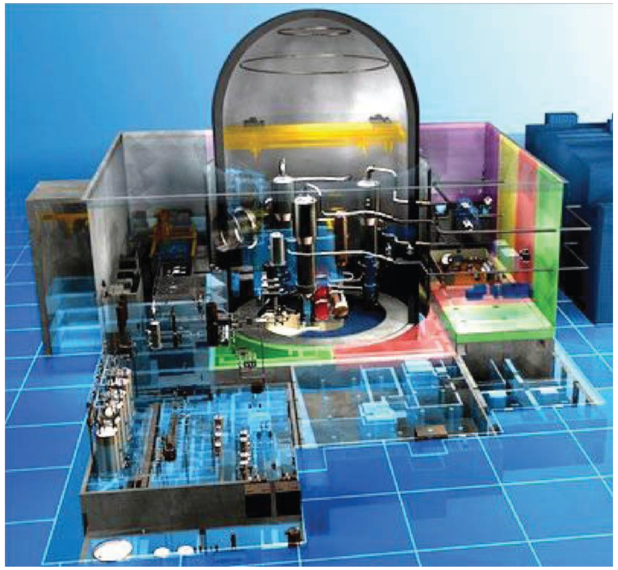
# Introduction to Conduction



**MUHAMMAD SYAHIR SARKAWI, PhD**
Nuclear Engineering Program
Energy Engineering Department
N01-273 | 0133274154
syahirsarkawi@utm.my

# Topic Outline

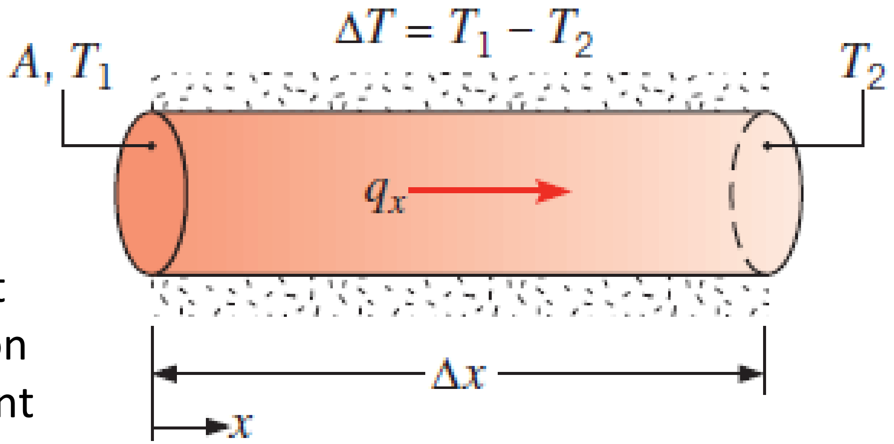
## CHAPTER 2 *Introduction to Conduction* 67

# The Conduction Rate Equation

**Recall**

**Conduction** is the **transport of energy** in a medium due to a **temperature gradient**, and the physical mechanism is one of **random atomic or molecular activity**

- Conduction heat transfer is governed by **Fourier's Law**
- Fourier's law is ***phenomenological***; that is, it is developed from observed phenomena rather than being derived from first principles.
- View the rate equation as a generalization based on much **experimental evidence.**




Steady-state heat conduction experiment

$$q_x \propto A\frac{\Delta T}{\Delta x}$$ Without consider material

$$q_x = -kA\frac{\Delta T}{\Delta x} \quad (1)$$ Consider property of material

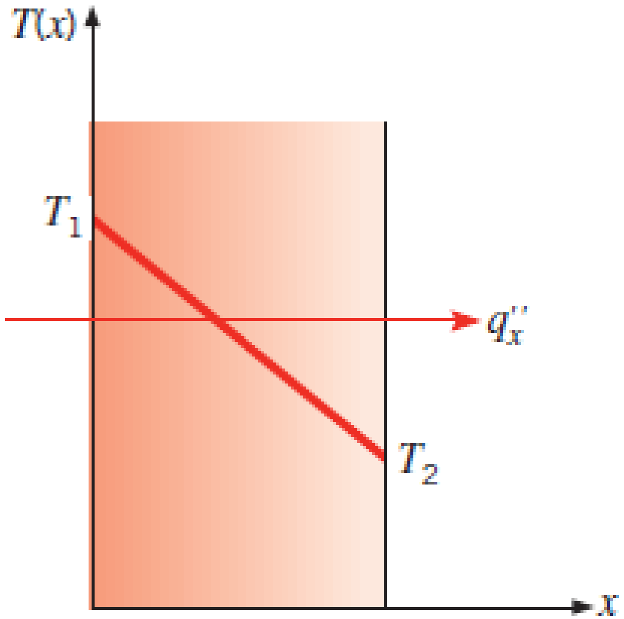
# The Conduction Rate Equation

**For heat flux**

$$q''_x = \frac{q_x}{A} = -k\frac{\Delta T}{\Delta x} \quad (2)$$

- Recall that the **minus sign (-eve)** is necessary because **heat** is always transferred in the direction of **decreasing temperature**.
- Based on Fourier's law equation above, Implies that the heat flux is a directional quantity.
- The direction of $q''_x$ is **normal** to the **cross-sectional area A**
- More generally, the direction of heat flow will always be normal to a surface of constant temperature, called an **isothermal surface**

# The Conduction Rate Equation




The relationship between coordinate system, heat flow direction, and temperature gradient in one dimension.

