SETTLING & SEDIMENTATION IN PARTICLE-FLUID SEPARATION

- **Particles are separated from the fluid by gravitation forces**
- **Particles - solid or liquid drops**
- **fluid - liquid or gas**
- **Applications:**

  Removal of solids from liquid sewage wastes

  Settling of crystals from the mother liquor

  Settling of a slurry from a soybean leaching process

  Separation of liquid-liquid mixture from a solvent-extraction stage

- **Purpose:**

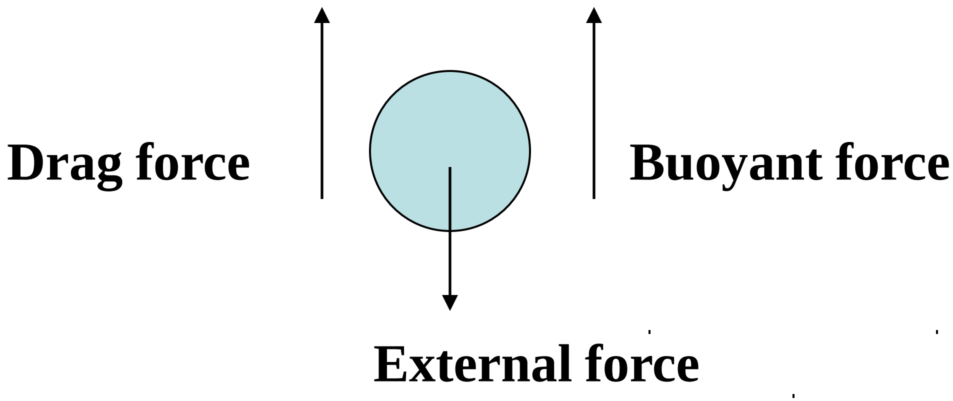
  Remove particles from the fluid (free of particle contaminant)

  Recover particles as the product

  Suspend particles in fluids for separation into different sizes or density

MOTION OF PARTICLES THROUGH FLUID

Three forces acting on a rigid particle moving in a fluid :



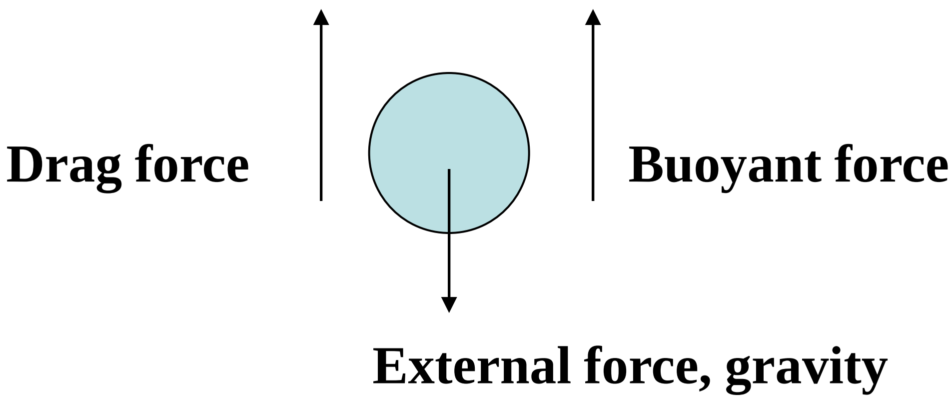
1. external force, gravitational or centrifugal

2. buoyant force, which acts parallel with the external force but in the opposite direction

3. drag force, which appears whenever there is relative motion between the particle and the fluid (frictional resistance)

Drag: the force in the direction of flow exerted by the fluid on the solid

Terminal velocity, u_t



The **terminal velocity** of a falling object is the velocity of the object when the sum of the drag force (*Fd*) and buoyancy equals the downward force of gravity (*FG*) acting on the object. Since the net force on the object is zero, the object has zero acceleration.

In fluid dynamics, an object is moving at its **terminal velocity** if its speed is constant due to the restraining force exerted by the fluid through which it is moving.

Terminal velocity, u_t

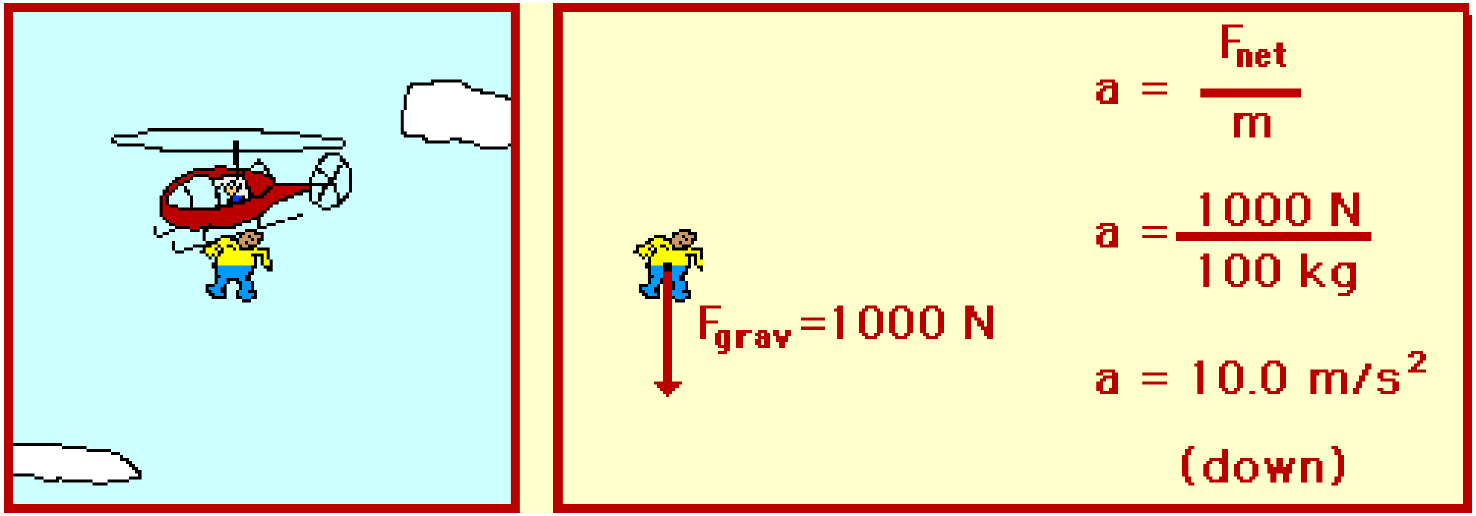
The terminal velocity of a falling body occurs during free fall when a falling body experiences zero acceleration.

This is because of the retarding force known as air resistance. Air resistance exists because air molecules collide into a falling body creating an upward force opposite gravity.

This upward force will eventually balance the falling body's weight. It will continue to fall at constant velocity known as the terminal velocity.

Terminal velocity, u_t

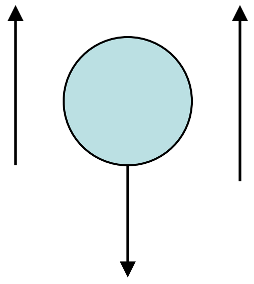
The terminal velocity of a falling body occurs during free fall when a falling body experiences zero acceleration.



ONE-DIMENSIONAL MOTION OF PARTICLE THRU' FLUID

F_D **Drag force**

F_b **Buoyant force**



F_e **External force**

$$m\frac{du}{dt}=F_e-F_b-F_D$$

where

$$F_e=ma$$

$$F_b=\frac{m\rho a}{\rho_p}$$

$$F_D=\frac{C_D u^2 \rho A_p}{2}$$

m **= mass of particle**

u **= velocity of particle relative to the fluid**

ρ, ρ_p **= densities of the fluid & particle, respectively**

a **= acceleration of the particle**

C_D **= drag coefficient (dimensionless)**

A_p **= projected area of the particle**

$$\frac{du}{dt}=a-\frac{\rho a}{\rho_p}-\frac{C_D u^2 \rho A_p}{2m}=a\frac{\rho_p-\rho}{\rho_p}-\frac{C_D u^2 \rho A_p}{2m}$$

ONE-DIMENSIONAL MOTION OF PARTICLE THRU' FLUID

- **Motion from gravitational force**

$$a = g$$

$$\frac{du}{dt} = g\frac{\rho_p - \rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m}$$

- **Motion in a centrifugal field**

$$a = r\omega^2$$

$$\frac{du}{dt} = r\omega^2\frac{\rho_p - \rho}{\rho_p} - \frac{C_D u^2 \rho A_p}{2m}$$

where

r = radius of path of particle

ω = angular velocity, rad/s

u is directed outwardly along a radius

TERMINAL VELOCITY (FREE SETTLING)

when a particle is at a sufficient distance from the walls of the container and from other particles, so that its fall is not affected by them

- period of accelerated fall (1/10 of a second)

