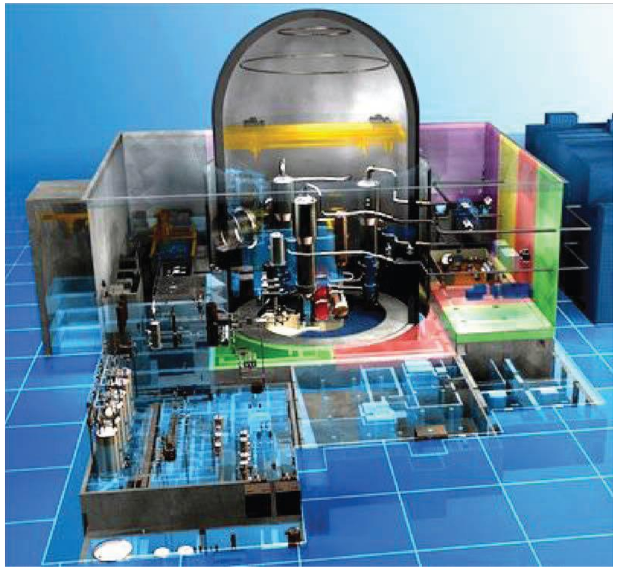
Heat Conduction in Reactor Elements
(Part 4)



MUHAMMAD SYAHIR SARKAWI, PhD
Nuclear Engineering Program
Energy Engineering Department
N01-273 | 0133274154
syahirsarkawi@utm.my

Heat Flow Out of Solid Cylindrical Fuel Elements

- **Unclad cylindrical fuel element**, or ***fuel rod*** or ***pin***, of a **diameter small** compared with that of the reactor core will be treated.

- **The neutron flux will be assumed not to change appreciably in either the axial or radial directions.**

- **The heat flow out of the fuel element will substantially be radial and will be equal in all directions.**

- **The equation here is the one-dimensional Poisson equation in cylindrical coordinates, given by**

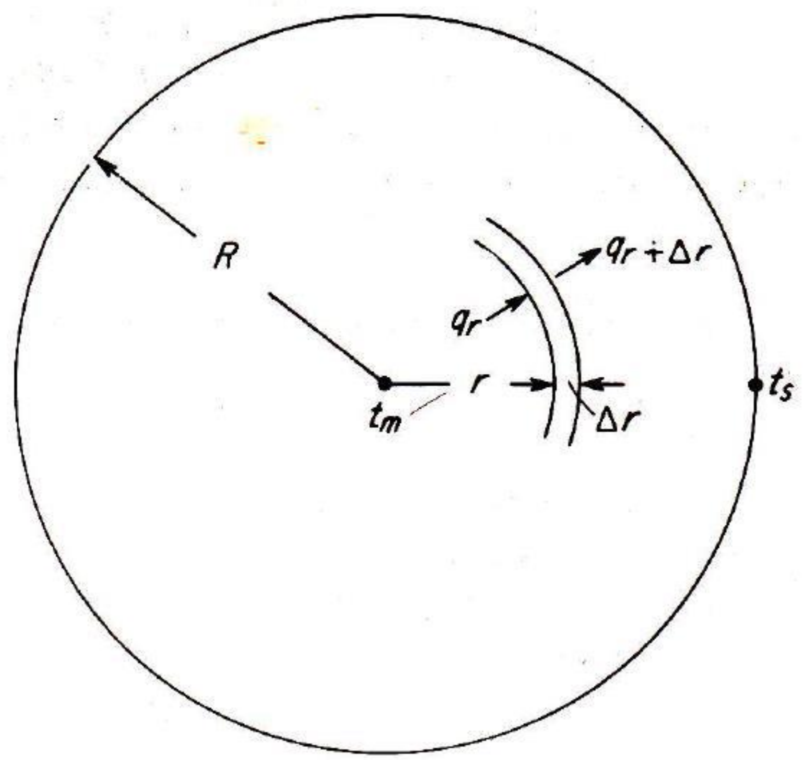
$$\nabla^2 T + \frac{q'''}{k} = 0 \quad \textbf{where} \quad \nabla^2 T = \left(\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right)$$