- The direction of heat flow $q''_x$ in a plane wall for which the **temperature** gradient $dT/dx$ is **negative**.
- Note that the **isothermal surfaces** are planes **normal** to the **x-direction.**
- Recognizing that the **heat flux** is a **vector quantity**.

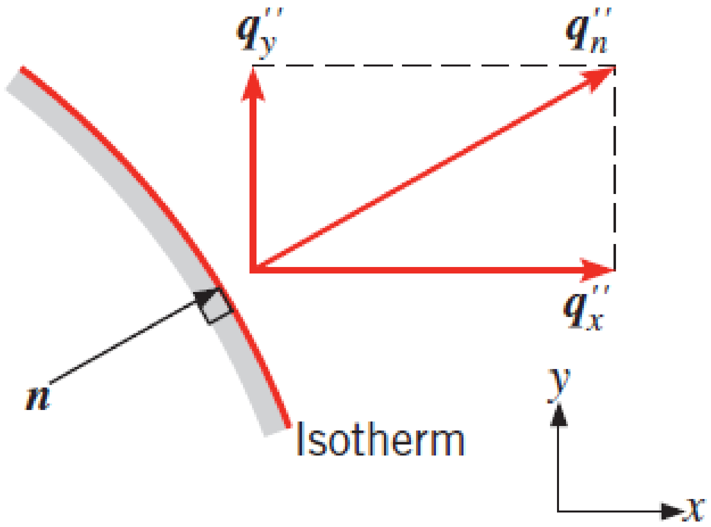
$$q''_x = -k\nabla T = -k\left(i\frac{\partial T}{\partial x} + j\frac{\partial T}{\partial y} + k\frac{\partial T}{\partial z}\right) \quad \textbf{(3)}$$

**∇** is the three-dimensional del operator

***T***(x,y,z) is the scalar temperature field

# The Conduction Rate Equation

- It is implicit in Equation (3) that the **heat flux vector** is in a direction **perpendicular** to the **isothermal surfaces**.




$$q''_x = q''_n \boldsymbol{n} = -k\frac{\partial T}{\partial n}\boldsymbol{n} \qquad (4)$$

$q''_n$ is the heat flux in a direction *n*, which is normal to an *isotherm*, and ***n*** is the unit normal vector in that direction.

- The heat transfer is **sustained** by a **temperature gradient** along ***n***

# The Conduction Rate Equation

- Note also that the **heat flux vector** can be resolved **into components** such that, in **Cartesian coordinates**, the general expression for $q''$

$$\boldsymbol{q}'' = \boldsymbol{i}q_x'' + \boldsymbol{j}q_y'' + \boldsymbol{k}q_z'' \qquad \textbf{(5)}$$

- where, from Equation (3), it follows that

$$q_x'' = -k\frac{\partial T}{\partial x} \qquad q_y'' = -k\frac{\partial T}{\partial y} \qquad q_z'' = -k\frac{\partial T}{\partial z} \qquad \textbf{(6)}$$

- Each of these expressions relates the **heat flux** across **a surface to the temperature gradient** in a direction **perpendicular to the surface**.

# The Conduction Rate Equation

- It is also implicit in Equation (3) that the **medium** in which the **conduction occurs** is **isotropic**.
- The value of the **thermal conductivity** is **independent** of the **coordinate direction**.

**Fourier's law is the cornerstone of conduction heat transfer, and its key features are summarized as follows.**

- It is not an expression that may be derived from first principles; it is instead a generalization based on **experimental evidence**.
- It is an expression that defies an important **material property**, the **thermal conductivity**.
- Fourier's law is a **vector expression** indicating that the **heat flux** is **normal** to an **isotherm** and in the direction of **decreasing temperature**.
- Note that Fourier's law **applies for all matter**, regardless of its state (solid, liquid, or gas).

# Thermal Conductivity

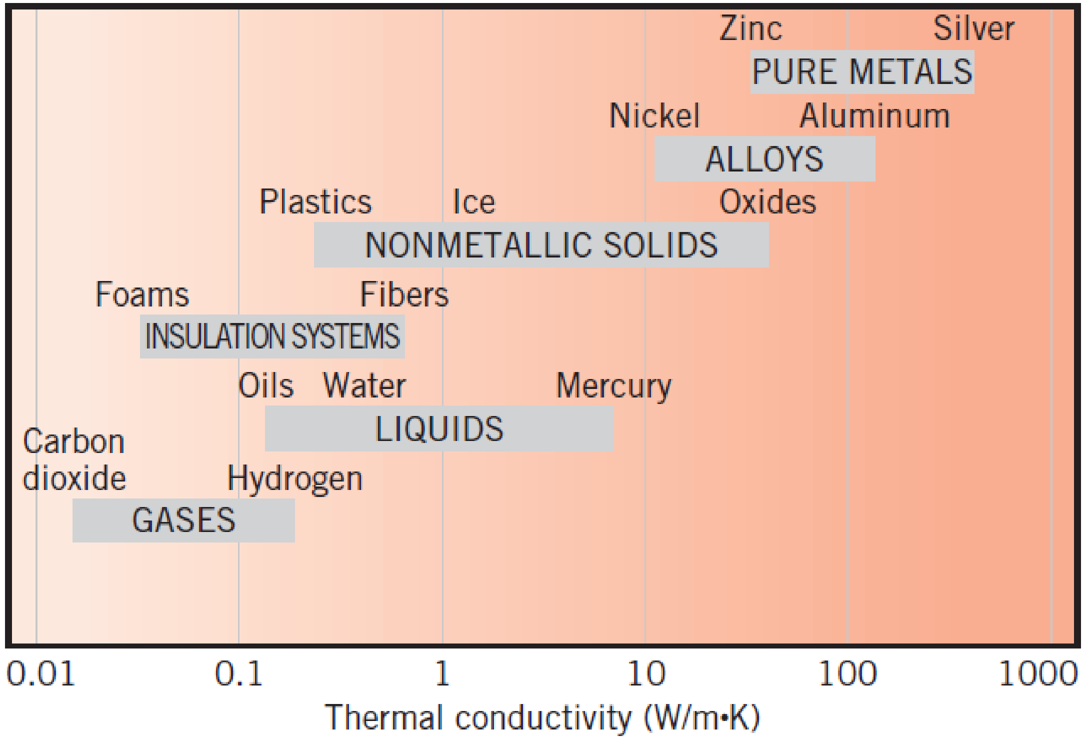
- To use Fourier's law, the **thermal conductivity** of the material must be **known**.
- Thermal conductivity is referred to as a **transport property**, provides an indication of the **rate** at which **energy is transferred** by the **diffusion process**.
- It depends on the **physical structure** of matter, atomic and molecular.

