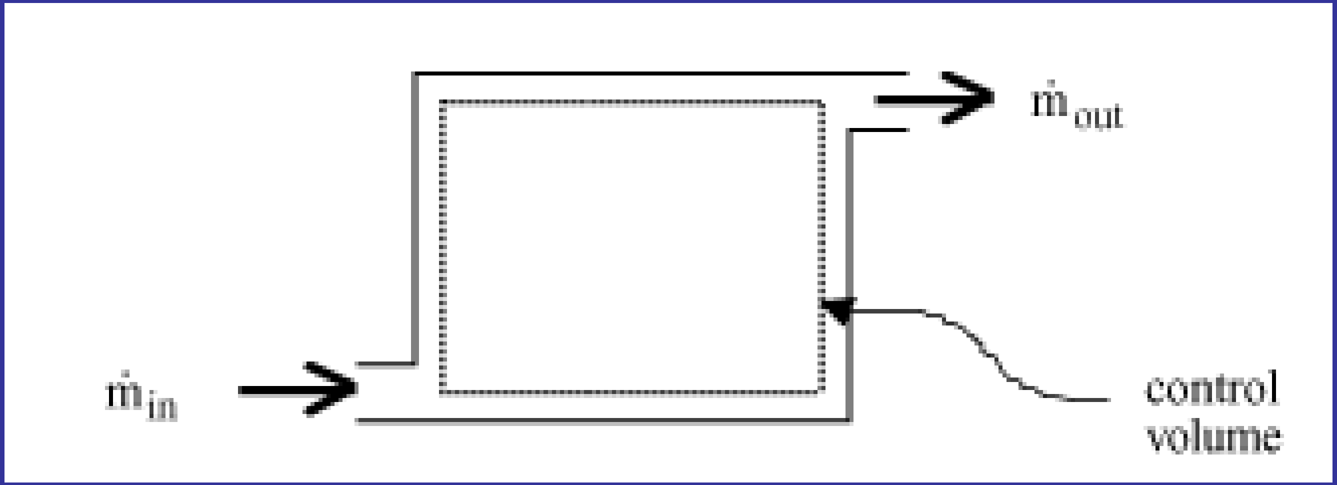
1^{st} Law Analysis of Control Volume (open system)

Chapter 6

In chapter 5, we did 1st law analysis for a control mass (closed system).

In this chapter the analysis of the 1st law will be on a control volume (open system) where there are flows of substance into or out of the system and both into and out of the system

Control volume – open system



Conservation of mass

$$\frac{dm_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\begin{pmatrix}\text{rate of change} \\ \text{of mass in c.v.}\end{pmatrix} = \begin{pmatrix}\text{mass flow} \\ \text{in}\end{pmatrix} - \begin{pmatrix}\text{mass flow} \\ \text{out}\end{pmatrix}$$

For steady state, $\frac{d}{dt} = 0$ therefore

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

CONTINUITY EQUATION

Flow inside a pipe

$\dot{V} = \nu A$ Volume flow rate (6.2)

$\dot{m} = \rho \dot{V} = \rho \nu A = \frac{\nu A}{v}$ Mass flow rate (6.3)

where ν is velocity (average velocity for the cross sectional area)

v is specific volume

A is cross sectional area

For steady flow and single inlet and single outlet, continuity equation becomes :

$0 = \dot{m}_i - \dot{m}_e$

$\rho_i \nu_i A_i = \rho_e \nu_e A_e$

Example 1 (Q1)

Refrigerant-134a at 200 kPa, 40% quality, flows through a 1.1-cm inside diameter, *d*, tube with a velocity of 50 m/s. Find the mass flow rate of the refrigerant-134a.

At *P* = 200 kPa, *x* = 0.4 we determine the specific volume from

$$\begin{aligned} v &= v_f + x v_{fg} \\ &= 0.0007533 + 0.4(0.0999 - 0.0007533) \\ &= 0.0404 \frac{m^3}{kg} \end{aligned}$$

$$\begin{aligned} \dot{m} &= \frac{\vec{V}_{ave} A}{v} = \frac{\vec{V}_{ave}}{v} \frac{\pi d^2}{4} \\ &= \frac{50\ m/s}{0.0404\ m^3/kg} \frac{\pi (0.011\ m)^2}{4} \\ &= 0.117 \frac{kg}{s} \end{aligned}$$

Example 1 (Q2)

Air at 100 kPa, 50^{o}C, flows through a pipe with a volume flow rate of 40 m^3/min. Find the mass flow rate through the pipe, in kg/s.

