force can also be determined directly from Equation [4.1],

$$\mathbf{dF = \tau_o\, dx = \rho U_o^2\, b d\theta}$$

Direct intergral produce.

$$\mathbf{F = \rho U_o^2\, b\theta(L)}$$

[2 marks

With θ{L] is dense shift momentum at y = L. From the definition of the displacement thickness and momentum,

$$\theta = \int_0^\delta \frac{u}{U_0}[1-\frac{u}{U_0}]dy = \int_0^\delta \left(\frac{y}{\delta}\right)^{1/8}\left[1-\left(\frac{y}{\delta}\right)^{1/8}\right]dy \qquad [4.4$$

Let say,

$$\eta = y/\delta \text{ so } dy = \delta\, d\eta \qquad [4.5$$

So, equation 4.4 would be

$$\theta = \delta \int_0^1 \eta^{1/8}[1-\eta^{1/8}]\, d\eta = \frac{8}{90}\delta \qquad [4.6$$

[3 marks

From equation [4.6],

$$\frac{d\theta}{dx} = \frac{8}{90}\frac{d\delta}{dx} = \frac{\tau_o}{\rho U_o^2}$$

or,

$$d\delta = (90/8)[0.0142\rho U_0^2(\nu/U_0\delta)^{1/5}/\rho U_o^2]dx$$

ie,

$$\delta^{1/5}\, d\delta = [0.1598/(\nu U_o)^{1/5}]\, dx$$

So,

$$\frac{5}{6}\delta^{6/5} = 0.1598\frac{x}{(\nu U_o)^{1/5}}\,\mathbf{x}$$

Which is,

$$\frac{\delta}{x} = \frac{0.252}{(\mathrm{Re})^{1/6}} \qquad \textbf{[4.7}$$

[3 marks]

With this, we can also show that,

$$C_F = \frac{0.045}{(\mathrm{Re}_L)^{1/6}} \qquad \textbf{[4.8}$$

[2 marks]

With this, Pers. [4.7] become,

$$\theta(x) = \frac{8}{90}\delta(x)$$

and,

$$\theta(L) = \frac{8}{90}\delta(L) = \frac{9}{90}\frac{0.252L}{(\mathrm{Re}_L)^{1/6}}$$

Then the force on the slider surface, is $F = \rho U_o^2 b\theta(L)$, or,

$$F = 0.0224\left[\frac{L}{\left(\frac{\rho U_o L}{\mu}\right)^{1/6}}\right.\rho U_o^2 b$$

Or else,

$$F = 0.0224bL^{5/6}\mu^{1/6}\rho^{5/6}U_o^{11/6} \qquad \textbf{[4.9}$$

[2 marks]

By substituting the parameters with the data given,

$L = 1$ m, $U_o = 16.6$ m, $\mu = 0.001$ kg/m, dan $\rho = 1.2 \times 10^3$ kg/m^3

[1 marks]

The force can also be determined from Pers. [4.2] dan [4.9],

$$F_1 = \frac{1}{2}\rho U^2 C_F A$$

with,

$$C_F = \frac{0.045}{(\mathrm{Re}_L)^{1/6}}$$

or,

$$F = 0.0225bL^{5/6}\rho^{5/6}\mu^{1/6}U_o^{11/6} \qquad \textbf{[4.10}$$

Note:

Note the similarity between Pers. [4.9] and [4.10]!

Question 5

a. From hydrodynamic analysis of a Pelton wheel, it can be shown that the hydraulic efficiency is a function of the ratio $\phi = u_b/V_J$ and the jet deflection angle $\beta = (180^o - \theta)$, that for ideal conditions,

$$\eta_h = 2\phi(1 - \phi)(1 + \mathrm{kos}\,\beta)$$

i. Show that η_h is maximum at $\phi = 0.5$,

ii. In the actual operating conditions, η_h become maximum for, $0.46 \le \phi \le 0.48$. Gives three [3] reasons.

iii. If $W_2 = kW_1$, the outlet relative velocity [2] reduced k times the inlet relative velocity [1], the maximum efficiency is also applied at $\phi = 0.5$.

