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Economic Lot Size For Production And Rotation Schedule With And Without Setup Time

[1]Nor Atikaf Rashid, [2]Dr Syarifah Zyurina Nordin and [3]Dr Nur Arina Bazilah Aziz

[1,2,3]Department of Mathematical Sciences
Faculty of Science, Universiti Teknologi Malaysia,
81310 Johor Bahru, Johor, Malaysia.

e-mail: [1]nor.atikaf@ymail.com, [2]szyurina@utm.my, [3]nurarina@utm.my,

Abstract In industry, company need to consider important things that can give them more profit. For this case, Economic Lot Size (ELS) is the important thing that can help the company minimizes the production cost. In this paper, the study concentrates on the inventory problem and production run for different items in a single machine. The objective is to determine the cycle length of the productions by minimizing the setup and holding costs. After calculating the cycle length, Economic Lot Size (ELS) for different items can be determined by using given formula in methodology. The sequences of the rotation schedule with and without setup time also will be defined using Travelling Salesman Problem (TSP) method. This study also conducts a case study including some different items to construct a feasible schedule from the optimal rotation schedule. .

Keywords Economic Lot Size (ELS); Travelling Salesman Problem (TSP); rotation schedule.

1 Introduction

Inventory is usually a company's largest asset and they become an important data to the budgeting system. In order to minimize the production cost, company needs to find Economic Lot Size. It refers to the best lot quantity to make the total cost minimum by considering the balance between ordering cost and inventory carrying cost, which are contradictory.

Another part to be considered is the customer's demand for each product. If there is no demand from customer, the inventory will have excess in stock and the holding costs occur for every item. Based on this situation, this study will solve the problem by finding the optimal cycle length and the Economic Lot Size that minimizes the production costs such as set up cost, holding cost and many more.

After getting the inventory information, company can make machine scheduling decisions using that data to see the sequence in production line based on cycle length and lot size for every item. The company can also calculate the amount of item produced in a year in order to minimize the production cost and they will get more profit in the future. In particular, total cost subject to capacity constraint can be minimized by constructing feasible production schedule because it satisfies all the constraints [1].

The study have the following objectives which are to determine the cycle length that minimizes setup costs and inventory holding cost, to calculate Economic Lot Size for different items and the last one is to determine rotation schedule without setup time and with setup times for different items in single machine.

2 Literature Review

In recent decades, process industry firms have thrived but changes in the markets are currently profitability at risk and growth. In this case, inventory management is progressively observed as an essential lever for creating a sustainable competitive advantage [2]. Demand is the quantity of a commodity or a service that people are willing or able to buy at a certain price, per unit of time. Demand curve can be obtained from the relationship of price and quantity demanded.

When the variability of demand is higher, variances becomes larger and the costs will be increased [3]. In manufacturing, a setup cost is incurred in order to get equipment such as machine to get ready. When setup depends only on the job to be processed, hence it is called sequence-independent and the next types is setup depends on both the job to be processed and the immediately preceding job, hence it is called sequence dependent [4].

Economic Lot Size likewise can be delegated the amount of material or units of a good that can be created or bought within the most reduced unit cost limit [5]. The lot size is most economic when total cost of ordering cost and inventory carrying cost is the lowest which is the minimum value. It is also called Economic Order Quantity (EOQ) [6]. Rotation schedule is a schedule to find production cycle which is identical that contain a single run for every item [7]. This TSP is equivalent to the sequencing problem which indicates city j as item j and the distance from city j to city k is the setup time needed when item k follows item j, s_{jk}. The algorithm naturally lends itself to implementation on a network of interconnected, analogue processing elements [8].

3 Methodology

In rotation schedule case, the task is determining the optimal cycle length by considering some assumptions which are, currently, the setup cost is sequence independent, the cycle lengths of the n items have to be identical, inventory level at the beginning of the production run of item j is zero, constant demands from customer, and the last one is, constant production rate.

