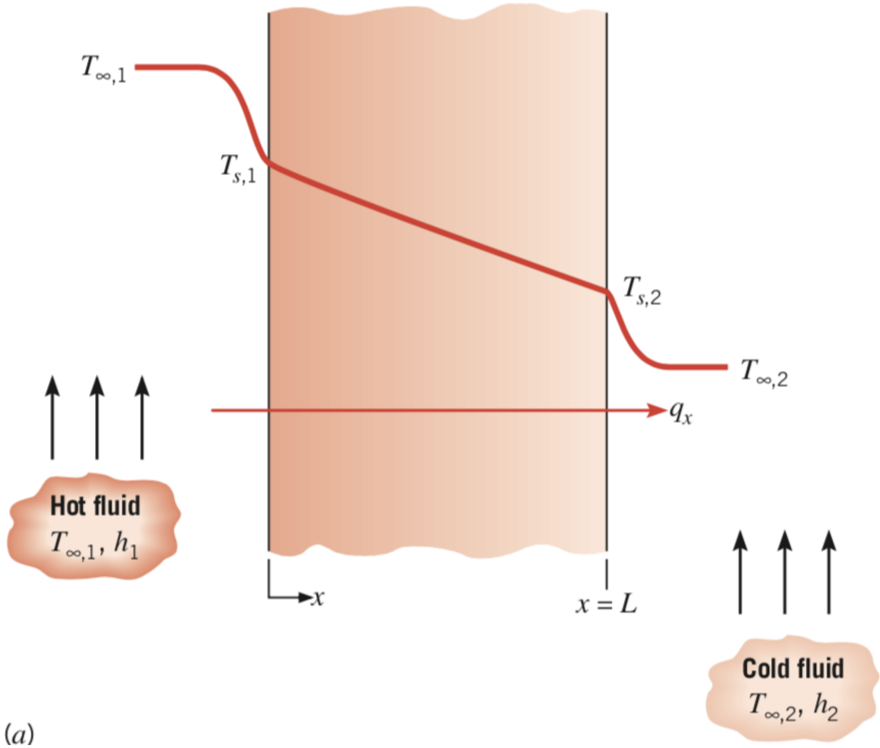
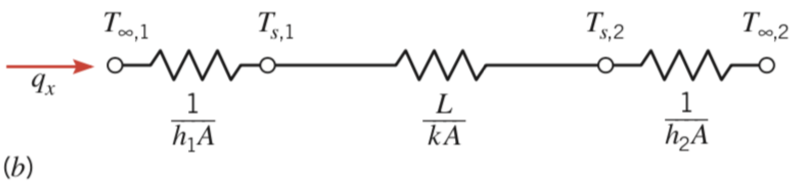
UTM
UNIVERSITI TEKNOLOGI MALAYSIA

1D STEADY STATE CONDUCTION

Plane Wall



1 Heat transfer through a
(a) Temperature distribution.



(b) Equivalent thermal circuit.

Temperature Distribution

- The temperature distribution in the wall can be determined by solving the heat equation with the proper boundary conditions.

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)=0 \qquad k\,\frac{d}{dx}\left(\frac{dT}{dx}\right) = 0$$

- For one-dimensional, steady-state conduction in a plane wall with no heat generation, the heat flux is a constant, independent of x.

- If the thermal conductivity of the wall material is assumed to be constant, the equation may be integrated twice to obtain the general solution:

$$\frac{d}{dx}\left(\frac{dT}{dx}\right)=0 \qquad \frac{dT}{dx}=C_1$$

$$T(x)=C_1x+C_2$$

Temperature Distribution

$$T(x) = C_1 x + C_2$$

- To obtain the constants of integration, C_1 and C_2, boundary conditions must be introduced.

- Applying the condition at x = 0 to the general solution, it follows that:

