SIZE REDUCTION

- **Large particles of solids are cut or broken into small pieces**

  in food-processing industry - eg. grind wheat to flour

  in ore-processing & cement industries - eg copper ores, nickel, cobalt ores & iron ores are ground before chemical processing

- **reasons:**

  1. occurs in sizes that are too large to be used

  2. so separation can be carried out

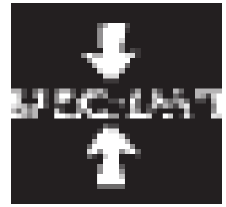
  3. increases the reactivity

  4. reduces bulk of materials for easier handling and for waste disposal

SIZE REDUCTION

- methods:

1. Compression or crushing

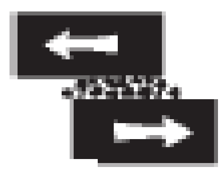


- coarse reduction of hard solids to coarse sizes

2. Impact/hammer

- gives coarse, medium or fine sizes

3. Attrition or rubbing

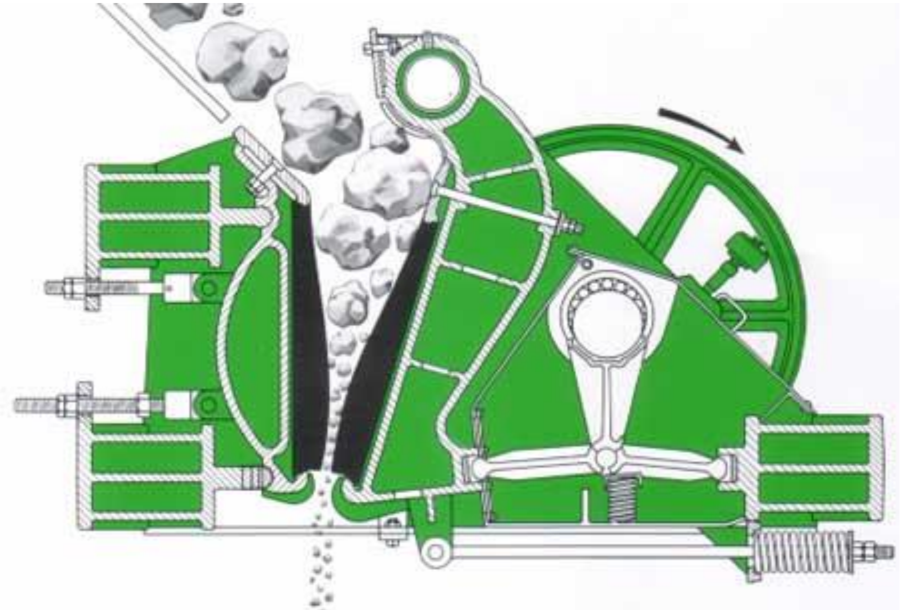


- yields very fine products

4. Cutting

- give definite sizes, sometimes a definite shape

SIZE REDUCTION



- An ideal crusher would:
  1. have a large capacity
  2. require a small power input per unit of product
  3. yield a product of the single size distribution desired

- <u>cost of power</u> is a major expense in size reduction equipment, so the factors that control this cost are important

EFFICIENCY

Crushing efficiency, η_C

$$\eta_C = \frac{\text{surface energy created by crushing}}{\text{energy absorbed by the solid}}$$

where

$$\eta_C = \frac{e_S(A_{wb} - A_{wa})}{W_n}$$

e_S = surface energy per unit area

W_n = energy absorbed by a unit mass

A_{wa}, A_{wb} = areas per unit mass of feed and product

Mechanical efficiency, η_m

$$\eta_m = \frac{\text{energy absorbed by the solid}}{\text{total energy input}}$$

$$\eta_m = \frac{W_n}{W} = \frac{e_S(A_{wb} - A_{wa})}{W_n \eta_C}$$

where **W = energy input =** $\dfrac{e_S(A_{wb} - A_{wa})}{\eta_m \eta_C}$

POWER REQUIRMENT

Power requirement by the size reduction machine is

$$P = W\dot{m} = \frac{6\dot{m}e_S}{\eta_C \eta_m \rho_p}\left(\frac{1}{\Phi_b \overline{D}_{sb}} - \frac{1}{\Phi_a \overline{D}_{sa}}\right)$$

where

P **= power required**

$\dot{m}$ **= feed rate**

$\overline{D}_{sa}, \overline{D}_{sb}$ **= volume-surface mean dia. of feed & product, respectively**