Assume air to be an ideal gas, so

$$v = \frac{RT}{P} = 0.287\frac{kJ}{kg \cdot K}\frac{(50+273)K}{100kPa}\frac{m^3kPa}{kJ}$$

$$= 0.9270\frac{m^3}{kg}$$

$$\dot{m} = \frac{\dot{V}}{v} = \frac{40m^3/\min}{0.9270m^3/kg}\frac{1\min}{60s}$$

$$= 0.719\frac{kg}{s}$$

Steady-state, steady-flow conservation of energy

Since the energy of the control volume is constant with time during the steady-state, steady-flow process, the conservation of energy principle becomes

$$\begin{bmatrix}\text{Sum of rate} \\ \text{of energy flowing} \\ \text{into control volume}\end{bmatrix} = \begin{bmatrix}\text{Sum of rate} \\ \text{of energy flowing} \\ \text{from control volume}\end{bmatrix}$$

or

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{system}^{\nearrow 0}}_{\substack{\text{Rate change in internal, kinetic,} \\ \text{potential, etc., energies}}} \qquad (kW)$$

or

$$\underbrace{\dot{E}_{in}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass} \\ \text{into the system}}} = \underbrace{\dot{E}_{out}}_{\substack{\text{Rate of energy transfer} \\ \text{by heat, work, and mass} \\ \text{from the system}}}$$

Considering that energy flows into and from the control volume with the mass, energy enters because heat is transferred to the control volume, and energy leaves because the control volume does work on its surroundings, the steady-state, steady-flow first law becomes

$$\dot{Q}_{in} + \dot{W}_{in} + \sum \underbrace{\dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{Q}_{out} + \dot{W}_{out} + \sum \underbrace{\dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

Often this result is written as

$$\dot{Q}_{net} - \dot{W}_{net} = \sum \underbrace{\dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}} - \sum \underbrace{\dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}}$$

where

$$\dot{Q}_{net} = \sum \dot{Q}_{in} - \sum \dot{Q}_{out}$$
$$\dot{W}_{net} = \sum \dot{W}_{out} - \sum \dot{W}_{in}$$

Steady-state, steady-flow (SSSF) for one entrance and one exit

A number of thermodynamic devices such as pumps, fans, compressors, turbines, nozzles, diffusers, and heaters operate with one entrance and one exit. The steady-state, steady-flow conservation of mass and first law of thermodynamics for these systems reduce to

$$\dot{m}_1 = \dot{m}_2 \qquad (kg/s)$$

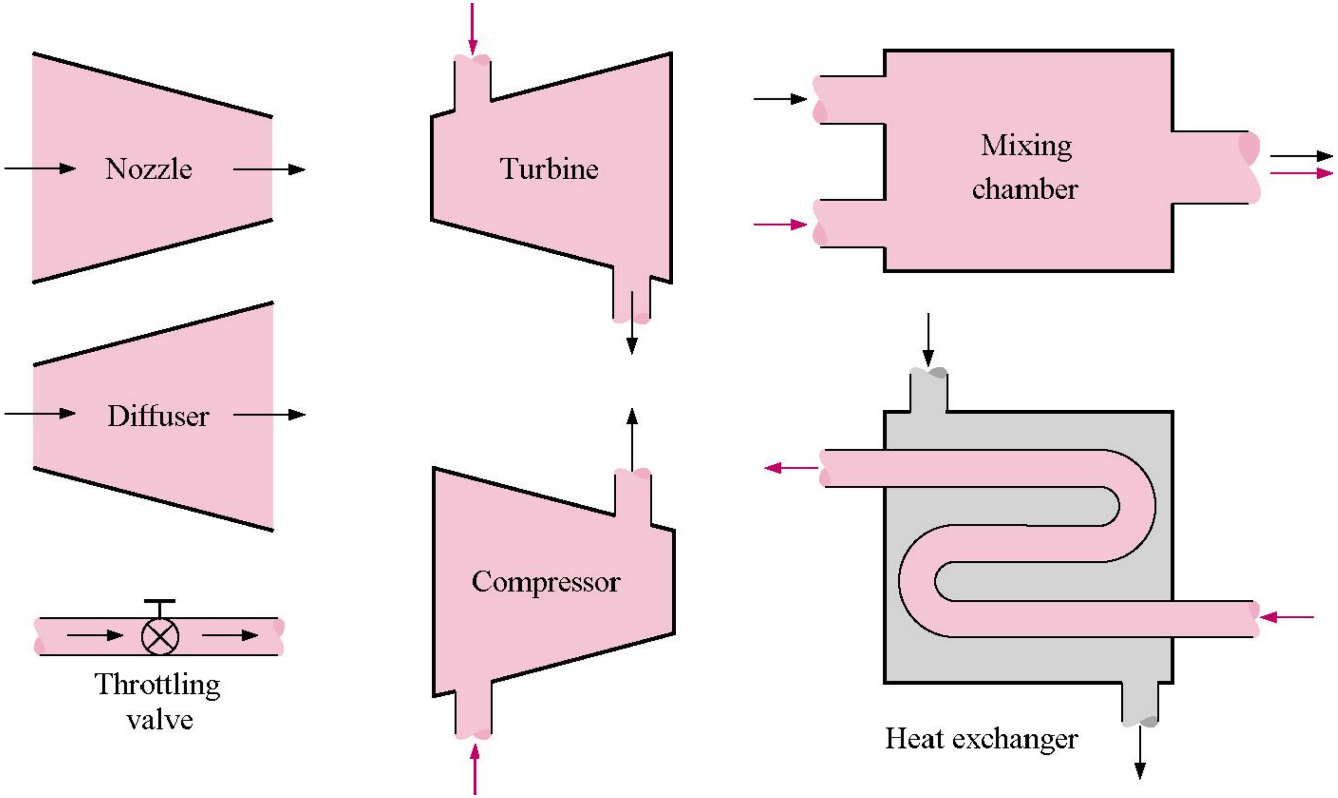
$$\frac{1}{v_1}\vec{V}_1 A_1 = \frac{1}{v_2}\vec{V}_2 A_2$$