The notations for the equation are shown below:

n = number of item
D_j = demand rate for item j (unit/day)
Q_j = capability of machine to produce item j (unit/day)
c_j = setup cost for item j (RM/for every setup)
L_j = length of production run of item j (day)
h_j = holding cost of item j (RM/days and unit)
x = the cycle length for all item
s_{jk} = the setup time needed when item k follows item j

In order to determine the optimal cycle length, the expression for the total cost per unit time as a function of the cycle length x need to be determined. The length of production run of item j, L_j is

$$L_j = \frac{D_j x}{Q_j} \quad \text{(Equation 1)}$$

During the production run, the level increases at rate $Q_j - D_j$ until it reaches maximum level of inventory level. The maximum of inventory level for item j, I_{max} can be calculated by using Equation 2.

$$I_{max} = \frac{(Q_j - D_j) D_j x}{Q_j} \quad \text{(Equation 2)}$$

In some production, there will be idle time which means no process occur for the machine. The items no longer produced by the machine. The idle time can be calculated using Equation 3.

$$\text{Idle time} = x\left(1 - \sum_{j=1}^{n} \frac{D_j}{Q_j}\right) \qquad \text{(Equation 3)}$$

During idle period (Equation 3), at rate D_j, the inventory decreases until it reaches zero. In this situation, the next production run starts to produce the item. The total average cost per unit time due to inventory holding costs and setup time can be calculated using Equation 4. In this case, the average cost per unit time (due to setup for item j) is considering as c_j / x.

$$T = \sum_{j=1}^{n}\left(\frac{1}{2} h_j\left(D_j x - \frac{D_j^2 x}{Q_j}\right) + \frac{c_j}{x}\right) \qquad \text{(Equation 4)}$$

Next step is, derive Equation 4 with respect to x and set it equal to zero as shown in Equation 5.

$$\sum_{j=1}^{n}\left(\frac{1}{2} h_j D_j \left(1 - \frac{D_j}{Q_j}\right)\right) - \frac{\sum_{j=1}^{n} c_j}{x^2} = 0 \qquad \text{(Equation 5)}$$

Finally, the optimal cycle length, x can be obtained when we arranged Equation 5 and make x as a subject.

$$x = \sqrt{\left(\sum_{j=1}^{n} \frac{h_j D_j \left(Q_j - D_j\right)}{2Q_j}\right)^{-1} \sum_{j=1}^{n} c_j} \qquad \text{(Equation 6)}$$

The quantity produced by every item, Economic Lot Size for every item j, $(\text{ELS})_j$ can be calculated by using Equation 7.

$$(ELS)_j = Q_j \times L_j \qquad \text{(Equation 7)}$$

Rotation schedule for all items can be constructed after finding all the values such as cycle length, idle time and maximum inventory level. The sequence of the item does not matter which is sequence independent since the optimal cycle length already obtained. In rotation schedule with set up time, the sequences of producing item will be sequence dependent. The sequencing of every item is important so that the sum of setup time can be minimized.

By using Travelling Salesman Problem (TSP) method, the minimum total of setup time for rotation schedule can be calculated. There are two types of TSP method which are asymmetric TSP (not the same in each opposite direction) and symmetric TSP (the same in each opposite direction). For asymmetric TSP, the possible sequences is *n!*.

4 Case Study

In this chapter, data are taken from previous journal with the title, 'The impact of batch shipments on the Economic Lot Scheduling Problem", which is data for the modified Eilon problem [9].

Table 4.1 : Data for Case Study

Item	Demand Rate (unit/day)	Production Rate (unit/day)	Holding Cost (RM/day and unit)	Setup Cost (RM/every setup
	D_j	Q_j	h_j	c_j
A	20	133	0.00461	3000
B	24	200	0.00312	1800
C	30	266	0.00651	3600
D	36	146	0.0118	1500
E	40	532	0.0119	6000
F	50	373	0.00847	30000

4.1 Data Analysis

The rotation schedule will include two cases which are with setup time and without setup time. These two cases will be calculated separately. The first calculation is to find cycle length, *x* using Equation 6, then the other values can be calculated such as:

a. Length of production run of item *j*, L_j (Equation 1)
b. Maximum of inventory level for item *j*, I_{max} (Equation 2)
c. Idle time (Equation 3)
d. Economic Lot Size for every item *j*, $(ELS)_j$ (Equation 7)

These values can be calculated using Microsoft Excel by substituting the values of holding cost, demand rate, capability of machine to produced item and the setup cost for item A, B, C, D, E and F.

