FRICTION FLOW IN PIPES

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Objectives of the Topic

- Have a deeper understanding of laminar and turbulent flow in pipes
- Calculate the major and minor losses associated with pipe flow in piping networks (series, parallel and branching)

Contents

- Introduction
- Energy losses in pipe
- Pipe in series
- Pipe in parallel
- Flow through branching pipes

Introduction

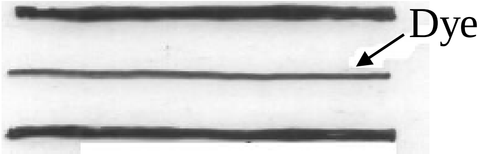
- Real fluid flow is much complex compared to perfect fluid flow
- Shear stress
  - Between fluid particles
  - Between fluid particles and pipe's wall

  μ fluid
- **Energy equilibrium principles** is used to solve the problems
- Partial differential equation (Euler's equation) has no general solution to solve problems
- Results from experiment (*analytical*) and semi-empirical method needs to be used to solve flow problems
- There are 2 types of steady flow of real fluid exists:

  1) Laminar flow   2) Turbulent flow

Introduction

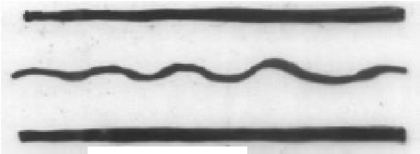
This one?



Laminar (viscous) flow

Smooth streamlines and highly ordered motion

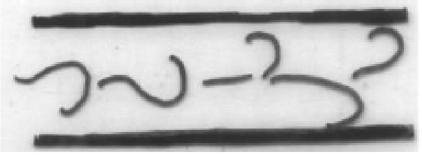
OR This one?



Transition flow

The flow fluctuates between laminar and turbulent flows.

OR This one?



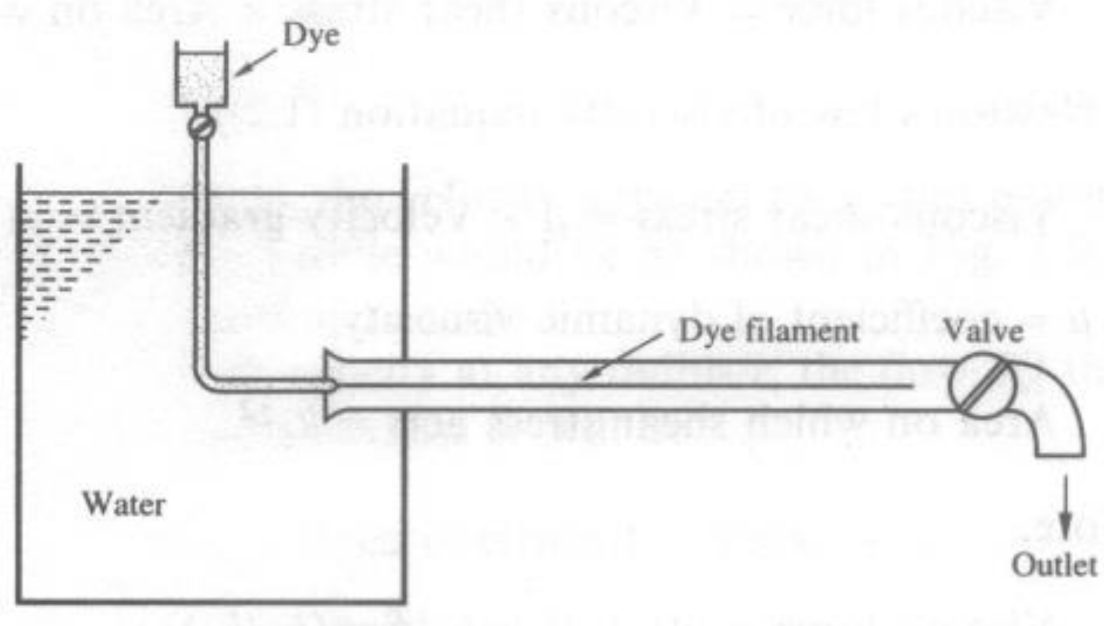
Turbulent flow

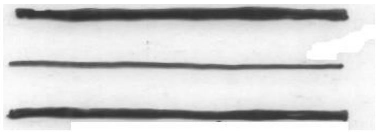
Velocity fluctuations and highly disordered motion

- All three types of flow actually do occurred in real fluid flow.
  - Laminar flow : $v\downarrow$
  - Turbulent flow : $v\uparrow$
- The problem is: what is $v\uparrow$ and $v\downarrow$. Why we need to know?

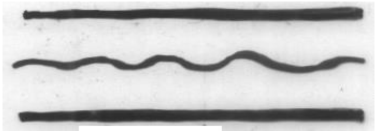
Introduction

- This phenomenon was first investigated in 1883 by Osborne Reynolds in an experiment which has a classic in fluid mechanic





Laminar flow



Transition flow

Turbulent flow

- After a few experiments, he found out a mathematical relationship

$$\frac{\rho v d}{\mu}$$

ρ = Fluid density
v = Fluid average velocity
d = Pipe diameter (internal)
μ = Fluid absolute viscosity

Introduction

- This mathematical relationship can be used to determine the types of flow

$$-\quad \frac{\rho v d}{\mu} < 2000 \qquad \text{laminar flow}$$

$$-\quad 2000 < \frac{\rho v d}{\mu} < 4000 \qquad \text{transition flow}$$

$$-\quad \frac{\rho v d}{\mu} > 4000 \qquad \text{turbulent flow}$$

- Subsequently until now, this mathematical relationship is known as **Reynolds number, Re**

$$\mathrm{Re} = \frac{\rho v d}{\mu} \Rightarrow \text{dimensionless}$$

$$\begin{aligned} &- \text{laminar flow} &&: \mathrm{Re} < 2000 \\ &- \text{transition flow} &&: 2000 < \mathrm{Re} < 4000 \\ &- \text{turbulent flow} &&: \mathrm{Re} > 4000 \end{aligned}$$

Introduction

$$\frac{\rho v d}{\mu}$$

Where:

ρ – fluid density
v – fluid average velocity
d – pipe inside diameter
μ – fluid absolute viscosity

- If kinematic viscosity, υ is inserted in the equation:

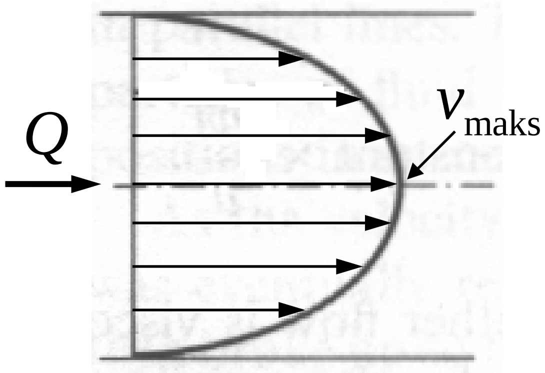
$$\upsilon = \frac{\mu}{\rho}$$

$$\mathrm{Re} = \frac{vd}{\upsilon}$$

Introduction

- Fluid velocity profile in a pipe:

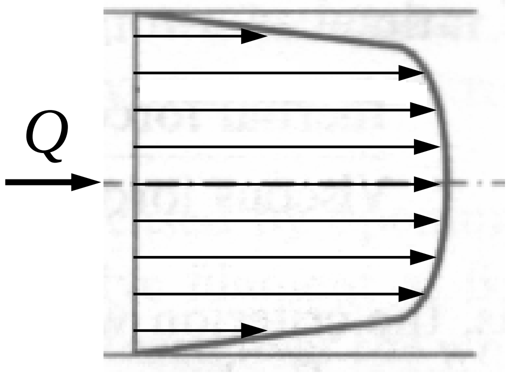
Laminar flow



$$v_{avg} = \tfrac{1}{2} v_{maks}$$

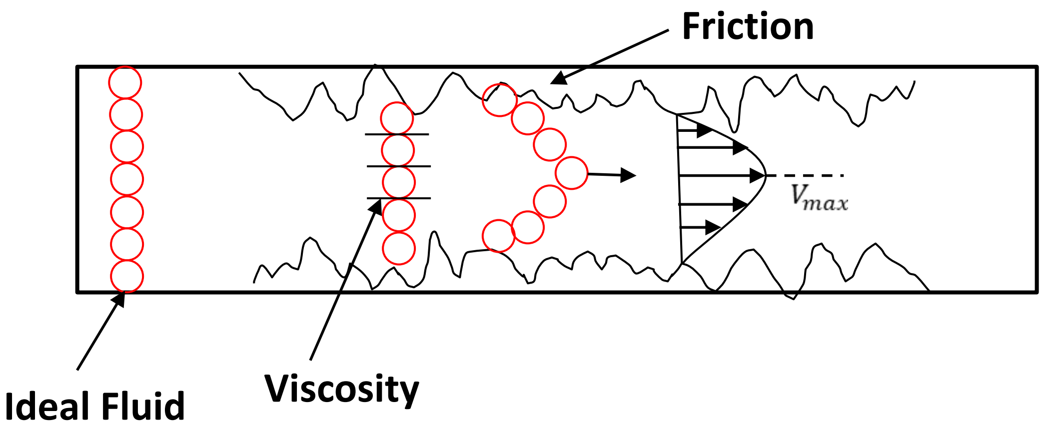
$$v_{avg} = \frac{Q}{A}$$

Turbulent flow

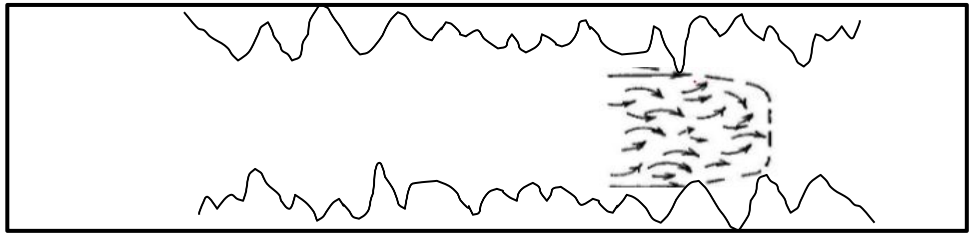


$$v_{avg} = \frac{Q}{A}$$

Fluid Velocity Profile



Laminar flow
Re < 2000



Turbulent flow
Re > 4000

Energy Losses In Pipe (Intro)

- ***Def***.: Any energy losses in closed conduits due to friction, H_L
- This types of losses can be divided into 2 main categories:
  - Major losses, $\boldsymbol{H_{L\text{-major}}}$
  - Minor losses, $\boldsymbol{H_{L\text{-minor}}}$
- From Bernoulli's equation:

$$\left(\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1\right)+H_A-H_L-H_E=\left(\frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2\right)$$

- Energy added to the system, $\boldsymbol{H_A}$, is frequently due to pump fluid head, $\boldsymbol{H_P}$, energy extracted, $\boldsymbol{H_E}$, is frequently due to turbine fluid head, $\boldsymbol{H_T}$, Bernoulli's equation can be simplify as:

$$\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1+H_P=\frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2+H_T+H_{L-\text{major}}+H_{L-\text{minor}}$$

Major Losses In Pipe

- ***Def***.: The head loss due to **friction** in long, straight sections of pipe
- The losses do happen in pipe, either in laminar or turbulent flow

a. Laminar flow

- Problem solved analytically → derived purely from mathematical relationship
- **Hagen-Porseuille** equation

$$\Delta p_f = \frac{32\mu v L}{d^2}$$

in the forms of head loss, H_L

$$H_L = \frac{32\mu v L}{\gamma d^2}$$

- **Darcy-Weisbach** equation

by replacing $\mathrm{Re} = \frac{\rho v d}{\mu}$ into Hagen-Porseuille equation $H_L = \frac{64}{\mathrm{Re}} \frac{L}{d} \frac{v^2}{2g}$

Major Losses In Pipe

b. Turbulent flow

- From **Darcy-Weisbach** equation for laminar flow

$$H_L = \frac{64}{\text{Re}}\frac{L}{d}\frac{v^2}{2g}$$

$$H_L = f\,\frac{L}{d}\frac{v^2}{2g}$$

f = friction factor or friction coefficient.

- Where, for laminar flow, $\left. f = \frac{64}{\text{Re}} \right\}$ a simple mathematical relationship
- For turbulent flow, f has to be solved empirically → experiment need to be done
- In laminar and turbulent flow, f is known as friction coefficient or friction factor

Friction Factor, f

a. Laminar flow

- Darcy-Weisbach equation

$$H_L = f \frac{L}{d}\frac{v^2}{2g} \qquad \text{where} \quad f = \frac{64}{\text{Re}}$$

b. Turbulent flow

- In the literature (from 1900's – current date), there are many studies that have been conceded by various researchers
- Blasius's equation (1913) → study for smooth pipes

$$f = \frac{0.079}{\text{Re}^{0.25}} \qquad \text{% error } \pm \text{ 5% for smooth pipes at Re up to 100 000}$$

- von Karman's equation modified by Prandtl

$$\frac{1}{\sqrt{f}} = 2\log(\text{Re}\sqrt{f}) - 0.8 \qquad \text{for smooth pipes}$$

$$\frac{1}{\sqrt{f}} = 2\log(\frac{\text{Re}}{\varepsilon}) - 1.74 \qquad \text{for rough pipes}$$

Friction Factor, *f*

- Nikuradse's equation (for smooth and rough pipes)

  The first researcher taken into account $\frac{\varepsilon}{d}$, Re

  f was expressed in the form of chart

- Colebrook-White equation (1940's)

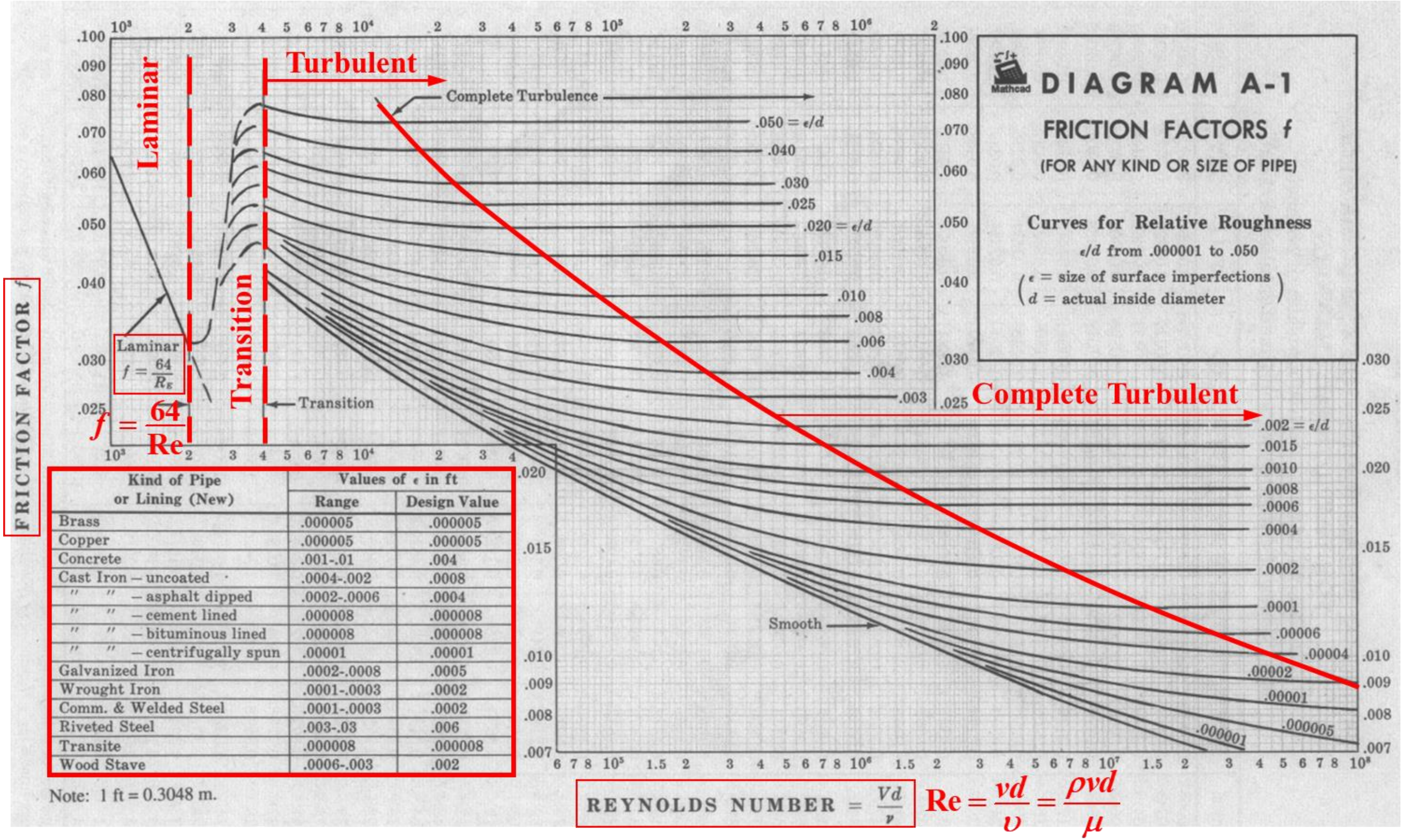
$$\frac{1}{\sqrt{f}} = -4\log\left[\frac{k_s}{3.71d} + \frac{1.26}{\mathrm{Re}\sqrt{f}}\right] \Rightarrow \frac{\varepsilon}{d}, \mathrm{Re} \text{ for various commercial pipes}$$