$$\dot{Q} - \dot{W} = \dot{m}\left[h_2 - h_1 + \frac{\vec{V}_2^{\,2} - \vec{V}_1^{\,2}}{2} + g(z_2 - z_1)\right] \qquad (kW)$$

where the entrance to the control volume is state 1 and the exit is state 2 and $\dot{m}$ is the mass flow rate through the device.

Some Steady-Flow Engineering Devices

Below are some engineering devices that operate essentially as steady-state, steady-flow control volumes.



Lets Do Exercise 2

Chapter 5 (5-4)

Read 2 Turbine and Compressor page 236

Read 3 Throttling valves page 239

Lets Do
Quiz 1
Quiz 3

Uniform-State, Uniform-Flow Problems (USUF)

During unsteady energy transfer to or from open systems or control volumes, the system may have a change in the stored energy and mass. Several unsteady thermodynamic problems may be treated as uniform-state, uniform-flow problems. The assumptions for uniform-state, uniform-flow are

- The process takes place over a specified time period.
- The state of the mass within the control volume is uniform at any instant of time but may vary with time.
- The state of mass crossing the control surface is uniform and steady. The mass flow may be different at different control surface locations.

To find the amount of mass crossing the control surface at a given location, we integrate the mass flow rate over the time period.

$$Inlets:\ m_i = \int_0^t \dot{m}_i\,dt \qquad Exits:\ m_e = \int_0^t \dot{m}_e\,dt$$

The change in mass of the control volume in the time period is

$$(m_2u_2 - m_1u_1)_{CV} = \int_0^t \left.\frac{dU}{dt}\right|_{CV} dt$$

The uniform-state, uniform-flow conservation of mass becomes

$$\sum m_i - \sum m_e = (m_2 - m_1)_{CV}$$

The first law for uniform-state, uniform-flow becomes

$$E_{in} - E_{out} = \Delta E_{CV}$$

$$Q - W = \sum m_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right) - \sum m_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right) + \left(m_2 e_2 - m_1 e_1 \right)_{CV}$$

When the kinetic and potential energy changes associated with the control volume and the fluid streams are negligible, it simplifies to

$$Q - W = \sum m_e h_e - \sum m_i h_i + \left(m_2 u_2 - m_1 u_1 \right)_{CV} \qquad (kJ)$$

Lets Do

Quiz 4

Quiz 5

Steady-State, Steady Flow Process (SSSF)

$$\dot{Q}_{c.v} + \sum \dot{m}_i (h_i + \frac{1}{2} V_i^2 + gZ_i) = \dot{W}_{c.v} + \sum \dot{m}_e (h_e + \frac{1}{2} V_e^2 + gZ_e)$$

Uniform State, Uniform Flow Process (USUF)

$$Q_{c.v}+\sum m_i\left(h_i+\frac{1}{2}V_i^2+gZ_i\right)=$$

$$\sum m_e\left(h_e+\frac{1}{2}V_e^2+gZ_e\right)+\left[m_2u_2-m_1u_1\right]_{c.v}+W_{c.v}$$

Try out problem in CENGEL

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