η_C **= crushing efficiency**

η_m **= mechanical efficiency**

ρ_p **= density of particle**

e_S **= surface energy per unit area**

Φ_a, Φ_b **= sphericity of feed and product, respectively**

$$\overline{D}_s \equiv \frac{6}{\Phi_s A_w \rho_p} = \frac{1}{\sum_{i=1}^{N}\left(\frac{x_i}{\overline{D}_{pi}}\right)}$$

and

$$A_W = \frac{6}{\Phi_s \rho_p}\sum_{i=1}^{N}\frac{x_i}{\overline{D}_{pi}}$$

SIZE REDUCTION

Highly energy intensive- 5% of all electricity generated used

Most inefficient unit operation in terms of energy

99% goes to heat and noise

1% goes to creating new interfacial area

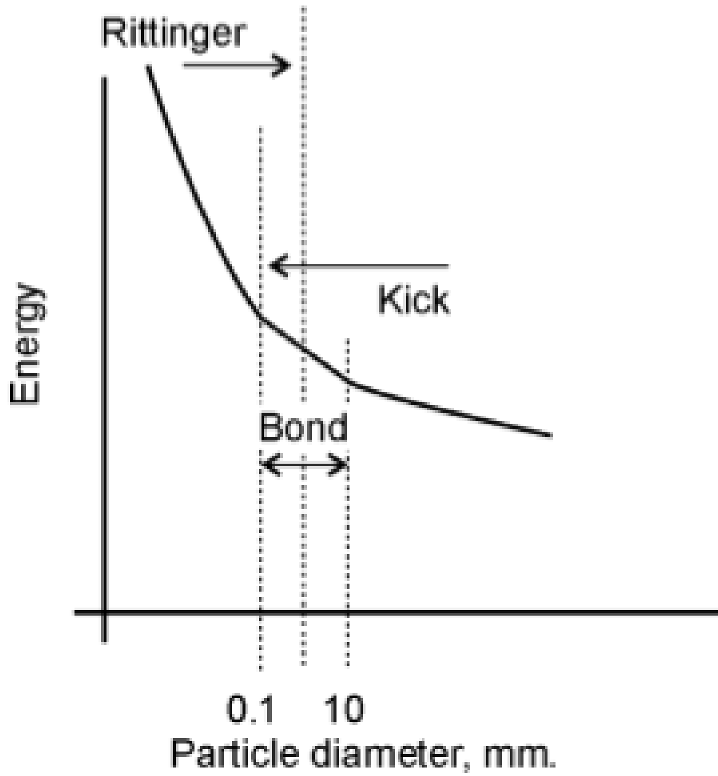
Finer sizes much more costly in term of energy

Equations to estimate energy due to :

Rittinger (1867)

Kick (1885)

Bond (1952)



Kick's law – better for larger particles

Rittinger's law – better for fine grinding

POWER REQUIRED IN SIZE REDUCTION

Rittinger's law :

work required in crushing is proportional to the new surface created

$$\frac{P}{\dot{m}} = K_r \left(\frac{1}{\bar{D}_{sb}} - \frac{1}{\bar{D}_{sa}} \right)$$

where

P = power required

K_r = Rittinger's coefficient

$$K_r = \frac{6e_s}{\eta_c \eta_m \rho_p} \left(\frac{1}{\Phi_b} - \frac{1}{\Phi_a} \right)$$

$\dot{m}$ = feed rate

$\bar{D}_{sa}, \bar{D}_{sb}$ = volume-surface mean dia. of feed & product, respectively

η_C = crushing efficiency

η_m = ratio of energy absorbed to energy input

ρ_p = density of particle

e_S = surface energy per unit area

Φ_a, Φ_b = sphericity of feed and product, respectively

Example 1

A certain crusher accepts a feed rock having a volume-surface mean diameter of 2 cm and discharges a product of volume-surface mean diameter of 0.5 cm. The power required to crush 10 ton/h is 8 HP. What should be the power consumption if the capacity is increased to 12 ton/h and the volume-surface mean diameter is reduced to 0.4 cm? Use Rittinger's law.

$$\frac{P}{\dot{m}} = K_r\left(\frac{1}{\bar{D}_{sb}} - \frac{1}{\bar{D}_{sa}}\right)$$

Example 2

A crusher was used to crush a material with a feed size of -5.08 cm +3.81 cm and the power required was 3.73 KW/ton. The screen analysis of the product was as follows: What would be the power required to crush 1 ton/h of the same material from a feed of - 4.44 cm + 3.81 cm to a product of average product size 0.051 cm? Use Rittinger's law.