Heat Flow Out of Solid Cylindrical Fuel Elements

$$\nabla^2 T + \frac{q'''}{k} = 0 \qquad \nabla^2 T = \left(\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right)$$

- **This equation can also be obtained by a heat balance on a cylindrical shell within the fuel element.**
- **The cross section of an unclad element having a radius R and length L**
- **A thin cylindrical shell at r of thickness Δr.**
- **A heat balance of steady-state heat transfer as follows:**

$$q'''(2\pi r \Delta r L) = q_{r+\Delta r} - q_r$$



Heat Flow Out of Solid Cylindrical Fuel Elements

$$q'''(2\pi r \Delta r L) = q_{r+\Delta r} - q_r \qquad (1)$$

where

$$q_r = -Ak_f \frac{dT}{dr} = -2\pi r L k_f \frac{dT}{dr} \qquad (2)$$

and

$$q_{r+\Delta r} = q_r + \frac{dq_r}{dr}\Delta r$$

$$q_{r+\Delta r} = q_r - 2\pi L k_f \Delta r \frac{d}{dr}(r\frac{dT}{dr})$$

$$q_{r+\Delta r} = q_r - 2\pi L k_f (r\frac{d^2T}{dr^2} + \frac{dT}{dr})\Delta r$$

$$q_{r+\Delta r} - q_r = -2\pi L k_f (r\frac{d^2T}{dr^2} + \frac{dT}{dr})\Delta r \qquad (3)$$

Heat Flow Out of Solid Cylindrical Fuel Elements

Substitute equation (2) and (3)in (1)

$$q'''(2\pi r\Delta r L) = -2\pi L k_f(r\frac{d^2T}{dr^2} + \frac{dT}{dr})\Delta r$$

$$q'''r = -k_f(r\frac{d^2T}{dr^2} + \frac{dT}{dr})$$

$$q'''r = -k_f\frac{d}{dr}(r\frac{dT}{dr}) \qquad (4)$$

Heat Flow Out of Solid Cylindrical Fuel Elements

$$\nabla^2 T + \frac{q'''}{k} = 0 \qquad \nabla^2 T = \left(\frac{dT^2}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right)$$

$$q'''r = -k_f\left(r\frac{d^2T}{dr^2} + \frac{dT}{dr}\right)$$

$$\frac{q'''}{k_f} = -\frac{1}{r}\left(r\frac{d^2T}{dr^2} + \frac{dT}{dr}\right)$$

$$\frac{q'''}{k_f} = -\left(\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right)$$

This reduces to

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{q'''}{k_f} = 0$$

Heat Flow Out of Solid Cylindrical Fuel Elements

$$q'''r = -k_f \frac{d}{dr}(r\frac{dT}{dr})$$

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{q'''}{k_f} = 0$$

$$\frac{d}{dr}(r\frac{dT}{dr}) + \frac{q'''}{k_f}r = 0$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{q'''}{k_f}r$$

Integrate 2 times

$$\left(r\frac{dT}{dr}\right) = -\frac{q'''}{2k_f}r^2 + C_1$$

$$\frac{dT}{dr} = -\frac{q'''}{2k_f}r + \frac{C_1}{r}$$

Heat Flow Out of Solid Cylindrical Fuel Elements

$$\frac{dT}{dr} = -\frac{q'''}{2k_f} r + \frac{C_1}{r}$$

$$T(r) = -q''' \frac{r^2}{4k_f} + C_1 \ln r + C_2 \qquad (5)$$

C_1 and C_2, the constants of the double integration. Hence, apply boundary condition as follows:

Bondary condition 1

$\frac{dT}{dr} = 0$ at $r = 0$ $\qquad 0 = -\frac{q'''}{2k_f}(0) + \frac{C_1}{r} \longrightarrow C_1 = 0$

Bondary condition 2

$T = T_m$ at $r = 0$ $\qquad T_m = T(0) = -q''' \frac{(0)^2}{4k_f} + (0)\ln(0) + C_2 \longrightarrow C_2 = T_m$

