# FLUID STATICS

**MUHAMMAD SYAHIR SARKAWI, PhD**
Nuclear Engineering Program
Energy Engineering Department
N01-273 | 0133274154
syahirsarkawi@utm.my

# Objectives of the Topic

- Calculate pressure from a given datum
- Apply the equal level or pressure principle on manometers.
- Calculate the magnitude and location of forces on plane and curve surfaces that are submerged in a fluid.
- Calculate the buoyant force on a body using Archimedes' principle.
- Demonstrate the stability of a floating body.

# Contents

## Introduction

- Involved the study of fluid under static conditions (rest), No shear stress act on the fluid

## Pressure

- Pressure at a point : Pascal's Law, Pressure head in fluid, Pressure transmitability, Types of pressure, Datum, Summary

## Manometer

- Piezometer Tube, U-Tube, Differential, Invert-Differential

## Fluid force on submerge bodies

- Introduction, Action of fluid pressure on a surface of submerged bodies, Resultant force and centre of pressure on a plane and curved surface immersed in a liquid

## Buoyancy and stability of floating bodies

- Introduction, Bodies completely submerged, Archimedes principal & centre of buoyancy, Floating bodies, Stability & stability determination of submerged & floating bodies, Metacentric height concepts for floating bodies

# Pressure

- Pressure at a point
- Absolute, gauge, and vacuum pressures
- Pressure head of fluid
- Variation of pressure with depth
- Scuba diving and hydrostatic pressure
- Transmission of fluid pressure
- Pascal's law

# Pressure

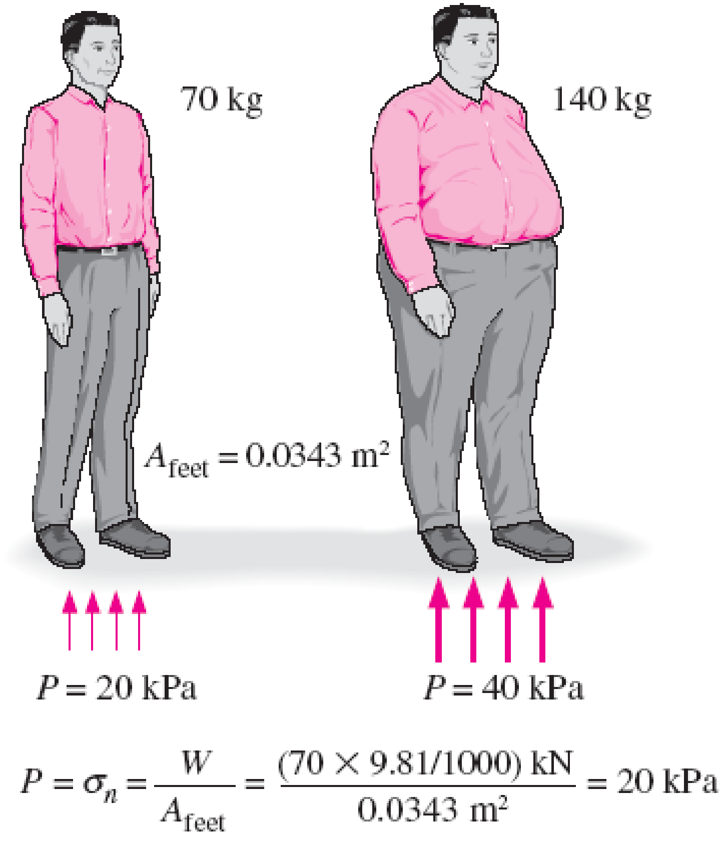
Pressure is defined as the **amount of force exerted on a unit area of a substance or on a surface.**

$$P = \frac{Force}{Area} = \frac{F}{A}$$

where:

**P = pressure, Pa ($N.m^{-2}$), $lb.ft^{-2}$, psi ($lb.in^{-2}$), atm(1 atm= 101,300 Pa = 2,116 $lb.ft^{-2}$), bar (1 bar = $10^5$Pa)**

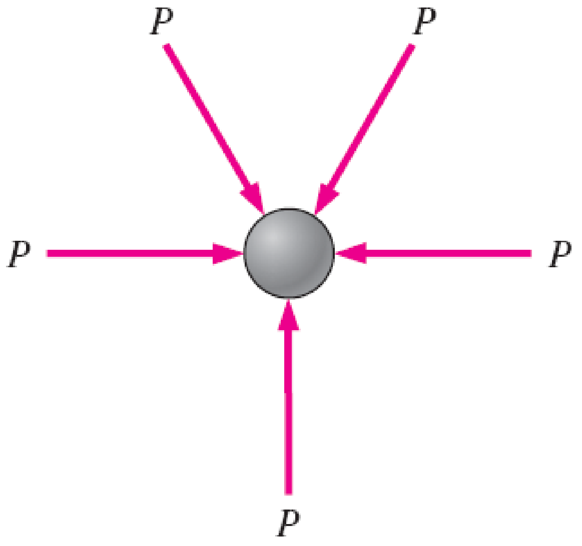
Some basic pressure gages.



# Pressure at a Point

- Pressure is the *compressive force* per unit area but it is not a vector. ***Pressure at any point in a fluid is the same in all directions [Pascal's Law].***

- Pressure has **magnitude** but **not** a specific **direction**, and thus it is a **scalar quantity.**



# Pressure (Absolute, Gauge and Vacuum)

- **Absolute pressure ($P_{abs}$)**

  Actual pressure at a give point

  It is measured relative to absolute vacuum (absolute zero pressure)

- **Atmospheric Pressure ($P_{atm}$)**

  Pressure due to weight of air above it

  Standard value: 1 atm equal to **101.3 kN/m$^2$**, 760 mmHg, 10.35 m$H_2O$ water (34 ft$H_2O$), **14.7 psi**

  Fluid pressure at free surface = $P_{atm}$

# Pressure (Absolute, Gauge and Vacuum)

- **Vacuum pressure ($P_{vac}$)**

  Pressure below atmospheric pressure:

$$P_{vac} = P_{atm} - P_{abs}$$

- **Gauge pressure ($P_g$)**

  Pressure that measured using pressure gauge

- **Most gauge** are calibrated to read zero in the atmosphere

$$P_{gauge} = P_{abs} - P_{atm}$$

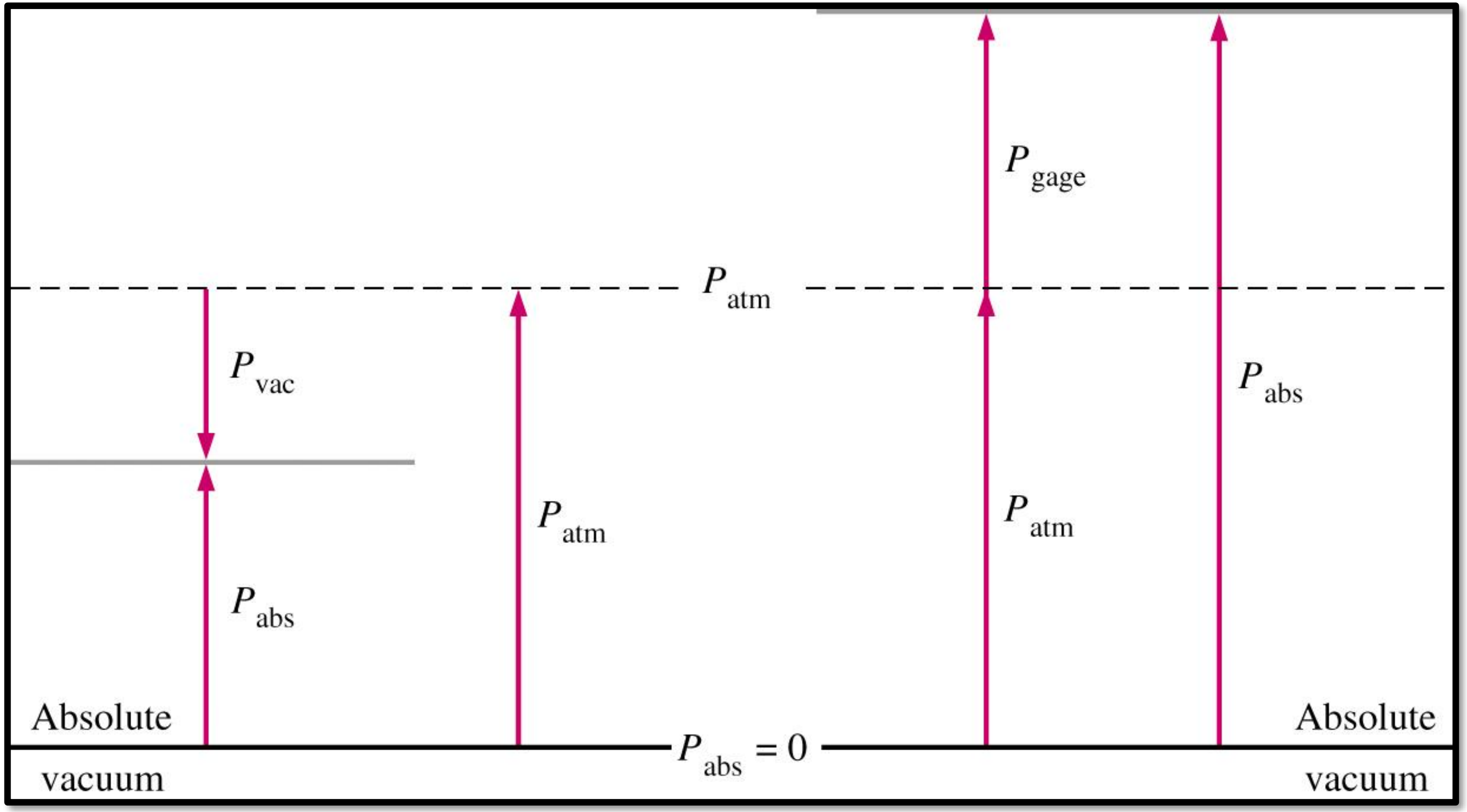
  +ve (above atm pressure)

  –ve (below atm pressure): suction pressure of vacuum pressure

  **Zero pressure = atmospheric pressure**

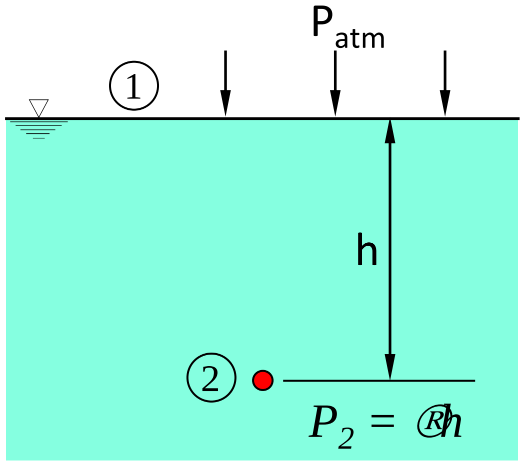
- Gauge pressure units: **$N/m^2$ gauge**, **psig**, **kPa gauge**, **barg**

# Pressure (Absolute, Gauge and Vacuum)



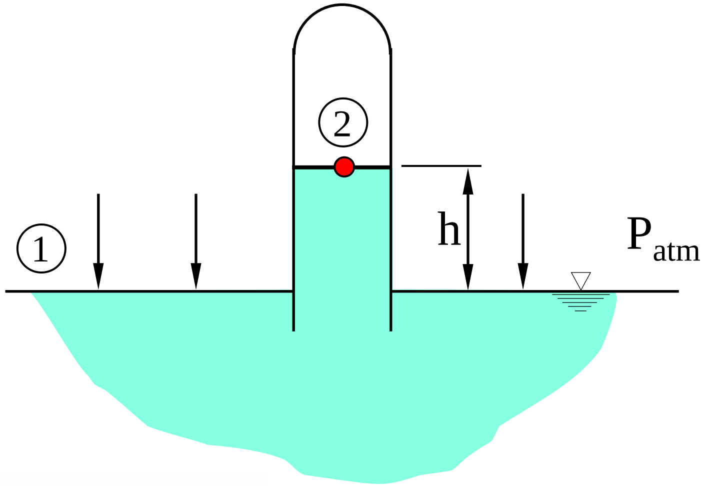
# Pressure (Absolute, Gauge and Vacuum)

- Case I




$$p_2 = \rho g h = \gamma h$$
$$p_a = p_{atm} + p_2$$

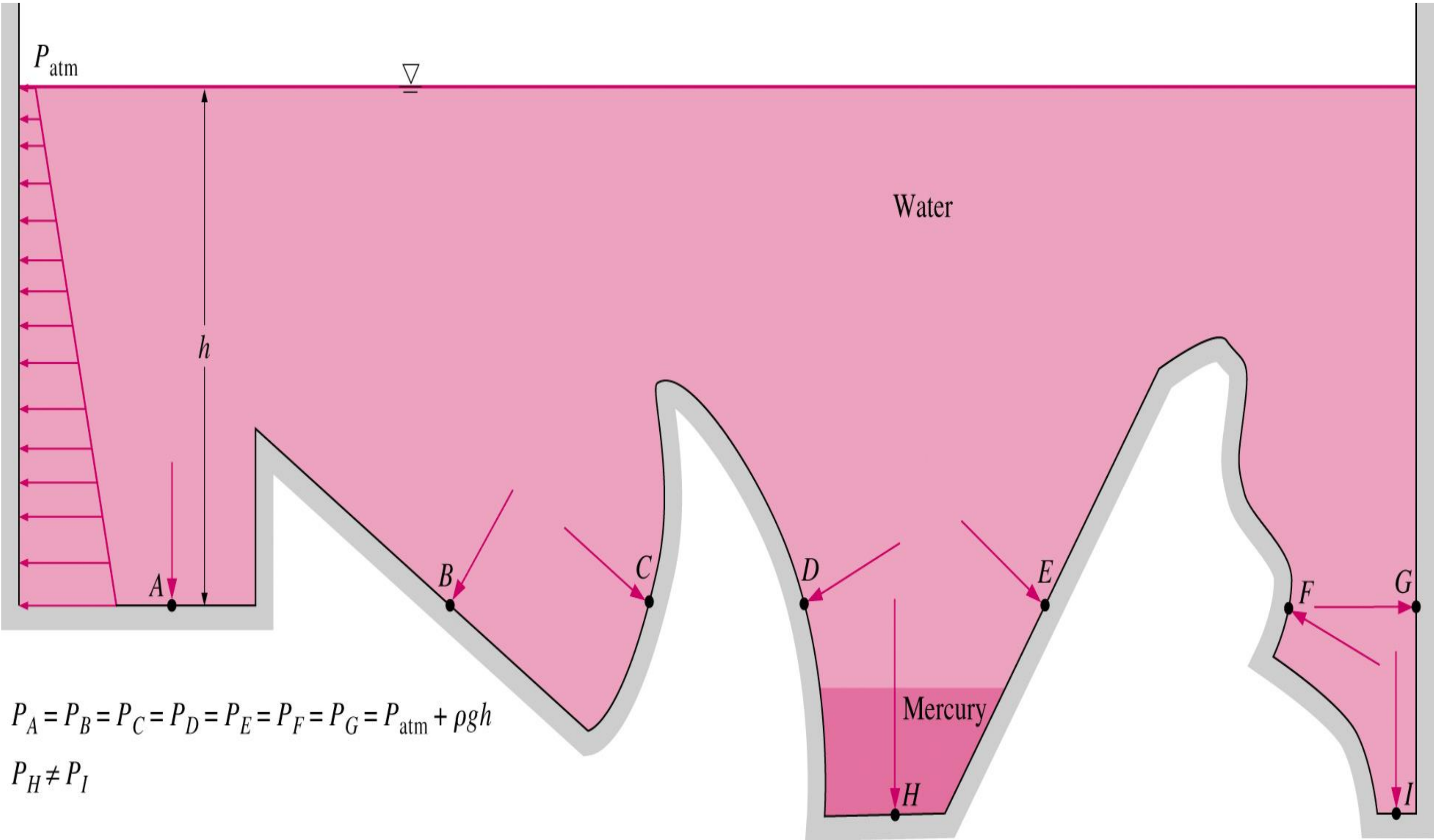
- Case 2




$$p_2 = \rho g h = \gamma h \quad \text{(-ve or +ve ????)}$$
$$p_a = p_{atm} + p_2$$

# Variation of Pressure with Depth

Pressure is the **same at all points** on a **horizontal plane** in a given fluid

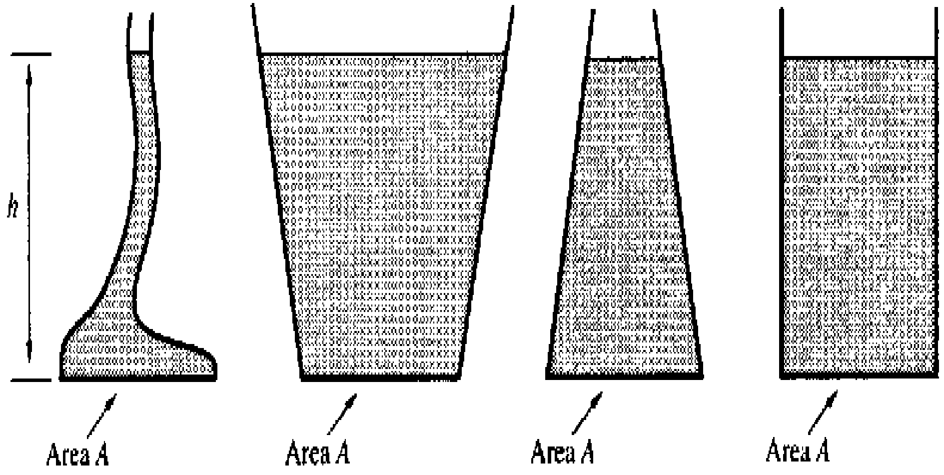



$$P_A = P_B = P_C = P_D = P_E = P_F = P_G = P_{atm} + \rho g h$$

$$P_H \neq P_I$$

# Pressure Head in Fluid

- Pressure is proportional to depth (P α h) :**regardless of shape of container**




- ✓ Equation:

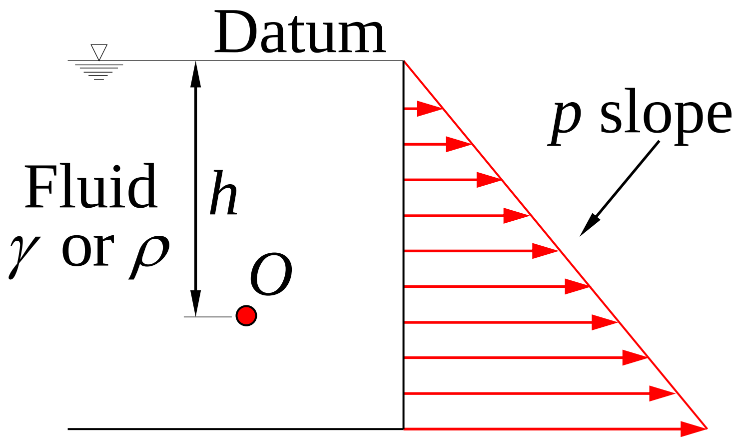
$$h = \frac{p_1 - p_2}{\gamma}$$

- ✓ Unit:

- **Pressure head:**

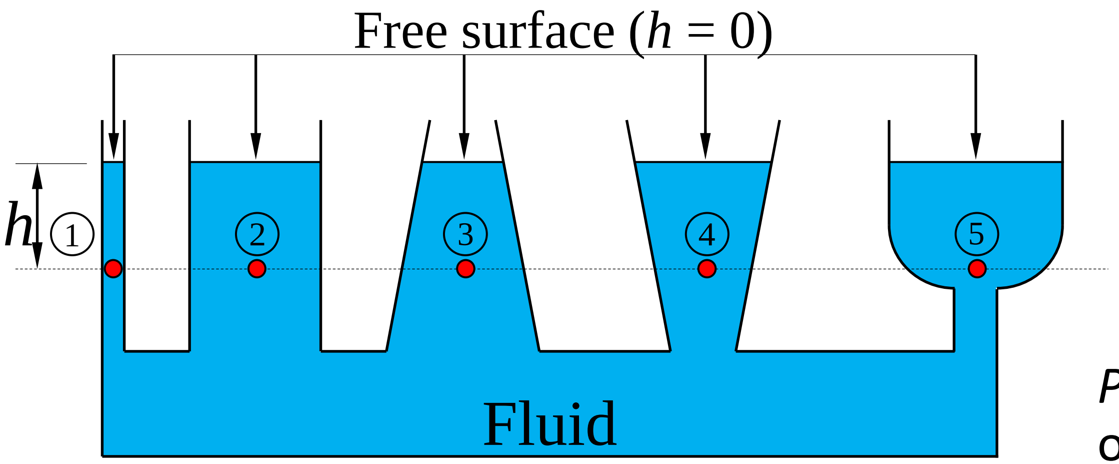
  The height of a column of fluid of **specific weight γ** required to give a pressure difference $P_1$-$P_2$

# Pressure Head in Fluid




Pressure at $O$

$$p_o = \rho g h = \gamma h$$

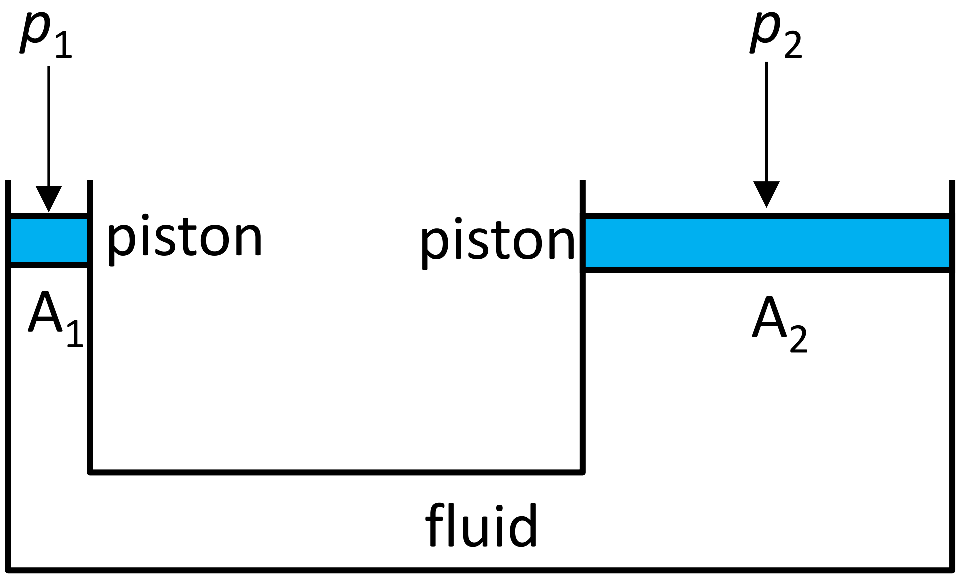



$$p_1 = p_2 = p_3 = p_4 = p_5$$

*P* exerted by a fluid is **dependent only on the vertical head, *h***, of the fluid; it is not affected by the size & shape of the vessels

# Pressure Transmissibility

- Pascal: ***"In a closed system, the variation of pressure at one point in the system will be transmitted to the whole system"***
- Basic **hydraulic system principles**



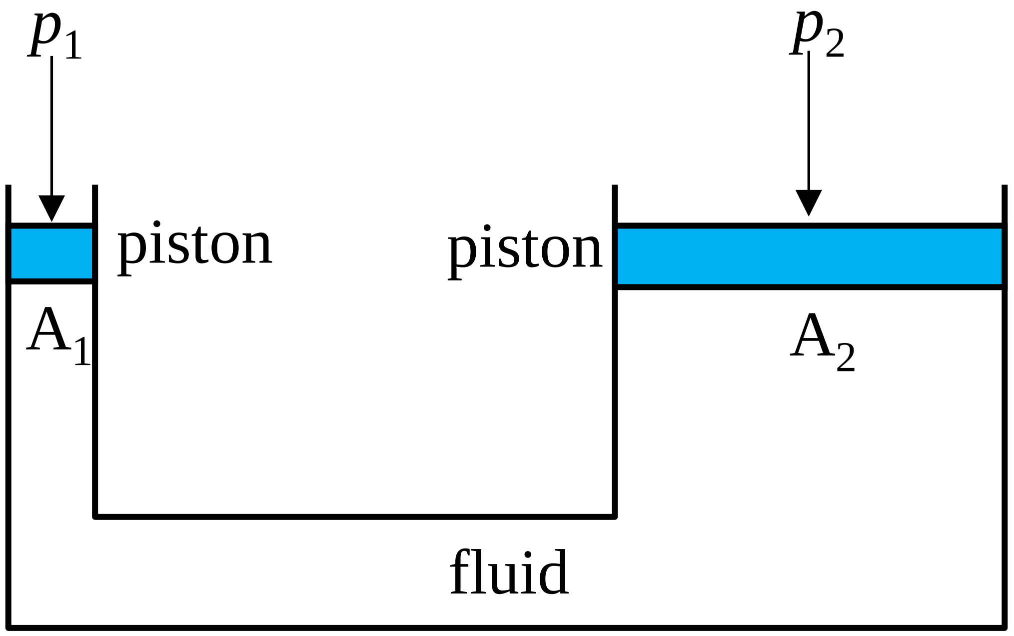

$p_1 = p_2$ (act on opposite direction)

# Pressure Transmissibility

- The transmission of fluid pressure throughout a stationary fluid is the principle upon which many **hydraulic devices** are based

**$P_1 = P_2$**

$F_1 = pA_1 \qquad F_2 = pA_2$

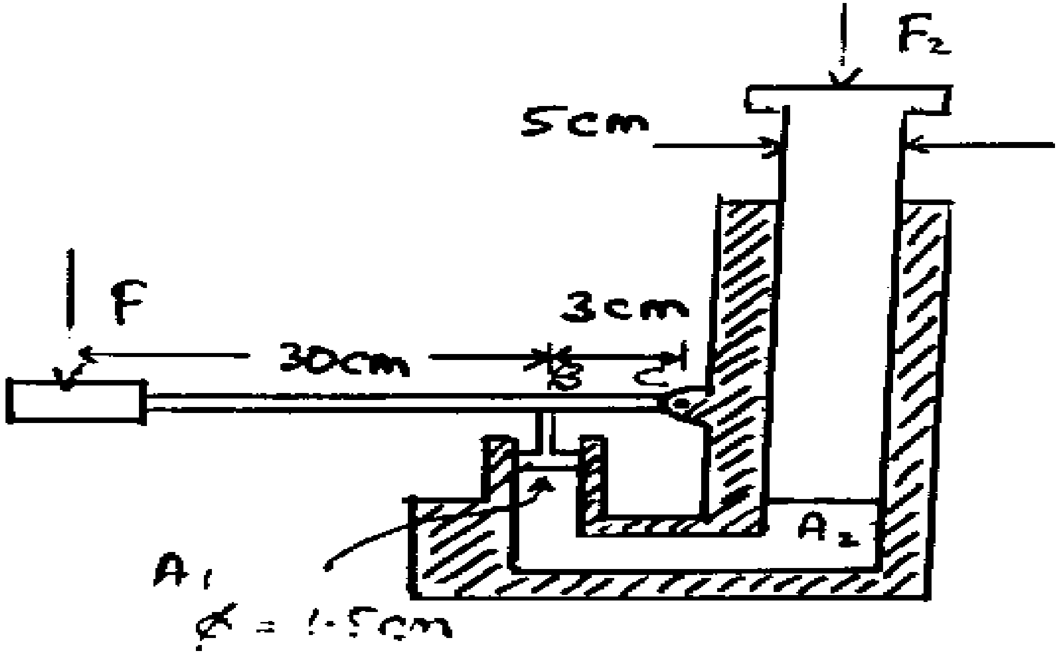



Note :

The pressure force exerted by the fluid is always normal to the surface at the specified points

# Pressure Transmissibility (Problem 2.1)

- Dimension of hydraulic jack is shown in figure below. If a force of 100 N applied onto the jet handle, determine a maximum force $F_2$ would be support.




$\phi$ (OD) = outer diameter = 1.5 cm

**HINT**

- FBD: Summing moment at point C
- Calculate $P_1$
- Principle of pressure transmissibility, calculate $F_2$

ANS: F2 = 12.22 kN

# Pressure Transmissibility (Problem 2.2)

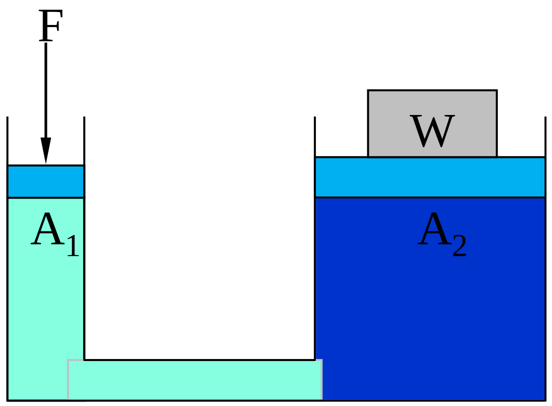
A diagram of a hydraulic jack is shown below. A force of 850 N is applied to the smaller cylinder of a hydraulic jack. The area of the small piston is 15 $cm^2$ and the area of the larger piston is 150 $cm^2$. What load *W* can be lifted on the larger piston

a) if the pistons are at the same level,

b) if the larger piston is 0.75 m below the smaller piston?

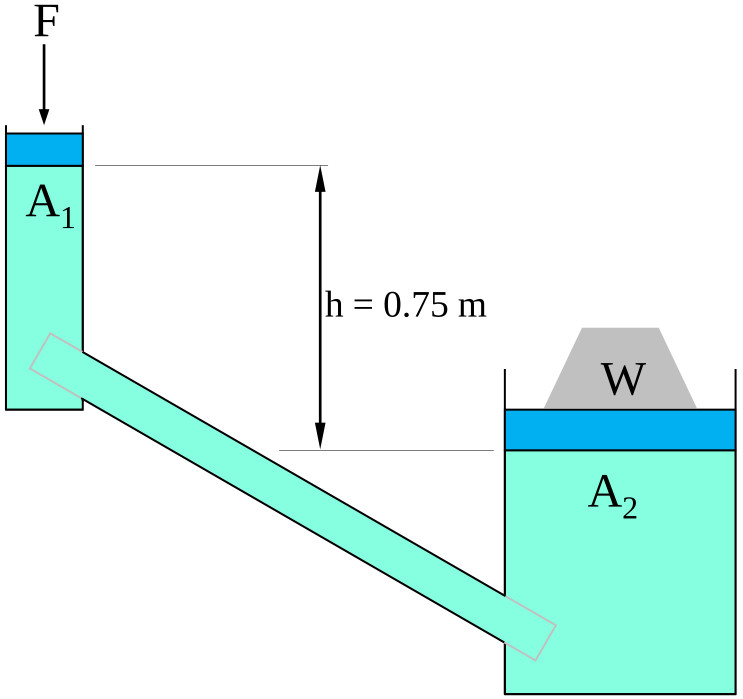
(***Note***: *The hydraulic fluid is water*)

a)




ANS: Load W = 866 kg

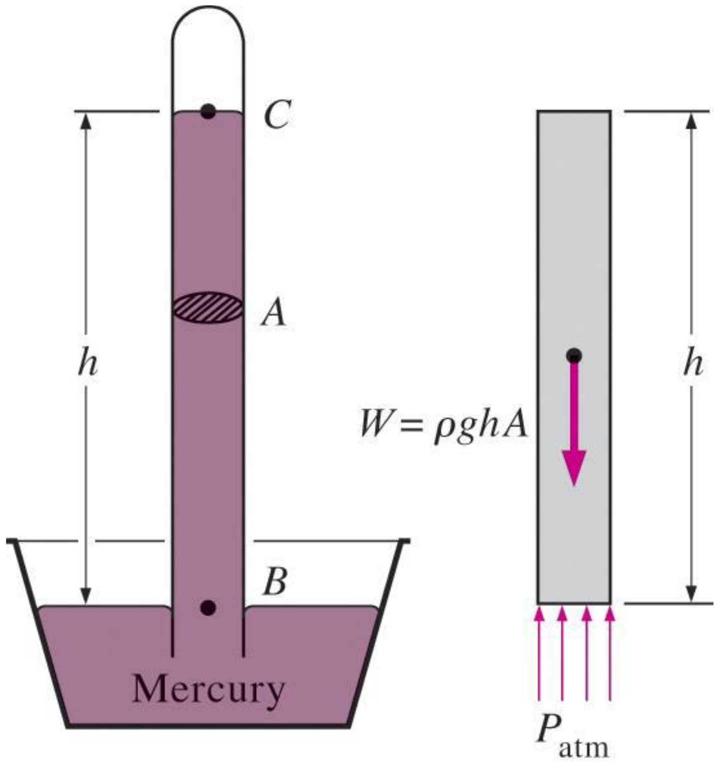
b)




ANS: Load W = 878 kg

# Barometer

- Atmospheric pressure is measured by a device called a **barometer**; thus, the atmospheric pressure is often referred to as the ***barometric pressure***.




**The basic barometer.**

- A frequently used pressure unit is the ***standard atmosphere***, which is defined as the pressure produced by a column of mercury 760 mm in height at 0°C ($\rho_{Hg}$= 13,595 kg/m3) under standard gravitational acceleration (*g* = 9.807 m/s2).

$$P_{atm} = \rho g h$$

# Manometer

- Used to measure pressure
- Only 4 types will be consider

  Piezometer Tube

  U-Tube Manometer

  Differential Manometer

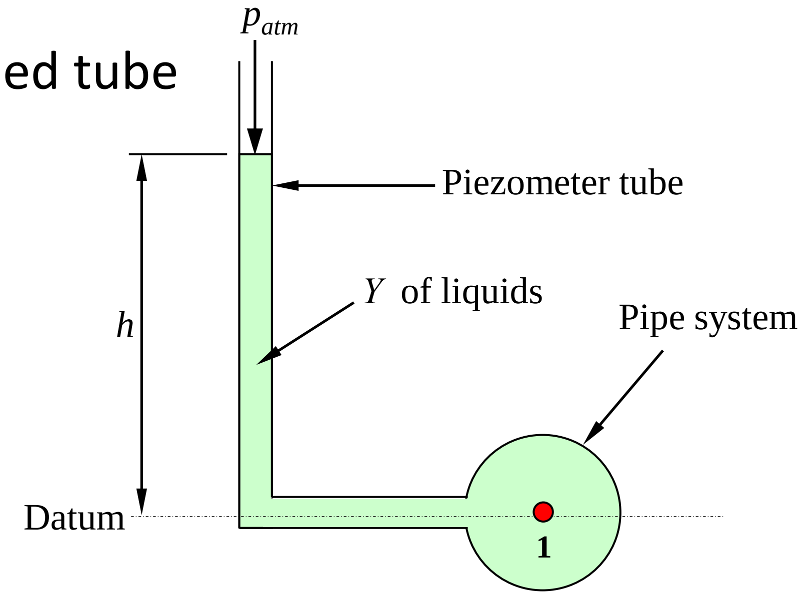
  Invert-Differential Manometer

- Generally, whatever shapes and/or orientation of the manometer, the basic concepts are still the same:

  pressure equilibrium concepts (needs to comprehend comprehensively)

# Manometer (Piezometer Tube)

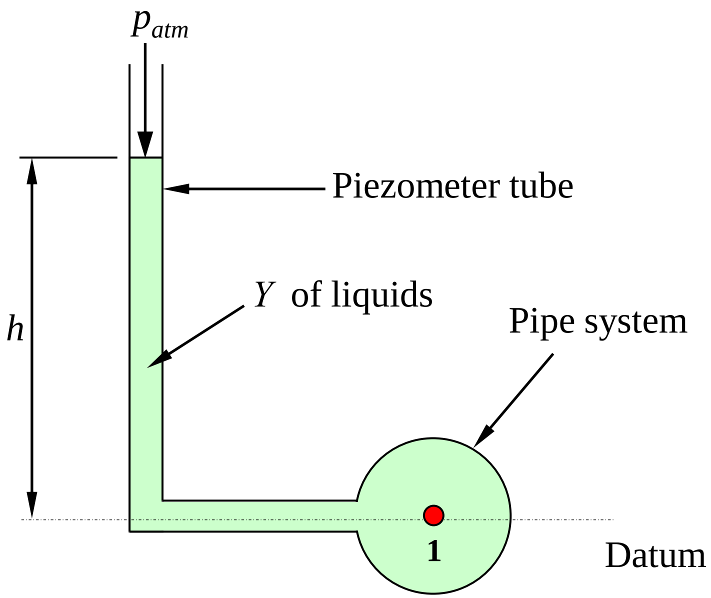
- Used the concepts of liquid head in vertical or inclined tube
- The simplest manometer




- Choose appropriate datum line
- Pressure at point 1(in the pipe system) is due to the weight of liquid in the Piezometer tube (above datum line): $p_1 = \rho g h = \gamma h$ (gauge)
- Absolute pressure: $p_1 = \gamma h + p_{atm}$ (absolute)

# Manometer_Piezometer Tube (Problem 2.3)

- Based on the diagram below, determine the maximum gauge pressure of water that can be measured by a piezometer tube 2 m high? (Water mass density = 1000 $kg/m^3$).