- Moody
  - plot f versus Re for various commercial pipes $\left(\frac{\varepsilon}{d}\right)$
  - This plot, later is known as Moody's Chart

$$- \ f = 0.001375\left[1 + \left(\frac{200k_s}{d} + \frac{10^6}{\mathrm{Re}}\right)^{1/3}\right]$$

- Barr's equation (1975)

$$- \ f = \left[-4\log\left(\frac{k_s}{3.71d} + \frac{5.1286}{\mathrm{Re}^{0.89}}\right)\right]^{-2}$$



Kind of Pipe or Lining (New)	Values of ε in ft	
	Range	Design Value
Brass	.000005	.000005
Copper	.000005	.000005
Concrete	.001-.01	.004
Cast Iron — uncoated	.0004-.002	.0008
" " — asphalt dipped	.0002-.0006	.0004
" " — cement lined	.000008	.000008
" " — bituminous lined	.000008	.000008
" " — centrifugally spun	.00001	.00001
Galvanized Iron	.0002-.0008	.0005
Wrought Iron	.0001-.0003	.0002
Comm. & Welded Steel	.0001-.0003	.0002
Riveted Steel	.003-.03	.006
Transite	.000008	.000008
Wood Stave	.0006-.003	.002

Moody Chart

Friction Factor, *f*

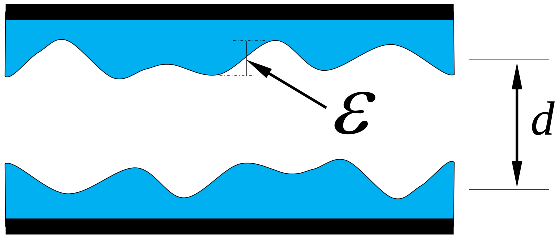
Normal Practice In Determination of *f*

1. Calculate Re to determine the types of flow.
2. H_L calculation: used Darcy-Weisbach equation.

$$H_L = f \frac{L}{d} \frac{v^2}{2g}$$

3. For laminar flow: $f = \dfrac{64}{\mathrm{Re}}$
4. For turbulent flow:
   a. Determine pipe relative roughness, $\dfrac{\varepsilon}{d}$

   Where:
   ε – pipes absolute roughness
   d – pipe internal diameter



ε is depend on pipe's material, normally is given in tabular forms

Material (new)	Absolute roughness, ε	
	ft	mm
Riverted steel	0.003 - 0.03	0.9 - 9.0
Concrete	0.001- 0.01	0.3 - 3.0
Wood stave	0.0006 - 0.003	0.18 - 0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Glass	0.0 (smooth)	0.0 (smooth)

   b. Obtain *f* from Moody chart, @ Re, $\dfrac{\varepsilon}{d}$

Friction Factor, f

Attention

1. **In this subject**, the f that we are using, is the American friction factor, $f_{American}$.

2. The value of $f_{American}$ is different to the one that used by the British

$$\underbrace{\underset{\frac{64}{\text{Re}}}{f_{American}} = 4\,\underset{\frac{16}{\text{Re}}}{f_{British}}}_{\text{needs to refer different Moody Chart}}$$

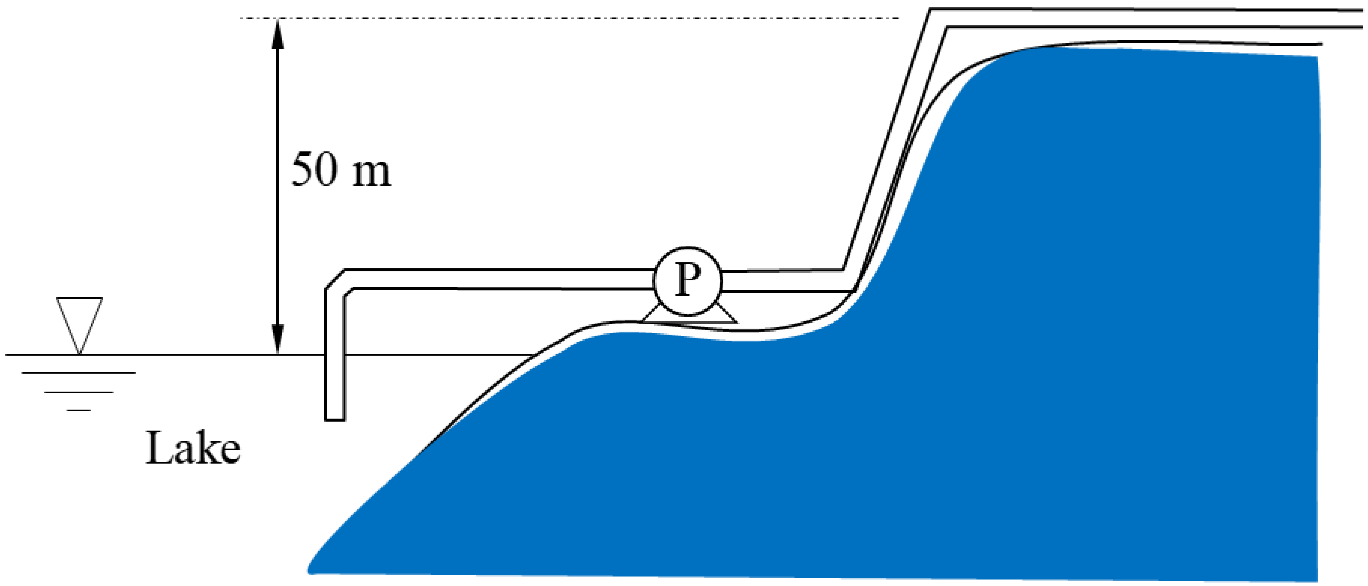
3. Sometimes: $\lambda = f_{American} = 4\,f_{British}$

Friction Factor, f (Problem 4.1)

The kinematic viscosity of a hydraulic oil is 0.538 X 10^{-3} ft^2/sec. If the oil is flowing at 40 ft/sec through 1 inch diameter 50 ft long commercial steel pipe, determine the losses due to the friction

Friction Factor, f (Problem 4.2)

A pump deliver water at 0.03 m^3/s from a huge lake to the hilltop through 80 mm diameter and 200 m long of cast iron pipe as shown in figure below. Calculate the power needed by the pump to deliver the water. Take kinematic viscosity of water as 1.12 x 10^{-6} m^2/s. Neglect minor losses



Minor Losses In Pipe

- Other head losses are generally categorized as "minor" head losses, $H_{L\text{-}minor}$
- They result when there is a significant change in flow pattern
- They occur in conduit contractions and enlargements (both sudden & gradual), valves, fittings, bends, etc., and entrance to or exit from a conduit
- In some cases, "minor" loss can be quite important
- Theoretical consideration of various minor losses are quite complicated
- They are usually evaluated by empirical methods
- They are commonly expresses in terms of the applicable velocity head
- In equation form:

$$\text{lost head (ft or m)} = K_L\left(\frac{v^2}{2g}\right)$$

Where:

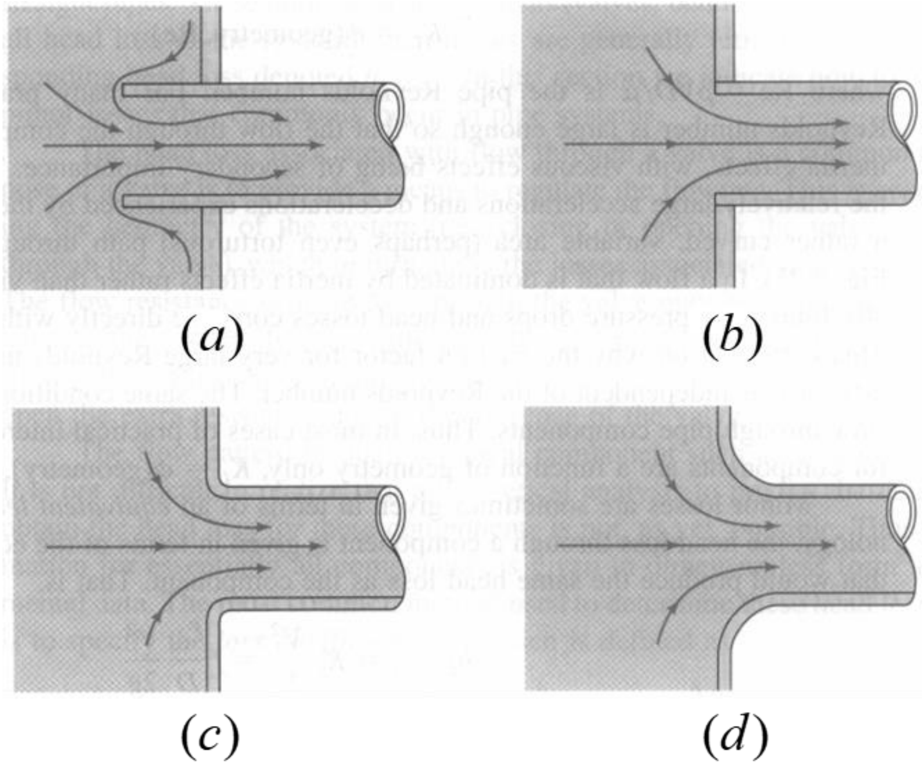
K_L = losses coefficient for divices

$\frac{v^2}{2g}$ = velocity head

Minor Losses In Pipe

a. Entrance losses

- Occur when a liquid enters a conduit from a huge tank or reservoir
- The amount of head loss is significantly dependent on the shape of the entrance
- If an entrance is well rounded, the entrance loss will be very small



Entrance flow conditions and loss coefficient.

(a) Reentrant, $K_L = 0.8$,

(b) sharp-edged, $K_L = 0.5$,

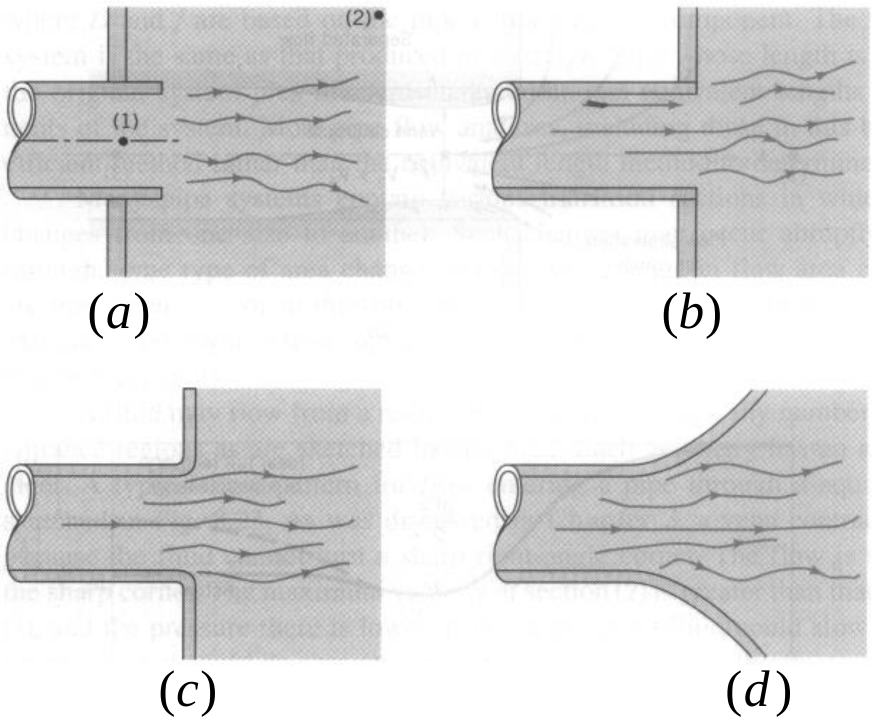
(c) slightly rounded, $K_L = 0.2$,

(d) well-rounded, $K_L = 0.04$

Minor Losses In Pipe

b. Exit losses

- Occur when a liquid exits a conduit and enters a huge tank or reservoir
- The amount of head loss is significantly dependent on the shape of the entrance
- If an exit is well rounded, the entrance loss will be very small



Exit flow conditions and loss coefficient.

(a) Reentrant, K_L = 1.0,

(b) sharp-edged, K_L = 1.0,

(c) slightly rounded, K_L = 1.0,

(d) well-rounded, K_L = 1.0

Minor Losses In Pipe

c. **Sudden contraction losses (Vena-Contraction)**: occur when there is an abrupt decrease in conduit size, and tank or reservoir

d. **Sudden expansion losses (Sudden enlargement)**: occur when there is an abrupt increase in conduit size

e. **Gradual expansion losses**: occur when there is a gradual increase in conduit size

f. **Gradual contraction losses**: occur when there is a gradual decrease in conduit size

g. **Other minor losses**

- The energy loss in valves, fitting, and bends are also considered as minor losses
- Similar to other types of minor losses, it is also have its own K_L

K_L Summary

Minor Losses In Pipe

Table 1

TYPICAL LOST HEAD ITEMS

(Subscript 1 = Upstream and Subscript 2 = Downstream)

	Item	Average Lost Head
Entrance	1. From Tank to Pipe, flush connection (entrance loss)	$0.50\frac{V_2^2}{2g}$
	projecting connection	$1.00\frac{V_2^2}{2g}$
	rounded connection	$0.05\frac{V_2^2}{2g}$
Exit	2. From Pipe to Tank (exit loss)	$1.00\frac{V_1^2}{2g}$
	3. Sudden Enlargement	$\frac{(V_1 - V_2)^2}{2g}$
	4. Gradual Enlargement (see Table 2)	$K\frac{(V_1 - V_2)^2}{2g}$
	5. Venturi Meters, Nozzles, and Orifices	$\left(\frac{1}{c_v^2} - 1\right)\frac{V_2^2}{2g}$
	6. Sudden contraction (see Table 3)	$\frac{v_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$
	7. Elbows, Fittings, Valves*	$K\frac{V^2}{2g}$

Some typical values of K are:

- 45° bend . . . . 0.35 to 0.45
- 90° bend . . . . 0.50 to 0.75
- Tees . . . . 1.50 to 2.00
- Gate valves (open) . . . . about 0.25
- Check valves (open) . . . . . . about 3.0

Table 2

VALUES OF K

Contractions and Enlargements

Sudden Contraction		Gradual Enlargement for Total Angles of Cone						
d_1/d_2	K_c	4°	10°	15°	20°	30°	50°	60°
1.2	0.08	0.02	0.04	0.09	0.16	0.25	0.35	0.37
1.4	0.17	0.03	0.06	0.12	0.23	0.36	0.50	0.53
1.6	0.26	0.03	0.07	0.14	0.26	0.42	0.57	0.61
1.8	0.34	0.04	0.07	0.15	0.28	0.44	0.61	0.65
2.0	0.37	0.04	0.07	0.16	0.29	0.46	0.63	0.68
2.5	0.41	0.04	0.08	0.16	0.30	0.48	0.65	0.70
3.0	0.43	0.04	0.08	0.16	0.31	0.48	0.66	0.71
4.0	0.45	0.04	0.08	0.16	0.31	0.49	0.67	0.72
5.0	0.46	0.04	0.08	0.16	0.31	0.50	0.67	0.72

Table 3: Loss coefficients for sudden contraction.

