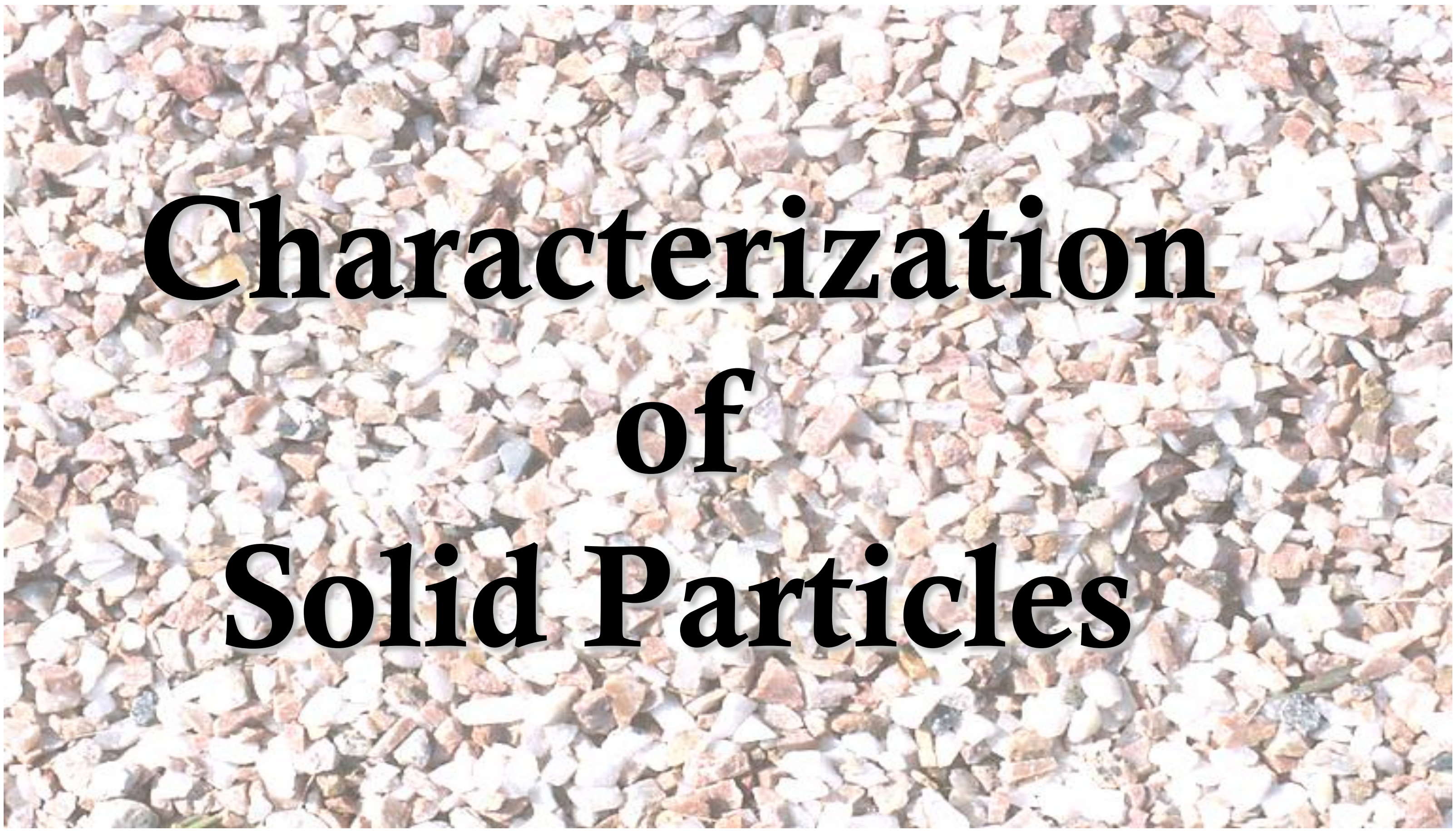
Chapter 1



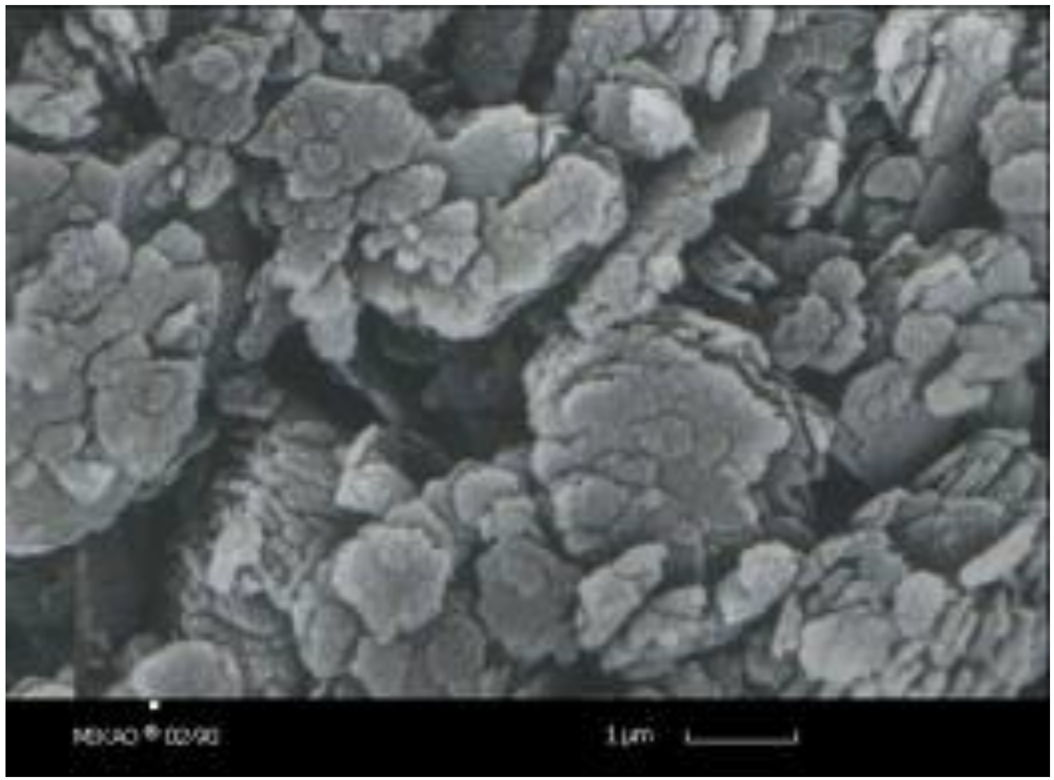
Characterization of Solid Particles

SUGAR – GULA

Very Fine sugar- Icing Sugar
Fine sugar – Granulated Sugar
Compact sugar – Gula Merah

What is the particle size of my powder???

Talc particles – as in baking powder



Irregular shape

In engineering, we wish to perform calculations using diameter; so we need simple basis for describing the irregularly shaped particle that can be used in communication and calculations

It is common practise to talk about particle size, which really means particle diameter

PARTICLE TECHNOLOGY

- **Techniques for processing & handling particulate solids**

Characterisation of solid particles

1. **Particle shape -sphericity Φ_S**
2. **Particle size - diameter D_p**
3. **Particle density- ρ**

Sphericity Φ_S

- **Independent of particle size**
- **Spherical particle of dia. D_p, $\Phi_S = 1$**
- **Non-spherical particle:** $\Phi_S = \frac{6v_p}{D_p S_p}$

D_p = equivalent diameter of particle (dia. of sphere of equal volume)

S_p = surface area of one particle

v_p = volume of one particle

SPHERICITY