$$\text{From Fourier's law Eq. (6)} \quad k_x = -\frac{q''_x}{\partial T/\partial x}$$

- **Similar** definitions are associated with thermal conductivities in the y- and z-directions $(k_y, k_z)$
- For an **isotropic material** the **thermal conductivity** is **independent** of the **direction** of transfer, $\boldsymbol{k_x = k_y = k_z = k}$.

# Thermal Conductivity

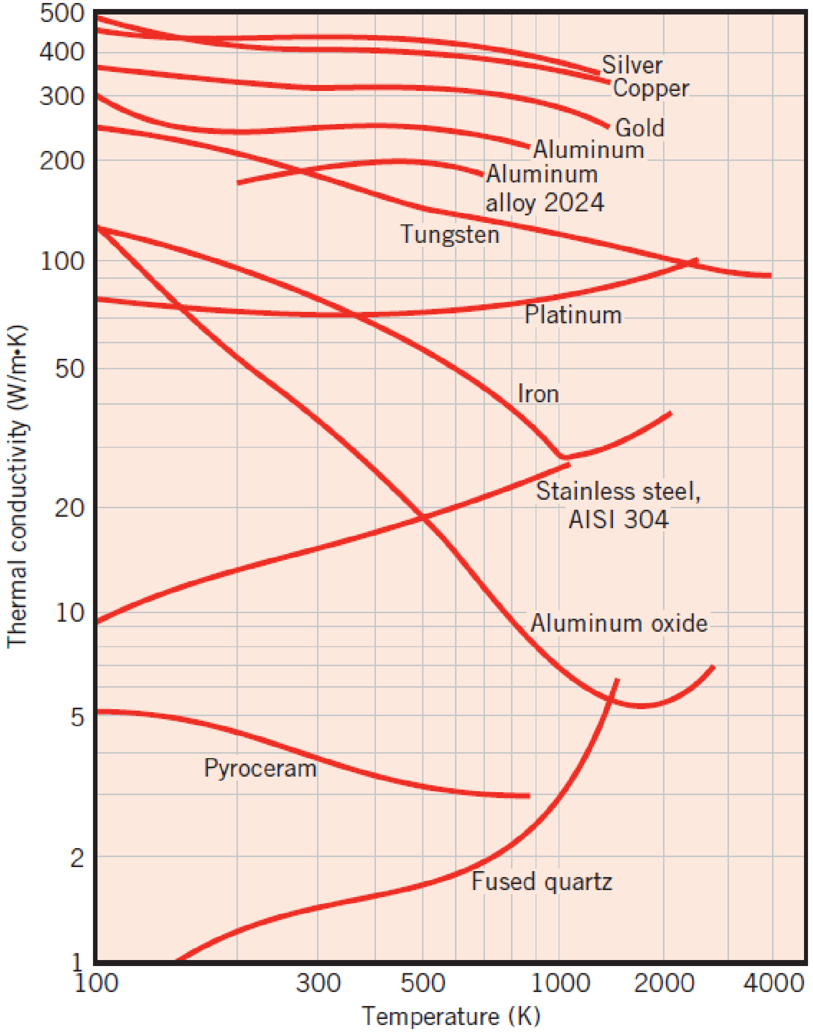
- The conduction heat flux increases with increasing thermal conductivity.
- In general, the **thermal conductivity** of a **solid** is **larger** than that of a **liquid**, which is **larger than** that of a **gas.**




- The thermal conductivity of a **solid** may be more than **four orders** of magnitude **larger** than that of a **gas.**
- This trend is due largely to **differences** in inter **molecular spacing** for the two states.

