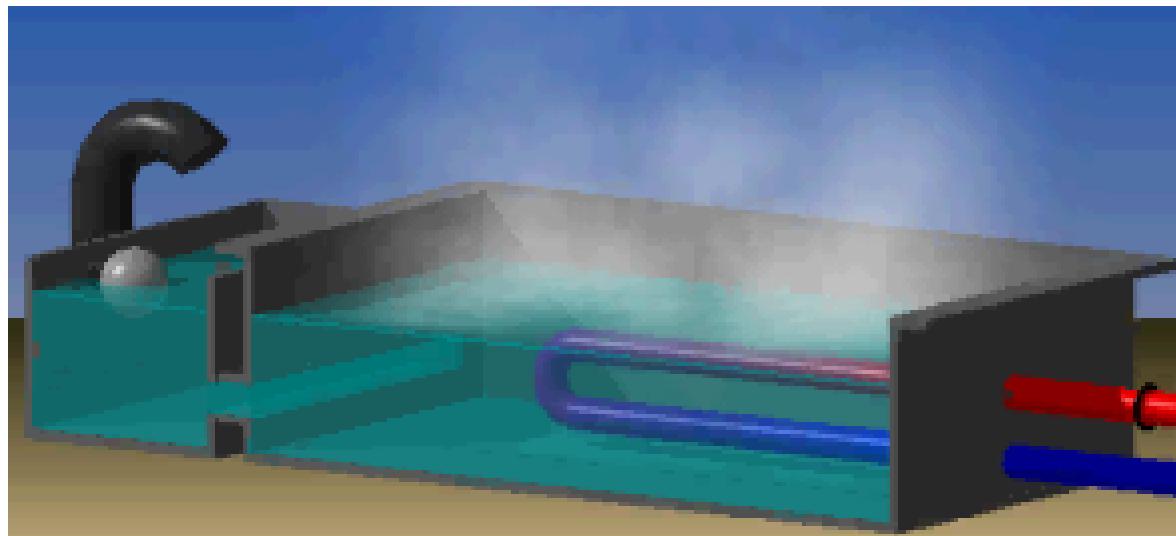


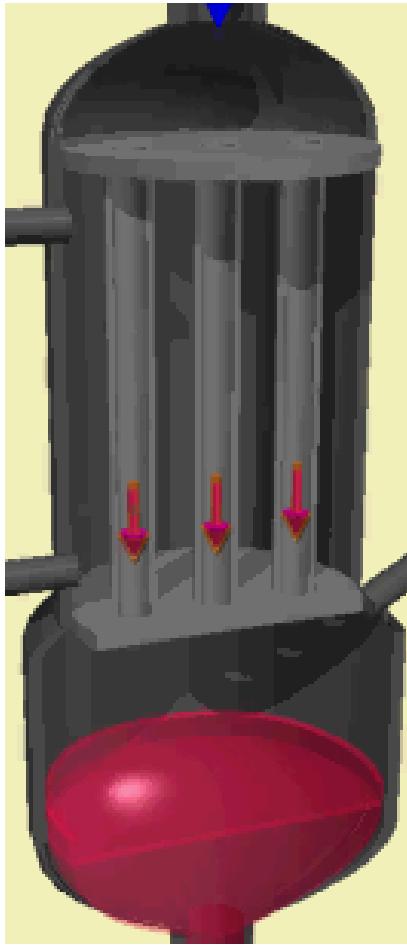
Mechanism of evaporation



- removing a liquid by boiling off some of the liquid
- thermal separation/concentration
- vapour removed as top product
- concentrated solution as bottom product
- desired products
 - concentrated solution eg. Milk
 - vapour usually water vapour
 - crystal eg. Salt crystal

Types of evaporators

3 Vertical-tube natural circulation evaporator

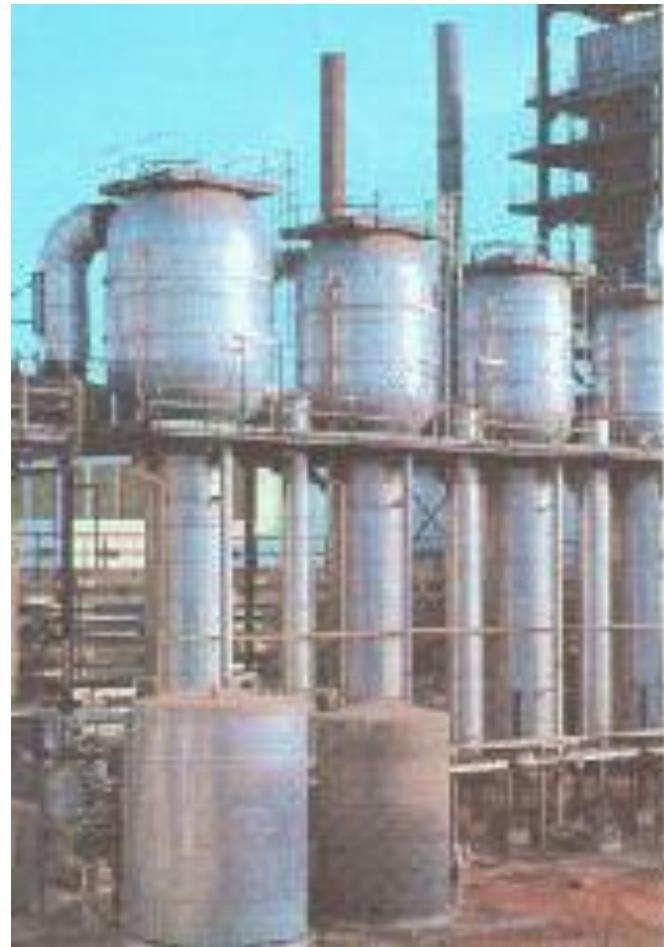


(a) Falling-film evaporator

(b) Climbing-film evaporator

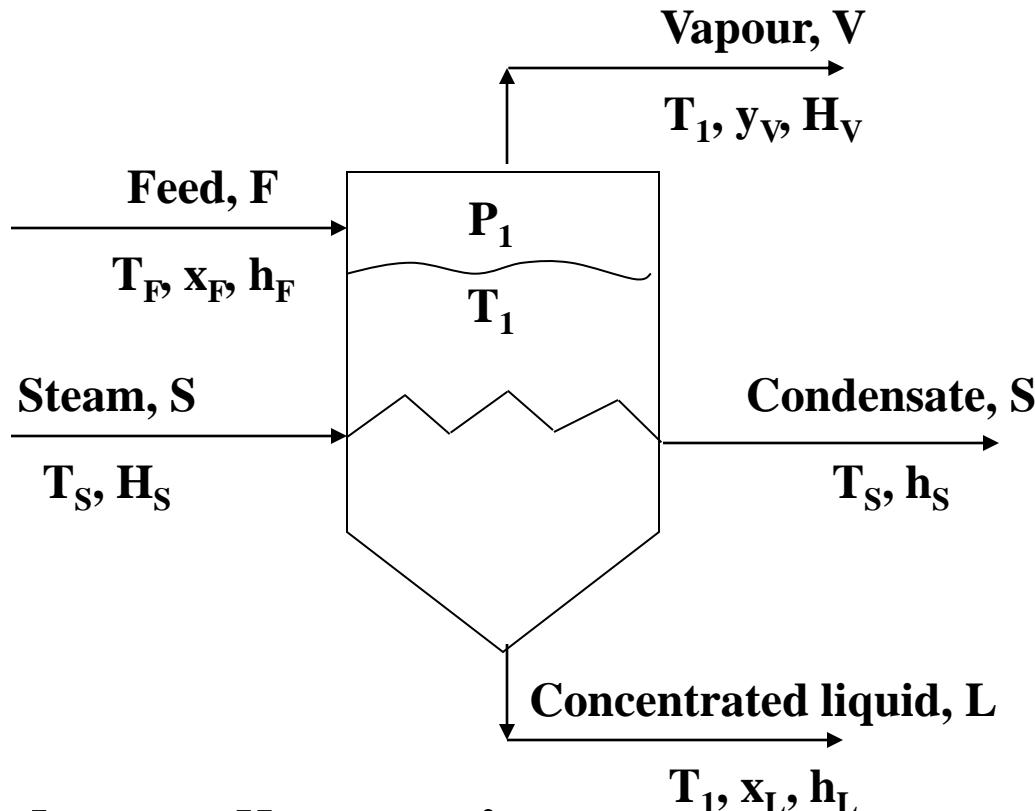


(a) Falling-film evaporator



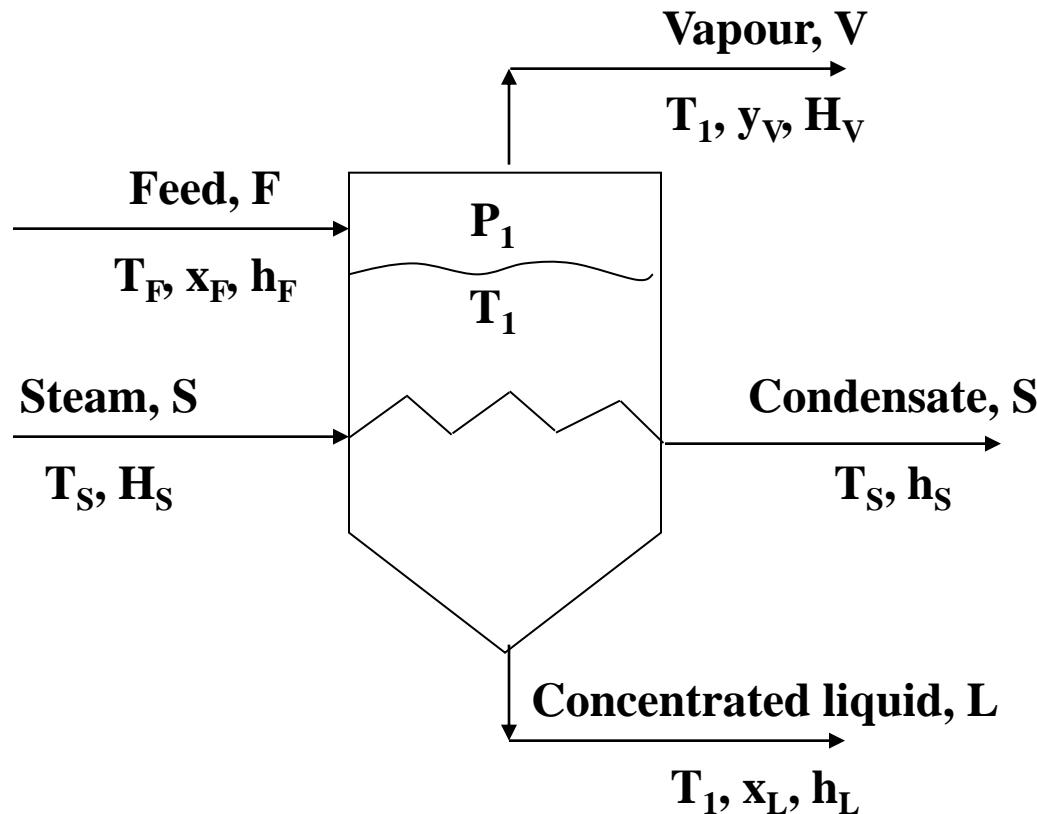
(b) Climbing-film evaporator

1. single-effect evaporator



- relatively small capacity
- cost of steam relatively cheap
- wasteful as latent heat of vapour is discarded
- $T_F = T_{bp}$, dilute solution, 1 kg steam = 1 kg vapour

Calculation methods for single-effect evaporator (dilute)

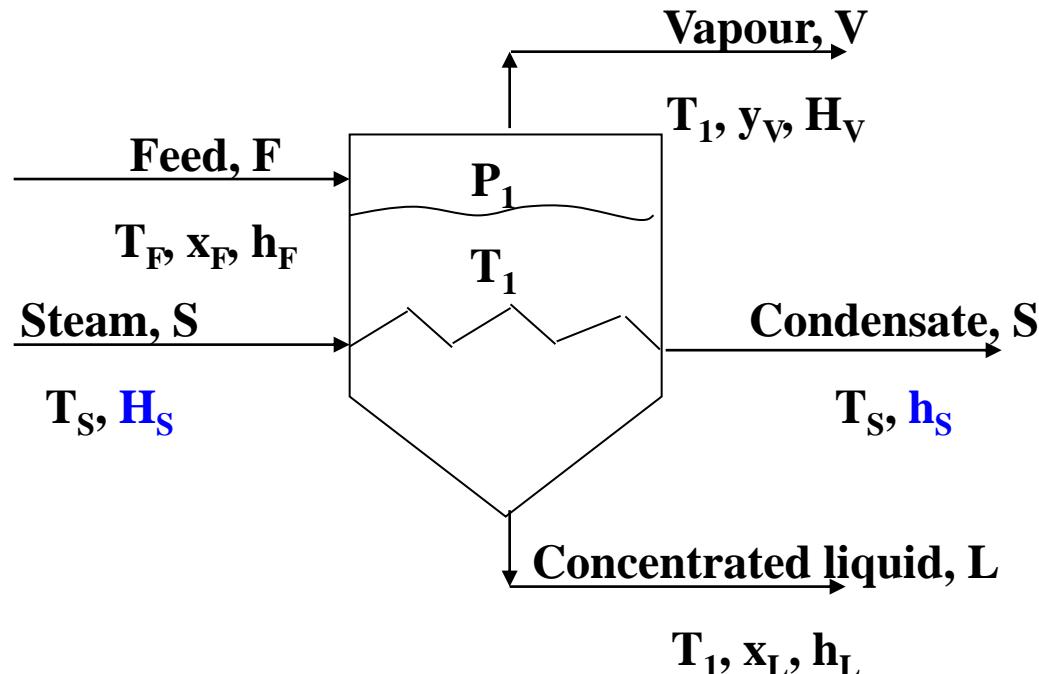


1. Vapour (V) & concentrated liquid (L) flowrates

Total material balance : $F = L + V$

Solute/solid balance: $F(x_F) = L(x_L)$ ($y_v = 0$ as vap has no salt)

Calculation methods for single-effect evaporator (dilute)



2. Heat-transfer surface area (A) & no boiling-point rise (BPR) :

$$\text{Energy balance: } Fh_F + S(H_S - h_S) = Lh_L + VH_V$$

$$\text{Heat transfer equation: } q = S\lambda = UA\Delta T = UA(T_S - T_1)$$

where $S(H_S - h_S) = S\lambda$

$$h = c_p(T - T_{\text{ref.}})$$

$$H_V = \text{latent heat at } T_1 \quad (\text{T ref.} = T_1)$$

$$c_p \text{ of inorganic salt in water} \approx c_p \text{ of water} = 4.14 \text{ kJ/kg.K}$$

