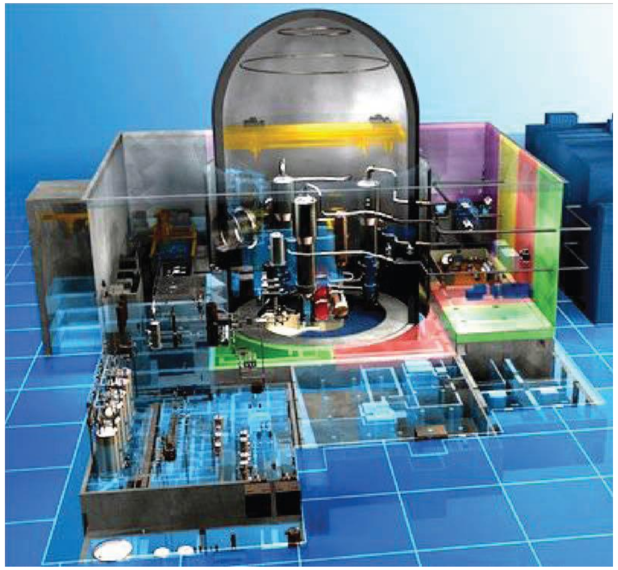
Heat Conduction in Reactor Elements
(Part 2)



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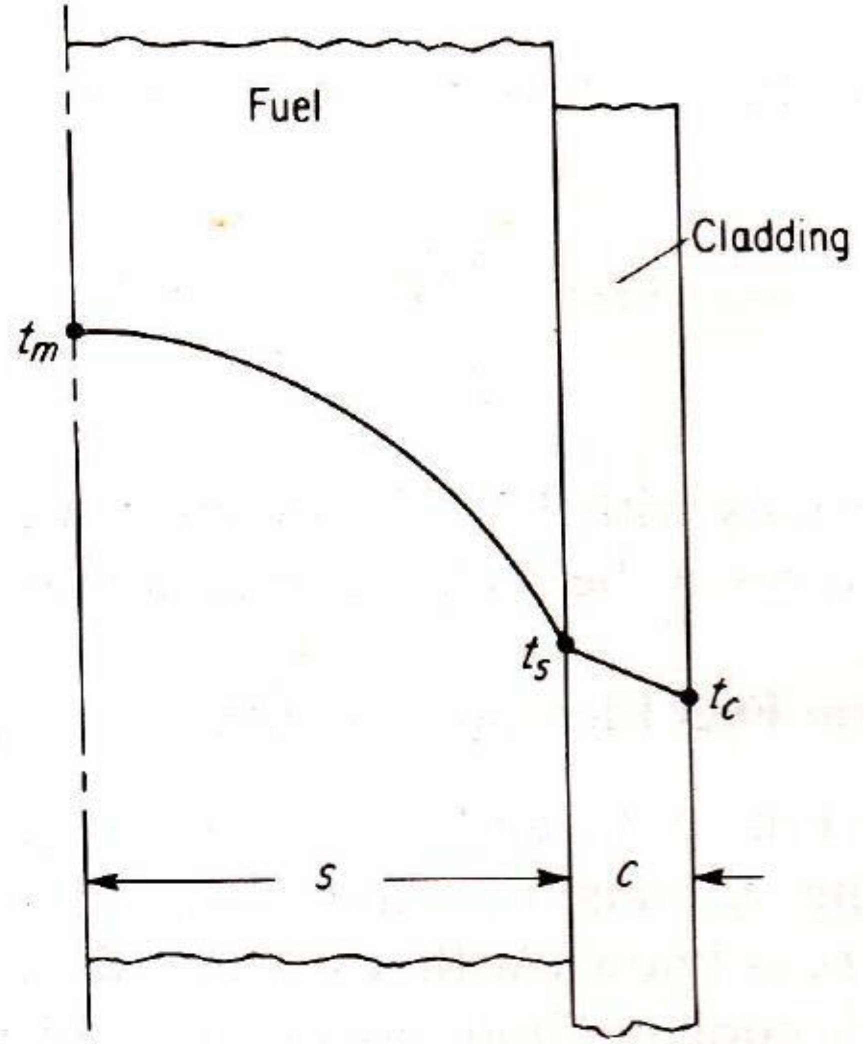
The Effect of Cladding

- System containing a plate-type fuel element of half thickness s with cladding of thickness, c and thermal conductivity k_c,

- The presence of the cladding, an extra, though usually small, resistance to heat conduction results in the decrease of q_s for the same overall temperature drop

- $T_m - T_c$ were the same as $T_m - T_s$ in the previous case



The Effect of Cladding

- In the steady-state since no heat generated in the cladding material, The amount of heat leaving surface s same as that leaving surface c.