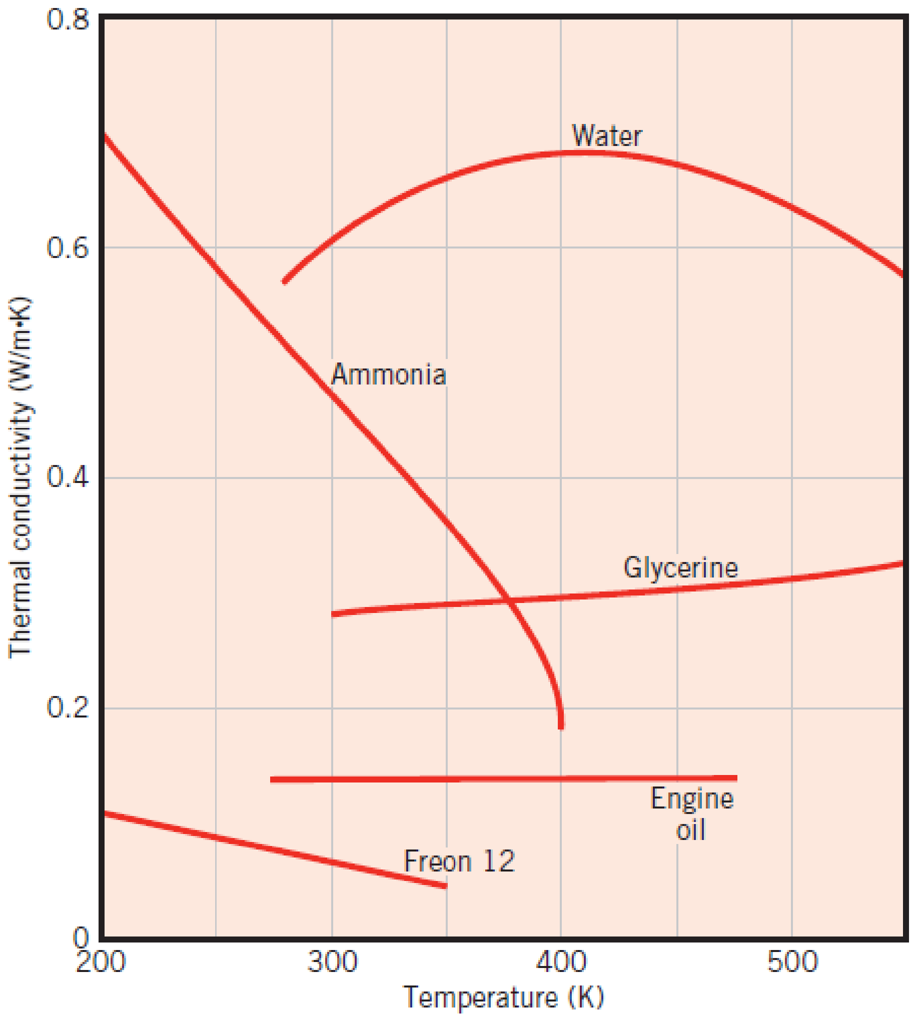
**Range of thermal conductivity for various states of matter at normal temperatures and pressure.**

# Thermal Conductivity (Solid State)




**The temperature dependence of the thermal conductivity of selected solids.**

# Thermal Conductivity (Liquid State)




- The **thermal conductivity** of **nonmetallic liquids** generally **decreases** with **increasing temperature**.
- The thermal conductivity of liquids is usually **insensitive** to **pressure except near** the **critical point**.
- Thermal conductivity generally **decreases** with **increasing** molecular **weight**.
- **Liquid metals** are **commonly used** in **high heat flux** applications, such as occur in **nuclear power plants**.

FIGURE 2.9 The temperature dependence of the thermal conductivity of selected nonmetallic liquids under saturated conditions.