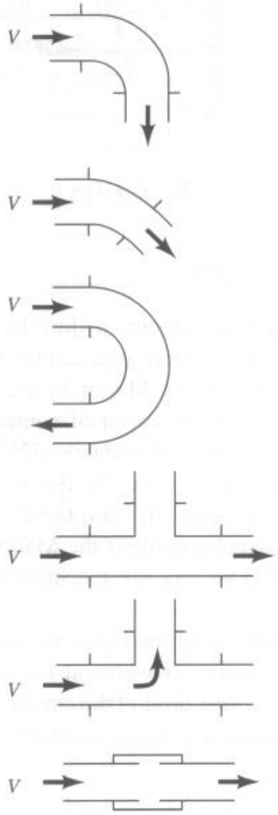
A_2/A_1	0.1	0.3	0.5	0.7	1.0
C_c	0.61	0.632	0.673	0.73	1.0
K	0.41	0.34	0.24	0.14	0

K_L Summary

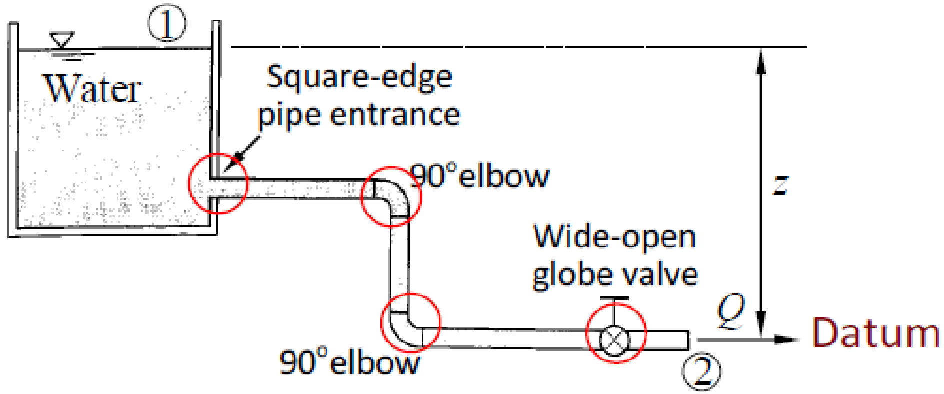
Minor Losses In Pipe

Loss Coefficients for Pipe Components $\left(h_L = K_L \frac{V^2}{2g}\right)$

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
***e. Valves**	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210



Solution Technique Involving $\mathbf{H_{L-minor}}$



$L =$ length of the pipe
$d =$ diameter of the pipe
$\varepsilon =$ absolute roughness of the pipe

Steps 1: Choose appropriate datum

Steps 2: Applying Bernoulli's equation at choosen datum

$$\frac{P_1}{\gamma}+\frac{v_1^2}{2g}+z_1+H_p=\frac{P_2}{\gamma}+\frac{v_2^2}{2g}+z_2+H_T+H_L$$

Steps 3: Simplify Bernoulli's equation

$$\cancelto{0}{\frac{P_1}{\gamma}}+\cancelto{0}{\frac{v_1^2}{2g}}+z_1+\cancelto{0}{H_p}=\cancelto{0}{\frac{P_2}{\gamma}}+\frac{v_2^2}{2g}+\cancelto{0}{z_2}+\cancelto{0}{H_T}+H_L$$

$$\therefore z_1=\frac{v_2^2}{2g}+H_L$$

Steps 4: Expands all losses

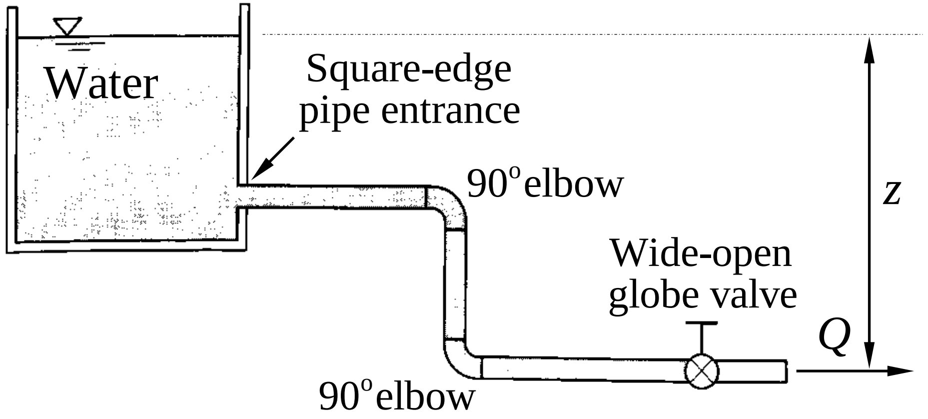
$v_1=v_2=v$ (due to no diameter changes)

$$\therefore H_L=f\frac{L}{d}\frac{v^2}{2g}+K_{L-entrance}\frac{v^2}{2g}+2K_{L-90^o}\frac{v^2}{2g}+K_{L-globevalve}\frac{v^2}{2g}$$

$$\therefore H_L=\frac{v^2}{2g}\left(f\frac{L}{d}+K_{L-ent.}\frac{v^2}{2g}+2K_{L-90^o}\frac{v^2}{2g}+K_{L-gv}\frac{v^2}{2g}\right)$$

Minor Losses In Pipe (Problem 4.3)

A water flows from a huge reservoir to 3 inches diameter and 50 ft long commercial steel pipe at 250 gpm, as shown in figure below. Compare the total minor losses to major losses. Take water kinematic viscosity as 1.21 X 10^{-5} ft^2/sec.

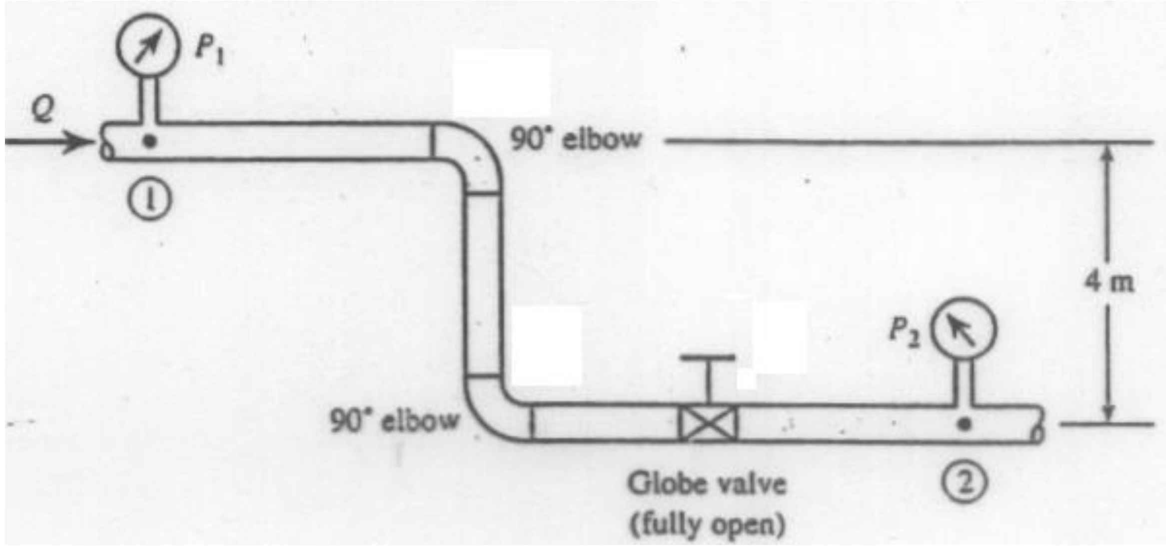


Minor Losses In Pipe (Problem 4.4)

Figure E3 shows a typical fluid (g = 9800 N/m^3 and n = 1.9 X 10^{-6} m^2/s) has been drawn from a piping system at the rate of 0.05 m^3/s. The pipe diameter is 100 mm and the height difference between point 1 and point 2 is 4 m. The total length of the cast iron pipe is 30 m from point 1 to point 2. Calculate:

a. The pressure difference between p_1 and p_2.
b. The pressure difference between p_1 and p_2 if the globe valve is replaced by gate valve which is fully open.