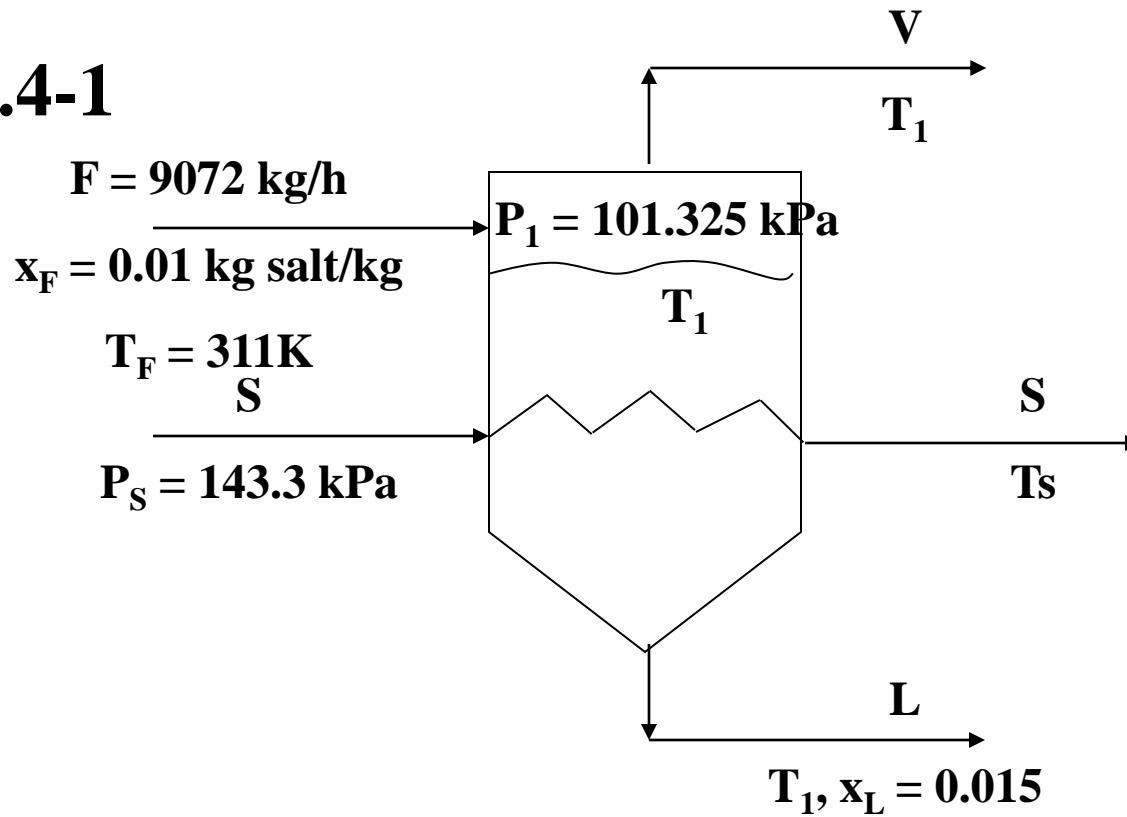
Example 8.4-1 (Geankoplis textbook)

A continuous single-effect evaporator concentrates 9072 kg/h of a 1.0 wt% salt solution entering at 311.0K (37.8°C) to a final concentration of 1.5 wt%. The vapor space of the evaporator is at 101.325 kPa (1.0 atm abs) and the steam supplied is saturated at 143.3 kPa. The overall coefficient $U = 1704 \text{ W/m}^2\cdot\text{K}$. Calculate

- i) the amounts of vapor and liquid product
- ii) the heat-transfer area required.

Assume that, since it is dilute, the solution has the same boiling point as water.

Example 8.4-1



Vapour (V) & concentrated liquid (L) flowrates

Total material balance : $9072 = F = L + V$ (1)

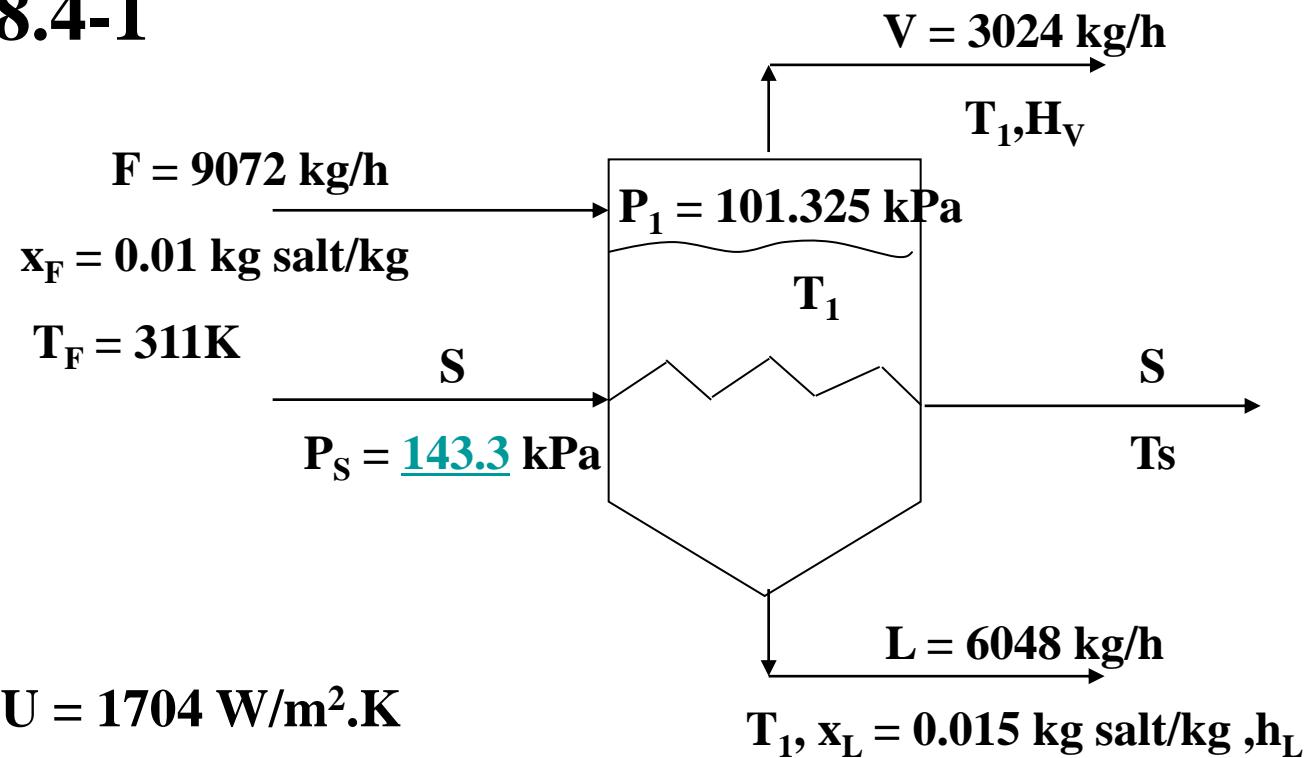
Salt balance: $F(x_F) = L(x_L)$

$$9072(0.01) = L(0.015)$$

$$L = \text{kg/h}$$

Substituting $L = \text{kg/h}$ into eq. (1), $V = \text{kg/h}$

Example 8.4-1



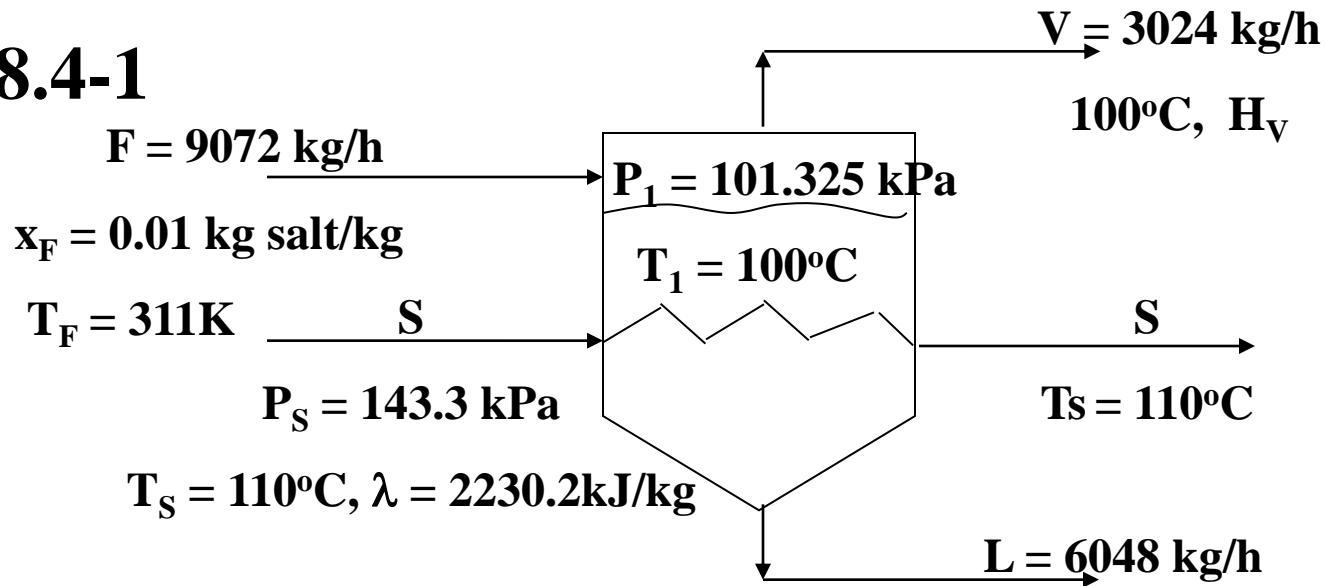
From the steam table (A.2-9) at $P_S = 143.3 \text{ kPa}$, $T_s = \quad {}^\circ\text{C}$,

$$H_s = \quad \text{kJ/kg}, h_s = \quad \text{kJ/kg}, \therefore \lambda = (\quad) = \quad \text{kJ/kg}$$

From the steam table (A.2-9) at $P_1 = 101.325 \text{ kPa}$, $T_1 = \quad {}^\circ\text{C}$,

$$H_{\text{sat. vapour}} = \quad \text{kJ/kg}, h_{\text{liquid}} = \quad \text{kJ/kg}$$

Example 8.4-1



From the steam table (A.2-9) at, $T_1 = 100^\circ\text{C}$, $H = 2676.1 \text{ kJ/kg}$, $h = 419.04 \text{ kJ/kg}$

$T_{\text{ref}} = 100^\circ\text{C}$ (same as T_1),

$$\text{Energy balance: } Fh_F + S(H_S - h_S) = Lh_L + VH_V$$

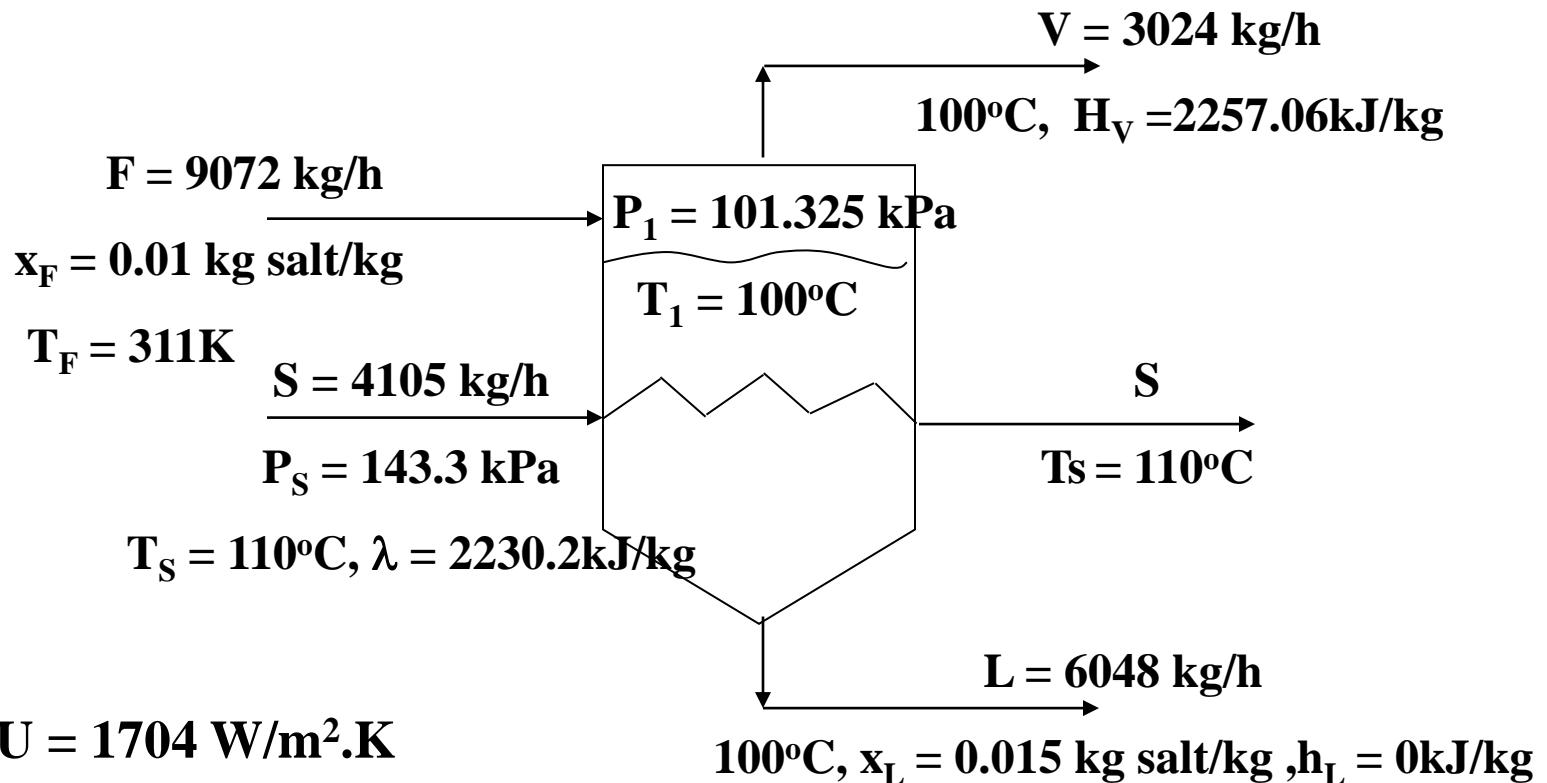
$$9072h_F + S(2230.2) = 6048(h_L) + 3024(H_V)$$

where $h_F = c_{PF}(T_F - T_{\text{ref}}) = \quad = \quad \text{kJ/kg}$

$h_L = c_{PL}(T_1 - T_{\text{ref}}) = \quad = \quad \text{kJ/kg}$

$H_V = [(H_{\text{sat.vapour}}) - h_{\text{liquid}}] = (\quad) = \quad \text{kJ/kg}$

$$S = \quad \text{kg/h}$$



Heat transfer equation:

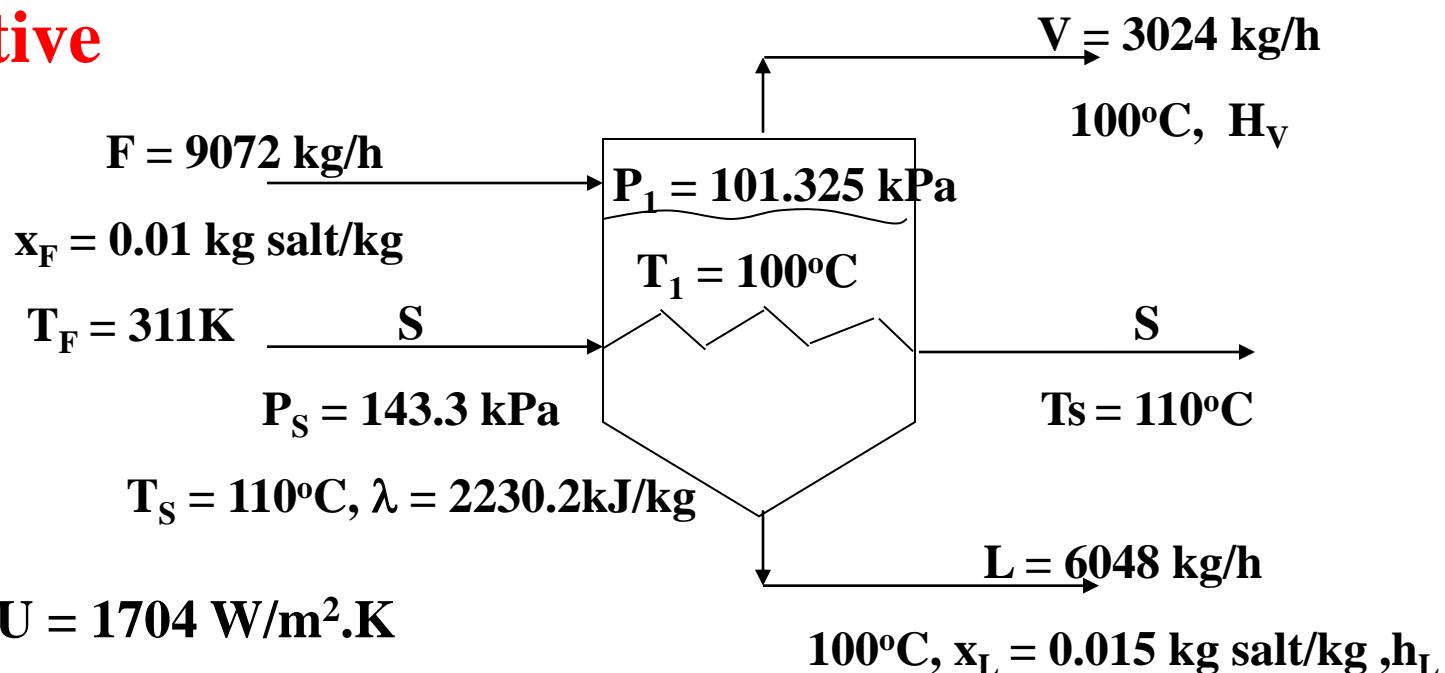
$$q = S\lambda = UA\Delta T = UA(T_S - T_1)$$

$$q = S\lambda = \frac{4105 \text{ kg}}{\cancel{\text{h}}} \left| \frac{2230.2 \text{ kJ}}{\cancel{\text{kg}}} \right| \frac{1\text{h}}{3600\text{s}} = 2543 \text{ kJ/s}$$

$$A = q/[U(T_S - T_1)]$$

$$A = \frac{2543.0 \times 10^3 \cancel{\text{W}}}{(110-100)\cancel{\text{K}}} \left| \frac{\text{m}^2.\text{K}}{1704 \cancel{\text{W}}} \right| = 149.3 \text{ m}^2$$

Alternative method



From the steam table (A.2-9) at, $T_1 = 100^\circ\text{C}$, $H = 2676.1 \text{ kJ/kg}$, $h = 419.04 \text{ kJ/kg}$

$T_{\text{ref}} = 0^\circ\text{C}$ (same as steam table),

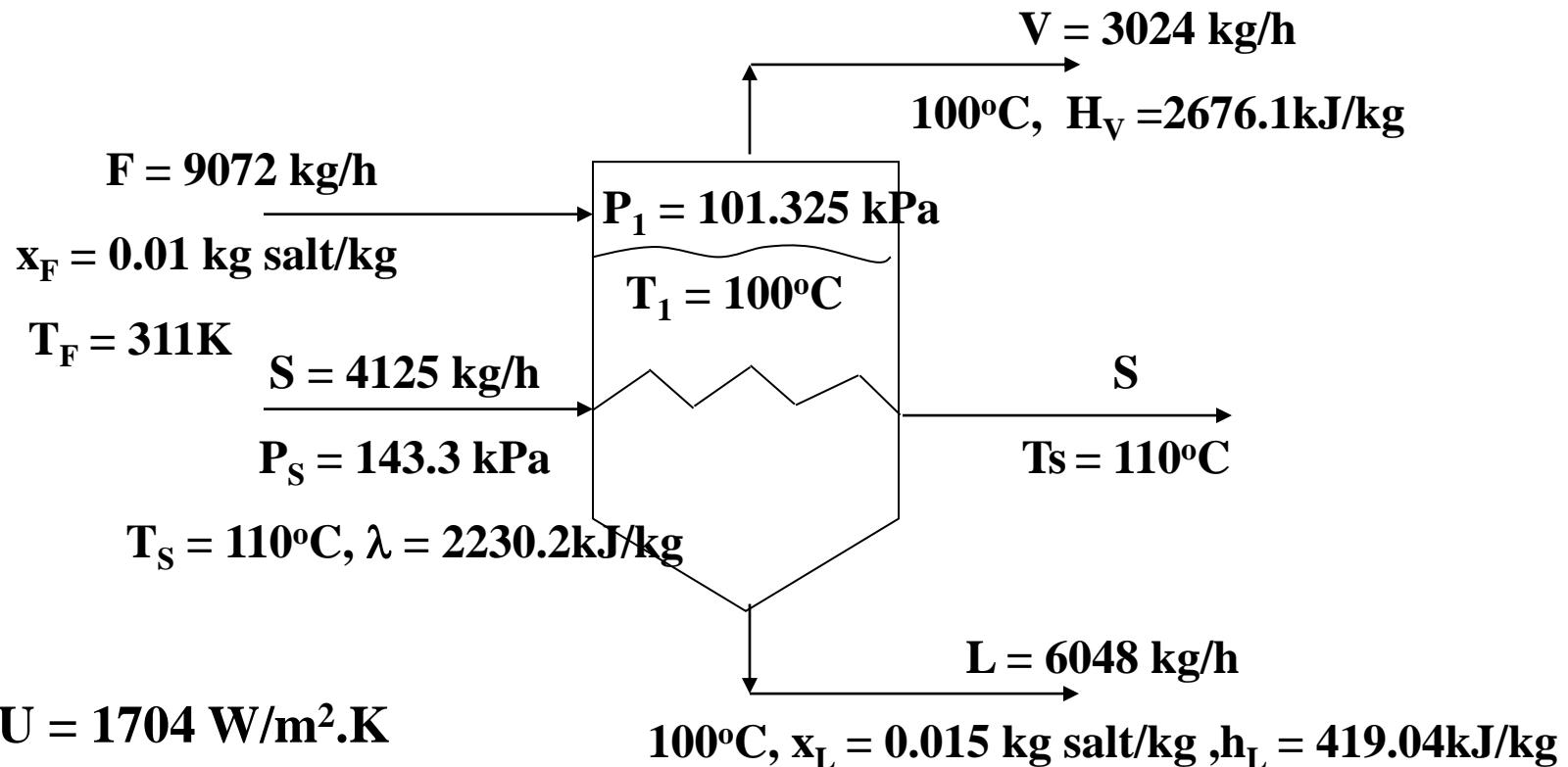
$$\text{Energy balance: } Fh_F + S(H_S - h_S) = Lh_L + VH_V$$

$$9072h_F + S(2230.2) = 6048(419.04) + 3024(2676.1)$$

where $h_F = 159.22 \text{ kJ/kg}$ (after interpolation) from steam table (A.2-9)

$$9072(159.22) + S(2230.2) = 6048(419.04) + 3024(2676.1)$$

$$S = 4125 \text{ kg/h} \text{ (compare to } 4108 \text{ kg/h})$$



Heat transfer equation:

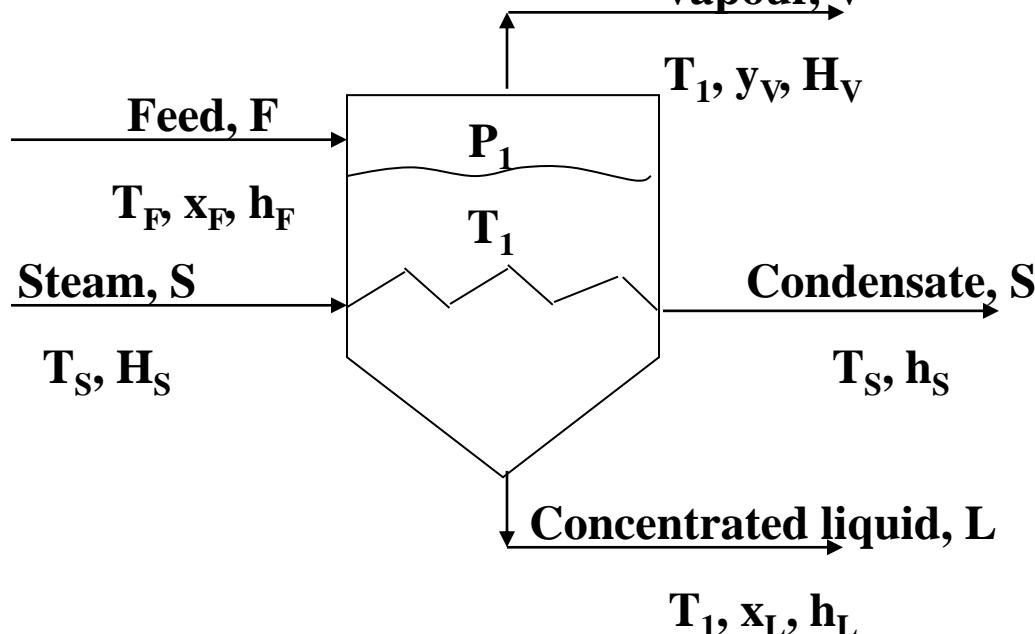
$$q = S\lambda = UA\Delta T = UA(T_s - T_1)$$

$$q = S\lambda = \frac{4125 \text{ kg}}{\cancel{h}} \left| \frac{2230.2 \text{ kJ}}{\cancel{kg}} \right| \frac{1\text{h}}{\cancel{3600s}} = 2555.4 \text{ kJ/s}$$

$$A = q/[U(T_s - T_1)]$$

$$A = \frac{2555.4 \times 10^3 \text{ W}}{(110-100) \cancel{\text{K}}} \left| \frac{\text{m}^2 \cdot \text{K}}{1704 \cancel{\text{W}}} \right| = 150 \text{ m}^2 \text{ (compare to } 149.3 \text{ m}^2)$$

Calculation methods for single-effect evaporator (concentrated)



2. Heat-transfer surface area (A) & with boiling-point rise (BPR) :

$$\text{Energy balance: } Fh_F + S(H_S - h_S) = Lh_L + VH_V$$

$$\text{Heat transfer equation: } q = S\lambda = UA\Delta T = UA(T_S - T_1)$$

where $T_1 = T_{\text{sat. at } P_1} + \text{BPR}$ or from fig. 8.4-2

h_F and h_L from fig. 8.4-3

$$H_V = H_{\text{sat. at } P_1} + 1.884(\text{BPR})$$

Boiling-point rise (BPR) of solution

Boiling-point elevation describes the phenomenon that the boiling point of a liquid (a solvent) will be higher when another compound is added, meaning that a solution has a higher boiling point than a pure solvent. This happens whenever a non-volatile solute, such as a salt, is added to a pure solvent, such as water.

Therefore, the boiling-point elevation means that when a solute is dissolved in a solvent, the boiling point of the resulting solution is higher than that of the pure solvent.