[10 marks]

b. A Pelton turbine with 2 jets produces 2 MW of power at 400 revolutions per minute rotation [rpm]. Wheel diameter is 1.5 m. Measured gross head at the nose of the nozzle to the surface of water in the reservoir is 200 m. Efficiency of power transmission through pipes and nozzles is 90%. Relative velocities out of the intake [or blade] were reduced by 10%. Blade deflecting water jet at an angle $\theta = 165^0$.

i. Sketch the velocity triangle of Pelton wheel,

ii. Determine the net head, H_B

iii. Determine the Euler's head, H_E

iv. Determine the hydraulic efficiency η_h, and

v. Determine the diameter of jet.

[10 marks]

Answer:

a. From equation

$$\eta_h = 2\phi(1 - \phi) (1 + kos\ \beta) \qquad [5.1$$

Maximum efficiency happen when $d\eta_h/d\phi = 0$. From Pers. [5.1], we find,

$$d\eta_h/d\phi = 0: \qquad 2 - 4\phi = 0, \text{ until } \phi = 1/2$$

[4 marks]

b. In a practical situation the maximum efficiency of a Pelton turbine not occurred at $\phi = ½$, but in a range of $0.46 \le \phi \le 0.48$. This occurrence can be seen from the definition, the velocity ratio $\phi = u_b/V_J$ with u_b = angular velocity of the wheel = $\pi ND/60$ and $V_J = C_V\ [2gH_{ef}]^{1/2}$, the velocity of the jet.

We see from here,ϕ only change when u_b changed, the changes of u_b is a rotation of the wheel, N. Three reasons that could change ϕ are:

i. air friction, between the wheel and the housing
ii. fluid friction, between the wheel and the housing,
iii. mechanical friction in the wheel bearings.

[3 marks]

iii. If relative velocity jet of water out of the blade is k times the relative velocity of the incoming water, then

$$W_2 = k\ W_1$$

Hence the expression hydraulic efficiency is,

$$\eta_h = 2\phi(1 - \phi) (1 + k\ kos\ \beta) \qquad [5.2$$

At max,

$$d\eta_h/d\phi = 0: \qquad [2 - 4\phi][1 + k\ kos\ \beta] = 0$$

Because of, $[1 + k\ kos\ \beta] \neq 0$, then,

$$\phi = ½$$

Obviously the friction on the surface of the blade does not change $\phi = ½$, where, $d\eta_h/d\phi = 0$.

[3 marks]

b. Given,

Total of jet on the wheels, n = 2,
Power derived, P = 2 MW
rotation of the wheel, N = 400 rpm
diameter of wheels, D = 1.5 m,
Height of the column from the nose of the nozzle, H_{kasar} = 200 m,
transmission efficiency, $\eta_{\ transmission}$ = 90%
k = 0.9 and $\theta = 165^o$ to $\beta = 15^o$

i. Figures show the triangle of a Pelton wheel are,

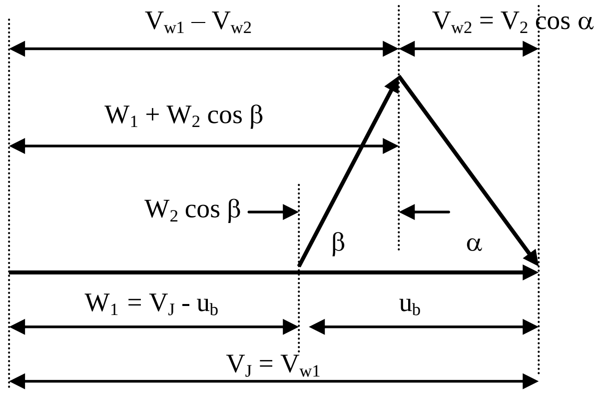


Figure J5.1

[2 marks]

ii. Net or effective head of staff,

$$H_B = \eta_{penghantaran} H_{kasar}$$

hence,

$$H_B = 0.9 \text{ x } 200 \text{ m} = 180 \text{ m}$$

[2 marks]

iii. The Euler head is,

$$H_E = [U_1V_{w1} – U_2V_{w2}]/g \qquad [5.3$$

For the Pelton's wheels, refer to Figure J5.1,

$$U_1 = U_2 = u_b = \pi ND/60$$
$$V_{w1} – V_{w2} = W_1 + W_2 \text{ kos } \beta = W_1 (1 + k \text{ kos } \beta)$$
$$W_1 = V_J - u_b$$