- period of constant-velocity fall

• maximum settling velocity (constant velocity) is called terminal/free settling velocity, u_t

$$u_t = \sqrt{\frac{2g(\rho_p - \rho)m}{A_p \rho_p C_D \rho}}$$

where

m = mass of particle

C_D = drag coefficient

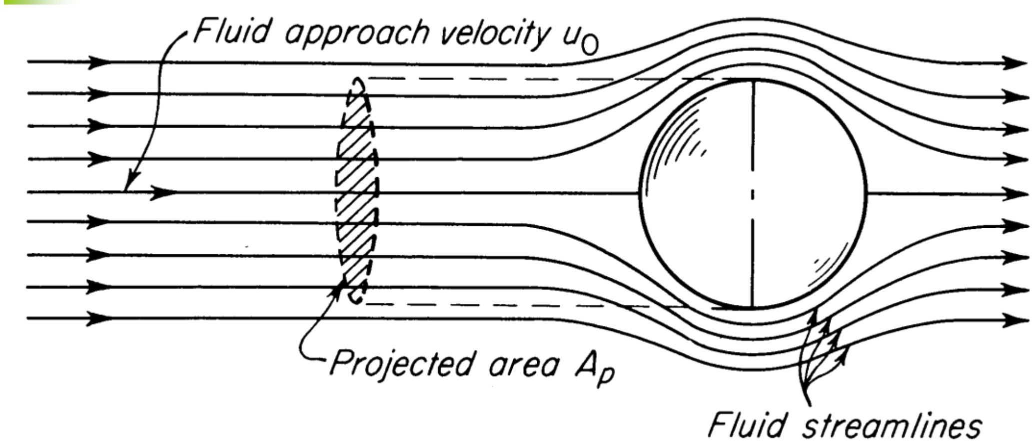
ρ, ρ_p = densities of the fluid & particle, respectively

D_p = equivalent dia. of particle

g = acceleration of the particle

A_p = projected area of the particle

MOTION OF SPHERICAL PARTICLES



$$m = \frac{1}{6}\pi D_p^3 \rho_p$$

$$A_p = \frac{1}{4}\pi D_p^2$$

Substituting *m* & A_p into

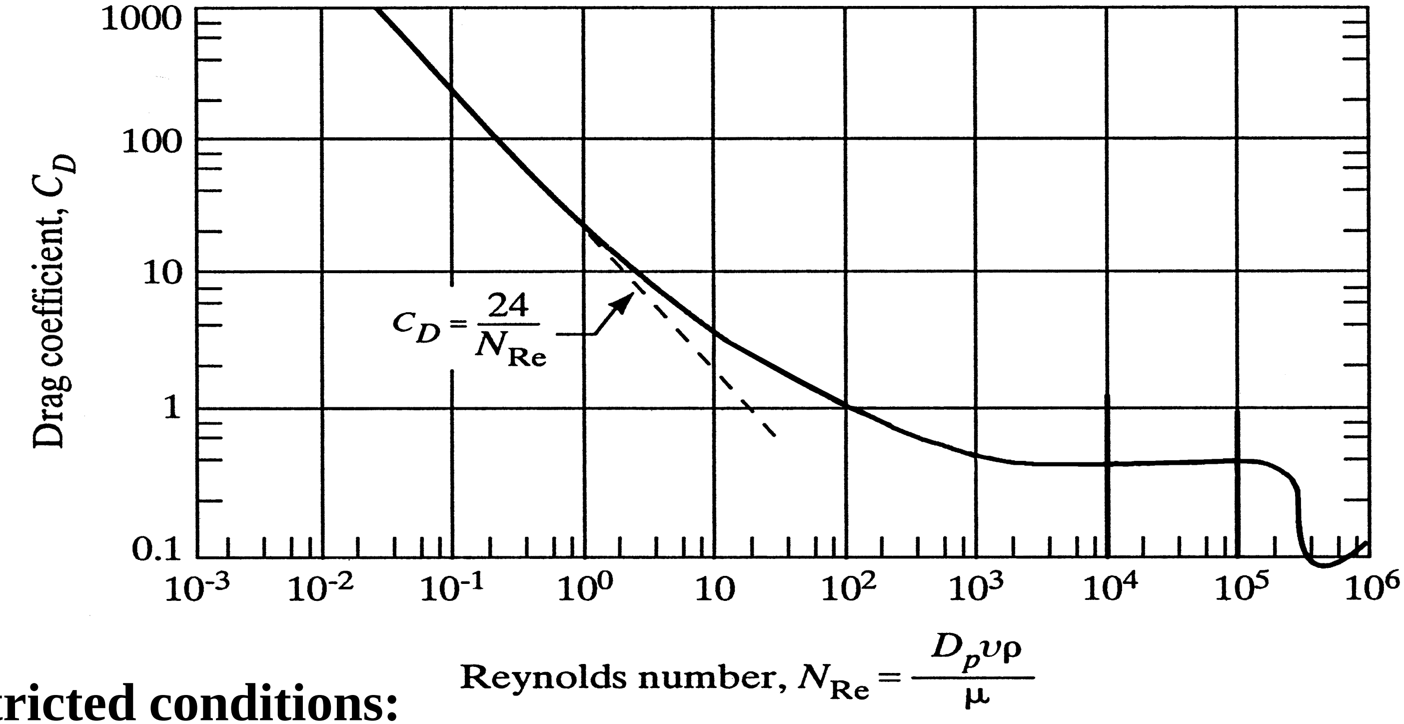
$$u_t = \sqrt{\frac{2g(\rho_p - \rho)m}{A_p \rho_p C_D \rho}}$$

∴ **terminal velocity, u_t:**

$$u_t = \sqrt{\frac{4g(\rho_p - \rho)D_p}{3C_D \rho}}$$

DRAG COEFFICIENT FOR RIGID SPHERES

- a function of Reynolds number



restricted conditions:

1) must be a solid sphere particle

2) far from other particles and the vessel wall (flow pattern around the particle is not distorted)

