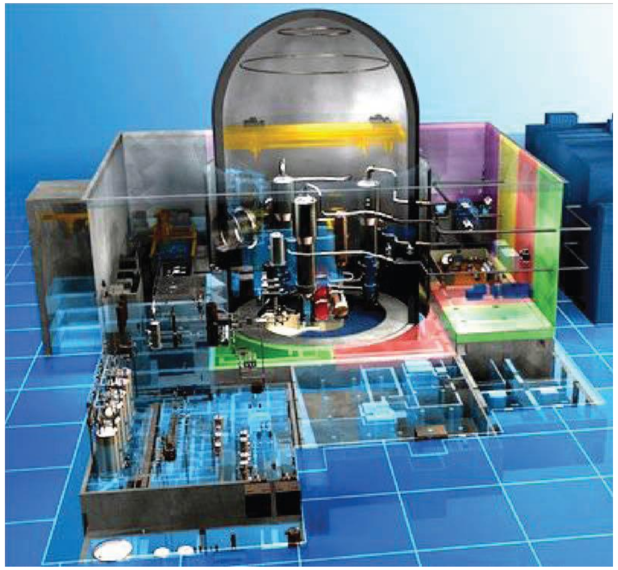
Reactor Heat Generation



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Objectives of topic

- Understand the fundamental heat production in nuclear reactors
- Calculate the heat generated by a single fuel and the total heat generated in nuclear reactor core
- Understand and calculate some parameters for heat generation from reactor shutdown and radioisotopes

Heat Production in Nuclear Reactors

- **Heat is produced in nuclear power plants by the process of *nuclear fission*.**
- **Most of this heat is deposited in the fuel, but a few percent of it is deposited in the coolant and the cladding.**
- **The fission neutrons are called prompt neutrons.**

Heat Production in Nuclear Reactors

- **The kinetic energy carried away by these neutrons is eventually converted into heat and this increases the temperature of the core.**

- **Coolant is then pumped through the core to remove this heat, and the heated coolant is eventually sent to the *nuclear steam supply system (NSSS)* to generate high-pressure steam.**

Measuring Nuclear Energy

- **Nuclear energy is measured in electron volts (eV), and most of the time, it is expressed in MeV.**

- **An electron volt is the amount of energy required by an electron falling through a potential difference of 1 V**

The number of Joules in an Electron Volt

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J or MeV} = 1.6 \times 10^{-13} \text{ J}$$

Measuring Nuclear Energy

- **When uranium or plutonium nucleus absorbs a neutron, if fission occur, approximately 200 MeV of *recoverable* kinetic energy is produced.**

- **Most of this energy is carried away by the fission products and by any alpha, beta, or gamma rays that are produced**

- **Fission neutrons carry away between 1% to 2% of the kinetic energy released**

Measuring Nuclear Energy

- **A single fission event creates about**

$$2 \times 10^{8} \times 1.6 \times 10^{-19} = 3.2 \times 10^{-11}\text{J}$$

- **Hence, about *30 billion fissions per second* are required to create 1 W of thermal power.**

- **If a NPP is 33% efficient, approximately *100 billion-billion fissions per second* are required to produce 3,000 MW of thermal power and 1,000 MW of electrical energy.**

Converting the Kinetic Energy of Nuclear Particles into Thermal Energy

- **Once an atomic nucleus *splits apart*, the particles that are produced move away from the nucleus at very high speeds**

- **Moreover, when the nucleus fissions, the *mass* of the outgoing particles is always less than the *mass* of the nucleus itself.**

- **The mass difference Δm between the mass of the original nucleus and the mass of the by-products is directly proportional to the kinetic energy of the by-products**

$$KE = \Delta m c^2$$

where c is the speed of light. Equation above is simply an alternative form of Einstein's equation of special relativity.

Converting the Kinetic Energy of Nuclear Particles into Thermal Energy

The kinetic energy of the particles produced by the fission process can then be written as

$$KE = \sum_{n=1}^{N} \left(\frac{1}{2} m_n v_n^2 \right) = \Delta m c^2$$

where N is the number of particles that are produced and v_n is their velocity.

- **This kinetic energy is equivalent to the thermal energy ΔE that is released when mass is converted into energy.**

Converting the Kinetic Energy of Nuclear Particles into Thermal Energy

and this is the energy that eventually shows up in the core in the form of heat, Q

$$Q = KE = \sum_{n=1}^{N} \left(\frac{1}{2} m_n v_n^2\right) = \Delta mc^2$$

When particles such as photons are also produced, an additional term must be added to Equation above to account for the presence of these particles. This can be done by extending the summation to read

$$Q = KE = \sum_{n=1}^{N} \left(\frac{1}{2} m_n v_n^2\right) + \sum_{m=1}^{M} hf_m = \Delta mc^2$$

where M is the number of photons, h is Planck's constant (a fundamental constant of nature), and f is their frequency of vibration (in cycles per second).