Hence equation [5.3] become,

$$H_E = (1/g)u_b[V_J – u_b][1 + k \cos \beta] \qquad [5.4$$

with V_J = velocity jet = $\sqrt{2gH_{bersih}}$

Hence,

$U_b = \pi ND/60 = 400 \text{ x } 1.5 \text{ x } \pi/60 = 31.4$ m/s
$V_J = [2 \text{ x } 9.81 \text{ x } 180]^{1/2} = [3531.6]^{1/2} = 59.4$ m/s

Hence, Euler column

$$H_E = [1/9.81][31.4][28.03 \ ][1 + 0.9 \cos 15^o] = 167.7 \text{ m}$$

[2 marks]

iv. hydraulic efficiency,

$$\eta_h = H_E/H_B = 167.7/180 = 0.93$$

or,

$$\eta_h = 2\phi[1 - \phi][1 + k \cos \beta]$$

with $\phi = u_b/V_J = 31.4/59.4 = 0.52$

[2 marks]

v. Jet diameter can be determined from the power received, which

$$P = \rho gQH_E = 20 \text{ MW} = 20 \text{ x } 10^6 \text{ kW}$$

Until the cross-flow turbine is,

$$Q = P/\rho gH_E$$

If the jet diameter is d_J, so

$n[\pi d_J^2/4]V_J = Q = P/\rho g H_E = 12.15\ m^3/s$
or,

$$d_J = \sqrt{\frac{4P}{n\pi\rho g H_E V_J}} = 6.5\ \text{cm}$$

[2 marks]

Question 6

a. From the hydrodynamic analysis, the hydraulic efficiency of a reaction turbine can be expressed as follows,

$$\eta_h = 1 - \frac{\sum h_L}{H_B} \qquad [6.1$$

With Σh_L and H_B were all head losses in the turbine and the net head d. If Σh_L minimum, obviously η_h is the maximum.

Obtain an expression for V_{w2} the swirl velocity at the exit rotor, if the total head losses in the turbine can be expressed as follows,

$$\sum h_L = k_1 \frac{W_2^2}{2g} + k_2 \frac{V_2^2}{2g} \qquad [6.2$$

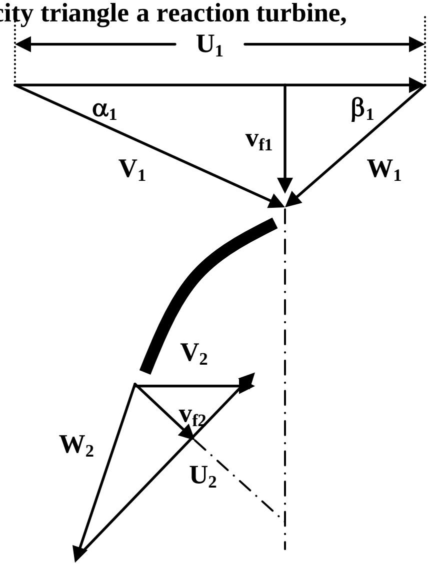
that can maximize the value of η_h In Eq. [6.2], W2 and V2 respectively - each is the relative velocity and absolute velocity at exit rotor, k_1 and k_2 are constants. **[10 marks]**

b In a reaction turbine, the rotor head loss is 5% of the kinetic head out rotor relative velocity, and the head loss in the draft tube is 20% of the absolute velocity kinetic head out rotor. Serentas turbine flow velocity is constant, which is 7.5 m / s. Around at the exit velocity is given high rotor, U2 = 15 m / s. Sketch the velocity triangle problem. Next at maximum efficiency; determine

i. Swirl velocity, V_{w2}, ii. Guide vanes angle, α_2, iii. Blade angle β_1 dan β_2, iv. The value of such a turbine hydraulic efficiency if **the net head is 90 m.**

[10 marks]

Answers:
Refer to the velocity triangle a reaction turbine,



[2 marks]

