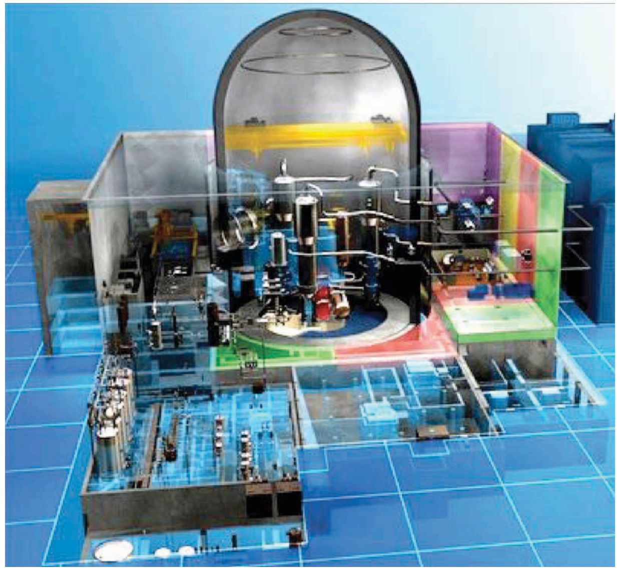
Neutron Flux Distribution in Cores



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Objectives of topic

Neutron conservation, Fission chain reaction, the diffusion of neutrons in media

Neutron flux distributions in cores

The Role of Energy Transport in Reactor Design

- **The heat generation at any point in a reactor fuel is primarily a function of the fission-reaction rate at that point.**

- **The neutrons are born whenever fuel is present in a core, and they travel or diffuse in random fashion and are slowed down at different rates.**

The Role of Energy Transport in Reactor Design

- **The procedure of determining the neutron flux distribution results in expressions for the critical dimensions and corresponding critical fuel mass of the core.**

- **These are the minimum dimensions and mass necessary for maintaining a chain reaction.**

Neutron Conservation

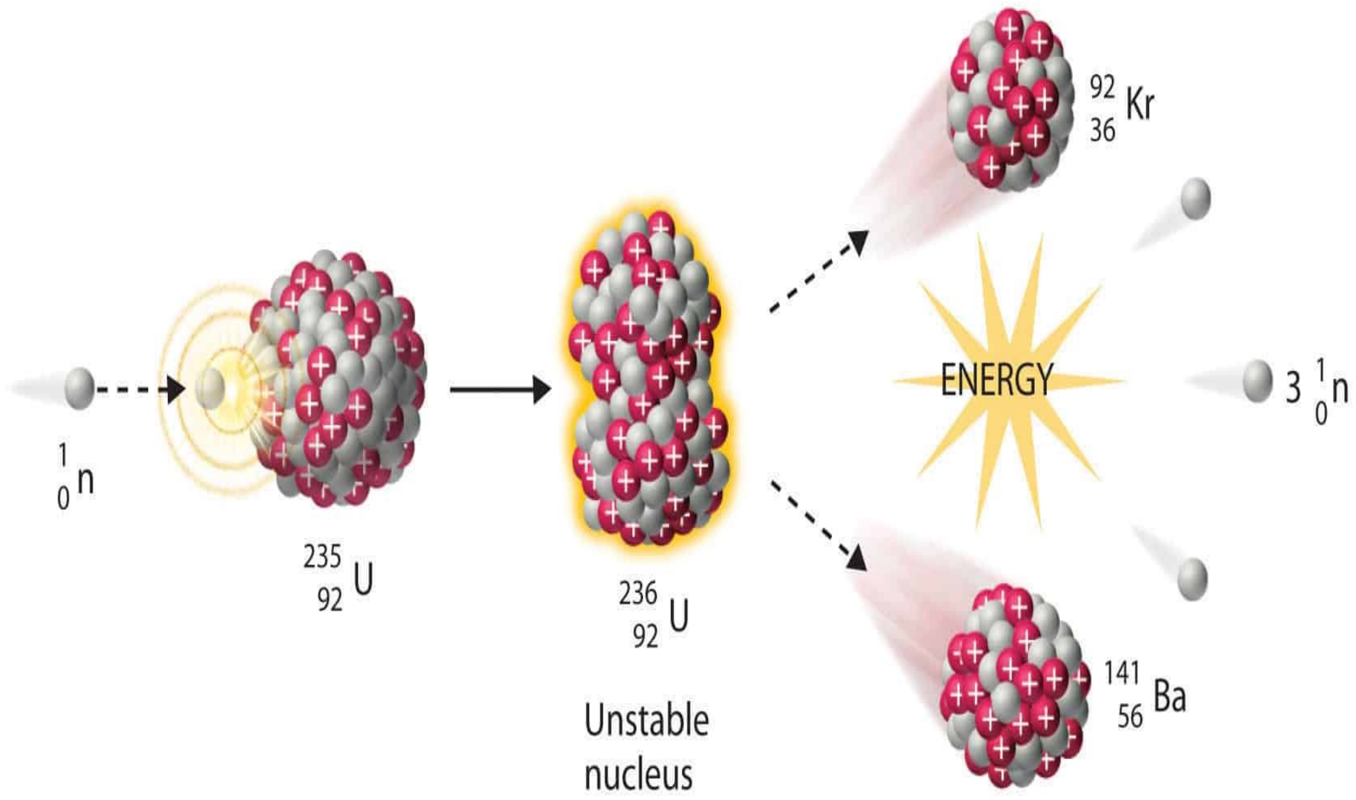
- U^{235} has to split into two (sometimes three and rarely four) nuclei.
- 40 % neutrons for bombarding other U^{235} nuclei, in a critical reactor capable of producing energy at a steady rate.
- 60 % fission neutrons are lost, either by escape to the outside of the fuel or by absorption in various materials not causing fission