# Thermal Conductivity (Gas State)

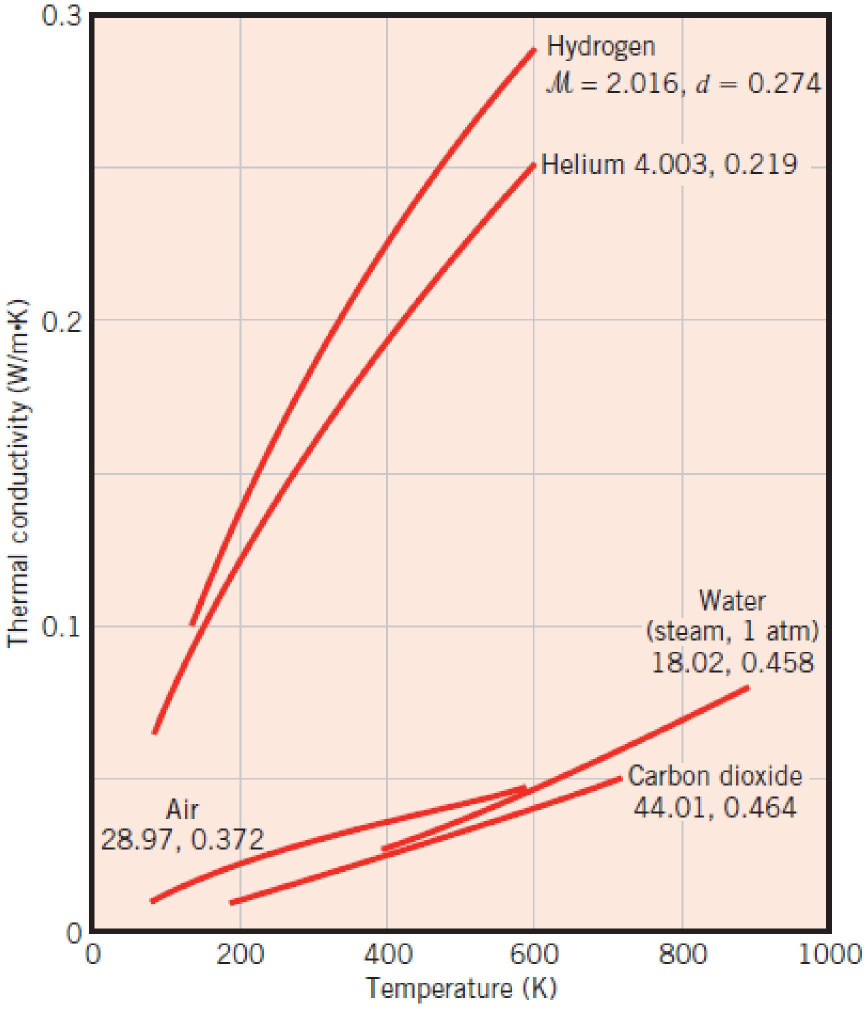
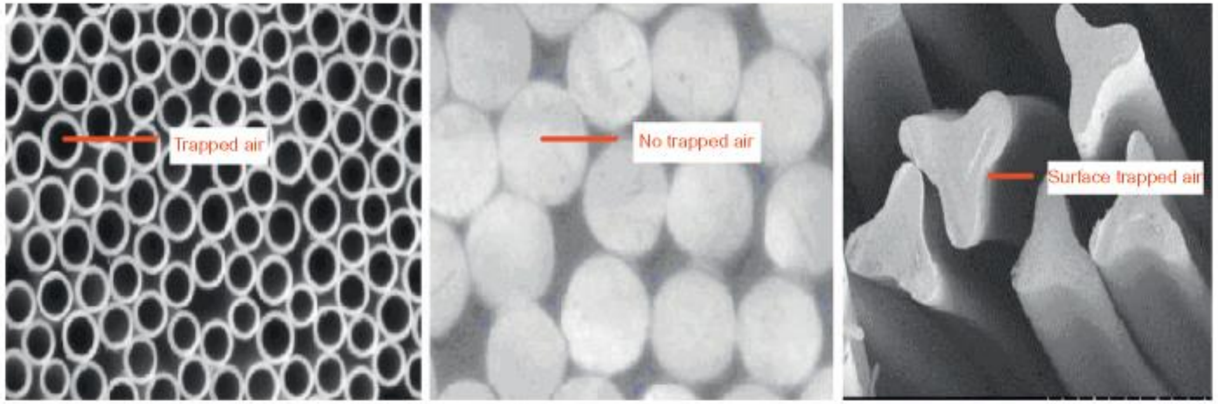



**FIGURE 2.8** The temperature dependence of the thermal conductivity of selected gases at normal pressures. Molecular diameters ($d$) are in nm [10]. Molecular weights ($\mathcal{M}$) of the gases are also shown.

# Thermal Conductivity (Insulation System)

- **Thermal insulations** consist of **low thermal conductivity** materials combined to achieve an even **lower system thermal conductivity.**
- In **conventional** fiber- , powder-, and flake -type insulations, the **solid material** is **finely dispersed** throughout an **air space**.
- ***Effective thermal conductivity***, which **depends** on the **thermal conductivity** and **surface radiative properties** of the solid material, as well as the **nature** and **volumetric fraction** of the **air or void space**.

# Thermal Conductivity (Insulation System)




Hollow fiber | Circular fiber | Non-circular fiber

Thermal protective clothing for firefighters

# Other Relevant Properties

- **Properties of matter** are generally referred to as **thermophysical properties** and include **two** distinct **categories**, **transport** and **thermodynamic** properties.

| Transport Properties | Thermodynamic Properties |
| --- | --- |
| Diffusion rate coefficients:<br>• Thermal conductivity, $k$ (heat transfer)<br>• Kinematic viscosity, $v$ (momentum transfer) | Pertain to the equilibrium state of a system.<br>• Density, $\rho$<br>• Specific heat, $c_p$<br>• **Volumetric heat capacity, $\rho c_p$ $(J/m^3 \cdot K)$** |

- **Densities** and **specific heats** are provided in the tables of **Appendix 1**

# Other Relevant Properties

- In heat transfer analysis, the **ratio** of the **thermal conductivity** to the **heat capacity** is an important property termed the ***thermal diffusivity***, which has units of $m^2/s$:

$$\alpha = \frac{k}{\rho c_p} \quad \textbf{(7)}$$

- It measures the **ability** of a **material** to **conduct thermal energy** relative to its **ability** to **store thermal energy**
- Materials of **large $\alpha$** will **respond quickly** to **changes** in their **thermal environment**, whereas materials of **small $\alpha$** will **respond** more **sluggishly,** taking **longer** to reach a new **equilibrium condition**.