Sphericity of miscellaneous materials[†]

Material	Sphericity	Material	Sphericity
Spheres, cubes, short cylinders ($L = D_p$)	1.0	Ottawa sand	0.95
Raschig rings ($L = D_p$)		Rounded sand	0.83
$L = D_o$, $D_i = 0.5D_o$	0.58[‡]	Coal dust	0.73
$L = D_o$, $D_i = 0.75D_o$	0.33[‡]	Flint sand	0.65
Berl saddles	0.3	Crushed glass	0.65
		Mica flakes	0.28

[†]By permission, from J. H. Perry (ed.), *Chemical Engineers' Handbook,* 6th ed., p. **5**-54, McGraw-Hill Book Company, New York, 1984.
[‡]Calculated value.

Note entries for cubes and cylinders. For convenience, some just calculate a nominal (average) diameter and assign a sphericity of unity.

For greatest contact area we want lower sphericity.

PARTICLE SIZE

- **equidimensional particles - diameter**

- **not equidimensional - second longest major dimension eg. Needle-like particle, D_p = thickness not the length**

- **Coarse particle : inches or millimetres**

- **Fine particle : screen size**

- **Very fine particles : micrometers or nanometers**

- **Ultra fine particle: surface area per unit mass, m^2/g**

SCREEN, SIEVES OR MESHES

- To remove the oversized particles.
- To break agglomerates or "de-lump".