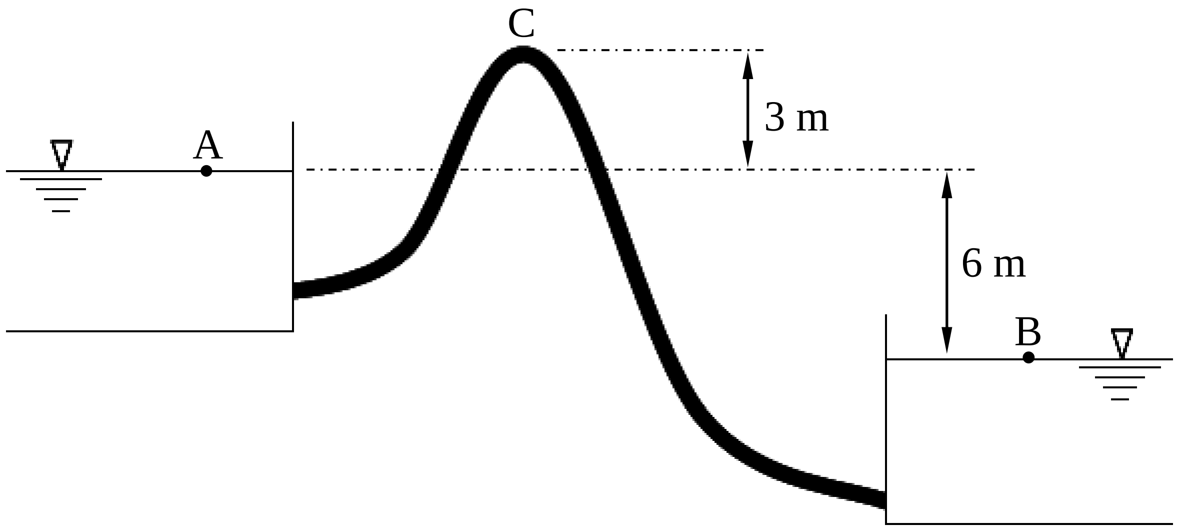
(*Notes: Use Darcy Weisbach's equation, $h_f = (flv^2)/(2dg)$.*)



Minor Losses In Pipe (Problem 4.5)

A pipeline connecting two reservoir having difference of level of 6 m is 720 m long, and rises to a height of 3 m above the upper reservoir at a distance of 240 m from the entrance before falling to the lower reservoir, as shown in figure below. If the pipe is 1.2 m in diameter and the frictional coefficient $f = 0.04$, what will be the discharge and the pressure at the highest point of the pipeline?

(Notes: Use Darcy Weisbach's equation, $h_f = (flv^2)/(2dg)$*).*



Equivalent Length for Pipe Fitting Loss Calculations

- A method which expressed head frictional loss to all devices (valve, fittings, bends, etc.) in terms of equivalent length of straight pipe, of same diameter
- This method is only applicable to pipeline system that in series
- Total frictional losses in pipeline system is defined as:

$$H_L = H_{L-\text{major}} + H_{L-\text{minor}}$$

$$\text{or, } H_{L_{total}} = \sum\left(f\frac{L}{d}\frac{v^2}{2g}\right) + \sum\left(K_L\frac{v^2}{2g}\right) = \sum H_{L-\text{major}} + \sum H_{L-\text{minor}}$$

- In equivalent methods,

$$H_{L-\text{major}} = H_{L-\text{minor}}$$

$$K_L\frac{v^2}{2g} = f\frac{L_e}{d}\frac{v^2}{2g}$$

$$\therefore L_e = K_L\frac{d}{f}$$

- Therefore, for a pipeline system, which is in series,

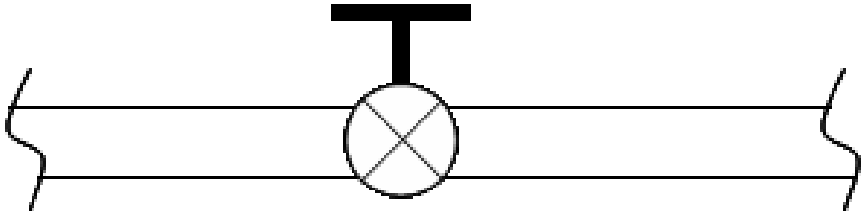
$$L_{e\,total} = \sum(L_{\text{major}}) + \sum(L_{\text{minor}}) = \sum(L_{pipeline}) + \sum(K_L\frac{d}{f})$$

$$\therefore H_{L_{total}} = f\frac{L_{e_{total}}}{d}\frac{v^2}{2g}$$

Equivalent Length for Pipe Fitting Loss Calculations (Problem 4.6)

A hydraulic oil with 0.9 specific gravity and 0.001 ft^2/sec kinematic viscosity is flowing through 1 inch in diameter and 100 ft long commercial steel pipe, as shown in figure below. If the flow rate is 50 gpm, calculate,

a. the equivalent length for fully open globe valve if the valve is put in the pipeline,
b. the total equivalent length for the pipeline and the valve, and
c. the total pressure drop across the pipeline.

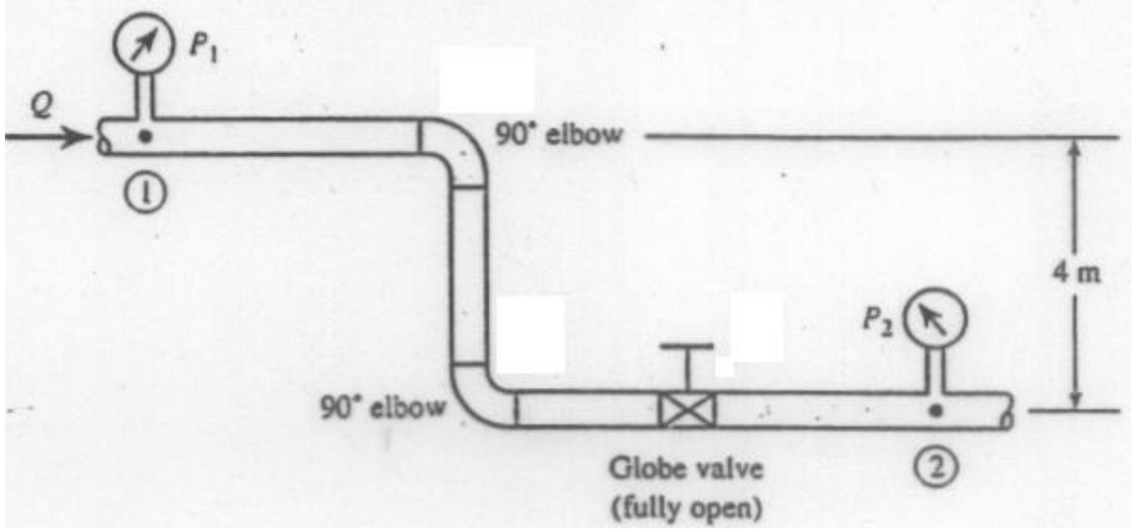


Equivalent Length for Pipe Fitting Loss Calculations (Problem 4.7)

Figure E3 shows a typical fluid (g = 9800 N/m^3 and n = 1.9 X 10^{-6} m^2/s) has been drawn from a piping system at the rate of 0.05 m^3/s. The pipe diameter is 100 mm and the height difference between point 1 and point 2 is 4 m. The total length of the cast iron pipe is 30 m from point 1 to point 2. By using equivalent length method, calculate:

a. The pressure difference between p_1 and p_2.
b. The pressure difference between p_1 and p_2 if the globe valve is replaced by gate valve which is fully open.