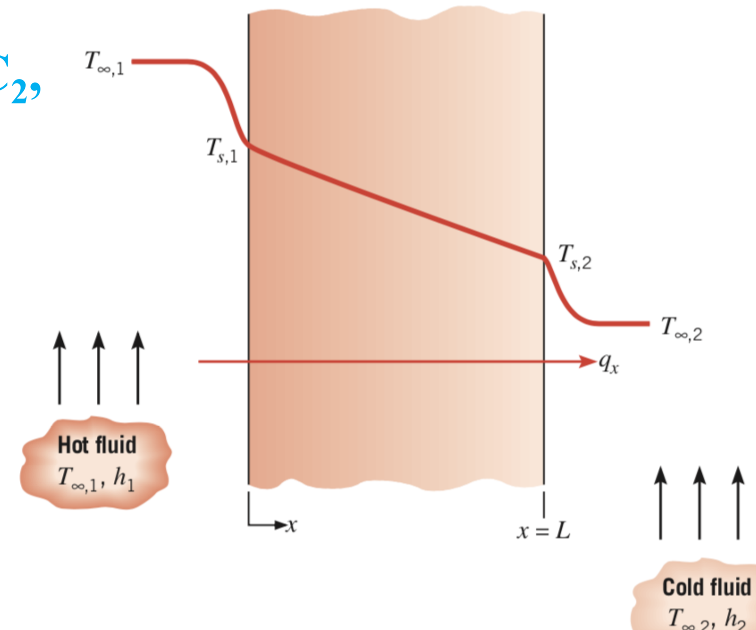
$$T_{s,1} = T(x=0) = C_1 * 0 + C_2 \quad \rightarrow \quad T_{s,1} = C_2$$

Similarly, at x = L,

$$T_{s,2} = T(x=L) = C_1 * L + C_2 \quad \rightarrow T_{s,2} = C_1 * L + T_{s,1}$$

in which case;

$$C_1 * L = T_{s,2} - T_{s,1} \quad \rightarrow C_1 = \frac{T_{s,2} - T_{s,1}}{L}$$



(a)

- Substituting into the general solution, the temperature distribution is then:

$$T(x) = \left(T_{s,2} - T_{s,1}\right)\frac{x}{L} + T_{s,1}$$

Thermal Resistance

- Just as an electrical resistance is associated with the conduction of electricity, a thermal resistance may be associated with the conduction of heat.

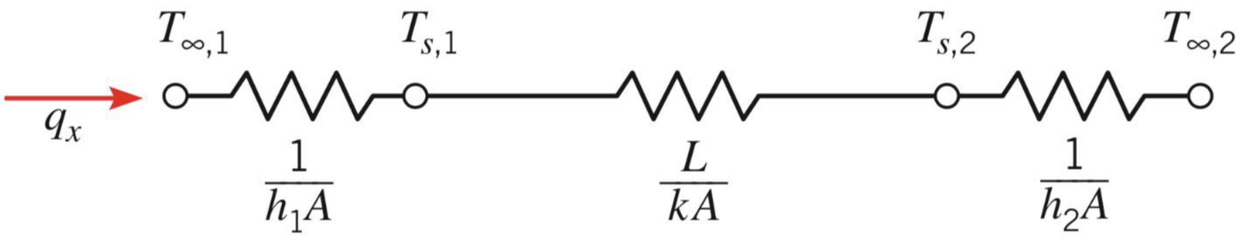
- Defining resistance as the ratio of a driving potential to the corresponding transfer rate, the thermal resistance for conduction in a plane wall is:

$$R_{t,\text{cond}} \equiv \frac{T_{s,1} - T_{s,2}}{q_x} = \frac{L}{kA}$$

- The thermal resistance for convection is then:

$$R_{t,\text{conv}} \equiv \frac{T_s - T_\infty}{q} = \frac{1}{hA}$$

Thermal Resistance



For ***one-dimensional, steady-state conduction in a plane wall with no heat generation, the heat flux is a constant, independent of x.***

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{1/h_1 A} = \frac{T_{s,1} - T_{s,2}}{L/kA} = \frac{T_{s,2} - T_{\infty,2}}{1/h_2 A}$$