Size of aperture (cm)	% product
0.63	-
0.38	26
0.203	18
0.076	23
0.051	8
0.025	17
0.013	8

$$\frac{P}{\dot{m}} = K_r \left(\frac{1}{\bar{D}_{sb}} - \frac{1}{\bar{D}_{sa}} \right)$$

POWER REQUIRED IN SIZE REDUCTION

Kick's Law :

Energy required to reduce a material in size was directly proportional to the size-reduction ratio

$$\frac{P}{\dot{m}} = K_k \ln \frac{\bar{D}_{sa}}{\bar{D}_{sb}}$$

where

P **= power required**

K_k is the Kick's coefficient

$\dot{m}$ = feed rate

$\bar{D}_{sa}, \bar{D}_{sb}$ = volume-surface mean dia. of feed & product, respectively

CRUSHING EFFICIENCY

Bond's Law :

work required using a large-size feed is proportional to the square root of the surface/volume ratio of the product

$$\frac{P}{\dot{m}} = K_b\left(\frac{1}{\sqrt{D_{pb}}} - \frac{1}{\sqrt{D_{pa}}}\right)$$

where

P = power required

K_b = constant

$\dot{m}$ = feed rate

If 80% of the feed passes a mesh size of D_{pa} mm and 80% of the product passes a mesh size of D_{pb} mm,

$$\frac{P}{\dot{m}} = 0.3162W_i\left(\frac{1}{\sqrt{D_{pb}}} - \frac{1}{\sqrt{D_{pa}}}\right)$$

where

W_i = work index

D_{pa},D_{pb} = dia. of feed & product, respectively (mm)

WORK INDEX

Gross energy (kW/h) required per ton of feed needed to reduce a very large feed ($\bar{D}_{pi}$ = ∞) to such a size that 80% of the product passes a 100 μm screen.

Include friction in the crusher & power

Material	Specific gravity	Work Index, W_i
Bauxite	2.20	8.78
Cement clinker	3.15	13.45
Cement raw material	2.67	10.51
Clay	2.51	6.30
Coal	1.4	13.00
Coke	1.31	15.13
Granite	2.66	15.13
Gravel	2.66	16.06
Gypsum rock	2.69	6.73
Iron ore (hematite)	3.53	12.84
Limestone	2.66	12.74
Phosphate rock	2.74	9.92
Quartz	2.65	13.57
Shale	2.63	15.87
Slate	2.57	14.30
Trap rock	2.87	19.32

Example 3

1. What is the power required to crush 100 ton/h of limestone if 80% of the feed pass a 2-in screen and 80% of the product a 1/8 in screen?

Solution:
The work index for limestone is 12.74.

$$\frac{P}{\dot{m}} = 0.3162 W_i \left(\frac{1}{\sqrt{D_{pb}}} - \frac{1}{\sqrt{D_{pa}}} \right)$$

$\dot{m} = 100 \text{ ton/h}$

$D_{pa} = 2 * 25.4 = 50.8mm$

$D_{pb} = 0.125 * 25.4 = 3.175mm$

The power required is :

$$P = 100 * 0.3162 * 12.74 \left(\frac{1}{\sqrt{3.175}} - \frac{1}{\sqrt{50.8}} \right)$$

$= 169.6kW$

Example 3

2. **80% of feed (ore) is less than 5.08 cm in size and the product size is such that 80% is less than 0.685 cm. The power required is 89.5 kW. What will be the power required using the same feed so that 80% is less than 0.3175 cm?**

$$\frac{P}{\dot{m}} = 0.3162 W_i \left(\frac{1}{\sqrt{D_{pb}}} - \frac{1}{\sqrt{D_{pa}}} \right)$$

Solution Example 3-Q2

- P_1 = 89.5 kW
- W_i = 19.32
- D_{pa1} = 50.8 mm
- D_{pb1} = 6.35 mm
- $m_{flowrate}$ = ?
- D_{pb2} = 3.18 mm
- P_2 = ?