- For a constant k_c , $dT\ /dx$ through the cladding is constant.

- Also neglecting resistance to heat flow at the fuel-cladding interface, we can write

$$q_s = q'''As = 2k_f A \frac{T_m - T_s}{s}$$

$$q_s = -Ak_c \frac{dT}{dx}\Big]_{clad} = Ak_c \frac{T_s - T_c}{c} \qquad (14)$$

The Effect of Cladding

$$q_s = q'''As = 2k_f A \frac{T_m - T_s}{s}$$

Solving for the temperature differences,

$$T_m - T_s = \frac{q_s s}{2k_f A} = \frac{q'''}{2k_f} s^2 \qquad (15)$$

$$T_s - T_c = \frac{q_s c}{k_c A} s = \frac{q'''}{k_c} sc \qquad (16)$$

Adding,

$$T_m - T_c = \frac{q_s}{A}\left(\frac{s}{2k_f} + \frac{c}{k_c}\right) = \frac{q''' s^2}{2k_f} + \frac{q''' sc}{k_c} \qquad (17)$$

The Effect of Cladding

Rearranging gives q_s in terms of the temperature

$$q_s = \frac{A(T_m - T_s)}{(\frac{s}{2k_f} + \frac{c}{k_c})} \qquad q_s = \frac{(T_m - T_c)}{(\frac{s}{2Ak_f} + \frac{c}{Ak_c})} \qquad (18)$$

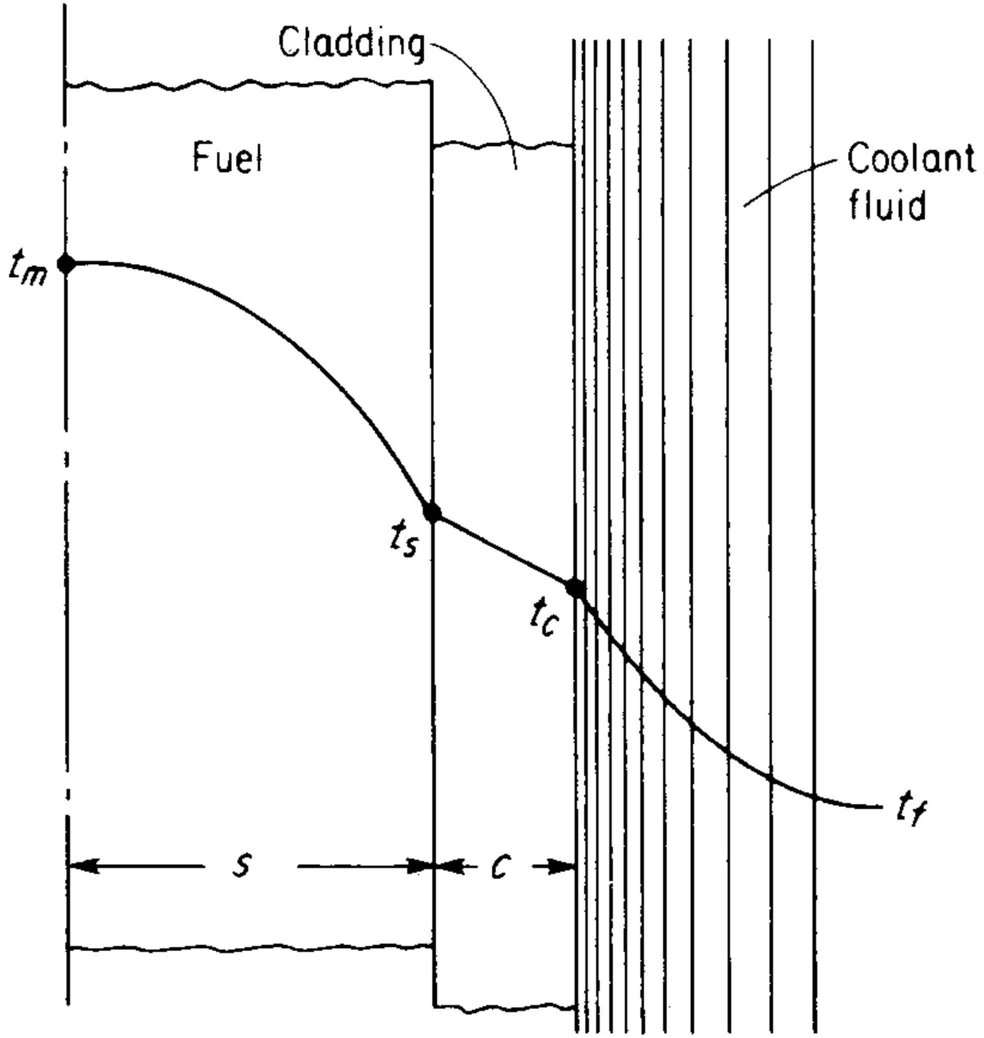
This formula gives the rate at which heat flows through one side of the fuel plate in terms of the difference in temperature between the center and surface of the plate.

$$R = \frac{s}{2Ak_f} + \frac{c}{Ak_c} \qquad (19)$$

R is the total thermal resistance of the fuel and cladding.