Boiling-point rise (BPR) of solution

Concentrated solution – thermal properties differ from water

Duhring line chart – estimate boiling-point rise for a solution

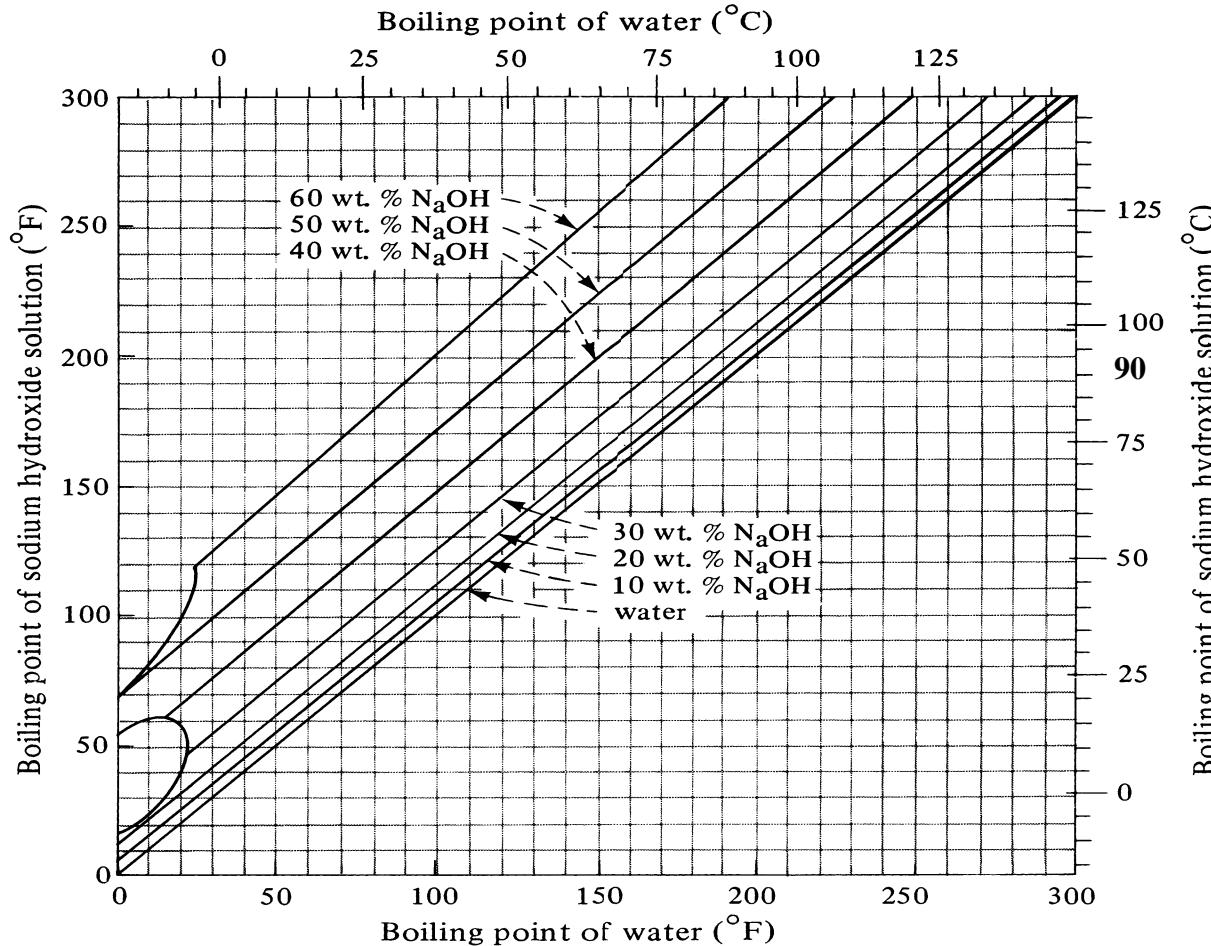


Figure 8.4-2. Duhring line chart for aqueous solutions of sodium hydroxide

Example 8.4-2 Use of Duhring Chart for BPR

As an example of the use of the chart, the pressure in an evaporator is given as 25.6 kPa (3.72 psia) and a solution of 30% NaOH is being boiled. Determine the boiling temperature of NaOH solution and the boiling point rise, BPR of the solution over that of water at the same pressure.

Steam table, bpt of water at 25.6 kPa = 65.5°C (150°F)

From Duhring chart, at 150°F and 30% NaOH, bpt of NaOH solution is 79.5°C (175°F)

$$\text{BPR} = (79.5 - 65.6)^\circ\text{C} = 13.9^\circ\text{C}$$

Boiling-point rise (BPR) of solution

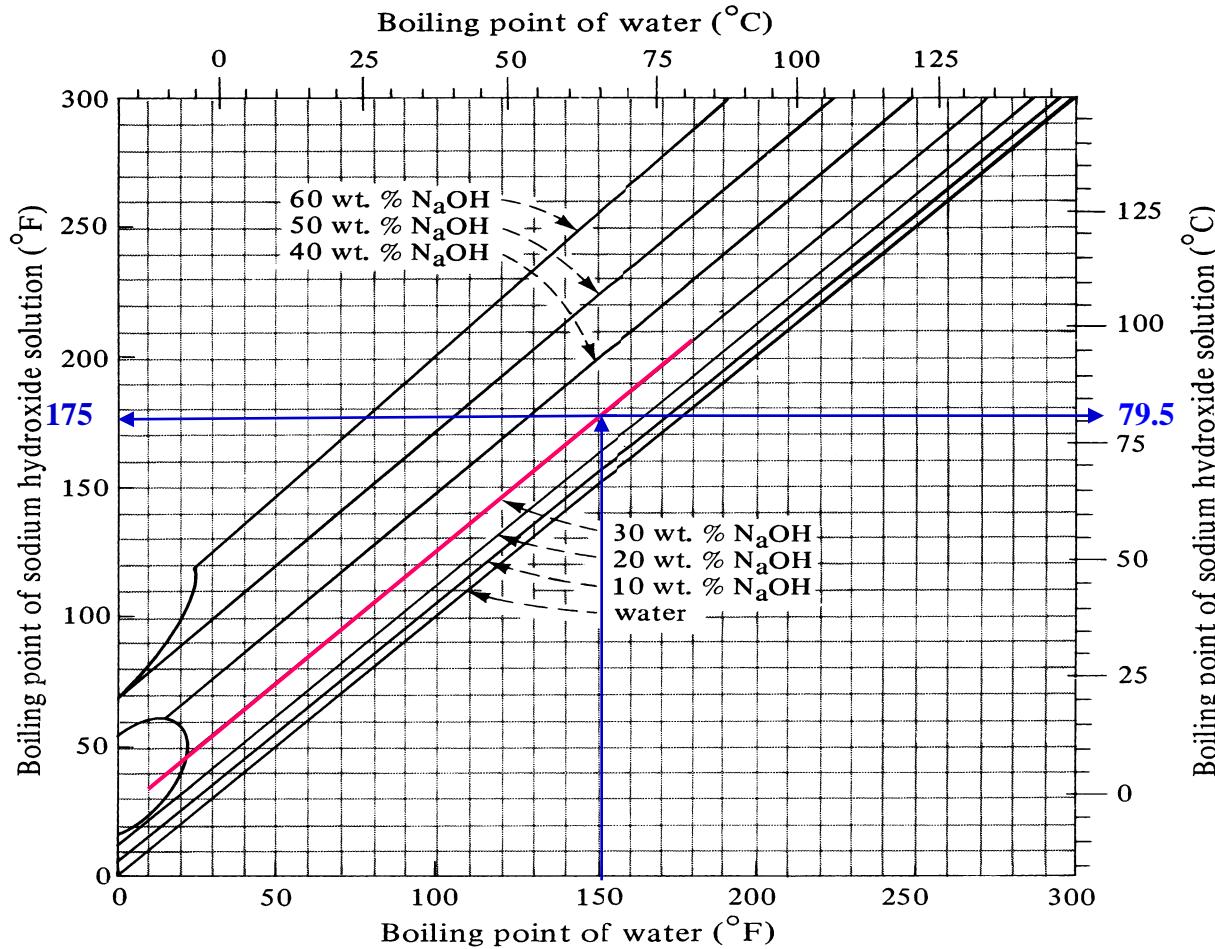


Figure 8.4-2. Duhring line chart for aqueous solutions of sodium hydroxide

Enthapy-concentration chart

enthalpy chart – $T_{ref.}$ = liquid water at 0°C

- used together with steam tables

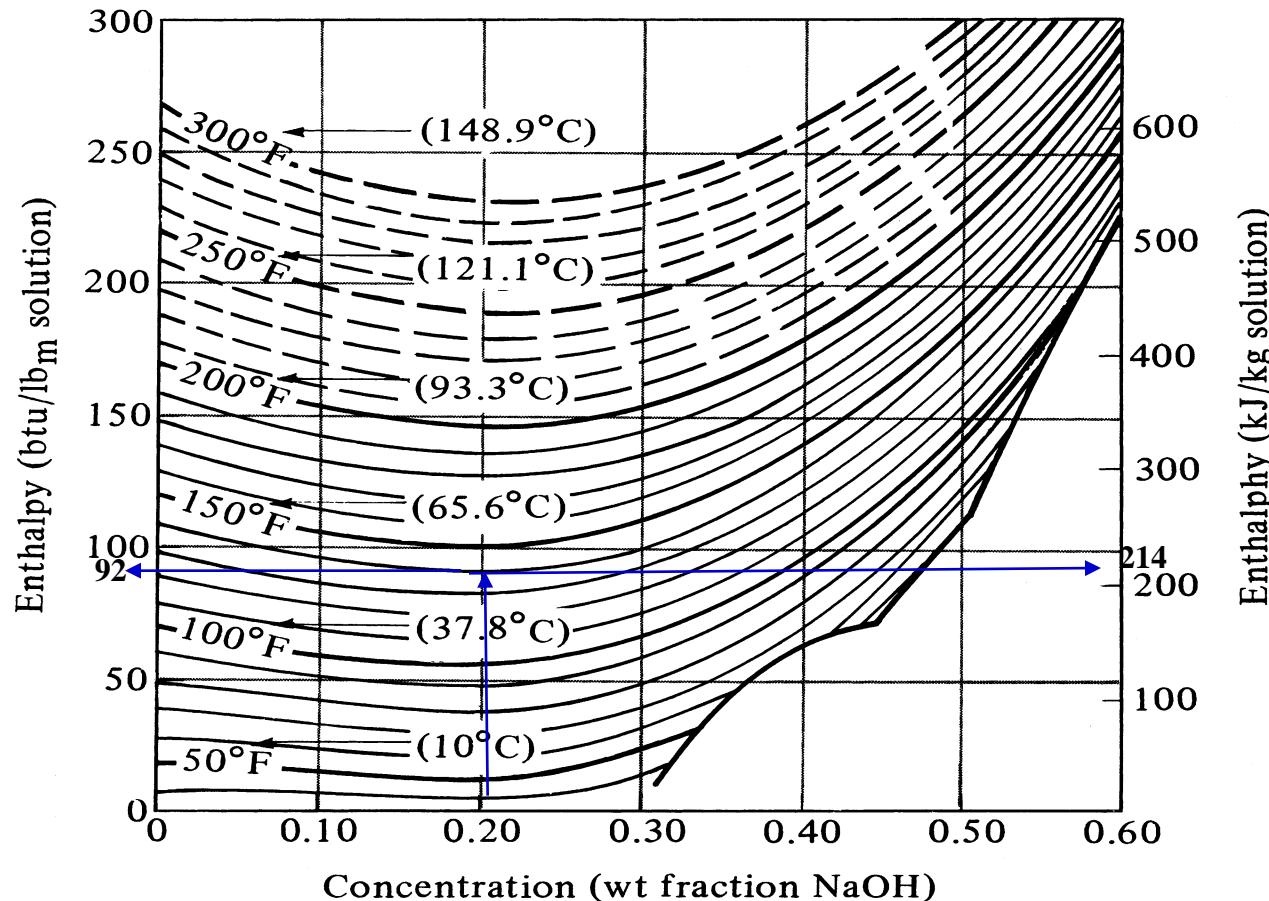


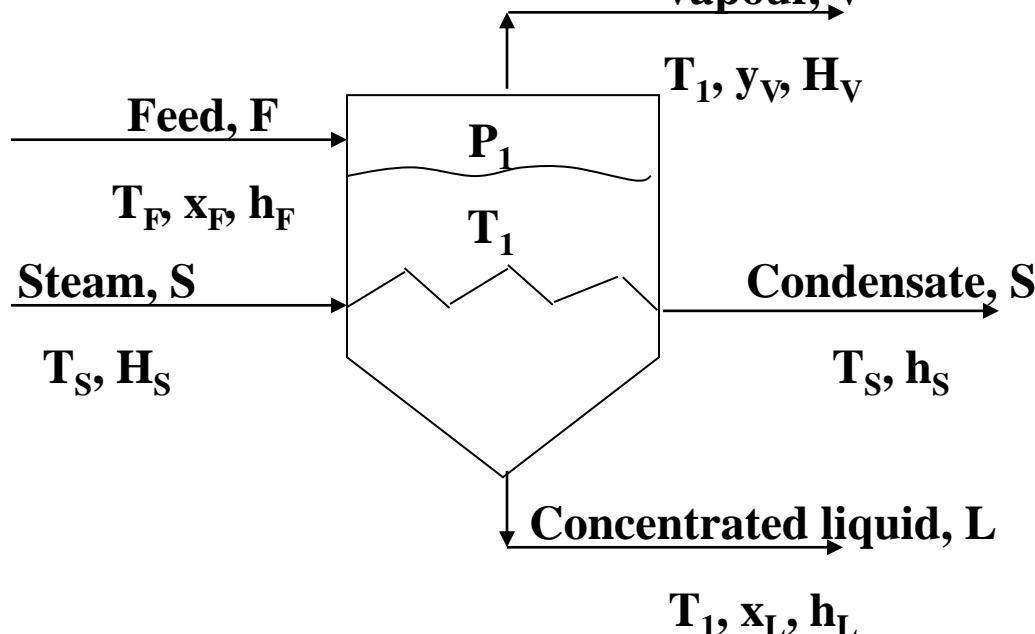
Figure 8.4-3. Enthalpy-concentration chart for the system NaOH-water

Example 8.4-3

An evaporator is used to concentrate 4536 kg/h (10,000 Ib_m/h) of a 30% solution of NaOH in water entering at 60°C (140°F) to a product of 40% solids. The pressure of the saturated steam used is 172.4 kPa (25 psia) and the pressure in the vapor space of the evaporator is 11.7 kPa (1.7 psia). The overall heat-transfer coefficient is 1560 W/m².K (275 btu/h.ft².°F).

Calculate the steam used, the steam economy in kg vaporized/kg steam used, and the heating surface area in m².