Heat Flow Out of Solid Cylindrical Fuel Elements

Hence,

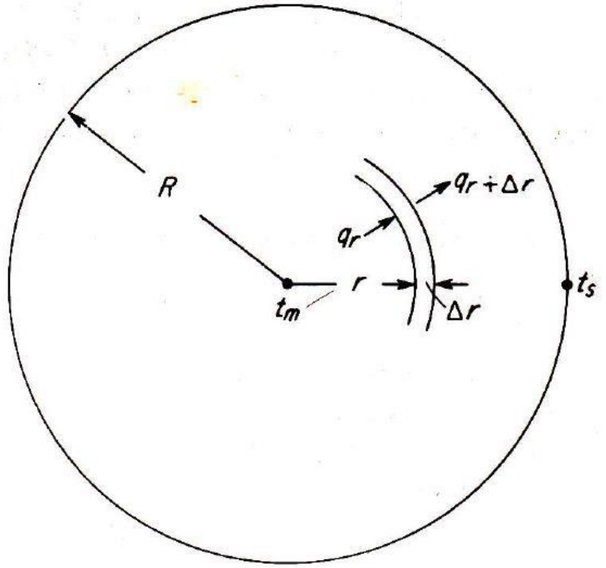
$$T(r) = -q''' \frac{r^2}{4k_f} + \mathrm{T_m}$$

Solution expressing T in terms of r is,

$$T(r) = \mathrm{T_m} - \frac{q''' r^2}{4k_f} \qquad (6)$$

The maximum temperature difference in the system may be obtained by putting $r = R$ and $T = T_s$

$$T_m - T_s = \frac{q''' R^2}{4k_f} \qquad (7)$$



Heat Flow Out of Solid Cylindrical Fuel Elements

The heat flow $q(r)$ at any radius r is equal to the total heat generated by the element within that radius. Thus

$$q(r) = \pi r^2 L q''' \qquad (8)$$

$$T_m - T_s = \frac{q''' R^2}{4k_f}$$

The heat conducted out of the periphery of the element q_s, is equal to the heat generated in the entire element And is given by:

$$q_s = \pi R^2 L q''' \qquad q''' = \frac{q_s}{\pi R^2 L} \qquad (9)$$

$$T_m - T_s = \frac{R^2}{4k_f} \times \frac{q_s}{\pi R^2 L} = \frac{1}{4k_f}\frac{q_s}{\pi L} \qquad q_s = 4\pi L k_f (T_m - T_s) \qquad (10)$$

Heat Flow Out of Solid Cylindrical Fuel Elements

Total heat generated is only a function of the maximum temperature drop $(T_m - T_s)$ and is independent of the volumetric