Planck's constant, h: $6.6 \times 10^{-34}\ kg \cdot m^2/s$ or $6.6260695 \times 10^{-34}\ J \cdot s = 4.1356675 \times 10^{-15}\ eV \cdot s$

Thus, a photon such as a 1 MeV gamma ray has a vibrational frequency of $f = E/h = 1 \times 10^6\ eV/4.1356675 \times 10^{-15}\ eV \cdot s \cong 2.42 \times 10^{20}\ Hz.$

Estimating the Total Energy Release

- In total, there are about 60 ways that a uranium or plutonium nucleus can split apart.
- Each way results in a slightly different amount of thermal energy being produced.
- Each fission event also produces different fission products with different atomic weights and different numbers of neutrons.

Estimating the Total Energy Release

For U-235 and Pu-239 nuclei, some of the most common ways in which a fission can occur are as follows:

Examples of the Nuclear Fission Process for Uranium-235 and Plutonium-239

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{140}_{55}Cs + {}^{93}_{37}Rb + 3\,{}^{1}_{0}n + Q'$$

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{139}_{56}Ba + {}^{94}_{36}Kr + 3\,{}^{1}_{0}n + Q''$$

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{142}_{54}Xe + {}^{92}_{38}Sr + 2\,{}^{1}_{0}n + Q'''$$

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n + Q''''$$

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{142}_{56}Ba + {}^{92}_{36}Kr + 2\,{}^{1}_{0}n + Q'''''$$

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{90}_{36}Kr + 2\,{}^{1}_{0}n + Q''''''$$

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{134}_{54}Xe + {}^{100}_{38}Sr + 2\,{}^{1}_{0}n + Q'''''''$$

$${}^{1}_{0}n + {}^{239}_{94}Pu \rightarrow {}^{240}_{94}Pu \rightarrow {}^{148}Ce_{58} + {}^{89}Kr_{36} + 3\,{}^{1}_{0}n + Q''''''''$$

where the variable Q represents the amount of kinetic energy that is released.

- **The value of Q varies between 190 and 220 MeV**

Estimating the Total Energy Release

- About 90% of the energy released by a nuclear fission (170-180 MeV) is released immediately in a fuel rod in the form of heat and light, and about 3% of the remaining 10% (about 6 out of 20 MeV) appears within the next 100s.
- The thermal energy that is released between 1 and 100 s is due to the decay of the *radioactive fission products* that are produced.
- The majority of this heat is deposited in the core by energetic beta particles and gamma rays.
- This thermal energy is sometimes called *decay heat* because it can be produced long after a reactor is shut down for maintenance or refueling

Estimating the Total Energy Release

TABLE 5.1
The Energy Released from the Thermal Fission of Uranium-235

Energy Released from the Fission of U-235	Average Energy Released (MeV)
Instantaneously Released Energy	
Kinetic energy of fission fragments	169.10
Kinetic energy of prompt neutrons	4.8
Energy carried by prompt γ-rays	7.0
Energy from Decaying Fission Products at a Later Time	
Energy of β-particles	6.5
Energy of delayed γ-rays	6.3
Energy released when those prompt neutrons that do not (re)produce fission are captured	6.8
Energy converted into heat in an operating thermal nuclear reactor	**202.5**
Energy of antineutrinos (unrecoverable energy)	6.8
Sum of all sources	**209.3**

Estimating the Total Energy Release

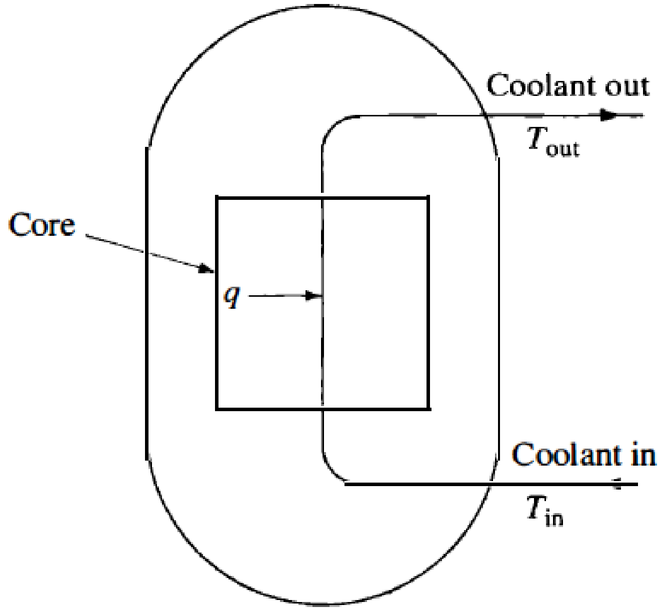
TABLE 5.2
The Energy Released from the Thermal Fission of Plutonium-239