3) moving at its terminal velocity with respect to the fluid

DRAG COEFFICIENT FOR RIGID SPHERES

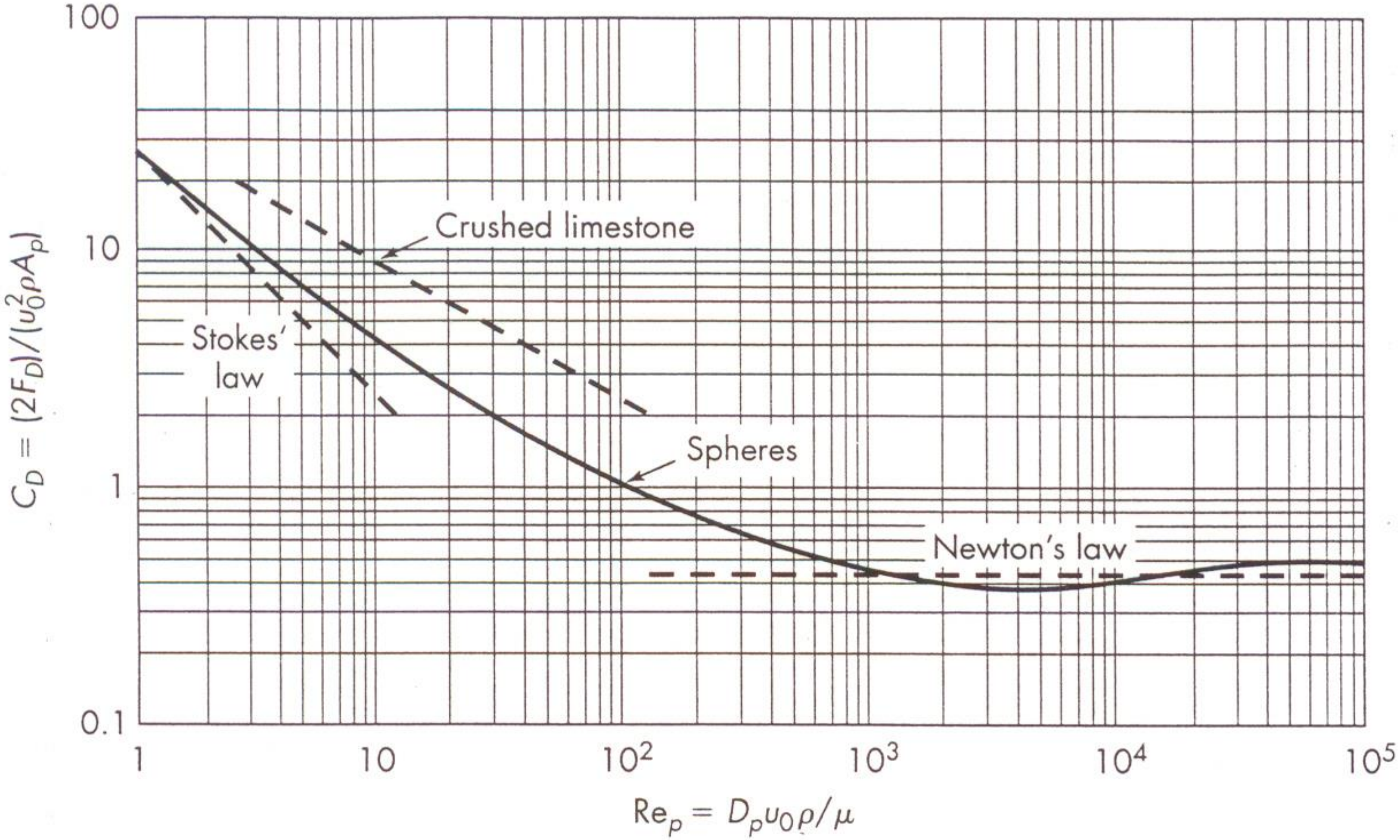
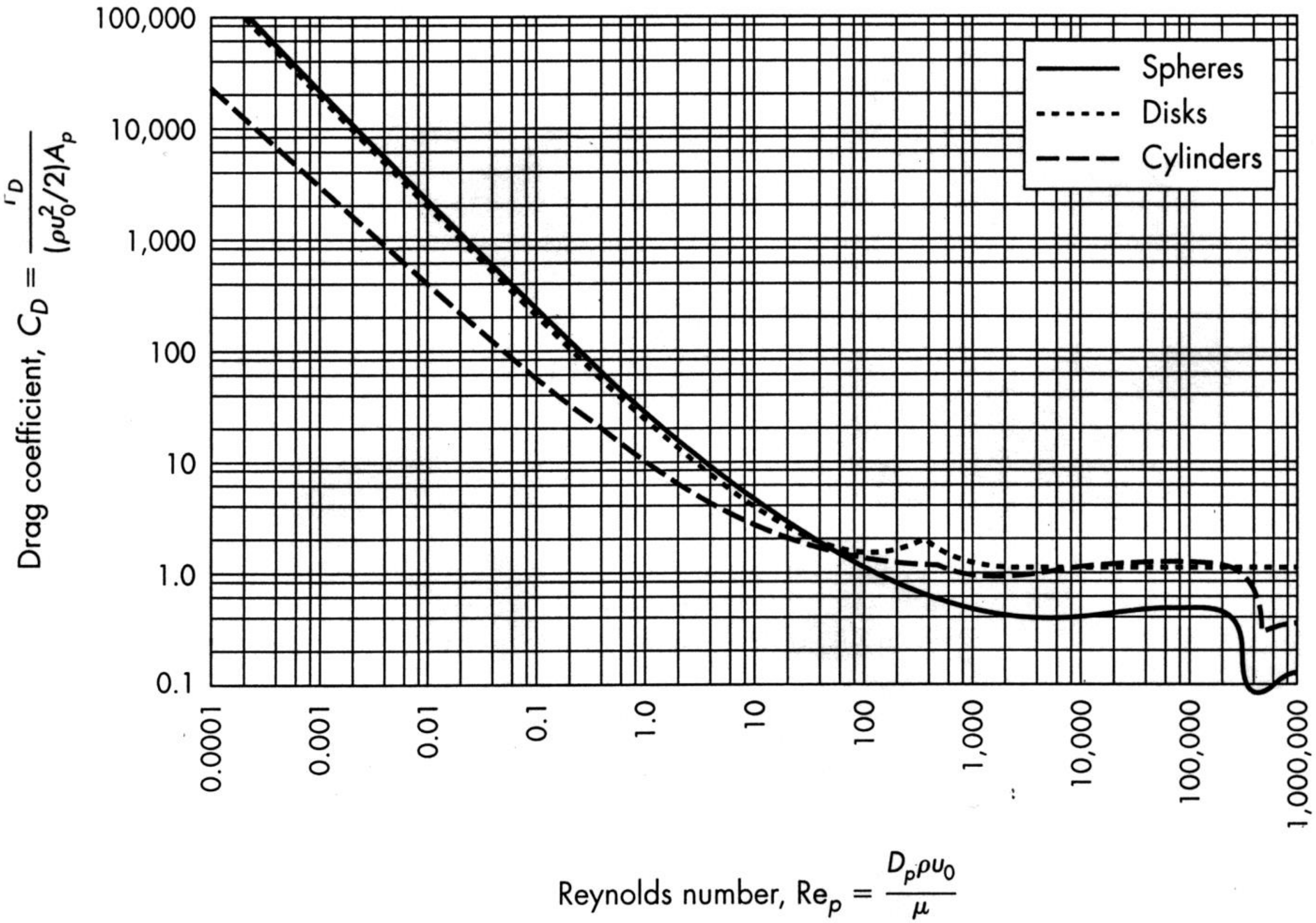


FIGURE 7.6
Drag coefficients for spheres and irregular particles.[3]

DRAG COEFFICIENT



STOKES' LAW (LAMINAR-FLOW REGION)

applies when $N_{Re} < 1.0$

$$C_D = \frac{24}{N_{\mathrm{Re},p}}$$

$$u_t = \frac{g D_p^2 (\rho_p - \rho)}{18\mu}$$

where

C_D = drag coefficient

N_{Re} = Reynolds number = $(D_p u_t \rho)/\mu$

F_D = total drag force

ρ, ρ_p = densities of the fluid & particle, respectively

μ = viscosity of fluid (Pa.s or kg/m.s)

D_p = equivalent dia. of particle

When $N_{\mathrm{Re},p} = 1$, C_D = 26.5

NEWTON'S LAW (TURBULENT-FLOW REGION)

$1000 < N_{Re,p} < 200{,}000$:

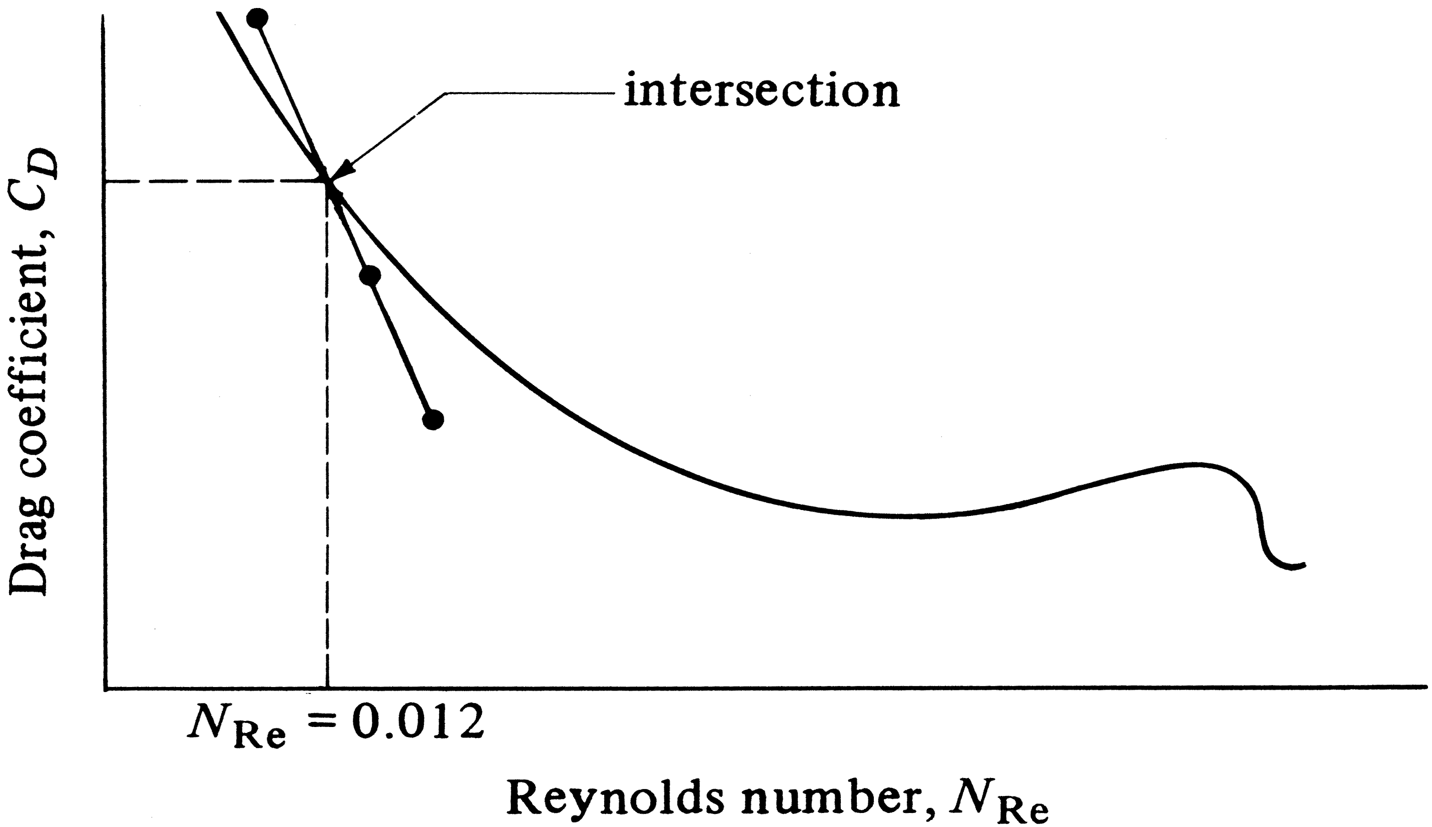
$$C_D = 0.44$$

$$u_t = 1.75\sqrt{\frac{gD_p\left(\rho_p - \rho\right)}{\rho}}$$

applies to fairly large particles falling in gases or low viscosity fluids

TERMINAL VELOCITY OF A PARTICLE

Terminal velocity can be found by trial and error by assuming various u_t to get calculated values of C_D & N_{Re} which are then plotted on the C_D vs N_{Re} graph to get the intersection on the drag-coefficient correlation line, giving the actual N_{Re}.