Figure J6.1

From Figure J6.1,

$$V_2^2 = V_{w2}^2 + v_{f2}^2$$

And,

$$W_2^2 = [U - V_{w2}]^2 + v_{f2}^2\ ]$$

Therefore,

$$\sum h_L = k_1 \frac{W_2^2}{2g} + k_2 \frac{V_2^2}{2g} \qquad [6.1$$

can be written as follows,

$$\Sigma h_L = (k_2/2g)[\ V_{w2}^2 + v_{f2}^2\ ] + (k_1/2g)[\ [U - V_{w2}]^2 + v_{f2}^2\ ] \qquad [6.2$$

[3 marks]

We note η is a maximum if, Σh_L is a minimum. Therefore,

$$d[\Sigma h_L]/dV_{w2} = 2(k_2/2g)\ V_{w2} + 2(k_1/2g)[\ (-1)(U_2 - V_{w2}] = 0$$

[2 marks]

Which,

$$V_{w2} = \frac{k_1}{k_1 + k_2} U_2 \qquad [6.3$$

[3 marks]

By these V_{w2} turbine efficiency will be maximum.

b. Data supplied are,
decrease in rotor, h_{Lr} = 5 % dari $W_2^2/2g$
decrease in the draft tube, h_{Ltd} = 20% to $V_2^2/2g$
turbine cross-flow velocity, v_f = 7.5 m/s
rotation velocity at the exit, U_2 = 15 m/s
Velocity triangle diagram is as follows:-

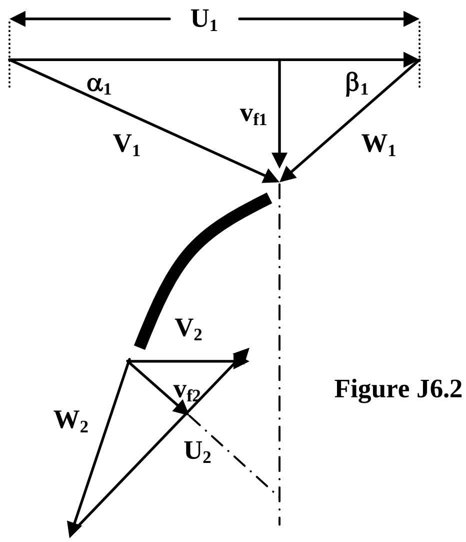


Figure J6.2

[2 marks]

i. Velocity momentum, V_{w2},
At maximum efficiency, the analysis Answer 6.a,

$$V_{w2} = \frac{k_1}{k_1 + k_2} U_2$$

From the above statement of the problem, we find, k_1 = 0.05, k_2 = 0.2 while U_2 = 15 m/s, then

$$V_{w2} = \frac{0.05}{0.25} \times 15 \text{ m/s} = 3.0 \text{ m/s}$$

[2 marks]

ii. Angle of outlet guide vanes, α_2,
From the diagram above velocity triangle,

$$\textbf{tan } \alpha_2 = \frac{v_{f2}}{V_{w2}} = \frac{7.5}{12} = 0.625$$

Therefore,

$\alpha_2 = 32.0^o$

[2 marks

iii. Blade angle, β_1 and β_2,
With reference to the diagram velocity triangles, above

$$\textbf{tan } \beta_2 = \frac{v_{f2}}{U_2 - V_{w2}} = \frac{7.5}{3}$$

Until,

$\beta_2 = 67.7^o$

[2 marks

$$\textbf{tan } \beta_1 = \frac{v_{f1}}{U_1 - V_{w1}}$$ **[ignore incomplete data]**

iv. The efficiency of the hydraulic turbine.
Efficiency is given by Eq. [6.1], which

$$\eta_h = 1 - \frac{\sum h_L}{H_B} \qquad [6.1$$

With H_B = net head, 90 m., and

$$\sum h_L = k_1 \frac{W_2^2}{2g} + k_2 \frac{V_2^2}{2g}$$

Here,

$\mathbf{V_2^2 = V_{w2}^2 + v_f^2 = 12^2 + 7.5^2 = 200.25\ m^2/s^2}$

Then, $\mathbf{V_2^2/2g = 10.21\ m,}$

$\mathbf{W_2^2 = v_{f2}^2 + [U_2 - V_{w2}]^2 = 7.5^2 + 3^2 = 65.25\ m^2/s^2}$

Which, $\mathbf{W_2^2/2g = 3.33\ m}$

So,

$\mathbf{\Sigma h_L = 0.20 \times 10.21 + 0.05 \times 3.33 = 1.176\ m}$

Hereby,

$\mathbf{\eta_h = 1 - 2.21/90 = 0.98\ [98\%]}$

[2 marks

Question 7

c. By using sketches, obtain an expression for a column manometer HM rotodynamic pump that pumps water across a pressure difference (p_h – p_s)/ρg, with a p_h and p respectively - each is the pressure head in the pipe to send and suction pipe.