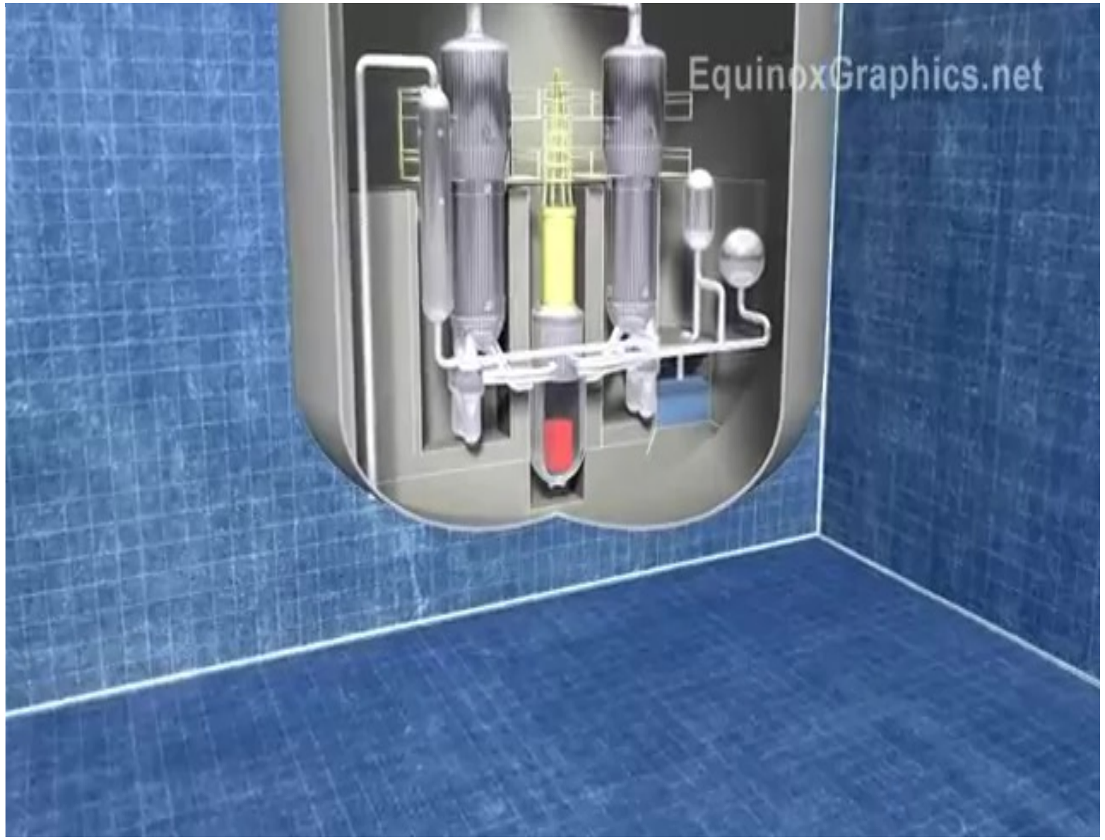
Neutron Conservation

- **The understanding of neutron conservation is, assumed all born at the same time, undergo scattering, leakage, absorption, and other reactions, attain the same energy levels, and finally cause fission simultaneously.**