CRITERION FOR SETTLING REGIME

criterion K :

$$K = D_p \left[\frac{g\rho(\rho_p - \rho)}{\mu^2} \right]^{1/3}$$

→ To determine whether regime is Stoke/Intermediate/Newton

K	Region	u_t
K < 2.6	Stokes' Law	$u_t = \frac{gD_p^2(\rho_p - \rho)}{18\mu}$
2.6 < K < 68.9	Intermediate Region	Trial and Error $u_t = \sqrt{\frac{4g(\rho_p - \rho)D_p}{3C_D\rho}}$
68.9 < K < 2360	Newton's Law	$u_t = 1.75\sqrt{\frac{gD_p(\rho_p - \rho)}{\rho}}$

TRIAL & ERROR METHOD

criterion *K* :

$$K = D_p \left[\frac{g\rho(\rho_p - \rho)}{\mu^2} \right]^{1/3}$$

K	Region	u_t
2.6 < K < 68.9	Intermediate Region	$u_t = \sqrt{\frac{4g(\rho_p - \rho)D_p}{3C_D\rho}}$

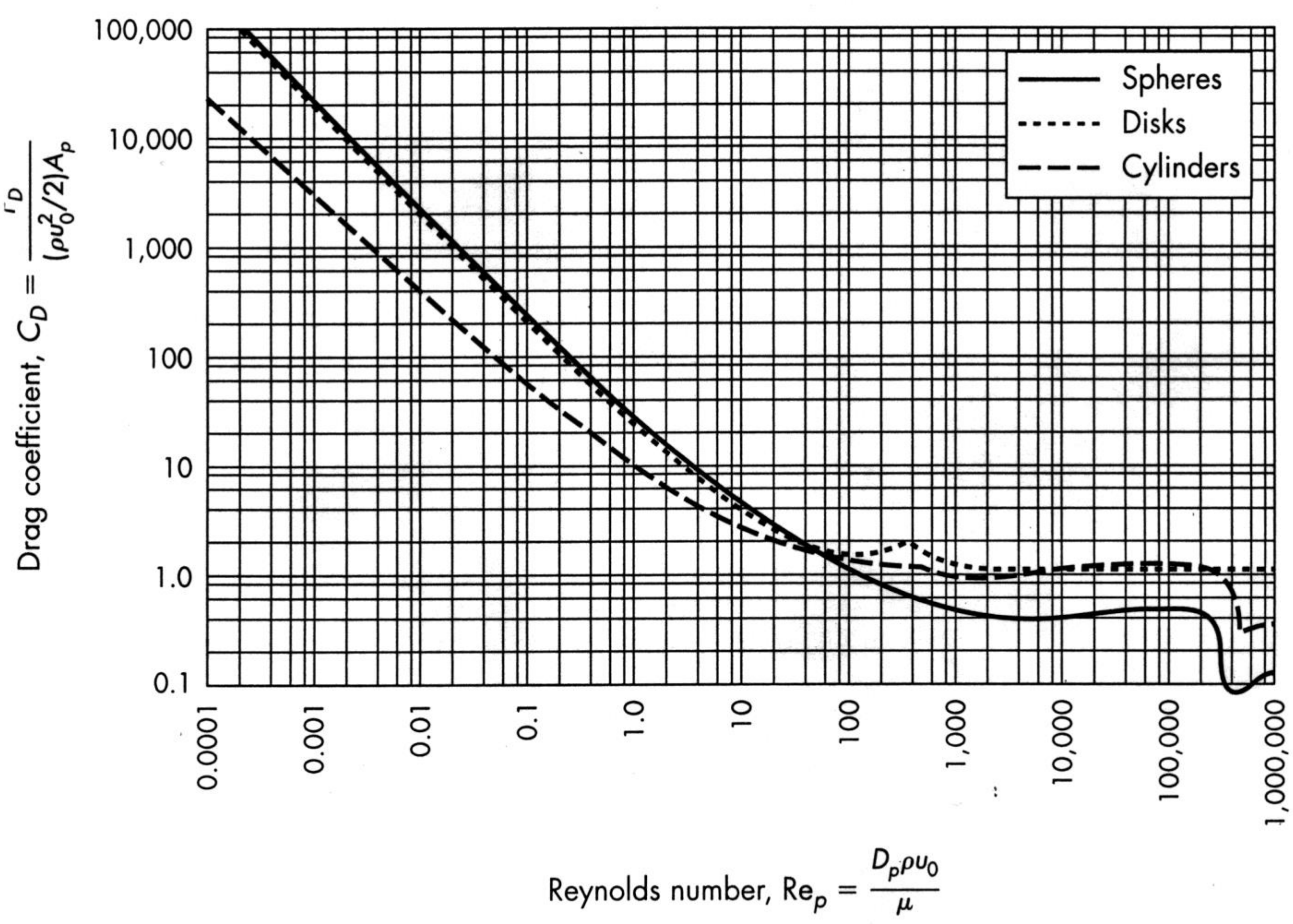
Terminal velocity can be found by trial and error by:

Step 1: Assume N_{Re} which then will give C_D from the C_D vs N_{Re} graph.

Step 2: Calculate u_t.

Step 3: Using the calculated u_t, the N_{Re} is checked to verify if it agrees with the assumed value.

DRAG COEFFICIENT



Example 1

Solid spherical particles of coffee extract from a dryer having a diameter of 400 μm are falling through air at a temperature of 422 K. The density of the particles is 1030 kg/m^3. Calculate the terminal settling velocity and the distance of fall in 5 s. The pressure is 101.32 kPa.

$$K = D_p \left[\frac{g\rho(\rho_p - \rho)}{\mu^2} \right]^{1/3}$$

Example 2

Oil droplets having a diameter of 20 mm are to be settled from air at 311K and 101.3 kPa pressure. The density of the oil droplets is 900 kg/m3. Calculate the terminal settling velocity of the droplets.