Calculation methods for single-effect evaporator (concentrated)



2. Heat-transfer surface area (A) & with boiling-point rise (BPR) :

$$\text{Energy balance: } Fh_F + S(H_S - h_S) = Lh_L + VH_V$$

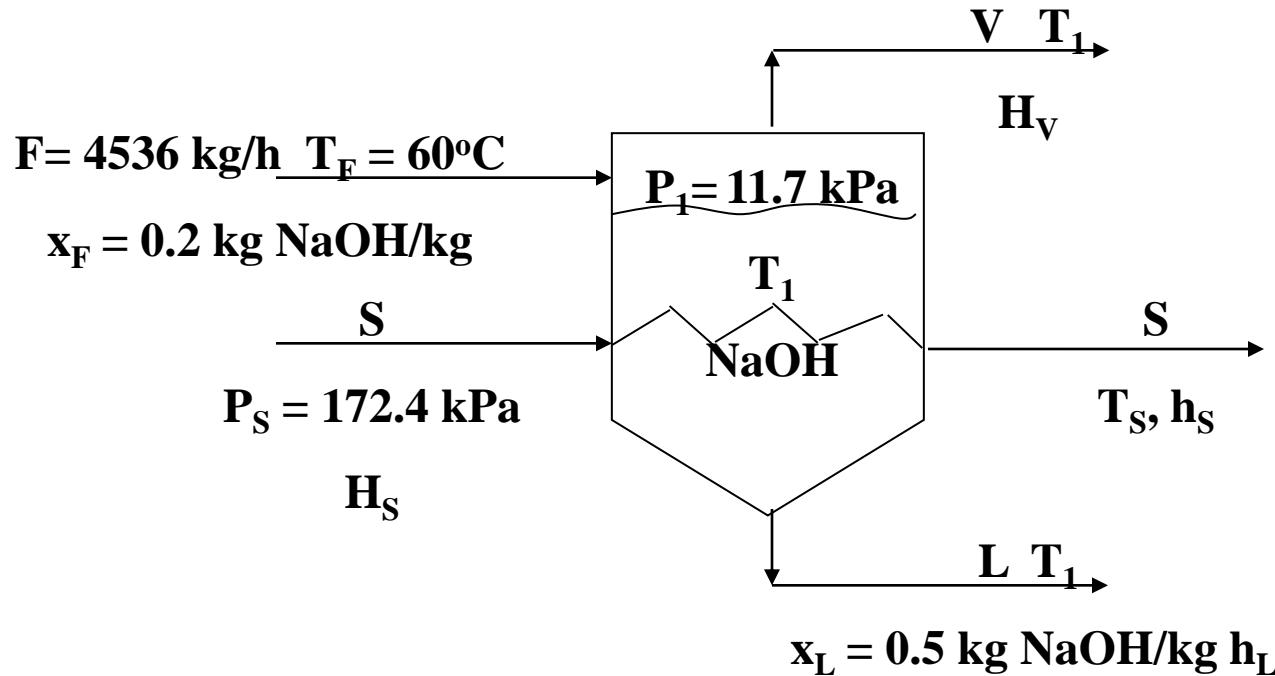
$$\text{Heat transfer equation: } q = S\lambda = UA\Delta T = UA(T_S - T_1)$$

where $T_1 = T_{\text{sat. at } P_1} + \text{BPR}$ or from fig. 8.4-2

h_F and h_L from fig. 8.4-3

$H_V = H_{\text{sat. at } P_1} + 1.884$ (BPR)

Example 8.4-3



Given : $U = 1560 \text{ W/m}^2.\text{K}$

Total material balance : $4536 = F = L + V$ (1)

Solute/solid balance: $F(x_F) = L(x_L)$

$$4536(0.2) = L(0.5)$$

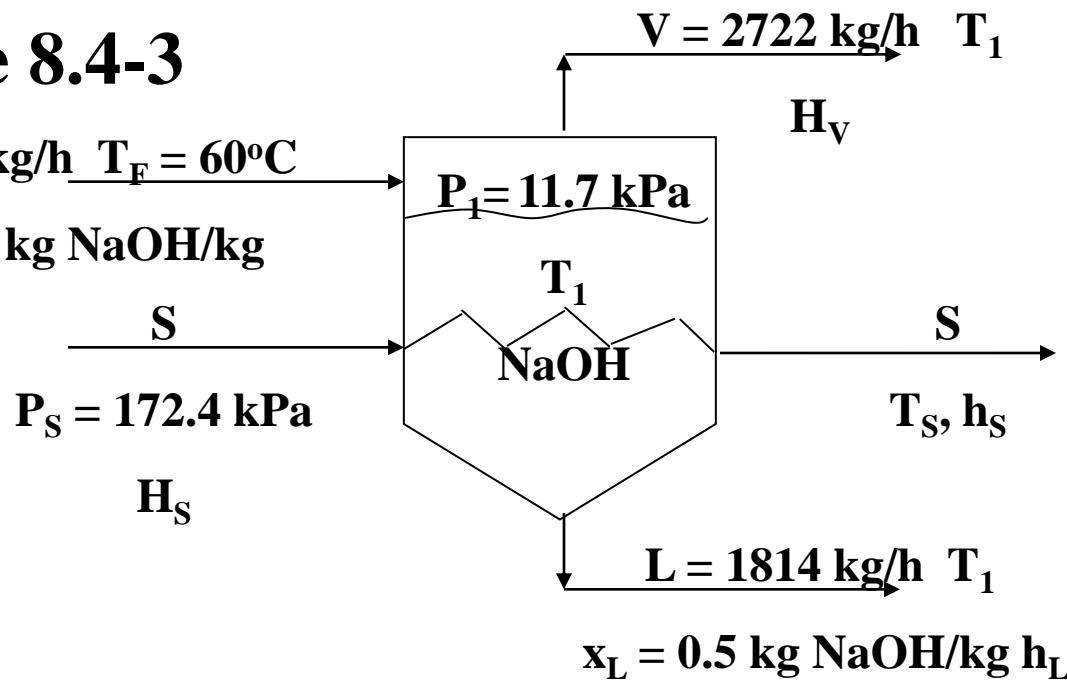
$$L = 1814 \text{ kg/h}$$

Substituting $L = 1814 \text{ kg/h}$ into eq. (1), $V = 2722 \text{ kg/h}$

Example 8.4-3

$$F = 4536 \text{ kg/h} \quad T_F = 60^\circ\text{C}$$

$$x_F = 0.2 \text{ kg NaOH/kg}$$



Given : $U = 1560 \text{ W/m}^2 \cdot \text{K}$

From the steam table (A.2-9) at $P_S = 172.4 \text{ kPa}$, after interpolation, $T_S = 115.6^\circ\text{C}$,

$$H_S = 2699.9 \text{ kJ/kg}, h_S = 485.03 \text{ kJ/kg}, \therefore \lambda = (2699.9 - 485.03) = 2214.87 \text{ kJ/kg}$$

From the steam table (A.2-9) at $P_1 = 11.7 \text{ kPa}$, after interpolation,

$$T_{\text{sat.}} = T_{\text{boiling point of water}} = 48.9^\circ\text{C}, H_{\text{sat. vapour}} = 2590 \text{ kJ/kg}$$

Example 8.4-3

$$F = 4536 \text{ kg/h} \quad T_F = 60^\circ\text{C}$$

$$x_F = 0.2 \text{ kg NaOH/kg}$$

$$\begin{aligned} S \\ P_S = 172.4 \text{ kPa} \\ T_S = 115.6^\circ\text{C} \end{aligned}$$

$$\lambda = 2214.87 \text{ kJ/kg}$$

$$V = 2722 \text{ kg/h} \quad T_1$$

$$H_V$$

$$P_1 = 11.7 \text{ kPa}$$

$$T_{\text{sat}} = 48.9^\circ\text{C}$$

$$T_1$$

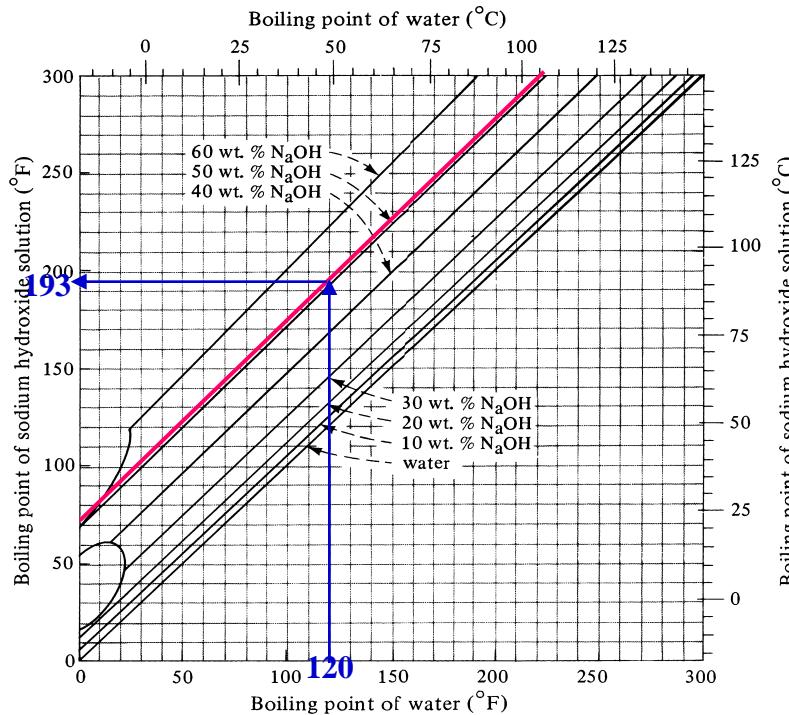
$$\text{NaOH}$$

$$\begin{aligned} S \\ T_S, h_S \end{aligned}$$

$$L = 1814 \text{ kg/h} \quad T_1$$

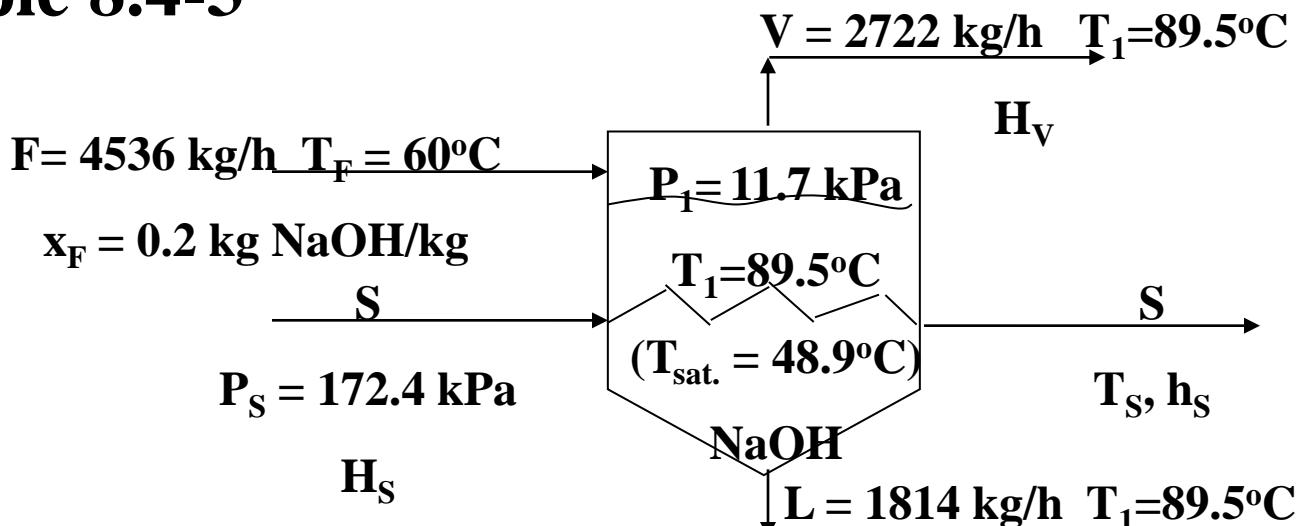
$$x_L = 0.5 \text{ kg NaOH/kg} \quad h_L$$

T_1 is obtained from Figure 8.4-2, at 50% NaOH and $T_{\text{bp water}} = 48.9^\circ\text{C}(120^\circ\text{F})$



$$\therefore T_1 = 193^\circ\text{F} (89.5^\circ\text{C})$$

Example 8.4-3



Given : $U = 1560 \text{ W/m}^2.\text{K}$

$$\text{BPR} = T_1 - T_{\text{sat.}} = (89.5 - 48.9)^\circ\text{C} = 40.6^\circ\text{C}$$

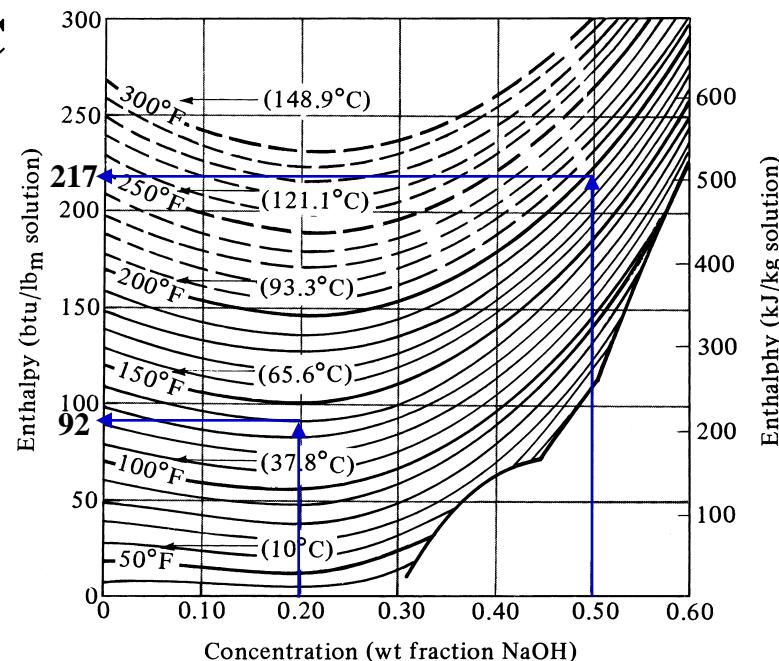
$T_{\text{ref.}} = 0^\circ\text{C}$ (same figure 8.4-3),

h_F at 60°C (140°F) & 20% NaOH from figure 8.4-3,

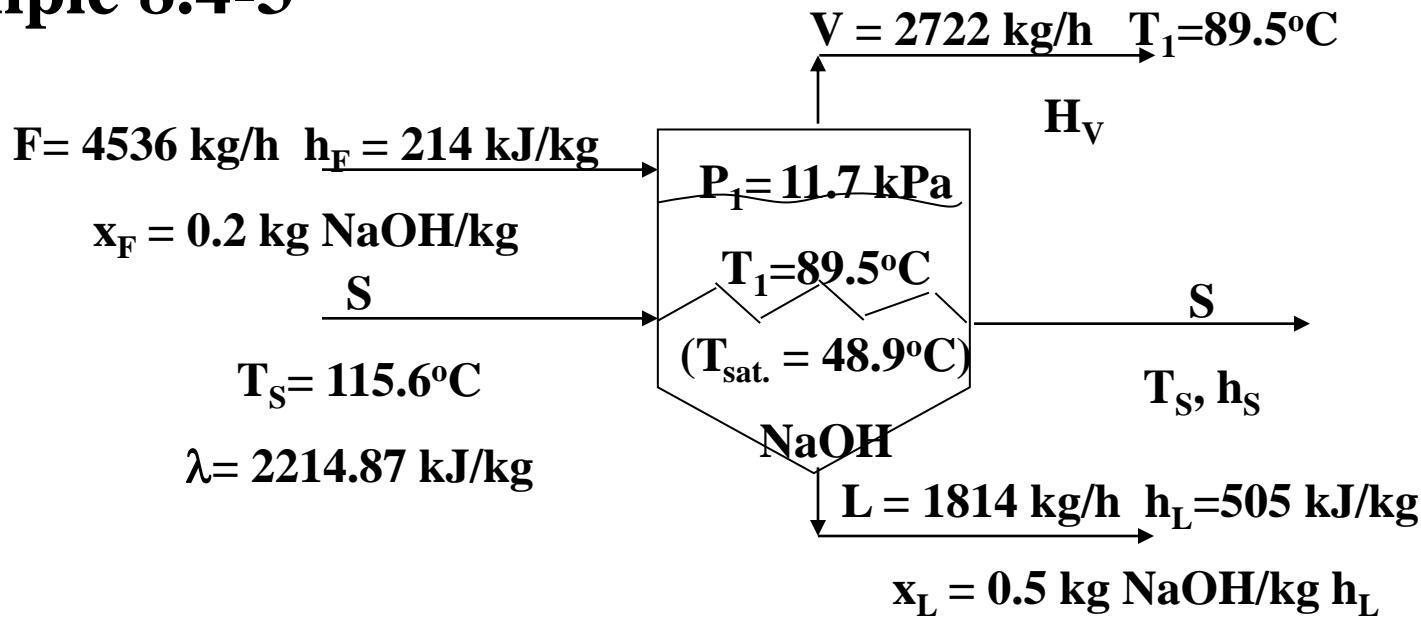
$$\therefore h_F = 92 \text{ btu/lb}_m \text{ (} 214 \text{ kJ/kg} \text{)}$$

h_L at 89.5°C (193°F) & 50% NaOH from figure 8.4-3,

$$\therefore h_L = 217 \text{ btu/lb}_m \text{ (} 505 \text{ kJ/kg} \text{)}$$



Example 8.4-3



Given : $U = 1560 \text{ W/m}^2 \cdot \text{K}$

$$\text{BPR} = T_1 - T_{\text{sat.}} = (89.5 - 48.9)^\circ\text{C} = 40.6^\circ\text{C}$$

$$T_{\text{sat.}} = 48.9^\circ\text{C}, H_{\text{sat. vapour}} = 2590 \text{ kJ/kg}$$