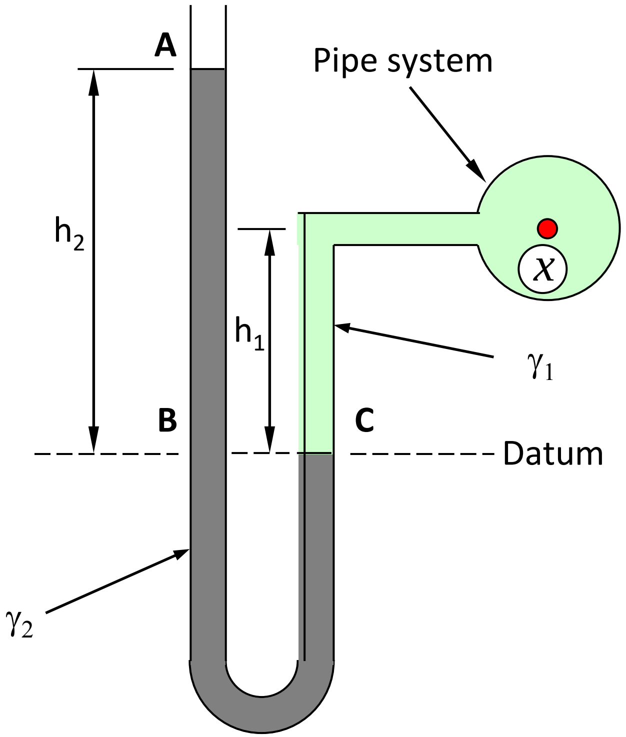

ANS: P1 = 20 kN/m^2

# Manometer (U-Tube)

- Fabricated from 'U' shaped tube with one open-ended
- Filled with manometer liquid which is of greater $\rho$ and is immiscible with the liquid system that to be measured

  **e.g. Hg/$H_2O$ Hg/Oil $H_2O$/Oil**

- First and foremost, choose the appropriate datum line
- **Rule of thumbs**: choose the lowest level of contact between the two liquids




**For equilibrium**

$$p_B = p_C$$

$$\begin{pmatrix}\text{Weight of liquid } \rho_2 \\ \text{above datum line}\end{pmatrix} = (p \text{ at } x) + \begin{pmatrix}\text{Weight of liquid } \rho_1 \\ \text{above datum line}\end{pmatrix}$$

$$\begin{pmatrix}h_2 \text{ head } p \text{ of} \\ \text{manometer liquid } \rho_2\end{pmatrix} = (p \text{ at } x) + \begin{pmatrix}h_1 \text{ head } p \text{ of} \\ \text{flowing liquid } \rho_1\end{pmatrix}$$

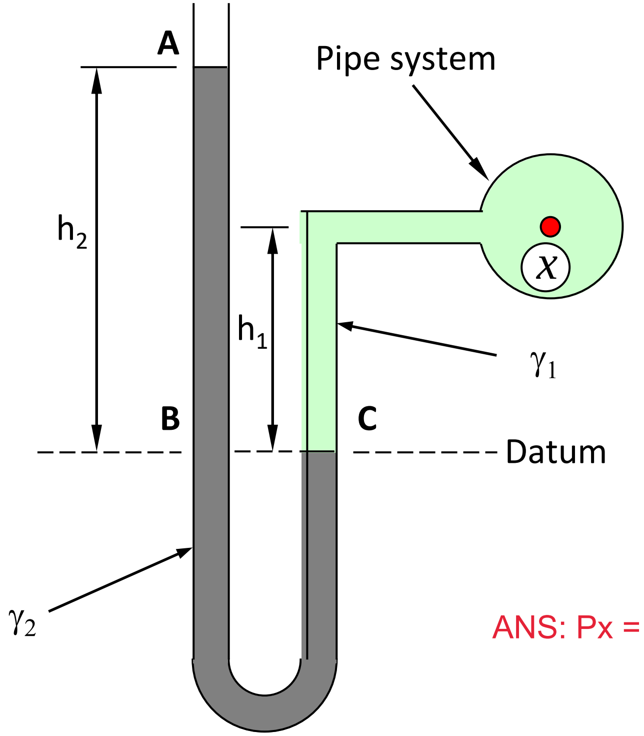
$$\rho_2 g h_2 = p_x + \rho_1 g h_1$$

$$p_x = \rho_2 g h_2 - \rho_1 g h_1$$

$$\therefore \quad p_x = g(\rho_2 h_2 - \rho_1 h_1) = \gamma_2 h_2 - \gamma_1 h_1 \quad \text{(gauge)}$$

# Manometer_U-Tube (Problem 2.4)

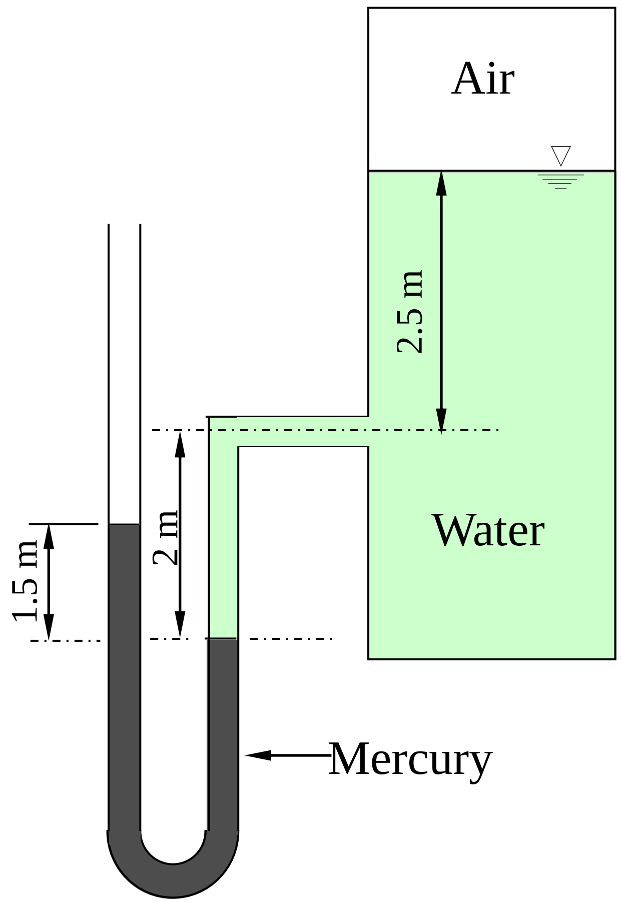
- A U-tube manometer as shown in figure below is used to measure the gauge pressure of fluid N of density $\rho_1$ = 800 kg/m$^3$. If the manometer fluid is mercury with a density of 13600 kg/m$^3$ ($\rho_2$), what will be the gauge pressure at x if $h_1$ = 0.5 m and $h_2$ = 0.9 m




ANS: Px = 116.15 kN/m^2

# Manometer_U-Tube (Problem 2.4)

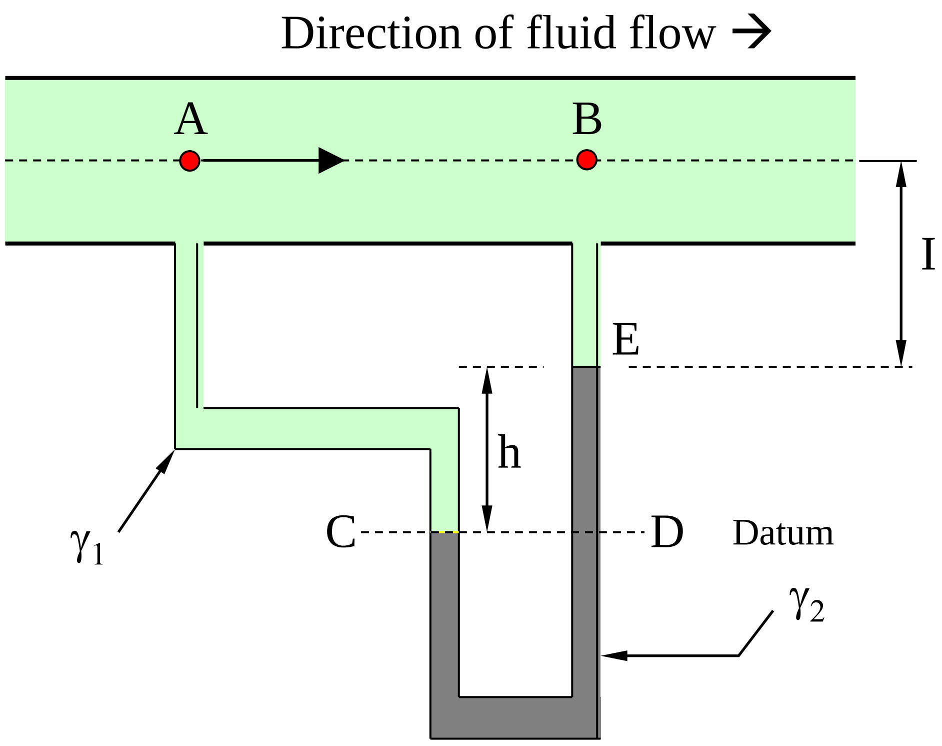
- Calculate the air pressure in the tank as shown in the figure below




ANS: Pair = 156 kN/m^2

# Manometer (Differential)

- Measure the pressure difference between 2 points in a pipeline
- Choose the lowest level of contact between the two liquids as datum




For equilibrium

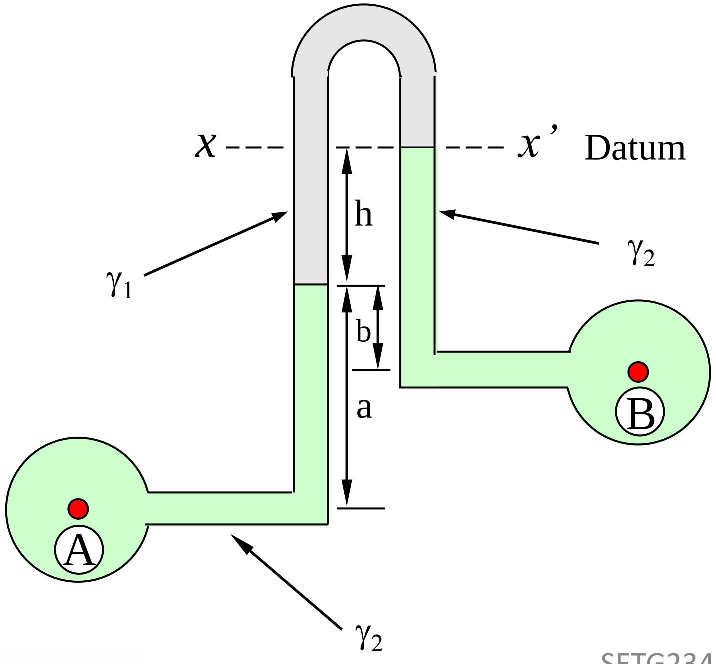
$$p_C = p_D$$

$$p_A + \rho_1 g(h + I) = p_B + \rho_2 gh + \rho_1 gI$$

$$\therefore \; p_A - p_B = gh(\rho_2 - \rho_1) = h(\gamma_2 - \gamma_1)$$

# Manometer (Inverted-Differential)

- Inverted-differential or inverted u-tube manometer
- As in the previous case, it is also measuring the pressure difference between 2 points in a pipeline
- Has a reverse orientation from an ordinary differential manometer
- Choose the lowest level of contact between the two liquids as a datum




For equilibrium

$$p_x = p_{x'}$$

$$P_A - \rho_2 g a - \rho_1 g h = P_B - \rho_2 g(b+h)$$

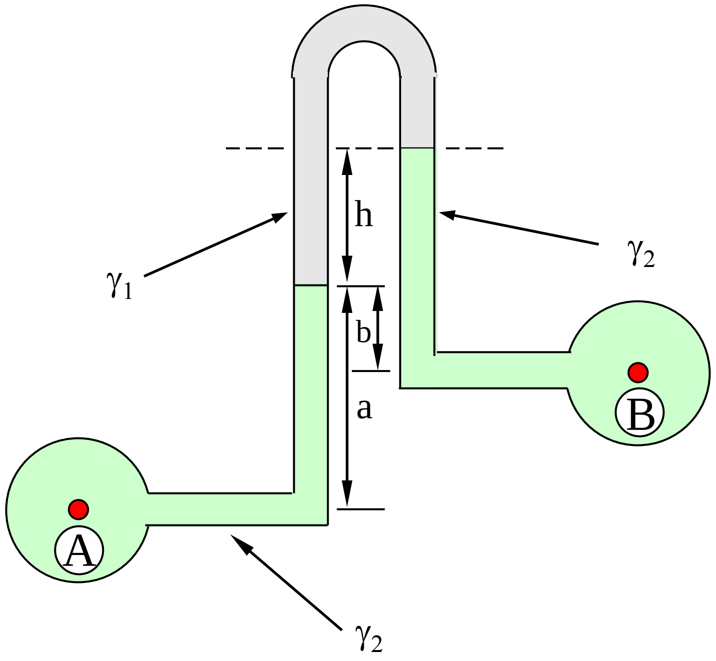
$$\therefore \; p_A - p_B = \rho_2 g(a-b) + gh(\rho_1 - \rho_2)$$

# Manometer_Inverted-Differential (Problem 2.6)

- An inverted U-tube manometer as shown in figure below is used to measure the gauge pressure. If the liquids at A and B are water with density *r* = 1000 kg/m$^3$, and the height differences are h = 0.3 m, a = 0.25 m and b = 0.15 m respectively. Calculate the pressure difference $p_A - p_B$ if the top of the manometer is filled with

  a. air ANS: Pair = -1962 N/m^2

  b. oil with SG = 0.8 ANS: Poil = 392.4 N/m^2



# Fluid Force on Submerged Bodies

- Introduction
- Action of fluid pressure on a surface of submerged bodies
- Resultant force and centre of pressure on a plane surface immersed in a liquid
- Resultant force and centre of pressure on a curved surface immersed in a liquid
- Buoyancy and stability of floating bodies

***p***

***R***

1) on immersed bodies in a liquid

2) plane & curved surfaces

# Action of Fluid Pressure on a Surface of Submerged Bodies

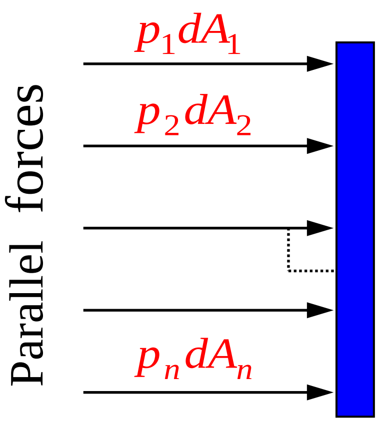
- Resultant force

$$R = p_1 dA_1 + p_2 dA_2 + \ldots\ldots\ldots\ldots + p_n dA_n = \Sigma\left(p_n dA_n\right)$$

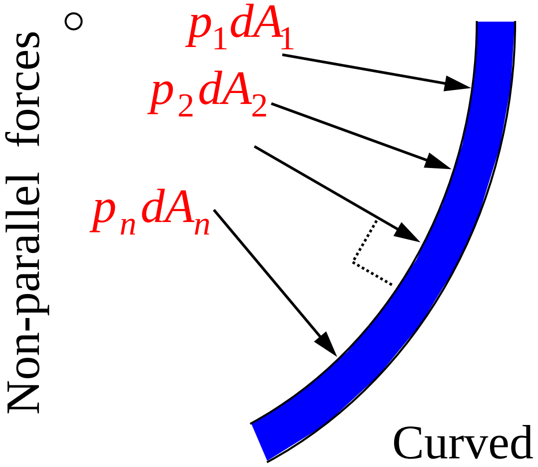
- Point which R acts ➡ centre of pressure/force
- If the boundary is a plane surface, all the forces act on it will be parallel

$$\therefore \quad R = \Sigma\left(p_n dA_n\right) \text{ sum of forces}$$

- If the boundary is a curved surface, all the forces will act perpendicular to the surface at each point ➡ non-parallel forces



Plane surface



Curved surface

# Action of Fluid Pressure on a Surface of Submerged Bodies

- For curved surface:

