UTM
UNIVERSITI TEKNOLOGI MALAYSIA

UNIVERSITI TEKNOLOGI MALAYSIA
FACULTY OF SCIENCE

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FINAL EXAMINATION
SEMESTER I SESSION 2017/2018

COURSE CODE	:	**SSCE 1693**
COURSE NAME	:	**ENGINEERING MATHEMATICS I**
PROGRAMME	:	**SKEE, SKEL, SMBE, SKMM, SKMP, SKMB, SKMI, SKMT, SKAW, SKTB, SKTG.**
LECTURER	:	**ASSOC. PROF. DR. ONG CHEE TIONG (C)** **PROF. DR ZUHAIMY ISMAIL** **ASSOC. PROF. DR. NOR'AINI ARIS** **ASSOC. PROF. DR. YUDARIAH YUSOF** **DR ZAITUL MARLIZAWATI ZAINUDDIN** **DR YEAK SU HOE** **DR ANI SHABRI** **DR MUHAMMAD FAUZEE HAMDAN** **DR NIKI ANIS ABD KARIM** **DR NUR ARINA BAZILAH AZIZ** **DR TAUFIQ KHAIRI AHMAD KHAIRUDDIN** **DR ZUHAILA ISMAIL** **TN HJ IBRAHIM JAIS** **PN NORASLINDA MOHAMED ISMAIL** **PN WAN RUKAIDA WAN ABDULLAH**
DATE	:	**2 JANUARY 2018**
DURATION	:	**3 HOURS**
INSTRUCTION	:	**ANSWER ALL QUESTIONS**

(THIS EXAMINATION BOOKLET CONSISTS OF 9 PRINTED PAGES)

QUESTION 1 (3 MARKS)

By using the definition of hyperbolic function, solve the equation

$$\sinh x + k = \cosh x$$

where $k > 0$ is a constant.

QUESTION 2 (3 MARKS)

Find $\frac{dy}{dx}$ if given

$$y = \tan^{-1}(\sinh 2x).$$

QUESTION 3 (7 MARKS)

Evaluate the integral

$$\int_0^{0.5} \tanh^{-1} x.$$

QUESTION 4 (7 MARKS)

Determine whether the following integral converges or diverges

$$\int_{0.5}^{1} \frac{dx}{x \ln x}.$$

If it converges find it's value

QUESTION 5 (10 MARKS)

a) Determine the convergence of

$$\sum_{r=1}^{\infty} (-1)^r \frac{r!}{\pi^r}.$$

(4 marks)

(b) Verify that the Maclaurin series for $\sin^2 x$ is given by

$$\sin^2 x = x^2 - \frac{1}{3}x^4 + \frac{2}{45}x^6 + \cdots$$

Hence, approximate the following definite integral correct to 4 decimal places

$$\int_0^{0.2} \frac{\sin^2 x}{x}\, dx.$$

(6 marks)

QUESTION 6 (15 MARKS)

a) Find the acute angle in degree between the planes defined by $8x + 10y + 3z = 4$ and $2x + 4y + 6z = 3$. (3 marks)

b) Find the shortest distance between the point A$(2, 5, -7)$ and the plane $3x - 5y + 6z = 9$.

(7 marks)

c) Determine whether the line with equation

$$\mathbf{r} = \begin{pmatrix} -3 \\ -6 \\ -11 \end{pmatrix} + t \begin{pmatrix} 22 \\ 1 \\ -11 \end{pmatrix}$$

lies in the plane that contains the points A$(2, 5, 6)$, B$(-7, 1, 4)$ and C$(6, -2 - 9)$.

(5 marks)

QUESTION 7 (15 MARKS)

a) Consider the following system of linear equations:

$$2x + 5y - 2z = 14,$$

$$x + 3y - 4z = 5,$$

$$x + 2y + 2z = 9.$$

i. Convert the above systems of linear equations into a matrix equation and by elementary row operations determine its rank to identify the type of solution. (4 marks)

ii. Solve the above matrix equation. (3 marks)

iii. What does the solution in part (ii) above represent?. (1 marks)

b) Given the matrix

$$\mathbf{P} = \begin{pmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{pmatrix}.$$

Find all the eigenvalues of $\mathbf{P}$. Hence obtain an eigenvector corresponding to the biggest eigenvalue of matrix $\mathbf{P}$.

(7 marks)

QUESTION 8 (20 MARKS)

a) Transform the polar equation $\left(\frac{2}{r}\right)^2 = \cos^2\theta + 2\sin^2\theta$ into its Cartesian form and sketch the graph. (4 marks)

b) Given the polar equation $r = 3\cos 2\theta$.

i. Test the symmetries of the above polar equation. (3 marks)

ii. Construct a table for (r, θ) with appropriate values and sketch the graph of $r = 3\cos 2\theta$ on the polar grid provided. (5 marks)

iii. Sketch the graph $r = 3\cos\theta$ on the same polar grid in part (ii). (3 marks)

iv. Find the intersection points between the curves $r = 3\cos 2\theta$ and $r = 3\cos\theta$. (5 marks)

QUESTION 9 (20 MARKS)

a) i. Show that

$$\frac{1+i\sqrt{3}}{1-i\sqrt{3}} = \frac{-1}{2} + i\frac{\sqrt{3}}{2}.$$

(2 marks)

ii. Find all the roots of $z^4 = \dfrac{1+i\sqrt{3}}{1-i\sqrt{3}}$. (4 marks)

iii. Sketch all the roots in an Argand diagram. (2 marks)

iv. If z_1 and z_3 are the roots in the first and third quadrant respectively, find w such that $\dfrac{1}{w} = \dfrac{1}{z_1} - \dfrac{1}{z_3}$. Give your answer in the form of $a + ib$. Deduce the modulus and argument of w such that $-\pi < \arg(w) \leq \pi$ (4 marks)

b) Using De Moivre's theorem show that

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1.$$

Hence obtain all the roots of the equation

$$12x^4 - 12x^2 + 1 = 0$$

correct to four decimal places.

(8 marks)