Energy balance: $Fh_F + S(H_S - h_S) = Lh_L + VH_V$

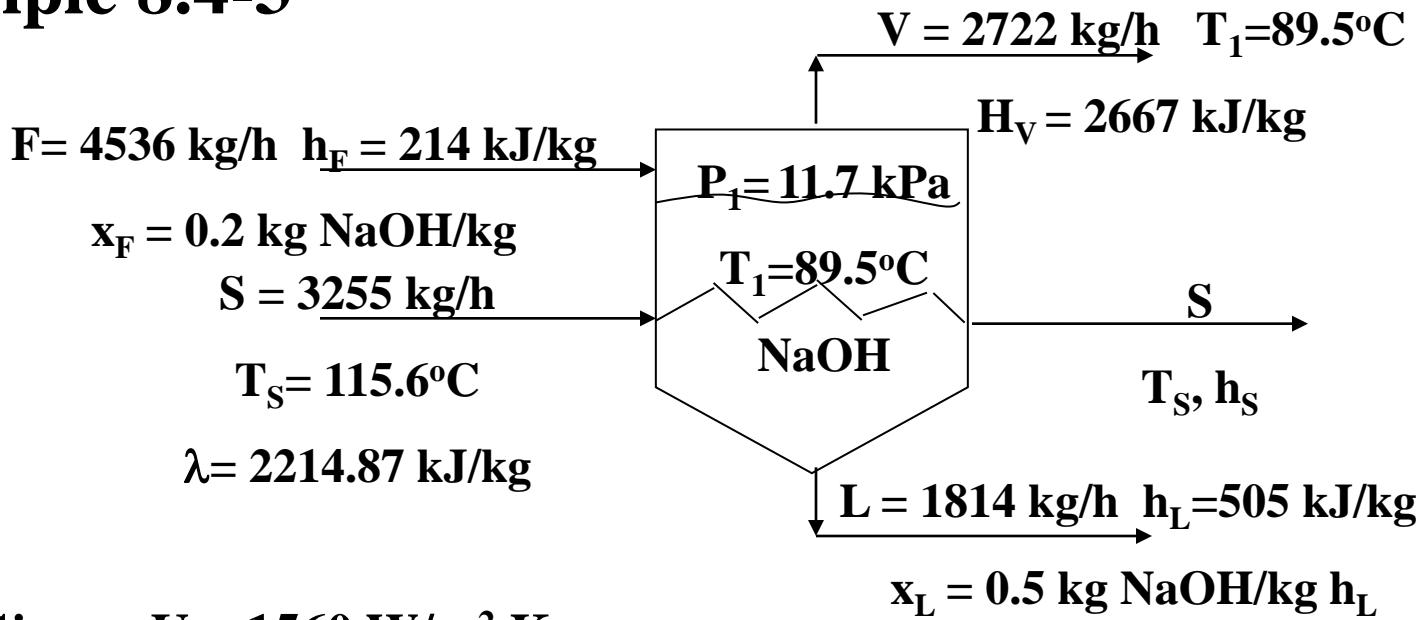
$$4536(214) + \textcolor{blue}{S}(2214.87) = 1814(505) + 2722\textcolor{blue}{H}_V$$

$$H_V = 2590 + 1.884(40.6) = 2667 \text{ kJ/kg}$$

$$4536(214) + \textcolor{blue}{S}(2214.87) = 1814(505) + 2722(2667)$$

$$\textcolor{blue}{S} = 3255 \text{ kg/h}$$

Example 8.4-3



Given : $U = 1560 \text{ W/m}^2 \cdot \text{K}$

Heat transfer equation:

$$q = S\lambda = UA\Delta T = UA(T_S - T_1)$$

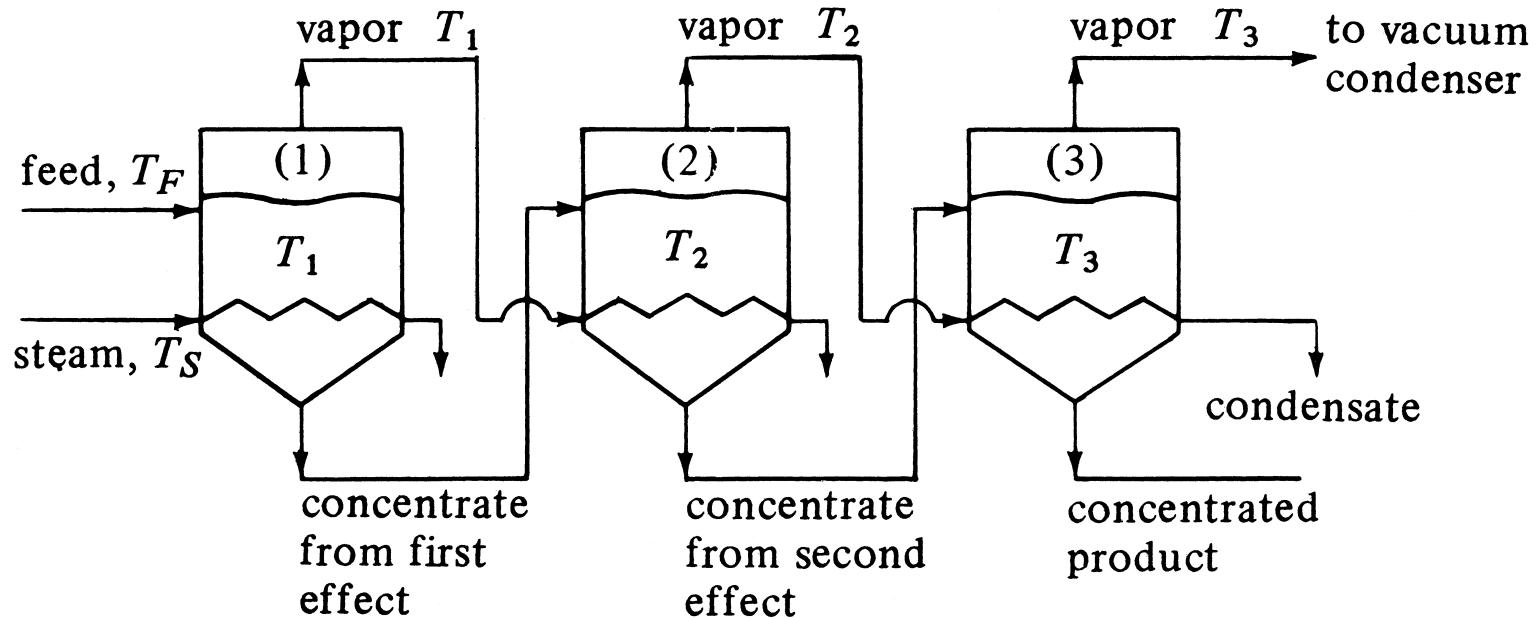
$$q = S\lambda = \frac{3255 \cancel{\text{kg}}}{\cancel{\text{h}}} \left| \frac{2214.87 \text{ kJ}}{\cancel{\text{kg}}} \right| \frac{1\cancel{\text{h}}}{3600\text{s}} = 2002 \text{ kJ/s}$$

$$A = \frac{2002 \times 10^3 \cancel{\text{W}}}{(115.6 - 89.5) \cancel{\text{K}}} \left| \frac{\text{m}^2 \cdot \text{K}}{1560 \cancel{\text{W}}} \right| = 49.2 \text{ m}^2$$

Steam economy = $V/S = 2722/3255 = 0.836 \rightarrow$

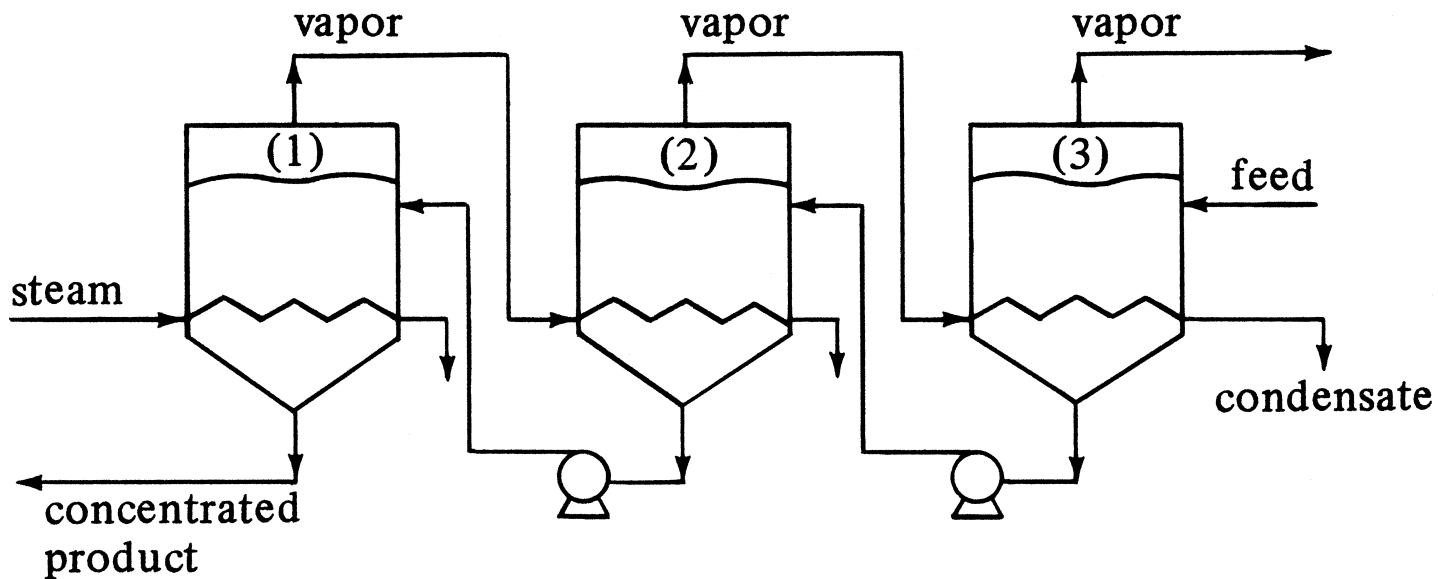
<1: not economical
>1: more economical

1. forward-feed multiple-effect evaporator



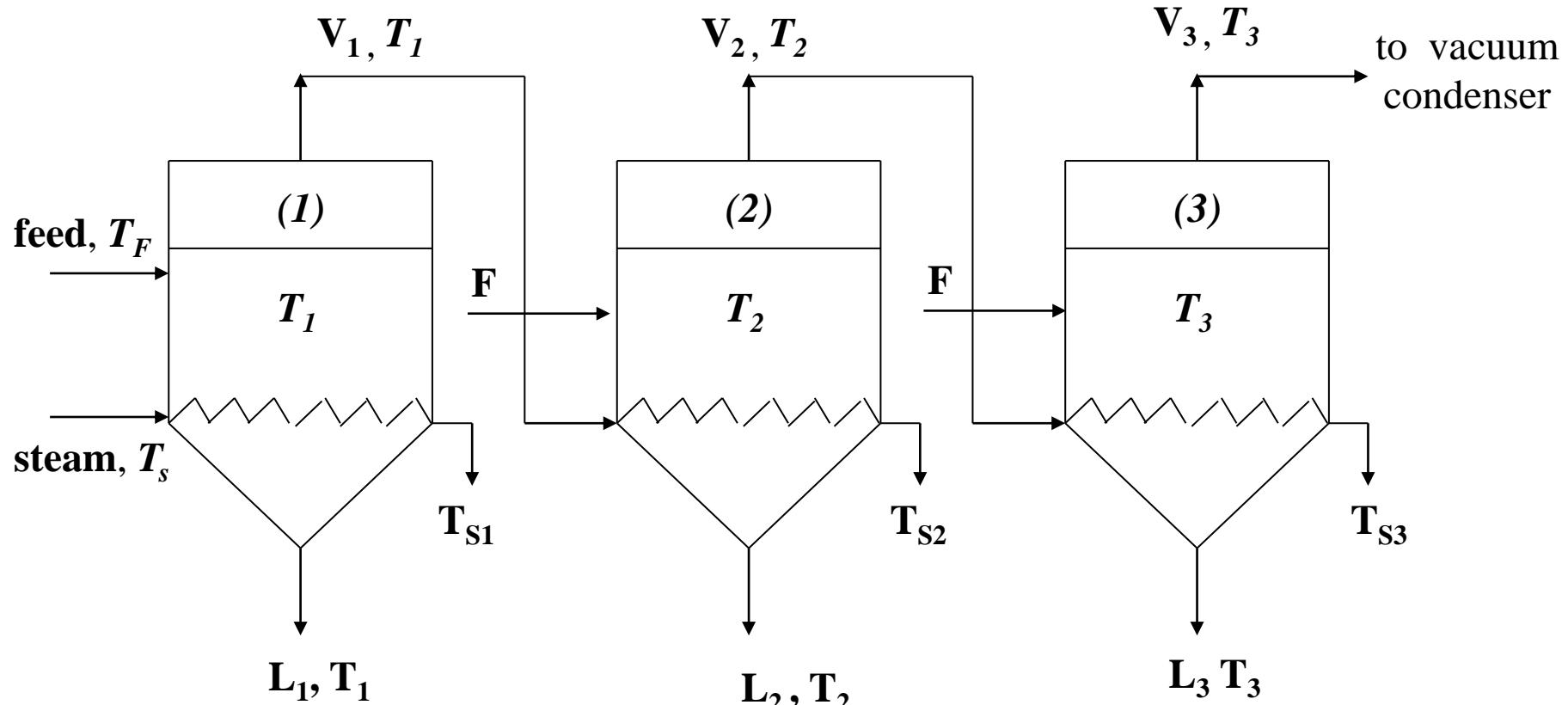
- fresh feed to 1st stage/effect
- conc. prod. from 1st effect to 2nd effect
- vapour from 1st effect as heating medium for 2nd effect
- co-current flow
- $T_F = T_{bp}$, dilute solution, 1 kg steam = 3 kg vapour
- feed is hot
- heat-sensitive final product