Energy Released from the Fission of Pu-239	Average Energy Released (MeV)
Instantaneously Released Energy	
Kinetic energy of fission fragments	175.8
Kinetic energy of prompt neutrons	5.9
Energy carried by prompt γ-rays	7.8
Energy from Decaying Fission Products at a Later Time	
Energy of β-particles	5.3
Energy of delayed γ-rays	5.2
Energy released when those prompt neutrons that do not (re)produce fission are captured	7.1
Energy converted into heat in an operating thermal nuclear reactor	**207.1**
Energy of antineutrinos (unrecoverable)	7.1
Sum of all sources	**214.2**

Exercise 1

Suppose that the average energy released from the fission of a U-235 nucleus is 200 MeV. How many fissions are required to produce 100 J of thermal energy?

- **From a thermodynamic point of view, a nuclear reactor is a device in which energy is produced and transferred to a moving fluid.**



- **heat is released in a reactor at the rate of q BTU/hr or watts and absorbed by the coolant, which enters the reactor at the temperature T_{in} and exits from the reactor at the temperature T_{out} passing through the system at the rate of $w\ lb/hr$ or kg/hr.**

Rate at which heat is absorbed by the coolant:

$$q = w(h_{out} - h_{in})$$

Schematic drawing of coolant flow through a reactor.

Heat Generated by a Single Fuel Element

- The **neutron flux** in a single fuel element **is not constant**.

- The axial variations in flux, and consequently **volumetric thermal source strength**, however, follow those in the core at the position where the fuel element is, and must be taken into account.

- If the variation of ϕ in the axial direction is a pure cosine function of z, the maximum value of ϕ and q''' occur in a single fuel element at its centre and will be designated ϕ_c and q_c'''

Heat Production in Fuel Elements (Fuel Rods)

Heat production per unit volume

$$q''' = G_f \Sigma_f \phi$$

G is the ***energy per reaction***

In the thermal reactor

$$q'''(z) = G_f \Sigma_f \phi(z)$$

The spatial dependence of the flux depends on the geometry and structure of the reactor.

For the case of sinusoidal flux

$$\phi(z) = \phi_c cos(\frac{\pi z}{H_e})$$

$$A = \phi_c = \frac{3.63P}{VE_r\Sigma_f}$$

Heat Production in Fuel Elements (Fuel Rods)

$$\phi(z) = \frac{3.63P}{VE_r\Sigma_f} cos(\frac{\pi z}{H_e})$$

P	is the *total power of the reactor*
E_r	is the *recoverable energy per fission in joules*
V	is the *fuel volume in cm^3*
H_e and R_e	are the *outer dimensions in cm* to the *extrapolated boundaries*

$$q'''(z) = \frac{3.63P}{VE_r\Sigma_f} G_f\Sigma_{fr} cos(\frac{\pi z}{H_e})$$

$$\Sigma_{fr} = \Sigma_{fr}\frac{n \cdot v}{V}$$

v is the reactor rod (πHa^2) and V is the reactor volume (πHR^2)

$$q'''(z) = \frac{3.63P}{VE_r\Sigma_{fr}\frac{n \cdot v}{V}} G_f\Sigma_{fr} cos(\frac{\pi z}{H_e})$$

Heat Production in Fuel Elements (Fuel Rods)

$$q'''(z) = \frac{3.63P}{E_r n(\pi H a^2)} G_f cos\left(\frac{\pi z}{H_e}\right) = \frac{1.16PG_f}{nE_r H a^2} cos(\frac{\pi z}{H_e})$$

The heat production occurs in the middle z=0 of the central rod (R=0)

$$q'''_{max} = \frac{1.16PG_f}{nE_r H a^2}$$

The maximum rate of heat production in a noncentral rod located at r not equal to zero

$$q'''(z) = q'''_c cos(\frac{\pi z}{H_e})$$

The total rate at which heat is produced in any fuel rod is given by the integral

