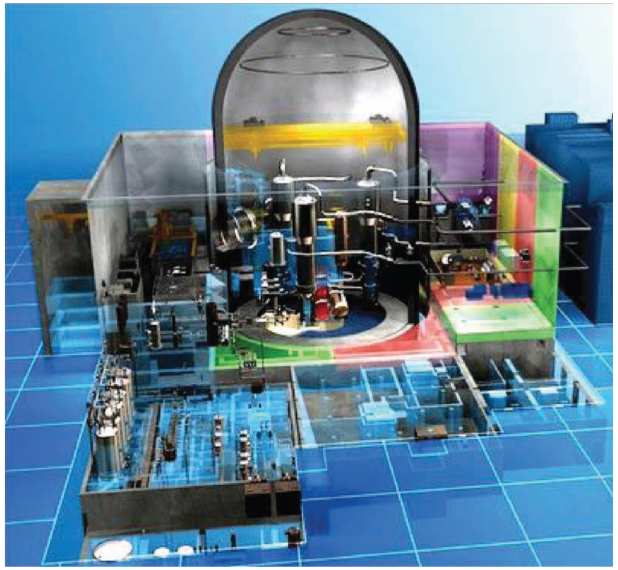
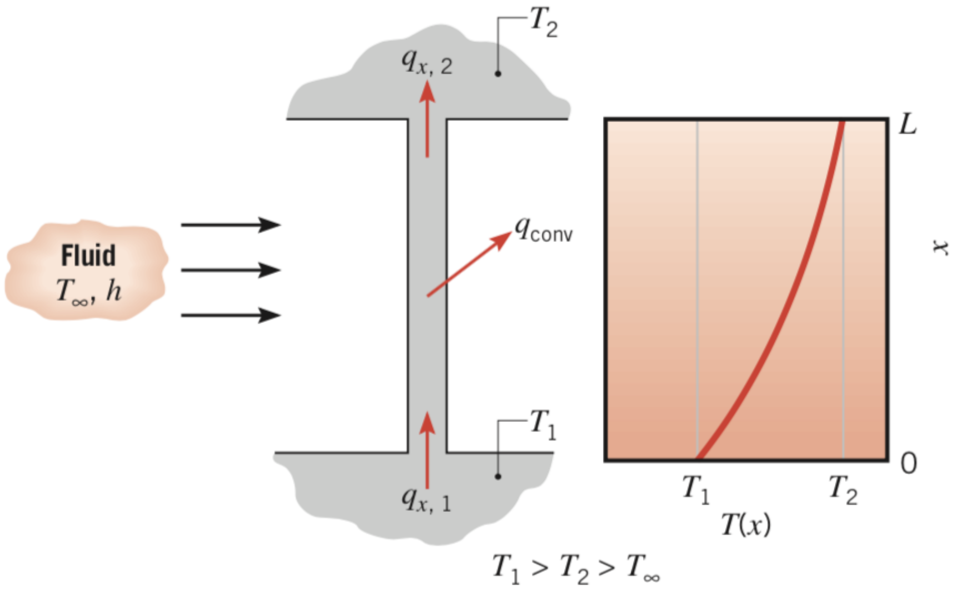
Heat Conduction in Reactor Elements
(Part 5)



MUHAMMAD SYAHIR SARKAWI, PhD
Nuclear Engineering Program
Energy Engineering Department
N01-273 | 0133274154
syahirsarkawi@utm.my

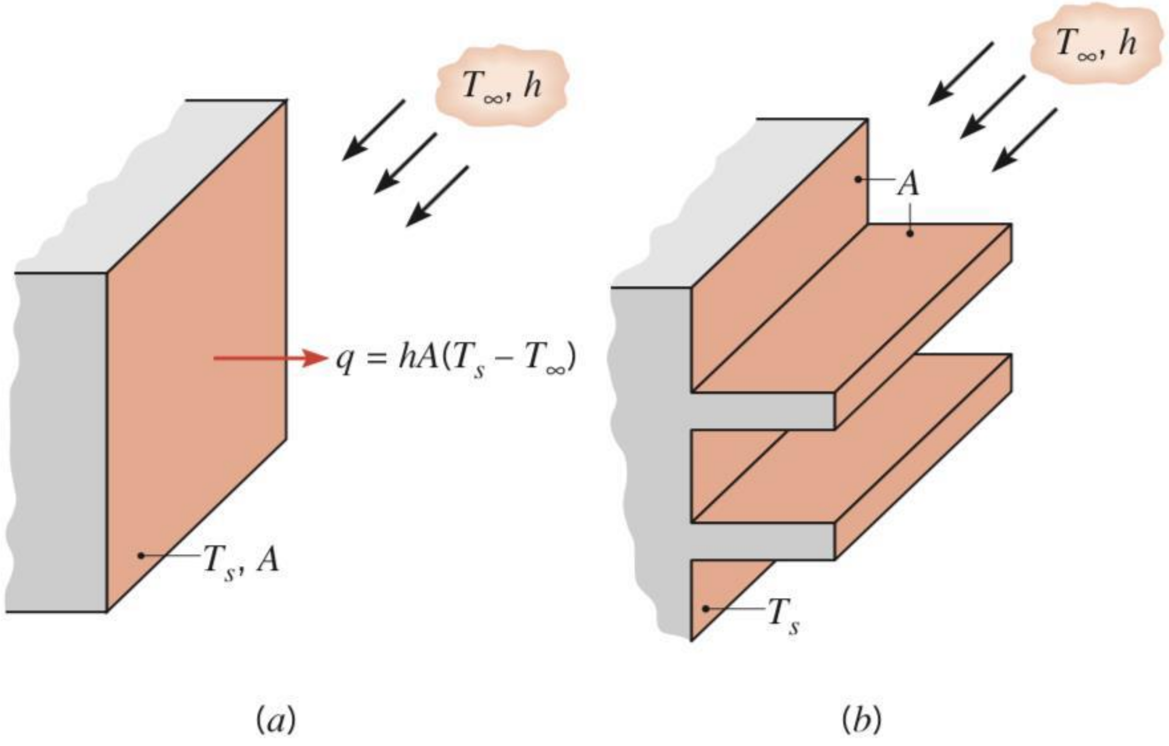
Heat Transfer from Extended Surfaces

- **The term *extended surface* is commonly used to depict an important special case involving heat transfer by conduction within a solid and heat transfer by convection (and/or radiation) from the boundaries of the solid.**



