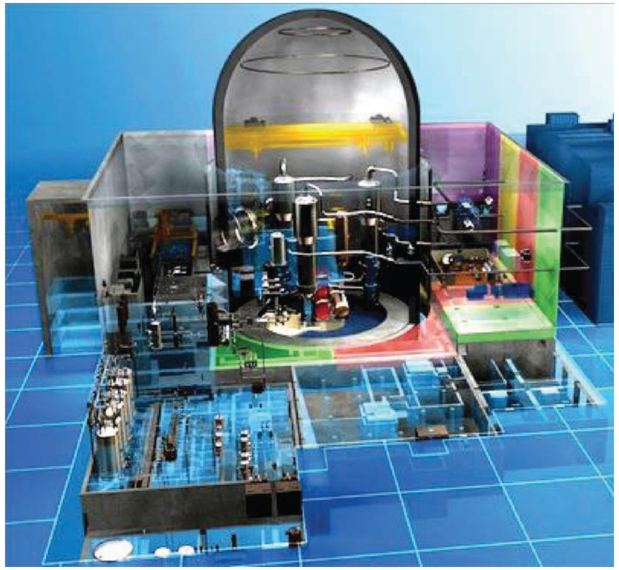
Heat Conduction in Reactor Elements (Part 1)



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General Assumptions

The problems of temperature distribution in and heat removal from these elements are important in evaluating reactor-core thermal performance. We will discuss the following

- **General heat-conduction mechanism.**
- **The equations will be derived for the temperature distribution in fuel elements of simple geometries, their cladding and coolant.**
- **Only one-dimensional steady-state heat flow with uniform heat generation is considered.**

General Assumptions

In treating the problems of heat removal from an individual fuel element,

- **The thermal conductivities of the fuel k_f and the cladding k_c *are constant.***
- **The physical properties of the coolant (density, viscosity, specific heat) are constant and independent of temperature.**
- **The heat-transfer coefficient, *h* between solid and coolant will therefore be considered constant also.**
- **The errors involved in these assumptions depend of course on the severity of the temperature gradients and the dependence of the above properties on temperature.**
- **The resistance to heat transfer in the contact areas between solid materials, such as between fuel and cladding, will either be neglected or taken into account via a conductance term.**

Thermal Conductivity, k_f of Some Fuel Materials $(\mathrm{Btu/hr \cdot ft \cdot °F})$

Temperature, °F	Uranium	UO_2	UC	PuO_2	Thorium	ThO_2
200	15.80	4.5	14.77	3.60	21.75	7.29
300	16.40	. . .	14.07	. . . . .	22.18	6.25
400	17.00	3.5	13.48	. . . . .	22.60	5.34
500	17.50	. . .	13.02	. . . . .	23.00	4.61
600	18.10	2.8	12.67	. . . . .	23.45	4.03
700	18.62	. . .	12.39	. . . . .	23.90	3.59
800	19.20	2.5	12.19	. . . . .	24.30	3.21
900	19.70	. . .	12.02	. . . . .	24.65	2.91
1000	20.25	2.2	11.91	. . . . .	25.75	2.68
1100	20.75	. . .	. . . . .	. . . . .	25.60	2.47
1200	21.20	2.0	11.82	. . . . .	26.13	2.30
1300	21.60	. . .	. . . . .	. . . . .	. . . . .	2.17
1400	22.00	1.6	11.76	1.57	. . . . .	2.07
1600	. . . . .	1.5	11.70	. . . . .	. . . . .	1.90
1800	. . . . .	1.4	11.67	. . . . .	. . . . .	1.80
2000	. . . . .	1.3	11.57	. . . . .	. . . . .	1.70
2200	. . . . .	1.2	. . . . .	. . . . .	. . . . .	1.69
2400	. . . . .	1.1	. . . . .	. . . . .	. . . . .	1.68
2600	. . . . .	1.1	. . . . .	. . . . .	. . . . .	. . . .
2800	. . . . .	1.1	. . . . .	. . . . .	. . . . .	. . . .
3000	. . . . .	1.1	. . . . .	. . . . .	. . . . .	. . . .
3200	. . . . .	1.1	. . . . .	. . . . .	. . . . .	. . . .