Table 4.2 : Calculation from Microsoft Excel for L, I_{max} and ELS for case study.

Item	**D (unit/ day)**	**Q (unit/ day)**	**h (RM/day and unit)**	**C (RM/every setup)**	**L (day)**	**I_{max} (unit)**	**ELS (unit)**
1	20	133	0.00461	3000	37.90889	4283.704159	5041.882
2	24	200	0.00312	1800	30.25129	5324.227272	6050.258
3	30	266	0.00651	3600	28.43166	6709.872886	7562.823
4	36	146	0.01180	1500	62.16019	6837.62064	9075.387
5	40	532	0.01190	6000	18.95444	9325.586045	10083.76
6	50	373	0.00847	30000	33.79277	10915.06601	12604.7

The cycle length for all items, *x* is equal to 252.09409 days calculated by using Equation 6. It is almost equal to 9 months. Next, the idle time for the production run is equal to 40.59485 days. The calculation of idle time is using Equation 3. The idle time is including in cycle length for all items, *x*.

The total production run for each items is 211.49924 days. The total idle time and production run for each items is 252.09409 days which is equal to the value of cycle length for all items, *x*. The quantity produced by every item, Economic Lot Size, $(ELS)_j$ are to fulfill the demand for each items for 252.09409 days. Based on values of $(ELS)_j$ in Table 4.2, it can be see that the production run for every items can produce the items based on the machine capacity and fulfill the demands for 252.09409 days. Next, based on Equation 2, inventory level can be constructed based on I_{max} in Table 4.2.

Rotation schedule for all items can be constructed after finding all the values such as cycle length, idle time and maximum inventory level. Since this case is rotation schedule without setup time, sequence of the item does not matter. It is called sequence independent since the optimal cycle length already obtained. Maximum inventory for Item A, Item B, Item C, Item D, Item E and Item F can be calculated by using Equation 2.

Table 4.3 : Data for Inventory Level

Time (days)	Inventory Level (units)	Item
0	0	A
37.9089	4283.7042	A
37.9089	0.0000	B
68.1602	5324.2273	B
68.1602	0.0000	C
96.5918	6709.8729	C
96.5918	0.0000	D
158.7520	6837.6206	D
158.7520	0.0000	E
177.7065	9325.5860	E
177.7065	0.0000	F
211.4992	10915.0660	F
252.0941	0.0000	A
252.0941	4283.7042	A
252.0941	0.0000	B
252.0941	5324.2273	B
252.0941	0.0000	C
252.0941	6709.8729	C
252.0941	0.0000	D
252.0941	6837.6206	D
252.0941	0.0000	E
252.0941	9325.5860	E
252.0941	0.0000	F
252.0941	10915.0660	F

Based on Table 4.3, at time 0 days, it shows the initial time for the machine of the production. At that time, inventory level for Item A, Item B, Item C, Item D, Item E and Item F are zero. The machine starts to produce Item A at time 0 days for 37.9089 days, then the machine stop from producing Item A. At time equal to 37.9089 days, machine shop from producing Item A because it reach maximum inventory level and the other item remain the same which are zero items.

In this rotation schedule problem, the other items cannot be produced by the machine until the previous items is not done yet. Production of Item A on the machine is stop when it reaches 4283.7042 units in inventory level. At time equal to 0 days until 37.9089 days, 758.1778 ≈ 759 units of Item A already delivered to the customer. Secondly, maximum inventory for Item

B is 5324.2273 units. During 37.9089th day until 68.1602th day, 726.0307 units already delivered to the customer.

Maximum inventory for Item C is 6709.8729 units. During period 68.1602th day until 96.5918th day, 852.9501 units already delivered to the customer. For Item D, the maximum inventory is 6837.6206 units and 2237.7664 units already delivered to the customer during period 96.5918th day until 158.7520th day.

Maximum inventory for Item E is 9325.5860 units. During period 158.7520th day until 177.7065th day, 758.174 units already delivered to the customer. The last one, the maximum inventory for Item F is 10915.0660 units and 1689.634 units already delivered to the customer during period 177.7065th day until 211.4992th day. All the values of units with decimal number will be taking the next integer number. Some of the items already delivered to customer during production run to avoid the holding cost of the inventory.