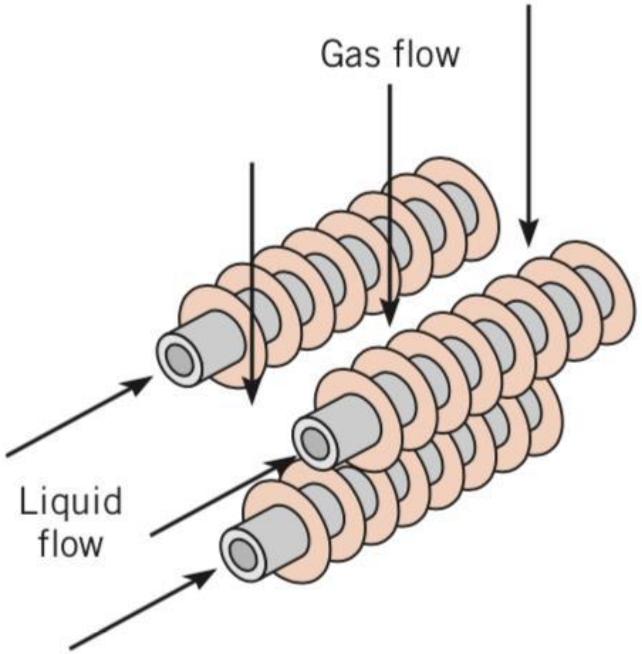
Combined conduction and convection in a structural element.

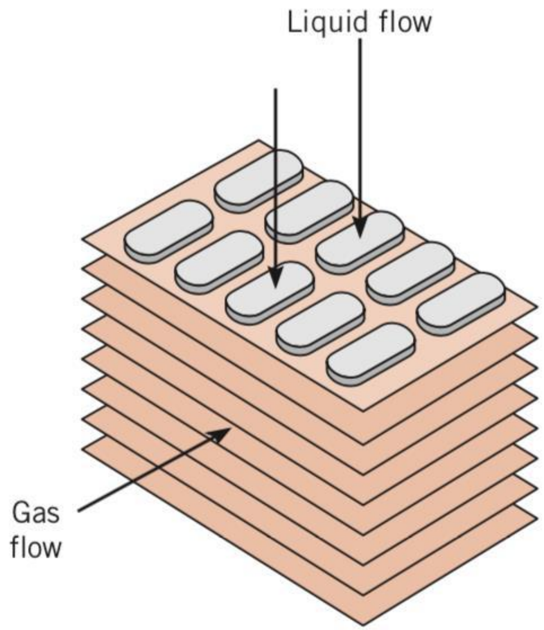
Heat Transfer from Extended Surfaces



Use of fins to enhance heat transfer from a plane wall.
(a) Bare surface. (b) Finned surface.

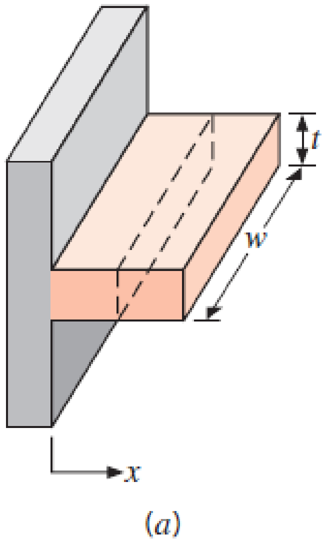
Heat Transfer from Extended Surfaces

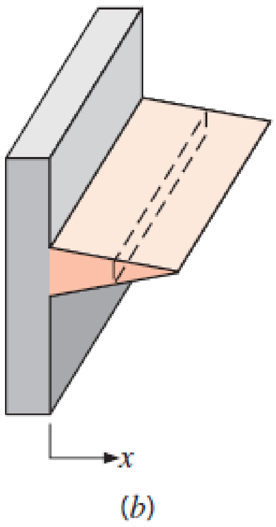


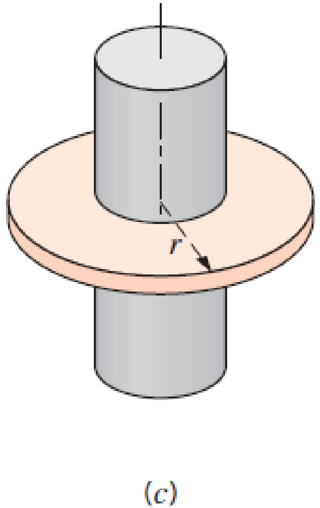


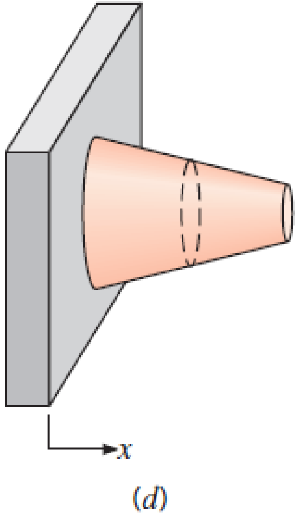
Schematic of typical finned-tube heat exchangers

General Conduction Analysis









(a) Straight fin of uniform cross section

(b) Straight fin of nonuniform cross section

(c) Annular fin

(d) Pin fin.