All forces are not parallel → must be combined as a vector

The forces will be divided to horizontal, $R_H$, and vertical, $R_V$, component of forces & then combined using Pythagorean theorem to determine the resultant force

$$R = F_T = \sqrt{R_H^2 + R_V^2}$$

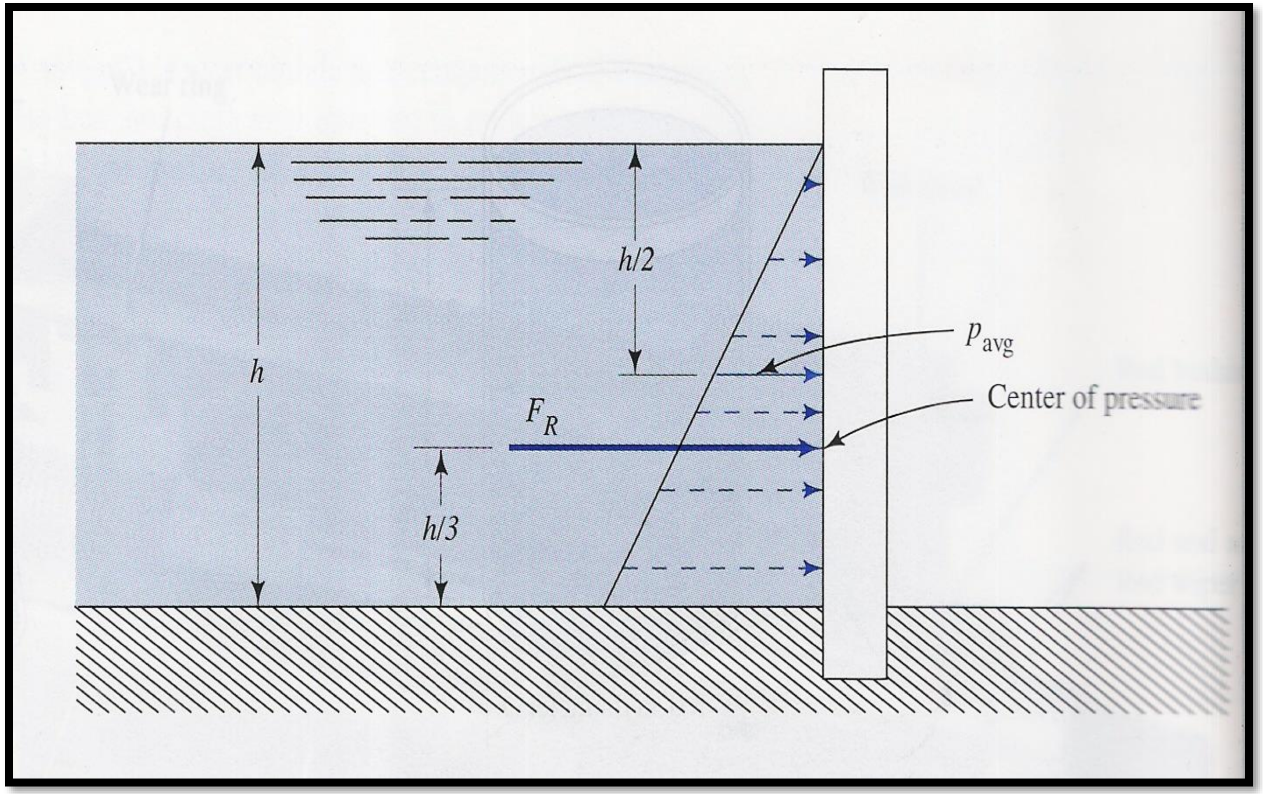
Resultant force direction, *q*

$$\theta = \tan^{-1}\left(\frac{R_V}{R_H}\right)$$

# Force Due to Statics Fluids

- Examples:-

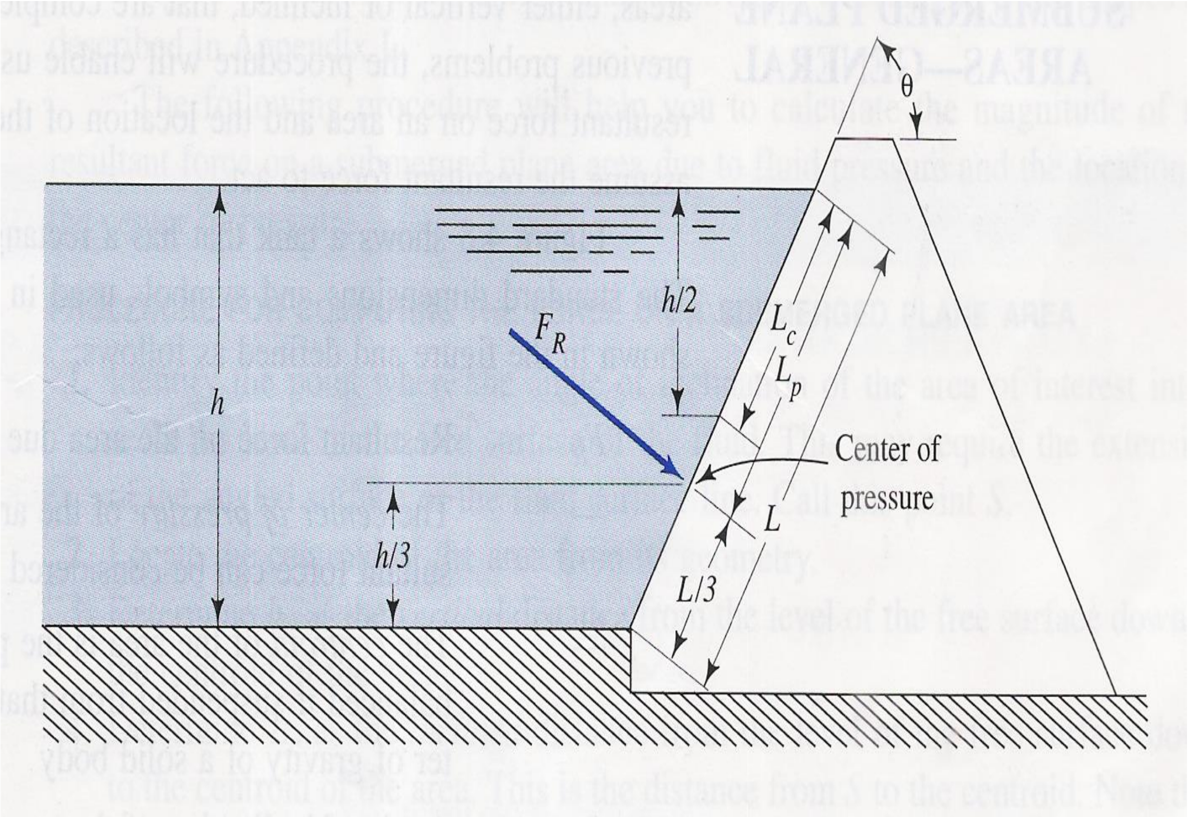
  1) vertical retaining wall



# Force Due to Statics Fluids

- Examples:-

    2) inclined wall (dam)




The figure shows a dam 30.5 m long that retains 8 m of fresh water and is inclined at an angle Ɵ of 60°C . Calculate the magnitude of the Resultant force on the dam and the location of the centre of pressure (CP)

# Application

**Itaipú Dam** on the Upper Paraná River, north of Ciudad del Este, Paraguay

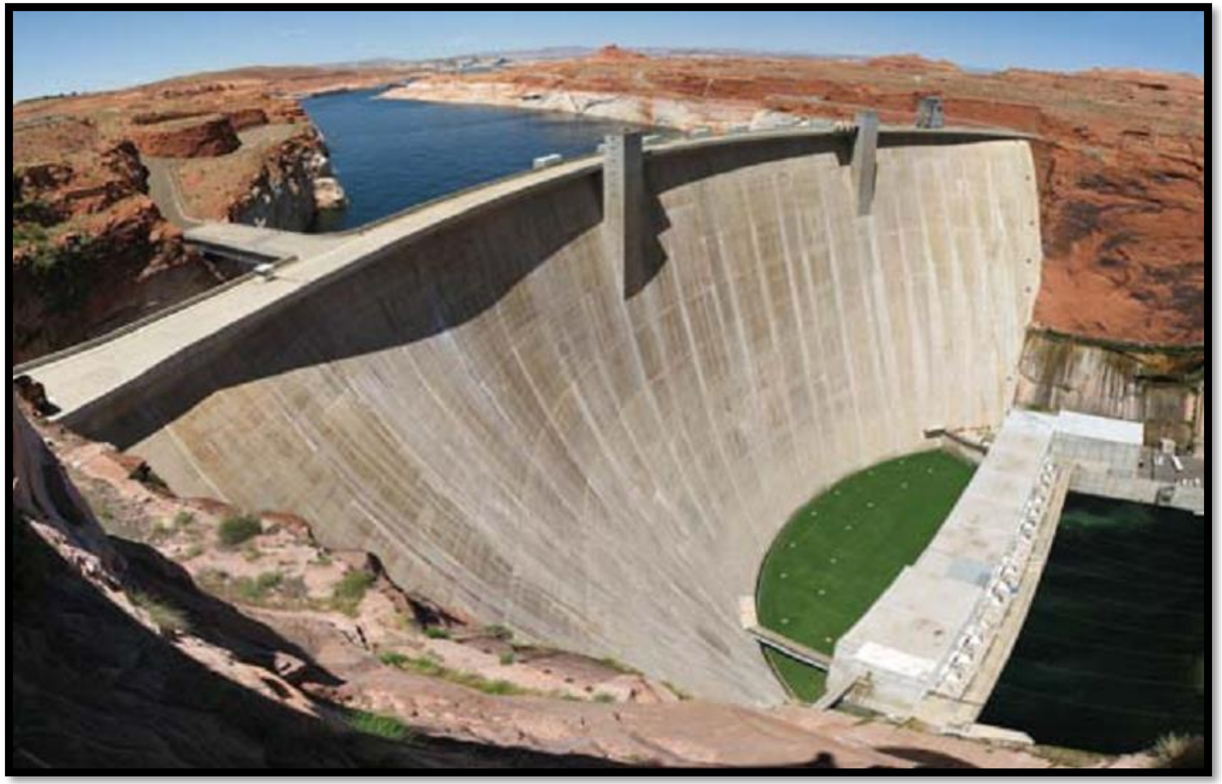
# Application

Aerial view of **Hoover Dam** on the Arizona-Nevada border

# Application

**Fort Peck Dam** on the Missouri River creates Fort Peck Lake, near Glasgow, northeastern Montana. Construction began in 1933 and was finished in 1940

# Application

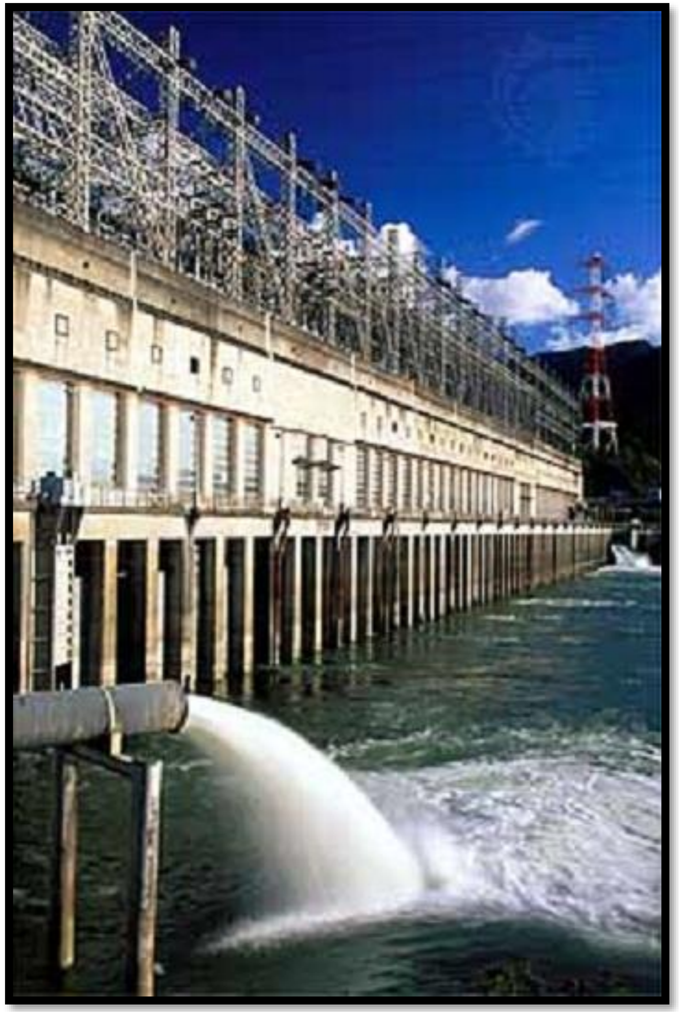


Construction of the **Glen Canyon Dam** on the Colorado River formed Lake Powell in Arizona

# Application

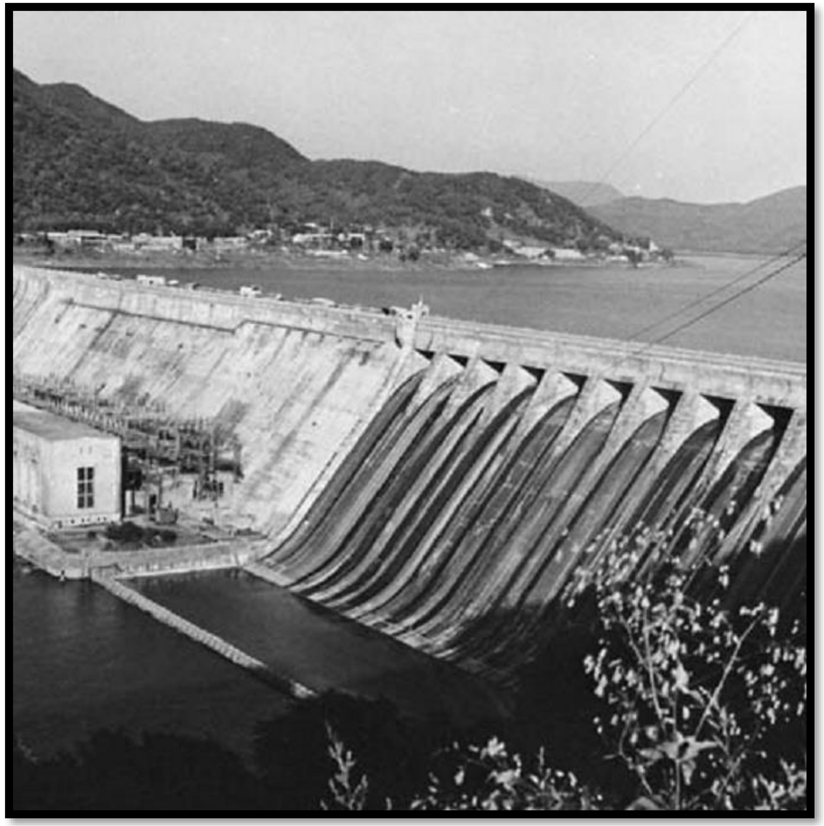
**Kariba Dam,** on the Zambezi River at the border between Zambia and Zimbabwe

# Application



**Bonneville Dam** stems the Columbia River in Washington and Oregon. The dam’s special fish ladders help salmon swim upstream to their spawning grounds

# Application

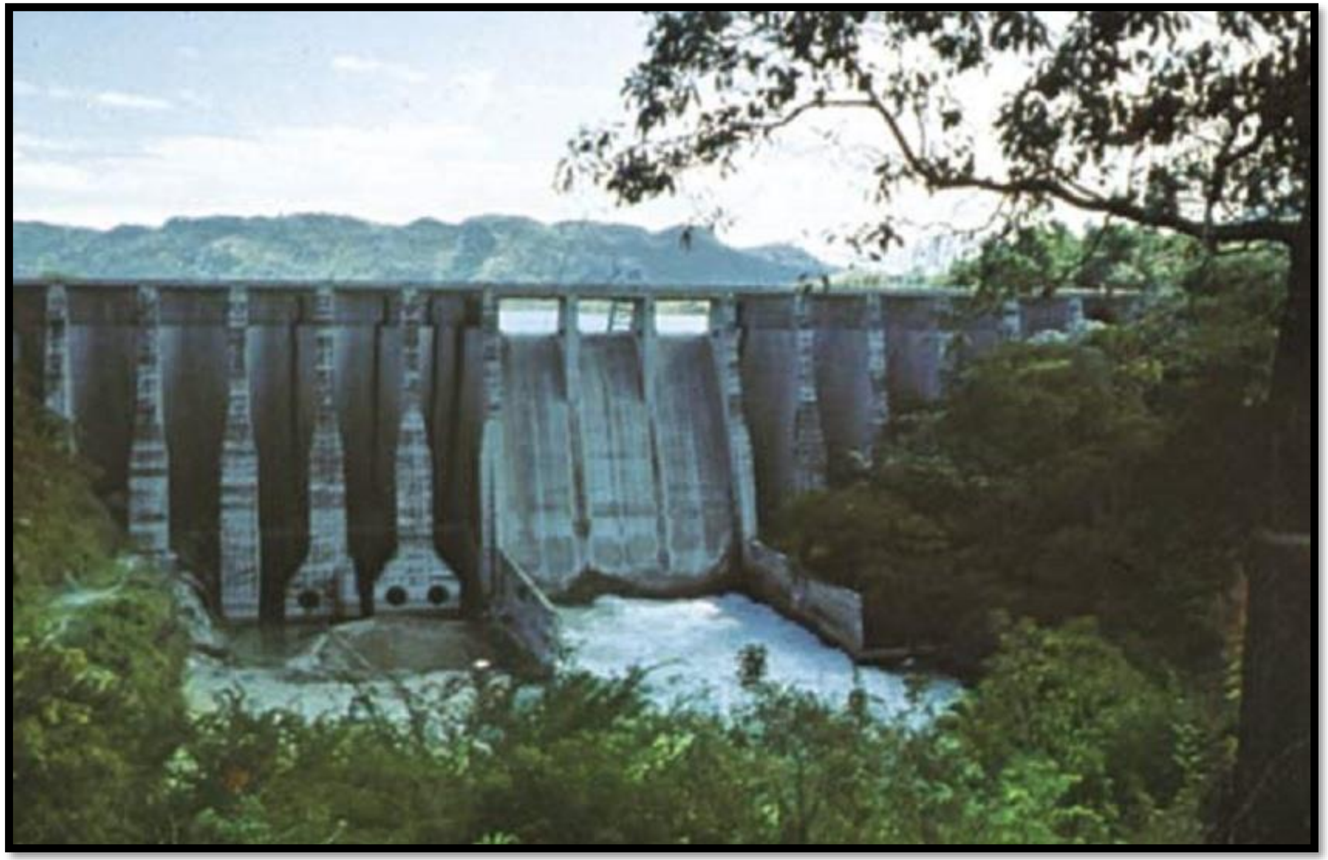


The **Fengman Dam** and hydroelectric power station on the Sungari (Songhua) River, Jilin province, northeastern China