# Other Relevant Properties

## THERMAL DIFFUSIVITY

How **_fast_** heat travel

$$\alpha = \frac{k}{\rho c_p}$$

Ability to **_conduct_** heat

Ability to **_store_** heat

$$\alpha_{aluminium\ alloy} = 7 \times 10^{-5}\ m^2/s$$

$$\alpha_{concrete} = 0.05 \times 10^{-5}\ m^2/s$$

# Other Relevant Properties

**TABLE A-3**

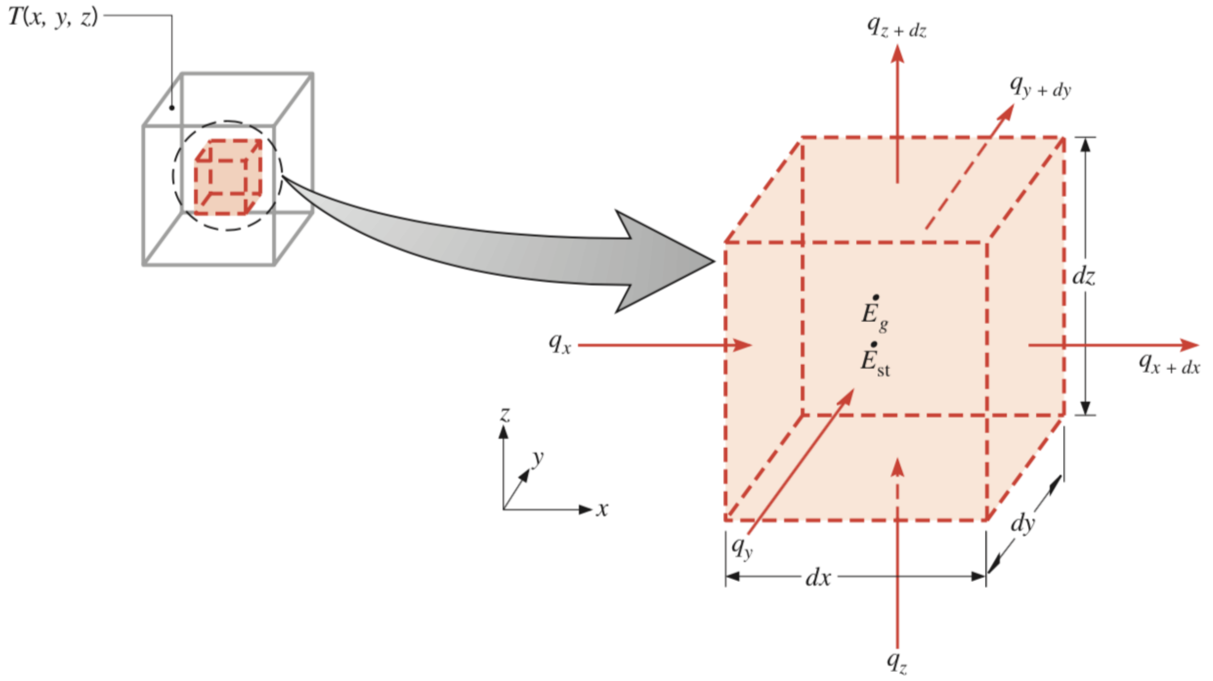
Properties of solid metals

| Composition | Melting Point, K | Properties at 300 K: $\rho$ kg/m$^3$ | $c_p$ J/kg · K | $k$ W/m · K | $\alpha \times 10^6$ m$^2$/s | Properties at Various Temperatures (K), $k$(W/m · K)/$c_p$(J/kg · K): 100 | 200 | 400 | 600 | 800 | 1000 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Aluminum: | | | | | | | | | | | |
| Pure | 933 | 2702 | 903 | 237 | 97.1 | 302 | 237 | 240 | 231 | 218 | |
| | | | | | | 482 | 798 | 949 | 1033 | 1146 | |
| Alloy 2024-T6 (4.5% Cu, 1.5% Mg, | 775 | 2770 | 875 | 177 | 73.0 | 65 | 163 | 186 | 186 | | |
| 0.6% Mn) | | | | | | 473 | 787 | 925 | 1042 | | |
| Alloy 195, Cast (4.5% Cu) | | 2790 | 883 | 168 | 68.2 | | | 174 | 185 | | |

**Linear Interpolation:**

$$\frac{y - y_0}{x - x_0} = \frac{y_1 - y_0}{x_1 - x_0},$$

# The Heat Diffusion Equation




$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

# The Heat Diffusion Equation

**Energy entering the system:**

$$q_x = -k\,dy\,dz\,\frac{\partial T}{\partial x}$$

$$q_y = -k\,dx\,dz\,\frac{\partial T}{\partial y}$$

$$q_z = -k\,dx\,dy\,\frac{\partial T}{\partial z}$$

**Energy exiting the system:**

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x}dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y}dy$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z}dz$$

# The Heat Diffusion Equation

- Within the medium there may also be an energy source term associated with the rate of thermal energy generation. This term is represented as:

$$\dot{E}_g = \dot{q}\, dx\, dy\, dz$$

- where q̇ (q dot) is the rate at which energy is generated per unit volume of the medium ($W/m^3$).

- Considering that the material is not experiencing a change in phase, latent energy effects are not pertinent, and the energy storage term may be expressed a:

$$\dot{E}_{st} = \rho c_p \frac{\partial T}{\partial t} dx\, dy\, dz$$

# The Heat Diffusion Equation

Fill all the terms in the energy balance equation:

$$\dot{E}_{\text{in}} + \dot{E}_g - \dot{E}_{\text{out}} = \dot{E}_{\text{st}}$$

This gives the “Heat Diffusion Equation”:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

This equation, often referred to as the heat equation, provides the basic tool for heat conduction analysis. From its solution, we can obtain the temperature distribution T(x, y, z) as a function of time. The first 3 terms relate to the net conduction heat flux into the control volume for the particular direction (x, y, z).

# The Heat Diffusion Equation

It is often possible to work with simplified versions of HDE. When the thermal conductivity is constant, the heat equation is reduced to:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

**Under steady-state conditions**, there can be no change in the amount of energy storage; hence HDE reduces to:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=0$$

# The Heat Diffusion Equation

- If the heat transfer is **one-dimensional** (e.g., in the x-direction) and there is no energy generation, HDE reduces to:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

- The important implication of this result is that, under steady-state, one-dimensional conditions with no energy generation, the heat flux is a constant in the direction of transfer.