General Conduction Analysis

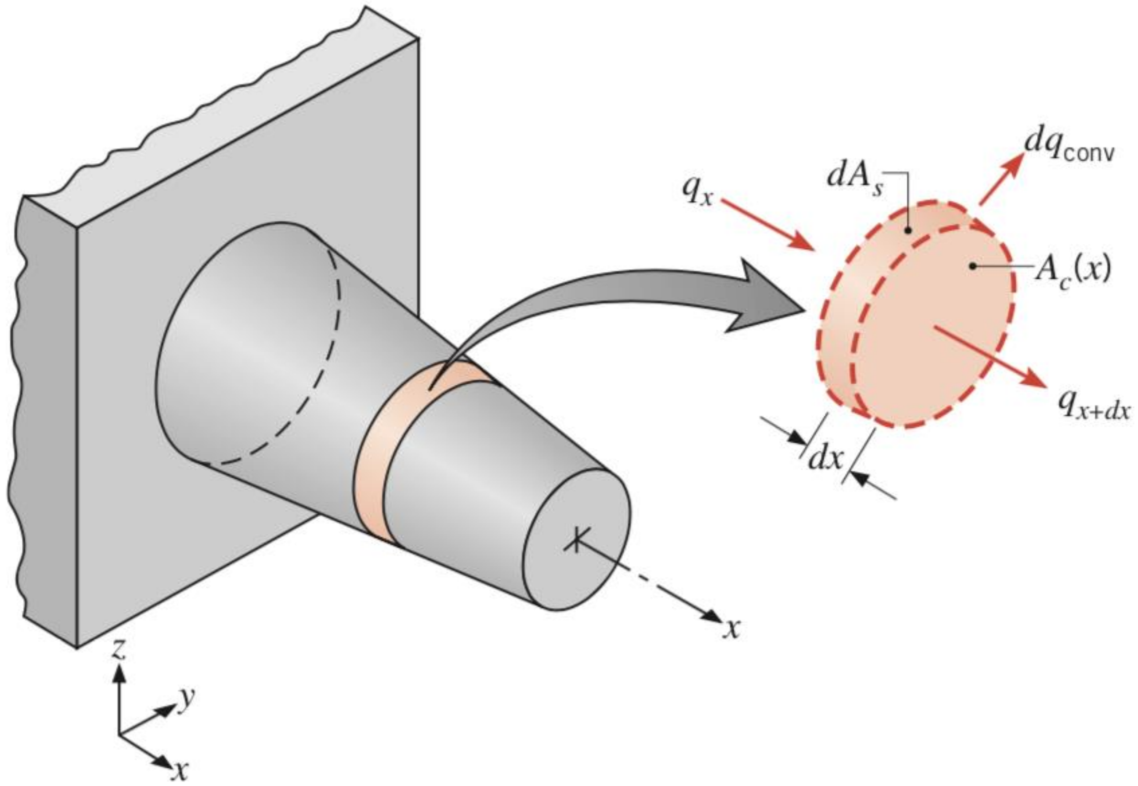


FIGURE 3.16 Energy balance for an extended surface.

$$q_x = q_{x+dx} + dq_{conv}$$

$$q_x = -kA_c \frac{dT}{dx}$$

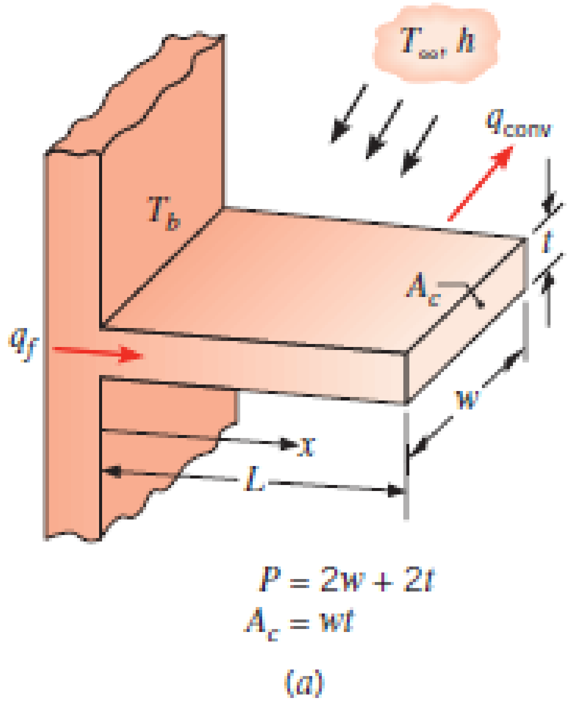
$$q_{x+dx} = q_x + \frac{dq_x}{dx} dx$$

HEAT DIFFUSION EQUATION

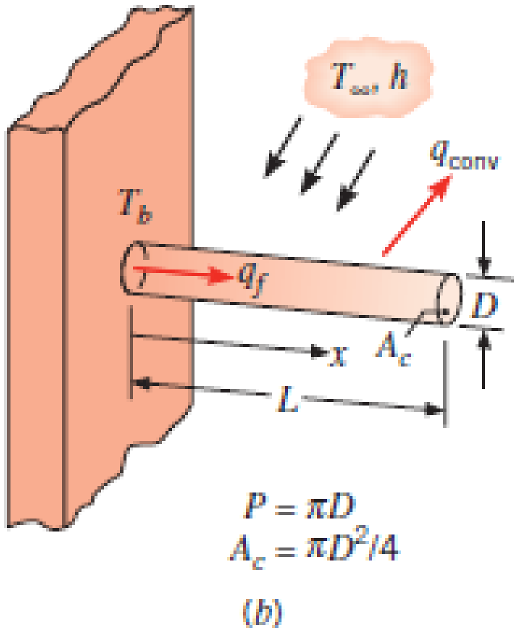
$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

***P* is the fin perimeter**

Fins of Uniform Cross-Sectional Area



(*a*) Rectangular fin



(*b*) Pin fin

Straight fins of uniform cross section.

Fins of Uniform Cross-Sectional Area

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -k d\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ (3.75)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.77)
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$ (3.80)	$M \tanh mL$ (3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.82)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.83)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx} (3.84)	M (3.85)

$\theta \equiv T - T_\infty$ $\qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\qquad M \equiv \sqrt{hPkA_c}\,\theta_b$