$$K = D_p \left[\frac{g\rho\left(\rho_p - \rho\right)}{\mu^2} \right]^{1/3}$$

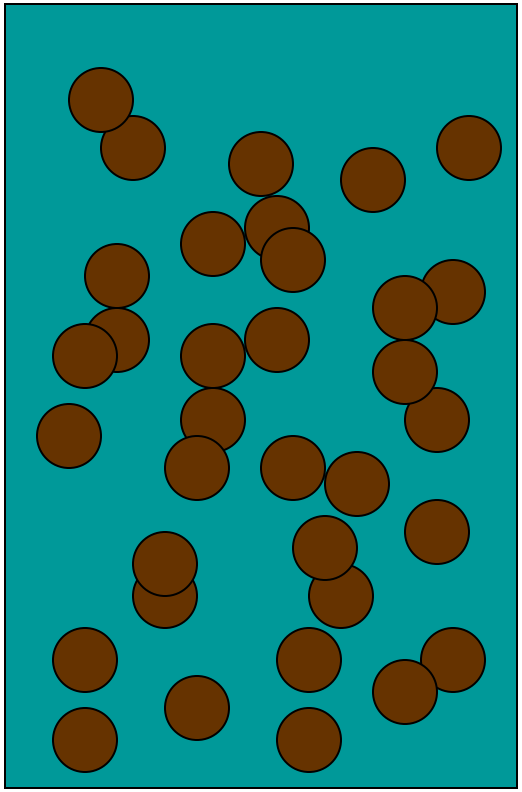
Physical Properties of Air

A.3-3 Physical Properties of Air at 101.325 kPa (1 Atm Abs), SI Units

T (°C)	T (K)	ρ (kg/m^3)	c_p ($kJ/kg \cdot K$)	$\mu \times 10^5$ ($Pa \cdot s$, or $kg/m \cdot s$)	k ($W/m \cdot K$)	N_{Pr}	$\beta \times 10^3$ ($1/K$)	$g\beta\rho^2/\mu^2$ ($1/K \cdot m^3$)
−17.8	255.4	1.379	1.0048	1.62	0.02250	0.720	3.92	2.79×10^8
0	273.2	1.293	1.0048	1.72	0.02423	0.715	3.65	2.04×10^8
10.0	283.2	1.246	1.0048	1.78	0.02492	0.713	3.53	1.72×10^8
37.8	311.0	1.137	1.0048	1.90	0.02700	0.705	3.22	1.12×10^8
65.6	338.8	1.043	1.0090	2.03	0.02925	0.702	2.95	0.775×10^8
93.3	366.5	0.964	1.0090	2.15	0.03115	0.694	2.74	0.534×10^8
121.1	394.3	0.895	1.0132	2.27	0.03323	0.692	2.54	0.386×10^8
148.9	422.1	0.838	1.0174	2.37	0.03531	0.689	2.38	0.289×10^8
176.7	449.9	0.785	1.0216	2.50	0.03721	0.687	2.21	0.214×10^8
204.4	477.6	0.740	1.0258	2.60	0.03894	0.686	2.09	0.168×10^8
232.2	505.4	0.700	1.0300	2.71	0.04084	0.684	1.98	0.130×10^8
260.0	533.2	0.662	1.0341	2.80	0.04258	0.680	1.87	0.104×10^8

HINDERED SETTLING

- **large number of particles are present**
- **velocity gradients around each particle are affected by the presence of nearby particles**
- **particle velocity relative to the fluid > the absolute settling velocity**



- **uniform suspension**

equation of Maude & Whitmore $$u_s = u_t\,(\,\varepsilon\,)^n$$

where

u_s **= settling velocity**

u_t **= terminal velocity for an isolated particle**

ε **= total void fraction (fluid fraction)**

n = exponent n from figure 7.8 (page 52 course notes)

HINDERED SETTLING

- **larger particles thru' a suspension of much finer solids:**

$$u_s = u_t (\varepsilon)^n$$

u_t calculated using the density and viscosity of the fine suspension

ε = volume fraction of the fine suspension, not the total void fraction

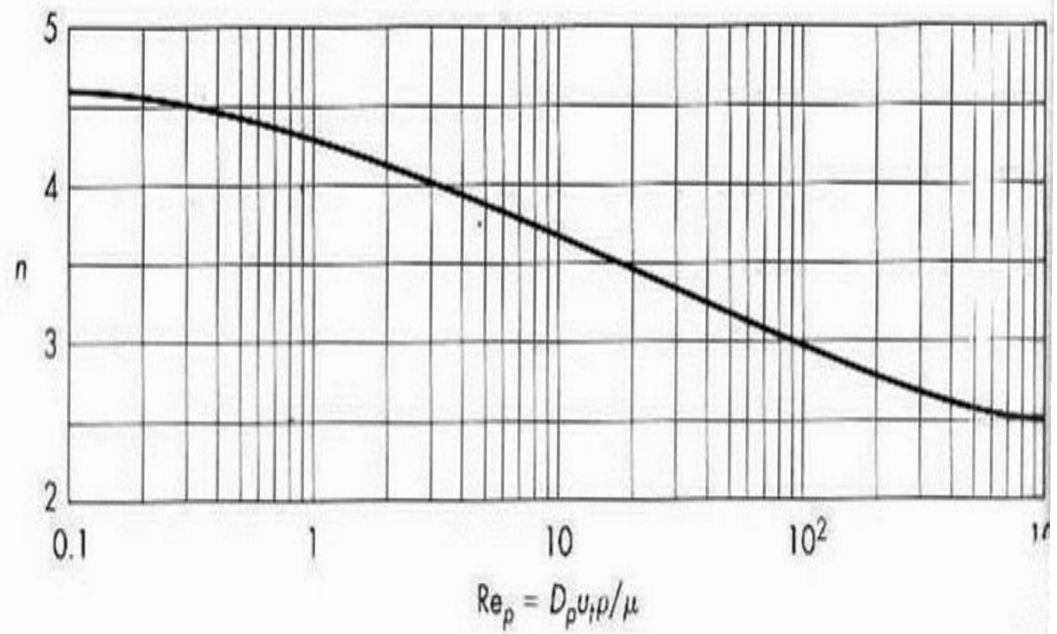
- **Suspensions of very fine sand in water :**

used in separating coal from heavy minerals

density of the suspension is adjusted to a value slightly greater than that of coal to make the coal particles rise to the surface, while the mineral particles sink to the bottom

HINDERED SETTLING

$$u_s = u_t\,(\varepsilon)^n$$



Example 3

1. **(a) Estimate the terminal velocity for 80-to-100 mesh particles of limestone (ρ_p = 2800 kg/m^3) falling in water at 30^{o}C.**

   (b) How much higher would the velocity be in a centrifugal separator where the acceleration is 50g?

*** Refer to page 57 for properties of water**

Physical Properties of Water

A.2-3 Density of Liquid Water

Temperature		*Density*		*Temperature*		*Density*	
K	*°C*	*g/cm^3*	*kg/m^3*	*K*	*°C*	*g/cm^3*	*kg/m^3*
273.15	0	0.99987	999.87	323.15	50	0.98807	988.07
277.15	4	1.00000	1000.00	333.15	60	0.98324	983.24
283.15	10	0.99973	999.73	343.15	70	0.97781	977.81
293.15	20	0.99823	998.23	353.15	80	0.97183	971.83
298.15	25	0.99708	997.08	363.15	90	0.96534	965.34
303.15	30	0.99568	995.68	373.15	100	0.95838	958.38
313.15	40	0.99225	992.25				

Source: R. H. Perry and C. H. Chilton, *Chemical Engineers' Handbook*, 5th ed. New York: McGraw-Hill Book Company, 1973. With permission.

A.2-4 Viscosity of Liquid Water

Temperature K	Temperature °C	Viscosity [(Pa · s) 10³, (kg/m · s) 10³, or cp]	Temperature K	Temperature °C	Viscosity [(Pa · s) 10³, (kg/m · s) 10³, or cp]
273.15	0	1.7921	323.15	50	0.5494
275.15	2	1.6728	325.15	52	0.5315
277.15	4	1.5674	327.15	54	0.5146
279.15	6	1.4728	329.15	56	0.4985
281.15	8	1.3860	331.15	58	0.4832
283.15	10	1.3077	333.15	60	0.4688
285.15	12	1.2363	335.15	62	0.4550
287.15	14	1.1709	337.15	64	0.4418
289.15	16	1.1111	339.15	66	0.4293
291.15	18	1.0559	341.15	68	0.4174
293.15	20	1.0050	343.15	70	0.4061
293.35	20.2	1.0000	345.15	72	0.3952
295.15	22	0.9579	347.15	74	0.3849
297.15	24	0.9142	349.15	76	0.3750
298.15	25	0.8937	351.15	78	0.3655
299.15	26	0.8737	353.15	80	0.3565
301.15	28	0.8360	355.15	82	0.3478
303.15	30	0.8007	357.15	84	0.3395
305.15	32	0.7679	359.15	86	0.3315
307.15	34	0.7371	361.15	88	0.3239
309.15	36	0.7085	363.15	90	0.3165
311.15	38	0.6814	365.15	92	0.3095
313.15	40	0.6560	367.15	94	0.3027
315.15	42	0.6321	369.15	96	0.2962
317.15	44	0.6097	371.15	98	0.2899
319.15	46	0.5883	373.15	100	0.2838
321.15	48	0.5683			

Source: Bingham, *Fluidity and Plasticity*. New York: McGraw-Hill Book Company, 1922. With permission.

- *Free Settling* – is when the fall of a particle is not affected by the boundaries of the container and from other particles (due to a sufficient distance between the particle-container and particle - particle*).*
- *Hindered Settling* – is when the fall is impeded by other particles because the particles are near to another.
- C_D hindered settling > C_D free settling

Hindered Settling

- In hindered settling, the velocity gradients around each particle are effected by the presence of nearby particles; so the normal drag correlations do not apply.
- Furthermore, the particles in settling displace liquid, which flows upward and make the particle velocity relative to the fluid greater than the absolute settling velocity, u_s.
- For uniform suspension, the settling velocity can be estimated from the terminal velocity for an isolated particle using the empirical equation of Maude and Whitmore :

$$u_s = u_t\,(\varepsilon)^n \qquad \text{----- Eq 7.46}$$

where ε is a total void fraction.