- The heat equation may also be expressed in **cylindrical** and **spherical** coordinates.

# The Heat Diffusion Equation

Heat Diffusion Equation for **Cylindrical Coordinates**:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

Heat Diffusion Equation for **Spherical Coordinates**:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

# The Heat Diffusion Equation

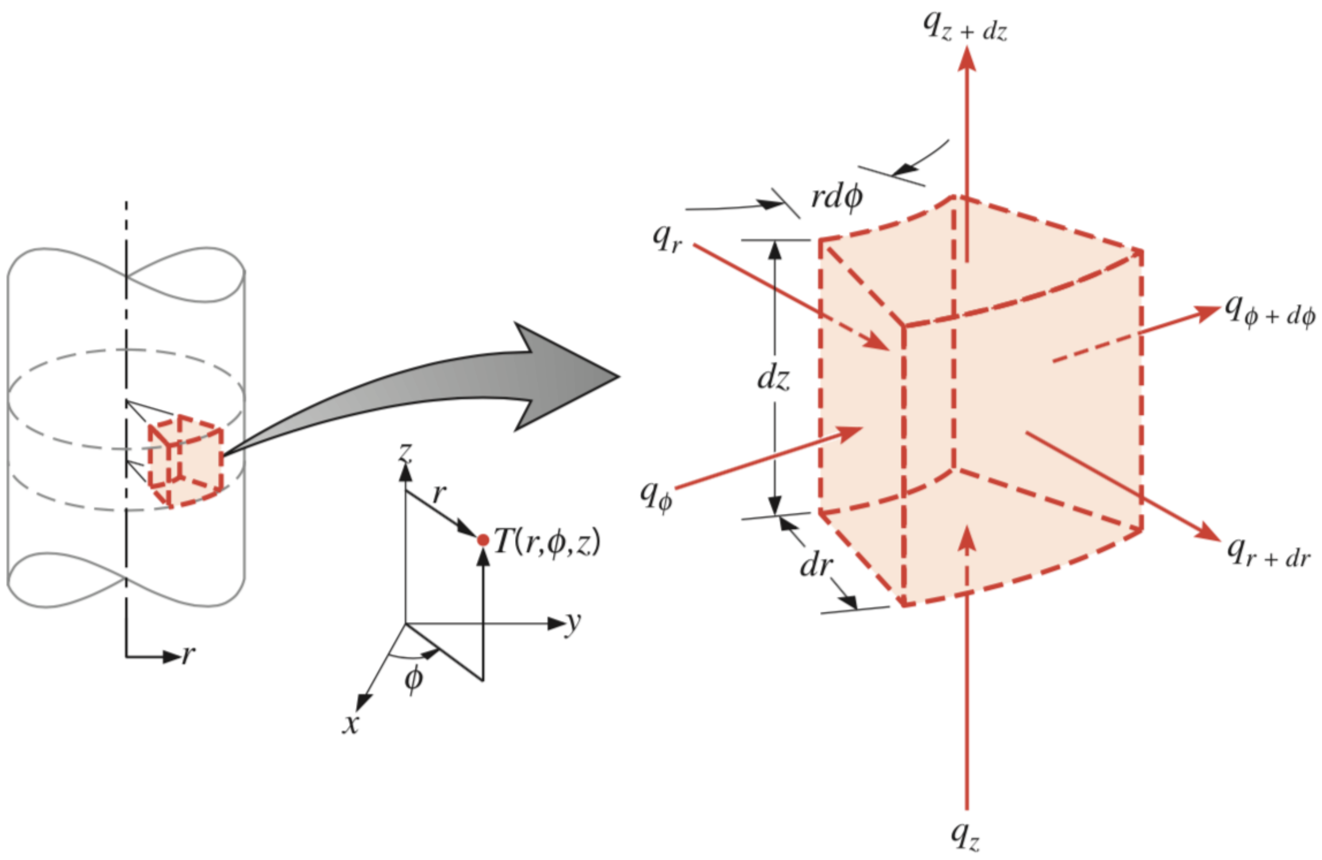



**FIGURE 2.12** Differential control volume, $dr \cdot r\, d\phi \cdot dz$, for conduction analysis in cylindrical coordinates $(r, \phi, z)$.