Fins Performance

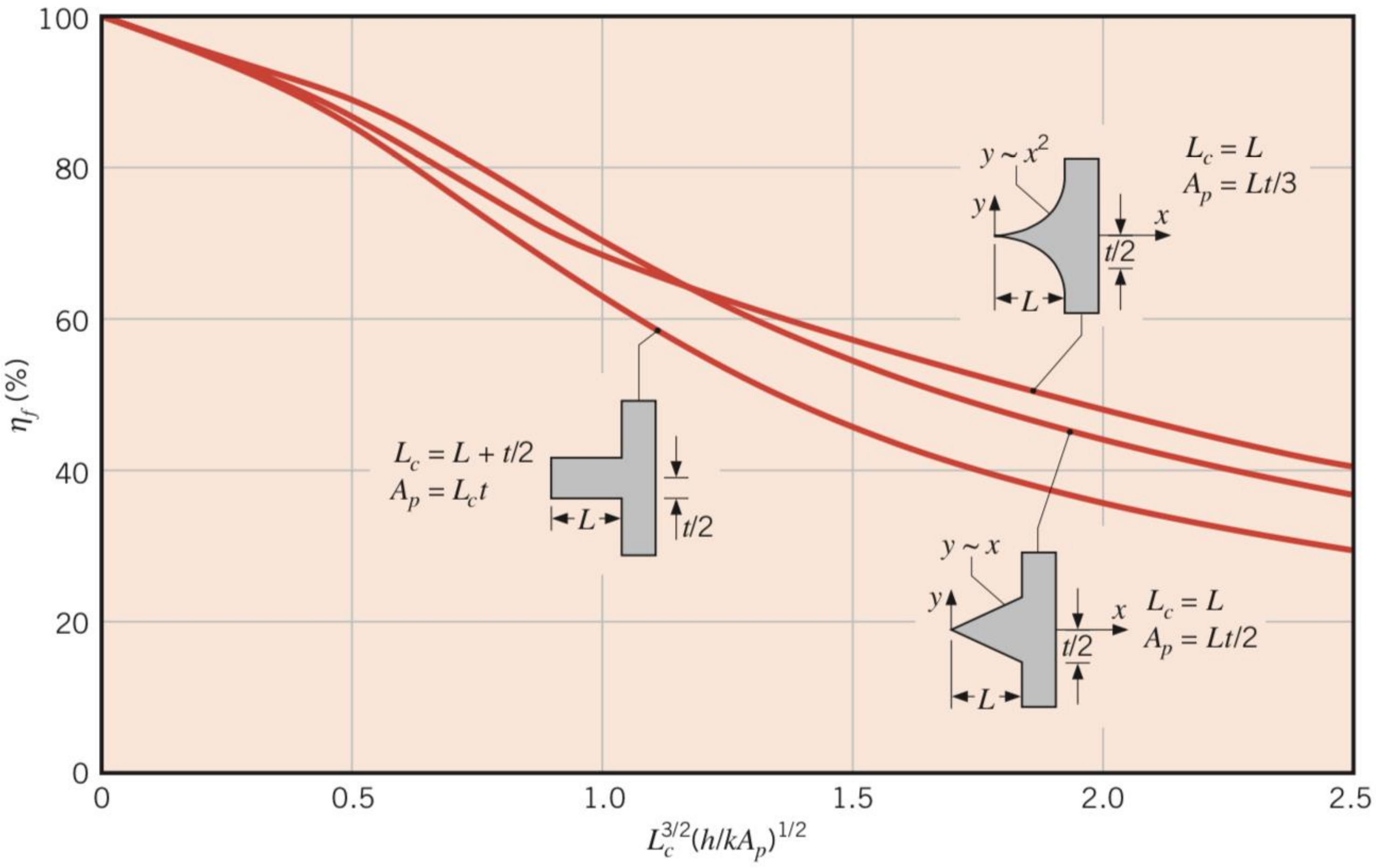


FIGURE 3.19 Efficiency of straight fins (rectangular, triangular, and parabolic profiles).

Fins Performance

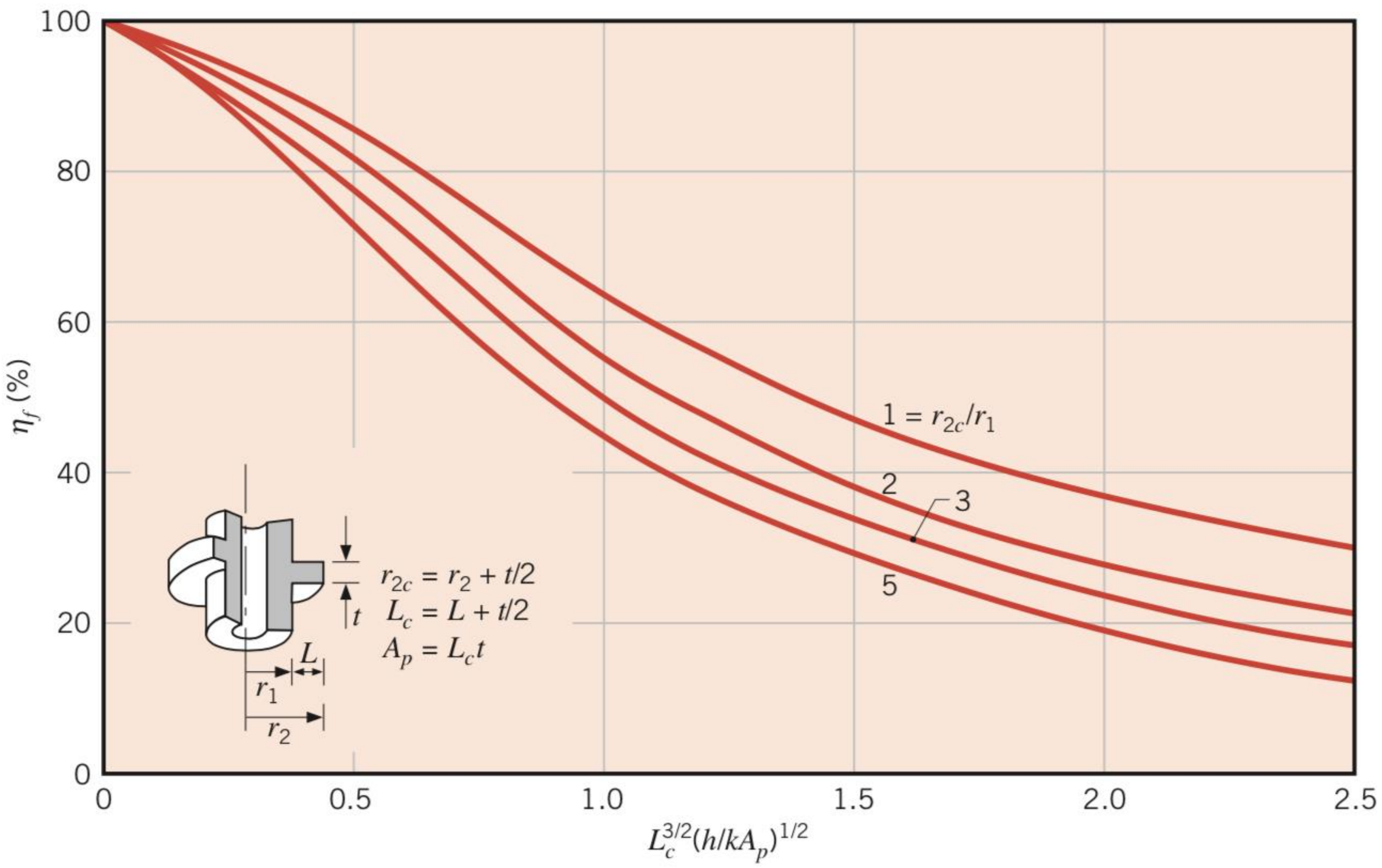


FIGURE 3.20 Efficiency of annular fins of rectangular profile.

Fins of Nonuniform Cross-Sectional Area

TABLE 3.5 Efficiency of common fin shapes

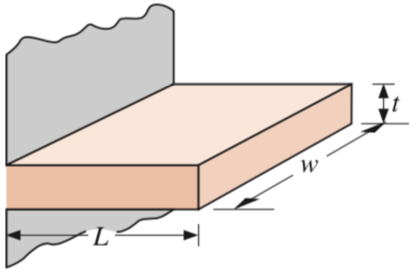
Straight Fins

Rectangular[a]

$A_f = 2wL_c$

$L_c = L + (t/2)$

$A_p = tL$

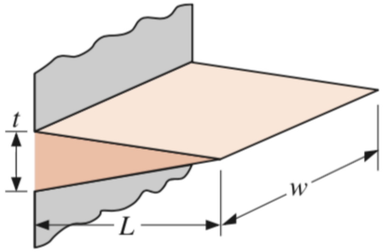


$$\eta_f = \frac{\tanh mL_c}{mL_c} \quad (3.94)$$

Triangular[a]

$A_f = 2w[L^2 + (t/2)^2]^{1/2}$

$A_p = (t/2)L$



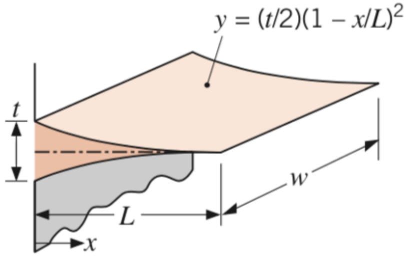
$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)} \quad (3.98)$$

