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Mathematical Model for Timetabling Problem in Maximizing the Preference Level

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Abstract Timetabling problem can be classified as an assignment problem which is very crucial in making sure all the events occur at the perfect place and time demanded. The main objective of this study is to investigate an optimal solution by maximizing the total preferences level on lecturer to course to time slot assignments. University course timetabling problem is the central focus in this study. Mixed integer linear programming (MILP) model is used to solve the problem and conducted using LINGO 16.0. Results obtained lead to a satisfaction for the lecturer and generate a conflict- free timetabling for all parties involved.

Keywords mathematical model; preference level; timetabling problem

1 Introduction

Timetabling is a set of information that will showing exactly when a particular events to take part. It is very vital in making sure all the events occur at the place and time required. In fact, timetabling is urgently important in areas such as: education, sports competitions, production and manufacturing, logistics and transport. In the usual form of the problem, a set of people need to be assign to a set of tasks. Among those, the University Course Timetabling Problem (UCTP) can be classified as a semiannual problem faced by the administration staffs as well as the lecturers. According to Burke et al. [1] timetabling problem is a thing that required a proper-organized and efficient schedule. Back then, timetables are constructed manually before the existence of computer technology use [2]. During 60's, it is the time for the first attempt timetable development are made with the application of computer aided tools [3].

Course timetabling problems are repeatedly faced practically by every school, college and university as well. Basically, a set of times must be assigned to a set of events where all of students can attend of their respective events [4]. The problem of constructing timetables for educational institutions is a classic combination problem that requires finding a schedule to determine which courses will be given by which lecturers and by which timeslots considering the lecturers and students preferences of courses to be allocated in specific timeslots to a number of hard and soft constraints.

The planning of an efficient weekly timetable has always been a fundamental challenge of every university administration. On the other hand, the effectiveness of a timetable is measured using factors such as: the total rate of lecturers and students preferences, exertion of resources, and total number of students conflicts. A pleased and supremacy university timetable is a timetable that can fulfill the user preferences while refrain clashes between courses taken by the same group of students and lecturers handling the courses and classroom being used [5]. The problem is farther intricate as there is usually an objective to be optimized. In the university course timetabling problem, the objective is relates to optimizing the total preferences of lecturers and students assigned to their desired slots [2].

The general university course timetabling problem is known to be NP-complete mainly due to the associated constraints have been studied [6]. In this study, a detailed analysis of the problem leads in creating a problem definition which allows the identification of all the process issues, constraints, restrictions and objectives. Generally, problems involving optimizing an objective function subject to a certain constraints can be solved using a mathematical programming approach [3]. Therefore, we concern to solved a more general timetabling problems in educational institutions more effectively using a mathematical approach.

The timetabling process is quite long and involved many stages before assigning lecturer to a certain course and at some particular timeslots. The data for timetabling problem is often collected in a table. The difficulty in this problem as to satisfying all the restrictions and requirements. The restrictions are related to the resources such as venue and time as well as conflicts, whereas the requirements are related to the preferences of customers and service providers.

Generally, this study is conducted in finding the optimal for university course timetabling problem which is assigning which lecturer will be teaching during a particular timeslot and which courses will be scheduled during the same timeslot. Furthermore, the consideration regarding the preferences of the lecturers will be highlighted in this study. In this case study, the rooms availability are fixed to always available and fit enough to hold the capacity of students. Thus, the problem is contributed to solved using LINGO 16.0 software with findings the optimal decision. The overall goal of the timetable is to provide students with a schedule that is not only feasible but also the one that conflict-free.

2 Problem Formulation

The Faculty of Science at Universiti Teknologi Malaysia (UTM) does not possess an automated timetabling system, as in any other faculties in UTM. At the starting of each academic semester, the administration staffs is doing the assignments in the pre-designed timetable with the consideration of new requirements. In Faculty of Science, UTM, there are combination of three main departments which are Department of Chemistry, Physics and Mathematical Sciences. This study will only cover one department as its case study only, that is Department of Mathematical Sciences.

The following information are taken from UTM for the Department of Mathematical Sciences undergraduate courses timetabling problem for semester 2 (2016/2017). Undergraduate programme for Department of Mathematical Sciences in Faculty of Science, UTM, consists of four years of studies and offers 23 different subjects with 2 or 3 credit hours per semester. A total of 104 class meeting should be assign to some specific timeslots by considering the preferences level on lecturer to courses at a timeslot. Each courses have freedom in select the elective subjects offered to cover up the credit requirements per semester. Therefore, some class meetings might assign to a same timeslot and same lecturer due to the criteria of same section.