The Heat Transfer from Fuel Element to Coolant

- **A clad fuel plate with coolant fluid passing parallel to it.**
- **At steady state, with no heat produced in cladding or coolant**



The Heat Transfer from Fuel Element to Coolant

If q_s, is the heat leaving the fuel element

$$q_s = q'''As = 2k_f A\frac{T_m - T_s}{s} = Ak_c\frac{T_s - T_c}{c} = hA(T_c - T_f) \qquad (20)$$

Solving for the temperature differences,

$$T_m - T_s = \frac{q_s s}{2k_f A} = \frac{q'''}{2k_f}s^2$$

$$T_s - T_c = \frac{q_s c}{k_c A}s = \frac{q'''}{k_c}sc$$

$$T_c - T_f = \frac{q_s}{hA} = \frac{q''' s}{h} \qquad (21)$$

The Heat Transfer from Fuel Element to Coolant

Adding,

$$T_m - T_f = \frac{q_s}{A}\left(\frac{s}{2k_f} + \frac{c}{k_c} + \frac{1}{h}\right) = \frac{q''' s^2}{2k_f} + q''' s\left(\frac{c}{k_c} + \frac{1}{h}\right) \qquad (22)$$

Which may be written as

$$q_s = \frac{(T_m - T_f)}{\left(\frac{s}{2k_f A} + \frac{c}{k_c A} + \frac{1}{hA}\right)} \qquad (23)$$

$$q''' = \frac{(T_m - T_f)}{\left(\frac{s^2}{2k_f} + \frac{cs}{k_c} + \frac{s}{h}\right)} \qquad (24)$$

Interdependence of Temperature, Heat Transfer and Heat (or Neutron) Flux

Equations (24) **may be used to explain some of the limitations on heat generation in nuclear reactors.**

$$q''' = \frac{(T_m - T_f)}{(\frac{s^2}{2k_f} + \frac{cs}{k_c} + \frac{s}{h})}$$

For any constant value of q''', if T_f is to be kept as high as possible for good plant thermal efficiency,

h has to be increased materially (since it affects only part of the equation) to keep the maximum fuel temperature T_m or the maximum cladding temperature T_s, from becoming excessively highly.

Interdependence of Temperature, Heat Transfer and Heat (or Neutron) Flux

- ***On the other hand***, if for metallurgical or other reasons these temperatures are limited to certain values, h
- The value of q''' may be regulated by changing the neutron flux with the help of the control rods.
- In essence, there is no limit to the quantity of generating, so long as adequate cooling is provided to keep the temperatures in the system from exceeding their safe limits.

Exercise 1

A ***plate-type*** **fuel element is made of 1.5 percent enriched uranium metal. The element is 4 ft long, 3.5 in. wide, and 0.2 in. thick. It is clad in Type 304l stainless steel with 0.005 in. thick. The effective thermal neutron flux at the point of maximum temperature (slightly above the centre of the core) is** 30×10^{13} **neutrons/sec cm²**. **For good plant efficiency, the coolant bulk temperature at that point should be no lower than 600 °F. Find**

Given:

$$G = 180 \text{ MeV}$$

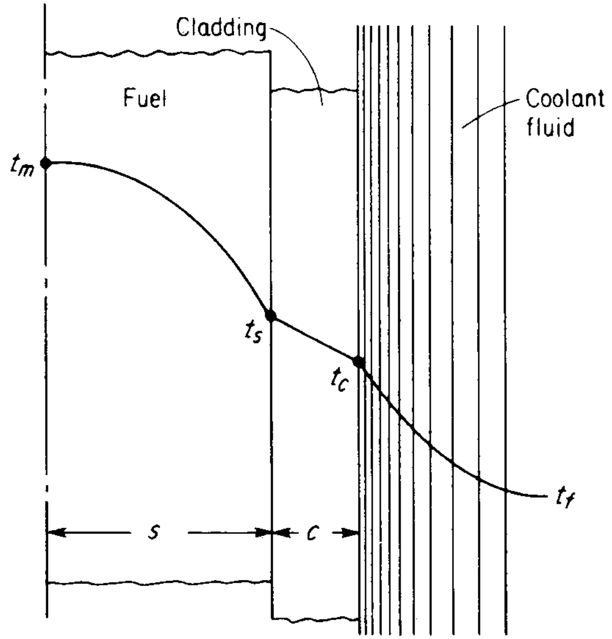
$$\text{The effective fission cross section}, \sigma_f = 364 \text{ b}$$