Parabolic[a]

$A_f = w[C_1L + (L^2/t)\ln(t/L + C_1)]$

$C_1 = [1 + (t/L)^2]^{1/2}$

$A_p = (t/3)L$



$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1} \quad (3.99)$$

Fins of Nonuniform Cross-Sectional Area

TABLE 3.5 *Continued*

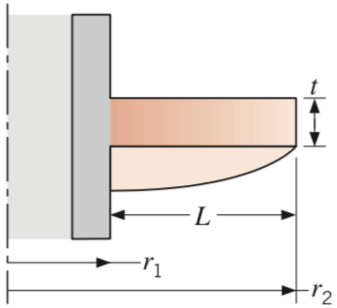
Circular Fin

Rectangular[a]

$A_f = 2\pi (r_{2c}^2 - r_1^2)$

$r_{2c} = r_2 + (t/2)$

$V = \pi (r_2^2 - r_1^2)t$



$$\eta_f = C_2 \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})} \quad (3.96)$$

$$C_2 = \frac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$$

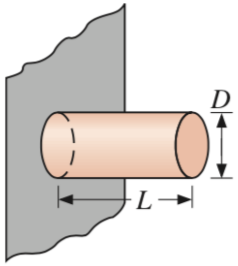
Pin Fins

Rectangular[b]

$A_f = \pi D L_c$

$L_c = L + (D/4)$

$V = (\pi D^2/4)L$



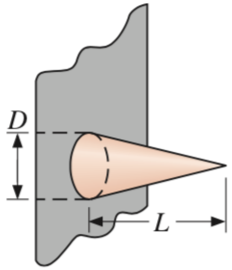
$$\eta_f = \frac{\tanh mL_c}{mL_c} \quad (3.100)$$

Fins of Nonuniform Cross-Sectional Area

Triangular[b]

$$A_f = \frac{\pi D}{2}\left[L^2 + (D/2)^2\right]^{1/2}$$

$$V = (\pi/12)D^2L$$



$$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)} \qquad (3.101)$$

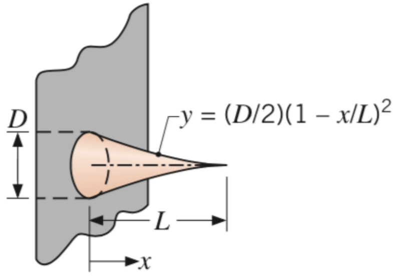
Parabolic[b]

$$A_f = \frac{\pi L^3}{8D}\left\{C_3C_4 - \frac{L}{2D}\ln\left[(2DC_4/L) + C_3\right]\right\}$$

$$C_3 = 1 + 2(D/L)^2$$

$$C_4 = \left[1 + (D/L)^2\right]^{1/2}$$

$$V = (\pi/20)D^2\,L$$



$$\eta_f = \frac{2}{\left[4/9(mL)^2 + 1\right]^{1/2} + 1} \qquad (3.102)$$

[a] $m = (2h/kt)^{1/2}$.
[b] $m = (4h/kD)^{1/2}$.

Overall Surface Efficiency

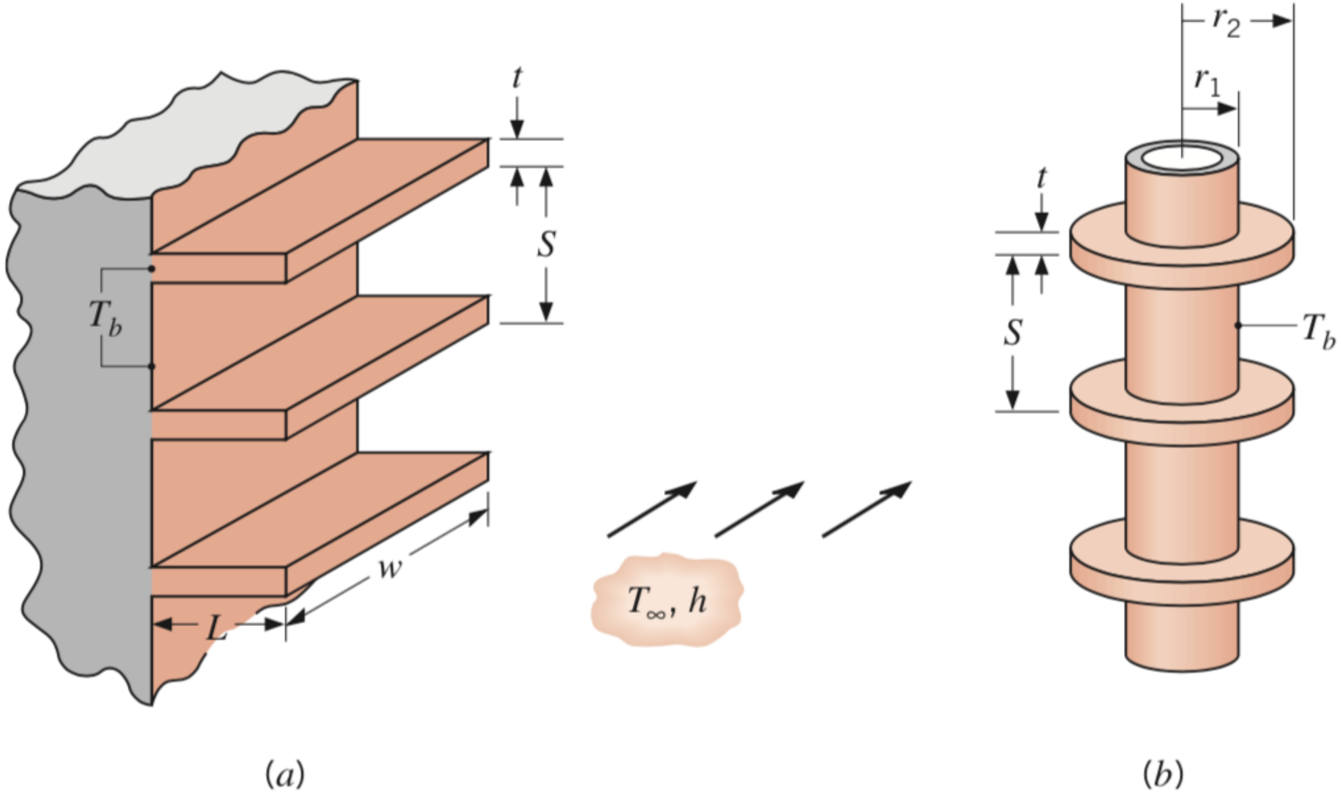


FIGURE 3.21 Representative fin arrays. (*a*) Rectangular fins. (*b*) Annular fins.