SCREEN, SIEVES OR MESHES

- size range between about 76mm and 38μm
- identified by meshes per inch, e.g. 30 mesh (590 μm), D_p=1\30=0.033 in.
- relates to the number of openings in the screen per inch
- area of openings in any one screen in the series = 2x area of openings in the next smaller screen
- arranged serially in a stack, with the smallest mesh at the bottom and the largest at the top (bottom screen is a solid pan)
- Materials are loaded at the top and then shacked & shaked using a shaker for a period of time
- 14/20 : through 14 mesh and on 20 mesh or -14+20
- Tyler standard or U.S. standard screen scale
- Separation can be either dry or wet
- Wet screening is more efficient, but drying of the product add cost

PARTICLE-SIZE-DISTRIBUTION DATA

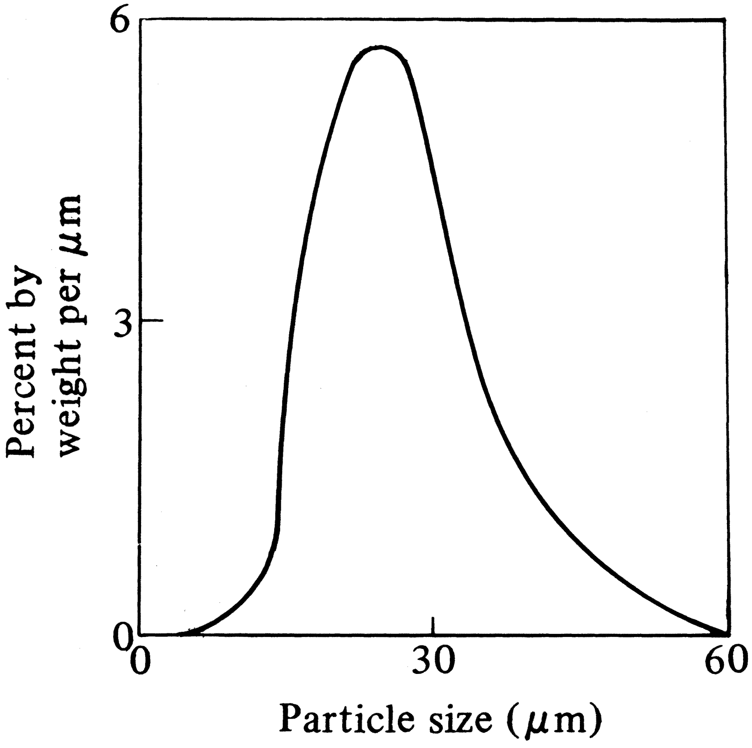
- presented in the form of a table

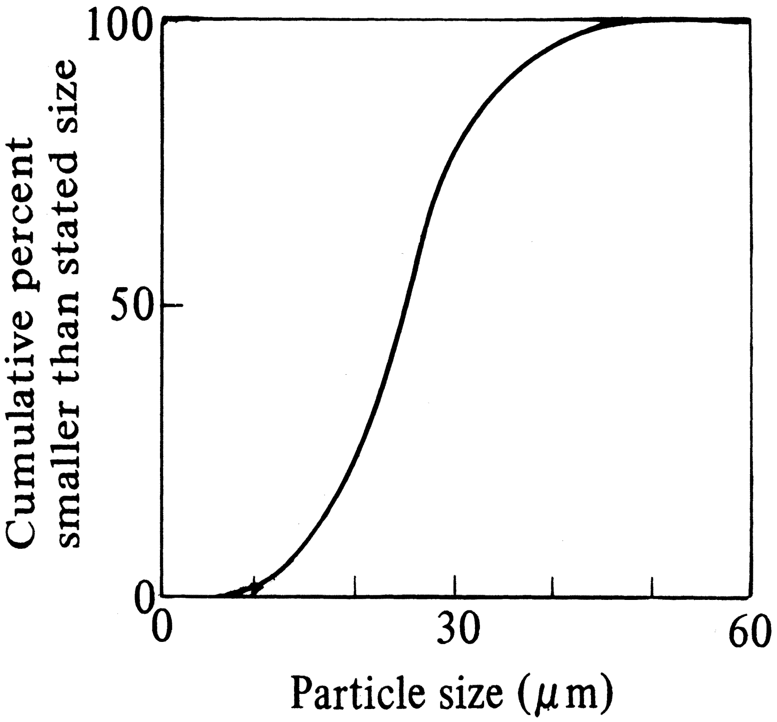
Mesh	Screen Opening, Dpi (mm)	Mass Retained on Screen, (g)
14	1.400	0.000
16	1.180	9.12
18	1.000	32.12
20	0.850	39.82
30	0.600	235.42
40	0.425	89.14
50	0.300	54.42
70	0.212	22.02
100	0.150	7.22
140	0.106	1.22
Pan	-	0.50
Total		491.00

- these results are not so informative as the exact size of the material sitting on each screen is unknown

SCREEN ANALYSIS

- **Differential & cumulative plots usually on a mass-fraction basis**





DIFFERENTIAL SCREEN ANALYSIS

- **arithmetic-average aperture for each mass fraction that passes thru' one screen but not the next screen**