Putting $2\pi RL = A_s$, the total heat generated

$$A_s = 2\pi RL \qquad \frac{A_s}{R} = 2\pi L \qquad \frac{2A_s}{R} = 4\pi L$$

$$q_s = 4\pi L k_f (T_m - T_s) \qquad q_s = \frac{2A_s}{R} k_f (T_m - T_s)$$

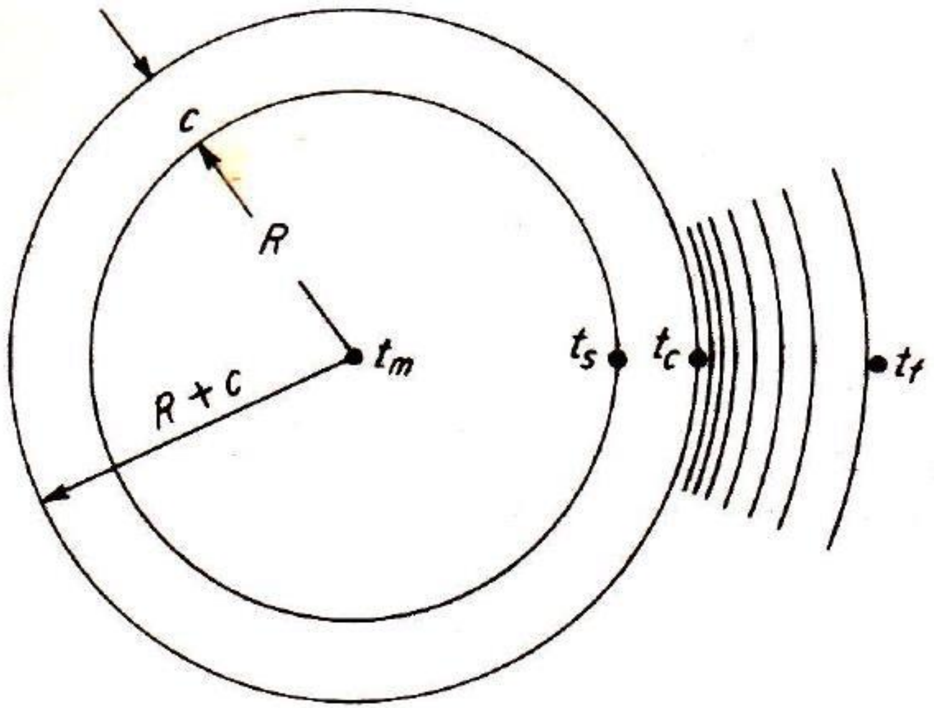
$$q_s = 2k_f A_s \left(\frac{T_m - T_s}{R}\right) \qquad (11)$$

Effect of Cladding and Coolant

- The cross section of a cylindrical fuel element of radius R and axial length L, having a center temperature T_m and surrounded by cladding of radial thickness c and coolant fluid having bulk temperature T_f

- In the steady state with no heat generated in cladding or coolant,

- The heat passing out of the fuel surface is the same as that passing through the cladding and into the coolant fluid



Effect of Cladding and Coolant

The heat conduction equation for the cladding is

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0 \qquad (12)$$

which has the solution

$$T(r) = C_1 \, ln\,(r) + C_2 \qquad (13)$$

Using the boundary conditions

B. C (1)

$$T(R) = T_s$$

B. C (2)

$$T(R + c) = T_c$$

to determine the constants C_1 and C_2 gives

$$T(r) = \frac{T_s \, ln\,(R + C) - T_c \, ln\,(R) - (T_s - T_c)\, ln\,(r)}{ln\,(1 + \frac{C}{R})} \qquad (14)$$

Effect of Cladding and Coolant

$$T(r) = \frac{T_s \ln(R + C) - T_c \ln(R) - (T_s - T_c) \ln(r)}{\ln(1 + \frac{C}{R})}$$

From Fourier's law, the total heat flowing out of the cladding is

$$q_s = -k_c A \frac{dT}{dr}$$

$$A_{R+c} = 2\pi L(R + c) \qquad (15)$$

$$q_s = -2\pi r L k_c \frac{dT}{dr} \qquad (16)$$

evaluated at $r = R + c$. Carrying out the differentiation of equation (14) yields

$$q_s = 2\pi L k_c \frac{(T_s - T_c)}{\ln(\frac{R + c}{R})} \qquad (17)$$

Effect of Cladding and Coolant

The expression for heat flow through the cladding is that for heat flow by conduction through a hollow cylinder.

$$q_s = -4\pi L k_f (T_m - T_s)$$

$$A_R = 2\pi L R$$

For cladding

$$q_s = 2\pi L k_c \frac{(T_s - T_c)}{\ln\left(\frac{(R+c)}{R}\right)}$$

$$A_m = \frac{2\pi L c}{\ln\left(\frac{(R+c)}{R}\right)} \qquad (18)$$

For coolant

$$q_s = 2\pi (R+c) L h (T_m - T_s)$$

$$A_{R+c} = 2\pi L (R+c) \qquad (19)$$

Rearrange the equation 17, 18 and 19

Effect of Cladding and Coolant

$$T_m - T_s = \frac{q_s}{4\pi K_f L} = \frac{q''' R^2}{4k_f} \qquad (20)$$

$$T_s - T_c = \frac{q_s \, ln(\frac{R+c}{R})}{2\pi K_c L} = \frac{q''' R^2 ln(\frac{R+c}{R})}{2k_c} \qquad (21)$$

$$T_c - T_f = \frac{q_s}{2\pi(R+c)Lh} = \frac{q''' R^2}{2(R+c)h} \qquad (22)$$

Add equations 20, 21 and 22

$$T_m - T_f = \frac{q''' R^2}{4k_f} + \frac{q''' R^2}{2}\left(\frac{\ln\left(\frac{R+c}{R}\right)}{k_c} + \frac{1}{(R+c)h}\right) \qquad (23)$$