(*Notes: Use Darcy Weisbach's equation, $h_f = (flv^2)/(2dg)$.*)



Multiple Pipes In Different Configuration

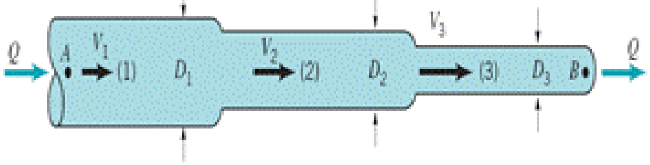
- Parallel
- Series
- Network

Multiple Pipes In Different Configuration (Series and Parallel)

Multiple-Path Systems
Series and Parallel Pipe System

Series

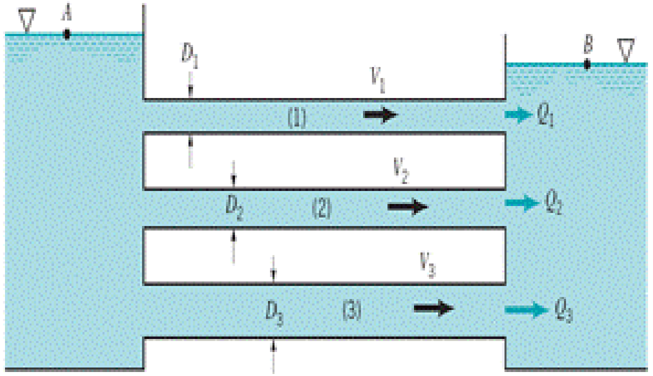
$$Q_1 = Q_2 = Q_3 \qquad h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3}$$



(a)

Parallel

$$Q = Q_1 + Q_2 + Q_3 \qquad h_{L_1} = h_{L_2} = h_{L_3}$$

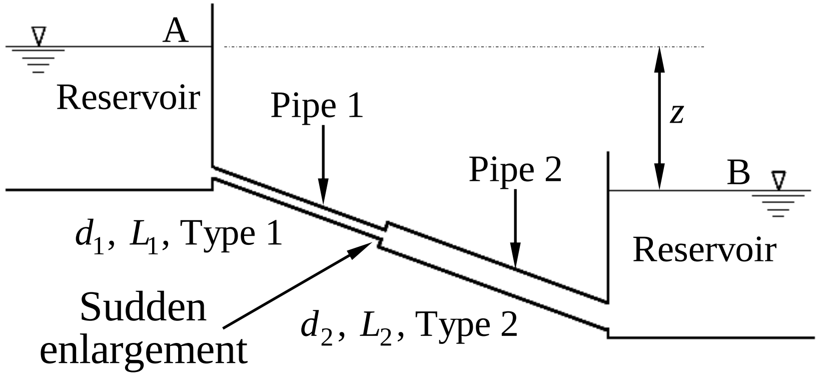


(b)

Pipe In Series

- ***Def.:*** Pipes of different diameters and/or different materials are connected end to end to form a pipeline, so that the fluid flows through each in turn
- The total loss of energy, or pressure loss, over the whole pipeline will be the sum of the losses for each pipe together with any separation losses as might occur at the junctions, entrance or exit

Solution technique for pipes in series



1. Apply Bernoulli's equation from A - B

$$\cancelto{0}{\frac{p_A}{\gamma}} + \cancelto{0}{\frac{v_A^2}{2g}} + z_A + \cancelto{0}{H_p} = \cancelto{0}{\frac{p_B}{\gamma}} + \cancelto{0}{\frac{v_B^2}{2g}} + \cancelto{0}{z_B} + \cancelto{0}{H_T} + H_L$$

2. Simplifying Bernoulli's equation becomes,

$$H_L = z_A$$

3. $$\begin{aligned} H_L &= H_{L-\text{major}} + H_{L-\text{minor}} \\ &= \left(H_{L-\text{major-1}} + H_{L-\text{major-2}}\right) \\ &\quad + \left(H_{L-\text{minor-ent.}} + H_{L-\text{minor-1\&2}} + H_{L-\text{minor-exit}}\right) \\ &= f_1 \frac{L_1}{d_1}\frac{v_1^2}{2g} + f_2 \frac{L_2}{d_2}\frac{v_2^2}{2g} \\ &\quad + K_{L-ent.}\frac{v_1^2}{2g} + \frac{(v_1 - v_2)^2}{2g} + K_{L-exit}\frac{v_2^2}{2g} \end{aligned} \quad ----(1)$$

4. Used continuity equation to solve for v_1 & v_2

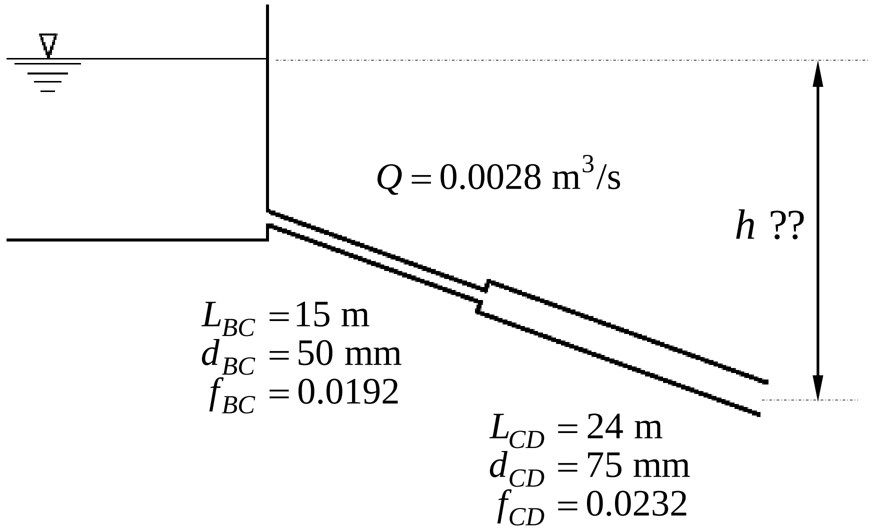
$$Q_1 = Q_2 \quad ----(2)$$

5. There are two equations, 1 & 2, in most cases simultaneous equation solution technique has to be applied.

Pipe In Series (Problem 4.8)

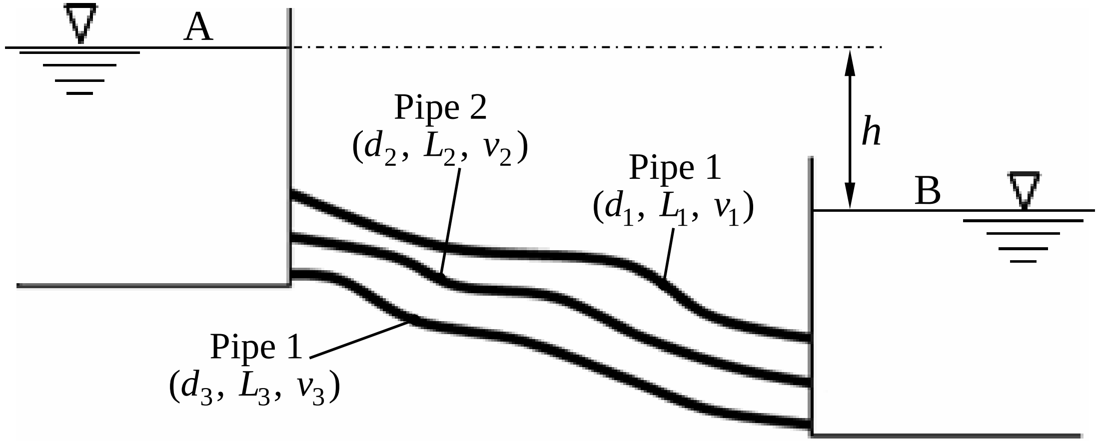
Water is discharged from a reservoir into the atmosphere through a pipe 39 m long. There is a sharp entrance to the pipe and the diameter is 50 mm for 15 m from the entrance. The pipe then enlarges suddenly to 75 mm in diameter for the remainder of its length. Taking into account the loss of head at entry and at the enlargement, calculate the difference of level between the surface of the reservoir and the pipe exit which will maintain a flow of 0.0028 m^3/s. Take f as 0.0192 for the 50 mm pipe and 0.0232 for the 75 mm pipe

(*Notes: Use Darcy Weisbach's equation, $h_f = (flv^2)/(2dg)$.*)



Pipe In Parallel

- When reservoirs are connected by two or more pipes in parallel, as shown in figure below, the fluid can flow from one to the other by a number of alternative routes
- The difference of head *h* available to produce flow will be the same for each pipe
- Each pipe can be considered separately, entirely independently of any other pipes running in parallel
- Bernoulli's equation can be applied for flow by each route and the total *Q* will be the sum of *Q* in each pipe (continuity equation)