Mesh	Screen Opening, Dpi (mm)	Mass Retained on Screen, (g)	% Mass Retained,
14	1.400	0.000	0.00
16	1.180	9.12	1.86
18	1.000	32.12	6.54
20	0.850	39.82	8.11
30	0.600	235.42	47.95
40	0.425	89.14	18.15
50	0.300	54.42	11.08
70	0.212	22.02	4.48
100	0.150	7.22	1.47
140	0.106	1.22	0.25
Pan	-	0.50	0.11
Total		491.00	100

Mesh Range	Ave. Particle Size, $\bar{D}_{pi}$ (mm)	Mass Fraction Retained, x_i
16	1.290	0.0186
18	1.090	0.0654
20	0.925	0.0811
30	0.725	0.4795
40	0.513	0.1815
50	0.363	0.1108
70	0.256	0.0448
100	0.181	0.0147
140	0.128	0.0025
pan	0.053	0.0011

Eg. mass fraction of 0.0186 passes thru' a screen of 1.4mm aperture but being retained at 1.180mm aperture, ave. of these two apertures =(1.4 + 1.18)/2 = 1.29mm

DIFFERENTIAL SCREEN ANALYSIS

Mesh Range	Ave. Particle Size, $\bar{D}_{pi}$ (mm)	Mass Fraction Retained, x_i
16	1.290	0.0186
18	1.090	0.0654
20	0.925	0.0811
30	0.725	0.4795
40	0.513	0.1815
50	0.363	0.1108
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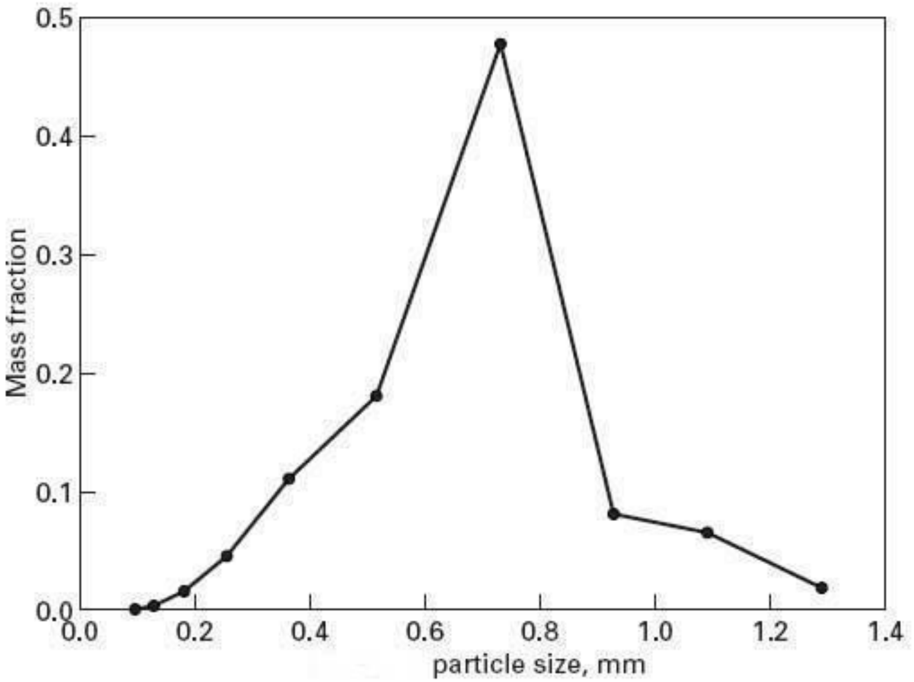


(a) Differential analysis

CUMULATIVE SCREEN ANALYSIS

- **Cumulative-weight-percent <u>oversize</u> (greater than D_{pi}) or cumulative-weight-percent <u>undersize</u> (smaller than D_{pi})**