Example 4

Granite rock is crushed with a uniform feed of 2 in-spheres. The screen analysis is given in the table below. The power required to crush this material is 500 kW; of this 20kW is needed to operate the empty mill. The feed rate is at 150 ton/hr. Calculate the power required for the second operation using:

a) Rittinger's Law
b) Kick's law

Mesh	x_1(%)	x_2 (%)
3/4	2.1	-
4/6	12.4	4.1
6/8	15.2	9.6
8/10	17.6	10.1
10/12	14.8	13.4
12/14	13.1	15.7
14/20	10.8	18.3
20/28	8.3	13.2
28/35	5.2	9.6
35/48	0.5	3.0
48/65	-	1.8
65/100	-	0.9
100/150	-	0.3

Change 50 to 150

Example 5

Trap rock is crushed in a gyratory crusher. The feed is nearly uniform 2-in. spheres. The differential screen analysis of the product is given in column (1) of Table 1 below The power required to crush this material is 400 kW. Of this 10 kW is needed to operate the empty mill. By reducing the clearance between the crushing head and the cone, the differential screen analysis of the product becomes that given in column (2) in Table1 below. From (a) Rittinger's law and (b) Kick's law, calculate the power required for the second operation. The feed rate is 110 ton/h.

Tyler Mesh size	Mesh size in μm	First grind product (%)	Second grind product (%)
-4 +6	4699 to 3327	3.1	-
-6 +8	3327 to	10.3	3.3
-8 +10	2362 to	20.0	8.2
-10 +14	1651 to	18.6	11.2
-14 +20	1168 to	15.2	12.3
-20 +28	833 to	12.0	13.0
-28 +35	589 to	9.5	19.5
-35 +48	417 to	6.5	13.5
-48 +65	295 to	4.3	8.5
-65		0.5	-
-65 +100	208 to	-	6.2
-100 +150	147 to	-	4.2
-150	104	-	0.3

Solution

Mesh	Product 1	Dpi_{av}	xi1	$xi1/Dpi_{av}$
4/6	3.1	4.013	0.031	0.007725
6/8	10.3	2.845	0.103	0.036204
8/10	20	2.007	0.2	0.099651
10/14	18.6	1.409	0.186	0.132009
14/20	15.2	1.001	0.152	0.151848
20/28	12	0.711	0.12	0.168776
28/35	9.5	0.503	0.095	0.188867
35/48	6.5	0.356	0.065	0.182584
48/65	4.3	0.252	0.043	0.170635
65	0.5	0.178	0.005	0.02809
	total 100			$\Sigma xi1/Dpi_{av}$ 1.166389

Mesh	Product 2	Xi2	Dpi_{av}	$xi2/Dpi_{av}$
6/8	3.3	0.033	2.845	0.011599
8/10	8.2	0.082	2.007	0.040857
10/14	11.2	0.112	1.409	0.079489
14/20	12.3	0.123	1.001	0.122877
20/28	13	0.13	0.711	0.182841
28/35	19.5	0.195	0.503	0.387674
35/48	13.5	0.135	0.356	0.379213
48/65	8.5	0.085	0.252	0.337302
65/100	6.2	0.062	0.178	0.348315
100/150	4	0.04	0.126	0.31746
150	0.3	0.003	0.089	0.033708
	total 100			$\Sigma xi2/Dpi_{av}$ 2.241335

- Rittinger's Law Kick's Law

$$\frac{P}{\dot{m}} = K_r \left(\frac{1}{\overline{\overline{D}}_{sb}} - \frac{1}{\overline{\overline{D}}_{sa}} \right)$$

$$\frac{P}{\dot{m}} = K_k \ln \frac{\overline{D}_{sa}}{\overline{\overline{D}}_{sb}}$$

- Dsb_{ave} = 1/1.166389 (produk 1) = 0.857347 mm
- Dsa_{ave} = 2 inci = 50.8 mm
- P = 390 kW/ton
- $m_{flow\ rate}$ = 110 ton/hr
- K_r = 3.039