Fins In Reactors

- Fuel element temperatures may exceed safe limits.
- This in the case of gases or organic liquids.
- One method of alleviating this problem is the use of finned cladding.
- Fins are recommended whenever there exists a large difference in heat-transfer coefficient between the two sides of a heat-transfer surface and are placed on the side of the low coefficient.
- The fins are usually made of the same material and are an integral part of the cladding.

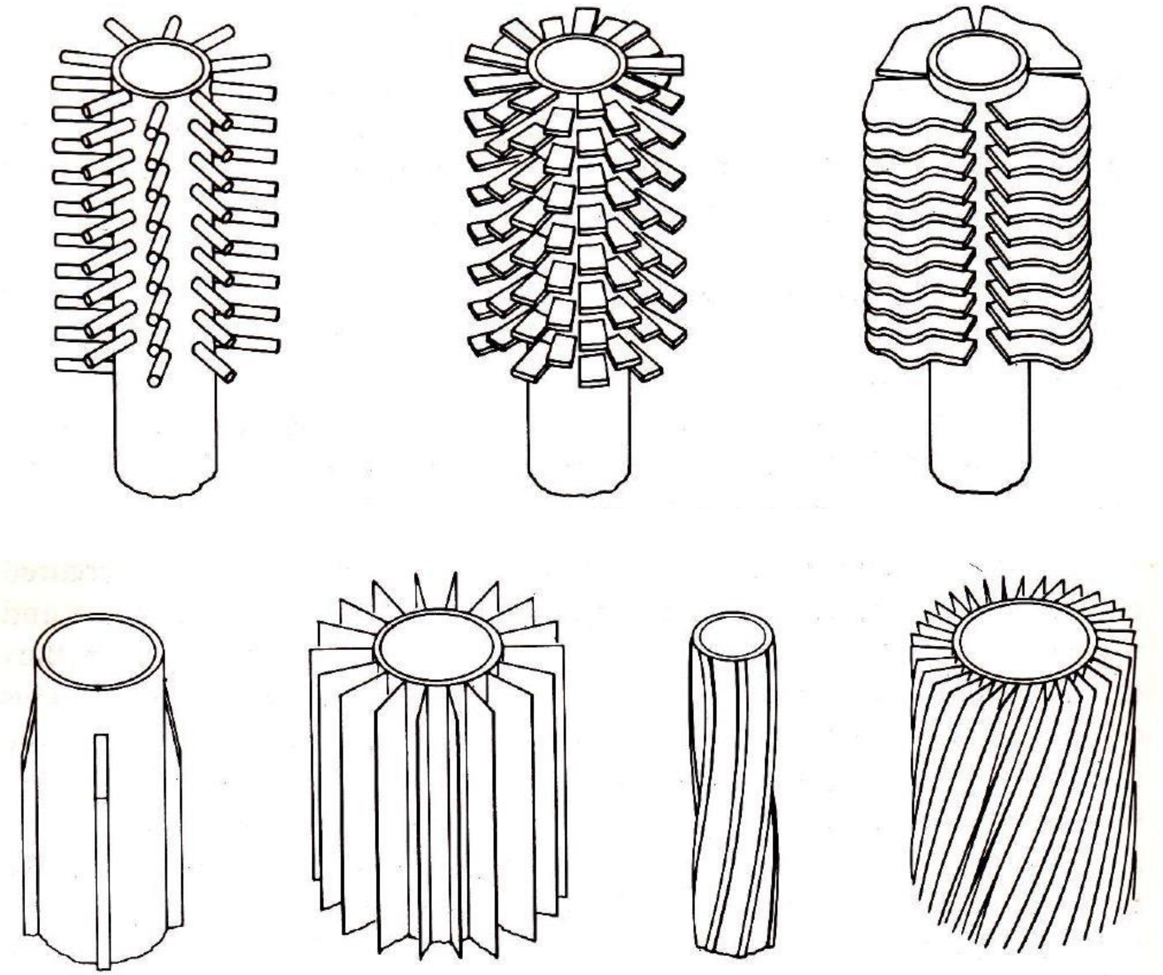
Various Types of Fins on Fuel Cladding

1) **Longitudinal** or **axial**,

2) **Circumferential** or **transverse**

(3) **Helical**, of **different pitch** and **height**

(4) **Pin** and **strip** type, in line or **staggered**



Fins In Reactors

- Fins on nuclear fuel elements while transferring fission heat to the coolant, generate some heat of their own because of the absorption of nuclear radiation
- They can consequently be assumed, with little error, to have uniform volumetric thermal source strengths $q''', \frac{Btu}{hr\,ft^3}$.
- The choice of fin material for a fuel element is a complex function of many variables.
  - A low neutron absorption cross sections are a primary requisite.
  - Good thermal conductivity k,

Some Properties of Some Fin Materials

Material	Relaxation length,* cm		Σ_a, cm^{-1} (thermal)	k, Btu/hr ft °F
	Fast neutrons	γ rays		
Beryllium	~9	18	0.00123	68 (600°F)
Aluminum	~10	13	0.01300	131 (600°F)
Magnesium			0.00254	91 (572°F)
Iron	~6	3.7	0.1900	27 (600°F)
Zirconium			0.00765	11 (480°F)