2. backward-feed multiple-effect evaporator



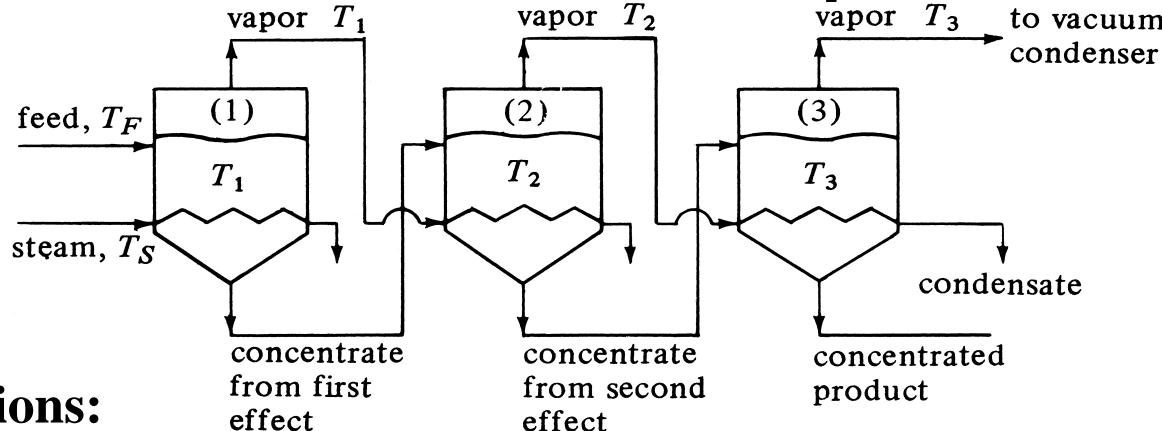
- fresh feed enter last stage/effect
- conc. prod. from 3st effect to 2nd effect
- vapour from 1st effect as heating medium for 2nd effect
- counter-current flow
- liquid pumps in each effect ($P_1 > P_2 > P_3$)
- cold feed as less liquid needs to be heated
- highly viscous conc. prod. as higher temp. reduce viscosity

3. parallel-feed multiple-effect evaporator



- **fresh feed and withdrawal conc. prod. at each effect**
- **vapour from 1st effect as heating medium for 2nd effect**
- **almost saturated feed**
- **solid crystals as product**

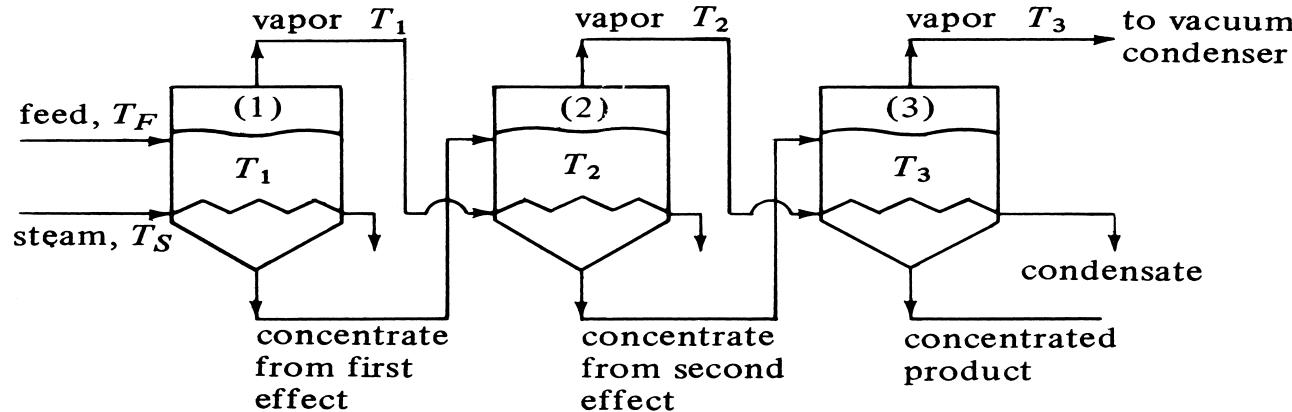
Triple-effect forward feed evaporators



Assumptions:

- latent heat of condensing steam = latent heat of vapour
- areas in all effects are equal ($A_1=A_2=A_3$)
- $q_1 = U_1 A_1 \Delta T_1$
- $q_1 = S \lambda_{S1}$
- $\Delta T_1 = T_{S1} - T_1$
- $\Delta T_1 = \sum \Delta T \frac{1/U_1}{1/U_1 + 1/U_2 + 1/U_3}$
- $\Delta T_2 = T_{S2} - T_2$
- No BPR: $\Delta T_2 = T_1 - T_2$
- No BPR: $\sum \Delta T = T_{S1} - T_{S4}$
- BPR: $\Delta T_2 = T_1 - BPR_1 - T_2$
- BPR: $\sum \Delta T = T_{S1} - T_{S4} - (BPR_1 + BPR_2 + BPR_3)$

Procedure for Triple-effect forward feed evaporators



Determine:

1. T_3 & BPR₃
2. L_3 & $V_1+V_2+V_3$. Assume $V_1=V_2=V_3$
3. L_1, L_2, x_1 & x_2
4. BPR₁ & BPR₂
5. $\sum \Delta T$, ΔT_1 , ΔT_2 & ΔT_3 . Readjust if necessary
6. T_1 , T_{S1} , T_2 , T_{S2} , T_3 & T_{S3}
7. c_p , enthalpies, L_1 , L_2 , V_1 , V_2 , V_3 & S
8. Compare ans. from step 7 with ans. from steps 2 & 3. If errors $> 10\%$, redo step 3.
9. If errors $< 10\%$, calculate q_1 , q_2 , q_3 , A_1 , A_2 , A_3 & A_{ave} . If errors $> 10\%$, revise ΔT_s .

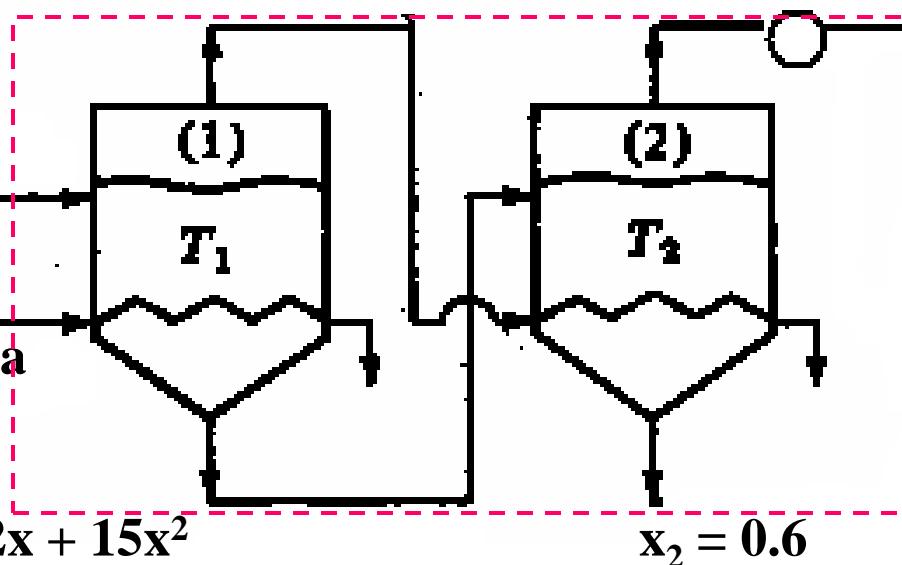
Example

$$F = 22680 \text{ kg/h}$$

$$x_F = 0.12$$

$$T_F = 93^\circ\text{C}$$

$$P_{s1} = 170 \text{ kPa}$$



$$\text{Given: BPR}^\circ\text{C} = 0.63 - 3.62x + 15x^2$$

$$c_p \text{ kJ/kg.K} = 4.19 - 2.93x$$

$$U_1 = 2271 \text{ W/m}^2.\text{K}, U_2 = 1420 \text{ W/m}^2.\text{K}$$

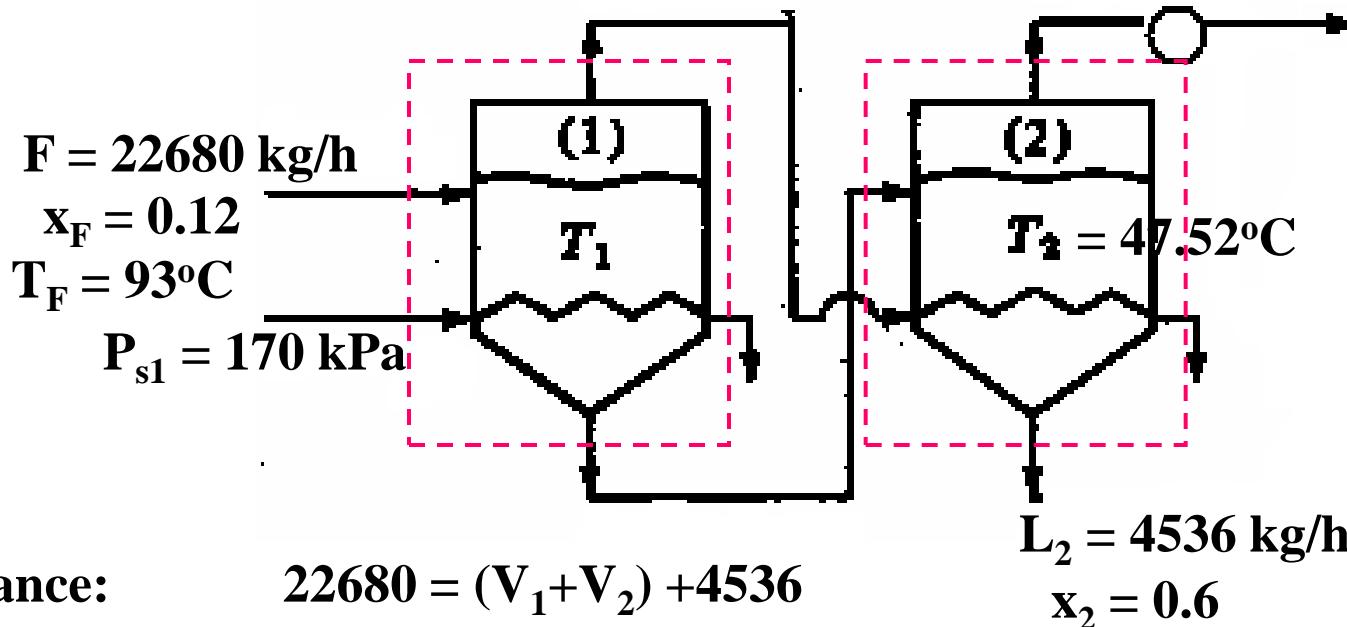
At 9 kPa, $T_{\text{sat}3.} = 43.66^\circ\text{C}$ (from A.2-9 after interpolation)

$$\text{BPR}_2 = 0.63 - 3.62(0.6) + 15(0.6)^2 = 3.858^\circ\text{C}$$

$$\therefore T_2 = T_{\text{sat}3.} + \text{BPR}_2 = 43.66 + 3.858 = 47.52^\circ\text{C}$$

$$\text{Total balance: } F = (V_1 + V_2) + L_2 = 22680 \text{ kg/h}$$

$$\text{Solid balance: } Fx_F = L_2 x_2 \quad 22680(0.12) = L_2(0.6) \quad \therefore L_2 = 4536 \text{ kg/h}$$



Total balance: $22680 = (V_1 + V_2) + 4536$

$$(V_1 + V_2) = 18\ 144 \text{ kg/h}$$

Assume $V_1 = V_2$ $V_1 = V_2 = 18\ 144 / 2 = 9072 \text{ kg/h}$

Effect 1: Total balance: $F = 22680 = V_1 + L_1$
 $22680 = 9072 + L_1$
 $L_1 = 13\ 608 \text{ kg/h}$