# Application

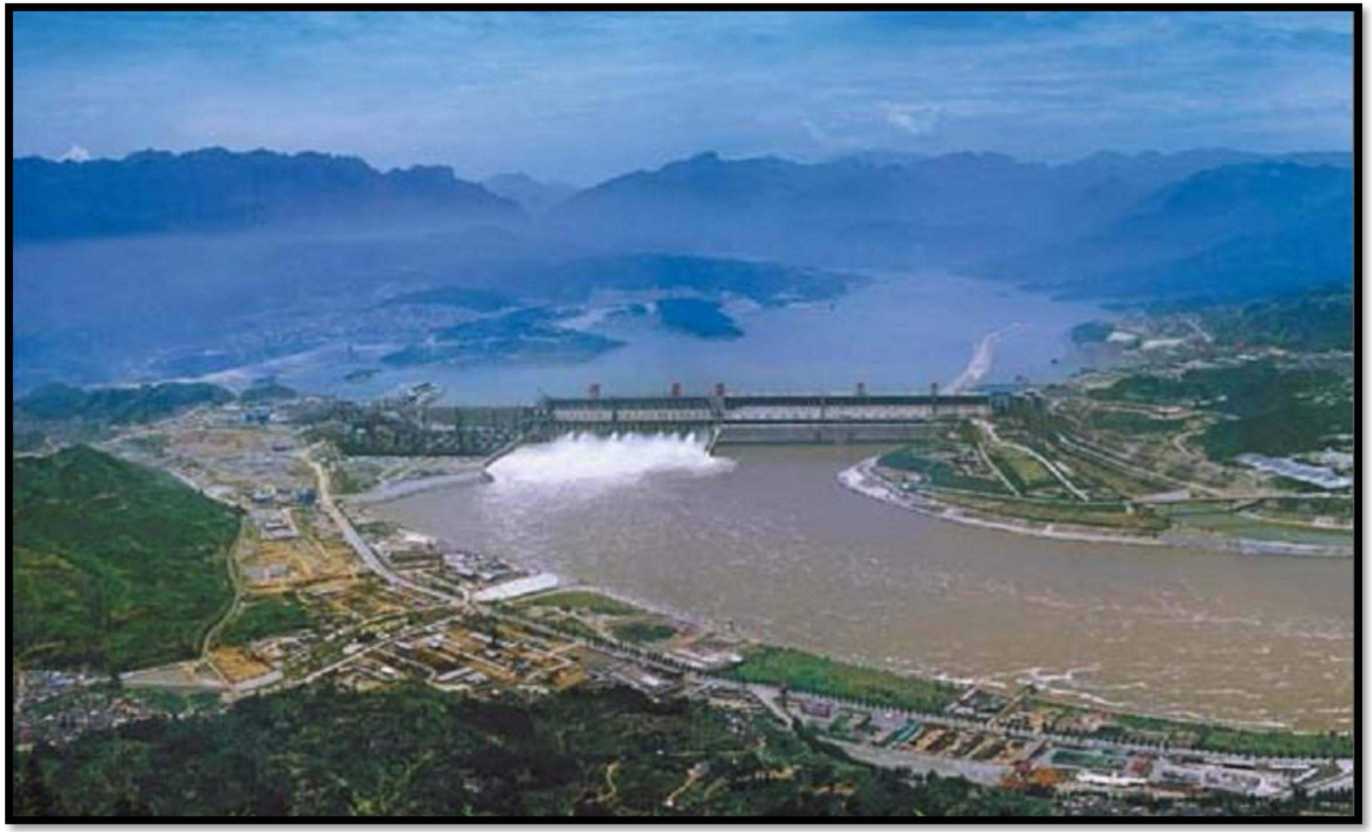
Dam on the **Karun River**, Iran

# Application



**Péligre Dam**, Artibonite River, Haiti

# Application



The **Three Gorges Dam** spanning the Yangtze River (Chang Jiang) near Yichang, Hubei province, China

# Application


The **Kenyir Dam** and hydroelectric power station on Kenyir Lake, Terengganu, Malaysia

(Average annual energy output is 1,600 GWh)

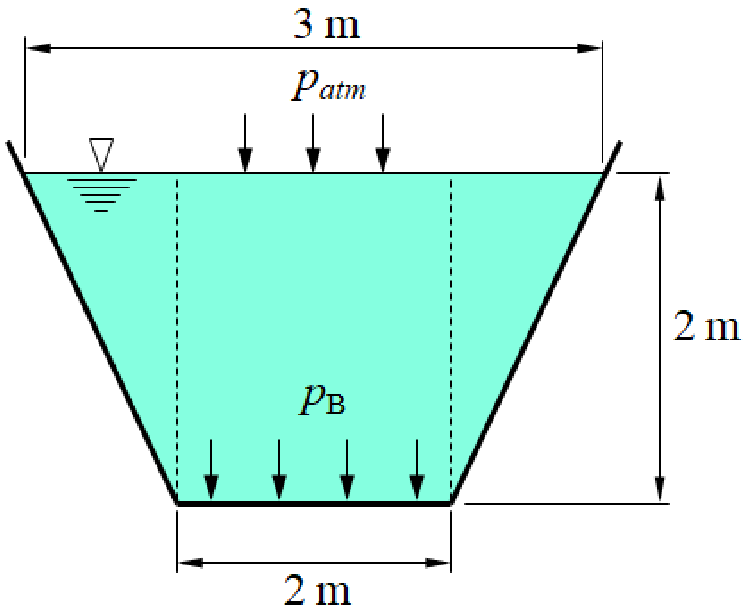
# Action of Fluid Pressure on a Surface of Submerged Bodies (Problem 2.7)

- Relationship between force and the weight of fluid

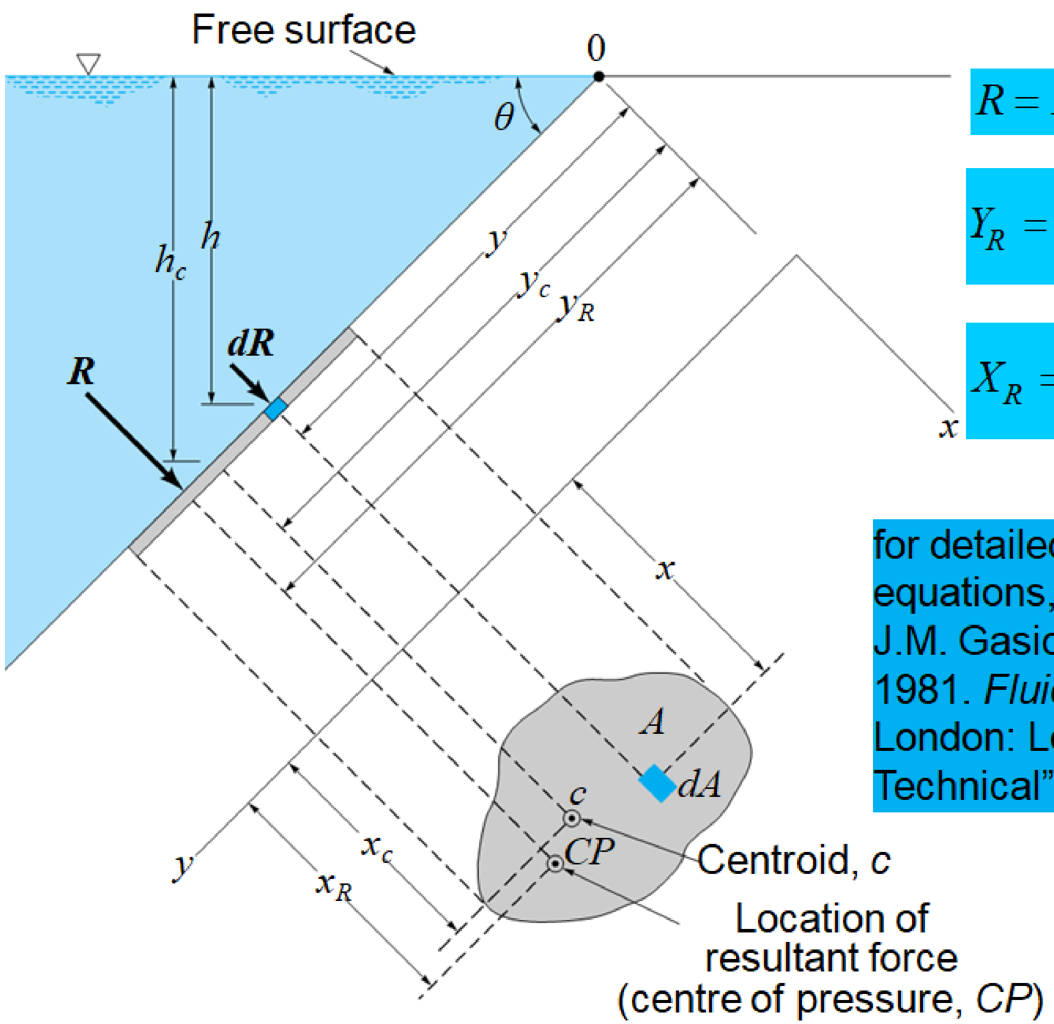
An opened trapezoidal shaped tank as shown below, length of 5 m, is filled with water. Calculate:

a) The weight of water in the tank ANS: Wwater = 245 kN

b) Resultant force which acts at the bottom of the tank ANS: R = 196 kN



# Resultant Force and Centre of Pressure on a Plane Surface Immersed in a Liquid




$$R = PA = \rho g h_C A = \gamma h_C A$$

$$Y_R = \frac{I_{XC}}{Y_C A} + Y_C$$

$$X_R = \frac{I_{XYC}}{Y_C A} + X_C$$

Coordinate of resultant force centre

for detailed derivation of the equations, please refer "Douglas, J.F., J.M. Gasiorek, and J.A. Swaffield. 1981. *Fluid Mechanics*. 3rd Edition. London: Longman Scientific and Technical", page 57-60

# Resultant Force and Centre of Pressure on a Plane Surface Immersed in a Liquid

- Symbols and meaning

$R =$ resultant force

$\rho =$ fluid density

$A =$ plane surface area

$h =$ depth

$h_C =$ depth of object centroid centre from horizontal axis, measured from free fluid surface

$h_R =$ depth of the centre of $R$ acts on an object from horizontal axis, measured from free fluid surface

$Y_C =$ object's centroid coordinate measured from $X$ axis perpendicular to object location (from free fluid surface)

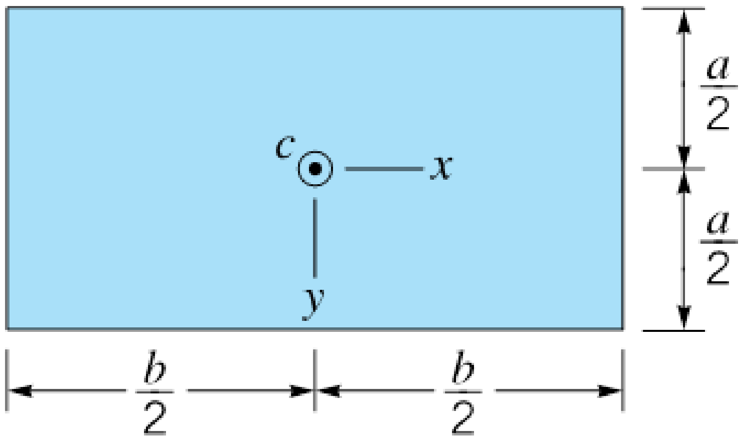
$Y_R =$ centre coordinate of $R$ acts on an object from $X$ axis perpendicular to object location (from free fluid surface)

$X_C =$ object's centroid coordinate measured from $Y$ axis perpendicular to object location (from free fluid surface)

$X_R =$ centre coordinate of $R$ acts on an object from $Y$ axis perpendicular to object location (from free fluid surface)

$I, I_{XC}, I_{XYC} =$ second moment of area / moment inertia

# Geometric Properties of some Common Shapes



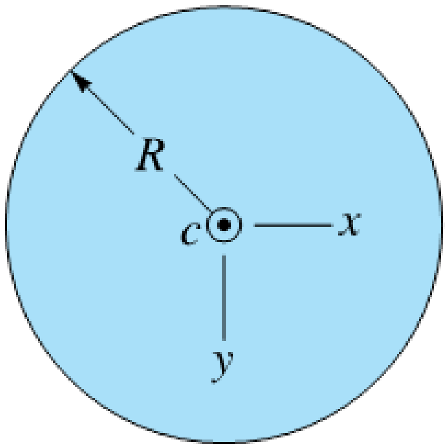

$$A = ba$$

$$I_{xc} = \frac{1}{12} ba^3$$

$$I_{yc} = \frac{1}{12} ab^3$$

$$I_{xyc} = 0$$

(*a*)



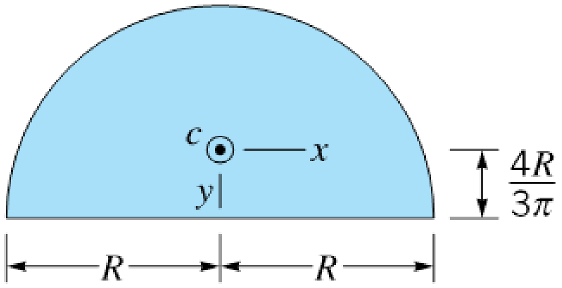

$$A = \pi R^2$$

$$I_{xc} = I_{yc} = \frac{\pi R^4}{4}$$

$$I_{xyc} = 0$$

(*b*)

# Geometric Properties of some Common Shapes



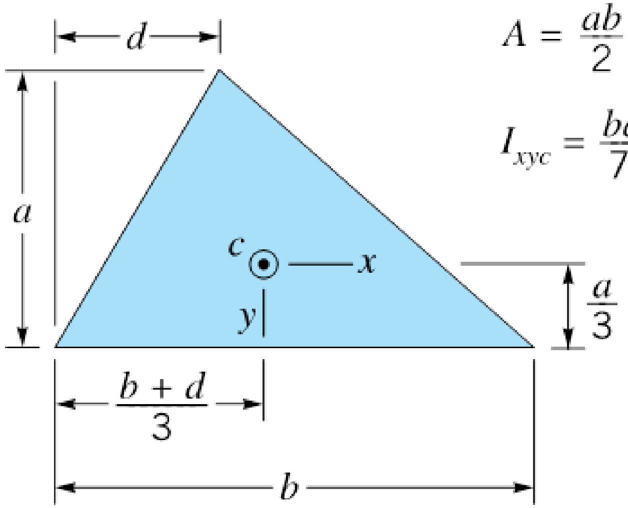

$$A = \frac{\pi R^2}{2}$$

$$I_{xc} = 0.1098R^4$$

$$I_{yc} = 0.3927R^4$$

$$I_{xyc} = 0$$

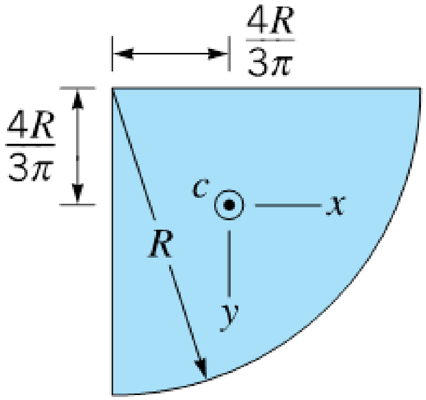
(c)




$$A = \frac{ab}{2} \qquad I_{xc} = \frac{ba^3}{36}$$

$$I_{xyc} = \frac{ba^2}{72}(b - 2d)$$

(d)



