Numerical Solution of Poisson Equation Using Finite Difference Method

Tay Bee Hong & Puan Shazirawati Mohd Puzi

In this study, our goal is to find the approximation solutions to differential equations, that is, to find a function (or some discrete approximation to this function) which satisfies a given relationship between various of its derivatives on some given region of space, along with some boundary conditions along the edges of this domain. In general this is a difficult problem and only rarely can an analytic formula be found for the solution. A finite difference method proceeds by replacing the derivatives in the differential equations by finite difference approximations. This gives a large algebraic system of equations to be solved in place of the differential equation, something that is easily solved on a computer. Before tackling this problem, we first consider the more basic question of how we can approximate the derivatives of a known function by finite difference formulas based only on values of the function itself at discrete points. Besides providing a basis for the later development of finite difference methods for solving differential equations, this allows us to investigate several key concepts such as the order of accuracy of an approximation in the simplest possible C++ algorithms. Hence, we will present a simple and efficient Poisson solver. In many physical problems, one often needs to solve the Poisson equation on a non-Cartesian domain, such as polar or cylindrical domains. Thus, in this thesis we are also providing a solution for Poisson equation in cylindrical domains.