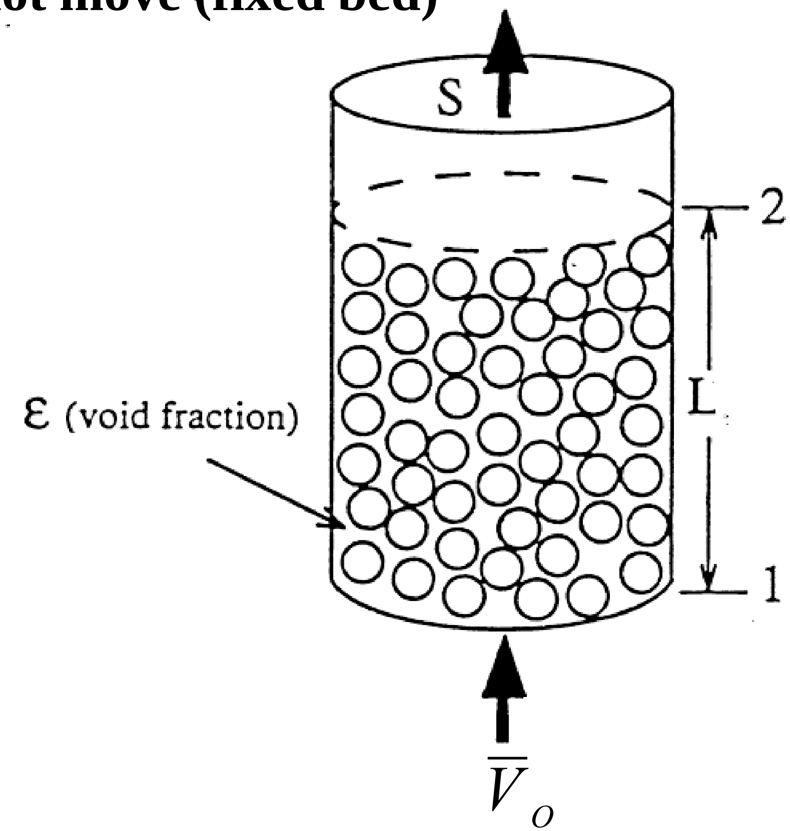
Solution

- $SG = \rho_p/\rho \Rightarrow \rho\, SG = \rho_p \Rightarrow \rho SG - \rho = \rho_p - \rho \Rightarrow \rho(SG-1) = \rho_p - \rho$
- D_p = 0.004 in = 0.004/12 ft
- 1 cP = 6.7197 x 10^{-4} lb/ft.s
- g = 32.174 ft/s^2
- Use Eq 7.40 to find u_t: $$u_t = \frac{gD_p^2(\rho_p - \rho)}{18\mu}$$
- Calculate R_{ep} using Eq 7.44: $$R_{ep} = \frac{D_p u_t \rho}{\mu}$$
- Use R_{ep} value to find exponent n from Fig 7.8.
- Use Eq 7.46 to find u_t in hindered settling

$$u_s = u_t (\varepsilon)^n$$

FLUIDIZATION

fluid is passed at a very low velocity up through a bed of solid, particles do not move (fixed bed)



	Fluid	Solids
fraction	ε	$(1-\varepsilon)$
volume	$\varepsilon(SL)$	$(1-\varepsilon)(SL)$
mass	$\varepsilon(SL)\rho$	$(1-\varepsilon)(SL)\rho_p$

At high enough velocity fluid drag plus buoyancy overcomes the gravity force so particle start to move/suspended and the bed expands (Fluidized Bed).

FLUIDIZATION

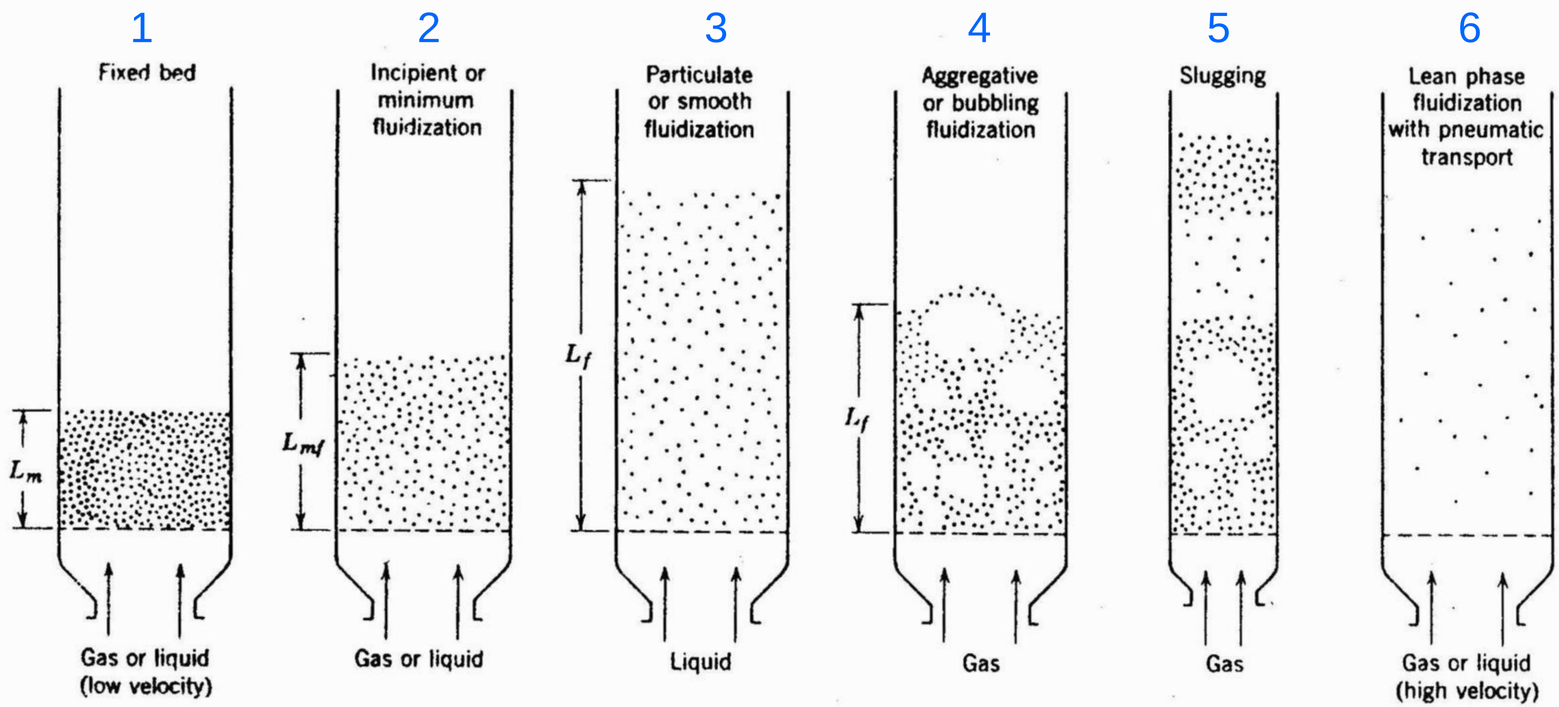


Figure 1 Various kinds of contacting of a batch of solids by fluid.

2 types of fluidization:

(1) particulate fluidization - bed remains homogeneous, intimate contact between gas & solid

(2) bubbling fluidization - bubbles with only a small % of gas passes in the spaces between particles, little contact between bubbles & particles

FLUIDIZATION

fully suspended particles & bed expands (the suspension behave like a dense fluid).

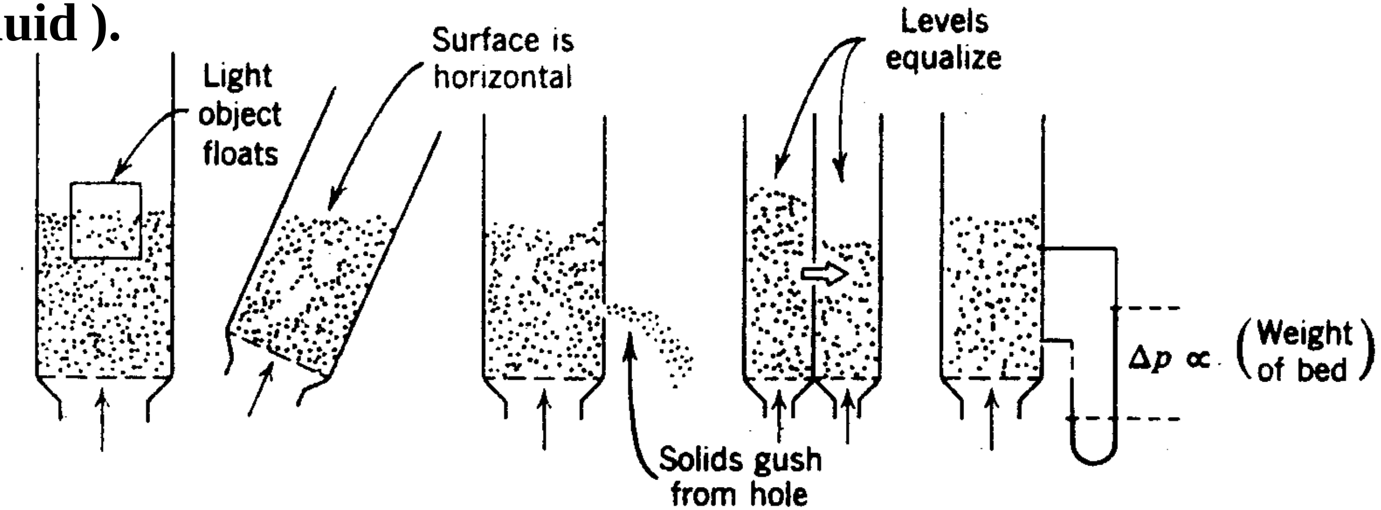


Figure 2 Liquidlike behavior of gas fluidized beds.