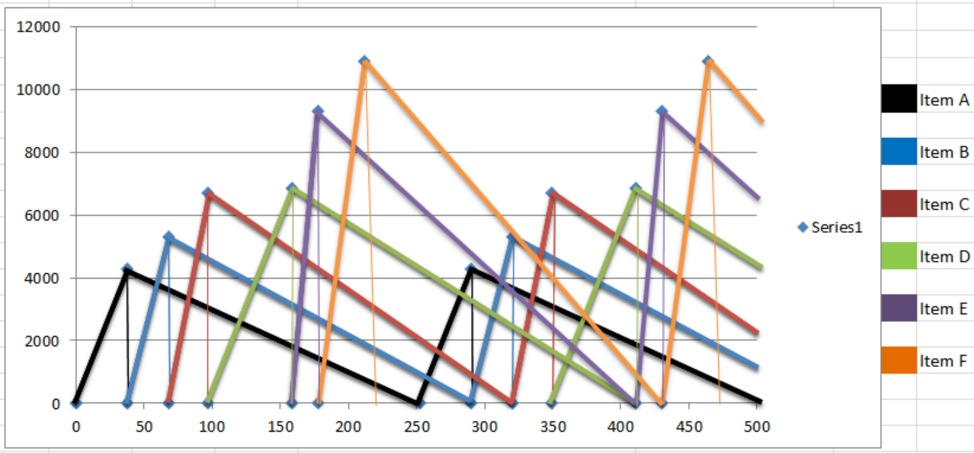

Figure 4.1 Rotation schedule for sequence (A-B-C-D-E-F) without setup time

Based on Figure 4.1, *y-axis* refers to inventory level and *x-axis* is time (days). From Figure 4.1, the sequence of items in the single machine are start from Item A followed by Item B, Item C, Item D, Item E and Item F. The time taken to produce all the items is 211.4992 days. After that, the production has the idle time for 40.59485 days. Then the second cycle for production will start with the same sequence. Besides, the second cycle will start to produce Item A and the other items after completing the first cycle including idle time for the machine. This cycle continues with the same sequence and constant cycle length for all items.

When the case includes setup time, using the same data in previous solution, the sequences of producing Item A, Item B, Item C, Item D, Item E and Item F will be sequence dependent. The sequencing is important so that the sum of setup time can be minimized and given the setup matrix is asymmetric.

Table 4.4 : Setup Time for case study

Setup time	A	B	C	D	E	F
SAk	-	3.325	5.868	2.842	7.953	7.410
SBk	2.153	-	3.430	6.292	2.990	2.563
SCk	4.028	6.899	-	2.761	7.073	6.729
SDk	7.105	5.283	6.582	-	6.865	6.067
SEk	6.844	4.354	7.346	2.870	-	2.793
SFk	4.872	5.208	5.912	4.546	4.362	-

The data for setup time are generated within range 2-8 days based on given setup time in the journal. In this study, these data have to generated separately because the given setup time in the journal are not asymmetric setup matrix. The range of data are taken from previous journal with the title, 'The impact of batch shipments on the Economic Lot Scheduling Problem", which is for the modified Eilon problem [9]. By using Travelling Salesman Problem (TSP) method, the minimum total of setup time for rotation schedule can be calculated. There are 6! =720 possible sequences can be constructed. All possible sequences are generated using Microsoft Visual Studio software. From Table 4.4, setup time can be calculated from the first item until the 6^{th} item, then go back to the first item.

For example, the sequence is A-B-C-D-E-F-A. The calculation of the total setup time (days) are calculated using Microsoft Excel for all the sequences.From the calculation of total setup time for every sequence, the sequence (A-B-F-E-D-C-A), (C-A-B-F-E-D-C), (D-C-A-B-F-E-D), (E-D-C-A-B-F-E), (F-E-D-C-A-B-F) have the same total setup time of the sequence which is 23.730 days which is the lowest compared with other sequences. The length of production run of item *j* remains the same, only the sequence has difference from the previous case by considering setup time. The set up time for those sequences (A-B-F-E-D-C-A), (C-A-B-F-E-D-C), (D-C-A-B-F-E-D), (E-D-C-A-B-F-E), (F-E-D-C-A-B-F) are optimal because it is less than idle time = 40.59485 days in the rotation schedule without setup time. So, the industry can choose between 5 of the sequences to get optimal sequence.