Meanwhile, there will be a total of five working days taken into consideration which are from Sunday until Thursday. The total working hours involved in a week is as much as 39 one hour timeslots, from 0800-1600. Depending the credit hours, some courses need to have at most two consecutive lecture hours per week while another one hour lecture if it is an 3 credit hours' subjects. This UCTP will be modelled as a MILP model and compute using LINGO 16.0 to generate a feasible solution. Hence, generate a conflict-free timetable for the semester. General statistics to briefly give the picture of the size for the case study is represented as below:

Table 1: General size of the case study

Requirements	**Day**	**Timeslots**	**Course**	**Lecturer**	**Max. Class Meeting**
Number	5	39	8	25	6

In order to strengthen the quality of the model, a few assumptions and requirements from the department should be taken as one priority. Therefore, the following are some assumptions that have been made regarding this case study:

- only 6 rooms available for one timeslot
- all rooms are always fit to hold a capacity of students
- at time 1300-1400 is reserved for rest time for all lecturer and students

The list of other requirements that are necessary to maintain the rules and regulations that have been fixed by the Department of Mathematical Science, FS, UTM are detailed as below:

1. all class meetings must be assigned to a specific timeslot and should be in between 0800 until 1700

2. a weekly work load of each lecturer must be between his/her lower and upper limits
3. each courses cannot having a same lecture more than 2 hours a day
4. two units of course taught by the same lecturer cannot be taught at the same timeslot
5. the number of the same timeslot assigned to the lecturer should always be bounded
6. a course can only attend as long a 8 hours of class meetings except during Thursday
7. not more than 6 class meetings should be schedule in one timeslot no lecture or tutorial can be scheduled during t_{22}, t_{23} and t_{24} since that timeslots are reserved for curricular's subjects

Then, MILP model is chosen to solve the case study. Moreover, this this the most preferable methods used in solving a timetabling problems. Besides that, this mixed integer linear programming also has been a successful method that have been solved in previous studies, but compilation in creating the model and the computational challenges due to the variety size of the problem lead in researcher moving towards another approaches [7].

Optimal solutions for a school and a university timetabling problem are able to achieve by presenting linear and integer programming model for the problem [8, 9]. Apart from that, Breslaw follow the lead by offering a solution for the faculty assignment problem via linear programming model [10]. McClure and Wells are also making an attempt to solve a same problem using the same approach.

3 MILP model

The MILP model will be designed based on the information obtained and described in this section. In order to construct the mathematical model, indices, sets of the model, parameters of the model, decision variable, objective function and the model constraints need to be obtained first.

3.1 Indices

l - index of lecturers, $l = 1,2, \dots, m$
k - index of courses, $k = 1,2, \dots, n$
t - index of timeslots, $t = 1,2, \dots, r$

3.2 Sets

L - Set of lecturers, $L = \{l: l = 1,2, \dots, m\}$
K - Set of courses, $K = \{k: k = 1,2, \dots, n\}$
T - Set of timeslots, $T = \{t: t = 1,2, \dots, r\}$

3.3 Parameters

h_k - Total number of credit hours for k_{th} course in a week

c_l	-	Maximum number of courses given to a l_{th} lecturer per semester
s_l	-	Lower bounds (in hours) of the l_{th} lecturer's work load
u_l	-	Upper bounds (in hours) of the l_{th} lecture's work load
d_t	-	Maximum number for consecutive timeslot, that might be assigned to a lecturer during the week
$p_{l,k,t}$	-	Preferences of l_{th} lecturer for the k_{th} course at timeslot, t

3.4 Decision variables

$$y_{l,k} = \begin{cases} 1, & if\ l_{th} \text{ lecturer is assigned to } k_{th} \text{ course} \\ 0, & \text{otherwise.} \end{cases}$$

$$z_{k,t} = \begin{cases} 1, & if\ k_{th} \text{ course is given on time, } t \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{l,k,t} = \begin{cases} 1, & if\ k_{th} \text{ course given by the } l_{th} \text{ lecturer at time, } t \\ 0, & \text{otherwise.} \end{cases}$$

