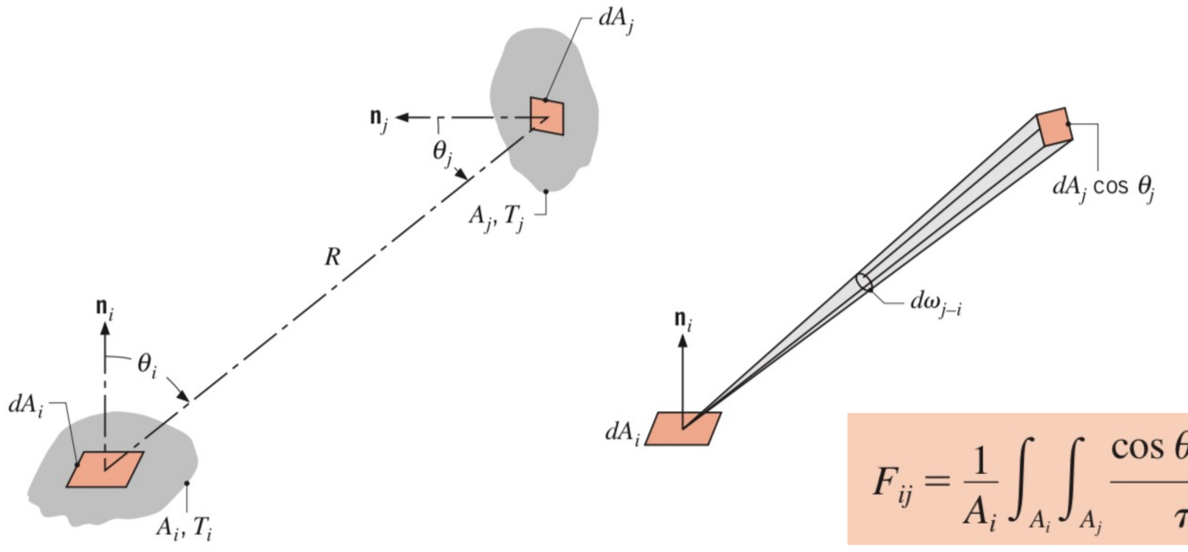
HEAT TRANSFER
SETN2223

RADIATION EXCHANGE BETWEEN SURFACES

THE VIEW FACTOR

13.1.1 The View Factor Integral

The view factor F_{ij} is defined as the *fraction of the radiation leaving surface i that is intercepted by surface j*. To develop a general expression for F_{ij}, we consider the arbitrarily oriented surfaces A_i and A_j of Figure 13.1. Elemental areas on each surface, dA_i and dA_j, are connected by a line of length R, which forms the polar angles θ_i and θ_j, respectively, with the surface normals $\mathbf{n}_i$ and $\mathbf{n}_j$. The values of R, θ_i, and θ_j vary with the position of the elemental areas on A_i and A_j.



$$F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2}\, dA_i\, dA_j$$

FIGURE 13.1 View factor associated with radiation exchange between elemental surfaces of area dA_i and dA_j.

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13.1.2 View Factor Relations

An important view factor relation is suggested by Equations 13.1 and 13.2. In particular, equating the integrals appearing in these equations, it follows that

$$A_i F_{ij} = A_j F_{ji} \qquad (13.3)$$

This expression, termed the *reciprocity relation*, is useful in determining one view factor from knowledge of the other.