a) **The volumetric thermal of element at that point,** q'''
b) **The minimum value of** h **if the fuel temperature should not exceed 700 °F**
c) **The maximum cladding and coolant temperatures**
d) **The minimum value of** h **if the fuel temperature should not exceed 700 °F, but for neutron fluxes of** 2×10^{13} **and** 4×10^{13}

Exercise 1 (Solution)

Solution. The physical properties of the fuel and cladding are evaluated at their respective mean temperatures. Since these are not known, the boundary temperatures are assumed as follows:

$T_m = 700°\text{F}$	**Given**
$T_s = 660°\text{F}$	**Assumed**
$T_c = 650°\text{F}$	**Assumed**
$T_f = 600°\text{F}$	**Given**



Mean fuel and cladding temperatures are 680 and 655 °F

ρ density of uranium metal at 680 °F = 18.71 gm/cm^3

k_f conductivity at same temperature= 18.5 Btu/hr ft °F

k_c of 304L stainless steer at 655 °F = 11.36 Btu/hr ft °F

From table in slide (Part 1)

Exercise 1 (Solution)

Therefore

(a) The volumetric thermal of element at that point, q'''

$$N_f = \frac{N_a \rho \nu (f)}{M} = \frac{(0.60225 \times 10^{24}\ \text{atoms/mole})(18.71\ \frac{\text{g}}{\text{cm}^3})(0.015)}{235.044\ g/mole} = 7.19 \times 10^{20}\ \text{nuclei/cm}^3$$

$$q''' = G N_f \sigma_f \phi$$

$$q''' = (180\ MeV)(7.19 \times 10^{20})(364 \times 10^{-24})(3 \times 10^{13}) = 1.413 \times 10^{15} \frac{MeV}{sec\ cm^3}$$

$$1.413 \times 10^{15} \frac{MeV}{sec\ cm^3} \times 1.548 \times 10^{-8} = 2.185 \times 10^{7} \frac{Btu}{hr \cdot ft^3}$$

Exercise 1 (Solution)

(b) The minimum value of h if the fuel temperature should not exceed 700 °F

$$q''' = 2.185 \times 10^7 \frac{Btu}{hr \cdot ft^3}, s = \frac{0.2}{2} in., c = 0.005\ in., T_f = 600°\text{F}, T_m = 700°\text{F}$$

$$T_m - T_f = \frac{q''' s^2}{2k_f} + q''' s\left(\frac{c}{k_c} + \frac{1}{h}\right)$$

$$700 - 600 = \frac{\left(2.185 \times 10^7 \frac{Btu}{hr \cdot ft^3}\right)\left(\frac{0.1}{12} ft\right)^2}{2\left(18.5 \frac{Btu}{hr \cdot ft \cdot °\text{F}}\right)} + (2.185 \times 10^7 \frac{Btu}{hr \cdot ft^3})(\frac{0.1}{12} ft)(\frac{\frac{0.005}{12} ft}{11.36 \frac{Btu}{hr \cdot ft \cdot °\text{F}}} + \frac{1}{h})$$

$$700 - 600 = 41.059 + (1.823 \times 10^5)(\frac{0.005\ ft}{12(11.36 \frac{Btu}{hr \cdot ft \cdot °\text{F}})} + \frac{1}{h})$$

$$h\left(for\ \phi = 3 \times 10^{13}\right) = 3488 \frac{Btu}{hr \cdot ft^2 \cdot °\text{F}}$$

Exercise 1 (Solution)