Heat Production in Fuel Elements (Fuel Rods)

$$q_t = \int_{-H/2}^{H/2} q'''(z) A_s dz$$

$$A_s = \pi a^2$$

$$q_t = A_s \int_{-H/2}^{H/2} q'''(z) dz$$

$$q_t = A_s \int_{-H/2}^{H/2} q_c''' \cos\left(\frac{\pi z}{H_e}\right) dz$$

$$q_t = A_s q_c''' \int_{-H/2}^{H/2} \cos\left(\frac{\pi z}{H_e}\right) dz$$

Heat Production in Fuel Elements (Fuel Rods)

$$q_t = A_s q_c''' \int_{-H/2}^{H/2} \cos\left(\frac{\pi z}{H_e}\right) dz$$

$$q_t = A_s q_c''' \frac{2H_e}{\pi} \sin\left(\frac{\pi H}{2H_e}\right)$$

Where the extrapolation lengths may be neglected, this equation reduces to

$$q_t = A_s q_c''' \frac{2H}{\pi}$$

This equation used to approximate the flux in a reactor where the fuel is contained in separate fuel rods,

$$q_c''' = \frac{1.16 P G_f}{n E_r H a^2} \qquad A_s = \pi a^2$$

$$q_t = \pi a^2 \times \frac{1.16 P G_f}{n E_r H a^2} \times \frac{2H}{\pi} \qquad q_t = \frac{2.32 P G_f}{n E_r}$$

Exercise 2

A small PWR plant operates at a power of 485 MWt. The core, which is approximately 75.4 in. in diameter and 91.9 in high, consists of a square lattice of 23,142 fuel tubes of thickness 0.021 in and inner diameter of 0.298 in. The tubes are filled with 3.40 w/o-enriched $U0^2$. The core is cooled by water, which enters at the bottom at 496°F and passes through the core at a rate of 34 x10^6 lb/hr at 2,015 psia. Compute

(a) the average temperature of the water leaving the core;

(b) the average power density in kW/liter;

(c) the maximum heat production rate, assuming the reactor core is bare.

$$n = 23{,}142 \ rods \ \& \ q = 485 \ MWt \ \& \ R = 75.4 \ in \ \& \ H = 91.9 \ in \ \& \ T_{in} = 496 \ {}^\circ F \ \&$$

$$G = 180 \ Mev \ \& \ E_r = 200 \ MeV \ \& \ a = 0.298 \ in \ \& \ w = 34 \times 10^6 \ Ib / hr$$

The Total Heat Generated in Core - General

The **individual fuel elements** in a **heterogeneous** core are subjected to **different values of**:

- **Neutron flux, ϕ**
- ***Volumetric* thermal source strengths, q_c'''**

In a homogeneous core, there are no individual fuel elements.

The Total Heat Generated in Core - General

However, the two cases can be treated in a like manner, provided that

1. **The number of fuel elements in the heterogeneous core is sufficiently large.**

2. **The fuel type and enrichment do not vary in the heterogeneous reactor**

The Total Heat Generated in Core - General

The total heat generated within the core is composed of the total heat generated by

- **The fuel in the core**
- **The structural components**
- **Coolant**
- **Moderator**
- **Other materials due to the absorption of radiations.**

The Total Heat Generated in Core - General

In a *thermal reactor*,

Additional heat is generated by the moderator due to the slowing down of neutrons (4 to 6 % of the total heat generated within the core).

In order to evaluate the heat generated by the fuel in any core, the flux distribution throughout the core should be known.

The Total Heat Generated in Core - General

In a homogeneous core

The type and concentration of the fissionable material are independent of location

The volumetric thermal source strength at any point in the core is given by

$$q''' = G_f N_f \sigma_f \phi$$

The value of G_f in a homogeneous core include the heat generated in the moderator which averages about 5 % of the total heat.

The value of 190 Mev/fission rather than the 180 in the fuel of a heterogeneous core) is used.

The Total Heat Generated in Core - General

To obtain the total heat generated in a given core, Q_t

The proper flux distribution should be substituted for geometry of reactor.

The required total heat generated is then obtained by integration over the core volume.

Exercise 3

Derive the equation for the total heat generated, Q_r (Btu/hr). in an un-reflected homogeneous reactor core of spherical shape and radius R (ft). Neglect the extrapolation length (that is, $R = R_e$).