$$F_{21} = \left(\frac{A_1}{A_2}\right) F_{12} = \left(\frac{A_1}{A_2}\right)$$

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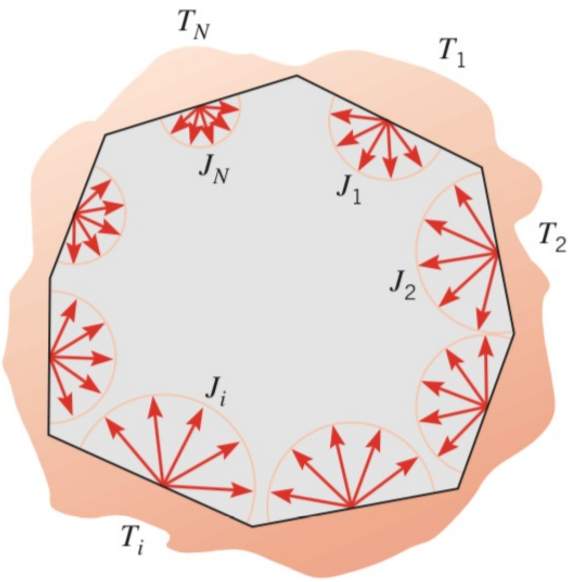

FIGURE 13.2 Radiation exchange in an enclosure.

Another important view factor relation pertains to the surfaces of an *enclosure* (Figure 13.2). From the definition of the view factor, the *summation rule*

$$\sum_{j=1}^{N} F_{ij} = 1 \qquad (13.4)$$

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$$\sum_{j=1}^{N} F_{ij} = 1 \qquad (13.4)$$

may be applied to each of the N surfaces in the enclosure. This rule follows from the conservation requirement that all radiation leaving surface i must be intercepted by the enclosure surfaces. The term F_{ii} appearing in this summation represents the fraction of the radiation that leaves surface i and is directly intercepted by i. If the surface is concave, it *sees itself* and F_{ii} is nonzero. However, for a plane or convex surface, $F_{ii} = 0$.

To calculate radiation exchange in an enclosure of N surfaces, a total of N^2 view factors is needed. This requirement becomes evident when the view factors are arranged in the matrix form:

$$\begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1N} \\ F_{21} & F_{22} & \cdots & F_{2N} \\ \vdots & \vdots & & \vdots \\ F_{N1} & F_{N2} & \cdots & F_{NN} \end{bmatrix}$$

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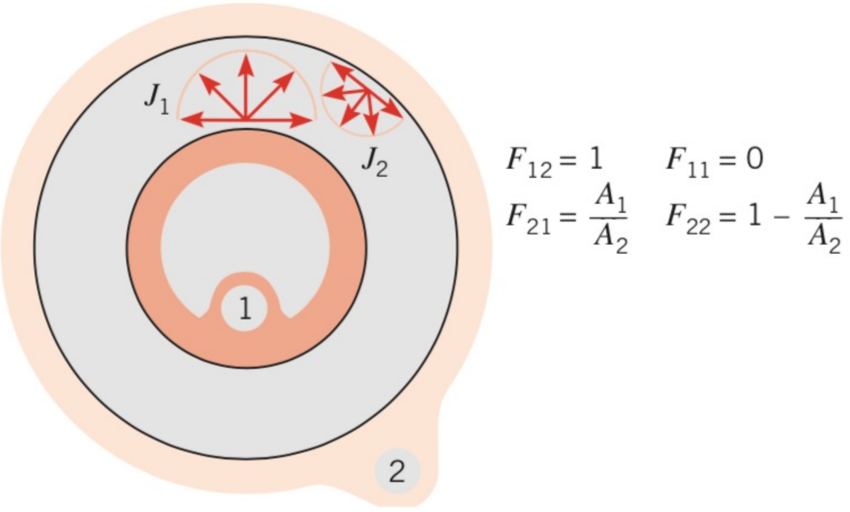


FIGURE 13.3 View factors for the enclosure formed by two spheres.

From the summation rule, we also obtain

$$F_{11} + F_{12} = 1$$

in which case $F_{11} = 0$, and

$$F_{21} + F_{22} = 1$$

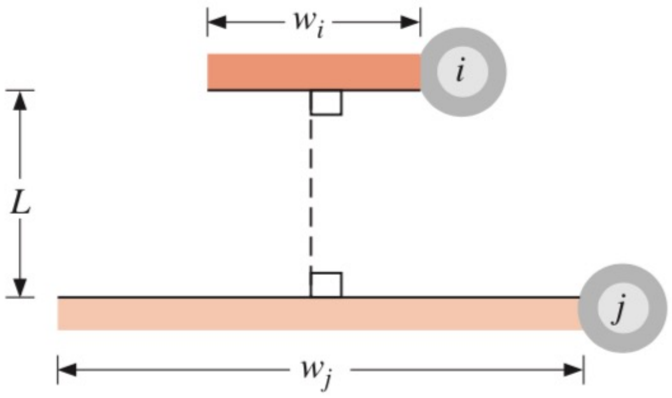
in which case

$$F_{22} = 1 - \left(\frac{A_1}{A_2}\right)$$

Table 13.1 View Factors for Two-Dimensional Geometries [4]

