FLUID DYNAMICS

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Objectives of the Topic

- Apply the concepts fluid dynamics and control volume
- Apply the concepts of conservation of mass and energy balance
- Solve fluid dynamic problems using continuity, Bernoulli, energy, momentum and Euler equations

Contents

Introduction

- Fluid motion → extremely complex by the introduction of μ

  → will create shear force → friction
- Qualitative analysis

  Normally done visually, e.g. analysis of fluid in motion
- Quantitative analysis

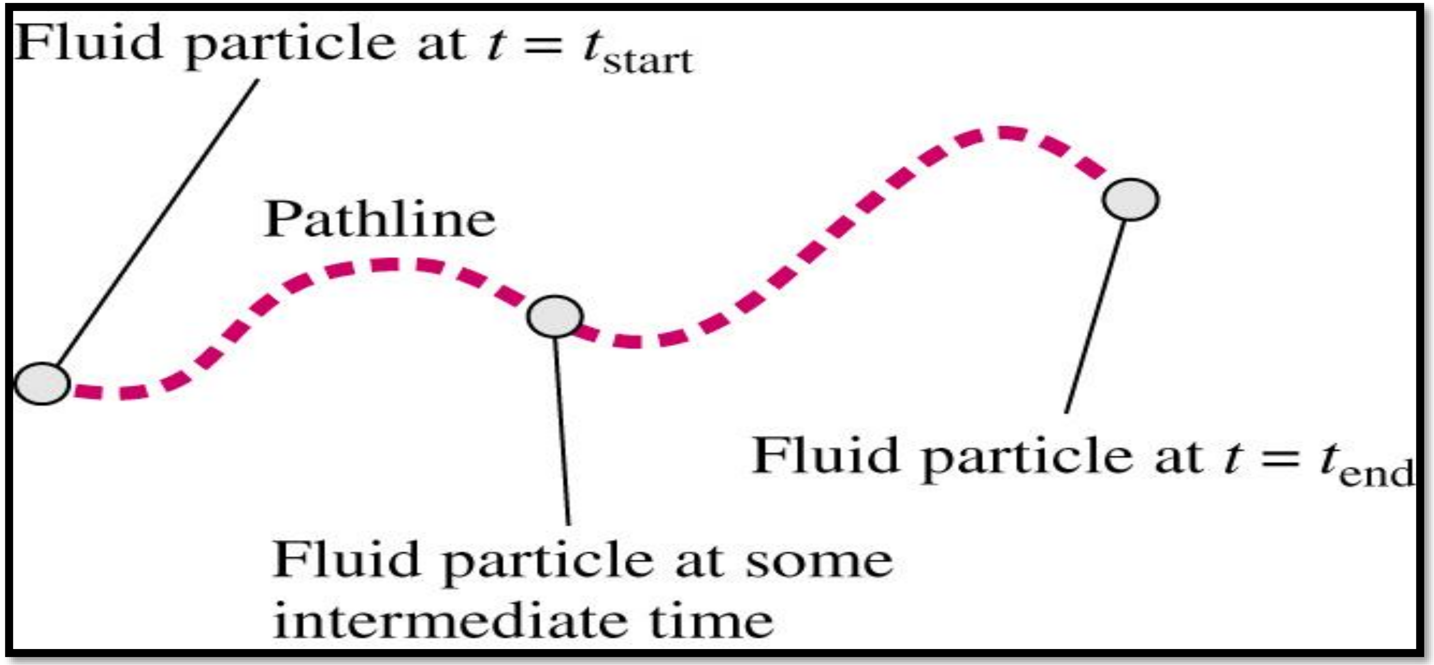
  Make some appropriate assumptions

  Neglecting some particular parameters

Introduction (Types of Flow Lines)

- **Path Lines**

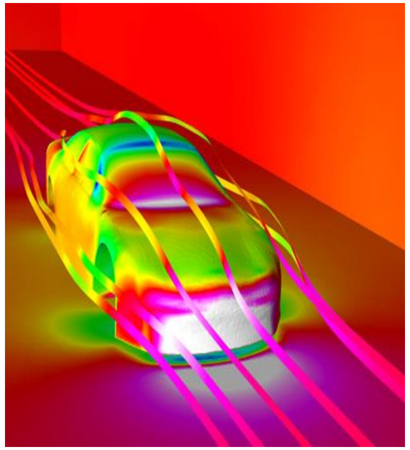
The **actual path traveled** by a fluid particle. The line shows the direction of a particle, for a certain period of time or between two given section



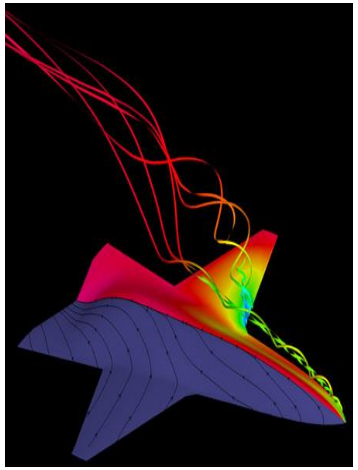
Introduction (Types of Flow Lines)

- **Stream Lines**

A curve (imaginary line) that is everywhere tangent to the **instantaneous** local velocity vector. An element of fluid, bounded by a **number of a stream lines**, which **confine the flow is called stream tube**. There is **no movement of fluid across a stream line**, therefore no fluid can enter or leave the stream tube except at the end. So that a stream tube behaves like a solid tube



NASCAR surface pressure contours and streamlines

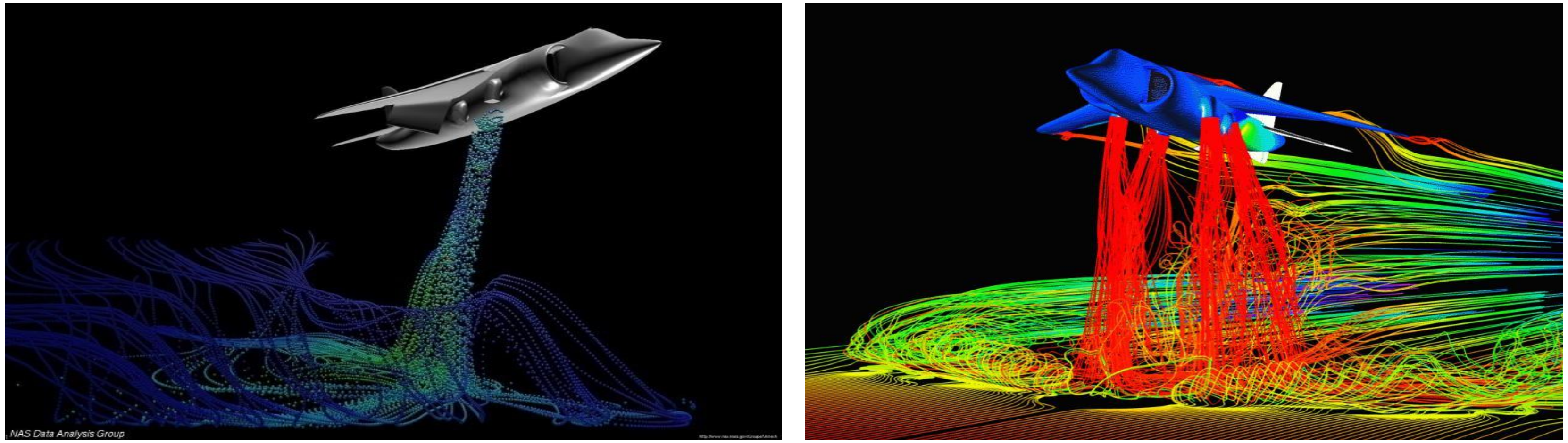


Airplane surface pressure contours, volume streamlines, and surface streamlines

Introduction (Types of Flow Lines)

- **Streak Lines**

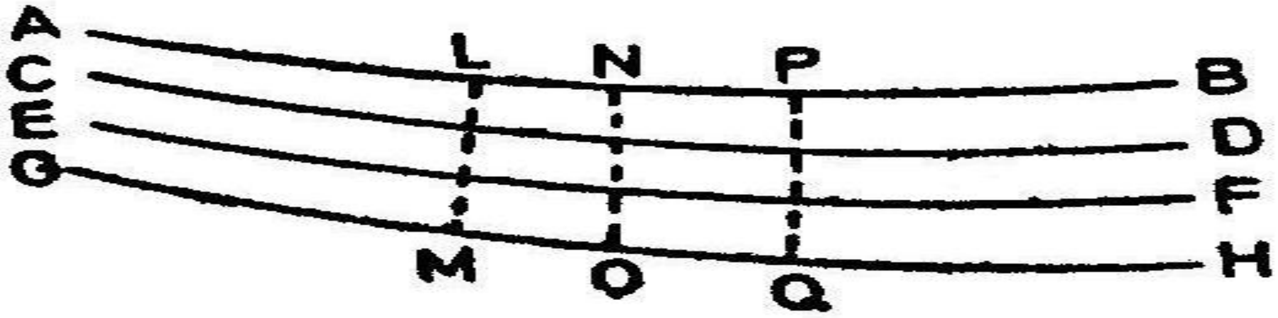
The locus of fluid particles that have passed sequentially through a prescribed point in the flow. Easy to generate in experiments: dye in a water flow, or smoke in an airflow



Introduction (Types of Flow Lines)

- **Potential Lines or Equi-potential Lines**

There is always a loss of head of the fluid particles as we proceed along the flow lines. If we draw **lines joining the points of equal potential** on adjacent flow lines we can get potential lines of equi-potential lines

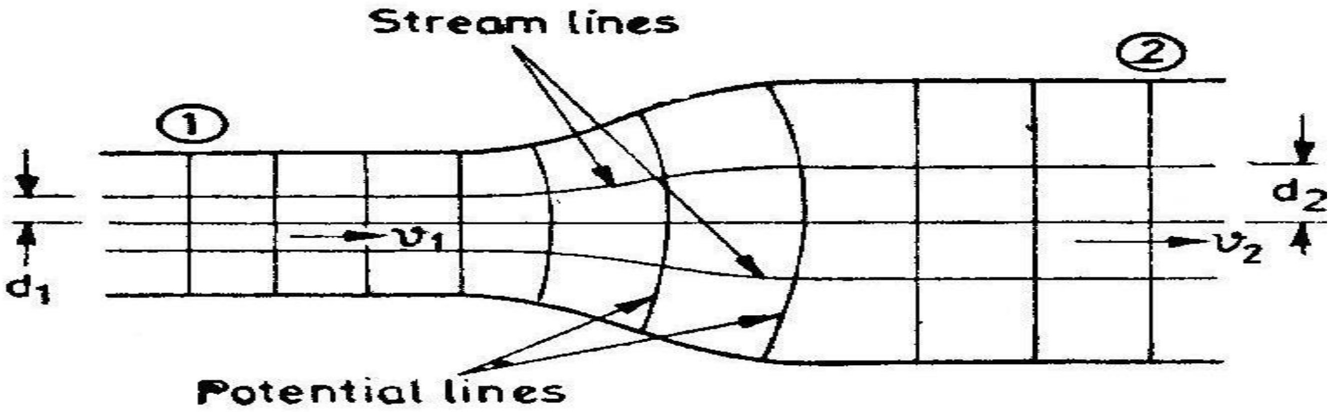


AB : Stream line
LM : Potential line

Introduction (Types of Flow Lines)

- **Flow Net**

  If we draw both the lines for a flow (stream lines and potential lines), the pattern obtained by the intersection of the two set of lines is called flow net. The flow net helps in depicting analyzing the behaviors of irrotational flow



Analysing Fluid Flow

- In analysing fluid flow, there are 2 main alternatives:

  As an element of fixed mass

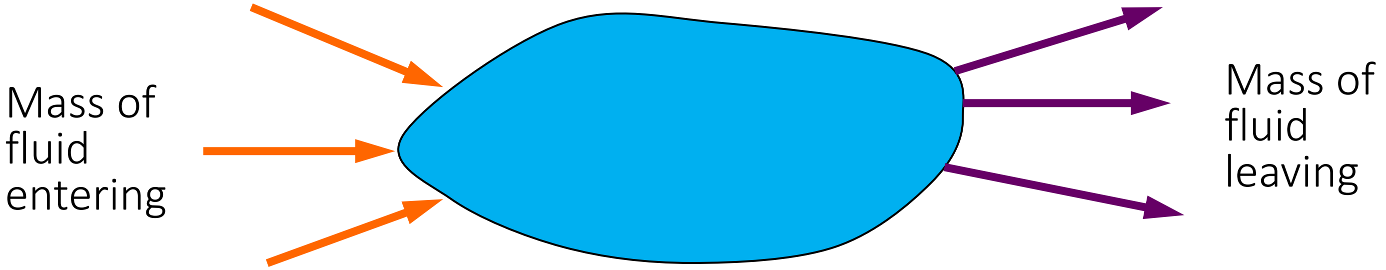
  assume as a closed system

  As an element of control volume

  open system

Continuity Equation

- Conservation of mass principle: "Matter is neither created not destroyed" ➡ can be applied to a flowing fluid
- Consider any fixed region in the flow constituting a control volume



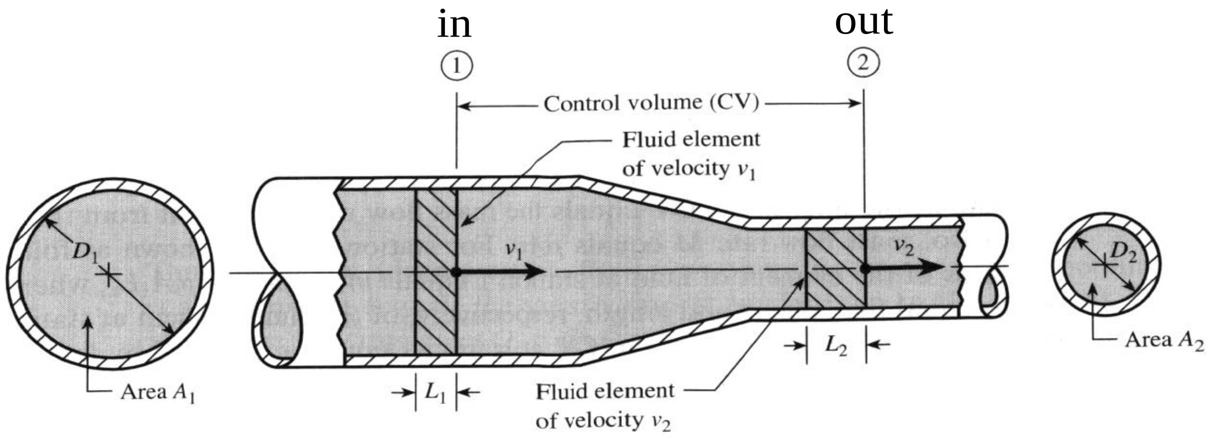
$$\begin{array}{c}\text{Mass of fluid}\\ \text{entering}\\ \text{per unit time}\end{array} = \begin{array}{c}\text{Mass of fluid}\\ \text{leaving}\\ \text{per unit time}\end{array} + \begin{array}{c}\text{Increase of mass of fluid}\\ \text{in the control volume}\\ \text{per unit time}\end{array}$$

- For **steady flow**, the mass of fluid in the control volume remains constant and the relation reduces to:

$$\begin{array}{c}\text{Mass of fluid entering}\\ \text{per unit time}\end{array} = \begin{array}{c}\text{Mass of fluid leaving}\\ \text{per unit time}\end{array}$$

Continuity Equation

- Other graphical methods in generating **equation of continuity** is illustrated below:



$$\text{mass}_{(in)} = \text{mass}_{(out)} \quad @ \quad (\text{mass rate of flow})_{(in)} = (\text{mass rate of flow})_{(out)}$$

$$\dot{m}_1 = \dot{m}_2$$

$$(\rho Q)_1 = (\rho Q)_2$$

$$(\rho A v)_1 = (\rho A v)_2$$

$$Q_1 = Q_2$$

$$Q = Av$$

Simplified form for an incompressible fluid, $\rho_1 = \rho_2$

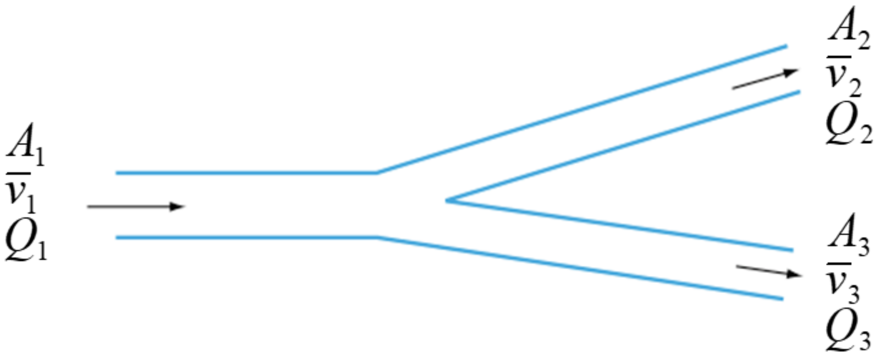
$$\frac{v_1}{v_2} = \left(\frac{D_2}{D_1}\right)^2$$

$$A = \frac{\pi}{4} D^2$$

$$\rho_1 = \rho_2$$

Continuity Equation

- The continuity equation can also be applied to determine the relation between the flows into and out of a junction. For steady conditions:



Total inflow to junction = Total outflow from junction

$$\rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3$$

- For an incompressible fluid, $\rho_1 = \rho_2 = \rho_3$, so that

$$Q_1 = Q_2 + Q_3$$

or $$A_1 v_1 = A_2 v_2 + A_3 v_3$$

- In general, if we consider flow towards the junction as +ve and flow away from the junction as –ve, then for incompressible steady flow at any junction will be: $\sum Q = 0$

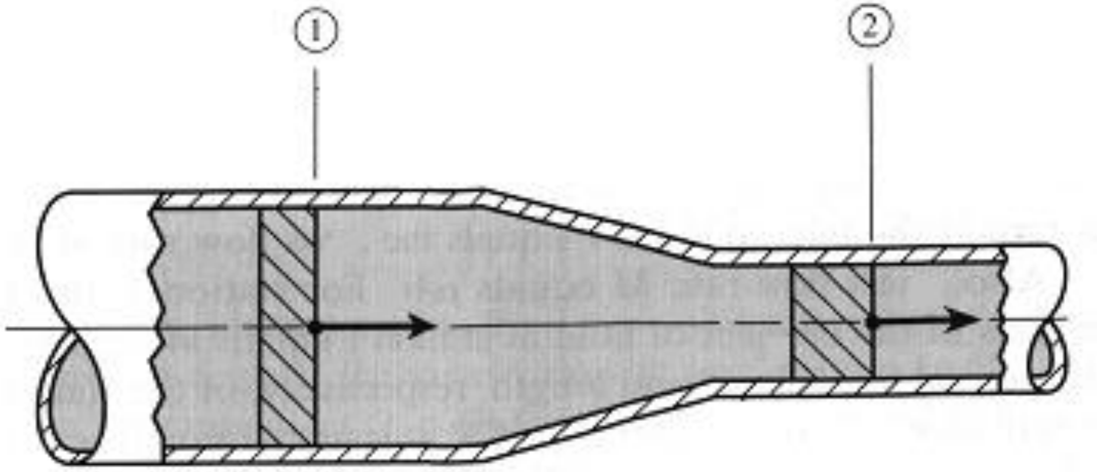
Continuity Equation (Problem 3.1)

Water flows through a pipe from point 1 to point 2, as shown in figure below. Given $D_1 = 4$ inches, $D_2 = 2$ inches and $v_1 = 4$ ft/s. Determine:

a) Volume rate of flow

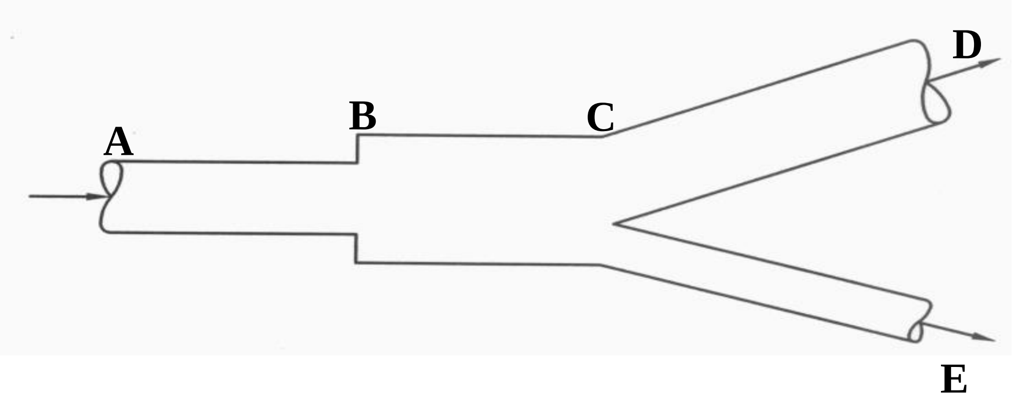
b) Fluid velocity at point 2

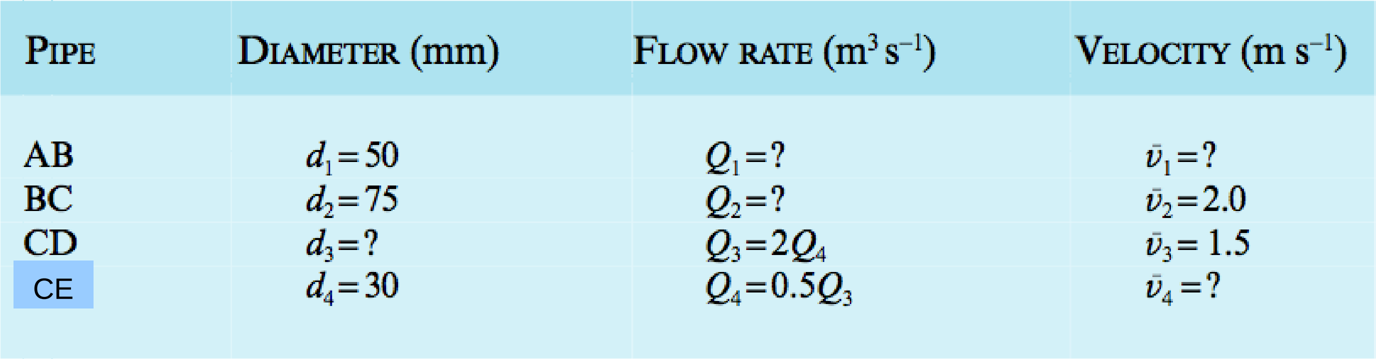
c) Mass rate of flow



Continuity Equation (Problem 3.2)

Water flows through a pipe AB (figure) of diameter d_1 = 50 mm, which is in series with a pipe BC of diameter d_2 = 75 mm in which the mean velocity v_2 = 2 m/s. At C the pipe forks an one branch CD is of diameter d_3 such that the mean velocity v_3 is 1.5 m/s. The other branch CE is of diameter d_4 = 30 mm and conditions are such that the discharge Q_2 from BC divides so that $Q_4 = 0.5Q_3$. Calculate the values of Q_1, v_1, Q_2, Q_3, d_3, Q_4 and v_4