Exercise 3 (Solution)

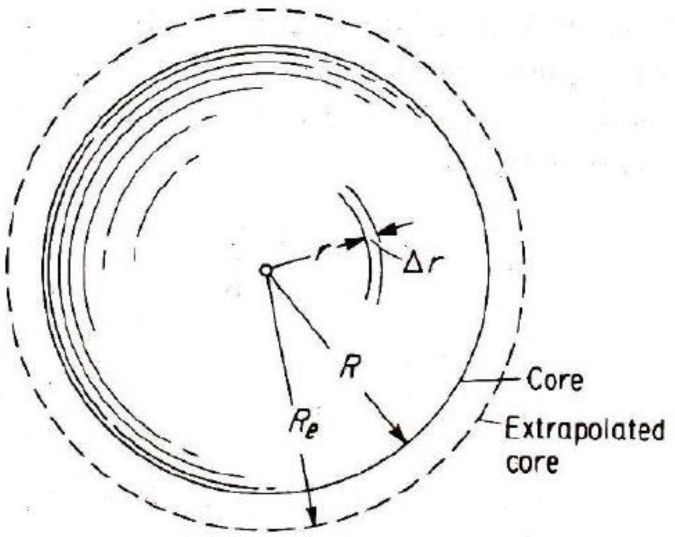
Derive the equation for the total heat generated, Q_r (Btu/hr). in an un-reflected homogeneous reactor core of spherical shape and radius R (ft). Neglect the extrapolation length (that is, $R = R_e$).

$$q''' = G_f \Sigma_f \phi$$

$$q''' = G_f \Sigma_f \phi(r)$$

$$\phi(r) = \frac{\phi_c}{\pi r / R} \sin\left(\frac{\pi r}{R}\right)$$

$$q''' = G_f \Sigma_f \frac{\phi_c}{\pi r / R} \sin\left(\frac{\pi r}{R}\right)$$



Exercise 3 (Solution)

For a spherical shell of radius *r* and thickness Δr

The heat generated in that shell, ΔQ_r , is equal to the above q''' multiplied by the volume of the shell, $4\pi r^2$. Thus

$$\Delta Q_r = q''' \, 4\pi r^2 \, \Delta r$$

$$\Delta Q_r = G_f \Sigma_f \frac{\phi_{co}}{\pi r / R} \sin\left(\frac{\pi r}{R}\right) 4\pi r^2 \, \Delta r$$

$$G_f = 190 \ Mev/\ fission$$

The conversion factor 1.5477 x 10^{-8} *to convert Mev/sec* cm^3 *to Btu/hr* ft^3

Exercise 3 (Solution)

$$\Delta Q_r = 4G_f \Sigma_f \phi_{co} Rr \sin\left(\frac{\pi r}{R}\right) \Delta r$$

The total heat generated in the core, Q_t, in Btu/hr, is now

$$Q_r = 4G_f \Sigma_f \phi_{co} R \int_0^R r \sin\left(\frac{\pi r}{R}\right) dr$$

This equation integration as follows:

$$Q_r = 4G_f \Sigma_f \phi_{co} R \int_0^R r \sin\left(\frac{\pi r}{R}\right) dr$$

$$\int_0^R x \sin ax \; dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a}\Bigg|_0^R$$

Exercise 3 (Solution)

$$Q_r = 4G_f\Sigma_f\phi_{co}R\left[\left(\frac{R}{\pi}\right)^2 \sin\frac{\pi r}{R} - \frac{R}{\pi} r\cos\frac{\pi r}{R}\Big|_0^R\right]$$

$$Q_r = 4G_f\Sigma_f\phi_{co}R\left[\left(\frac{R}{\pi}\right)^2 \sin\frac{\pi r}{R} - \frac{R}{\pi} r\cos\frac{\pi r}{R}\Big|\right]$$

$$Q_r = 4G_f\Sigma_f\phi_{co}R\left[\frac{R}{\pi}R\right]$$

$$Q_r = \frac{4G_f\Sigma_f\phi_{co}R^3}{\pi}$$

The Case of the Heterogeneous Reactor with a Large Number of Fuel Elements

In the case of large number of fuel elements, as is usually the case in a power reactor, Q_t can be obtained, with little error, by *homogenizing* the core .

q''' would be modified by multiplying it by the ratio of the fuel volume to the core volume:

$$q_h''' = q''' \frac{\text{fuel volume}}{\text{core volume}}$$

Where q_h''' is the homogenized volumetric thermal source strength.

A differential equation can now be written for the geometry of the core and integrated

Exercise 4

Derive the equation for the total heat generated, Q_t (Btu/hr), in a cylindrical reactor core of radius *R* and height *H* containing *n* vertical fuel elements. Assume a normal neutron-flux distribution.

