Chapter 8

Entropy

A Measure of Disorder or Randomness
Entropy Definition and Change

**Define a property** $S = \text{Entropy}$

\[ \int_{1_{\text{rev}}}^{2} \frac{\delta Q}{T} = S_2 - S_1 \]
\[ \int_{2_{\text{irrev}}}^{1} \frac{\delta Q}{T} + S_2 - S_1 \leq 0 \]
\[ \int_{2_{\text{irrev}}}^{1} \frac{\delta Q}{T} + ds \leq 0 \]
\[ ds \geq \int_{2_{\text{irrev}}}^{1} \frac{\delta Q}{T} \]
\[ ds \geq \int_{\text{irrev}}^{\text{rev}} \frac{\delta Q}{T} \]
\[ ds = \int_{\text{rev}}^{\text{irrev}} \frac{\delta Q}{T} \]
\[ \delta Q = \int Tds \]
The temperature-entropy diagrams for water

$T_1 \frac{P_1}{s_1} \approx s_f \text{ at } T_1$

Compressed liquid

Saturated liquid–vapor mixture

Superheated vapor

$T_2 \frac{x_2}{s_2} = s_f + x_2 s_f g$
Entropy for a Pure Substance

Values of specific entropy are given in tables of thermodynamic properties.
The tables are the same as for u, h, and v

\[ s = (1 - x)s_f + s_g \]

OR

\[ s = s_f + xs_{fg} \]
### Example 8-1

Find the entropy and/or temperature of steam at the following states:

<table>
<thead>
<tr>
<th>$P$</th>
<th>$T$</th>
<th>Region</th>
<th>$s$ kJ/(kg K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 MPa</td>
<td>120$^\circ$C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 MPa</td>
<td>50$^\circ$C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.8 MPa</td>
<td>400$^\circ$C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 kPa</td>
<td>Quality, $x = 0.9$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 kPa</td>
<td></td>
<td></td>
<td>7.1794</td>
</tr>
</tbody>
</table>
### Answer to Example 8-1

<table>
<thead>
<tr>
<th>$P$</th>
<th>$T$</th>
<th>Region</th>
<th>$s$ kJ/kg·K</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 MPa</td>
<td>120°C</td>
<td>Compressed Liquid and in the table</td>
<td>1.5233</td>
</tr>
<tr>
<td>1 MPa</td>
<td>50°C</td>
<td>Compressed liquid but not in the table</td>
<td>$s = s_f$ at 50°C = 0.7038</td>
</tr>
<tr>
<td>1.8 MPa</td>
<td>400°C</td>
<td>Superheated</td>
<td>7.1794</td>
</tr>
<tr>
<td>40 kPa</td>
<td>$T=T_{\text{sat}}$</td>
<td>Quality, $x = 0.9$ Saturated mixture</td>
<td>$s = s_f + x s_{fg} = 7.0056$</td>
</tr>
<tr>
<td>40 kPa</td>
<td>$T=T_{\text{sat}}$</td>
<td>$s_f &lt; s &lt; s_g$ at $P$ Saturated mixture</td>
<td>7.1794</td>
</tr>
<tr>
<td></td>
<td>75.87°C</td>
<td>$X = (s - s_f)/s_{fg}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 0.9262$</td>
<td></td>
</tr>
</tbody>
</table>
Temperature Entropy Property Diagrams

\[ dS = \int \frac{\delta Q}{T} \]

for constant \( T \), \( Q = T \int dS = T_{\text{constant}} \Delta S \)

\[ Q_{\text{in}} = T_H \Delta S \]
\[ Q_{\text{out}} = T_L \Delta S \]
\[ W_{\text{net}} = Q_{\text{in}} - Q_{\text{out}} = (T_H - T_L) \Delta S \]
The Thermodynamic Property Relation

From First Law for a closed system,

\[ \partial Q - \partial W = dU + dKE + dPE \]
\[ \partial Q - \partial W = dU \]