Geometry | **Relation**

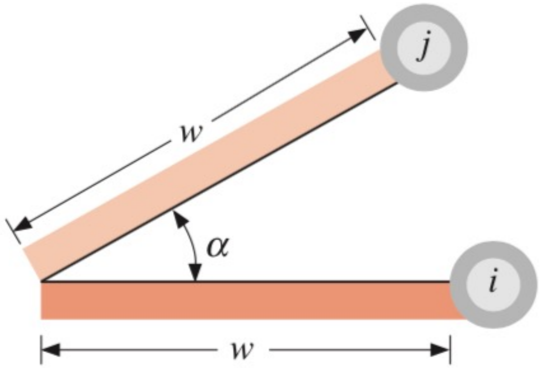
Parallel Plates with Midlines Connected by Perpendicular



$$F_{ij} = \frac{[(W_i + W_j)^2 + 4]^{1/2} - [(W_j - W_i)^2 + 4]^{1/2}}{2W_i}$$

$$W_i = w_i/L, \; W_j = w_j/L$$

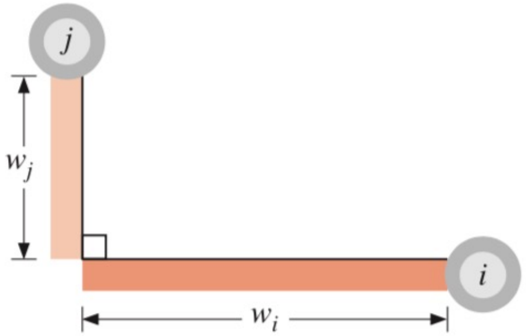
Inclined Parallel Plates of Equal Width and a Common Edge



$$F_{ij} = 1 - \sin\left(\frac{\alpha}{2}\right)$$

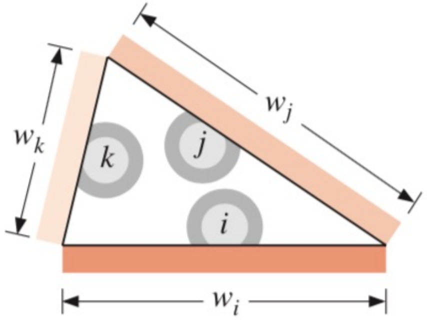
(*continues*)

Perpendicular Plates with a Common Edge



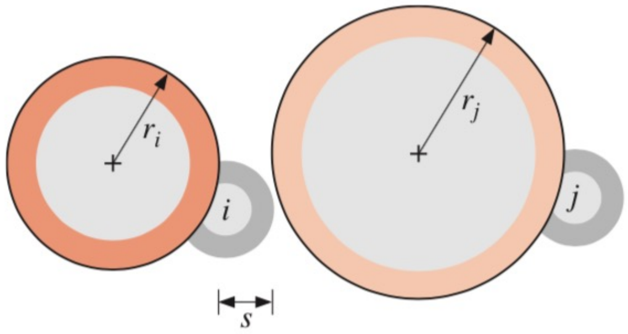
$$F_{ij} = \frac{1 + (w_j/w_i) - [1 + (w_j/w_i)^2]^{1/2}}{2}$$