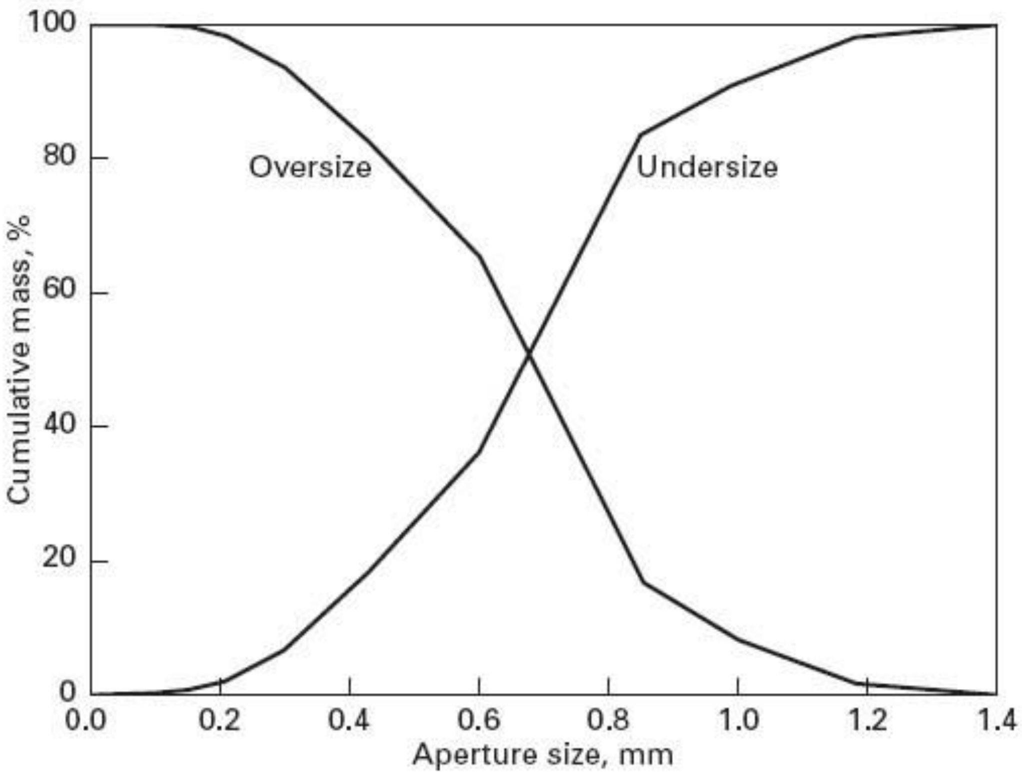
Mesh	Screen Opening, Dpi (mm)	Mass Retained on Screen, (g)	% Mass Retained
14	1.400	0.000	0.00
16	1.180	9.12	1.86
18	1.000	32.12	6.54
20	0.850	39.82	8.11
30	0.600	235.42	47.95
40	0.425	89.14	18.15
50	0.300	54.42	11.08
70	0.212	22.02	4.48
100	0.150	7.22	1.47
140	0.106	1.22	0.25
Pan	-	0.50	0.11
Total		491.00	100

Mesh	Screen Opening, Dpi (mm)	Cumulative wt% undersize	Cumulative wt% oversize
14	1.400	100	0.00
16	1.180	98.14	1.86
18	1.000	91.60	8.40
20	0.850	83.49	16.51
30	0.600	35.54	64.46
40	0.425	17.39	82.61
50	0.300	6.31	93.69
70	0.212	1.83	98.17
100	0.150	0.36	99.64
140	0.106	0.11	99.89

- **bec. 0.11 wt% particle retained on the pan, cumulative wt% undersize ≠0 & cumulative wt% oversize ≠ 100**

CUMULATIVE SCREEN ANALYSIS

Mesh	Screen Opening, Dpi (mm)	Cumulative wt% undersize	Cumulative wt% oversize
14	1.400	100	0.00
16	1.180	98.14	1.86
18	1.000	91.60	8.40
20	0.850	83.49	16.51
30	0.600	35.54	64.46
40	0.425	17.39	82.61
50	0.300	6.31	93.69
70	0.212	1.83	98.17
100	0.150	0.36	99.64
140	0.106	0.11	99.89



(c) Cumulative analysis

- two curves (mirror images of each other) cross at a median size where 50wt. % is larger in size & 50wt.% is smaller
- A log scale for the cumulative wt% preferred if an appreciable fraction of the data points lie below 10%

Example 1 (page 11 in note)

Mesh	Screen Opening, Dpi (mm)	Mass Fraction Retained, x_i
4	4.699	0.000
6	3.327	0.0251
8	2.362	0.1250
10	1.651	0.3207
14	1.168	0.2570
20	0.833	0.1590
28	0.589	0.0538
35	0.417	0.0210
48	0.295	0.0102
65	0.208	0.0077
100	0.147	0.0058
150	0.104	0.0041
200	0.074	0.0031
Pan	-	0.0075

Calculate (a) average particle diameter, (b) cumulative fraction smaller than D_{pi} (c) plot the graph cumulative analysis of part (b)

Solution for Example 1

Mesh	Screen Opening, Dpi (mm)	Mass Fraction Retained, x_i	Ave Particle Diameter, $\bar{D}_{pi}$ (mm)	Cumulative fraction smaller than D_{pi}
4	4.699	0.000	-	1.000
6	3.327	0.0251		
8	2.362	0.1250		
10	1.651	0.3207		
14	1.168	0.2570		
20	0.833	0.1590		
28	0.589	0.0538		
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65	0.208	0.0077		
100	0.147	0.0058		
150	0.104	0.0041		
200	0.074	0.0031		
Pan	-	0.0075		