3.5 Objective function

Objective function are constructed by maximizing the preference level of each lecturer on the assignments. The corresponding preferences input can be scalarized as a level from 1,2…,9 and value 0 represents a non-availability. The constructed objective function can be seen below:

Maximize

$$\sum_{l=L}\sum_{k=K}\sum_{t=T} p_{l,k,t}x_{l,k,t}$$

Subject to

$$\sum_{t=T} z_{k,t} = h_k \qquad \forall\ k \in K \tag{3.1}$$

$$\sum_{k=K} x_{l,k,t} \leq d_k \qquad \forall\ l \in L, t \in T \tag{3.2}$$

$$\sum_{k=K} y_{l,k} \leq c_l \qquad \forall\, l \in L \tag{3.3}$$

$$s_l \leq \sum_{k=K} y_{l,k} h_k \leq u_l \qquad \forall\, l \in L \tag{3.4}$$

$$\sum_{t=T} x_{l,k,t} \leq y_{l,k} \qquad \forall\, l \in L, k \in K \tag{3.5}$$

$$\sum_{l=L} x_{l,k,t} \leq z_{k,t} \qquad \forall\, k \in K, t \in T \tag{3.6}$$

The objective function is to maximize the timeslot preference, $p_{l,k,t}$ of allocating k_{th} course given by the l_{th} lecturer. Based on the approach of assigning values of the parameter, $p_{l,k,t}$ all lecturers will provide different level of preferences for their desired timeslots. Therefore, this information is a pre-processing data obtain from the lecturer before the process of timetabling begins.

The constraint set (3.1) ensure that the number of a course assign to the timeslots must be equal to h_k. Meanwhile, constraint (3.2) ensure that the number of timeslot, t assigned to each lecturer does not exceed d_t. Similarly, the constraint set (3.3) guarantees that the number of courses assigned to a lecturer will not exceed c_l. Then, constraint set (3.4) will make sure the weekly work load of the lecturer is between his/her lower and upper limits. If any assignment of lecturer to a course at some timeslot, constraint set (3.5) will forces $y_{l,k}$ = 1. However, if there is no lecturer been assign to a course at some particular timeslot, the left-hand side of the inequality in constraint (3.5) will be set as zero, which is $y_{l,k}$ = 0. Correspondingly, if any pair of course assigned at a timeslot, t to a respective lecturer, the constraint set (3.6) will set $z_{k,t}$ = 1. Otherwise, if no pair of course-timeslot assigned, then $z_{k,t}$ = 0.

4 Results and Disussion

Second Semester course unit timetable of Department of Mathematical Sciences, Faculty of Science, UTM has been modeled and optimal solutions were found for the four years of studies. The data was taken from the administrations staff at Academic Office of Faculty of Science, UTM. A MILP model has been defined for each of the eight courses and solved using LINGO 16.0 software, since the model involved thousand of variables and constraints.

Before reach a conclusion stated whether the model is such a success and efficient, there are few materials that should be looking at from the derived solution are:

1. lecturers' preference level in assign to a class meeting at a timeslot
2. existence of class-clashing or parellel lecturers' incharge
3. CPU time on getting a solution

The first criteria that should be achieved is whether the models able to maximize the lecturers' preference level in the assignments. Then, are there any existence of class-clashing and the time taken of the CPU to generate a solution will lead to a meaning of its a time-saving model for UCTP.

An overall timetable for the eight courses in Second Semester (2016/2017) is given in **Table 2**, while **Table 3** shows an overall of the lecturer assign to that specific timeslot and courses. Since this study intends to develop a model which assigns each courses to a lecturer and at its most desirable timeslot, **Table 2** and **Table 3** generally shown not just a feasible solution, but also with a complete conflicts free semester timetable.