(c) The maximum cladding and coolant temperatures

$$q''' = 2.185 \times 10^{7}\,\frac{Btu}{hr \cdot ft^{3}}\,,\ s = \frac{0.2}{2}\,in.\,,\ c = 0.005\,in.\,,\ T_f = 600°\mathrm{F}\,,\ T_m = 700°\mathrm{F}$$

$$\mathbf{700 - T_s = \frac{(2.185 \times 10^{7}\,\frac{Btu}{hr \cdot ft^{3}})}{2(18.5\,\frac{Btu}{hr \cdot ft \cdot °F})}\left(\frac{0.1}{12}\,ft\right)^{2}}$$

$$\mathbf{T_s = 700 - 41.3 = 658.9°F}$$

Exercise 1 (Solution)

$$t_c - t_f = \frac{q'''s}{h}$$

$$t_c - 600 = \frac{(2.185 \times 10^7 \frac{Btu}{hr \cdot ft^3})(\frac{0.1}{12} ft)}{3488 \frac{Btu}{hr \cdot ft^2 \cdot °F}}$$

$$T_c = 600 + 52.3 = 652.3°F$$

$$T_s = 700 - 41.3 = 658.9°F$$

$T_s = 660°F$ Assumed
$T_c = 650°F$ Assumed

T_c is the highest coolant temperature

At this point these calculated boundary temperatures are compared with the ones assumed at the beginning of the solution.

The differences are sufficiently small, and the answers are therefore sufficiently accurate.

Exercise 1 (Solution)

(d) The minimum value of h if the fuel temperature should not exceed 700 °F, but for neutron fluxes of 2×10^{13} and 4×10^{13}

Repeat the same method used in part (b)

$$h\left(for\ \phi = 2 \times 10^{13}\right) = 1783\,\frac{Btu}{hr \cdot ft^2 \cdot °F}$$

$$h\left(for\ \phi = 4 \times 10^{13}\right) = 6682\,\frac{Btu}{hr \cdot ft^2 \cdot °F}$$

h the minimum necessary to keep the maximum fuel temperature within 700°F, increase rapidly with neutron flux

Maximum Neutron Flux

This theoretical maximum value of flux is obtained by putting h infinite and evaluating a corresponding value of

$$T_m - T_f = \frac{q'''s^2}{2k_f} + q'''s(\frac{c}{k_c} + \frac{1}{h})$$

$$q'''_{max} = \frac{(T_m - T_f)}{(\frac{s^2}{2k_f} + \frac{sc}{k_c})}$$

In effect, this makes the temperature of the coolant equal to the temperature of the cladding's outer surface, T_c

Maximum Neutron Flux

ϕ_{max} can be then evaluated from q'''_{max}

$$\phi_{max} = \frac{q'''_{max}}{G N_f \sigma_f}$$

To increase ϕ beyond ϕ_{max} for a given fuel and cladding configuration,

- **It is necessary to lower T_f *(with its undesirable* effect on plant thermodynamic efficiency)**
- **To increase T_m *(at the expense* of reduced fuel burnup or otherwise by fuel alloying or using ceramic fuels), or both.**

Exercise 2

Calculate the maximum possible neutron flux corresponding to the data from Exercise 1

$q''' = 2.185 \times 10^7 \frac{Btu}{hr \cdot ft^3}$, $\phi = 3 \times 10^{13}\ neutrons/\sec cm^2$, $s = \frac{0.2}{2} in.$, $c = 0.005\ in.$, $T_f = 600°\text{F}$, $T_m = 700°\text{F}$

Given:

$\mathbf{G = 180\ MeV}$

The effective fission cross section, $\boldsymbol{\sigma_f = 364\ b}$

Exercise 2 (Solution)

Calculate the maximum possible neutron flux corresponding to the data from Exercise 1

$q''' = 2.185 \times 10^7 \frac{Btu}{hr \cdot ft^3}$, $\phi = 3 \times 10^{13}\ neutrons/\sec cm^2$, $s = \frac{0.2}{2} in.$, $c = 0.005\ in.$, $T_f = 600°F$, $T_m = 700°F$, $T_c = 652.3°F$

$$q'''_{max} = \frac{(T_m - T_{f_{max}})}{(\frac{s^2}{2k_f} + \frac{sc}{k_c})}$$

$$q'''_{max} = \frac{(700 - 652.3)}{(\frac{(\frac{0.1}{12})^2}{2(18.5)} + \frac{\left(\frac{0.1}{12}\right)(\frac{0.005}{12})}{11.36})} = 4.06 \times 10^7 \frac{Btu}{hr \cdot ft^3}$$

$$q'''_{max} = \frac{4.06 \times 10^7}{1.548 \times 10^{-8}} = 2.62 \times 10^{15} \frac{MeV}{sec \cdot cm^3}$$

Exercise 2 (Solution)

$$\phi_{max} = \frac{q'''_{max}}{G N_f \sigma_f}$$

$$\phi_{max} = \frac{2.62\times10^{15}}{(180)(7.19\times10^{20})(364\times10^{-24})} = 5.57 \times 10^{13}\ \frac{Neutrons}{sec \cdot cm^2}$$

Thank You

Stay safe!