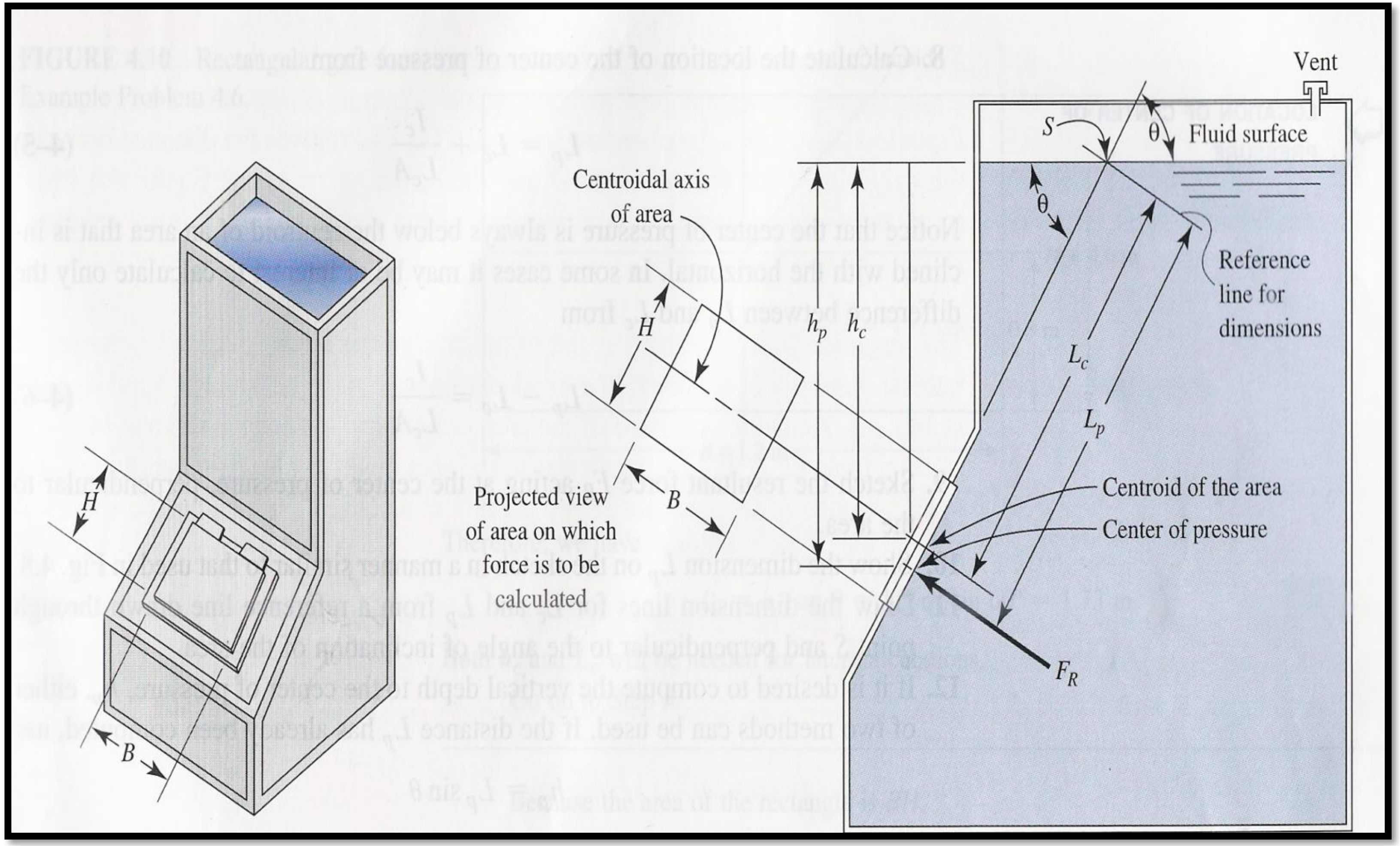

$$A = \frac{\pi R^2}{4}$$

$$I_{xc} = I_{yc} = 0.05488R^4$$

$$I_{xyc} = -0.01647R^4$$

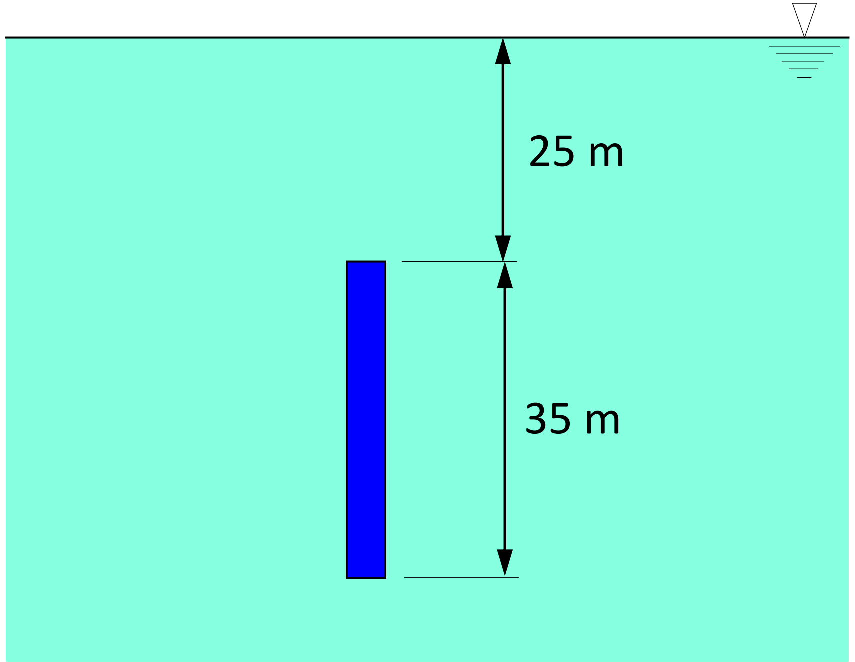
(e)

# Total Submerged Plane Areas



# Resultant Force and Centre of Pressure on a Plane Surface Immersed in a Liquid (Problem 2.8)

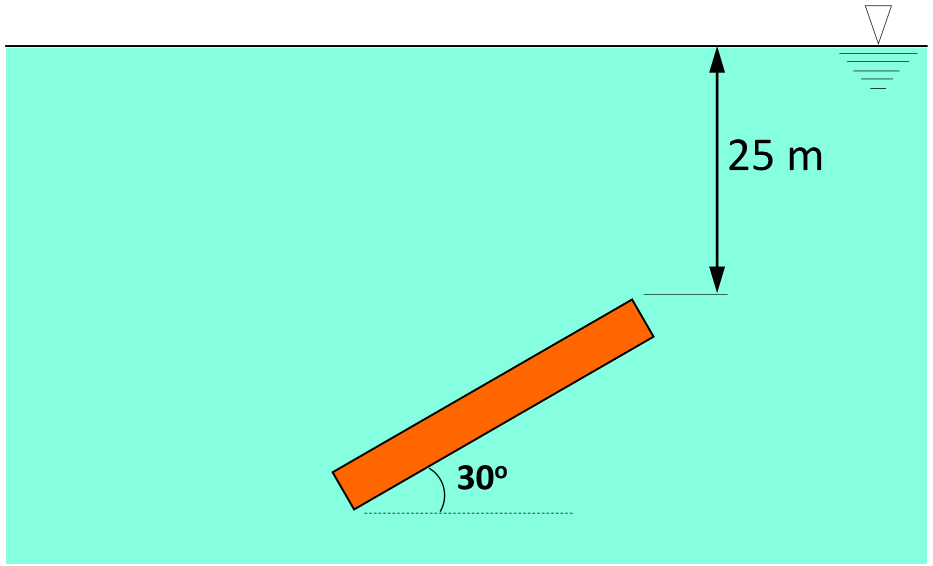
A rectangular shaped plate with a dimension of 10 m wide by 35 m long, is immersed 25 m deep vertically in a water as shown in figure below. Find the resultant force that acts on the plate and its centre of action from a surface




ANS: R = 146 MN
$h_R$ = 44.90 m

# Resultant Force and Centre of Pressure on a Plane Surface Immersed in a Liquid (Problem 2.9)

If the same plate (Problem 2.9) is tilted to 30° angle from the liquid surface (as shown below), determine R and $Y_R$




ANS: R = 115.88 MN
h_R = 34.58 m

# Resultant Force and Centre of Pressure on a Curved Surface Immersed in a Liquid

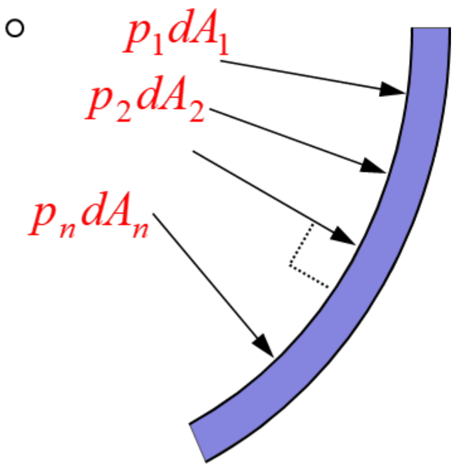
- Forces are not parallel
- Forces act on the curved surface have to be combine vectorially
- There are 2 components of the resultant force **R**

1. **Horizontal component, $R_H$**
   - This force acts horizontally from vertical axis
   - Mathematically

$$R_H = \rho g h_C A = \gamma h_C A$$

2. **Vertical component, $R_V$**
   - This force acts vertically from horizontal axis due to the weight of fluid above the curved surface
   - Mathematically

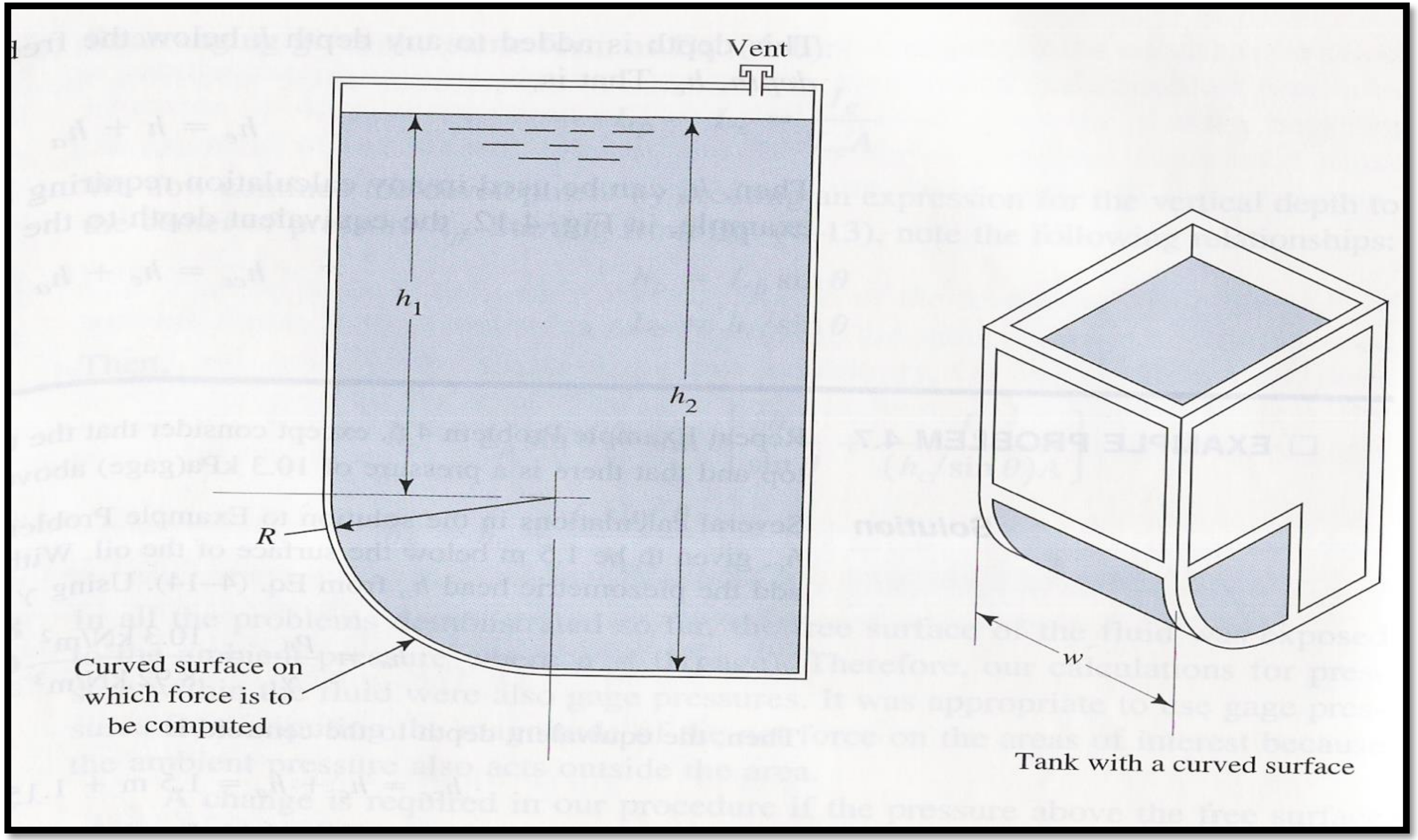
$$R_V = \rho g V = \gamma V$$



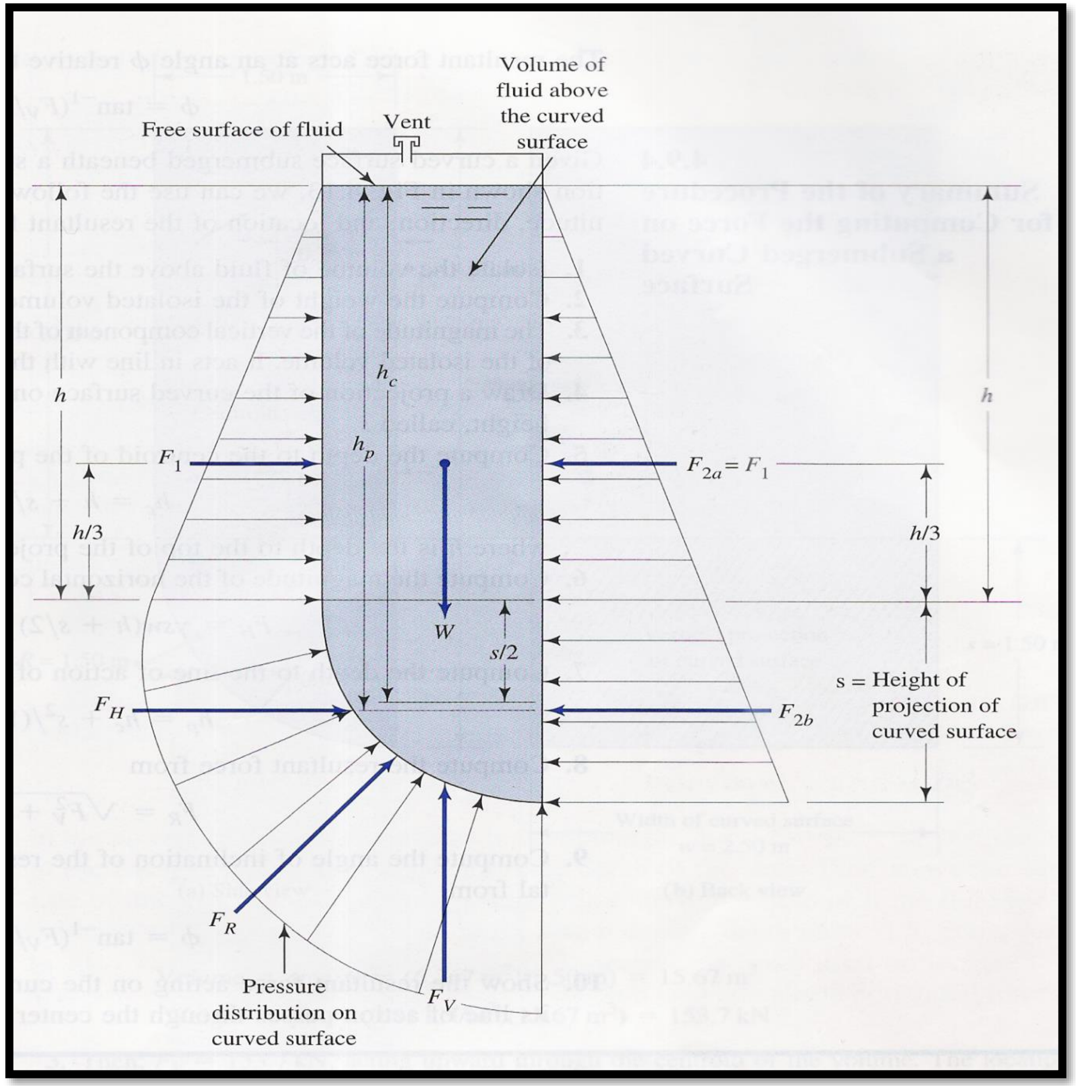
Curved surface

- Forces are combined using Pythagorean theorems $R = \sqrt{R_H^2 + R_V^2}$
- The direction of R, $\theta$, is solved by using trigonometry law $\theta = \tan^{-1}\left(\frac{R_V}{R_H}\right)$

# Distribution of Force on a Submerged Curve Surface



# Distribution of Force on a Submerged Curve Surface

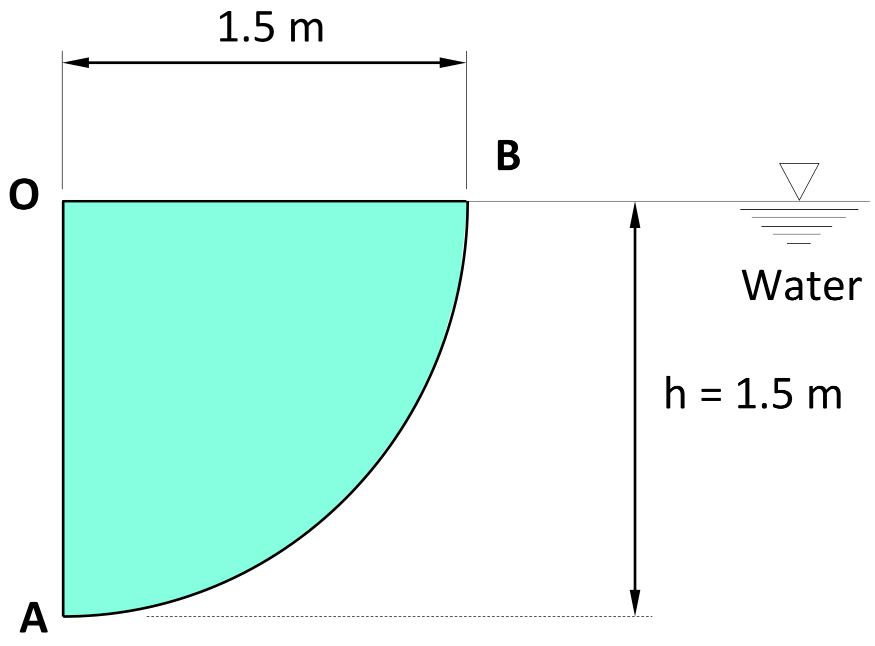



## Aims

To determine the horizontal force $F_H$ and the vertical force $F_V$ exerted on the fluid by the curved surface and their resultant force $F_R$

# Resultant Force and Centre of Pressure on a Curved Surface Immersed in a Liquid (Problem 2.10)

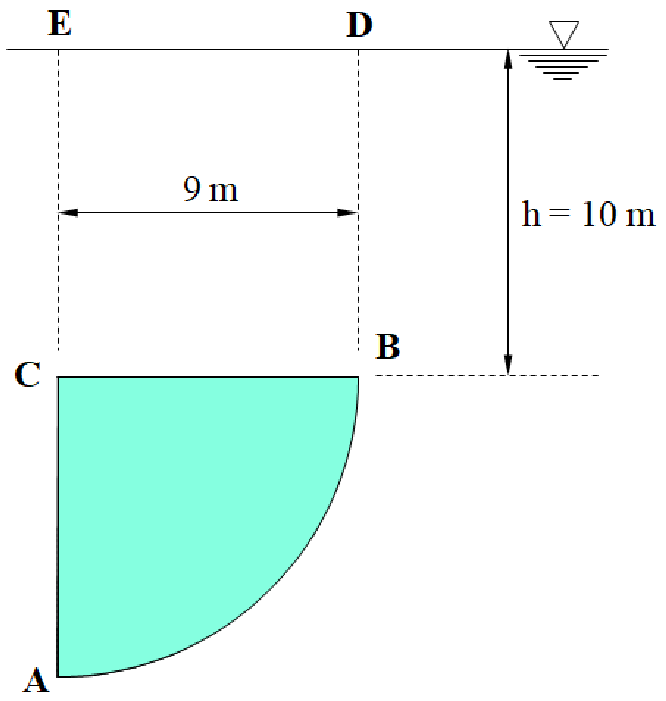
A sluice gate consists of quadrant of a circle of radius 1.5 m pivoted at O. Determine the magnitude and direction of the resultant force on the gate. The width of the gate is 3 m




ANS: R = 61.65 kN
Thetha = 57.52 deg.