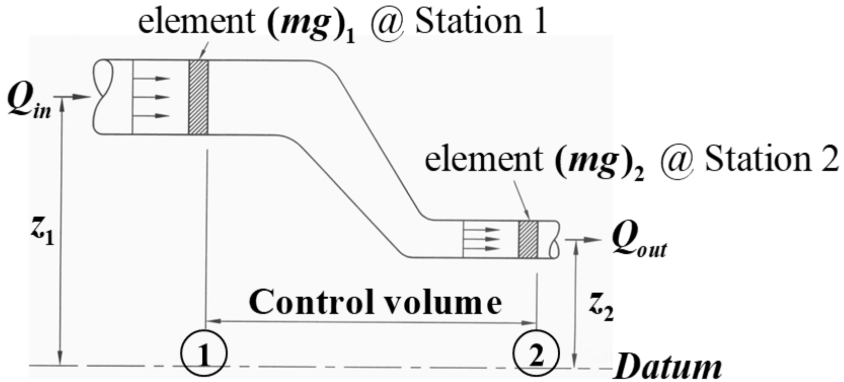




PIPE	DIAMETER (mm)	FLOW RATE ($m^3 s^{-1}$)	VELOCITY ($m s^{-1}$)
AB	$d_1=50$	$Q_1=?$	$\bar{v}_1=?$
BC	$d_2=75$	$Q_2=?$	$\bar{v}_2=2.0$
CD	$d_3=?$	$Q_3=2Q_4$	$\bar{v}_3=1.5$
CE	$d_4=30$	$Q_4=0.5Q_3$	$\bar{v}_4=?$

Bernoulli’s Equation

- Bernoulli’s theorem: The total energy of each particle of a body of fluid is the same provided that no energy enters or leaves the system at any point
- There are 3 types of energy involved: potential, pressure, and kinetic energies
- The division of these energies may vary, but the total remains constant
- Consider the diagram below:



For an element of weight mg

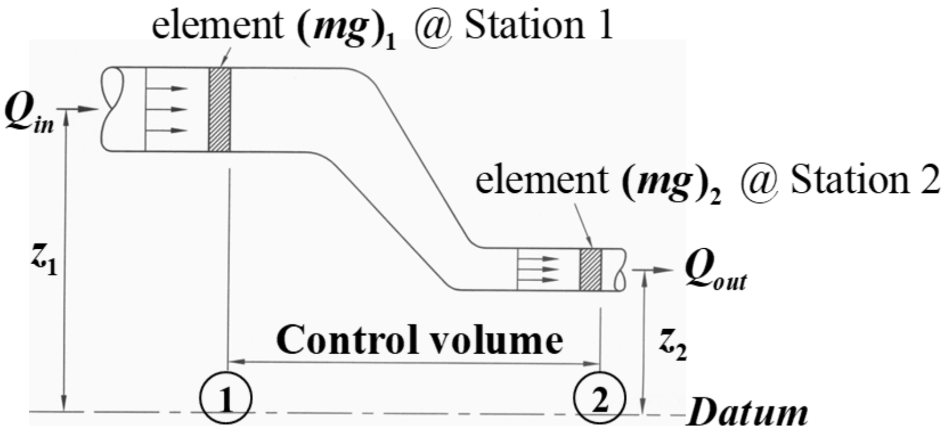
$$\text{Potential energy}\left(\begin{array}{c}\text{possess due to } z\\ \text{above datum}\end{array}\right) = mgz \rightarrow \text{Potential head}\left(\begin{array}{c}\text{potential energy}\\ \text{per unit weight}\end{array}\right) = z$$

$$\text{Kinetic energy}\left(\begin{array}{c}\text{possess due to}\\ \text{its velocity } v\end{array}\right) = \frac{1}{2}mv^2 \rightarrow \text{Kinetic head}\left(\begin{array}{c}\text{Kinetic energy}\\ \text{per unit weight}\end{array}\right) = \frac{v^2}{2g}$$

$$\text{Pressure energy}\left(\begin{array}{c}\text{possess due to fluid flow}\\ \text{generates a force}\end{array}\right) = \frac{pmg}{\rho g} = \frac{pm}{\rho} \rightarrow \text{Pressure head}\left(\begin{array}{c}\text{pressure energy}\\ \text{per unit weight}\end{array}\right) = \frac{p}{\rho g}$$

In head term (unit height → m or ft)

Bernoulli's Equation



- Based on Bernoulli's Theorem

$$\sum E_1 = \sum E_2$$

$$(\text{PE} + \text{KE} + \text{FE})_1 = (\text{PE} + \text{KE} + \text{FE})_2$$

- By simplifying the above relationship

$$z_1 + \frac{v_1^2}{2g} + \frac{p_1}{\rho g} = z_2 + \frac{v_2^2}{2g} + \frac{p_2}{\rho g}$$

Potential head

Velocity head

Pressure head

This equation, then, is known as Bernoulli's equation

Bernoulli’s Equation

- By simplifying the above relationship

$$\underbrace{z_1}_{\text{Potential head}} + \underbrace{\frac{v_1^2}{2g}}_{\text{Velocity head}} + \underbrace{\frac{p_1}{\rho g}}_{\text{Pressure head}} = z_2 + \frac{v_2^2}{2g} + \frac{p_2}{\rho g}$$

This equation, then, is known as Bernoulli’s equation

- Bernoulli’s Equation also can be written as

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2$$

where, $\gamma = \rho g$

Bernoulli’s Equation

$$\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2$$

- Assumption

1) The fluid flow is **incompressible,** and g is constant

2) The fluid is ideal (has **no *m***), therefore, there is **no energy losses due to friction**

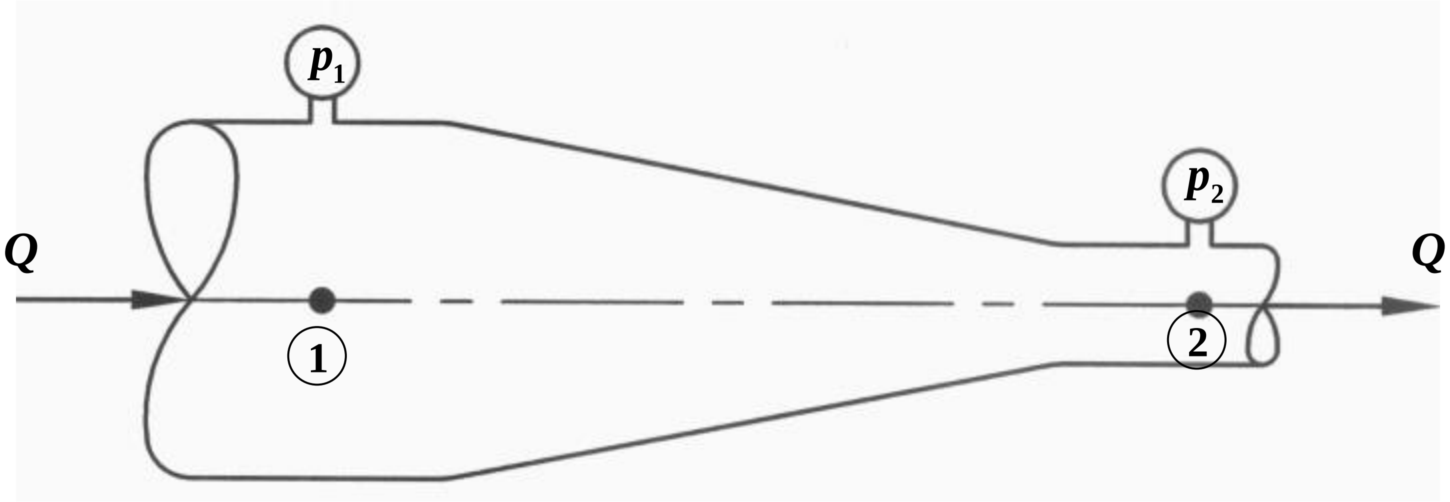
3) There is no mechanical devices in the system, lie between the two considered point, such as pump or turbine, which will increase or decrease the energy to or from the system

4) The fluid is under **steady state flow conditions**

5) There is **no heat being transfer** (increase or decrease) from or to point 1 to point 2 in the fluid system

Bernoulli’s Equation (Problem 3.3)

For a pipe system in the figure below, determine p_2 if p_1 = 20 psig, d_1 = 2 in and d_2 = 1.5 in, and Q_{water} = 200 gpm



Energy Equation

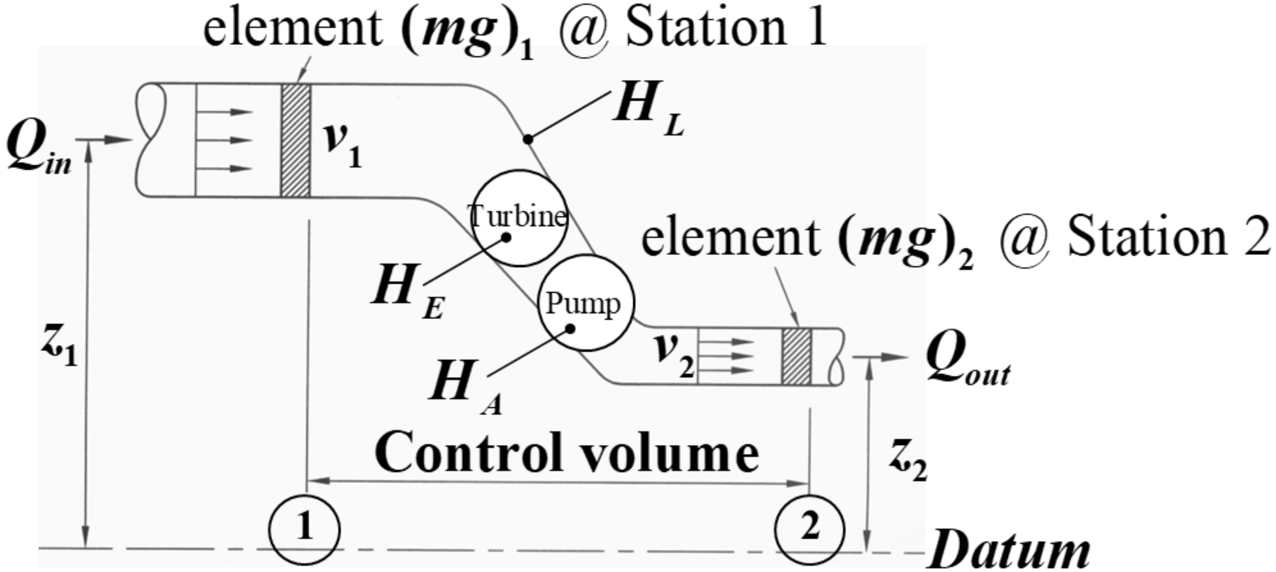
- Introduction
- Mechanical energy of a flowing fluid
- Pumps and turbines
- Pumps and turbines efficiency

Energy Equation (Introduction)

- Also fundamental to the study and prediction of fluid flow phenomena
- Apply the same principal as Bernoulli's equation → conservation of energy to fluid flow
- The equation accommodate energy losses due to friction (μ → to boundaries, fittings, valves, etc.), mechanical devices (e.g. fans, pump, turbine), suitable to steady incompressible flow
- Widely used in solving practical problems

Mechanical Energy of a Flowing Fluid

- Consider the situation below:



- The energy possessed by a flowing fluid consists of internal energy and energies due to pressure, velocity, and position:

$$\begin{matrix}\text{energy at}\\ \text{section 1}\end{matrix} + \begin{matrix}\text{energy}\\ \text{added}\end{matrix} - \begin{matrix}\text{energy}\\ \text{lost}\end{matrix} - \begin{matrix}\text{energy}\\ \text{extracted}\end{matrix} = \begin{matrix}\text{energy at}\\ \text{section 2}\end{matrix}$$

- This equation, for steady flow of incompressible fluids in which the change in internal energy is negligible, simplifies to:

$$\left(\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1\right) + H_A - H_L - H_E = \left(\frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2\right)$$

Mechanical Energy of a Flowing Fluid

$$\left(\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1\right)+H_A-H_L-H_E=\left(\frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2\right)$$

- This equation is known as the Energy equation or popularly known as Bernoulli equation (with respects to the pioneer)
- The units used are: ft *OR* m
- Many problems dealing with flow of liquids used this equation as the basis of solution
- Flow of gases, in many instances, involves principles of thermodynamics and heat transfer, that are beyond the scope of this subject

H_A : Any additional mechanical devices that can add energy to the system such as pump, H_P, etc.