- **The series of events that it undergoes from birth until a new generation is born by fission is called a life cycle**

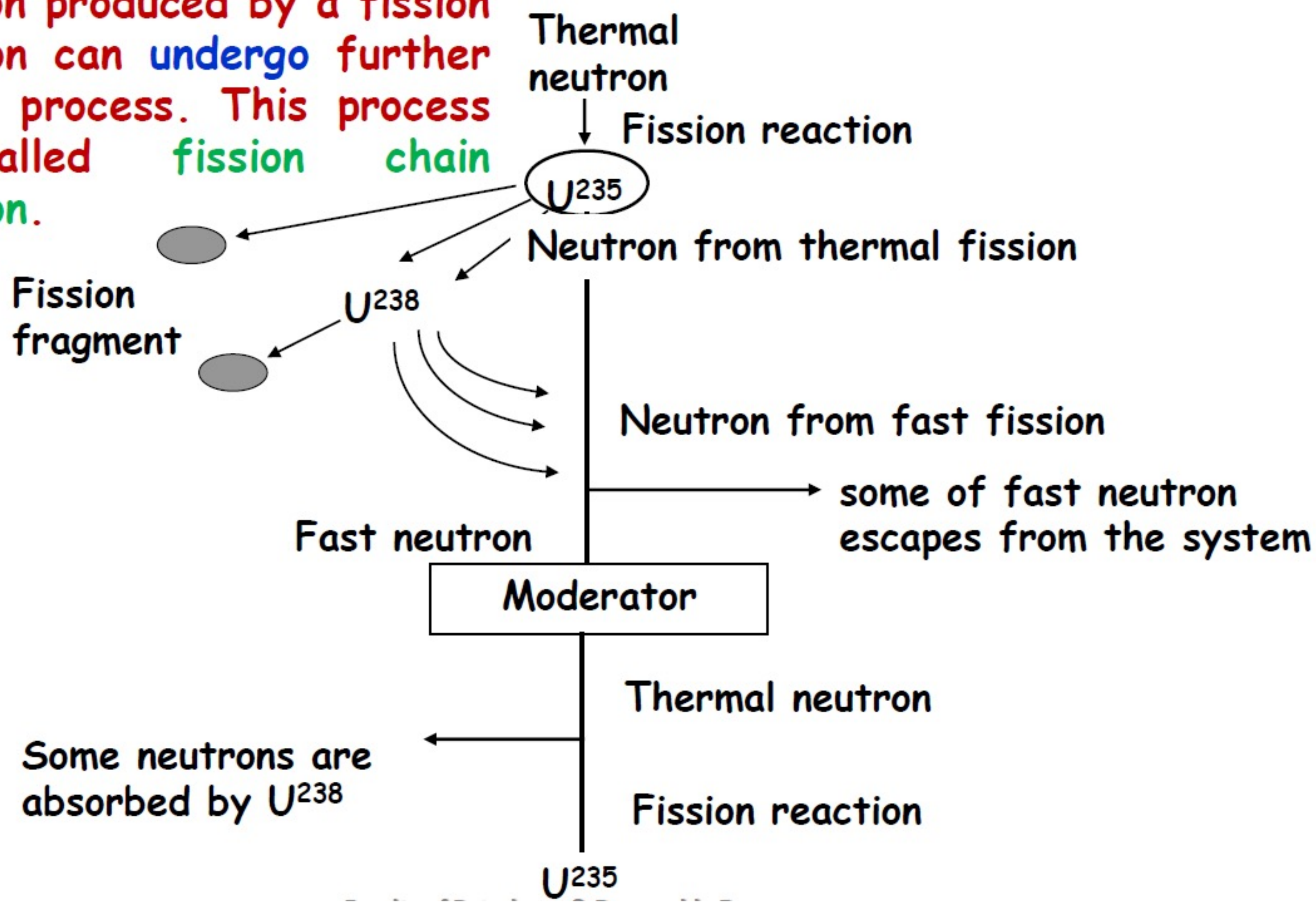
Nuclear Fission



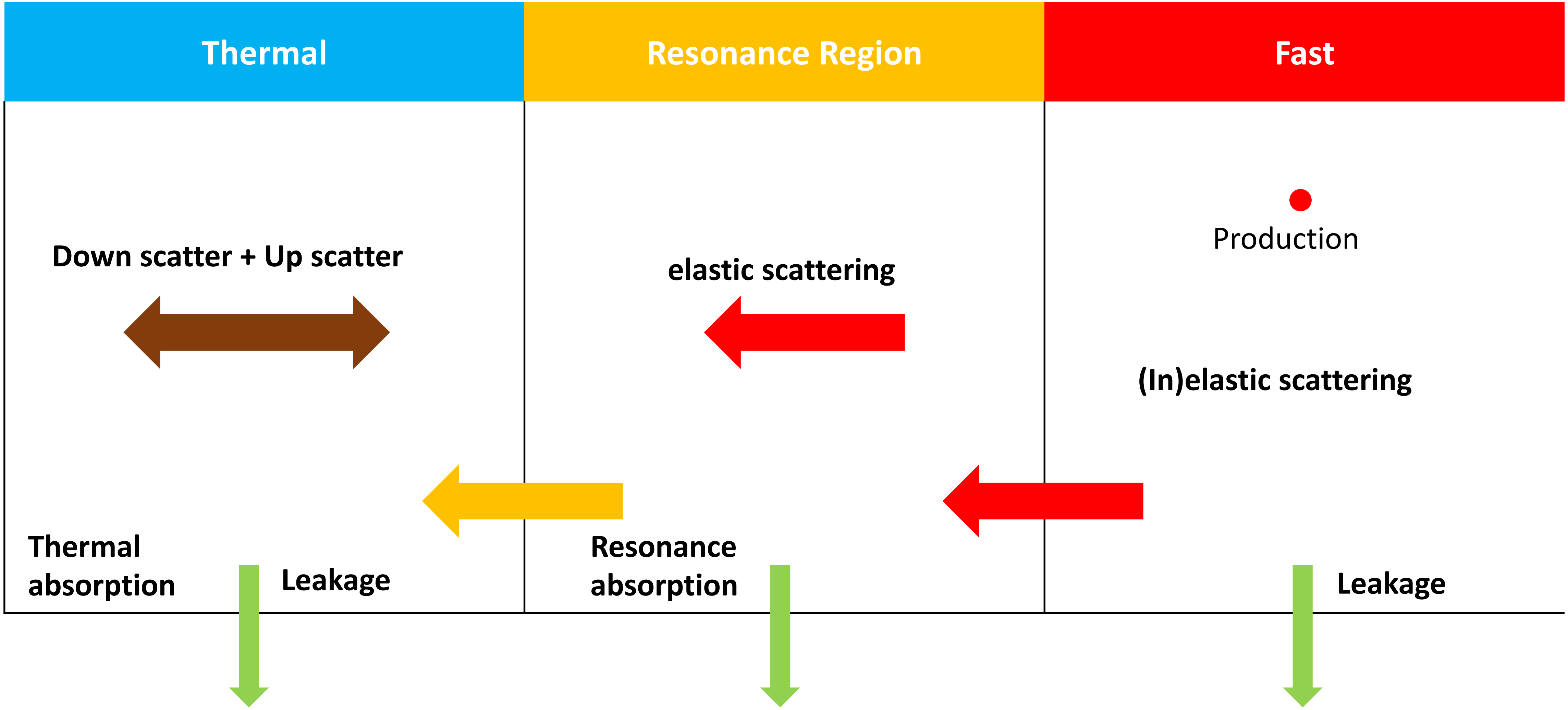


Fission Chain Reaction

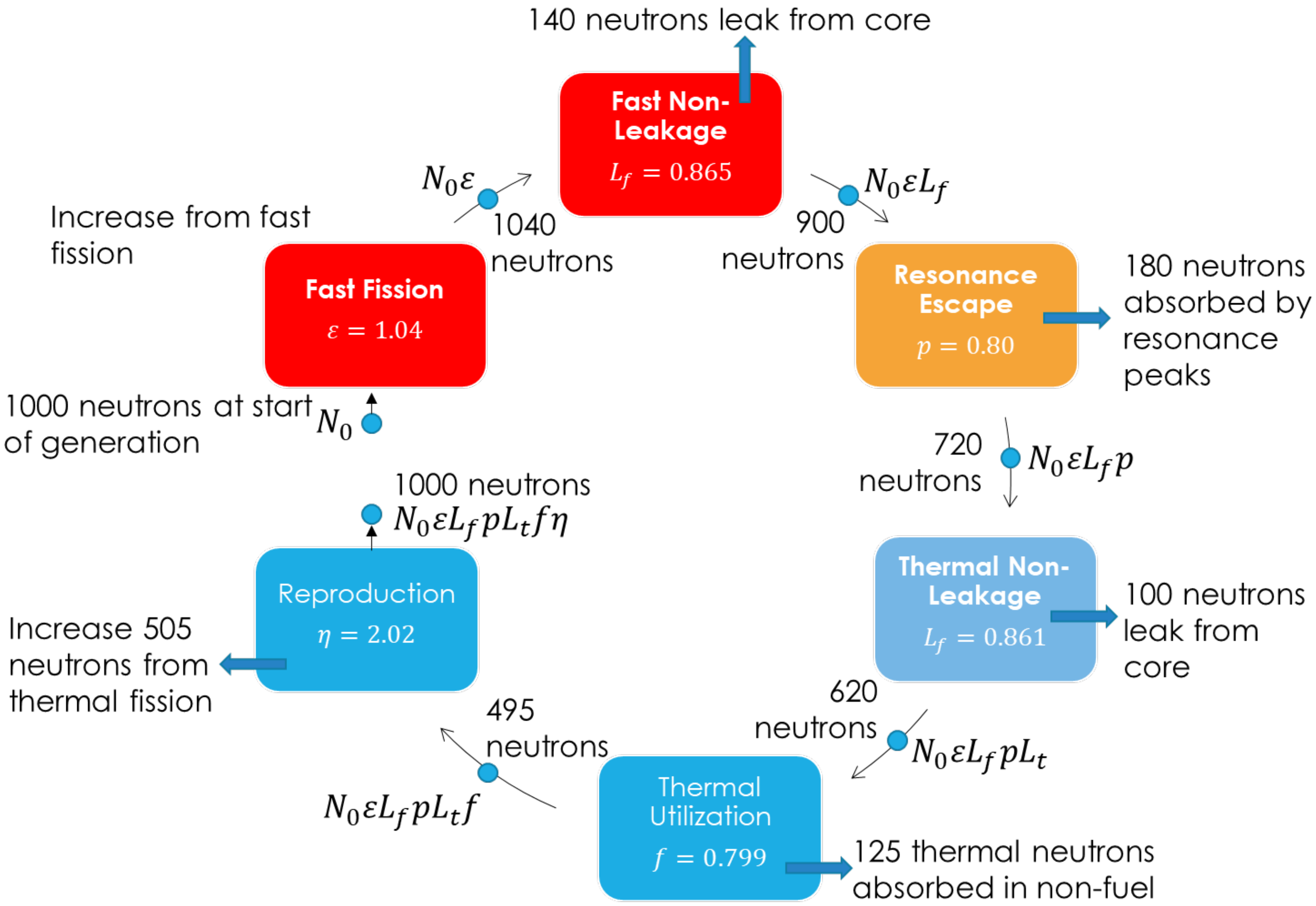
Neutron produced by a fission reaction can undergo further fission process. This process is called fission chain reaction.