For reversible process,

\[ \partial Q_{rev} = dU + \partial W_{rev} \]
\[ TdS = dU + PdV \]
\[ dU = TdS - PdV \] (8.5)
\[ du = Tds - PdV \quad (8.5) \]

From definition of enthalpy,
\[ H = U + PV \]

Differential,
\[ dH = dU + PdV + VdP \]

Replaced into Eq(8.5),
\[ TdS = dH - VdP \]

OR
\[ dH = TdS + VdP \]
Entropy Change for Solid and Liquid

\[ du = Tds - Pdv \]
\[ dh = Tds + vdp \]

For liquid and solid, \( v \) is almost negligible; \( dv \approx 0 \) for all process. If \( dP \) is small,
\[ du = Tds \]
\[ dh = Tds \]

From Eq.(5.17) for solid and liquid,
\[ dh \approx du \approx CdT \quad ( \text{no phase change}) \]

And \( Cp \approx Cv \approx C \)
• So,

\[ ds \approx C \frac{dT}{T} \]

• If \( C \) is constant,

\[ \Delta s = s_2 - s_1 = C \ln \frac{T_2}{T_1} \]

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Entropy Change for Ideal Gas

- From  \( du = Tds - Pdv \)
- For ideal gas,

\[
C_{vo} \, dT = Tds - \frac{RT}{v} \, dv
\]

\[
ds = C_{vo} \, \frac{dT}{T} + R \, \frac{dv}{v}
\]

\[
\Delta s = \int C_{vo} \, \frac{dT}{T} + R \ln \frac{v_2}{v_1}
\]

\[
\Delta s = C_{vo} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}
\]

8.22
• From \( dh = Tds + vdP \)
• For ideal gas,

\[ C_{po} dT = Tds + RT \frac{dP}{P} \]

\[ ds = C_{po} \frac{dT}{T} - R \frac{dP}{P} \]

\[ \Delta s = \int C_{po} \frac{dT}{T} - R \ln \frac{P_2}{P_1} \]

\[ \Delta s = C_{po} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \]
Reversible, adiabatic process (isentropic)

For ideal gas which is isentropic (\(ds = 0\))

\[ du = Tds - Pdv \]

\[ du = -Pdv \]

\[ k = \frac{C_p}{C_v} \quad \text{and} \quad C_p = C_v + R \]

\[ C_v = \frac{R}{k-1} \quad \text{and} \quad C_p = \frac{kR}{k-1} \]

\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{(k-1)}{k}} = \left( \frac{v_1}{v_2} \right)^{k-1} \]
Entropy Generation

\[ dS = \frac{\partial Q}{T} + \partial S_{\text{gen}} \]  
(8.11)

where \( \partial S_{\text{gen}} \geq 0 \)

\( \partial S_{\text{gen}} \) is entropy generation due to irreversibility of the process. It is a path function.

\[ \partial S_{\text{gen}} = 0 \quad \text{For reversible process} \]
\[ \partial S_{\text{gen}} > 0 \quad \text{For irreversible process} \]
From Eq. (8.11), the conclusion can be drawn,

<table>
<thead>
<tr>
<th>$dS_{sys}$</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increased</td>
<td>1. Heat is transferred into the system</td>
</tr>
<tr>
<td>(+)</td>
<td>2. Irreversible process</td>
</tr>
<tr>
<td>Decreased</td>
<td>Heat is transferred from the system</td>
</tr>
<tr>
<td>(-)</td>
<td></td>
</tr>
<tr>
<td>Unchanged</td>
<td>Adiabatic and reversible process</td>
</tr>
<tr>
<td>(0)</td>
<td></td>
</tr>
</tbody>
</table>
Entropy equation

\[ dS = \frac{\partial Q}{T} + \partial S_{\text{gen}} \]  \hspace{1cm} (8.11)

where \( \partial S_{\text{gen}} \geq 0 \)

\[ s_2 - s_1 = \frac{1}{T_o} \left( \frac{Q_2}{1} + S_2 \right) \]

Normally we need to calculate \( S_2 \) (Sgen)