H_E : Any additional mechanical devices that can extract energy from the system such as turbine, H_T, etc.

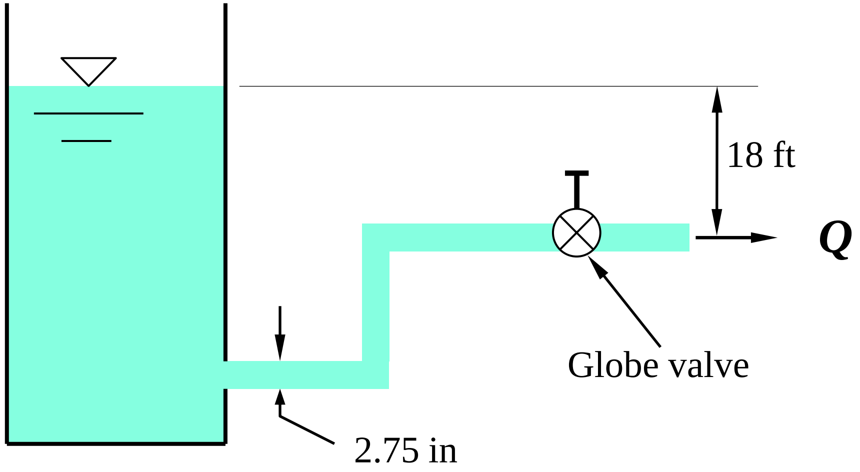
H_L : Any energy losses due to friction. Can be further divided into two, major losses, $H_{L\text{-}major}$, and minor losses, $H_{L\text{-}minor}$

$H_{L\text{-}major}$: Any energy losses due to friction between fluid and solid boundaries (pipe's wall) along the flow system (piping system)

$H_{L\text{-}minor}$: Any energy losses due to friction between fluid and bends, valves, filters, elbows, junctions, etc.

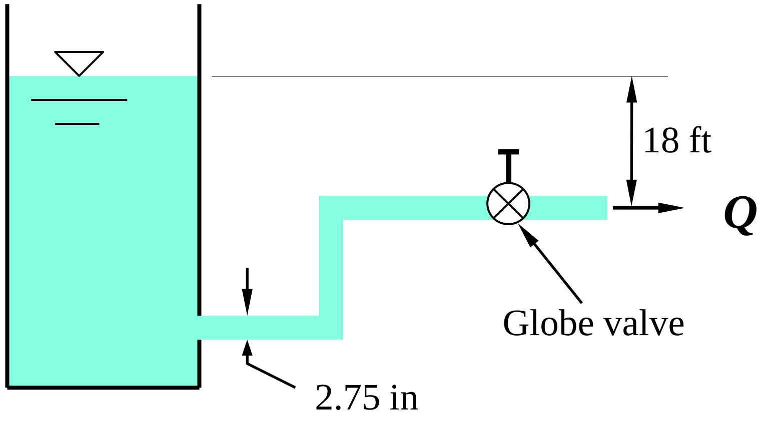
Bernoulli's Equation (Problem 3.4)

Water flows from a huge reservoir with 1.0 ft^3/sec flow rate, as shown in figure below. Losses occurred in the flow system are due to the pipe sudden contractions from the reservoir, two 90^o bends, and globe valve. Calculate all head losses, H_L



Bernoulli’s Equation (Problem 3.5)

Calculate the flow rate in Problem 3.4 if friction losses are ignored



Pumps and Turbines

- Pump will add energy to the fluid flow system, whereas turbine will extract energy from the fluid flow system
- This energy is called **motor fluid hydraulic power (fluid hydraulic power) or motor fluid power (fluid power)**

$$\left[\begin{array}{c}\text{Fluid hydraulic}\\ \text{power}\end{array}\right] = \left[\begin{array}{l} DpQ = gHQ \;\Rightarrow\; \text{SI Unit: } \dfrac{\text{Nm}}{\text{s}} = \dfrac{\text{joule}}{\text{s}} = watt \\ DpQ = gHQ \;\Rightarrow\; \text{BC Unit: } \;\Rightarrow \dfrac{\left(\text{ft.lb}/\text{sec}\right)}{550} = \text{Horse power } (HP) \end{array}\right]$$

$$\text{where: } \left[\begin{array}{l} \text{for SI Unit: } \gamma \to \dfrac{\text{N}}{\text{m}^3},\;\; H \Rightarrow \text{e.g. } H_p \text{ or } H_T = \begin{array}{c}\text{pump or turbine}\\ \text{hydrostatic head}\end{array} = H_A \text{ or } H_E \to \text{m},\;\; Q \to \dfrac{\text{m}^3}{\text{s}} \\ \text{for BC unit: } \gamma \to \dfrac{\text{lb}}{\text{ft}^3},\;\; H \Rightarrow \text{e.g. } H_p \text{ or } H_T = \begin{array}{c}\text{pump or turbine}\\ \text{hydrostatic head}\end{array} = H_A \text{ or } H_E \to \text{ft},\;\; Q \to \dfrac{\text{ft}^3}{\text{sec}} \end{array}\right]$$

- Pump possesses an energy which will be added to the fluid flow system, whereas turbine gains its energy from fluid flow system
- This energy is called **motor mechanical power**

$$\left[\begin{array}{c}\text{Motor mechanical}\\ \text{power}\end{array}\right] = \left[\begin{array}{l} \text{For pumps or any other devices that will add energy to} \\ \text{the system, normally it is given as } HP \text{ or } watt. \\ \\ \text{For turbines or any other devices that will extract energy from} \\ \text{the system, normally it is also given as } HP \text{ or } watt. \end{array}\right]$$

Pumps and Turbines Efficiency

- For pumps or any other devices that will add energy to the system

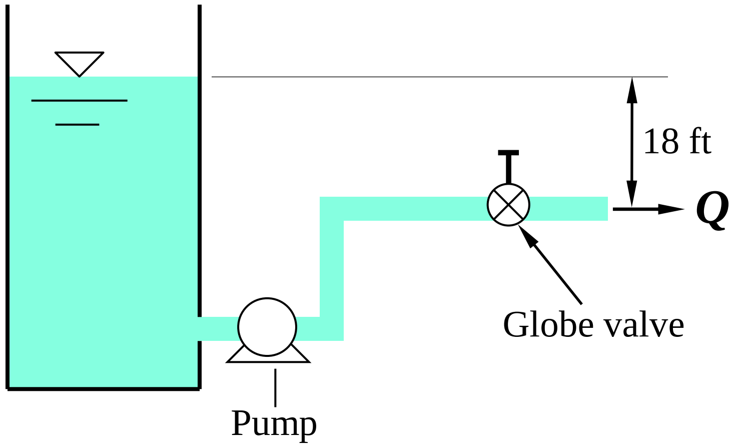
$$h_{(\%)} = \frac{\text{motor fluid hydraulic power}}{\text{motor mechanical power}} \text{ X } 100\% = \frac{g h_A Q}{HP \text{ or watt}} \text{ X } 100\%$$

- For turbines or any other devices that will extract energy from the system

$$h_{(\%)} = \frac{\text{motor mechanical power}}{\text{motor fluid hydraulic power}} \text{ X } 100\% = \frac{HP \text{ or watt}}{g h_E Q} \text{ X } 100\%$$

Pumps and Turbines (Problem 3.6)

If a pump is included in the system of Example 3.4, to increase the flow rate from 1ft^3/sec to 1.25ft^3/sec, and friction head losses increases to 12 ft due to the increasing of flow rate, calculate the pump head, H_p

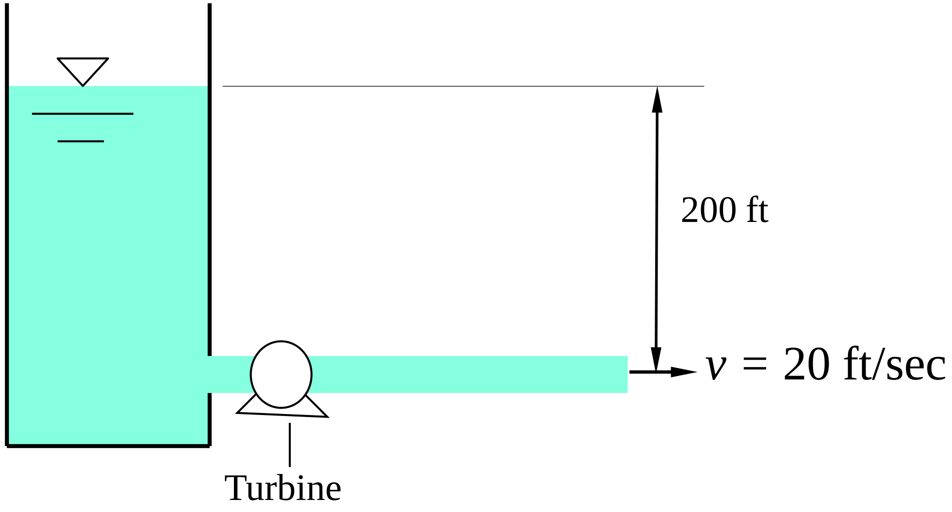


Pumps and Turbines (Problem 3.7)

What is the pump fluid power to deliver a water in a pipeline, if the pump need to increase pressure to 500 kPa and maintaining flow rate at 0.05 m^3/s?

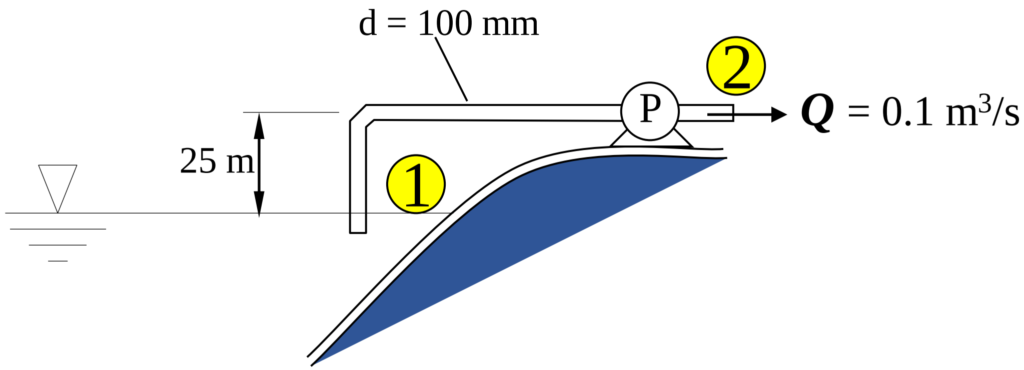
Pumps and Turbines (Problem 3.8)

A hydroelectric turbine receives water from a huge reservoir and jetting it out to atmosphere through a 18 inches pipe, as shown in figure below. If the velocity of water in the pipe is 20 ft/sec, calculate the fluid power, which is being transferred from the water by the turbine. Ignore any losses due to friction



Pumps and Turbines (Problem 3.9)

A pump deliver water at 0.1 m^3/s from a huge lake to a factory, as shown in figure below. Friction head losses from point 1 to point 2 is $H_L = C(v^2/2g)$, where C is friction losses coefficient (10 for this system) and p_2 is the atmospheric pressure. Calculate the power needed to kick off the pump, if pump efficiency is 75 %



Moment Equation

- Introduction
- Momentum and fluid flow ➡ force exerted by a jet
- Momentum equation for 2D-flow
  - ➡ Flow in branching pipe
  - ➡ Flow exerted by a jet: Angular plate

Moment Equation (Introduction)