Neutron Life Cycle in a Nuclear Reactor Core



Neutron Life Cycle in a Nuclear Reactor Core



Fast Fission Factor, ε

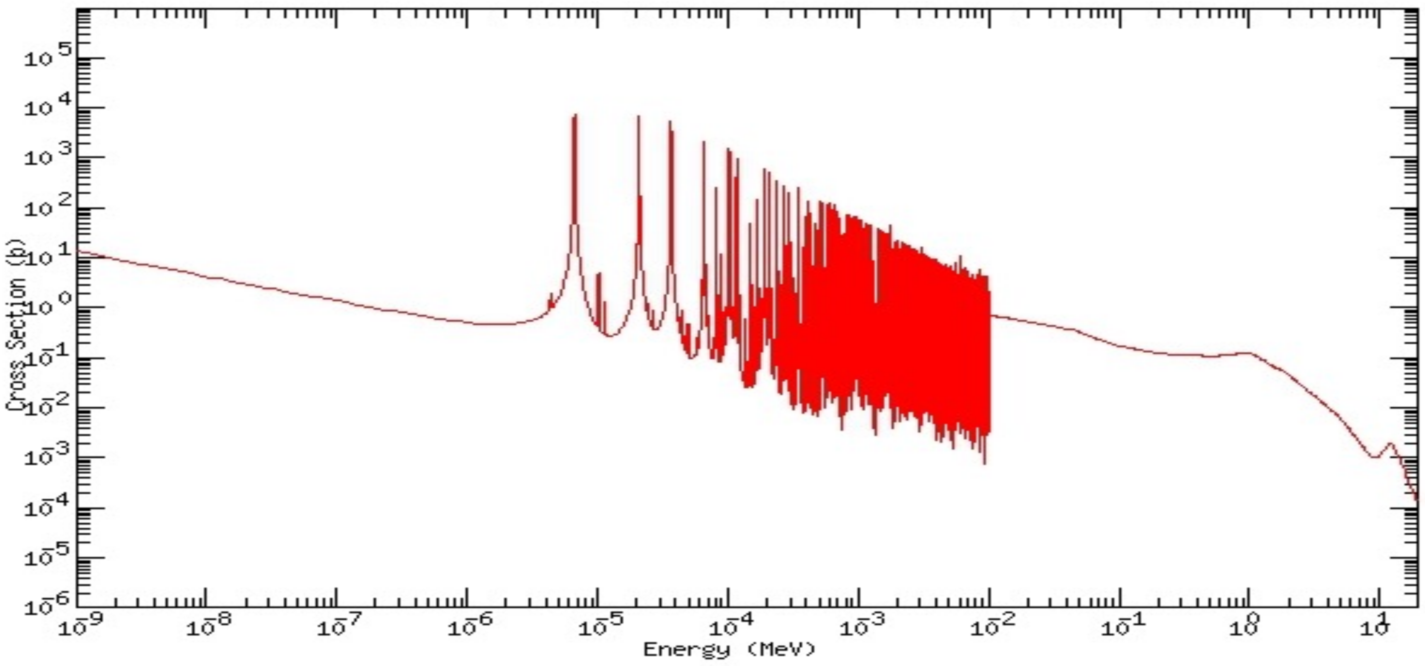
$$\varepsilon = \frac{\text{Total no. of fission neutrons}}{\text{No. of neutrons from thermal fission}}$$

$$\varepsilon = \exp\left(-\frac{N(U)\, I}{\xi \Sigma_s(m)}\right)$$

Resonance Escape Probability, p

$$p = \frac{\text{No. of neutrons leaving resonance energy range}}{\text{No. of neutrons entering resonance energy range}}$$

$$p = \frac{1 + 0.690\left(\frac{N_{238}}{N_w}\right)}{1 + 0.563\left(\frac{N_{238}}{N_w}\right)}$$



Uranium-238 Absorption Cross Section

Thermal Utilization Factor, f

$$f = \frac{\text{Rate of absorption of thermal neutrons bt the fuel}}{\text{Rate of absorption of thermal neutrons by all reactor materials}}$$

$$f = \frac{\Sigma_a(f)\, v(f)}{\Sigma_a(f)\, v(f) + \Sigma_a(M)\, v(M) + \Sigma_a(O)\, v(O)}$$

Regeneration Factor, η

Regeneration factor is also known as **thermal fission** and **reproduction factor**

$$\eta = \frac{\text{Total no.of fission neutrons (neutrons produced)}}{\text{No.of neutrons absorption}} = \upsilon \frac{\Sigma_f\,(f)}{\Sigma_a\,(f)}$$

$$\eta = \frac{\upsilon\; v(U^{235})\,\Sigma_f(U^{235})}{v(U^{235})\,\Sigma_a(U^{235}) + v(U^{238})\,\Sigma_a(U^{238})}$$

υ – Number of neutrons produce per neutron engaged in fission

Non-Leakage Probability

L_f = Fraction of **fast neutron** not leaking from reactor

L_t = Fraction of **thermal neutron** not leaking from reactor

Multiplication Factor

Six-factor formula:

$$k_{eff} = \varepsilon p f \eta L_f L_t$$

Finite reactor core

$$k_{eff} < 0 \; subcritical$$
$$k_{eff} = 0 \; critical$$
$$k_{eff} > 0 \; supercritical$$

Four-factor formula:

$$k_{\infty} = \varepsilon p f \eta$$

Infinite reactor core

The factors ε, p and f depend on both fuel and core configuration and materials, are called lattice constants.