$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}}$$

$$R_{tot} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

Composite Wall

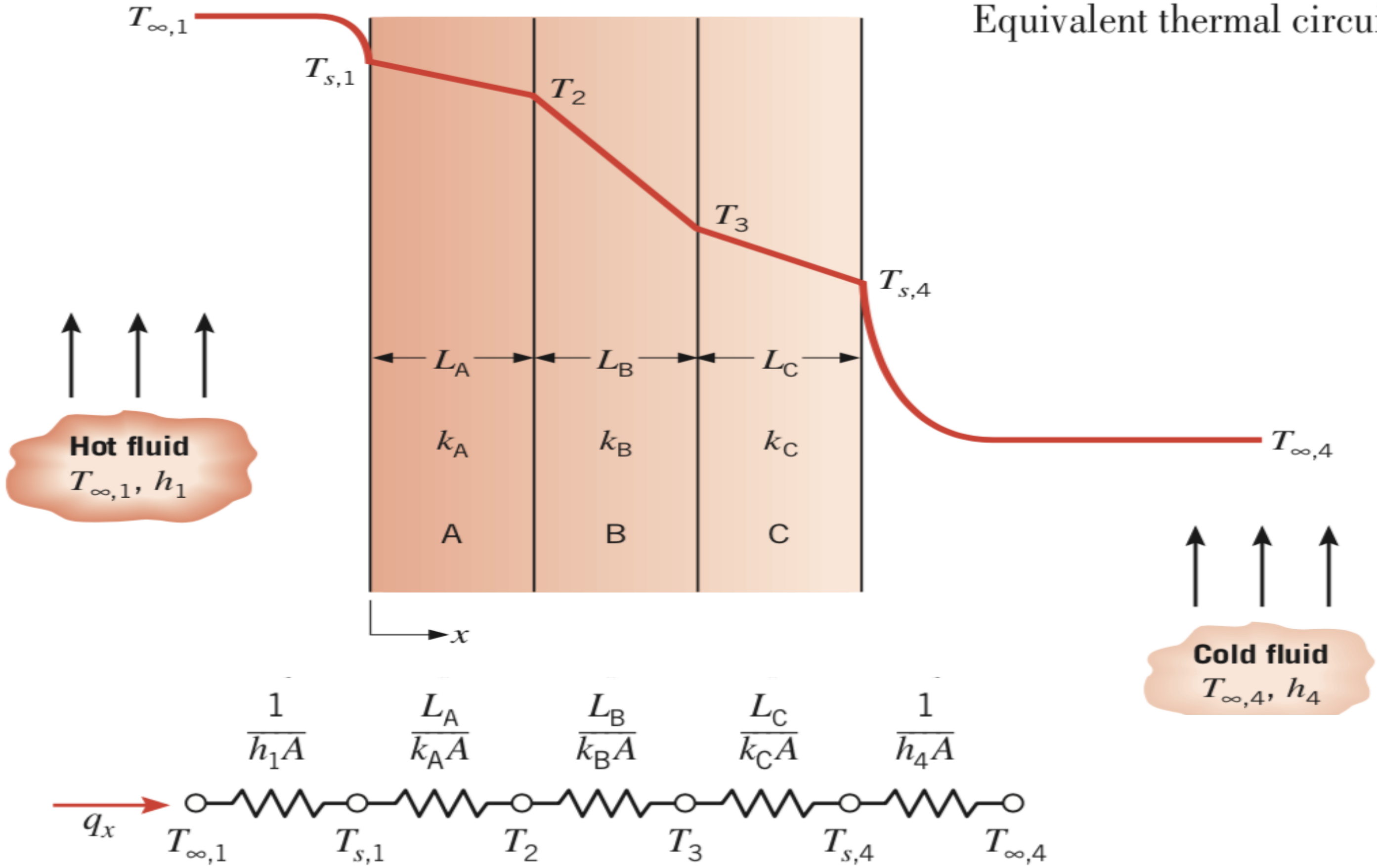
- Equivalent thermal circuits may also be used for more complex systems, such as composite walls.
- Such walls may involve any number of series and parallel thermal resistances due to layers of different materials.
- The one-dimensional heat transfer rate for this system may be expressed as:

$$q_x = \frac{T_{\infty,1} - T_{\infty,4}}{\Sigma R_t}$$

- Where $T_{\infty,1} - T_{\infty,4}$ (free stream) is the overall temperature difference, and the summation includes all thermal resistances.

Composite Wall

Equivalent thermal circuit for a series composite wall.



$$q_x = \frac{T_{\infty,1} - T_{\infty,4}}{[(1/h_1A) + (L_A/k_AA) + (L_B/k_BA) + (L_C/k_CA) + (1/h_4A)]}$$