fluidized solids can be drained from the bed through pipes and valves just as a liquid can

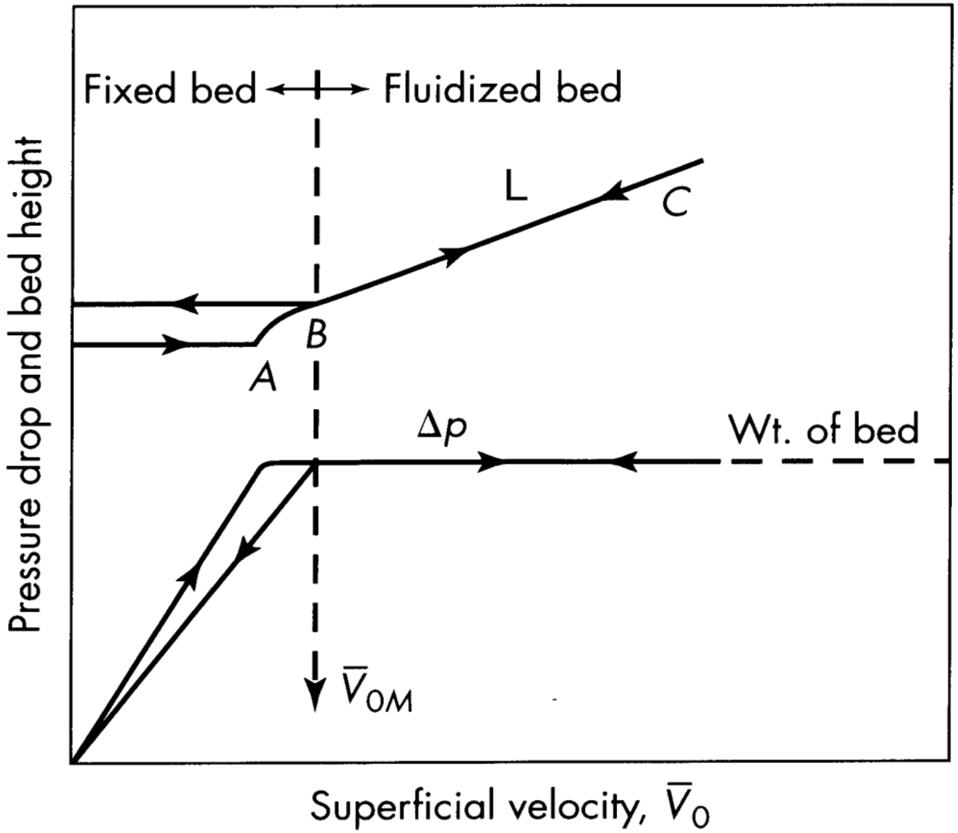
Applications:

Fluidized bed drying

Fluidized bed combustion

Fluidized bed reactions

FLUIDIZATION



Until onset of fluidization Δp increases, then becomes constant.

L is constant until onset of fluidization and then begins to increase.

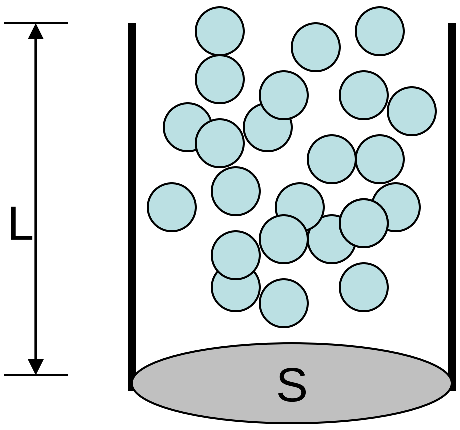
MINIMUM FLUIDIZATION VELOCITY

pressure drop across the bed equal to the weight of the bed per unit area :

$$\Delta P = g(1-\varepsilon)(\rho_p - \rho)L$$

pressure drop given by Ergun Eq :

$$\frac{\Delta P}{L} = \frac{150\bar{V}_o \mu}{g\Phi_s^2 D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75\rho\bar{V}_o^2}{g\Phi_s D_p}\frac{1-\varepsilon}{\varepsilon^3}$$



At the point of incipient/beginning fluidization :

$$\frac{150\mu\bar{V}_{OM}}{\Phi_s^2 D_p^2}\frac{(1-\varepsilon_M)}{\varepsilon_M^3} + \frac{1.75\rho\bar{V}_{OM}^2}{\Phi_s D_p}\frac{1}{\varepsilon_M^3} = g(\rho_p - \rho)$$

where

$\bar{V}_{OM}$ **= minimum fluidization velocity (fluid vel. at which fluidization begins)**

ε_M **= minimum bed porosity/void fraction**

MINIMUM FLUIDIZATION VELOCITY

At the point of incipient/beginning fluidization :

$$\frac{150\mu\bar{V}_{OM}}{\Phi_s^2 D_p^2}\frac{(1-\varepsilon_M)}{\varepsilon_M^3}+\frac{1.75\rho\bar{V}_{OM}^2}{\Phi_s D_p}\frac{1}{\varepsilon_M^3}=g(\rho_p-\rho)$$

Void Fraction at Min. Fluidization

ε_M depends on the shape of the particles. For spherical particles ε_M is usually 0.4 – 0.45.

Type of Particles	*Particle Size, D_p (mm)*			
	0.06	0.10	0.20	0.40
	Void fraction, ε_{mf}			
Sharp sand ($\phi_S = 0.67$)	0.60	0.58	0.53	0.49
Round sand ($\phi_S = 0.86$)	0.53	0.48	0.43	(0.42)
Anthracite coal ($\phi_S = 0.63$)	0.61	0.60	0.56	0.52

Example 4

A bed of ion-exchange beads 8 ft deep is to be backwashed with water to remove dirt. The particle have a density of 1.24g/cm^3 and an average size of 1.1 mm. What is the minimum fluidization velocity using water at 20^{o}C. The beads are assumed to be spherical ($\Phi_s = 1$) and ε_M is taken as 0.4.

$$\frac{150\mu\overline{V}_{OM}}{\Phi_s^2 D_p^2}\frac{(1-\varepsilon_M)}{\varepsilon_M^3}+\frac{1.75\rho\overline{V}_{OM}^2}{\Phi_s D_p}\frac{1}{\varepsilon_M^3}=g(\rho_p-\rho)$$

A.2-3 Density of Liquid Water

Temperature		Density		Temperature		Density	
K	*°C*	*g/cm³*	*kg/m³*	*K*	*°C*	*g/cm³*	*kg/m³*
273.15	0	0.99987	999.87	323.15	50	0.98807	988.07
277.15	4	1.00000	1000.00	333.15	60	0.98324	983.24
283.15	10	0.99973	999.73	343.15	70	0.97781	977.81
293.15	20	0.99823	998.23	353.15	80	0.97183	971.83
298.15	25	0.99708	997.08	363.15	90	0.96534	965.34
303.15	30	0.99568	995.68	373.15	100	0.95838	958.38
313.15	40	0.99225	992.25				

Source: R. H. Perry and C. H. Chilton, *Chemical Engineers' Handbook*, 5th ed. New York: McGraw-Hill Book Company, 1973. With permission.