# Resultant Force and Centre of Pressure on a Curved Surface Immersed in a Liquid (Problem 2.11)

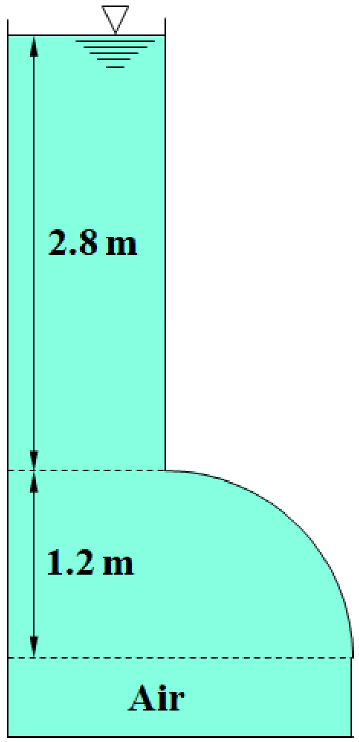
The figure below shows a part of the water tank of a quadrant circle of 9 m radius. Calculate the magnitude and direction of the resultant force on the curved surface from the horizontal axis. (Given $g$ = 14715 N/m$^3$)




ANS: R =2.87 MN
Thetha = 49.65 deg.

# Resultant Force and Centre of Pressure on a Curved Surface Immersed in a Liquid (Problem 2.12)

Calculate $R_H$, $R_V$, R and $\theta$ act on the curved surface of the quarter circle. The length of the object is 1.5 m




ANS: R = 80.7 kN
Thethe = 42 deg.

# Buoyancy and Stability of Floating Bodies

- Introduction
- Bodies completely submerged
- Archimedes principal & centre of buoyancy
- Floating bodies
- Stability of submerged bodies
- Stability determination of submerged bodies
- Stability of floating bodies
- Metacentric height concepts for floating bodies

# Introduction

- Previous sections ➡ **R** determination which acts on any surface of submerged bodies
- Another important aspect need to consider in fluid static is **hydrostatic pressure**, which is the **neat force, which acts on any submerged or floating bodies in a fluid**
- This force is known as **buoyancy force**, occurred due to the increase of $p$ with $h$
- The neat horizontal $p$ on a body of completely submerged or floating is equal to zero ➡ due to pressure forces on each side are equal

# Body Completely Submerged

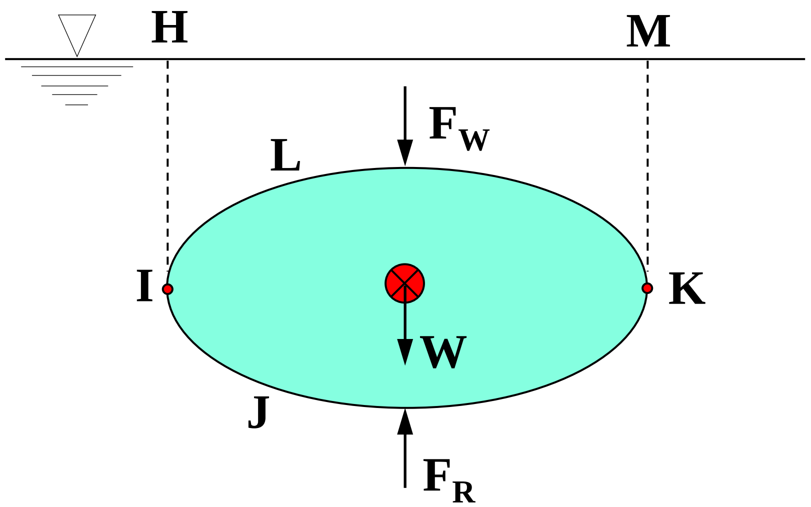
- Weight of submerged body, $w$ ➡ $V_{IJKLI}$
- Force due to weight of fluid, $\mathbf{F_w}$

$$\mathbf{F_W} = \gamma_{fluid} V_{fluid} = \gamma_{fluid} V_{HILKMH}$$

- Upthrust force acts on the surface body, $\mathbf{F_R}$

$$\mathbf{F_R} = \gamma_{fluid} V_{fluid} = \gamma_{fluid} V_{HIJKMH}$$

- $\mathbf{F_R} > \mathbf{F_W}$, therefore, the neat vertical pressure force $(\mathbf{F_R} - \mathbf{F_W})$ acts on the submerged bodies



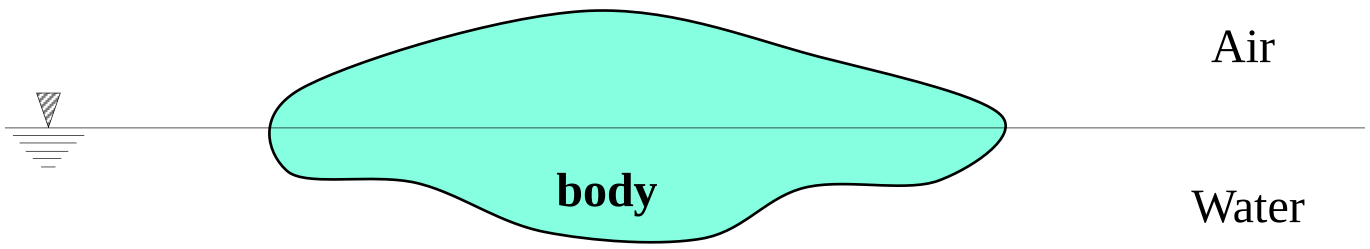

$$\therefore \quad F_B = F_R - F_W = \gamma_{fluid} V_{body}$$

Neat vertical force @ buoyancy force

# Body Completely Submerged

- Archimedes principle: **The upthrust force (upward vertical force due to the fluid) on a body immersed in a fluid is equal to the weight of fluid displaced**
- The buoyancy force will act through the centre of centroid of the displaced fluid which is known as centre of buoyancy
- Generally, buoyancy force does not equal to the weight of immersed body due to that immersed body has upward or downward acceleration which depends on *g* of the body either bigger or smaller than *g* of the fluid
- For floating body, there is no upward acceleration. Buoyancy force for floating body is equal to its own weight. In other words, a floating body replaces its own weight with the weight of fluid when it floats

# Floating Bodies



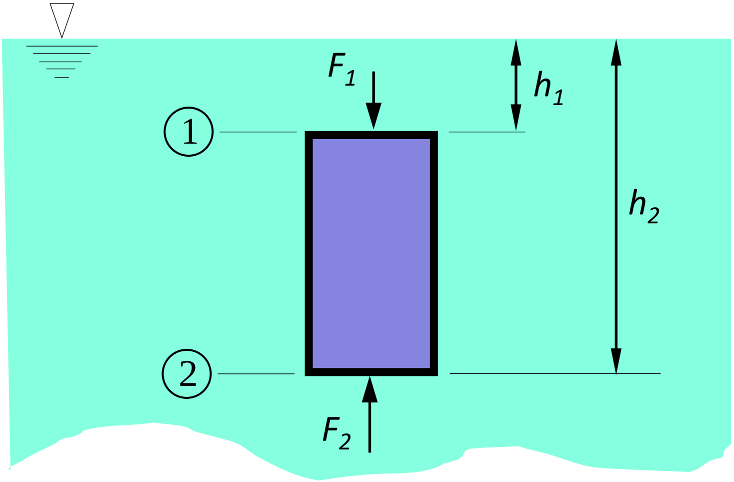

$$\text{Body apparent weight } = \gamma_{\text{body}} V_{\text{body}} - \gamma_{\text{air}} V_{\text{air}}$$

In engineering calculations, $g_{air}$ can be neglected

$$\text{Body apparent weight } = \text{Body's weight} = \gamma_{\text{body}} V_{\text{body}}$$

# Floating Bodies (Problem 2.13)

By using pressure-depth relationships ($p = \gamma h$), shows that buoyancy force on a solid cylinder which is completely immersed is equal to the weight of fluid replaced by the cylinder. (Neglect the weight of the cylinder)



# Floating Bodies (Problem 2.13)

- The top of the cylinder is marked with **1** & at the bottom is marked with **2**, the cross-sectional area is *A*, and volume is $V_{cylinder}$

$$\text{Buoyancy force} = \begin{bmatrix}\text{Upthrust}\\ \text{force}\end{bmatrix} - \begin{bmatrix}\text{Downthrust}\\ \text{force}\end{bmatrix}$$

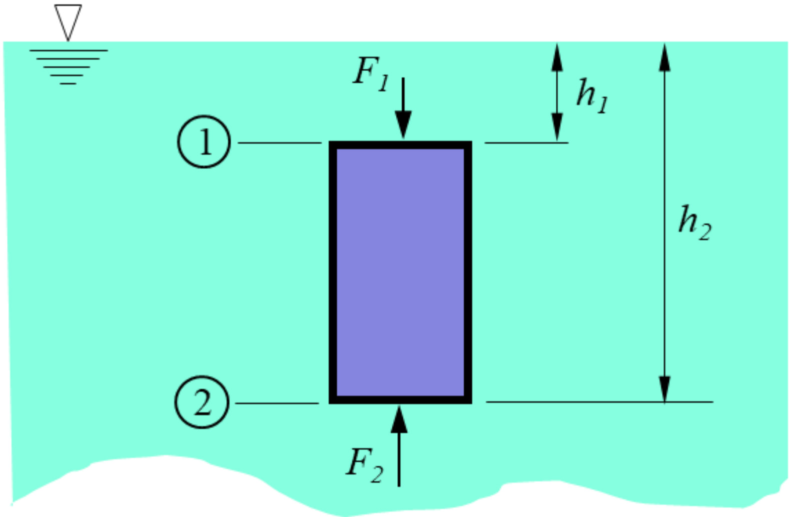
$$F_B = F_2 - F_1 = p_2 A_2 - p_1 A_1 = A(p_2 - p_1)$$

$A_2 = A_1$

$$\Rightarrow p = \gamma h$$

$$\therefore\ F_B = A[(p_{atm} + \gamma h_2) - (p_{atm} + \gamma h_1)] = \gamma (h_2 - h_1) A$$

where $\gamma = \gamma_{fluid}$

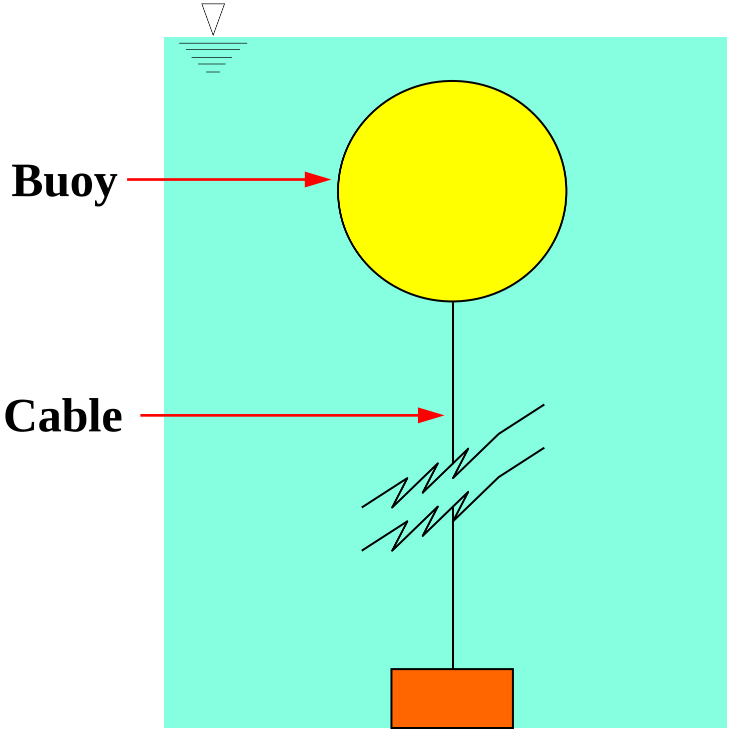



$$\Rightarrow (h_2 - h_1) A = V_{cylinder} \qquad \text{(volume of cylinder)}$$

$$\therefore\ F_B = \gamma_{fluid} V_{cylinder}$$

# Floating Bodies (Problem 2.14)

A spherical buoy is fastened to the bottom of the river (fresh water) with a cable, as shown in figure below. The buoy was designed to float in water. At one time, the river water level has risen and submerges the buoy. If the buoy is 1 m diameter and 750 kg/m$^3$ density, calculate the tension force that acts on the cable




ANS: T = 1285 N

# Stability of Submerged Bodies

- Submerged body is said stable:

  When small amount of **F** (disturbance) applied on a body during its stability state, the body will turn back to its previous position

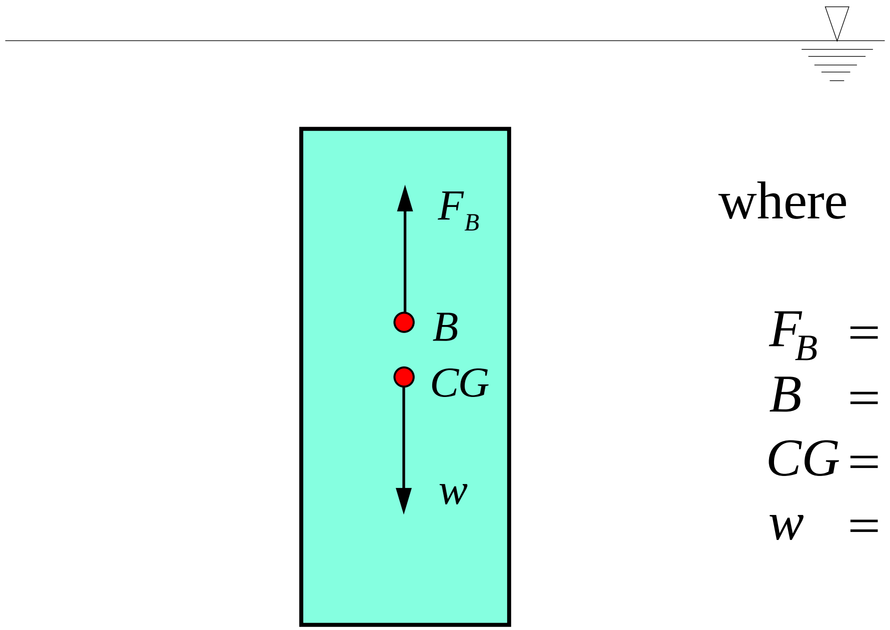
- **Example**
  - Air balloon (which is completely floats/submerged in the air) → wind blow disturbance)
  - Submarine (which is completely submerged in the water) → current disturbance