[5 marks

d. A centrifugal pump is driven by an electric motor at a speed of 1450 revolutions per minute, ppm send up to 0.3 m3 / s. The diameter of the impeller blade width and angle at the exit respectively was 600 mm, 400 mm and 300. The diameter of the impeller blade width and angle at the entrance respectively is 300 mm, 80 mm and 20^0. Level-term pressure mounted, on tap send and suction pipe, respectively - showed a positive reading of 13.5 bar and negative reading of 0.5 bar.

With assume diameter suction pipe and send the same size, determine the

iv. column manometer, H_M
v. manometric efficiency, η_h
vi. power must be supplied by the motor if the mechanical efficiency is 98%.

[15 marks

Answer :
Based on the diagram.

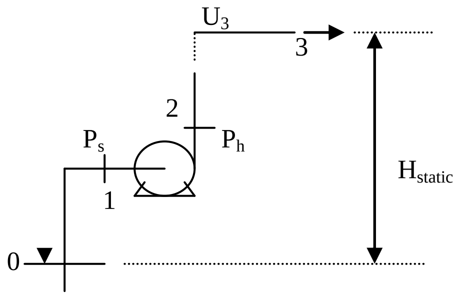


Diagram J7.1

Apply equation Energy from 0 to 1,

$$P_0/\rho g + U_o^2/2g + z_o = P_1/\rho g + U_1^2/2g + z_1 + h_{L01}$$

Clearly, $p_o = 0$ [gauge], $U_o = 0$,

So $z_o = P_1/\rho g + U_1^2/2g + z_1 + h_{L01}$

Namely,

$$P_1/\rho g = (z_o - z_1) - [U_1^2/2g + h_{L01}] \qquad [a$$

Apply equation Energy from 2 to 3,

$$P_2/\rho g + U_2^2/2g + z_2 = P_3/\rho g + U_3^2/2g + z_3 + h_{L23}$$

Here , $P_3 = 0$ [gauge], then

$$P_2/\rho g + U_2^2/2g + z_2 = U_3^2/2g + z_3 + h_{L23}$$

Namely,

$$P_2/\rho g = [U_3^2/2g - U_2^2/2g] + [z_3 - z_2] + h_{L23} \qquad [b$$

Equation [b] Reduced Equation [a],

Form

$$[P_2/\rho g - P_1/\rho g] = U_3^2/2g + (U_1^2 - U_2^2)/2g + (z_3 - z_0) + h_{L01} + h_{L23}$$

But , $p_1 = p_s$ and $p_2 = p_h$, so

$$[P_2/\rho g - P_1/\rho g] = [U_1^2/2g - U_2^2/2g] + U_3^2/2g + H_s + [h_{L01} + h_{L23}] \qquad [c$$

Usually we known the last equation as head manometers , H_m. this is first definition. So ,

$$H_m = = [U_1^2/2g - U_2^2/2g] + U_3^2/2g + H_s + [h_{L01} + h_{L23}] \quad [d$$

with H_s = static head, $[h_{L01} + h_{L23}]$ = head decrease at angle and pipeline.

If we applied equation energy *serentas* pump, namely from point 1 to point` 2, so we will get,

$$P_1/\rho g + U_1^2/2g + z_1 + H_E = P_2/\rho g + U_2^2/2g + z_2 + h_{L12}$$

Namely,

$$P_2/\rho g - P_1/\rho g = H_E + (U_1^2/2g - U_2^2/2g) - h_{L12}$$

Or , manometer head H_M according to the definition, namely

$$H_M = H_E + (U_1^2/2g - U_2^2/2g) - h_{L12} \qquad [e$$

If we assume the diameter of sent pipe and suction pipe is the same, so $U_1 = U_2$. With this, Equation [d] become,