# The Heat Diffusion Equation

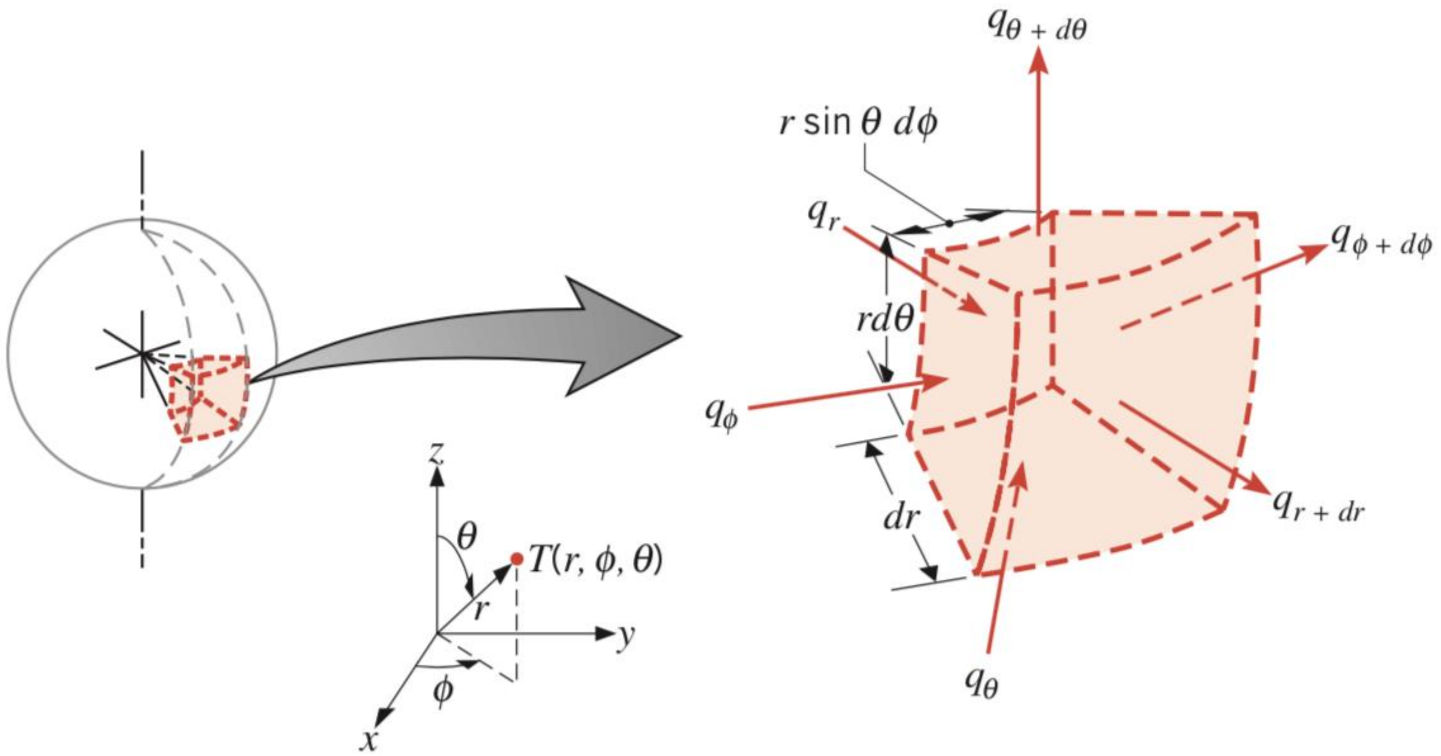


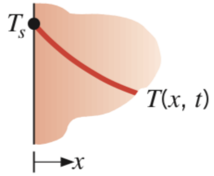

**FIGURE 2.13** Differential control volume, $dr \cdot r \sin\theta \, d\phi \cdot r \, d\theta$, for conduction analysis in spherical coordinates $(r, \phi, \theta)$.


# Boundary Conditions

**TABLE 2.2 Boundary conditions for the heat diffusion equation at the surface ($x = 0$)**

1. Constant surface temperature

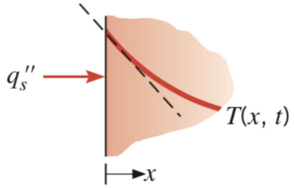
$$T(0, t) = T_s \qquad (2.31)$$




2. Constant surface heat flux

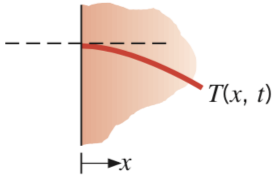
(a) Finite heat flux

$$-k \frac{\partial T}{\partial x}\bigg|_{x=0} = q_s'' \qquad (2.32)$$



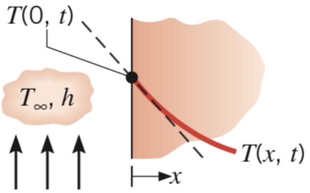

(b) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0 \qquad (2.33)$$



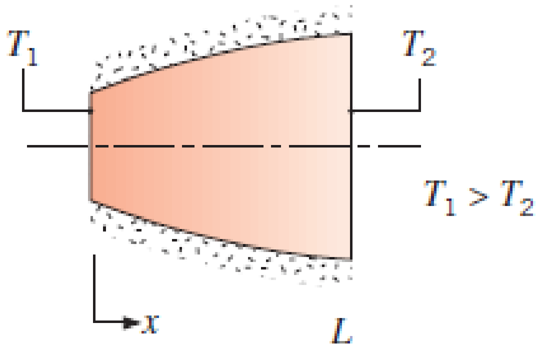

3. Convection surface condition

$$-k \frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0, t)] \qquad (2.34)$$



# Example 1

Assume steady-state, one-dimensional heat conduction through the axisymmetric shape shown below.




Assuming constant properties and no internal heat generation, sketch the temperature distribution on *T* - *x* coordinates. Briefly explain the shape of your curve.

# Example 2

The temperature distribution across a wall 0.3 m thick at a certain instant of time is $T(x) = a + bx + cx^2$, where $T$ is in degrees Celsius and $x$ is in meters, $a = 200°\text{C}$, $b = -200\ °\text{C}/m$, and $c = 30\ °\text{C}/m$. The wall has a thermal conductivity of $1\ W/m \cdot K$.

(a) On a unit surface area basis, determine the rate of heat transfer into and out of the wall and the rate of change of energy stored by the wall.

# Thank You

Stay safe!