# Stability Determination of Submerged Bodies

- For completely submerged body, to be stable

body's centre of gravity | must lie directly below | body's centre of buoyancy

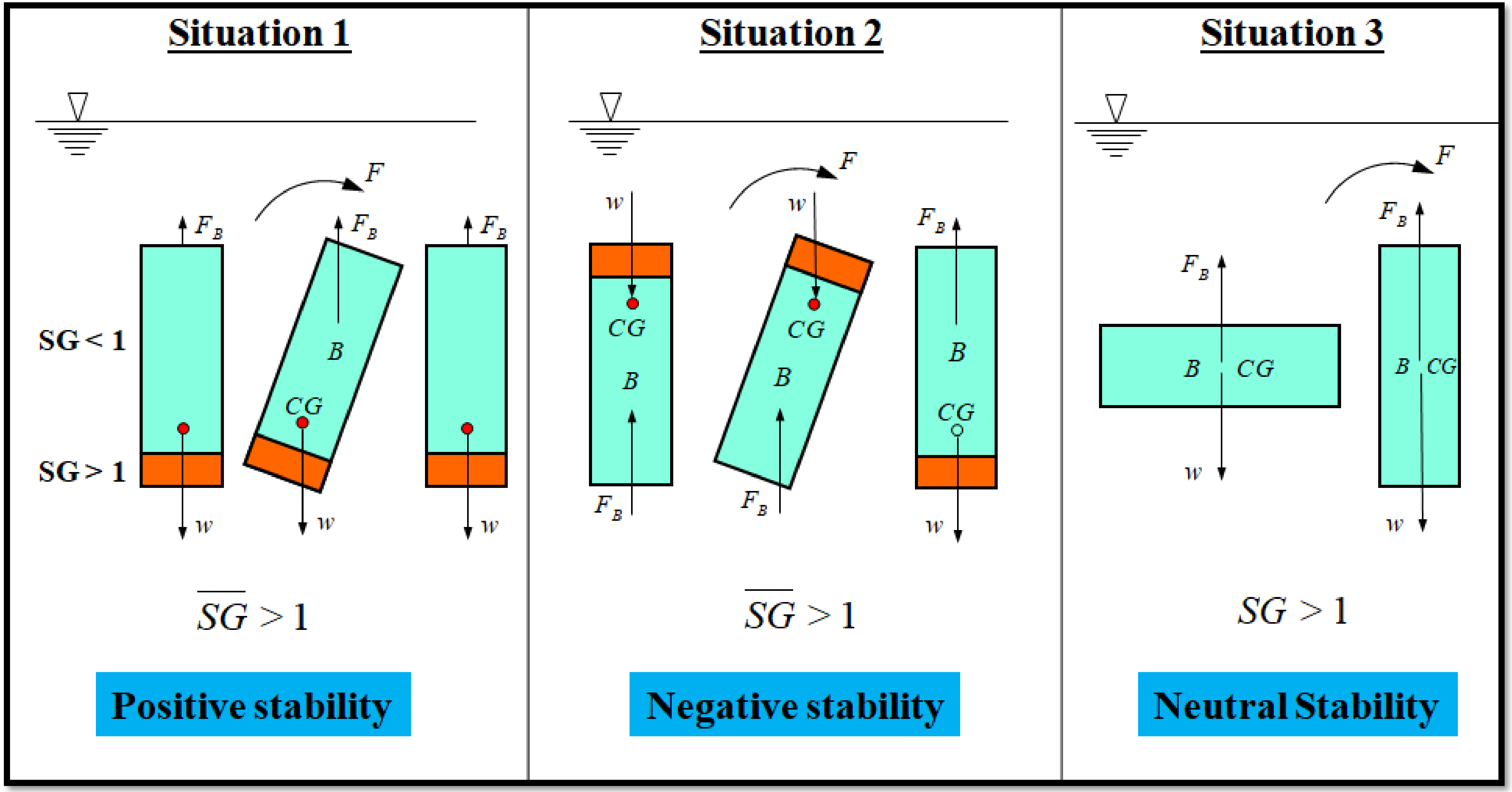
- Graphically




where

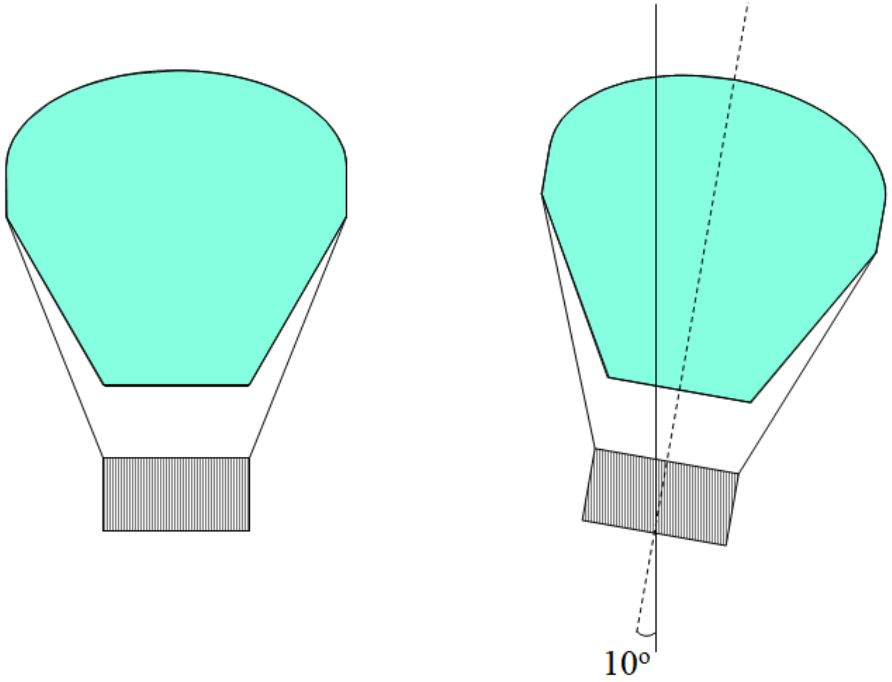
$F_B$ = buoyancy force
$B$ = body's centre of buoyancy
$CG$ = body's centre of gravity
$w$ = body's weight

# Stability Determination of Submerged Bodies



# Stability Determination of Submerged Bodies (Problem 2.15)

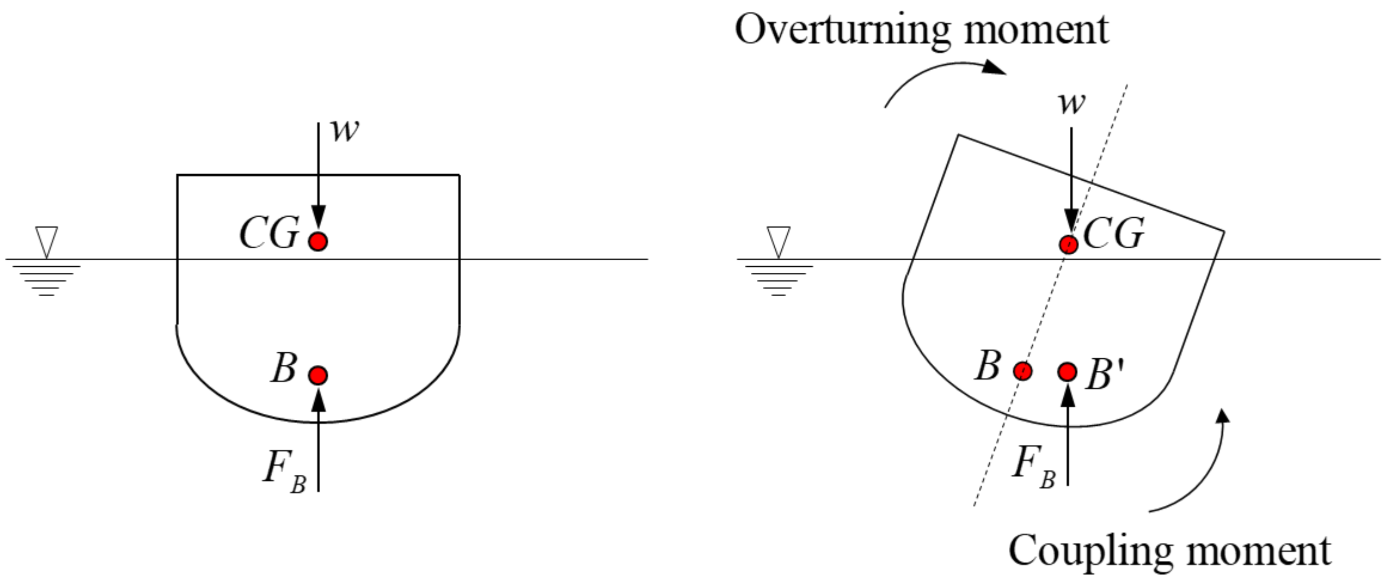
- An air balloon system is shown in figure below. Due to a cross wind blowing, the air balloon is turning clockwise through the 10° angle from stability state. If the weight of the air balloon system is 2500 lb and the distance, *L*, between the gravity centre and buoyancy centre is 18 ft, determine the magnitude and direction of the coupling moment produced




ANS: M_coupling = 7830 ft.lb anticlockwise

# Stability of Floating Bodies

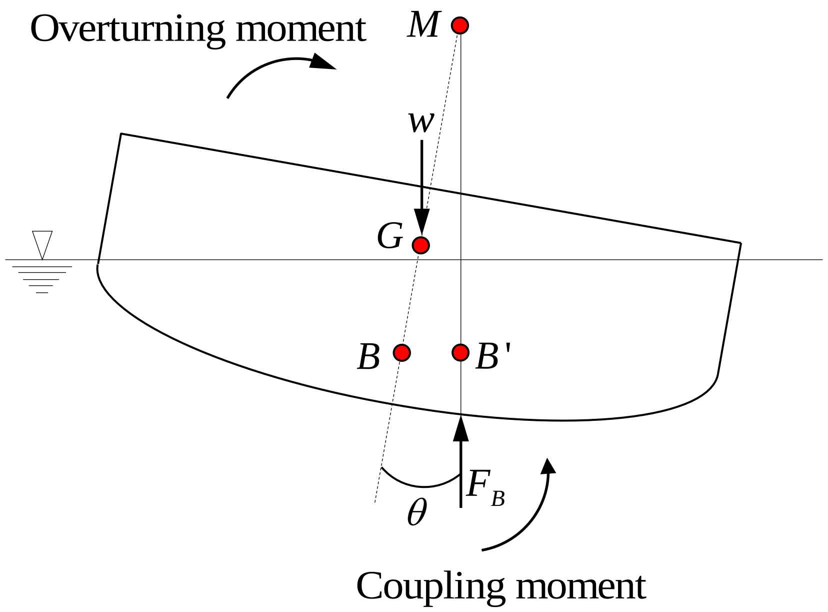
- The analysis is more complex compared to completely submerged bodies
  - The buoyancy centre position will shift with the body shapes when turning force is applied
  - Generally, to be more stable, the body's buoyancy centre must lie below the centre of gravity. (Note: It's depends on the shape of the object. Will be discuss in Metacentric height concepts)
- Consider the situation below:



# Metacentric Height Concepts for Floating Bodies

- Metacentric height:

  Used as a parameter to measure the stability of floating bodies ➡ quantitatively




where

- G - body's centre of gravity
- B - body's centre of buoyancy
- B' - body's modified centre of buoyancy
- M - meta centre
- $F_B$ - buoyancy force
- $w$ - body's weight
- $\theta$ - angle of roll
- $\overline{GM}$ - metacentric height

if $\overline{GM} > 0$ (M lie above G)
→ body is stable

if $\overline{GM} < 0$ (M lie below G)
→ body is unstable

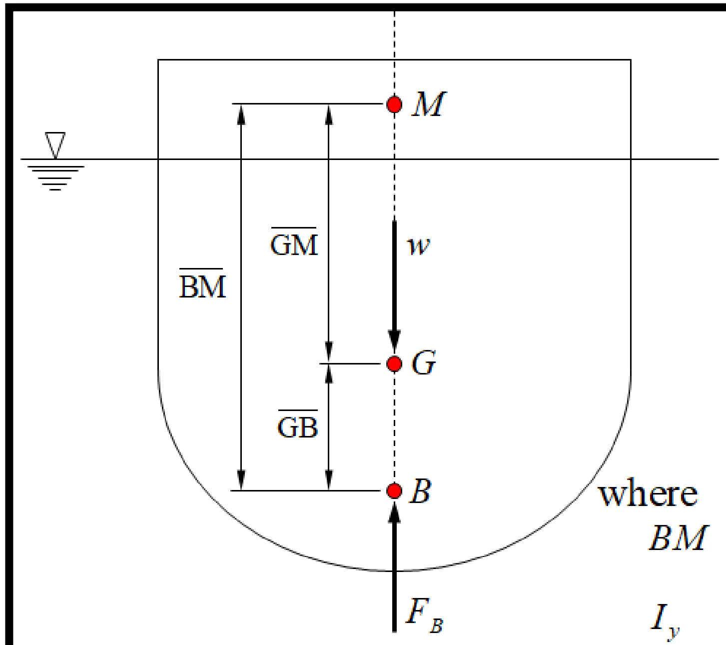
if $\overline{GM} = 0$ (M coincide with G)
→ body has neutral stability

$\overline{GM}$ – Directly measure the stability of floating bodies due to coupling moment produced is directly proportional to the height of meta centre

$$M_{coupling} = w\overline{GM}\sin\theta$$

# Metacentric Height Concepts for Floating Bodies

- Determine magnitude of GM ➡ $M_{coupling}$




$$\overline{BM} = \frac{I_y}{V}$$

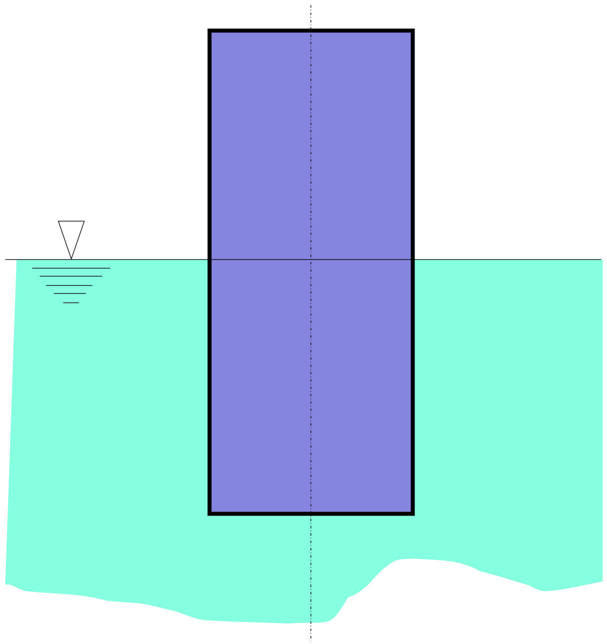
$$\overline{GM} = \overline{BM} \pm \overline{GB}$$

where

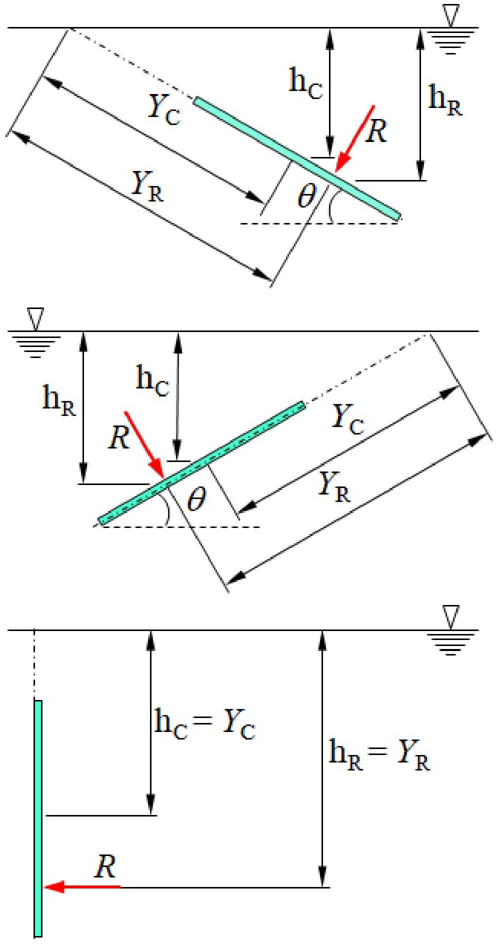
| | | |
|---|---|---|
| $BM$ | – | distance between body's centre of buoyancy to the centre of meta |
| $I_y$ | – | inertia moment of body's cross-section parallel to fluidline with average $Y$ axis |
| $V$ | - | displaced volume of fluid |
| +ve sign | – | if G lie below B |
| –ve sign | – | if G lie above B |

# Metacentric Height Concepts for Floating Bodies (Problem 2.16)

A cylindrical block of wood which has a dimension of 1 ft long and 3 inches diameter, is placed vertically in water as shown in figure below. If the wood has a specific gravity of 0.5, determine whether the cylindrical block of wood is stable or not?



# Summary (Fluid Force on Submerged Bodies- Plane Surface)




$h_c$

$$R = \gamma h_c A$$

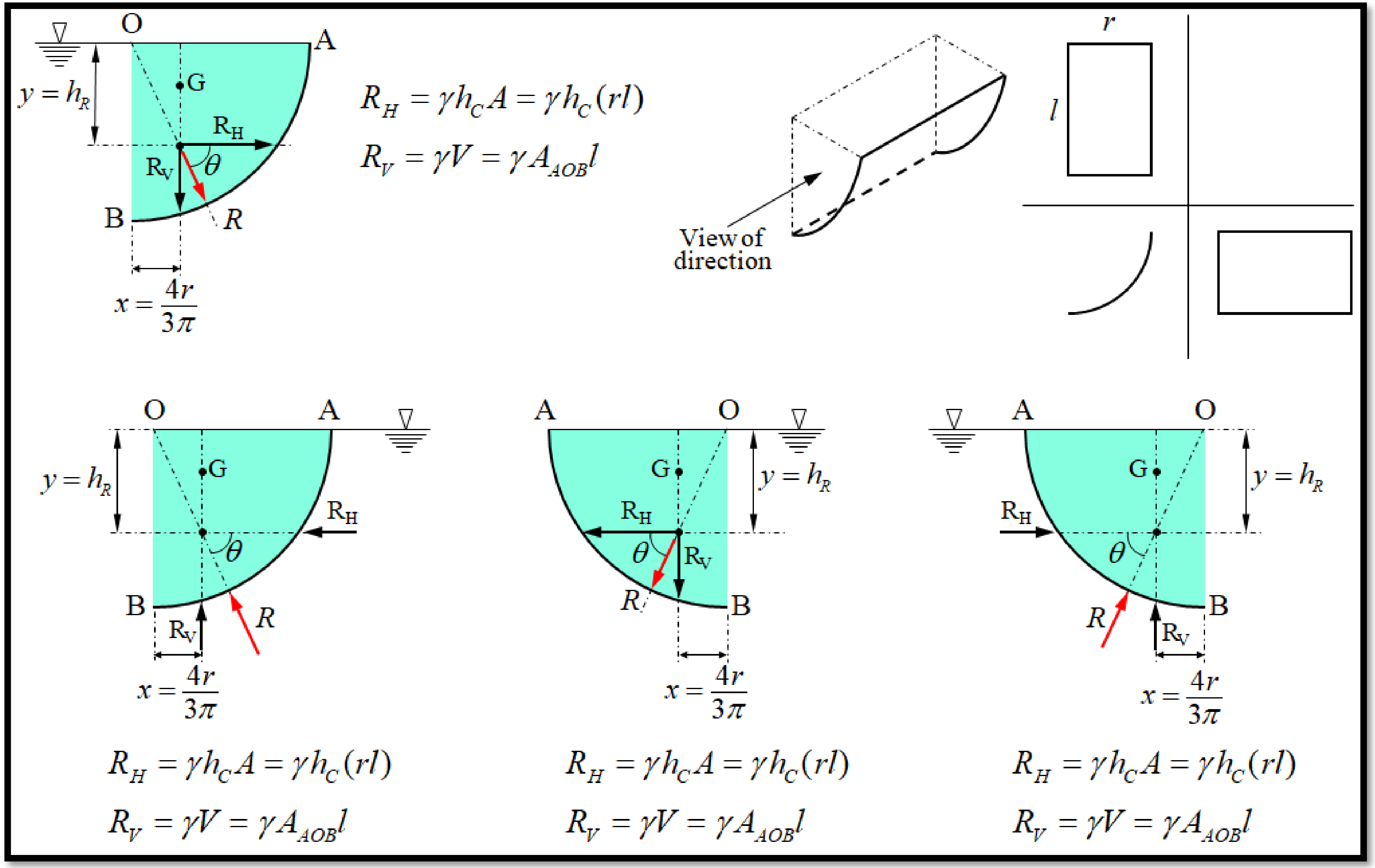
$$Y_C = \frac{h_c}{\sin\theta}$$

If ( = 90°
$Y_C = h_C$

$$Y_R = \frac{I_{XC}}{Y_C A} + Y_C$$

If ( = 90°
$Y_R = h_R$

# Summary (Fluid Force on Submerged Bodies-Curve Surface)



# THANK YOU

Stay safe!