Effect of Cladding and Coolant

Or could be written,

$$T_m - T_f = \frac{q_s}{4\pi K_f L} + \frac{q_s \ln\left(\frac{R+c}{R}\right)}{2\pi K_c L} + \frac{q_s}{2\pi(R+c)Lh}$$

$$T_m - T_f = q_s\left(\frac{R}{2K_f A_R} + \frac{c}{K_c A_m} + \frac{1}{hA_{R+c}}\right)$$

$$q_s = \frac{T_m - T_f}{\left(\frac{R}{2K_f A_R} + \frac{c}{K_c A_m} + \frac{1}{hA_{R+c}}\right)} \qquad (24)$$

Effect of Cladding and Coolant

$$q_s = \frac{T_m - T_f}{\left(\frac{R}{2K_f A_R} + \frac{c}{K_c A_m} + \frac{1}{hA_{R+c}}\right)} \qquad (24)$$

where,

$$A_R = 2\pi LR$$

$$A_m = \frac{2\pi Lc}{ln\left(\frac{(R+c)}{R}\right)}$$

$$A_{R+c} = 2\pi L(R+c)$$

In the usual case, where c is very small compared to R, A_m may be taken, with little error, to be the arithmetic mean area of the cladding

$$A_m = \frac{(A_R + A_{R+c})}{2} = [2\pi R + 2\pi(R+c)]\frac{L}{2}$$

Example 1

A fuel rods 0.158 in diameter, 30.5 in long. The fuel rods are 26 w/o enriched uranium clad in 0.005-in stainless steel. The effective thermal neutron flux at the point of maximum temperature (slightly above the centre of the core) is $5.4\ x\ 10^{13}\ neutrons/sec \cdot cm^2$. The temperature at the center of the fuel rod where q'' is the largest, is 1,220 °F. Calculate the temperatures at the fuel-cladding interface at the outer surface of the cladding. The effective fission cross section σ_f is 364 b.

$D = 0.158$ in

$c = 0.005$ in

$L = 30.5$ in

$\rho = 18.3\ g/cm^3$

$\varphi = 5.4 \times 10^{13}\ n/see\ cm^2$

$T_m = 1220\ °F$

$\sigma_f = 364$ b

$k_f \approx 20$ Btu / hr ft °F

$k_c \approx 11$ Btu / hr ft °F

Example 1 (Solution)

$$q''' = G N_f \sigma_f \phi$$

$$N_f = \frac{0.60225 \times 10^{24}}{235.044} \times 18.3 \times 0.26 = 1.219 \times 10^{22}$$

$$q''' = G N_f \sigma_f \phi = 180 \times (1.219 \times 10^{22}) \times (364 \times 10^{-24}) \times (5.4 \times 10^{13})$$

$$q''' = 4.313 \times 10^{16} \times 1.55 \times 10^{-08}$$

$$q''' = 6.676 \times 10^{08} \text{ Btu/hr ft}^3$$

Example 1 (Solution)

$$T_m - T_s = \frac{q'''}{4k_f} R^2$$

$$T_s = T_m - \frac{q'''}{4k_f} R^2$$

$$T_s = 1220 - \frac{6.676 \times 10^{08} \left(\frac{0.158}{2 \times 12}\right)^2}{4 \times 20}$$

$$T_s = 1220 - \frac{3.709 \times 10^{08} \left(\frac{0.158}{2 \times 12}\right)^2}{2 \times 21.20} = 858^\circ F$$