Three-Sided Enclosure



$$F_{ij} = \frac{w_i + w_j - w_k}{2w_i}$$

Parallel Cylinders of Different Radii

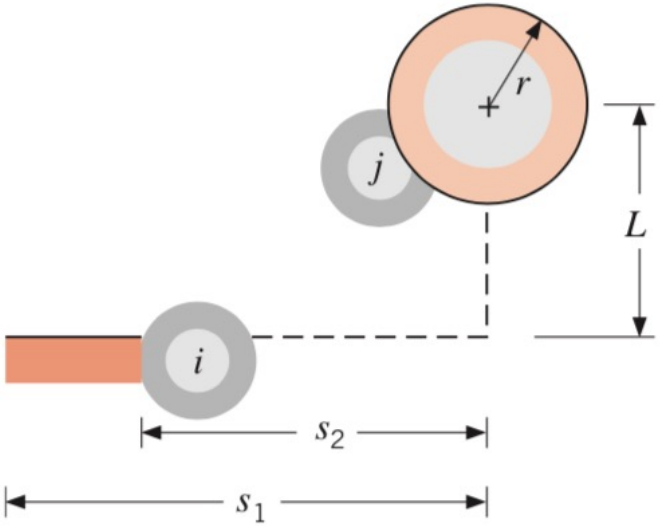


$$F_{ij} = \frac{1}{2\pi}\Bigg\{\pi + [C^2 - (R+1)^2]^{1/2} - [C^2 - (R-1)^2]^{1/2} + (R-1)\cos^{-1}\left[\left(\frac{R}{C}\right) - \left(\frac{1}{C}\right)\right] - (R+1)\cos^{-1}\left[\left(\frac{R}{C}\right) + \left(\frac{1}{C}\right)\right]\Bigg\}$$

$R = r_j/r_i,\ S = s/r_i$

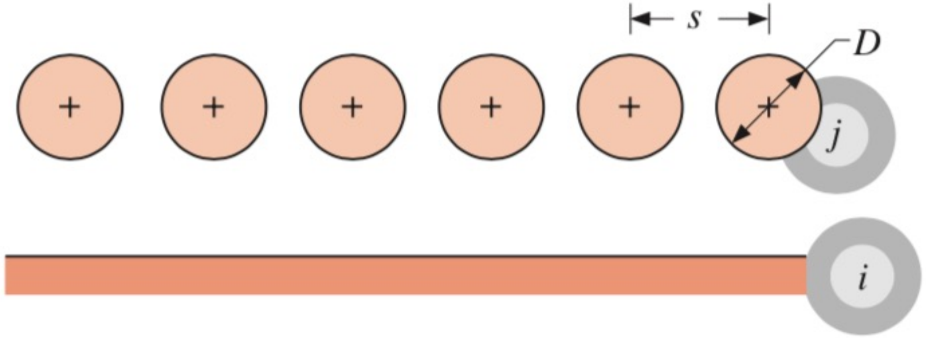
$C = 1 + R + S$

Cylinder and Parallel Rectangle



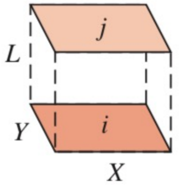
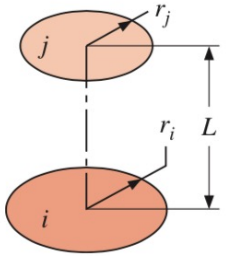
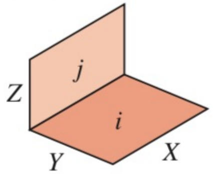
$$F_{ij} = \frac{r}{s_1 - s_2}\left[\tan^{-1}\frac{s_1}{L} - \tan^{-1}\frac{s_2}{L}\right]$$

Infinite Plane and Row of Cylinders



$$F_{ij} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \left(\frac{D}{s}\right)\tan^{-1}\left[\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}\right]$$

TABLE 13.2 View Factors for Three-Dimensional Geometries [4]

Geometry	Relation
Aligned Parallel Rectangles **(Figure 13.4)** 	$\bar{X} = X/L,\ \bar{Y} = Y/L$ $F_{ij} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$
Coaxial Parallel Disks **(Figure 13.5)** 	$R_i = r_i/L,\ R_j = r_j/L$ $S = 1 + \frac{1+R_j^2}{R_i^2}$ $F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$
Perpendicular Rectangles with a Common Edge **(Figure 13.6)** 	$H = Z/X,\ W = Y/X$ $F_{ij} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$