- Knowledge of the forces exerted by moving fluids is very importance in the analysis and design of such objects as pumps, turbine, airplane, rocket, propellers, ship and many other hydraulic devices
- The energy relationship is not sufficient to solve most of these problems
- Momentum principle: One additional tool of mechanics

  ➡ most important
- Momentum equation **relates the sum of forces acting on a fluid element to its acceleration or rate of change of momentum in the direction of the resultants force**

Moment Equation (Momentum and Fluid Flow)

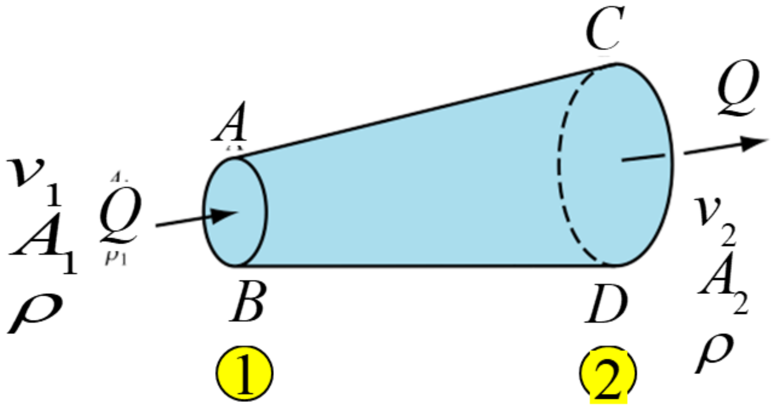
- Momentum of a particle or object: **the product of its mass *m* and its velocity *v***

$$\text{Momentum} = mv$$

- The particle of a fluid stream will possesses momentum, and, whenever the *v* of the stream is changed in magnitude or direction, there will be a corresponding change in the momentum of the fluid particles

Moment Equation (Momentum and Fluid Flow)

- Rate of change of momentum determination in a fluid stream



Consider a control volume ABCD

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 = \dot{m}$$

- Rate of change of momentum across the control volume

$$(\rho_1 A_1 v_1) v_1 - (\rho_2 A_2 v_2) v_2$$

- or from the continuity of mass flow equation

$$\rho_1 A_1 v_1 (v_2 - v_1) = \dot{m}(v_2 - v_1) = \text{Mass flow per unit time X Change of velocity}$$

Moment Equation (Momentum and Fluid Flow)

$$\rho_1 A_1 v_1 \left(v_2 - v_1\right) = \dot{m}\left(v_2 - v_1\right) = \text{Mass flow per unit time X Change of velocity}$$

- This is the *increase* of momentum per unit time in the direction of motion: according to Newton’s 2nd law will be caused by a force **F**

$$F = \dot{m}\left(v_2 - v_1\right)$$

- This is the resultant force (or the rate of change of momentum) acting on the fluid element ABCD in the direction of motion in 1-D problems

Moment Equation (Momentum and Fluid Flow)

$$F = \dot{m}\left(v_2 - v_1\right)$$

or

$$F = \dot{m}\left(v_{out} - v_{in}\right)$$

- This resultant force, **F**, is caused by:

  contacts between fluid and solid body (e.g. rotor blades, pipe bends, pipe sudden contractions, etc)

  or any other forces, which act with each other.
- **F** and ***v*** are vectors and assumed +ve in the direction of flow
- Some forces are dynamic (due to fluid motions) and some forces are static (due to pressure in the fluid)

Moment Equation (Momentum and Fluid Flow)

- For any control volume

$$F = F_1 + F_2 + F_3 = \dot{m}\left(v_{out} - v_{in}\right)$$

where:

F_1 = force exerted *in the given direction* on the fluid in the control volume by any *solid body* within the control volume or coinciding with the boundaries of the control volume (**force exerted by boundary area to the fluid**)

F_2 = force exerted *in the given direction* on the fluid in the control volume by *body forces such as gravity* (**force exerted by the weight of the fluid**)

F_3 = force exerted *in the given direction* on the fluid in the control volume **by the fluid outside the control volume**

- The force *R exerted by* the fluid on the solid body inside or coinciding with the control volume in the given direction will be equal and opposite to F_1 (Newton 3rd law) so that:

$$R = -F_1$$

Moment Equation (Momentum and Fluid Flow)

- Important equation in solving momentum problem:

Momentum equation

$$F = \dot{m}\left(v_2 - v_1\right)$$

Continuity equation

$$Q_1 = Q_2$$
$$Q_1 = Q_2 + Q_3$$

$$\dot{m}_1 = \dot{m}_2$$
$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

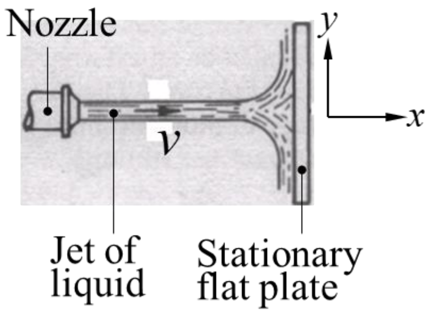
Bernoulli’s equation

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2$$

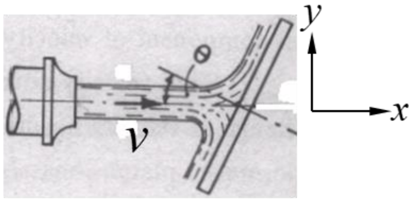
Force Exerted by a Jet

- Consider the situation below:

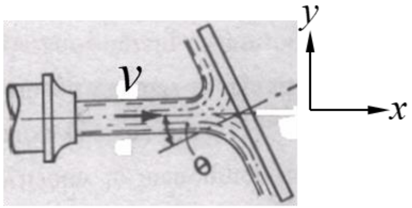
①



②



③



Case 1:

$$F = F_1 + \overset{0}{\cancel{F_2}} + \overset{0}{\cancel{F_3}} = \dot{m}\left(\overset{0}{\cancel{v_2}} - v_1\right)$$

$$\therefore\ F_1 = -\dot{m} v_1$$

$$R = -F_1$$

$$\boxed{\therefore\ R = \dot{m} v_1}$$

Case 2:

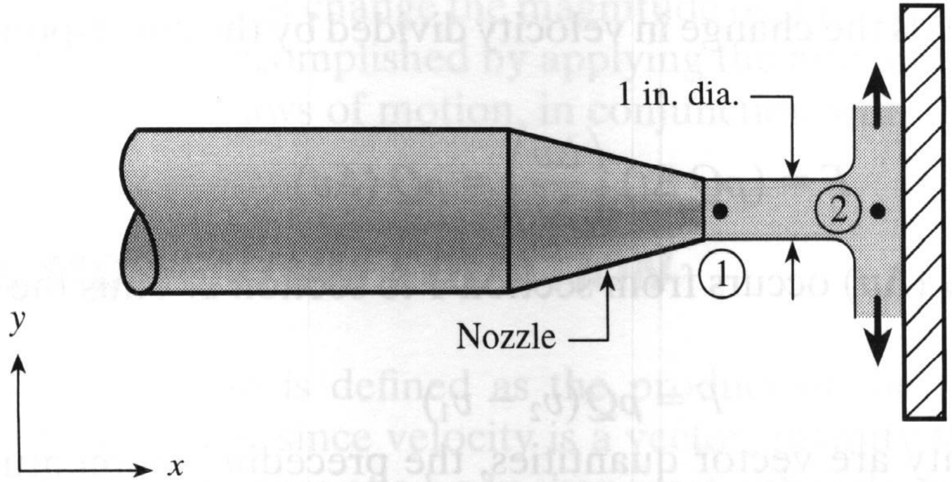
Case 3:

$$\boxed{\therefore\ R = \dot{m} v_1 \cos\theta}$$

1-Dimensional

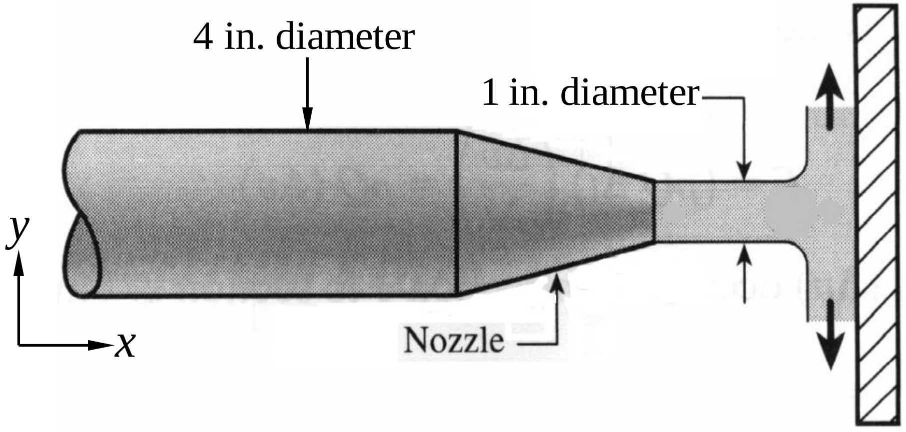
Force Exerted by a Jet (Problem 3.10)

A jet of water from 1 inch diameter nozzle strikes with a velocity of 30 ft/sec on a stationary flat plate as shown in figure below. Calculate the force exerted by the water on the plate



Force Exerted by a Jet (Problem 3.11)

A 1 inch diameter nozzle is connected to a horizontal pipe of 4 inches in diameter, as shown in figure below. The water is flowing through the pipe and jetting out at 200 gpm and strike a stationary plate. If the nozzle outlet is exposed to atmosphere, determine the forces exerted by the water on the plate and in the nozzle

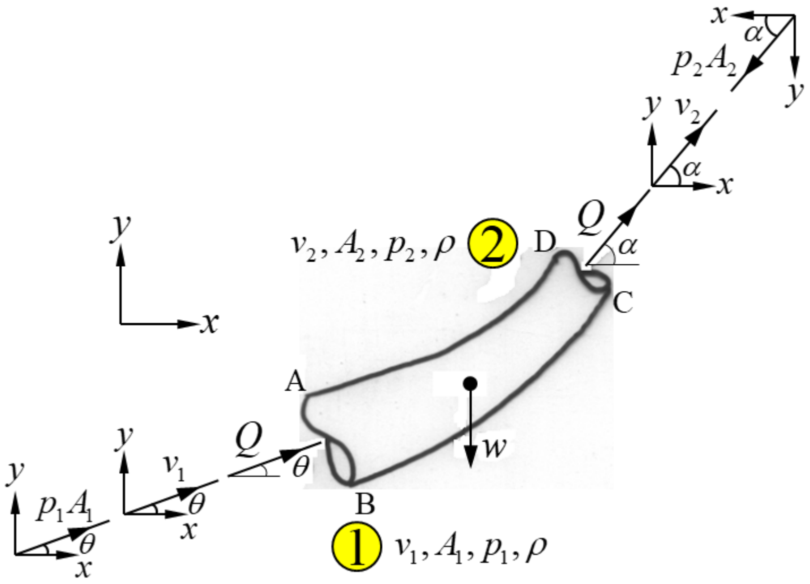


Force Exerted by a Jet (Notes)

- If p_1, & p_2 are given, just put it in the derived equation, an then solve it directly
- If either one of p is not given, use Bernoulli's equation together with continuity equation to solved the problem (z has to be given in this case, or the nozzle has to be in horizontal position)
- If p_1, & p_2 are not given, use iteration technique ➡ computing time will be too long

Momentum Equation for 2D-Flow

- Consider the situation below:



$$F = F_1 + F_2 + F_3 = \dot{m}(v_2 - v_1)$$

General momentum equation for 1-D

$$(F_1 + F_2 + F_3)_x = \dot{m}(v_{2x} - v_{1x})$$
$$(F_1 + F_2 + F_3)_y = \dot{m}(v_{2y} - v_{1y})$$