A.2-4 Viscosity of Liquid Water

Temperature		Viscosity [(Pa · s) 10^3, (kg/m · s) 10^3, or cp]	Temperature		Viscosity [(Pa · s) 10^3, (kg/m · s) 10^3, or cp]
K	°C		K	°C	
273.15	0	1.7921	323.15	50	0.5494
275.15	2	1.6728	325.15	52	0.5315
277.15	4	1.5674	327.15	54	0.5146
279.15	6	1.4728	329.15	56	0.4985
281.15	8	1.3860	331.15	58	0.4832
283.15	10	1.3077	333.15	60	0.4688
285.15	12	1.2363	335.15	62	0.4550
287.15	14	1.1709	337.15	64	0.4418
289.15	16	1.1111	339.15	66	0.4293
291.15	18	1.0559	341.15	68	0.4174
293.15	20	1.0050	343.15	70	0.4061
293.35	20.2	1.0000	345.15	72	0.3952
295.15	22	0.9579	347.15	74	0.3849
297.15	24	0.9142	349.15	76	0.3750
298.15	25	0.8937	351.15	78	0.3655
299.15	26	0.8737	353.15	80	0.3565
301.15	28	0.8360	355.15	82	0.3478
303.15	30	0.8007	357.15	84	0.3395
305.15	32	0.7679	359.15	86	0.3315
307.15	34	0.7371	361.15	88	0.3239
309.15	36	0.7085	363.15	90	0.3165
311.15	38	0.6814	365.15	92	0.3095
313.15	40	0.6560	367.15	94	0.3027
315.15	42	0.6321	369.15	96	0.2962
317.15	44	0.6097	371.15	98	0.2899
319.15	46	0.5883	373.15	100	0.2838
321.15	48	0.5683			

Source: Bingham, *Fluidity and Plasticity*. New York: McGraw-Hill Book Company, 1922. With permission.

MINIMUM FLUIDIZATION VELOCITY USING N_{RE}

Minimum fluidization Reynolds number :

$$N_{\mathrm{Re}M} = \frac{D_p \overline{V}_{OM} \rho}{\mu}$$

At the point of incipient/beginning fluidization :

$$\frac{150\mu\overline{V}_{OM}}{\Phi_s^2 D_p^2}\frac{(1-\varepsilon_M)}{\varepsilon_M^3} + \frac{1.75\rho\overline{V}_{OM}^2}{\Phi_s D_p}\frac{1}{\varepsilon_M^3} = g(\rho_p - \rho)$$

In term of minimum fluidization Reynolds number:

$$\frac{150(1-\varepsilon_M)(N_{\mathrm{Re}M})}{\Phi_s^2\varepsilon_M^3} + \frac{1.75(N_{\mathrm{Re}M})^2}{\Phi_s\varepsilon_M^3} = \frac{D_p^{\ 3}\rho g(\rho_p - \rho)}{\mu^2}$$

MINIMUM FLUIDIZATION VELOCITY

- **very small particles ($N_{Re,p}$ <1) :**

$$\overline{V}_{OM} \approx \frac{g(\rho_p - \rho)}{150\mu} \frac{\varepsilon_M^3}{1-\varepsilon_M} \Phi_s^2 D_p^2$$

only the laminar-term is significant

ratio of u_t/V_{OM} :

$$\frac{u_t}{\overline{V}_{OM}} = \frac{gD_p^2(\rho_p - \rho)}{18\mu} \frac{150\mu}{g(\rho_p - \rho)\Phi_s^2 D_p^2} \frac{1-\varepsilon_M}{\varepsilon_M^3} = \frac{8.33(1-\varepsilon_M)}{\Phi_s^2 \varepsilon_M^3}$$

MINIMUM FLUIDIZATION VELOCITY

- **Larger Particles ($N_{Re,p}$ > 1000, larger than 1 mm) :**

$$\overline{V}_{OM} \approx \left[\frac{\Phi_s D_p g(\rho_p - \rho)\varepsilon_M^3}{1.75\rho} \right]^{1/2}$$

ratio of u_t/V_{OM} :

$$\frac{u_t}{\overline{V}_{OM}} = 1.75 \left[\frac{g D_p (\rho_p - \rho)}{\rho} \right]^{1/2} \left[\frac{1.75\rho}{g D_p (\rho_p - \rho)\varepsilon_M^3} \right]^{1/2} = \frac{2.32}{\varepsilon_M^{3/2}}$$

u_t = terminal settling velocity of the particles (maximum allowable velocity)

MINIMUM FLUIDIZATION VELOCITY

If ε_M & Φ_S are unknown:

$$\Phi_S \varepsilon_M^3 = \frac{1}{14}$$

$$\frac{1-\varepsilon_M}{\Phi_S^2 \varepsilon_M^3} = 11$$

Substituting into the minimum fluidization velocity eq. :

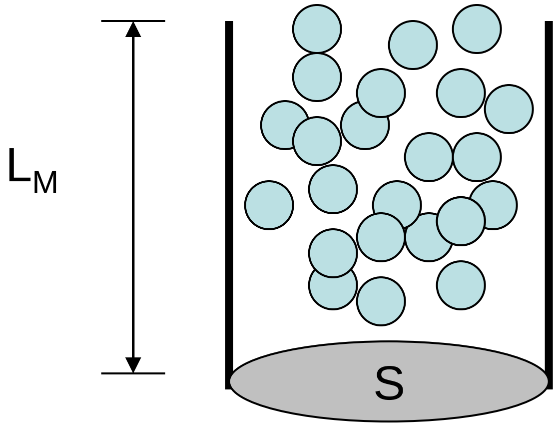
$$N_{\mathrm{Re}M} = \left[(33.7)^2 + 0.0408 \frac{g D_p^3 (\rho_p - \rho)\rho}{\mu^2} \right]^{1/2} - 33.7$$

Holds for $0.001 \leq N_{re} \leq 4000$

Reasonable estimate (± 25%)

BED LENGTH AT MINIMUM FLUIDIZATION

Bed height is needed in order to size the vessel



$$L_M = \frac{m}{S(1-\varepsilon_M)\rho_p}$$

where

L_M = minimum bed height at onset of fluidization

m = mass of particles

S = cross-sectional area of fluidized bed

ε_M = void fraction at minimum fluidization ρ

ρ_p = density of particle

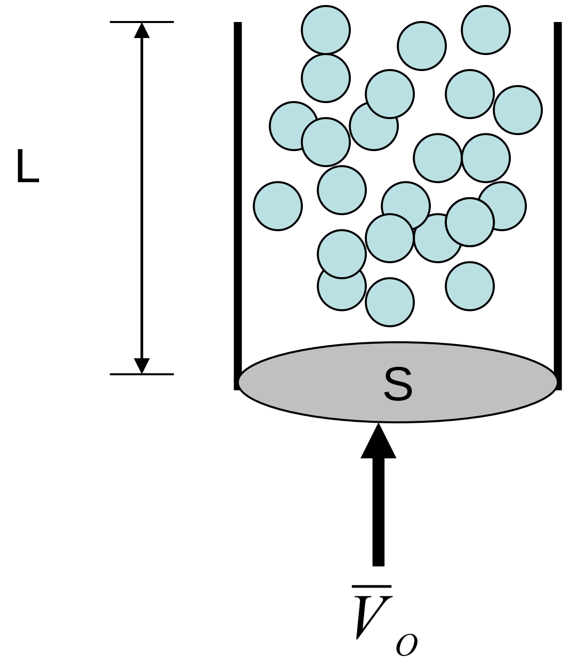
EXPANSION OF FLUIDISED BEDS

Particulate fluidization

Small particles & $N_{Re,p}$ < 20 :

$$\overline{V}_O \approx \frac{D_p^2 g\left(\rho_p - \rho\right)\Phi_S^2}{150\mu}\frac{\varepsilon^3}{1-\varepsilon} = K_1 \frac{\varepsilon^3}{1-\varepsilon}$$

$$L = L_M \frac{\left(1-\varepsilon_M\right)}{1-\varepsilon}$$



where

$\overline{V}_O$ **= operating velocity**

L = expanded bed height

ε = void fraction at operating velocity

Example 5

Solid particles having a size of 0.12 mm, a shape factor Φ_s of 0.88, and a density of 1000 kg/m^3 are to be fluidized using air at 2 atm abs and 25^{o}C. The voidage at minimum fluidizing conditions ε_M is 0.42.

a. **If the cross section of the empty bed is 0.3 m^2 and the bed contains 300 kg of solid, calculate the minimum height of the fluidized bed.**

b. **Calculate the presure drop at minimum fluidizing conditions.**

c. **Calculate the minimum velocity for fluidization.**

d. **Assuming that data for Φ_s and ε_m are unavailable, calculate the minimum fluidization velocity**

Example 5

$$L_M = \frac{m}{S(1-\varepsilon_M)\rho_p}$$

$$\Delta P = g(1-\varepsilon)(\rho_p - \rho)L$$

$$\frac{150\mu \overline{V}_{OM}}{\Phi_s^2 D_p^2}\frac{(1-\varepsilon_M)}{\varepsilon_M^3} + \frac{1.75\rho \overline{V}_{OM}^2}{\Phi_s D_p}\frac{1}{\varepsilon_M^3} = g(\rho_p - \rho)$$

$$N_{\mathrm{Re}M} = \left[(33.7)^2 + 0.0408\frac{gD_p^3(\rho_p - \rho)\rho}{\mu^2}\right]^{1/2} - 33.7$$