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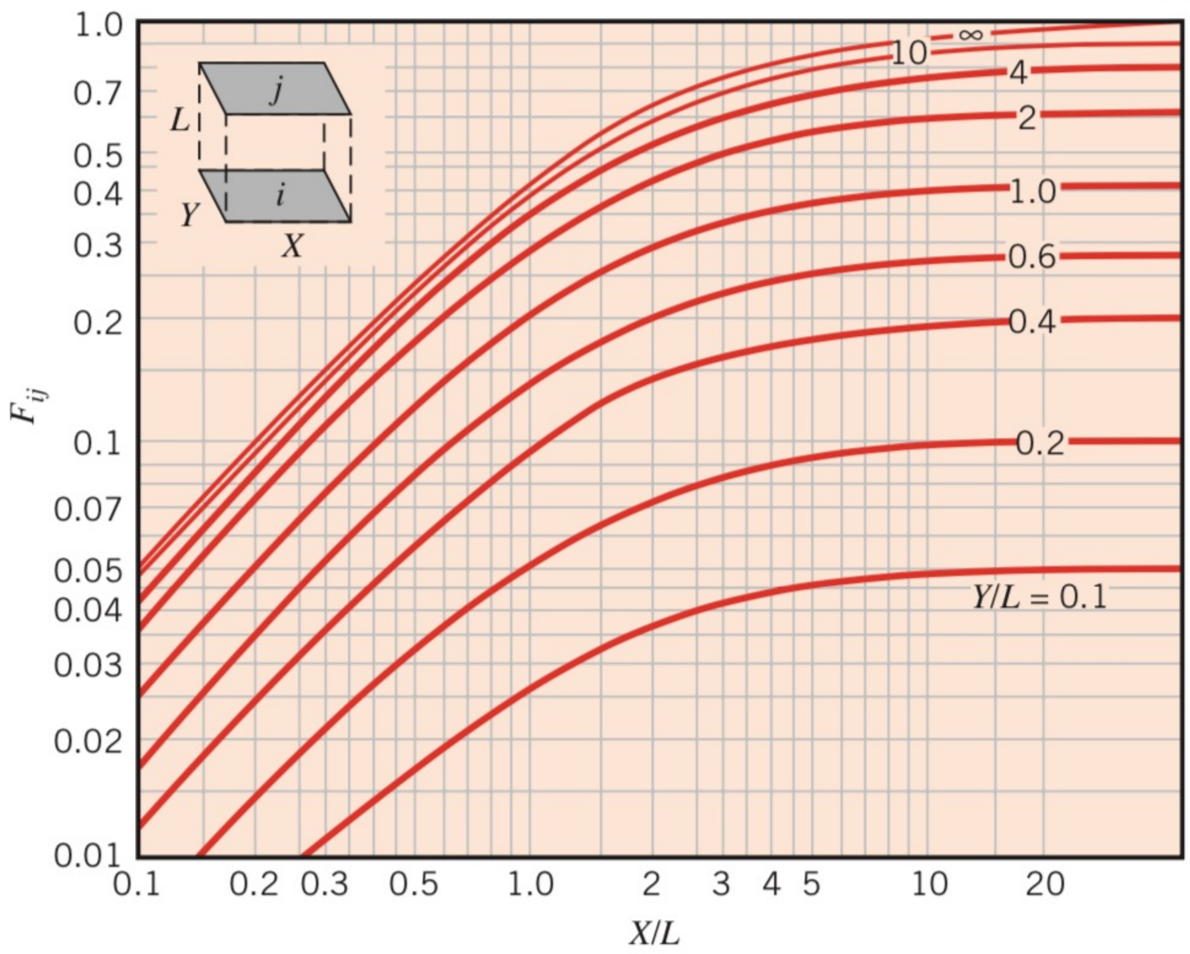

FIGURE 13.4 View factor for aligned parallel rectangles.