2-Dimensional Steady State Conduction

Introduction

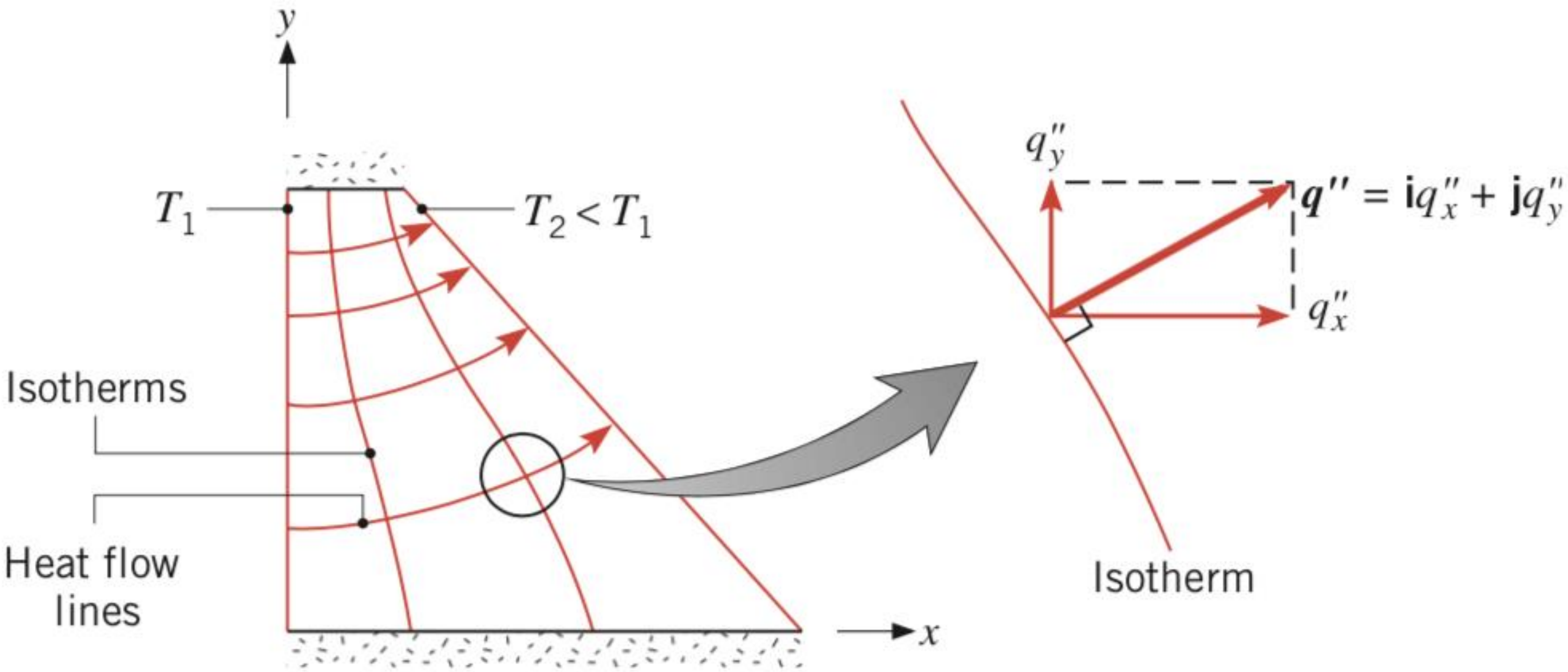


FIGURE 4.1 Two-dimensional conduction.

Adiabats and Isotherms Lines

HEAT FLUX PLOT

Isotherms – constant temperature lines
Adiabats – heat flow lines

Basic Rules:

1. Identify lines of both geometric and thermal symmetrics.
2. Adiabats and isotherms are normal in the interiors of the body.
3. Isotherms intersects the adiabats at right angle (90°).
4. Adiabats intersects the isotherms at right angle (90°).
5. Adiabats bisects isothermal corners.
6. Isotherms-Adiabats intersections form curvilinear squares.

Adiabats and Isotherms Lines

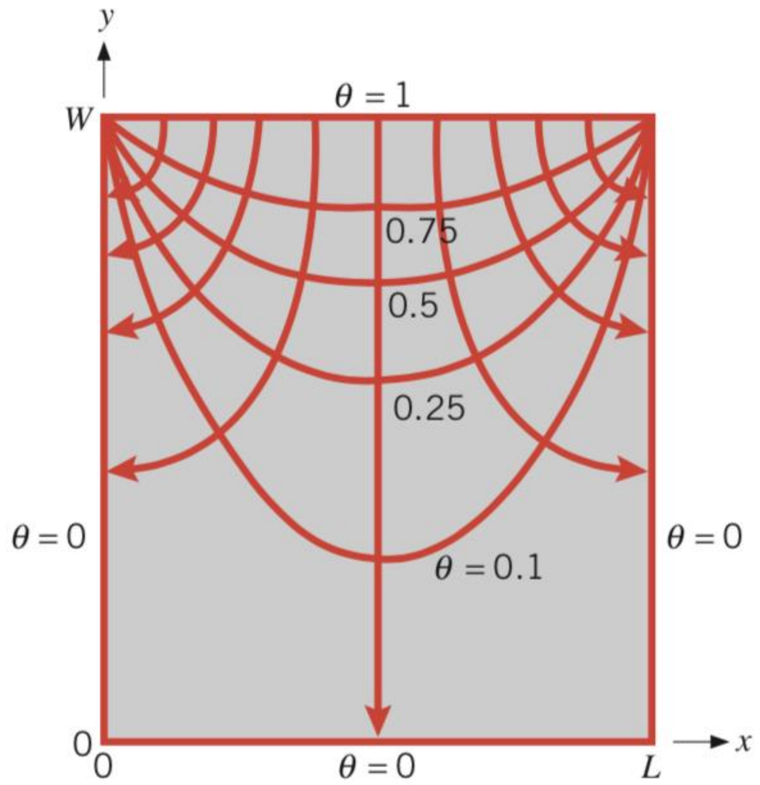
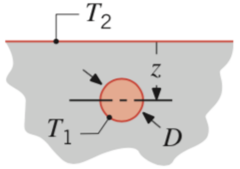
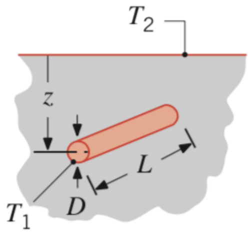
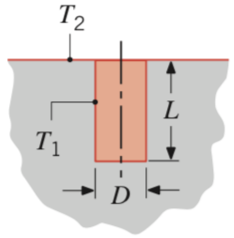


FIGURE 4.3 Isotherms and heat flow lines for two-dimensional conduction in a rectangular plate.