Based on data analysis, there are 5 sequences that have the same setup time which is 40.59485 days. The sequences will give the optimal sequence for the single machine to run all the items. From the result of setup time, it shows that there are no sequence starting with Item B gives the minimum setup time but the others 5 items has one sequence that optimal for production run. The setup time of the sequence must less than idle time, to avoid extra time to setup the machine. If the setup time for the machine is larger than the idle time, it will disturb all the production process such as the cycle length for all items.

5 Conclusion

As a conclusion, all the objectives in this study have been achieved and all the result already obtained. The first objective of the study is to determine the cycle length that minimizes setup costs and inventory holding cost. From example of the small size production, the calculation of cycle length can be calculated using the given formula in Equation 6, but for the case study, all the calculation become easier using Microsoft Excel.

All the values such as of production rates, demands rates, holding costs and setup costs are substituted in Microsoft Excel and applying all the given equation to find L, I_{max} and ELS. Besides that, the possible sequences for the rotation schedule in case study becomes more complicated because there are up to 720 possible sequences compared to the possible sequences in small example. This is solved by generating the sequences using C++ Programming. All the possible sequences can be analysing based on its setup time in the second case which is rotation schedule with setup time.

The second objective of this study also has been achieved successfully when the Economic Lot Size (ELS) for all items have been calculated. The value of ELS of every items fulfil all the demands for the given time based on the production run which is the cycle length for all items.

The last objective of this study, which is to determine rotation schedule without setup time and with setup times for different items in single machine becomes easier using Microsoft Excel. In case study, we have to consider 720 possible sequences and it setup time find the minimum setup time so that the optimal sequence can be obtained.

6 Recommendation

In real industry, there are more than 6 item used in ELSP. So, for further study, to determine the optimal sequences with minimum setup time, a special coding can be built so that the optimal sequences can be obtained with simplest analysis. Besides, to do analysis about the rotation with setup time, suitable software of production run of the machine can be used to see the long term of production in the industry. Lastly, the transportation cost for complete lots or batch shipment should be considered so that the industry can minimize the profit.

7 References

[1] Rossi, T., Pozzi, R., & Testa, M. (2017). EOQ-based inventory management in single machine multi-item systems. *Omega, 71*, 106-113. Doi:10.1016/j.omega.2016.10.002

[2] Moser, P., Isaksson, O. H. D., & Seifert, R. W. (2017). Inventory dynamics in process industries: An empirical investigation. *International Journal of Production Economics, 191*(Supplement C), 253-266. doi:https://doi.org/10.1016/j.ijpe.2017.06.019

[3] Ridder, A., Van Der Laan, E., & Salomon, M. (1998). How larger demand variability may lead to lower costs in the newsvendor problem. Operations Research, 46(6), 934-936.

[4] Allahverdi, A., Gupta, J. N., & Aldowaisan, T. (1999). A review of scheduling research involving setup considerations. *Omega*, *27*(2), 219-239.

[5] Economic Lot Size. BusinessDictionary.com. Retrieved February 16, 2018, from http://www.businessdictionary.com/definition/economic-lot-size.html

[6] The Economic Lot Size Model. The Economic Lot Size Model ...{Strategos}. Retrieved February 16, 2018, from www.strategosinc.com/economic_lot_sizing.htm.

[7] Sule, D. R. (2008). *Production Planning and Industrial Scheduling: Examples, Case Studies and Applications*. United States of America: CRC Press

[8] Angeniol, B., Vaubois, G. D. L. C., & Le Texier, J. Y. (1988). Self-organizing feature maps and the travelling salesman problem. *Neural Networks*, *1*(4), 289-293.

[9] Beck, F. G. and C. H. Glock (2016). "The impact of batch shipments on the Economic Lot Scheduling Problem." Computers & Industrial Engineering 96: 126-139.