Exercise 4 (Solution)

The flux distribution in a cylindrical core is given by

$$\phi_c = \phi_{co} J_0\left(\frac{2.405r}{R_e}\right)\cos\left(\frac{\pi z}{H_e}\right) \qquad (1)$$

where J_0 is the Bessel function of the first kind. zero order (Appendix C). z = 0 represents the middle plane of the core where all fuel elements have their maximum neutron flux ϕ_c

$$\phi_c = \phi_{co} J_0\left(\frac{2.405r}{R_e}\right) \qquad (2)$$

Exercise 4 (Solution)

Similarly,

$$q_c''' = q_{co}''' J_0\left(\frac{2.405r}{R_e}\right) \qquad (3)$$

ϕ_c and q_c''' are the maximum (midpoint) neutron flux and volumetric thermal source strength for a fuel element at a radial distance r from the center

ϕ_{co} and q_{co}''' are the neutron flux and volumetric thermal source strength geometrical center of the core

A'= reactor cross-sectional area per fuel clement will now be defined

$$A' = \frac{\pi R^2}{n} \qquad (4)$$

n is the total number of fuel elements in the core.

Exercise 4 (Solution)

q_t will now be modified to give q_t', the heat generated per unit area A'

$$q_t' = \frac{q_t}{A'} = \frac{n}{\pi R^2} q_t$$

$$q_t = A_s q_c''' \frac{2H_e}{\pi} \sin\left(\frac{\pi H}{2H_e}\right)$$

$$q_t' = \frac{2n}{\pi^2 R^2} A_s H_e \sin\left(\frac{\pi H}{2H_e}\right) q_c''' \qquad (5)$$

or, where the extrapolation lengths may be neglected,

$$q_t' = \frac{2n}{\pi^2 R^2} A_s q_{max}''' H \qquad (6)$$

These equations, in which all components are constant except q_c''' , can be combined with Eq. 3 to give q_t' as a function of *r*.

$$q_t'(r) = \frac{2n}{\pi^2 R^2} A_s H_e \sin\left(\frac{\pi H}{2H_e}\right) q_{co}''' J_0\left(\frac{2.405r}{R_e}\right) \qquad (7)$$

Exercise 4 (Solution)

Taking a differential cylindrical element of width Δr at radius r from the axis of the core the total heat generated in the core, Q,(Btu/hr), can be computed from the equation

$$Q_t = \int_0^R q_t'(r)\, 2\pi r dr$$

$$Q_t = \int_0^R \frac{2n}{\pi^2 R^2} A_s H_e \sin\left(\frac{\pi H}{2H_e}\right) q_{co}''' J_0\left(\frac{2.405r}{R_e}\right) 2\pi r dr$$

$$Q_t = \frac{4n}{\pi R^2} A_s H_e \sin\left(\frac{\pi H}{2H_e}\right) q_{co}''' \int_0^R r J_0\left(\frac{2.405r}{R_e}\right) dr \qquad (8)$$

Exercise 4 (Solution)

where the extrapolation length may be neglected

$$\int_0^R rJ_0\left(\frac{2.405r}{R_e}\right)dr = \frac{R_e}{2.405}\left[rJ_1\left(\frac{2.405r}{R_e}\right)\right]_0^R$$

$$J_1(0) = 0$$

$$Q_t = \frac{4n}{\pi R^2} A_s H_e \sin\left(\frac{\pi H}{2H_e}\right) q'''_{co} \frac{R_e R}{2.405} J_1\left(\frac{2.405R}{R_e}\right) \qquad (9)$$

$$R_e = R \ and \ H_e = H$$

$$Q_t = \frac{4n}{2.405\pi} A_s H q'''_{co} J_1(2.405)$$

$$J_1(2.405) = 0.519$$

Exercise 4 (Solution)

This quantity can then be substituted in Equation (9)

$$Q_t = 0.275 n A_s H q'''_{co}$$

The above Q_t represents the total heat generated in the solid fuel alone.

To obtain the total heat generated in the core, allowances must be made for the heat generated in the moderator and other reactor materials.

A reasonable approach is to multiply the above Q_t, by 1.05 giving

$$Q_t = 0.289 n A_s H q'''_{co}$$

Reactor Shutdown Heat Generation

After a few days of reactor operation, the β and γ radiation emitted from decaying fission products amounts to about 7% of the total thermal power output of the reactor.

In reactor shutdown, the reactor power does not immediately drop to zero but falls off rapidly according to a negative period.