Conduction Shape Factor

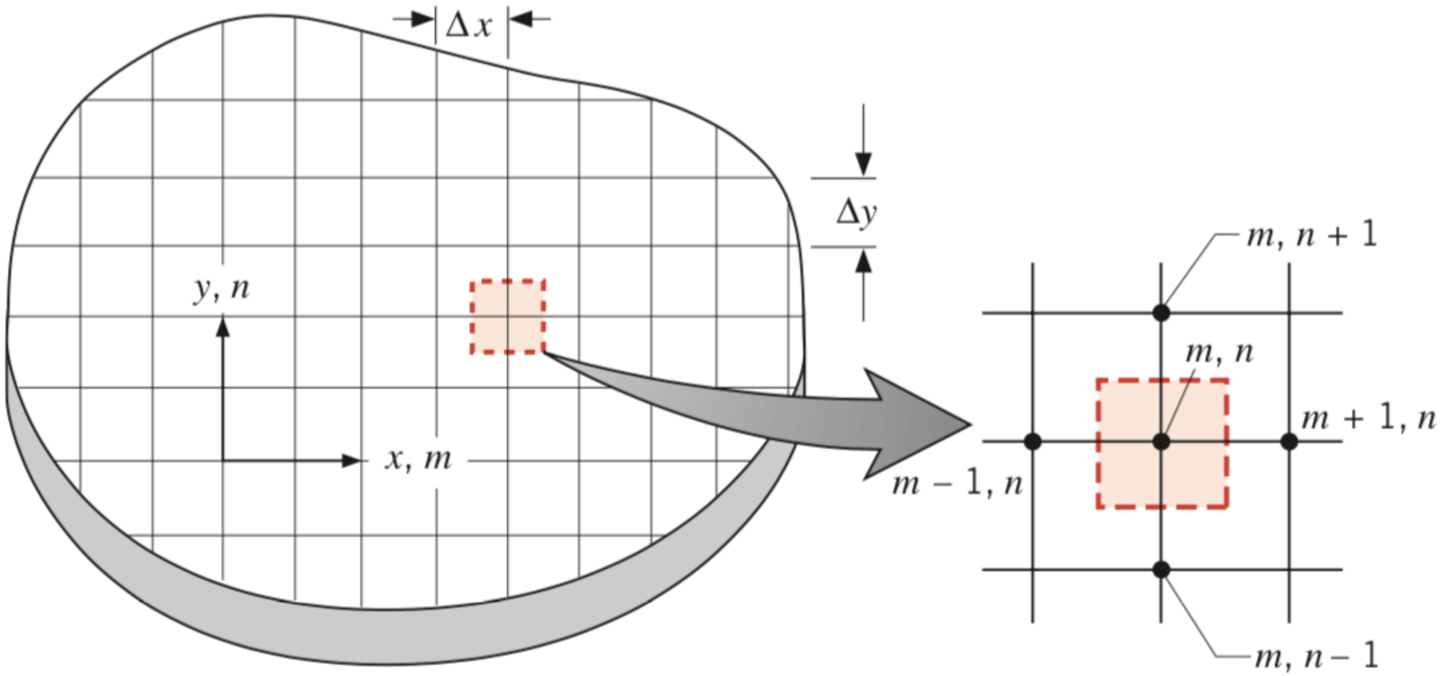
TABLE 4.1 Conduction shape factors and dimensionless conduction heat rates for selected systems.

(a) Shape factors [$q = Sk(T_1 - T_2)$]

System	Schematic	Restrictions	Shape Factor
Case 1 Isothermal sphere buried in a semi-infinite medium		$z > D/2$	$\dfrac{2\pi D}{1 - D/4z}$
Case 2 Horizontal isothermal cylinder of length L buried in a semi-infinite medium		$L \gg D$ $L \gg D$ $z > 3D/2$	$\dfrac{2\pi L}{\cosh^{-1}(2z/D)}$ $\dfrac{2\pi L}{\ln(4z/D)}$
Case 3 Vertical cylinder in a semi-infinite medium		$L \gg D$	$\dfrac{2\pi L}{\ln(4L/D)}$

$$q = Sk\Delta T_{1-2} \qquad R_{t,\text{cond(2D)}} = \frac{1}{Sk}$$

Finite-Difference Equations

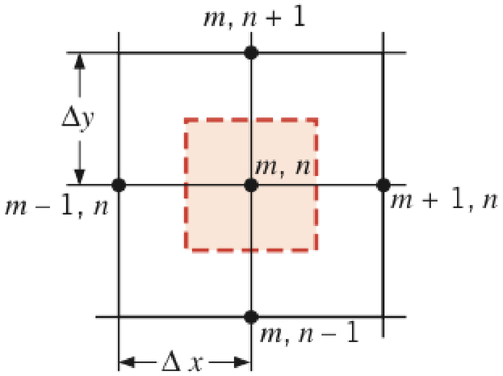
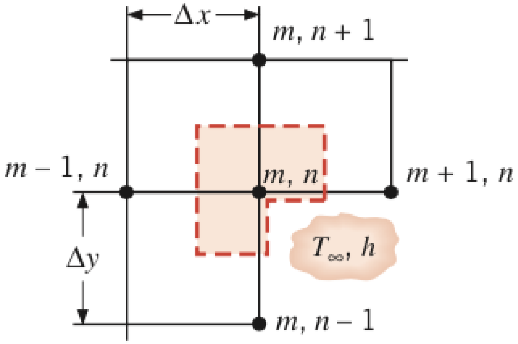


$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0$$

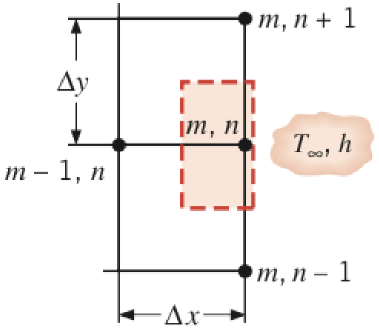
FIGURE 4.4 Two-dimensional conduction. (*a*) Nodal network.

Finite-Difference Equations

TABLE 4.2 Summary of nodal finite-difference equations

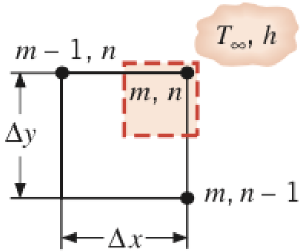
Configuration	Finite-Difference Equation for $\Delta x = \Delta y$	
	$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0$ **Case 1.** Interior node	(4.29)
	$2(T_{m-1,n} + T_{m,n+1}) + (T_{m+1,n} + T_{m,n-1}) + 2\frac{h\,\Delta x}{k}T_{\infty} - 2\left(3 + \frac{h\,\Delta x}{k}\right)T_{m,n} = 0$ **Case 2.** Node at an internal corner with convection	(4.41)

Finite-Difference Equations



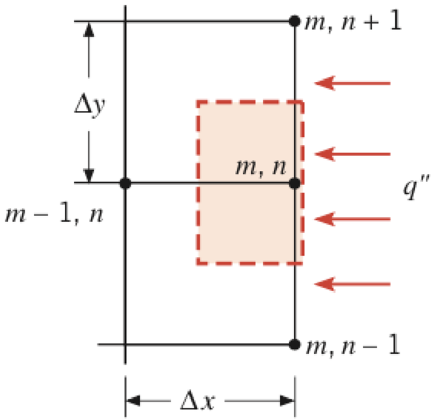
$$(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + \frac{2h\,\Delta x}{k}T_\infty - 2\left(\frac{h\,\Delta x}{k} + 2\right)T_{m,n} = 0 \qquad (4.42)^a$$

Case 3. Node at a plane surface with convection



$$(T_{m,n-1} + T_{m-1,n}) + 2\frac{h\,\Delta x}{k}T_\infty - 2\left(\frac{h\,\Delta x}{k} + 1\right)T_{m,n} = 0 \qquad (4.43)$$

Case 4. Node at an external corner with convection



$$(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + \frac{2q''\,\Delta x}{k} - 4T_{m,n} = 0 \qquad (4.44)^b$$

Case 5. Node at a plane surface with uniform heat flux

[a,b] To obtain the finite-difference equation for an adiabatic surface (or surface of symmetry), simply set h or q'' equal to zero.

Thank You

Stay safe!