Table 2: Courses' general timetable model for Semester 2 (1026/2017)

Time / Day	0800-0900	0900-1000	1000-1100	1100-1200	1200-1300	1400-1500	1500-1600	1600-1700
Sunday	1SSCM 2SSCE 3SSCM 3SSCE	1SSCM 2SSCM 2SSCE 3SSCM 3SSCE 4SSCM	1SSCE 2SSCM 3SSCE 4SSCM	1SSCE 3SSCE	1SSCM 1SSCE	2SSCM 2SSCE 3SSCM 4SSCM 4SSCE	2SSCM 2SSCE 3SSCM 4SSCM 4SSCE	4SSCE
Monday	1SSCE 3SSCM 3SSCE	1SSCM 3SSCM 3SSCE	1SSCM 4SSCE	3SSCM 4SSCE 4SSCE	4SSCM 4SSCE		1SSCE 2SSCM 2SSCE	1SSCE 2SSCM 2SSCE 3SSCM 3SSCE
Tuesday	2SSCM 3SSCE	1SSCE 2SSCM 3SSCM 3SSCE	2SSCE 3SSCM	1SSCM 2SSCE	1SSCM			
Wednesday	1SSCM 2SSCE 4SSCM	1SSCM 2SSCE 3SSCM 3SSCE 4SSCM	1SSCM 1SSCE 2SSCE 3SSCE	1SSCE 3SSCM 3SSCE 4SSCE	2SSCM 3SSCM 4SSCE	2SSCE 4SSCE		
Thursday	2SSCE 4SSCE	1SSCE 4SSCM	2SSCM 4SSCM 4SSCE	2SSCM 2SSCE 4SSCM 4SSCE	1SSCE 3SSCM 3SSCE	1SSCM 1SSCE 2SSCM 2SSCE 3SSCE 4SSCM	1SSCM 2SSCM	

As what can be seen from the table, there are no clashing class meetings between the courses. Based on the approach of this study, assigning values of the preferences in the objective function, a level of preferences has been generated randomly using Excel Solver.

As for the satisfaction level, each assignments of a lecturer to a course at a timeslot have been analyzed and it shows that the model successful in that objective

Table 3: Lecturer's general timetable model for Semester 2 (1026/2017)

Time / Day	0800-0900	0900-1000	1000-1100	1100-1200	1200-1300	1400-1500	1500-1600	1600-1700
Sunday	l_5 l_2 l_{18}	l_5 l_{11} l_2 l_{18} l_{25}	l_2 l_9 l_{16} l_{25}	l_2 l_{16}	l_1 l_4	l_{12} l_{11} l_{13} l_{22}	l_2 l_{11} l_{13} l_{23} l_1	l_1
Monday	l_6 l_{17}	l_4 l_{17}	l_4 l_{20}	l_{21} l_{23} l_{20}	l_{23} l_{21}		l_7 l_3	l_7 l_3 l_{18}
Tuesday	l_9 l_{19}	l_7 l_9 l_{15} l_{19}	l_{10} l_{15}	l_5 l_{10}	l_8			
Wednesday	l_4 l_{12} l_{24}	l_1 l_{12} l_{13} l_{14} l_{24}	l_1 l_4 l_{11} l_{14}	l_4 l_{21} l_{14}	l_{12} l_{21}	l_{12} l_{20}		
Thursday	l_2 l_1	l_2 l_{24}	l_2 l_{22}	l_2 l_{12} l_{22}	l_6 l_{17}	l_8 l_6 l_{10} l_{11} l_{14} l_{25}	l_8 l_{11}	

Furthermore, from **Table 3**, there's no redundant in terms of a lecturer teaching 2 classes during one timeslot except those which from a same section. Just as it shown in **Table 4** below, the model achieved computational time of 0.47 seconds for the case study. In other words, it is remarkably a time-saving. Hence, this already gives satisfaction in terms of CPU time taken to generate an optimal solution.

Table 4: Summary of case study's result from LINGO 16.0 software

Elapsed runtime	0.47 *sec*
Objective value	932
Total variables	8312
Total constraints	1570

To sum things up, the application of contructed model to the case study adapted from Department of Mathematical Sciences FS, UTM is such a success. Moreover, the results obtained are in fact considered excellent.

5 Conclusions

In short, the model was able to maximize the preferences level of lecturer on the assignments of lecturer to a course at some particular timeslots. In addition an optimal solution also able to be generated and a timetable model was deduced from the analyzed results. Further, the model still successful in obey the requirements fixed by the Department of Mathematical Sciences where there is not more than 6 class meeting being assign to a timeslot. This is because of the limitations criteria of the rooms availability. Together with all the consideration of assumptions and requirements, its slightly not indicate any inefficiency of the developed model. As a suggestion it would be encourageable if any additional features to be implemented in the same model by further analysis.

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