Properties of Some Cladding Materials

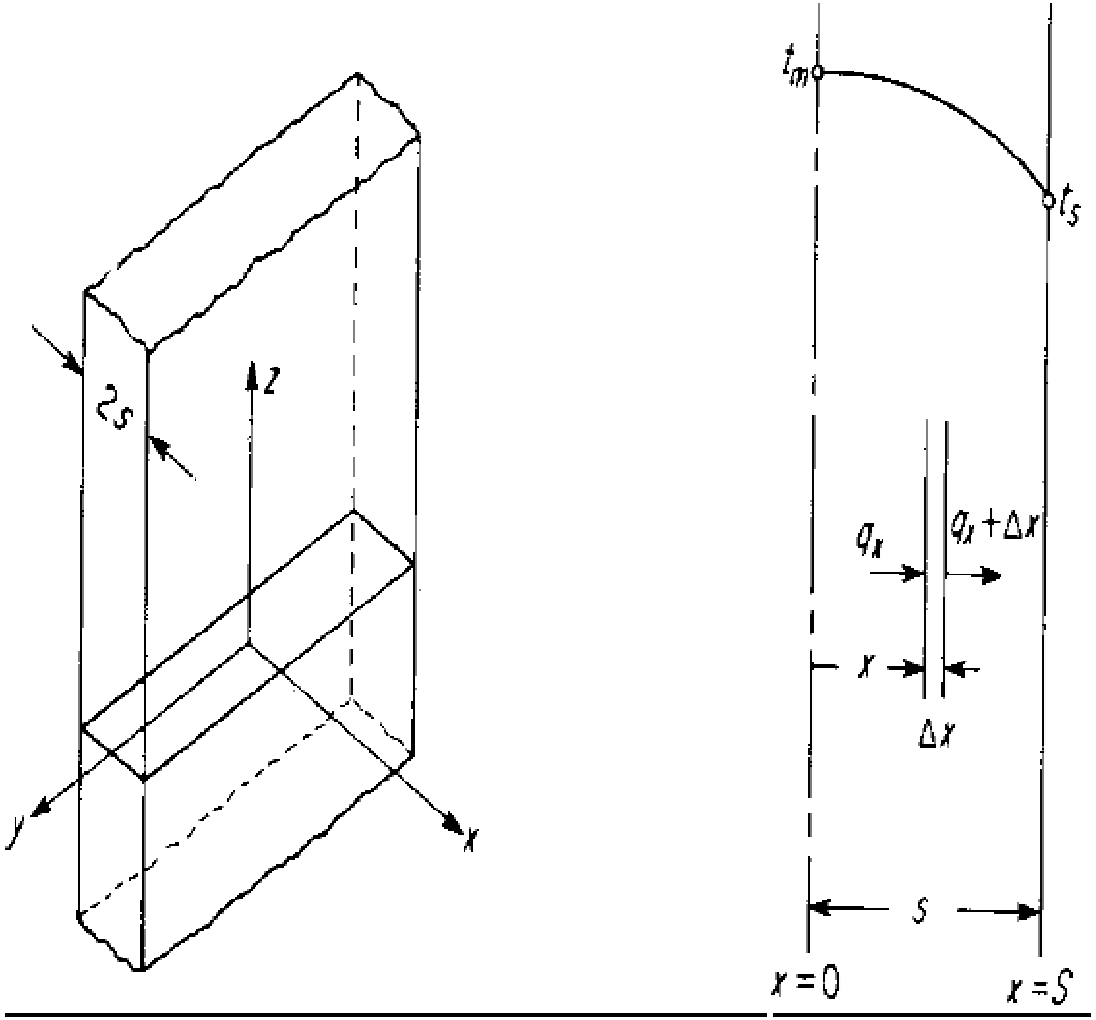
Material	Atomic Number	ρ, Density lb_m/ft^3	N, no. of Nuclei/cm^3	Microscopic Absorption Cross Section σ_a, Barns (thermal)	Macroscopic Absorption Cross Section Σ_a, cm^{-1} (thermal)	k_c, Thermal Conductivity, Btu/ft hr °F	Specific Heat, c_p, Btu/lb_m °F	Melting Point, °F
Beryllium	4	115 (77°F)	0.12×10^{24}	0.010	0.00123	128 (77°F) 82.2 (200°F)	0.506 (261°F)	2330
Magnesium	12	109	0.0431×10^{24}	0.059	0.00254	97 (200°F) 94 (390°F) 91 (570°F)	......	1205
Aluminum	13	169 (68°F)	0.0603×10^{24}	0.215	0.0130	132 (68°F) 131 (390°F) 131 (750°F)	0.230 (212°F) 0.245 (572°F)	1040
Zirconium	40	406 (75°F)	0.0423×10^{24}	0.18 ± 0.02	0.00765	12.1 (120°F) 11.8 (200°F) 11.5 (300°F) 11.0 (500°F) 10.6 (750°F)	0.0739 (260°F) 0.0815 (800°F)	3370
Zircaloy 2 (1.2–1.7% Sn. 0.07–0.2% Fe, 0.05–0.15% Chr, 0.03–0.06% Ni), also Zircaloy 4	...	409	....	....	....	Less than 10	....	3310
Types 304 and 304L stainless steel, 18–20% Cr, 8–12% Ni, 2% max. Mn, 0.08% max. C, 1% max. Si, 0.03% max. C (L)	...	501	....	....	....	9.4 (200°F) 12.4 (930°F)	0.12 (32–212°F)	2550–2650
Type 347 stainless steel, 17–19% Cr, 9–12% Ni, 0.8% Cb, 2% max. Mn, 1% max. Si, 0.08% max. C	...	....	....	....	....	9.3 (200°F) 12.8 (930°F)	0.12 (32–212°F)	2550–2600