- Dsb_{ave} = 1/2.241335 (produk 2) = 0.446163 mm
- Dsa_{ave} = 2 inci = 50.8 mm
- $m_{flow\ rate}$ = 110 ton/hr
- K_r = 3.039
- P = 751 kW
- Total Power required =

751 kW + 10 kW = 761 kW

SIZE REDUCTION EQUIPMENTS

selection of equipments:

1) **input size**
2) **product size**
3) **hardness**
4) **brittleness**
5) **plasticity**
6) **flammability**

major types:

crushers, ultrafine grinders, grinders & cutting machines

CRUSHERS

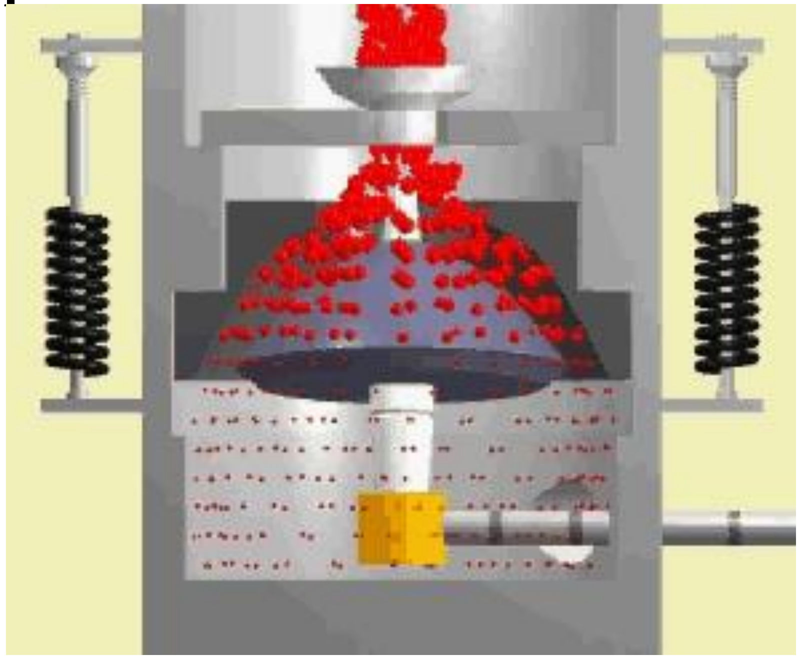
- Slow-speed machine for coarse reduction of large quantities of solids
- break large pieces of solid material into small lumps

Primary crusher - accepts anything from mine & breaks into 150 - 250 mm

Secondary crusher - reduces lumps into 6 mm

main types:

1) Jaw crushers
2) Gyratory crushers
3) smooth-roll crushers
4) toothed-roll crushers



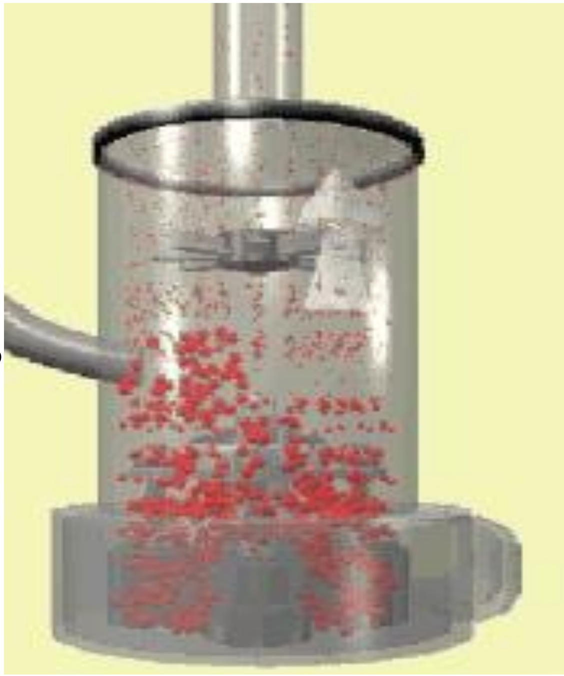
Gyratory crusher

GRINDERS

- for intermediate duty (from crushers to grinders for further reduction)
- reduce crushed feed to powder
- product from intermediate grinder might pass a 40-mesh screen
- product from fine grinder would pass a 200-mesh screen (74μm screen)

commercial grinders:

1) Hammer mills & impactor
2) rolling-compression machines
3) Attrition mills
4) Tumbling mills



Spin mill

hammer

SIZE REDUCTION EQUIPMENTS

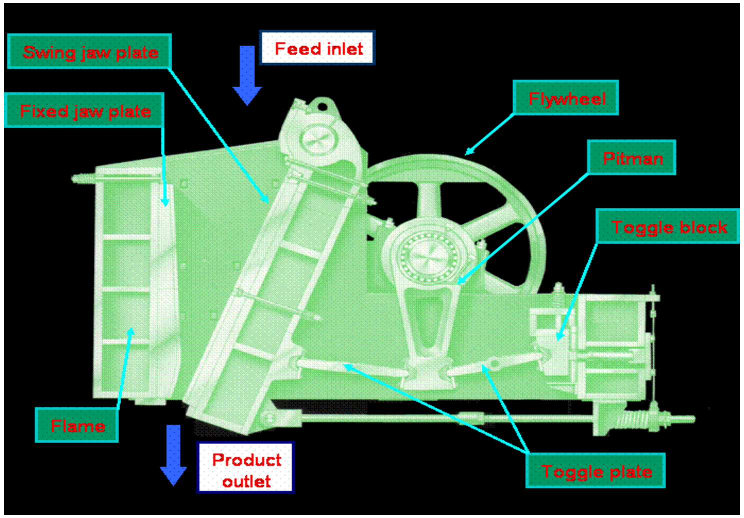
- **Crushers (coarse and fine)**
  - Jaw crushers
  - Gyratory crushers
  - Crushing rolls

- **Grinders (intermediate and fine)**
  - Hammer mills; impactors
  - Rolling-compression mills
  - Attrition mills
  - Tumbling mills

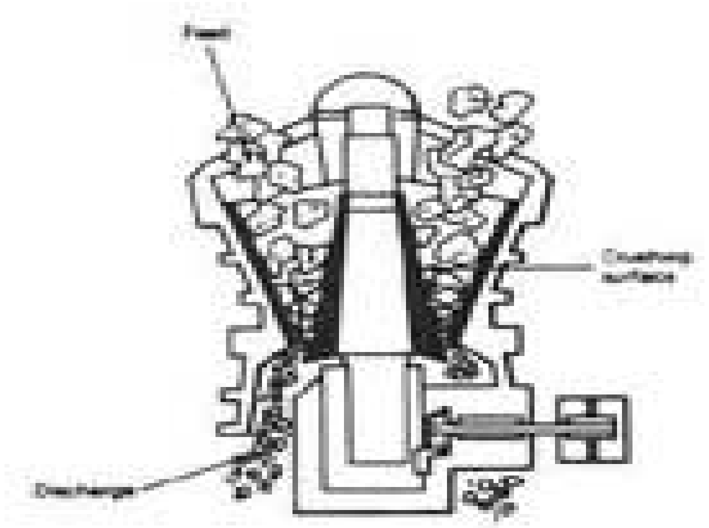
- **Ultrafine grinders**
  - hammer mills with internal classification
  - Fluid-energy mills
  - Agitated mills