General Momentum equation for 2-D

$\xrightarrow{F \ +\text{ve}}$

x direction

$$R_x = -(F_1)_x$$
$$F_{2x} = 0 \left(\begin{array}{l}\text{due to no fluid's weight} \\ \text{component in } x \text{ direction}\end{array}\right)$$
$$F_{3x} = p_1A_1\cos\theta - p_2A_2\cos\alpha$$
$$v_{1x} = v_1\cos\theta$$
$$v_{2x} = v_2\cos\alpha$$

Combine all the equation together, becomes:

$$R_x = p_1A_1\cos\theta - p_2A_2\cos\alpha - \dot{m}(v_2\cos\alpha - v_1\cos\theta)$$

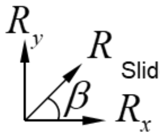
y direction $\uparrow$ F +ve

$$R_y = -(F_1)_y$$
$$F_{2y} = -w$$
$$F_{3y} = p_1A_1\sin\theta - p_2A_2\sin\alpha$$
$$v_{1y} = v_1\sin\theta$$
$$v_{2y} = v_2\sin\alpha$$

Combine all the equation together, becomes:

$$R_y = p_1A_1\sin\theta - p_2A_2\sin\alpha - w - \dot{m}(v_2\sin\alpha - v_1\sin\theta)$$

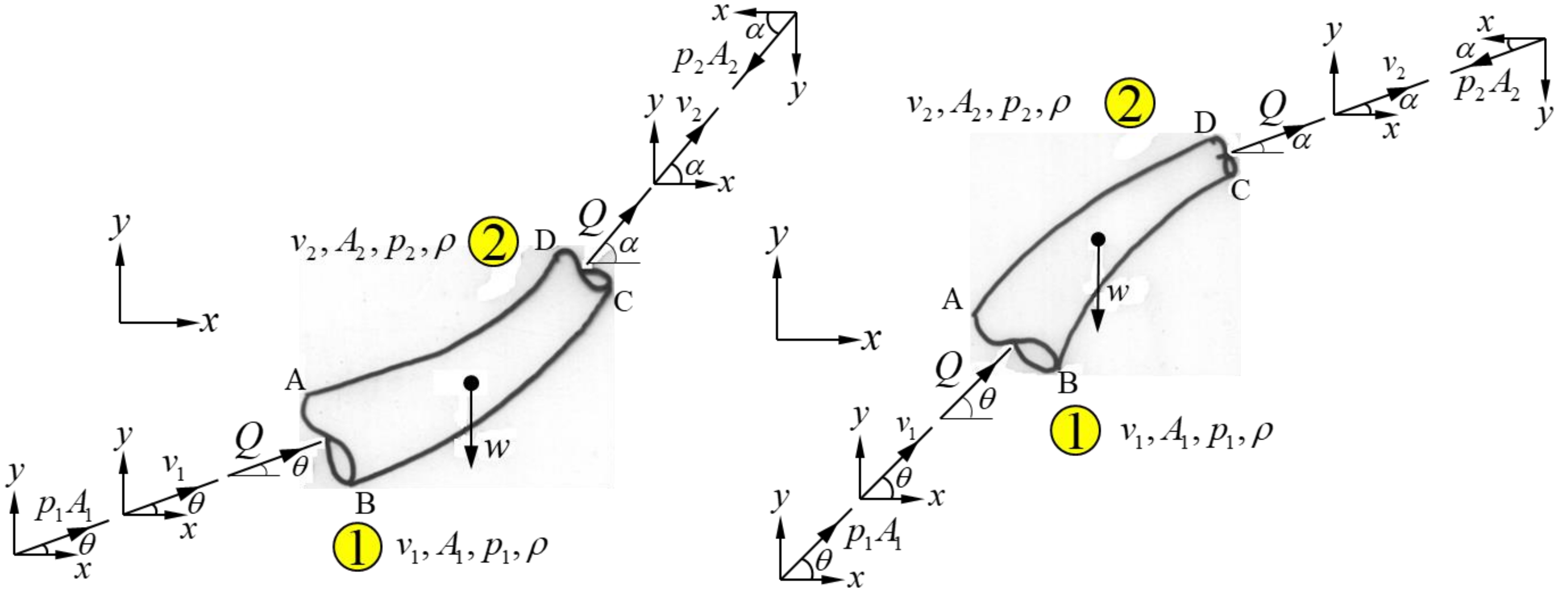
$$R = \sqrt{R_x^2 + R_y^2}, \quad \beta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



Slide 60

Momentum Equation for 2D-Flow

- Summary

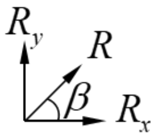


$$R_x = p_1A_1\cos\theta - p_2A_2\cos\alpha - \dot{m}\left(v_2\cos\alpha - v_1\cos\theta\right)$$

$$R_y = p_1A_1\sin\theta - p_2A_2\sin\alpha - w - \dot{m}\left(v_2\sin\alpha - v_1\sin\theta\right)$$

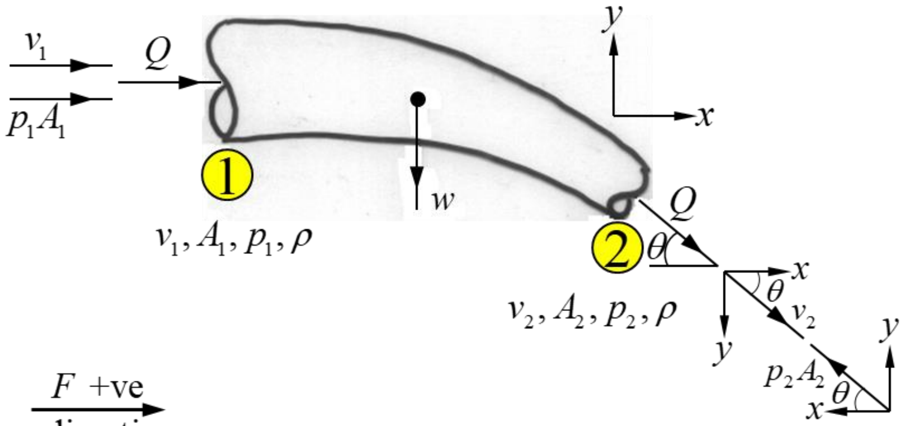
$$R = \sqrt{R_x^2 + R_y^2}$$

$$\beta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



Momentum Equation for 2D-Flow

- Consider the situation below:



$\xrightarrow{F\ +\text{ve}}$
x direction

$$(F_1 + F_2 + F_3)_x = \dot{m}(v_{2x} - v_{1x})$$

$$R_x = -(F_1)_x$$

$$F_{2x} = 0 \begin{pmatrix}\text{due to no fluid's weight} \\ \text{component in } x \text{ direction}\end{pmatrix}$$

$$F_{3x} = p_1A_1 - p_2A_2\cos\theta$$

$$v_{1x} = v_1$$

$$v_{2x} = v_2\cos\theta$$

Combine all the equation together, becomes:

$$R_x = p_1A_1 - p_2A_2\cos\theta - \dot{m}(v_2\cos\theta - v_1)$$

y direction $\uparrow$ F +ve

$$(F_1 + F_2 + F_3)_y = \dot{m}(v_{2y} - v_{1y})$$

$$R_y = -(F_1)_y$$

$$F_{2y} = -w$$

$$F_{3y} = \overset{0}{\cancel{p_1A_1}} + p_2A_2\sin\theta = p_2A_2\sin\theta$$

$(\text{due to no } p_1A_1 \text{ component in } y \text{ direction})$

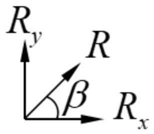
$$v_{1y} = 0 \begin{pmatrix}\text{due to no } v_1 \text{ component} \\ \text{in } y \text{ direction}\end{pmatrix}$$

$$v_{2y} = -v_2\sin\theta$$

Combine all the equation together, becomes:

$$R_y = p_2A_2\sin\theta - w + \dot{m}v_2\sin\theta$$

$$R = \sqrt{R_x^2 + R_y^2}, \quad \beta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



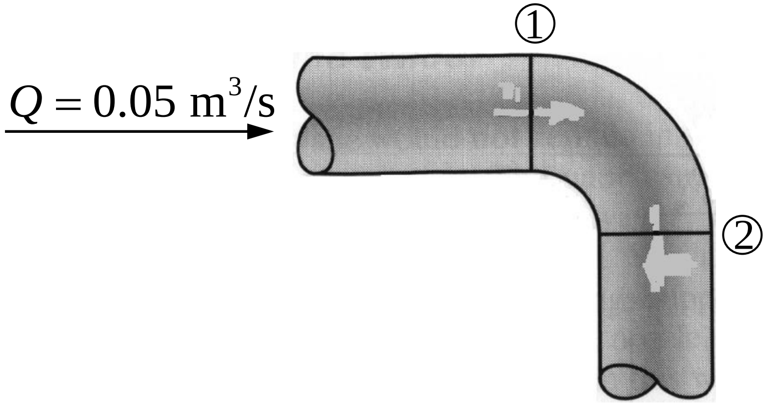
Momentum Equation for 2D-Flow

Notes:

- If p_1, & p_2 are given, just put it in the derived equation, an then solve it directly
- If either one of *p* is not given, use Bernoulli's equation together with continuity equation to solved the problem (*z* has to be given in this case, or the nozzle has to be in horizontal position)
- If the bend is in horizontal position, ignore *w* (*x* & *y*), if it is in vertical position, take *w* into account
- If p_1, & p_2 are not given, use iteration technique → computing time will be too long

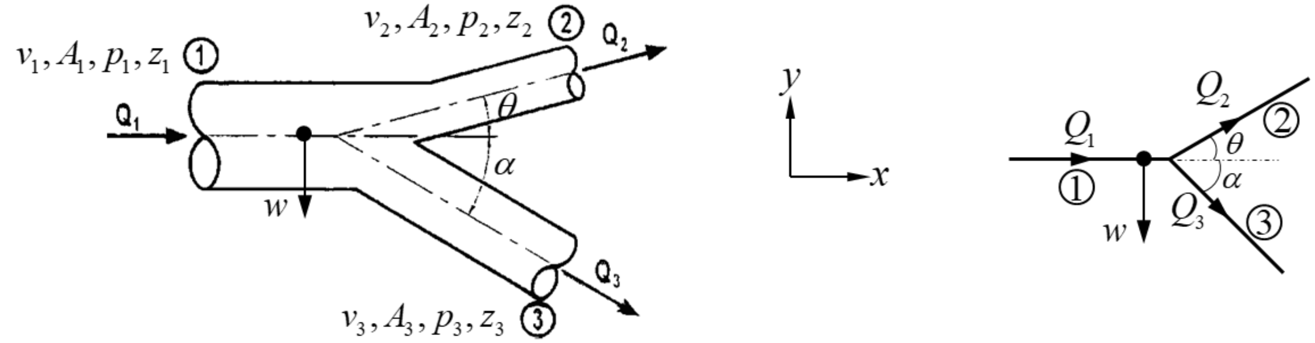
Momentum Equation for 2D-Flow (Problem 3.12)

An oil of 0.9 specific gravity flows at 0.05 m^3/s through an 90° elbow which is laid horizontally and connected to 100 mm pipe, as shown in figure below. If the pressure at 1 is 200 kPa gauge, determine the force exerted by the water to the elbow