$$T_S - T_C = \frac{q_s \ln|(R+c)/R|}{2 \pi k_C L} = \frac{q''' R^2 \ln|(R+c)/R|}{2 k_c}$$

$$T_c = T_s - \frac{q''' R^2 \ln\left|\left(\frac{R+c}{R}\right)\right|}{2 k_c}$$

$$T_c = 858 - \frac{3.709 \times 10^{08} \times \left(\frac{0.158}{2 \times 12}\right)^2 \times \ln\left|\left(\frac{0.079 + 0.005}{0..079}\right)\right|}{2 \times 11}$$

$$T_c = 858 - 44.84 = 778\ ^\circ F$$

Heat Flow Out of Spherically Shaped Fuel

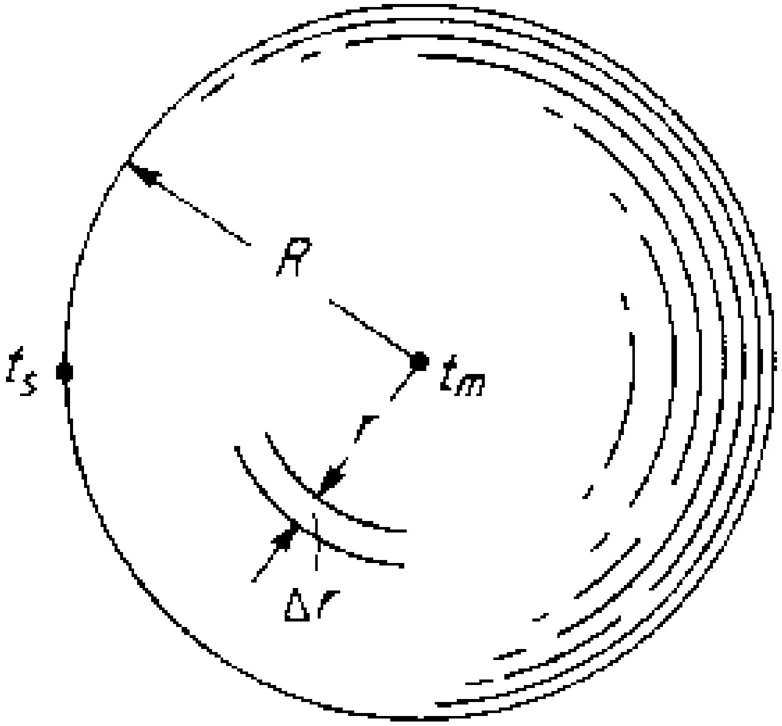
Poisson equation in spherical coordinates, also obtained by combining equation (25) and (26) to give

$$\nabla^2 T + \frac{q'''}{k} = 0 \qquad (25)$$

$$\nabla^2 T = \left(\frac{d^2T}{dr^2} + r^2\frac{dT}{dr}\right)$$

$$\frac{d^2T}{dr^2} + r^2\frac{dT}{dr} + \frac{q'''}{k_f} = 0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{q'''}{k_f} = 0 \qquad (26)$$



Heat Flow Out of Spherically Shaped Fuel

Integration of this equation for the boundary conditions $T(r)$ at r,

B. C (1)

$T = T_m \quad$ at $r = 0$

B. C (2)

$T = T_s \quad$ at $r = R$

$$T(r) = \mathrm{T_m} - \frac{q'''r^2}{6k_f} \qquad (27)$$

$$T_m - T_s = \frac{q'''R^2}{6k_f} \qquad (28)$$

The total heat generated, q_s, is given by the expression

$$q_s = \frac{4}{3}\pi R^3 q''' \qquad (29)$$

Heat Flow Out of Spherically Shaped Fuel

Combining equation (28) and (29)

$$q_s = 8\pi R k_f (T_m - T_s) \qquad (30)$$

Putting $4\pi R^2 = A_s$ the total peripheral area of the spherical element, and rearranging give

$$q_s = 2 k_f A_s \left(\frac{T_m - T_s}{R}\right) \qquad (31)$$

Thank You

Stay safe!