Solution for Example 1

Mesh	Screen Opening, Dpi (mm)	Mass Fraction Retained, x_i	Ave Particle Diameter, $\bar{D}_{pi}$ (mm)	Cumulative fraction smaller than D_{pi}
4	4.699	0.000	-	1.000
6	3.327	0.0251	4.013	0.9749
8	2.362	0.1250	2.845	0.8499
10	1.651	0.3207	2.007	0.5292
14	1.168	0.2570	1.409	0.2722
20	0.833	0.1590	1.001	0.1132
28	0.589	0.0538	0.711	0.0594
35	0.417	0.0210	0.503	0.0384
48	0.295	0.0102	0.356	0.0282
65	0.208	0.0077	0.252	0.0205
100	0.147	0.0058	0.178	0.0147
150	0.104	0.0041	0.126	0.0106
200	0.074	0.0031	0.089	0.0075
Pan	-	0.0075	0.037	0.000

AVERAGE PARTICLE SIZE

- **Volume-surface mean diameter (most used average particle size):**

$$\overline{D}_s \equiv \frac{6}{\Phi_s A_w \rho_p} = \frac{1}{\sum_{i=1}^{N}\left(\frac{x_i}{\overline{D}_{pi}}\right)} \quad \text{or} \quad \overline{D}_S = \frac{\sum_{i=1}^{N} N_i \overline{D}_{pi}^{3}}{\sum_{i=1}^{N} N_i \overline{D}_{pi}^{2}}$$

- **Arithmetic mean diameter:**

$$\overline{D}_N = \frac{\sum_{i=1}^{N}\left(N_i \overline{D}_{pi}\right)}{\sum_{i=1}^{N} N_i} = \frac{\sum_{i=1}^{N}\left(N_i \overline{D}_{pi}\right)}{N_T} = \frac{\sum_{i=1}^{N}\left(\frac{x_i}{\overline{D}_{pi}^{2}}\right)}{\sum_{i=1}^{N}\left(\frac{x_i}{\overline{D}_{pi}^{3}}\right)}$$

where

N_T = number of particles in the entire sample

AVERAGE PARTICLE SIZE

- **Mass mean diameter :**

$$\bar{D}_W = \sum_{i=1}^{N} x_i \bar{D}_{pi}$$

- **Volume mean diameter:**

$$\bar{D}_V = \left[\frac{1}{\sum_{i=1}^{N} \frac{x_i}{\bar{D}_{pi}^{3}}} \right]^{1/3}$$

Example 2

Mesh	Screen Opening, Dpi (mm)	Mass Fraction Retained, x_i	Ave Particle Diameter, $\bar{D}_{pi}$ (mm)	Cumulative fraction smaller than D_{pi}
4	4.699	0.000	-	1.000
6	3.327	0.0251	4.013	0.9749
8	2.362	0.1250	2.845	0.8499
10	1.651	0.3207	2.007	0.5292
14	1.168	0.2570	1.409	0.2722
20	0.833	0.1590	1.001	0.1132
28	0.589	0.0538	0.711	0.0594
35	0.417	0.0210	0.503	0.0384
48	0.295	0.0102	0.356	0.0282
65	0.208	0.0077	0.252	0.0205
100	0.147	0.0058	0.178	0.0147
150	0.104	0.0041	0.126	0.0106
200	0.074	0.0031	0.089	0.0075
Pan	-	0.0075	0.037	0.000

Determine from mesh 4 to mesh 200 :

(a) arithmetic mean diameter

(b)Volume -surface mean diameter

(c) Mass mean diameter

(d) Volume mean diameter

Formula

Calculate (a) arithmetic mean diameter (b)Volume-surface mean diameter (c) Mass mean diameter (d) Volume mean diameter

(a) arithmetic mean

$$\bar{D}_N = \sum_{i=1}^{N}\left(\frac{x_i}{\bar{D}_{pi}^{2}}\right) \Big/ \sum_{i=1}^{N}\left(\frac{x_i}{\bar{D}_{pi}^{3}}\right)$$

(b)Volume-surface mean diameter

$$\bar{D}_S = \frac{\sum_{i=1}^{N} N_i \bar{D}_{pi}^{-3}}{\sum_{i=1}^{N} N_i \bar{D}_{pi}^{-2}}$$

(c) Mass mean diameter

$$\bar{D}_W = \sum_{i=1}^{N} x_i \bar{D}_{pi}$$

(d) Volume mean diameter

$$\bar{D}_V = \left[\frac{1}{\sum_{i=1}^{N} \frac{x_i}{\bar{D}_{pi}^{3}}}\right]^{1/3}$$

Example 3

Calculate (a) arithmetic mean diameter (b)Volume-surface mean diameter (c) Mass mean diameter (d) Volume mean diameter