Flow in Branching Pipe

- Consider the situation below:



$$F = F_1 + F_2 + F_3 = \dot{m}\left(v_{out} - v_{in}\right)$$

General momentum equation for 1-D

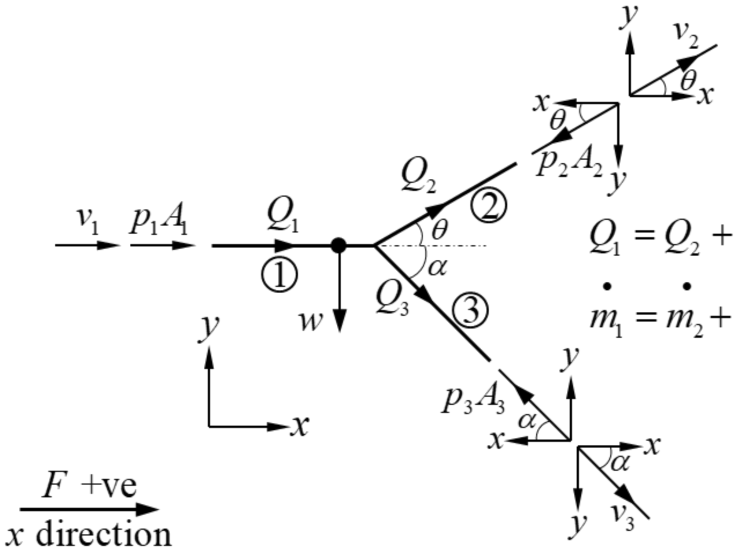
$$\left(F_1 + F_2 + F_3\right)_x = \left(\overbrace{(\dot{m}_3 v_3 + \dot{m}_2 v_2)}^{out} - \overbrace{(\dot{m}_1 v_1)}^{in}\right)_x$$

General Momentum equation for 2-D at *x* direction

$$\left(F_1 + F_2 + F_3\right)_y = \left(\overbrace{(\dot{m}_3 v_3 + \dot{m}_2 v_2)}^{out} - \overbrace{(\dot{m}_1 v_1)}^{in}\right)_y$$

General Momentum equation for 2-D at *y* direction

Flow in Branching Pipe



$$Q_1 = Q_2 + Q_3$$

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

F +ve

x direction

$$(F_1 + F_2 + F_3)_x = \left((\dot{m}_3\, v_3 + \dot{m}_2\, v_2) - (\dot{m}_1\, v_1) \right)_x$$

$$R_x = -(F_1)_x$$

$$F_{2x} = 0 \left(\begin{array}{l}\text{due to no fluid's weight component in} \\ x \text{ direction}\end{array} \right)$$

$$F_{3x} = p_1A_1 - p_2A_2\cos\theta - p_3A_3\cos\alpha$$

$$v_{1x} = v_1$$

$$v_{2x} = v_2\cos\theta$$

$$v_{3x} = v_3\cos\alpha$$

Combine all the equation together, becomes:

$$R_x = p_1A_1 - p_2A_2\cos\theta - p_3A_3\cos\alpha + \dot{m}_1 v_1 - \dot{m}_2\, v_2\cos\theta - \dot{m}_3\, v_3\cos\alpha$$

y direction $\uparrow$ F +ve

$$(F_1 + F_2 + F_3)_y = \left((\dot{m}_3\, v_3 + \dot{m}_2\, v_2) - (\dot{m}_1\, v_1) \right)_y$$

$$R_y = -(F_1)_y$$

$$F_{2y} = -w$$

$$F_{3y} = \overset{0}{\cancel{p_1}A_1} - p_2A_2\sin\theta + p_3A_3\sin\alpha$$

(due to no p_1A_1 component in y direction)

$$v_{1y} = 0 \left(\begin{array}{l}\text{due to no } v_1 \text{ component in} \\ y \text{ direction}\end{array} \right)$$

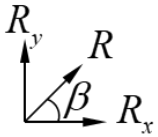
$$v_{2y} = v_2\sin\theta$$

$$v_{3y} = -v_3\sin\alpha$$

Combine all the equation together, becomes:

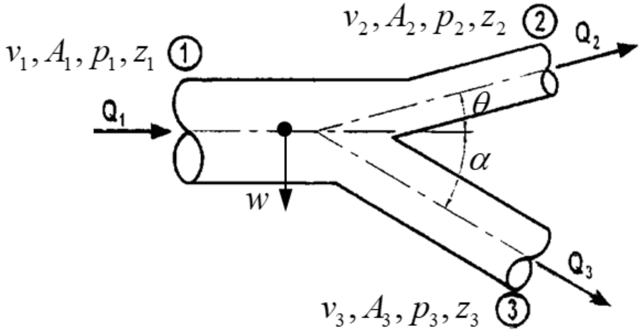
$$R_y = -p_2A_2\sin\theta + p_3A_3\sin\alpha - w - \dot{m}_2\, v_2\sin\theta + \dot{m}_3\, v_3\sin\alpha$$

$$R = \sqrt{R_x^2 + R_y^2}, \quad \beta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



Flow in Branching Pipe

Summary & Notes

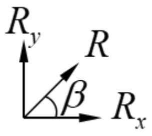


$$R_x = p_1A_1 - p_2A_2\cos\theta - p_3A_3\cos\alpha + \dot{m}_1v_1 - \dot{m}_2\,v_2\cos\theta - \dot{m}_3\,v_3\cos\alpha$$

$$R_y = -p_2A_2\sin\theta + p_3A_3\sin\alpha - w - \dot{m}_2\,v_2\sin\theta + \dot{m}_3\,v_3\sin\alpha$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\beta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



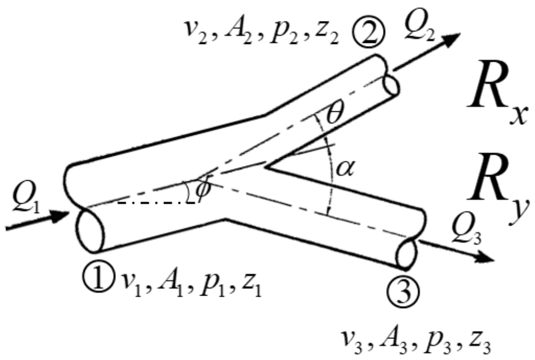
- p_1, p_2 & p_3, solved using Bernoulli's equation (branching pipe problem)
- v_1, v_2 & v_3, solved using continuity equation
- Solution can be made directly, or by using simultaneous equation, or by using iteration method, depending on the known parameters available.

Example of Cases

- $v_2 = v_3$, $p_2 = p_3 = p_{atm}$, pipe is in horizontal position, Q_1 is given, v_1 can be solved using continuity equation, p_1 can be solved using Bernoulli's equation, $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ → the easiest one.
- too many unknown parameter, needs iteration method to solved each one → the hardest one: computing time too long.

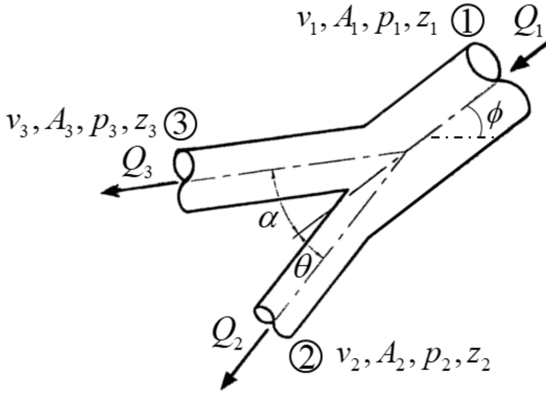
Flow in Branching Pipe

- Food for thought:



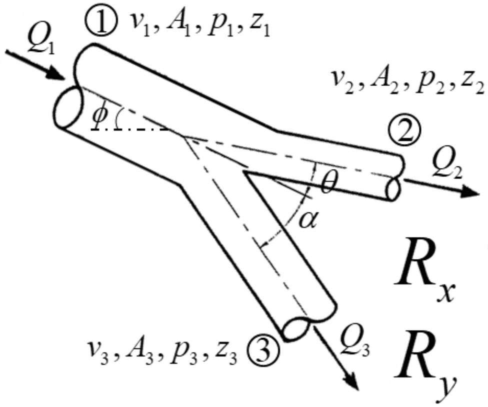
R_x ????

R_y ????



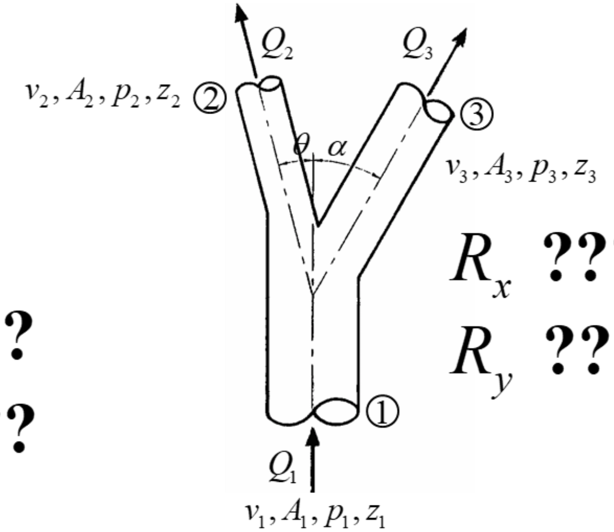
R_x ????

R_y ????



R_x ????

R_y ????

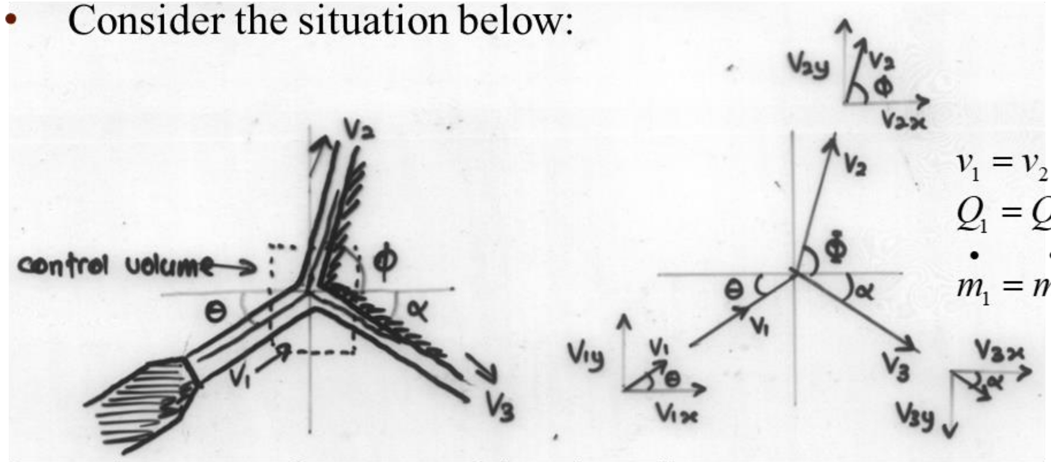


R_x ????

R_y ????