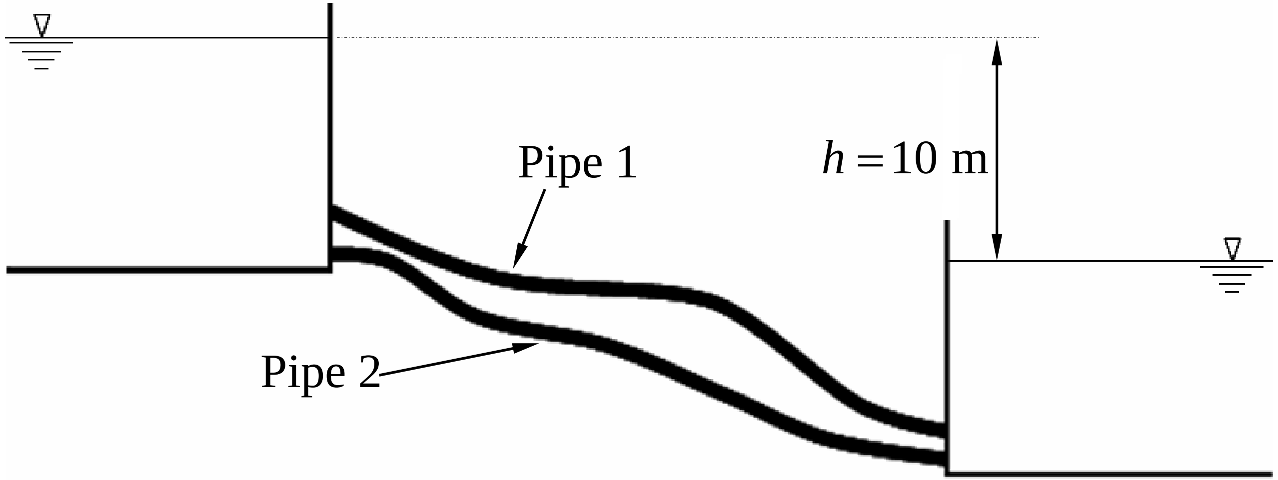


Pipe In Parallel (Problem 4.9)

Two sharp-ended pipes of diameter d_1 = 50 mm, and d_2 = 100 mm, each of length 100 m, are connected in parallel between two reservoirs which have a difference of level h = 10 m, as shown in figure below. If the friction coefficient, f = 0.032 for each pipe, calculate:

a. the rate of flow for each pipe
b. the diameter d of a single pipe 100 m long which would give the same flow if it was substituted for the original two pipes

(Notes: Use Darcy Weisbach's equation, $h_f = (flv^2)/(2dg)$)

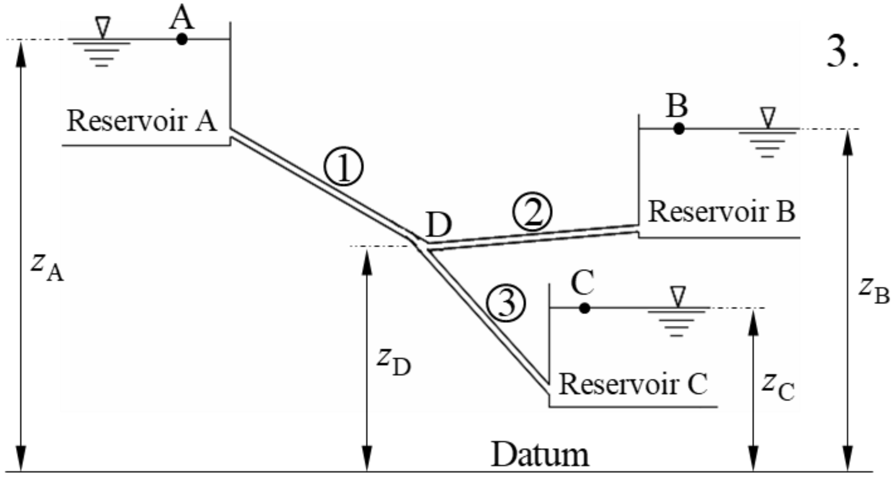


Flow Through Branching Pipes

- Combination of series & parallel pipe problem plus branching flow problem from single pipe to two or more pipes
- In the case of flow through branching pipes, there is no general solution. The problem has to be dealt with using basic principle of fluid flow
  a. Bernoulli's equation (or Energy equation)
  b. Continuity equation
- Solution, normally too complex
- In practice: used appropriate computer software
- For manual solution needs to make few assumptions and eliminating minor losses
- Solution using Al-Jabar method is too cumbersome (sometime, nearly impossible). Try & error + iteration + interpolation techniques are normally used

Flow Through Branching Pipes

The Three-Reservoir Problem



Assumptions

1. Fluid flow pathway

Fluid from Reservoir A $\xrightarrow{\text{Pipe 1 \& Pipe 3}}$ Reservoir C

Fluid from Reservoir B $\xrightarrow{\text{Pipe 2 \& Pipe 3}}$ Reservoir C

Continuity equation at D: $Q_1 + Q_2 = Q_3$

2. Neglect minor losses

3. Used Bernoulli's equation separately
   a. Bernoulli's equation A - C
   b. Bernoulli's equation B - C

4. Check again using continuity equation at D

$$Q_1 + Q_2 = Q_3 \quad ???$$

5. If in procedure 4, the continuity equation does not tally, make a new assumption of fluid flow pathway, and repeat the process of 2 - 4.

e.g.

Fluid from Reservoir A $\xrightarrow{\text{Pipe 1 \& Pipe 2}}$ Reservoir B

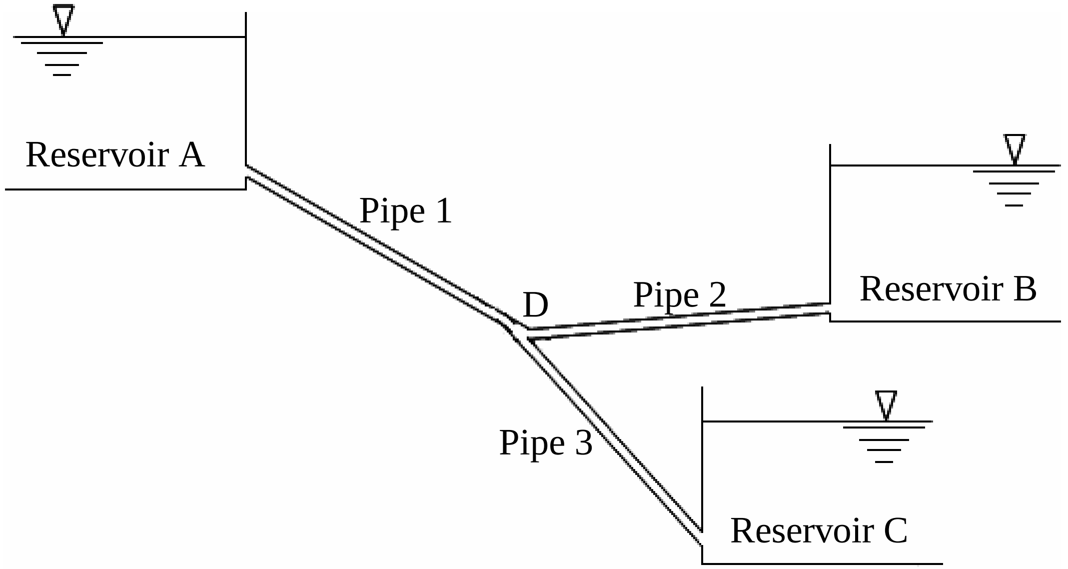
Fluid from Reservoir A $\xrightarrow{\text{Pipe 1 \& Pipe 3}}$ Reservoir C

Continuity equation at D: $Q_1 = Q_2 + Q_3$

Flow Through Branching Pipes (Problem 4.10)

Water flows from a Reservoir A (figure below) through a pipe of diameter d_1 = 120 mm and length L_1 = 120 m to a junction at D, from which a pipe of diameter d_2 = 75 mm and length L_2 = 60 m leads to Reservoir B in which the water level is 16 m below that in Reservoir A. A third pipe, of diameter d_3 = 60 mm and length L_3 = 40 m, leads from D to Reservoir C, in which the water level is 24 m below that in Reservoir A. Taking friction coefficient, f = 0.04 for all the pipes and neglecting all losses other than those due to friction, determine the volume rates of flow in each pipe

(*Notes: Use Darcy Weisbach's equation,* $h_f = (flv^2)/(2dg)$)



THANK YOU

Stay safe!