Eventually determined by the half-life of the longest-lived delayed neutron group

After shutdown, a reactor therefore continues to generate power, P_{ts}, a function of time.

Reactor Shutdown Heat Generation

The amount of such power generation depends on

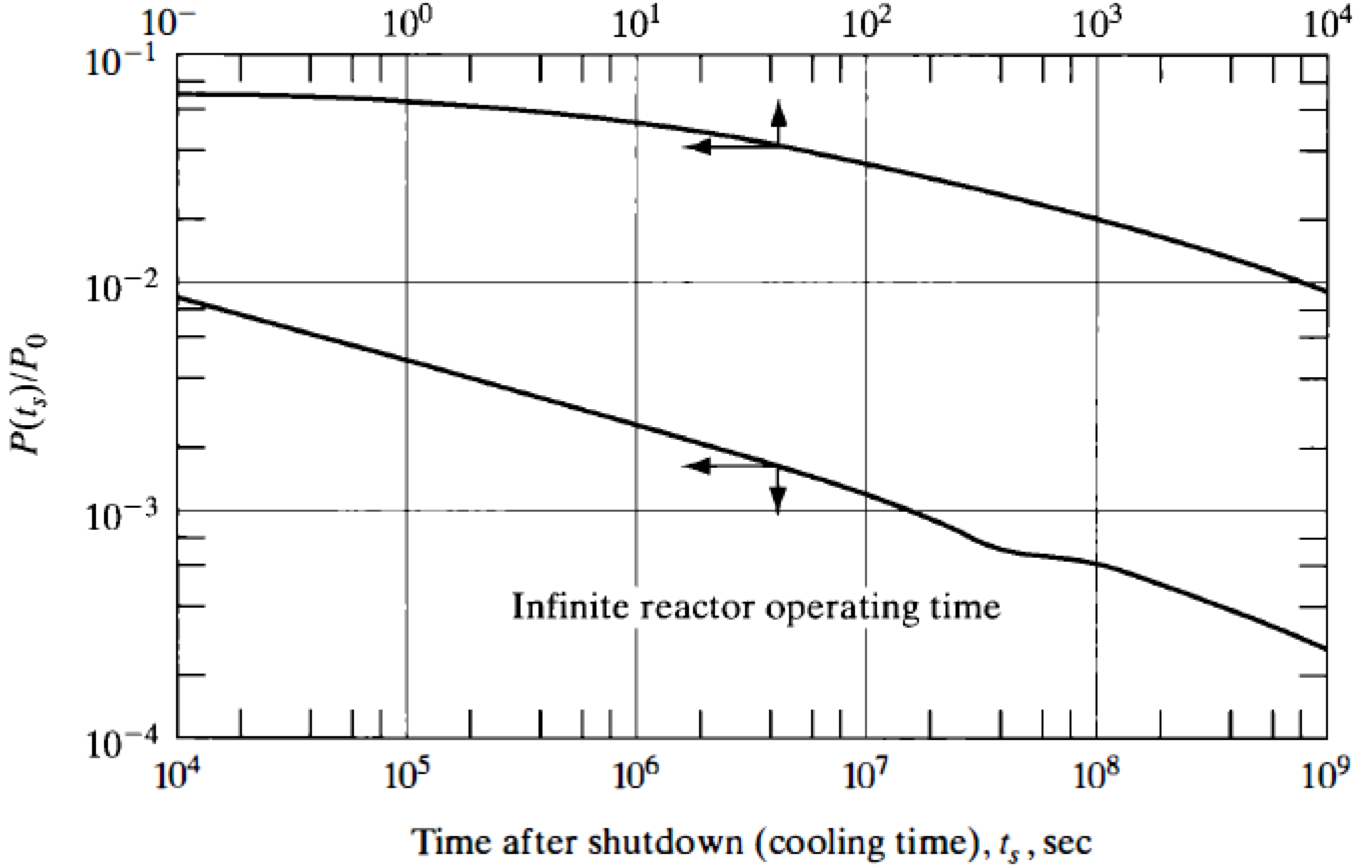
(a) The level of power before shutdown, P_{ts}
(b) The length of time, t_o

The ratio of the volumetric thermal source strength after shutdown q_{t_s}''' *to that before shutdown* q_{t_0}''' *as*

$$\frac{q_{t_s}'''}{q_{t_0}'''} = \frac{P_{t_s}}{P_{t_0}}$$

Reactor Shutdown Heat Generation

$$\frac{P(t_0, t_s)}{P_0} = \frac{P(t_s)}{P_0} - \frac{P(t_0 + t_s)}{P_0}$$



The ratio $\frac{P(t_s)}{P_0}$ of the fission product decay power to the reactor operating power as a function of time t_s after shutdown.