Multiplication Factor

Reactivity determines the time dependency of the neutron flux and power

$$\rho = \frac{k_{eff} - 1}{k_{eff}}$$

$$\rho > 0 \; neutron\ flux\ increase$$
$$\rho < 0 \; neutron\ flux\ decrease$$

Only reactivity is zero, the reactor is critical and the neutron flux constant

Exercise 1

Calculate the thermal-fission factor for 2,200-m/ sec neutrons for (a) natural uranium, (b) 2 percent, (c) 20 percent, and (d) fully enriched uranium (e) fully 238-U

where $\sigma_a(U^{235})$ = 687 barn, $\sigma_f(U^{235})$ = 587 barn, $\sigma_a(U^{238})$ = 2.73 barn.

$$\eta(\mathrm{U}) = \frac{\upsilon\ \nu(\mathrm{U}^{235})\Sigma_f(\mathrm{U}^{235})}{\nu(\mathrm{U}^{235})\Sigma_a(\mathrm{U}^{235}) + \nu(\mathrm{U}^{238})\Sigma_a(\mathrm{U}^{238})}$$

$$= \frac{\upsilon}{\dfrac{\sigma_a(\mathrm{U}^{235})}{\sigma_f(\mathrm{U}^{235})} + \dfrac{\nu(\mathrm{U}^{238})}{\nu(\mathrm{U}^{235})}\dfrac{\sigma_a(\mathrm{U}^{238})}{\sigma_f(\mathrm{U}^{235})}}$$

Exercise 2

Calculate the infinite multiplication factor for homogeneous mixture of Fe-55 and Pu-239 is to be the shape of a bare sphere of this fast reactor.

material	σ_f (barns)	σ_a (barns)	ν	η	σ_{tr} (barns)
Fe-55	—	0.006	—	—	2.7
Pu-239	1.85	2.11	3.0	2.6	6.8

The Diffusion of Neutrons in Media

- The transport theory, sometimes called the Boltzmann transport theory because of its similarity to Boltzmann's theory of diffusion of gases.

- Transport theory reduces to the neutron diffusion theory which holds for most of a reactor core.

- Fick's law used into the neutron diffusion equation.

- Neutron diffusion equation expressions for the desired neutron flux distribution in the core but also for the *critical* core dimensions

Reactor Equation

In solids, a *neutron conservation equation* of the volume in the core is given by

$$\frac{\partial n}{\partial t} = -(\text{leakage rate}) - (\text{elimination rate}) + (\text{production rate})$$

where n is the neutron density, $\mathbf{neutrons/cm^3}$
and t is time, sec. Mathematically.

$$\frac{\partial n}{\partial t} = -\nabla \cdot J - \Sigma_{elim}\phi + S$$

Reactor Equation

And in the steady state

$$-\nabla \cdot J - \Sigma_{elim}\phi + S = 0$$

J = neutron current, neutrons/sec · cm^2
Σ_{elim} = elimination macroscopic cross section
ϕ = neutron flux
S = neutron source, neutrons/sec · cm^3

The diffusion term $\nabla \cdot J$ for example, is given in terms of a *diffusion coefficient*, D, in the form given by *Fick's law* of diffusion gases as

$$J = -D\nabla.\phi$$

Reactor Equation

So that leakage rate becomes $-\nabla.(D\nabla.\phi)$, and

$$\nabla^2\phi - \frac{\Sigma_{elim}}{D}\phi + \frac{S}{D} = 0$$

Equation above is called the diffusion equation.
D **is evaluated in terms of a slowing-down length (L,), or a thermal diffusion length (L),**

For all energy groups, Eq above

$$\nabla^2\varphi + B^2\varphi = 0$$

This equation called the reactor equation

B^2 **is called the material buckling, and sometimes written as** B^2**m.**

Reactor Equation

- **It is a number that depends on the properties of the material of a particular reactor core.**
- **It has the dimensions (length)^-2**
- **The name buckling came the similarity of equation to the-well known equation of a loaded column encountered in strength of materials studies.**

Neutron-flux Distribution in Reactor Cores

- **Reactor equation expresses the flux distribution as a function of the space coordinates as independent variables.**

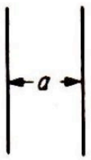
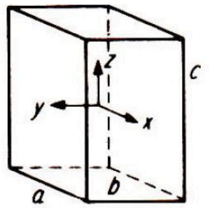
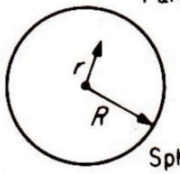
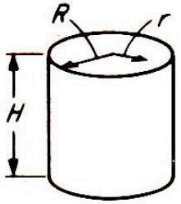
- **It is easily solved for various geometries, for homogeneous bare (unreflected) reactor cores.**