$$H_m = U_3^2/2g + H_s + [h_{L01} + h_{L23}] \qquad [f$$

And Equation [e] is,

$$H_M = H_E - h_{L12} \qquad [g$$

[5 marks]

b. Given data:-

rotation speed N = 1450 rpm,

Diameter, width and angle, at inlet [1], and at outlet,

D_1 = 300 mm, D_2 = 600 mm,

b_1 = 80 mm, b_2 = 400 mm,

$\beta_1 = 20^o$ $\beta_2 = 30^o$

Pressure at suction pipe [s] and sent pipe is,

p_s = - 0.5 bar = - 0.5 x 100 kPa = - 50 kPa

p_h = 13.5 bar = 13.5 x 100 kPa = 1350 kPa

flow rate, Q = 0.3 m^3/s.

i. manometer head, H_M

According to the definition of the manometric head is

$$H_M = [p_h - p_s]/\rho g = [1350 - (-50)]/(10^3 \times 9.81)$$

$$= 142.7 \text{ m}$$

[2 marks]

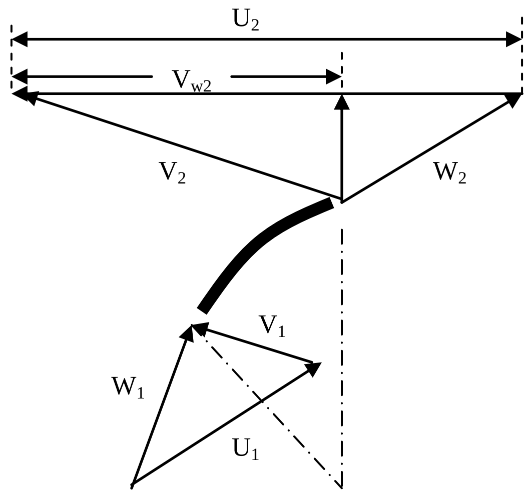
ii. manometric efficiency, η_h

According to the definition,

$$\eta_h = [V_{w2}U_2 - V_{w1}U_1] / gH_M$$

[2 marks]

To determine U_2 and V_{w2} we consider the triangle diagram in terms of the speed of a centrifugal pump , as follow :



[2 marks]

diagram J7.1

Revolution speed,

$$U_1 = \pi N D_1/60 = 1450\pi \times 0.3/60 = 22.8 \text{ m/s}$$

$$U_2 = \pi N D_2/60 = 1450\pi \times 0.6/60 = 45.6 \text{ m/s}$$

[2 marks]

According equation continuity,

$$Q = \pi D b\, v_f$$

So ,

$$V_{f2} = Q/\pi D_1 b_1 = 0.3/[\pi \times 0.3 \times 0.08] = 3.97 \text{ m/s}$$

$$V_{f1} = 0.3/[\pi \times 0.6 \times 0.4] = 0.39 \text{ m/s}$$

[2 marks]

From diagram J7.1,

$$V_w = U - v_f \text{ kot } \beta$$

so,

$$V_{w2} = U_2 - v_{f2} \cot \beta_2$$

$$= 45.6 - 3.97 \cot 30^o = 38.72 \text{ m/s}$$

$$V_{w1} = 22.8 - 0.39 \cot 20^o = 21.73 \text{ m/s}$$

[2 marks

With this pump hydraulic efficiency is,

$$\eta_h = [V_{w2}U_2 - V_{w1}U_1] /gH_M$$

$$= [38.72 \text{ x } 45.6 - 21.73 \text{ x } 22.8]/(142.7g)$$

$$\mathbf{= 0.91 \ [91\%]}$$

[1 mark]

i. Power must be supplied by the motor if the mechanical efficiency is 98%.

The power required by the pump, ideally, is

$$\mathbf{P = \rho g Q H_M}$$

$$\mathbf{= 9.81 \text{ x } 0.3 \text{ x } 142.7 \text{ x } 10^3}$$

$$\mathbf{= 419.97 \text{ x } 10^3 \text{ Watt}}$$

[1 mark]

The mechanical efficiency of the motor power supply is only 98%, and then the actual power generated by the motor is,

$$P = 419.97/0.98 = 428.5 \text{ kW}$$

[1 mark]