Mesh Range	Ave. Particle Size, Dpi (mm)	Mass Fraction Retained, x_i
14	1.290	0.0186
16	1.090	0.0654
18	0.925	0.0811
20	0.725	0.4795
30	0.513	0.1815
40	0.363	0.1108
50	0.256	0.0448
70	0.181	0.0147
100	0.128	0.0025
140	0098	0.0011

(a) arithmetic mean

$$\bar{D}_N = \sum_{i=1}^{N}\left(\frac{x_i}{\bar{D}_{pi}^2}\right) \Big/ \sum_{i=1}^{N}\left(\frac{x_i}{\bar{D}_{pi}^3}\right)$$

(b) Volume-surface mean diameter

$$\bar{D}_S = \frac{\sum_{i=1}^{N} N_i \bar{D}_{pi}^{3}}{\sum_{i=1}^{N} N_i \bar{D}_{pi}^{2}}$$

(c) Mass mean diameter

$$\bar{D}_W = \sum_{i=1}^{N} x_i \bar{D}_{pi}$$

(d) Volume mean diameter

$$\bar{D}_V = \left[\frac{1}{\sum_{i=1}^{N} \frac{x_i}{\bar{D}_{pi}^{3}}}\right]^{1/3}$$

Solution Example 3

Solution:

Mesh Range	Ave. Particle Size, $\overline{D}_{pi}$ (mm)	Mass Fraction Retained, x_i	$x_i/\overline{D}_{pi}$	$x_i\overline{D}_{pi}$	$x_i/\overline{D}_{pi}^2$	$x_i/\overline{D}_{pi}^3$
14	1.290	0.0186	0.0144	0.0240	0.0112	0.0087
16	1.090	0.0654	0.0600	0.0713	0.0550	0.0505
18	0.925	0.0811	0.0877	0.0750	0.0948	0.1025
20	0.725	0.4795	0.6614	0.3476	0.9122	1.2583
30	0.513	0.1815	0.3538	0.0931	0.6897	1.3444
40	0.363	0.1108	0.3052	0.0402	0.8409	2.3164
50	0.256	0.0448	0.1750	0.0115	0.6836	2.6703
70	0.181	0.0147	0.0812	0.0027	0.4487	2.4790
100	0.128	0.0025	0.0195	0.0003	0.1526	1.1921
140	0.098	0.0011	0.0112	0.0001	0.1145	1.1687
Σ		1.0000	1.7695	0.6658	4.0032	12.5909

(a) arithmetic mean diameter = 0.318 mm

(b) Volume-surface mean diameter = 0.565 mm

(c) Mass mean diameter = 0.666 mm

(d) Volume mean diameter = 0.430 mm

MIXED PARTICLE SIZE & SIZE ANALYSIS

Uniform particles of diameter D_p:

Total number of particle in sample , $N = \dfrac{m}{\rho_p v_p}$

Total surface area of the particles, $A = NS_p = \dfrac{6m}{\Phi_S \rho_p v_p}$

where

m **= mass of the sample**

ρ_p **= density of one particle**

S_p **= surface area of one particle**

v_p **= volume of one particle**

AVERAGE PARTICLE SIZE

- **Volume-surface mean diameter (most used average particle size):**

$$\overline{D}_s \equiv \frac{6}{\Phi_s A_w \rho_p} = \frac{1}{\sum_{i=1}^{N} \left(\frac{x_i}{\overline{D}_{pi}} \right)}$$

or

$$\overline{D}_S = \frac{\sum_{i=1}^{N} N_i \overline{D}_{pi}^{\,3}}{\sum_{i=1}^{N} N_i \overline{D}_{pi}^{\,2}}$$

- **Arithmetic mean diameter:**

$$\overline{D}_N = \frac{\sum_{i=1}^{N} \left(N_i \overline{D}_{pi} \right)}{\sum_{i=1}^{N} N_i} = \frac{\sum_{i=1}^{N} \left(N_i \overline{D}_{pi} \right)}{N_T} = \frac{\sum_{i=1}^{N} \left(\frac{x_i}{\overline{D}_{pi}^{2}} \right)}{\sum_{i=1}^{N} \left(\frac{x_i}{\overline{D}_{pi}^{3}} \right)}$$

where

N_T = number of particles in the entire sample

MIXED PARTICLE SIZE & SIZE ANALYSIS

Mixture of particles of various size & densities sorted into fractions, each of constant density & approx. constant size

