LISTS OF FORMULAE

Trigonometric

$$\cos^2 x + \sin^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$\cot^2 x + 1 = \text{cosec}^2 x$$
$$\sin 2x = 2 \sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$
$$2 \sin x \cos y = \sin(x + y) + \sin(x - y)$$
$$2 \sin x \sin y = -\cos(x + y) + \cos(x - y)$$
$$2 \cos x \cos y = \cos(x + y) + \cos(x - y)$$

Hiperbolic

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$1 - \tanh^2 x = \text{sech}^2 x$$
$$\coth^2 x - 1 = \text{cosech}^2 x$$
$$\sinh 2x = 2 \sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$= 2\cosh^2 x - 1$$
$$= 1 + 2\sinh^2 x$$
$$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x}$$
$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$
$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$
$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$

Logarithm

$$a^x = e^{x \ln a}$$
$$\log_a x = \frac{\log_b x}{\log_b a}$$

Inverse Hiperbolic

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}), \ -\infty < x < \infty$$
$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}), \ \ x \geq 1$$
$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right), \ -1 < x < 1$$

Differentiations	**Integrations**
$\frac{d}{dx}[k] = 0, \quad k$ constant	$\int kdx = kx + C$
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$
$\frac{d}{dx}[\ln\lvert x\rvert] = \frac{1}{x}$	$\int \frac{dx}{x} = \ln\lvert x\rvert + C$
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x\, dx = -\cos x + C$
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\, dx = \sin x + C$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}[\cot x] = -\text{cosec}^2 x$	$\int \text{cosec}^2 x\, dx = -\cot x + C$
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\, dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\, x] = -\text{cosec}\, x \cot x$	$\int \text{cosec}\, x \cot x\, dx = -\text{cosec}\, x + C$
$\frac{d}{dx}[e^x] = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}[\cosh x] = \sinh x$	$\int \sinh x\, dx = \cosh x + C$
$\frac{d}{dx}[\sinh x] = \cosh x$	$\int \cosh x\, dx = \sinh x + C$
$\frac{d}{dx}[\tanh x] = \text{sech}^2 x$	$\int \text{sech}^2 x\, dx = \tanh x + C$
$\frac{d}{dx}[\coth x] = -\text{cosech}^2 x$	$\int \text{cosech}^2 x\, dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\, x] = -\text{sech}\, x \tanh x$	$\int \text{sech}\, x \tanh x\, dx = -\text{sech}\, x + C$
$\frac{d}{dx}[\text{cosech}\, x] = -\text{cosech}\, x \coth x$	$\int \text{cosech}\, x \coth x\, dx = -\text{cosech}\, x + C$
$\frac{d}{dx} \ln\lvert \sec\ x + \tan\ x\rvert = \sec\ x$	$\int \sec\ x\, dx = \ln\lvert \sec\ x + \tan\ x\rvert + C$
$\frac{d}{dx} \ln\lvert \text{cosec}\, x + \cot\ x\rvert = -\text{cosec}\, x$	$\int \text{cosec}\, x\, dx = -\ln\lvert \text{cosec}\, x + \cot\ x\rvert + C$

Differentiations of Inverse Functions
$\frac{d}{dx}[\sin^{-1} u] = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx},\ \lvert u\rvert < 1.$
$\frac{d}{dx}[\cos^{-1} u] = \frac{-1}{\sqrt{1-u^2}} \cdot \frac{du}{dx},\ \lvert u\rvert < 1.$
$\frac{d}{dx}[\tan^{-1} u] = \frac{1}{1+u^2} \cdot \frac{du}{dx}.$
$\frac{d}{dx}[\cot^{-1} u] = \frac{-1}{1+u^2} \cdot \frac{du}{dx}.$
$\frac{d}{dx}[\sec^{-1} u] = \frac{1}{\lvert u\rvert\sqrt{u^2-1}} \cdot \frac{du}{dx},\ \lvert u\rvert > 1.$
$\frac{d}{dx}[\mathrm{cosec}^{-1} u] = \frac{-1}{\lvert u\rvert\sqrt{u^2-1}} \cdot \frac{du}{dx},\ \lvert u\rvert > 1.$
$\frac{d}{dx}[\sinh^{-1} u] = \frac{1}{\sqrt{u^2+1}} \cdot \frac{du}{dx}$
$\frac{d}{dx}[\cosh^{-1} u] = \frac{1}{\sqrt{u^2-1}} \cdot \frac{du}{dx},\ \lvert u\rvert > 1.$
$\frac{d}{dx}[\tanh^{-1} u] = \frac{1}{1-u^2} \cdot \frac{du}{dx},\ \lvert u\rvert < 1.$
$\frac{d}{dx}[\coth^{-1} u] = \frac{1}{1-u^2} \cdot \frac{du}{dx},\ \lvert u\rvert > 1.$
$\frac{d}{dx}[\mathrm{sech}^{-1} u] = \frac{-1}{u\sqrt{1-u^2}} \cdot \frac{du}{dx},\ 0 < u < 1.$
$\frac{d}{dx}[\mathrm{cosech}^{-1} u] = \frac{-1}{\lvert u\rvert\sqrt{1+u^2}} \cdot \frac{du}{dx},\ u \neq 0.$

Integrations Resulting in Inverse Functions
$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C.$
$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C.$
$\int \frac{dx}{\lvert x\rvert\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C.$
$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C,\ a > 0.$
$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C,\ x > 0.$
$\int \frac{dx}{a^2-x^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert < a, \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert > a. \end{cases}$
$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\mathrm{sech}^{-1}\left(\frac{x}{a}\right) + C,\ 0 < x < a.$
$\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\mathrm{cosech}^{-1}\left\lvert\frac{x}{a}\right\rvert + C,\ 0 < x < a.$

Laplace Transforms $F(s) = \int_0^\infty f(t)e^{-st}dt$

$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n,\ n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t),\ n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^n F}{ds^n}$
$\frac{d^n f(t)}{dt^n},\ n = 1, 2, 3, \ldots$	$s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$
$f(at),\ a > 0$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\frac{f(t)}{t}$	$\int_s^\infty F(u)\, du$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\delta(t-a)$	e^{-as}
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\int_0^t f(u)g(t-u)\, du$	$F(s)G(s)$
$\int_0^t f(u)\, du$	$\frac{F(s)}{s}$
$f(t) = f(t+T)$	$\frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt$

Convolution Theorem $\mathcal{L}^{-1}\left\{F(s)G(s)\right\} = \int_0^t f(u)g(t-u)\, du$