- **Cutting machines**
  - Knife cutters; dicers; slitters

CRUSHER

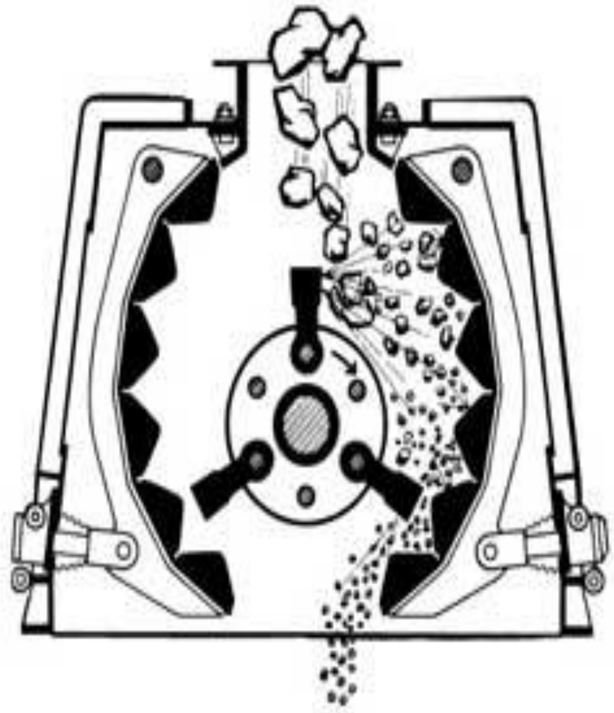


Blake jaw crusher

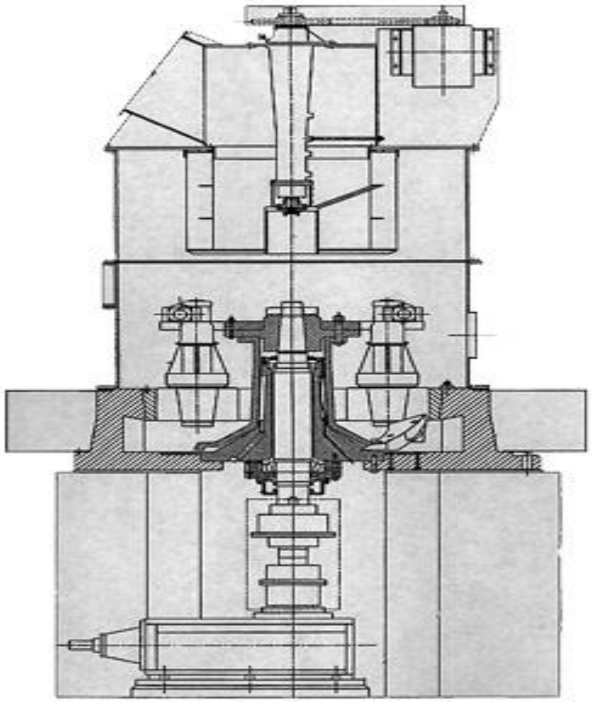


Gyratory Crusher

GRINDERS



Impactor



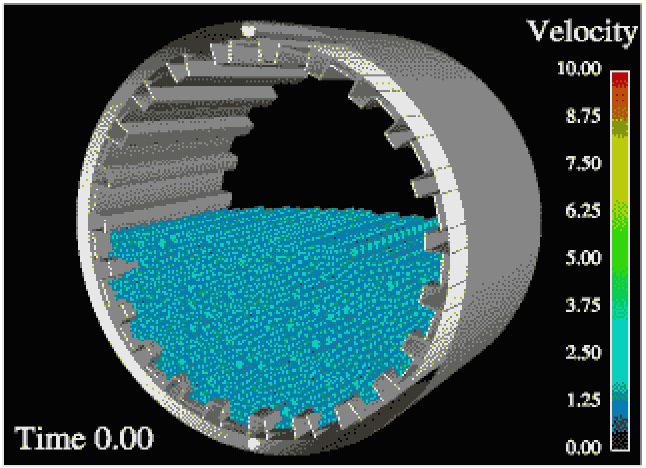
Roller mill

CUTTER

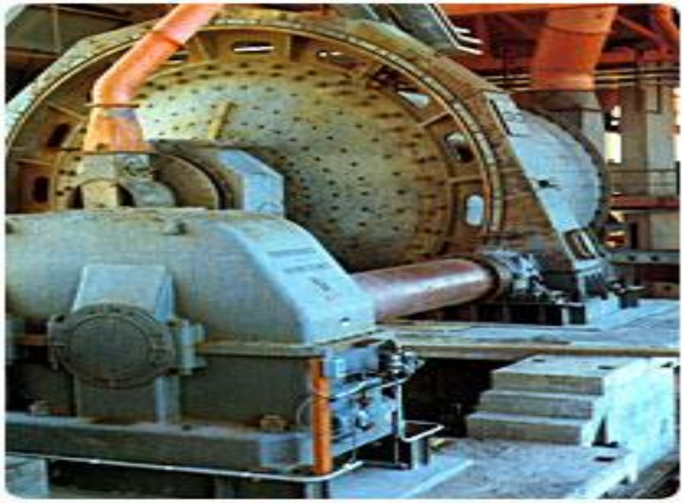
Rotary knife cutter

SIZE REDUCTION EQUIPMENTS

Ball Mills:



Ball mill in Mining Industry



In Cement Industry