Composite Wall

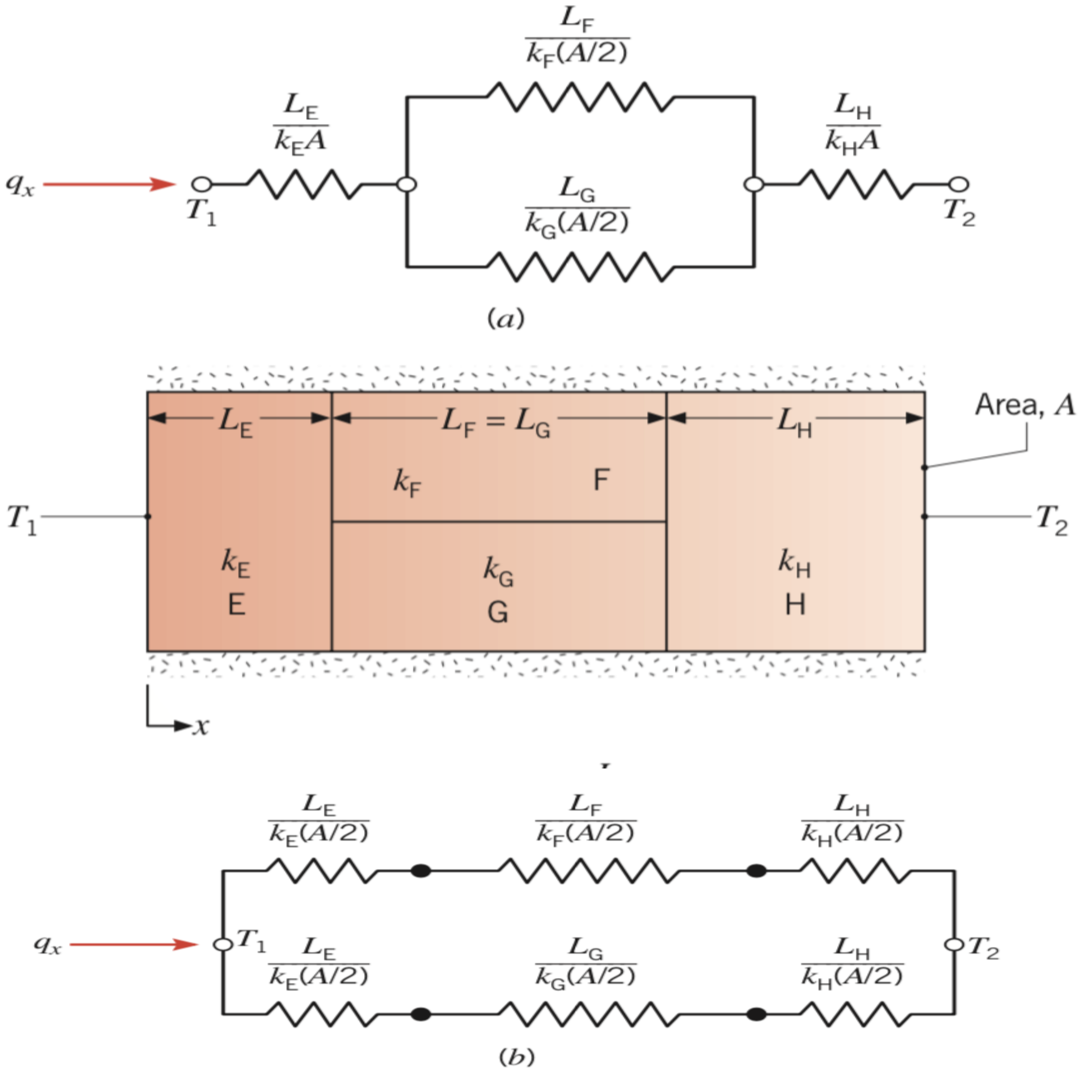


FIGURE 3.3 Equivalent thermal circuits for a series–parallel composite wall.

SUMMARY

TABLE 3.3 One-dimensional, steady-state solutions to the heat equation with no generation

	Plane Wall	Cylindrical Wall[a]	Spherical Wall[a]
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Heat flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\ln(r_2/r_1)}$	$\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$
Thermal resistance ($R_{t,\mathrm{cond}}$)	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$

[a]The critical radius of insulation is $r_{cr} = k/h$ for the cylinder and $r_{cr} = 2k/h$ for the sphere.

Thermal Resistance

- For some cases, it is convenient to express the net radiation heat exchange in the form:

$$q_{\mathrm{rad}} = h_r A(T_s - T_{\mathrm{sur}})$$

- Where h_r, is the ***radiation heat transfer coefficient***:

$$h_r \equiv \varepsilon\sigma(T_s + T_{\mathrm{sur}})(T_s^2 + T_{\mathrm{sur}}^2)$$

- Therefore, thermal resistance for radiation may be defined by:

$$R_{t,\mathrm{rad}} = \frac{T_s - T_{\mathrm{sur}}}{q_{\mathrm{rad}}} = \frac{1}{h_r A}$$