Reactor Shutdown Heat Generation

If a ^{235}U-fueled reactor contains substantial quantities of ^{238}U, as many of these reactors do, the decay of ^{239}U and ^{239}Np , formed by the absorption of neutrons in the ^{238}U, must also be taken into account.

$$\frac{P_{239_U}}{P_\circ} = 2.28\times10^{-3}\, C\left(\frac{\bar{\sigma}a_{235_U}}{\bar{\sigma}f_{235_U}}\right)\left[1-e^{-4.91\times10^{-4}t_0}\right]e^{-4.91\times10^{-4}t_s}$$

$$\frac{P_{239_{Np}}}{P_o} = 2.17\times10^{-3}\, C\left(\frac{\bar{\sigma}a_{235_U}}{\bar{\sigma}f_{235_U}}\right)\left[\left(1-e^{-3.41\times10^{-6}t_0}\right)e^{-3.41\times10^{-6}t_s} - 7.0\times10^{-3}\left(1-e^{-4.91\times10^{-4}t_0}\right)e^{-4.91\times10^{-4}t_s}\right]$$

Reactor Shutdown Heat Generation

$P_{^{239}U}$ and $P_{^{239}Np}$ the decay powers of ^{239}U and ^{239}Np , respectively.

C is the conversion factor for the reactor

$\overline{\sigma}_{a\,^{235}U}$ and $\overline{\sigma}_{f\,^{235}U}$ are the effective thermal cross-sections of ^{235}U

Exercise 5

A BWR power plant operating for 1 .5 years at an efficiency of 34% has an electrical output of 1,101 MW.

a) **What is the maximum fission product decay energy in the reactor at shutdown?**
b) **What is the decay energy 6 months after shutdown**

Heat Generation by Radioisotopes

Radioisotopic fuels are widely used in small power devices to generate heat.

The heat is usually then directly converted to electrical energy in compact power devices by thermoelectric.

In the United States, such systems are called Systems for Nuclear Auxiliary Propulsion [SNAP].

Heat Generation by Radioisotopes

These SNAP devices are given odd numbers.

For example,

SNAP 17,

- **used to power communications satellites**
- **uses strontium-90 as fuel.**
- **weighs 30 lb_m**
- **produces 30 watts of electricity**
- **has a design life of 3-5 years**

Heat Generation by Radioisotopes

SNAP 27

- **A plutonium-238-fueled thermoelectric generator**
- **Produces 67 watts**

Another different category of compact power sources,

- **using small fission reactors rather than radioisotopes**
- **referred to as SNAP given (even numbers.)**

There are generally two types of fuel used:

- **Those processed from spent reactor fuels**
- **Those a emitters prepared by irradiation in a reactor.**

Radioisotope Fuels

Type	Radioisotope	Activity	Half-life	Probable Fuel Material	Fuel Material Density, g_m/cm^3	Power Density $w(t)/cm^3$
1	Sr^{90}	β	28 yr	$SrTiO_3$	4.8	0.54
	Cs^{137}	β	33 yr	$CsCl$	3.9	1.27
	Ce^{144}	β	285 d	CeO_2	6.4	12.5
	Pm^{147}	β	2.5 yr		6.6	1.1
2	Po^{210}	α	138.4 d	Po	9.3.	132.0
	Pu^{238}	α	86 yr	PuC	12.5	6.9
	Cm^{242}	α	163 yr	CmC	11.75	1,169

Heat Generation by Radioisotopes

Heat generation in a radioisotope is due to the exothermic decay reactions and is uniform throughout a fuel element.

The volumetric thermal source strength q''' is obtained by evaluating the mass defect per decay reaction and the energy associated with it.

Exercise 6

A SNAP generator is fuelled with 475 gm of Pu^{238} C, 100 % enriched in Pu^{238} . Calculate the volumetric thermal source strength in Btu/hr ft^3, and the total thermal power generated in watts.

Density of PuC= 12.5 gm/ cm^3 •

Temperature Conversion

Fahrenheit	Celsius	C = (F - 32) / 1.8
Fahrenheit	kelvin	K = (F + 459.67) / 1.8
Fahrenheit	Rankine	Ra = F + 459.67
Fahrenheit	Réaumur	Re = (F - 32) / 2.25

Thank You

Stay safe!