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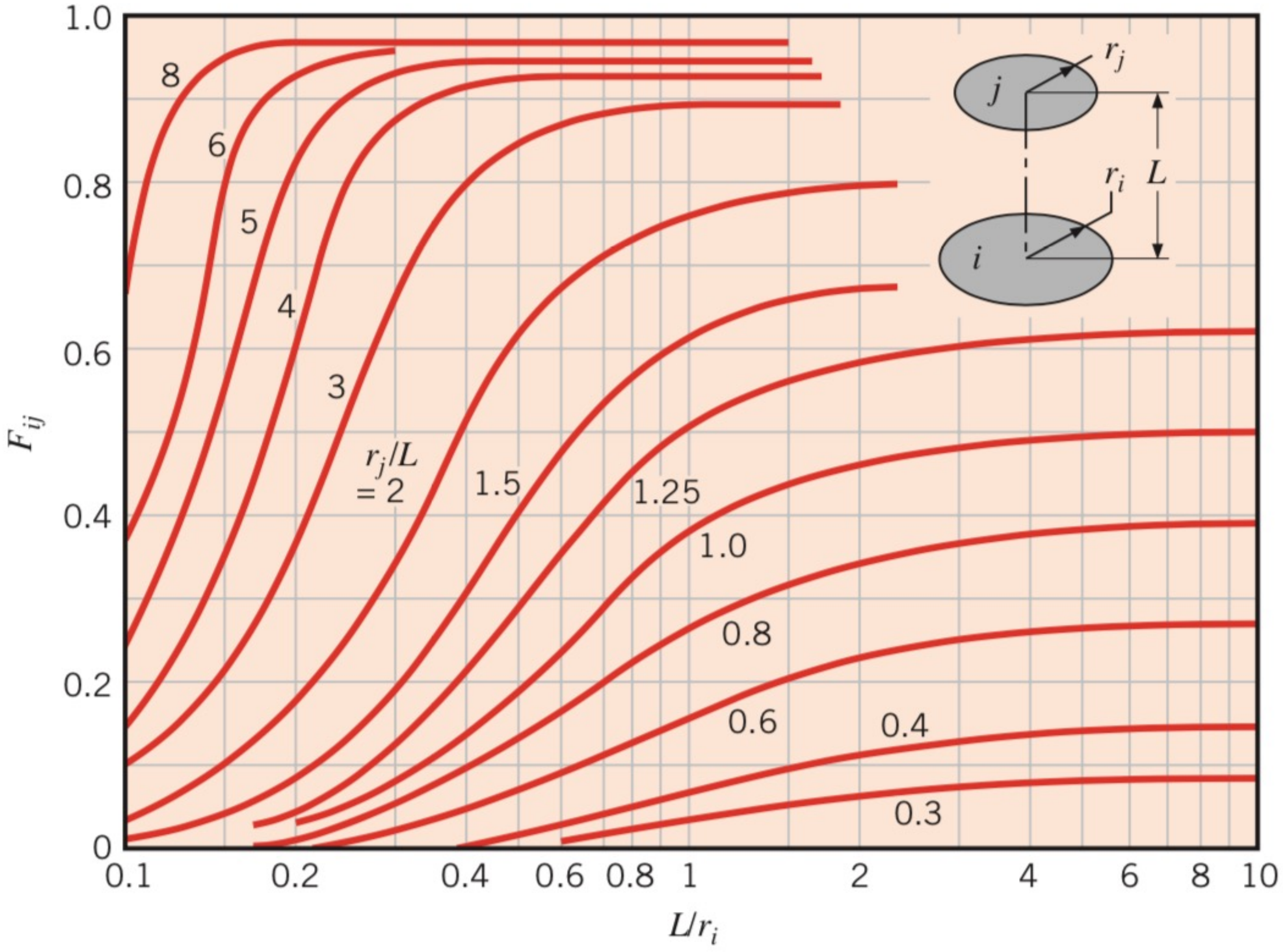

FIGURE 13.5 View factor for coaxial parallel disks.