- **The extended dimensions of a core are called the extrapolated dimensions.**

- **For example, the actual radius and height of cylindrical core R and H, plus the extrapolation lengths are called the extrapolated radius R_e and the extrapolated height H_e,.**

$$R_e = R + \lambda_e \qquad\qquad H_e = H + \lambda_e$$

Neutron-flux Distribution in Reactor Cores

Geometry of Reactor	Buckling, B^2	Minimum critical Volume	Flux Distribution
 Infinite slab	$\left(\frac{\pi}{a_e}\right)^2$	∞	$\varphi_{co} \cos \frac{\pi x}{a_e}$
 Parallelepiped	$\left(\frac{\pi}{a_e}\right)^2 + \left(\frac{\pi}{b_e}\right)^2 + \left(\frac{\pi}{c_e}\right)^2$	$\frac{161}{B^3}$	$\varphi_{co} \cos \frac{\pi x}{a_e} \cos \frac{\pi y}{b_e} \cos \frac{\pi z}{c_e}$
 Sphere	$\left(\frac{\pi}{R_e}\right)^2$	$\frac{130}{B^3}$	$\frac{\varphi_{co}}{\pi r / R_e} \sin \frac{\pi r}{R_e}$
 Finite cylinder	$\left(\frac{\pi}{H_e}\right)^2 + \left(\frac{2.405}{R_e}\right)^2$	$\frac{148}{B^3}$	$\varphi_{co} \cos \frac{\pi z}{H_e} J_0 \left(\frac{2.405 r}{R_e}\right)$

Buckling, Minimum Critical Volume, and Flux Distribution in some core shapes

Neutron-flux Distribution in Reactor Cores

Geometry	Dimensions	Buckling (B^2)	Flux	A	ϕ_{max}/ϕ_{av}
Infinite slab	*Thickness a*	$\left(\frac{\pi}{a}\right)^2$	$A \cdot \cos\left(\frac{\pi x}{a}\right)$	$\frac{1.57 \cdot P}{a \cdot E_R \cdot \Sigma_f}$	1.57
Rectangular parallelepiped	$a \cdot b \cdot c$	$\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$	$A \cdot \cos\left(\frac{\pi x}{a}\right) \cdot \cos\left(\frac{\pi y}{b}\right) \cdot \cos\left(\frac{\pi z}{c}\right)$	$\frac{3.87 \cdot P}{V \cdot E_R \cdot \Sigma_f}$	3.88
Infinite cylinder	*Radius R*	$\left(\frac{2.405}{R}\right)^2$	$A \cdot J_0 \cdot \left(\frac{2.405 r}{R}\right)$	$\frac{0.738 \cdot P}{R^2 \cdot E_R \cdot \Sigma_f}$	2.32
Finite cylinder	*Radius R* *Height H*	$\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$	$A \cdot J_0 \cdot \left(\frac{2.405 r}{R}\right) \cdot \cos\left(\frac{\pi z}{H}\right)$	$\frac{3.63 \cdot P}{V \cdot E_R \cdot \Sigma_f}$	3.64
Sphere	*Radius R*	$\left(\frac{\pi}{R}\right)^2$	$A \cdot \frac{1}{r} \cdot \sin\left(\frac{\pi r}{R}\right)$	$\frac{P}{4 \cdot R^2 \cdot E_R \cdot \Sigma_f}$	3.29

Buckling, Minimum Critical Volume, and Flux Distribution in some core shapes

Exercise 3

A thermal heterogenous reactor has a cylindrical bare core whose height equals its diameter. It uses 1.3 percent enriched uranium metals as fuel and light water as moderator. The fuel and lattice constants are follows:

Fast fission factor, $\boldsymbol{\varepsilon = 1.0558}$

Resonance escape probability, $\boldsymbol{P = 0.830}$

Thermal utilization factor, $\boldsymbol{f = 0.870}$

Regeneration factor, $\boldsymbol{\eta = 1.40}$

Transport mean free path, $\boldsymbol{\lambda = 0.45\ cm}$

$\boldsymbol{\Sigma_a = 0.0197\ cm^{-1}}$

$\boldsymbol{D_{H_2O} = 0.16\ cm^{-1}}$

Calculate the minimum critical dimension of the core.

Exercise 4

A large research reactor consists of a cubical array of natural uranium rods in a graphite moderator. The research reactor is 25 ft on a side and operates at a power of 20 MW. The average value of Σ_f is $2.5\times 10^{-3}\ \mathrm{cm}^{-1}$.

a) **Calculate the buckling.**
b) **What is the maximum value of the thermal flux?**
c) **What is the average value of the thermal flux?**

Thank You

Stay safe!