ρ_p & Φ_s **are known**

specific surface, A_w (mm²/g) :

$$A_w = \frac{6x_1}{\Phi_s \rho_p \bar{D}_{p1}} + \frac{6x_2}{\Phi_s \rho_p \bar{D}_{p2}} + \ldots + \frac{6x_n}{\Phi_s \rho_p \bar{D}_{pn}} = \frac{6}{\Phi_s \rho_p} \sum_{i=1}^{N} \frac{x_i}{\bar{D}_{pi}}$$

where

x_i = mass fraction in a given increment

$\bar{D}_{pi}$ **= average diameter**

NUMBER OF PARTICLES IN MIXTURE

Volume of any particle :

$$v_p = aD_p^3$$

where

a = volume shape factor (a =0.5236 for sphere, 0.785 for a short cylinder (height = dia.), 1.0 for a cube)

Assuming that a is independent of size

Total population in the sample (particles/g), N_w :

$$N_w = \frac{1}{a\rho_p}\sum_{i=1}^{N}\frac{x_i}{\overline{D}_{pi}^3} = \frac{1}{a\rho_p \overline{D}_v^3}$$

Example 5

The density of the particles is 2650 kg/m^3 (0.00265 g/mm^3) and the shape factors are a = 0.8 and Φ_S = 0.571. For the material between 4-mesh and 200-mesh in particle size, calculate

(a) A_W (mm^2/g)

(b) N_W (particles/g)

(c) N_i for the 150/200-mesh increment (particles/g)

(d) fraction of the total number of particles in the 150/200-mesh increment

Mesh	Screen Opening, Dpi (mm)	Mass Fraction Retained, x_i	Ave Particle Diameter, $\bar{D}_{pi}$ (mm)	Cumulative fraction smaller than $\bar{D}_{pi}$
4	4.699	0.000	-	1.000
6	3.327	0.0251	4.013	0.9749
8	2.362	0.1250	2.845	0.8499
10	1.651	0.3207	2.007	0.5292
14	1.168	0.2570	1.409	0.2722
20	0.833	0.1590	1.001	0.1132
28	0.589	0.0538	0.711	0.0594
35	0.417	0.0210	0.503	0.0384
48	0.295	0.0102	0.356	0.0282
65	0.208	0.0077	0.252	0.0205
100	0.147	0.0058	0.178	0.0147
150	0.104	0.0041	0.126	0.0106
200	0.074	0.0031	0.089	0.0075
Pan	-	0.0075	0.037	0.000

Solution Example 5

mesh	Dpi	xi	Ave Dpi	xi/Ave Dpi	xi/(Ave Dpi)^	xi(Ave Dpi)	xi/(Ave Dpi)^
4	4.699	0		0	0	0	0
6	3.327	0.0251	4.013	0.00625467	0.000388388	0.1007263	0.0015586
8	2.362	0.125	2.845	0.04393673	0.005428292	0.355625	0.01544349
10	1.651	0.3207	2.007	0.15979073	0.039669511	0.6436449	0.07961671
14	1.168	0.257	1.409	0.18239886	0.091875591	0.362113	0.12945271
20	0.833	0.159	1.001	0.15884116	0.158523952	0.159159	0.15868248
28	0.589	0.0538	0.711	0.07566807	0.149683343	0.0382518	0.10642486
35	0.417	0.021	0.503	0.0417495	0.165011928	0.010563	0.083001
48	0.295	0.0102	0.356	0.02865169	0.226073771	0.0036312	0.08048226
65	0.208	0.0077	0.252	0.03055556	0.481159542	0.0019404	0.1212522
100	0.147	0.0058	0.178	0.03258427	1.028414015	0.0010324	0.18305769
150	0.104	0.0041	0.126	0.03253968	2.049614672	0.0005166	0.25825145
200	0.074	0.0031	0.089	0.03483146	4.39735648	0.0002759	0.39136473
pan		0.0075	0.037	0.2027027	148.0662547	0.0002775	5.47845142
From mesh 4 to mesh 200 only)				0.82780239	8.793199485	1.6774795	1.60858818

Vol.-surface mean dia. = 1/0.8278 = 1.208 mm

Arith. Mean dia. = 1.609/8.793 0.183 mm

Mass mean dia. = 1.677 mm

Vol. mean dia. = (1/8.793)^(1/3) = 0.485 mm

Aw = (6/(0.571x0.00265))(0.8278) = 3.31 mm2/g

Nw = (1/(0.8x0.00265))(8.793) = 4148 particles/g

Ni at 150/200 mesh = (1/0.8x0.00265))(4.397) = 2074 particles/g

Fraction of total no. of particles in 150/200 mesh = 2074/4148 = 0.5

Example 4

Solve for the unknowns:

Mesh	Screen Opening (mm)	Mass fraction retained, x_i	Ave. particle dia. $\bar{D}_p$ (mm)	Cumulative fraction smaller than Dp_i
14	A	0.2355	1.409	0.2722
20	0.833	0.1595	0.999	B
28	0.585	0.0535	C	D
35	E	F	0.503	0.0382
48	0.294	G	H	0.0283
65	0.205	I	J	0.0202