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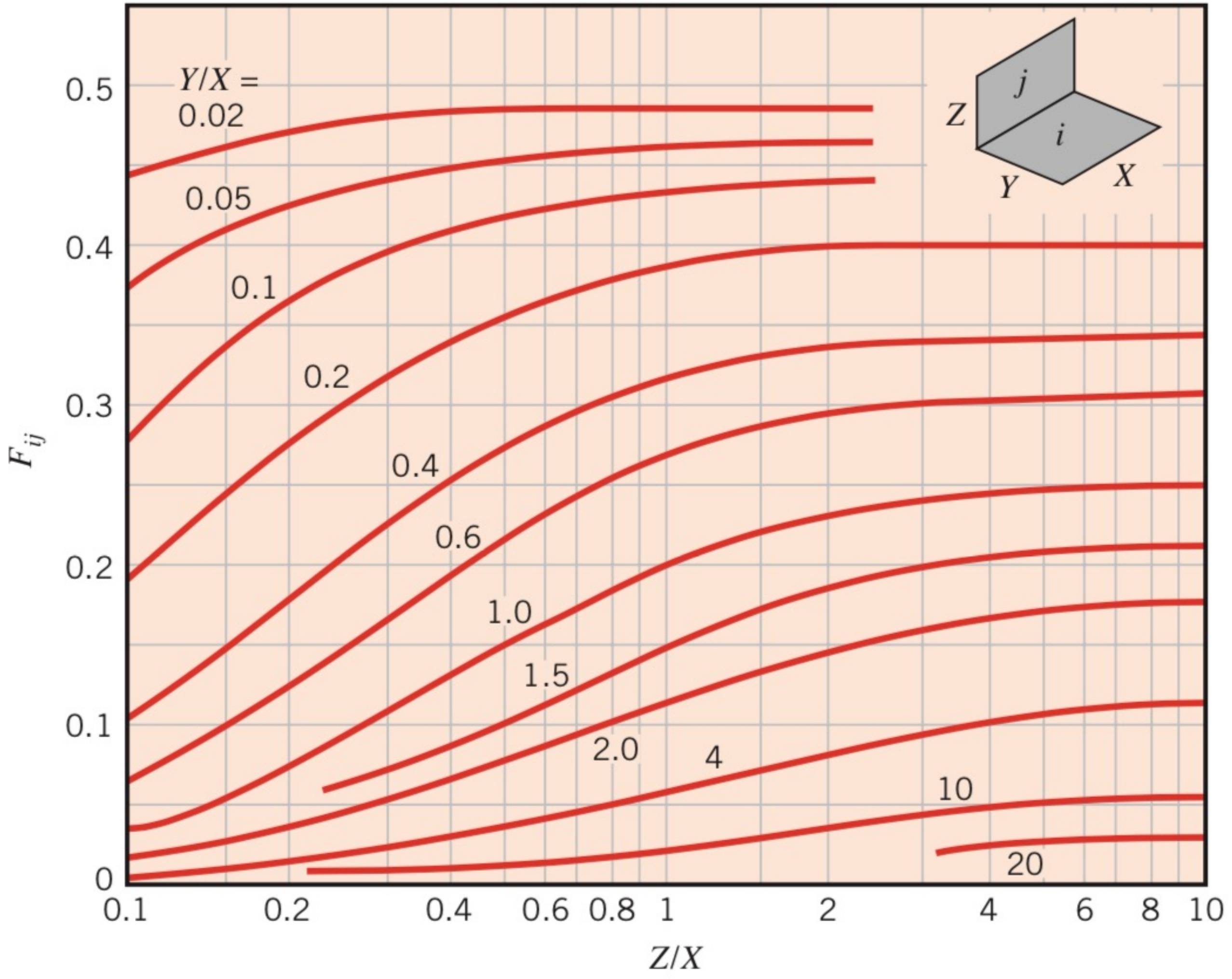

FIGURE 13.6 View factor for perpendicular rectangles with a common edge.

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