Forces Exerted by a Jet: Angular Plate

- Consider the situation below:



$v_1 = v_2 = v_3 = v$ (friction losses is ignored)

$$Q_1 = Q_2 + Q_3$$

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

In previous cases, force exerted by a jet strike on flat stationary plate.

$$F = \dot{m}(v_{out} - v_{in}) = \dot{m}\,v_{out} - \dot{m}\,v_{in} = -\dot{m}\,v_{in}$$

In cases, where force exerted by a jet strike on angular flat stationary plate.

$$F = \dot{m}(v_{out} - v_{in}) = \dot{m}(v'_2 - v_1) = \dot{m}\,v'_2 - \dot{m}\,v_1$$

$\xrightarrow{F\ +\text{ve}}$

x direction

$$\left(F_1 + F_2 + F_3\right)_x = \left(\overbrace{(\dot{m}_2\, v_2 + \dot{m}_3\, v_3)}^{out} - \overbrace{(\dot{m}_1\, v_1)}^{in}\right)_x$$

$$\left(F_1 + F_2 + F_3\right)_x = (\dot{m}_2\, v_2 + \dot{m}_3\, v_3 - \dot{m}_1\, v_1)_x$$

In the control volume,
$F_{2x} = 0$ (ignore because the w effect is too small)
$F_{3x} = 0$ (atmospheric pressure)

$$\therefore\ (F_1)_x = (\dot{m}_2\, v_2 + \dot{m}_3\, v_3 - \dot{m}_1\, v_1)_x$$

$$R_x = -(F_1)_x$$

$$\therefore\ R_x = -(\dot{m}_2\, v_2 + \dot{m}_3\, v_3 - \dot{m}_1\, v_1)_x$$

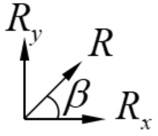
By using similar technique, for y direction.

y direction $\uparrow$ F +ve

$$\therefore\ R_y = -(\dot{m}_2\, v_2 + \dot{m}_3\, v_3 - \dot{m}_1\, v_1)_y$$

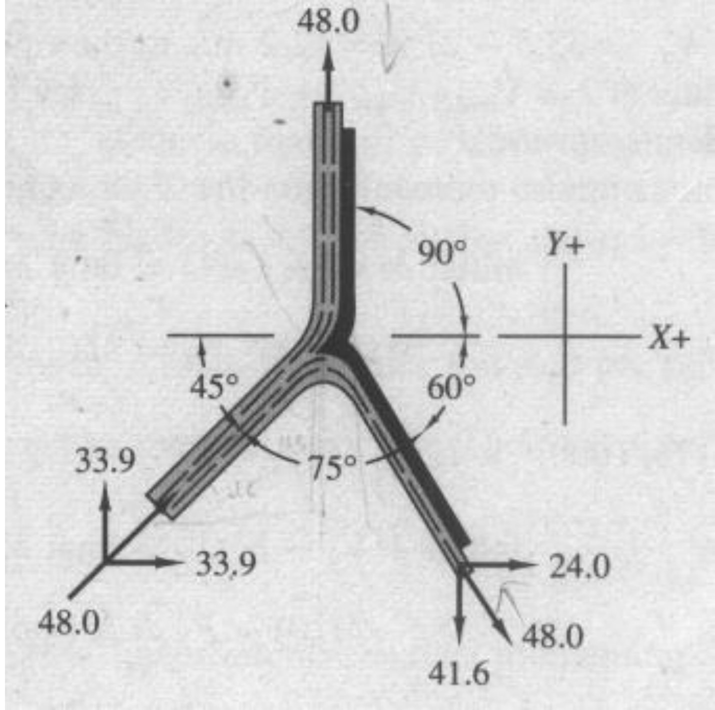
$$R = \sqrt{R_x^2 + R_y^2}$$

$$\beta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



Forces Exerted by a Jet: Angular Plate (Problem 3.13)

The fixed surface as shown in figure below, divides the jet so that 1.00 ft^3/sec goes in each direction. For an initial velocity of 48.0 ft/sec, find the values of R_x and R_y components to keep in equilibrium (assuming no friction), also find the forces exerted by the water on the surface



Forces Exerted by a Jet: Angular Plate

General equation for Example 3.13 (or for similar cases)

$\xrightarrow{F\ +\text{ve}}$

x direction

$$R_x = -(\dot{m}_2\, v_2 + \dot{m}_3\, v_3 - \dot{m}_1\, v_1)_x \qquad v_1 = v_2 = v_3 = v$$

$$R_x = -(\dot{m}_2\, v \cos\phi + \dot{m}_3\, v \cos\alpha - \dot{m}_1\, v \cos\theta)_x$$

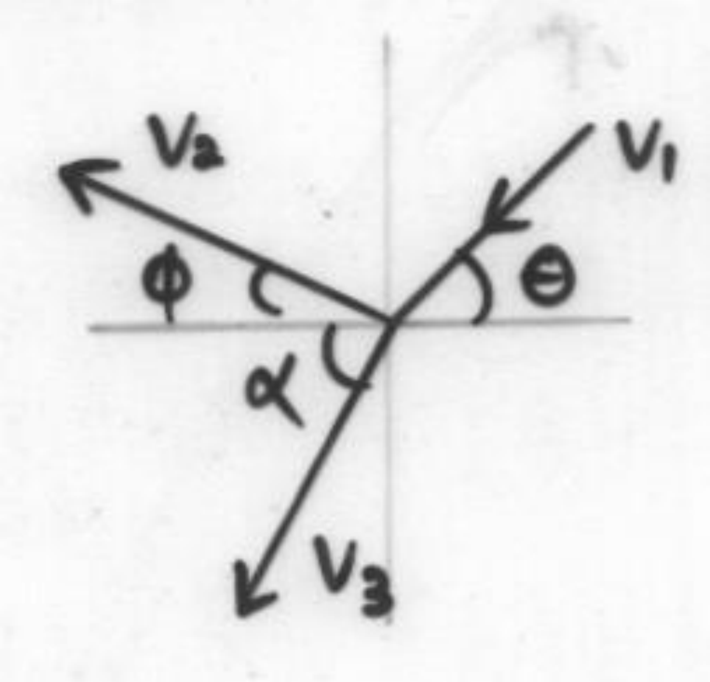
y direction $\uparrow$ F +ve

$$R_y = -(\dot{m}_2\, v_2 + \dot{m}_3\, v_3 - \dot{m}_1\, v_1)_y \qquad v_1 = v_2 = v_3 = v$$

$$R_y = -(\dot{m}_2\, v \sin\phi - \dot{m}_3\, v \sin\alpha - \dot{m}_1\, v \sin\theta)_y$$

Forces Exerted by a Jet: Angular Plate

Food for thought

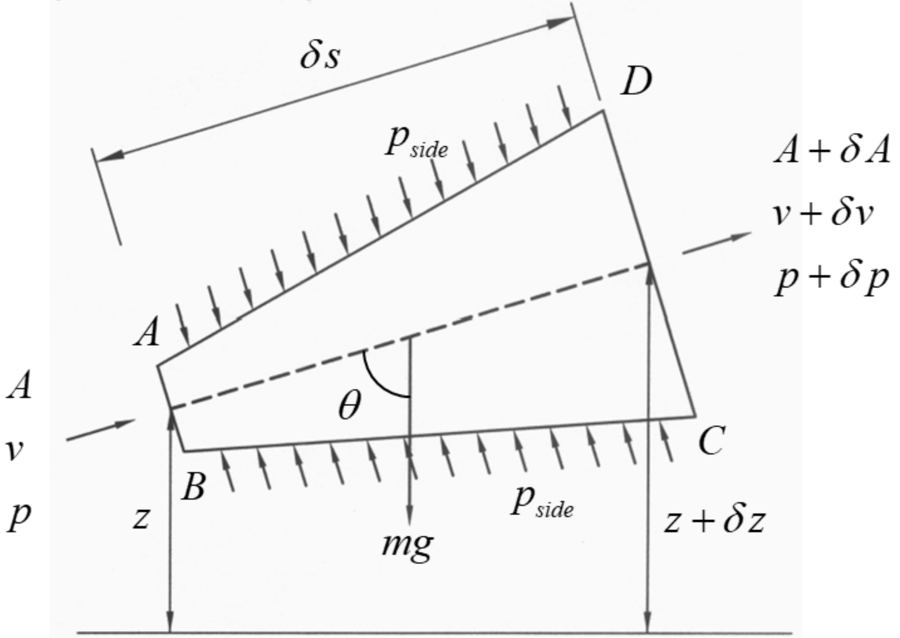


R_x ????

R_y ????

Euler's Equation

- From consideration of the rate of change of momentum from point to point along a streamline and the forces acting due to the effect of the surrounding pressure and changes of elevation, → possible to derive a relationship between p, v, ρ and z along a streamline



for detailed derivation of the equations, please refer "Douglas, J.F., J.M. Gasiorek, and J.A. Swaffield. 1981. *Fluid Mechanics*. 3rd Edition. London: Longman Scientific and Technical", page 134-136

$$\frac{1}{\rho}\frac{dp}{ds}+v\frac{dv}{ds}+g\frac{dz}{ds}=0$$

Euler's equation

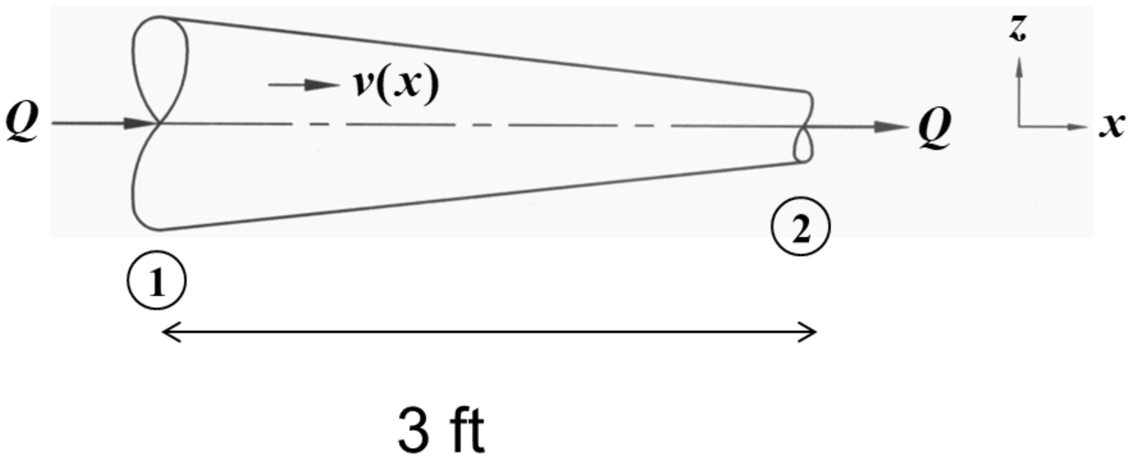
- If the relationship between p, v, ρ and z is known, the equation can be solved by integration → it is very cumbersome

Euler’s Equation (Problem 3.14)

A steady flow of water through a pipe which changes its diameter, is shown in figure below. The relationship between velocity and the distance of flow is given by $v = 10(1+x)$ ft/s, where x in ft

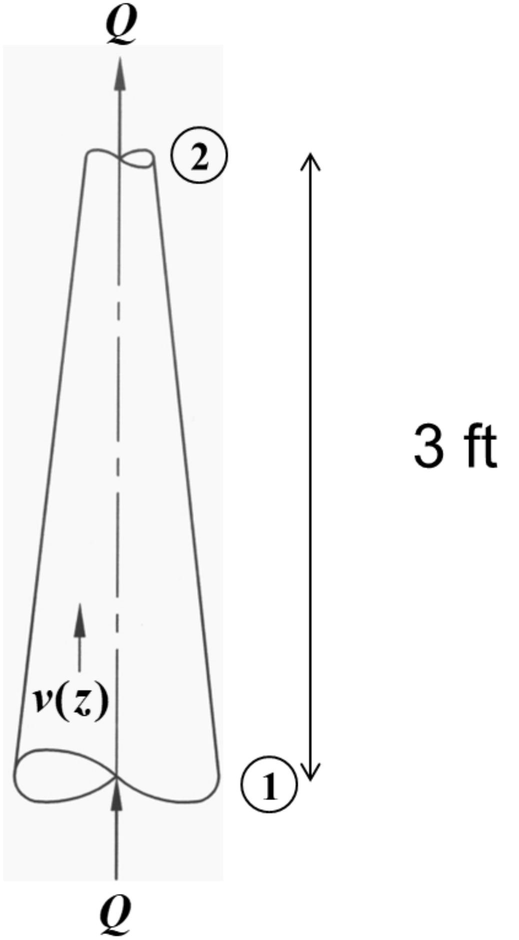
a. Determine the pressure slope, *dp/dx,* required to generate the flow

b. If the pressure at point 1 is 50 psi, determine the pressure at point 2 by integrating *dp/dx* from a.



Euler's Equation (Problem 3.15)

Repeat Example 3.a if the position of the pipe and direction of flow are vertical



Euler’s Equation (Problem 3.16)

What is the pressure slope along streamline that is required to accelerate water upward in vertical pipe at the rate of 30 ft/sec^2. What is the answer if the flow is downward?

THANK YOU

Stay safe!