Differential Equations of Temperature in Heat Conduction

Equation name	Conduction Mode	Equation
General conduction	Transient with heat generation	$\nabla^2 T + \frac{q'''}{k} = \frac{1}{\alpha}(\frac{dT}{dt})$
Poisson	Steady state with heat generation	$\nabla^2 T + \frac{q'''}{k} = 0$
Fourier	Transient with no heat generation	$\nabla^2 T = \frac{1}{\alpha}(\frac{dT}{dt})$
Laplace	Steady state with no heat generation	$\nabla^2 T = 0$
Helmholtz	Steady state with linear function of temperature term	$\nabla^2 T + B^2 T = 0$

Heat Flow Out of Solid-Plate-Type Fuel Elements

- A thin bare (unclad) plate-type fuel element of thermal conductivity k_f and constant cross-sectional area.
- The dimensions of the element are large in the y and z directions compared with that in the x-direction, so that heat flow can be considered one-dimensional
- ϕ and q''' are constant over the element cross section
- So, heat is conducted equally in the +x and –x directions



Heat Flow Out of Solid-Plate-Type Fuel Elements

- **The heat-conduction equation for this case is the one-dimensional Poisson equation**

$$\nabla^2 T + \frac{q'''}{k_f} = 0 \qquad (1)$$

- **The same equation can be obtained by a heat balance. It is instructive to do this.**

$$(Heat\ crossing\ plane\ x + \Delta x) = Heat\ crossing\ plane\ x + (Heat\ generated\ in\ layer)$$

Thus

$$q_{x+\Delta x} = q_x + q''' A \Delta x \qquad (2)$$

Heat Flow Out of Solid-Plate-Type Fuel Elements

But

$$q_x = -Ak_f \frac{dT}{dx} \quad (3)$$

And

$$q_x + \Delta x = q_x + \frac{d}{dx} q_x \Delta x \quad (4)$$

$$q_x + \Delta x = (-Ak_f \frac{dT}{dx}) + (-Ak_f \frac{d^2T}{dx^2} \Delta x) \quad (5)$$

where *A* is the area of the layer in the yz plane, perpendicular to the direction of heat flow Equation (2) can now be written

$$q_{x+\Delta x} - q_x = q''' A \Delta x \quad (6)$$

Heat Flow Out of Solid-Plate-Type Fuel Elements

$$q'''A\Delta x = -Ak_f \frac{d^2T}{dx^2}\Delta x$$

Divide the equation $\mathbf{A}\ \Delta x$ and rearrange it

$$q''' = -k_f \frac{d^2T}{dx^2}$$

And

$$\frac{d^2T}{dx^2} = -\frac{q'''}{k_f} \qquad (7)$$

Equation (7) double integrated

$$\frac{dT}{dx} = -\frac{q'''}{k_f}x + c_1$$

Heat Flow Out of Solid-Plate-Type Fuel Elements

$$T = -\frac{q'''}{2k_f}x^2 + c_1 + c_2 \qquad (8)$$

Where c_1 and c_2 are the constants of integration

The boundary conditions are:

- **Because of symmetry a round the mid-plane and there is equal and opposite and consequently no net heat flow at the midplane, T_m,**
- **The temperature at the midpoint, is the maximum temperature in the section.**

Heat Flow Out of Solid-Plate-Type Fuel Elements

$$\frac{dT}{dx} = 0 \qquad \text{at } x = 0$$

$$T = T_m \qquad \text{at } x = 0$$

Thus $c_1 = 0$ and $c_2 = T_m$ Substituting the values for c_1 and c_2 into equation (8)

$$\frac{dT(x)}{dx} = -\frac{q'''}{k_f}x \qquad (9)$$

$$T(x) = -\frac{q'''}{2k_f}x^2 + T_m \qquad (10)$$

The *temperature at the surface*, Ts, can be obtained by putting *x = s*, where s is equal to half the element thickness in the *x* direction. Thus

$$T_s = -\frac{q'''}{2k_f}s^2 + T_m \qquad (11)$$

Heat Flow Out of Solid-Plate-Type Fuel Elements

q_x, the heat conducted past any plane x, is equal to the total heat generated between $x = 0$ and x. Thus

$$q_x = q'''A_x \qquad (12)$$

q_s, the heat conducted out of one surface (x = s), equal to the heat generated from one-half of the element, is given by

$$q_s = q'''A_s \qquad q''' = \frac{q_s}{A_s} \qquad (13)$$

By combining equation (11) and (13) and rearranging to give

$$T_s = -\frac{q_s}{2k_f A_s}s^2 + T_m$$

Heat Flow Out of Solid-Plate-Type Fuel Elements

$$q_s = 2k_f A \frac{(T_m - T_s)}{s} \qquad q_s = \frac{(T_m - T_s)}{s/2k_f A} \qquad (14)$$

This expression may be viewed as the heat transfer analogue of Ohm's law in electricity-namely,.

$$I = \frac{V}{R}$$

In the present case,

q **corresponds to** I

$T_m - T_s$ **is analogous to the potential difference, and**

$s/2k_f A$ **is called the thermal resistance**

Thank You

Stay safe!