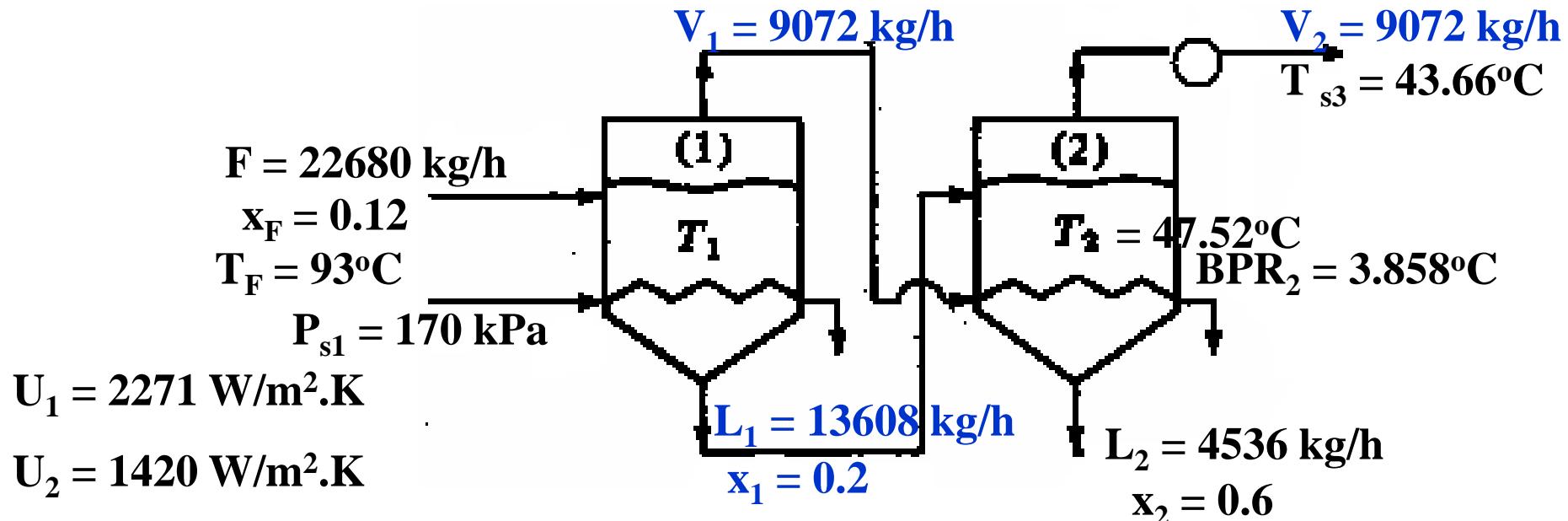
Solid balance: $Fx_F = L_1 x_1$

$$22680(0.12) = 13\ 608 x_1$$

$$x_1 = 0.2$$

Effect 2: Total balance: $L_1 = V_2 + L_2$ $13608 = 9072 + 4536$

✓



$$BPR = 0.63 - 3.62x + 15x^2$$

$$BPR_1 = 0.63 - 3.62(0.2) + 15(0.2)^2 = 0.506^\circ\text{C}$$

At 170 kPa, $T_s = 115^\circ\text{C}$

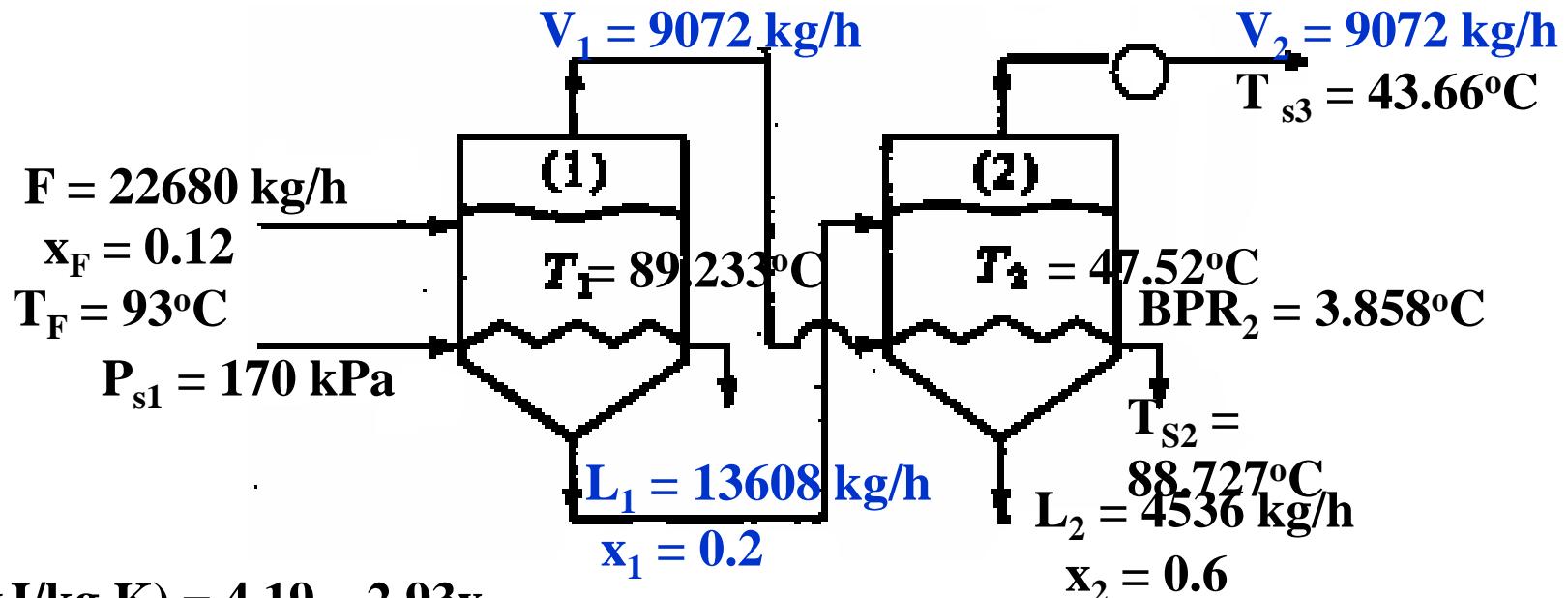
$$\sum \Delta T = T_{s1} - T_{s3} - (BPR_1 + BPR_2) = 115 - 43.66 - (3.858 + 0.506) = 66.976^\circ\text{C}$$

$$\Delta T_1 = \sum \Delta T \frac{1/U_1}{1/U_1 + 1/U_2} = 66.976 \frac{1/2271}{1/2271 + 1/1420} = 25.767^\circ\text{C}$$

$$\Delta T_1 = T_{s1} - T_1 \quad 25.767 = 115 - T_1 \quad \therefore T_1 = 89.233^\circ\text{C}$$

$$\Delta T_2 = \sum \Delta T - \Delta T_1 = 66.976 - 25.767 = 41.209^\circ\text{C}$$

$$\Delta T_2 = T_{s2} - T_2 \quad 41.209 = T_{s2} - 47.52 \quad \therefore T_{s2} = 88.727^\circ\text{C}$$



Given: $c_p \text{ (kJ/kg.K)} = 4.19 - 2.93x$

$$c_{pF} = 4.19 - 2.93x_F = 4.19 - 2.93(0.12) = 3.838 \text{ kJ/kg.K}$$

$$c_{p1} = 4.19 - 2.93x_1 = 4.19 - 2.93(0.2) = 3.604 \text{ kJ/kg.K}$$

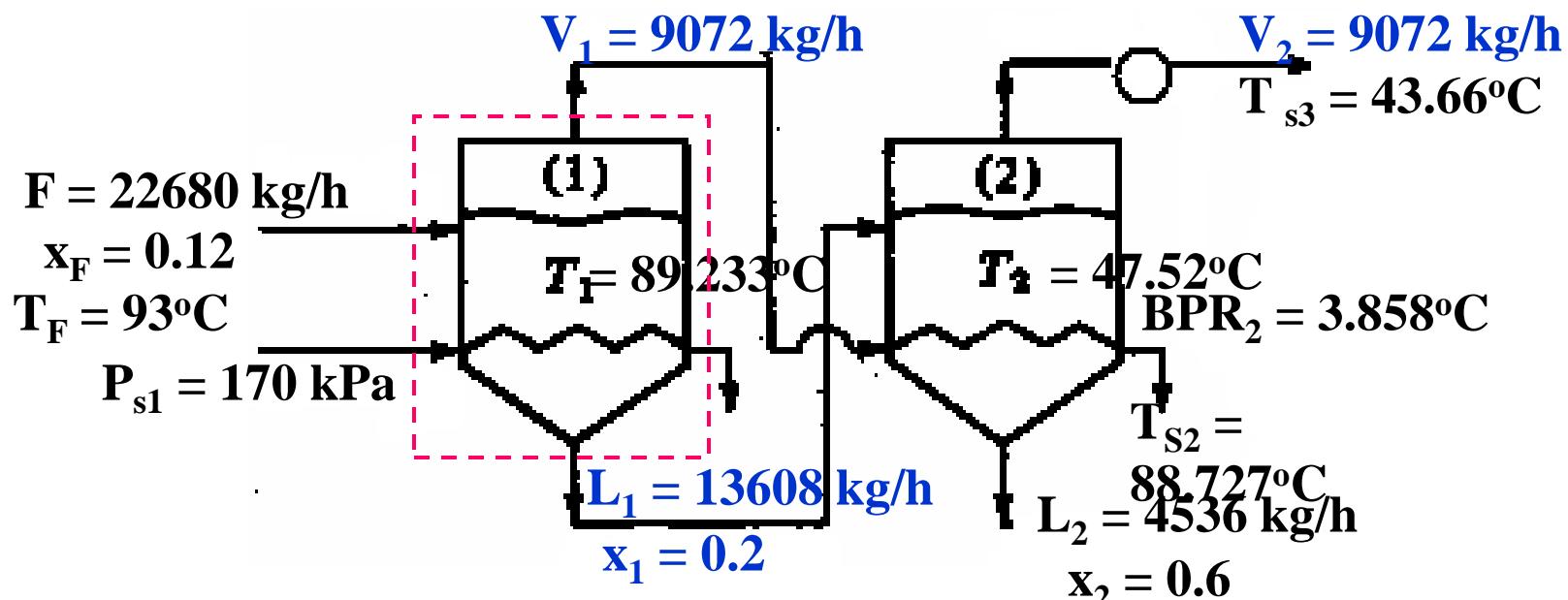
$$c_{p2} = 4.19 - 2.93x_2 = 4.19 - 2.93(0.6) = 2.432 \text{ kJ/kg.K}$$

$T_{ref.} = 0^\circ\text{C}$ (liquid solution),

$$h_F = c_{pF}(T_F - T_{ref.}) = 3.838(93 - 0) = 356.97 \text{ kJ/kg}$$

$$h_1 = c_{p1}(T_1 - T_{ref.}) = 3.604(89.233 - 0) = 321.6 \text{ kJ/kg}$$

$$h_2 = c_{p2}(T_2 - T_{ref.}) = 2.432(47.52 - 0) = 115.56 \text{ kJ/kg}$$



$$h_F = 356.97 \text{ kJ/kg} \quad h_1 = 321.6 \text{ kJ/kg} \quad h_2 = 115.56 \text{ kJ/kg}$$

Tref. = 0°C , liquid solution (same as steam table),

Effect 1: Energy balance: $Fh_F + S(H_{S1} - h_{S1}) = L_1h_1 + V_1H_1$

$$22680h_F + S\lambda_{S1} = L_1h_1 + V_1H_1$$

Material balance: $22680 = V_1 + L_1$

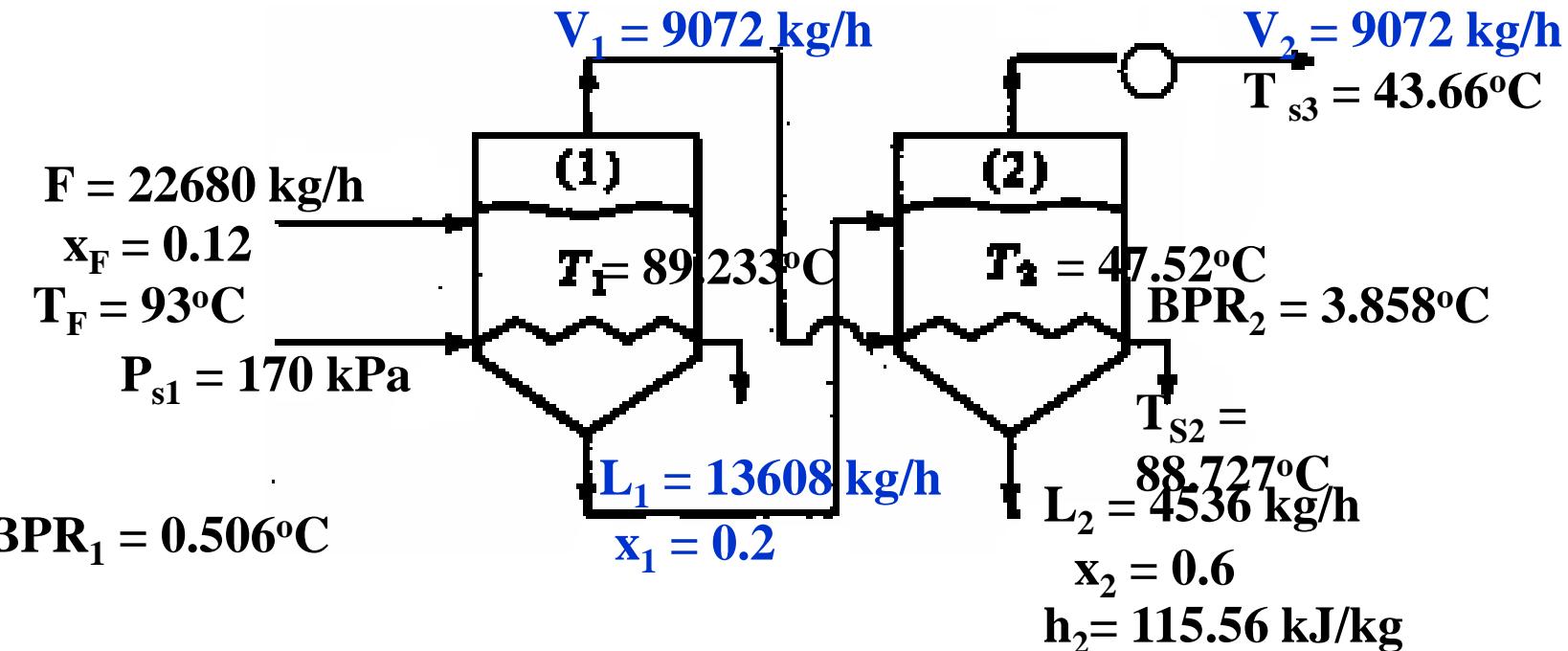
$$V_1 = 22680 - L_1$$

$$22680h_F + S\lambda_{S1} = L_1h_1 + (22680 - L_1)H_1$$

$$22680(356.97) + S\lambda_{S1} = L_1(321.6) + (22680 - L_1)H_1$$

$$T_{S1} = 115^\circ\text{C}, H_{S1} = 2699 \text{ kJ/kg} \text{ & } h_{S1} = 482.48 \text{ kJ/kg.}$$

$$\therefore \lambda_{S1} = (2699 - 482.48) = 2216.52 \text{ kJ/kg}$$



Tref. = 0°C (same as steam table),

$$\text{Effect 1: } 22680(356.97) + S(2216.52) = L_1(321.6) + (22680 - L_1)H_1$$

$$H_1 = H_{s2} + 1.884(BPR_1)$$

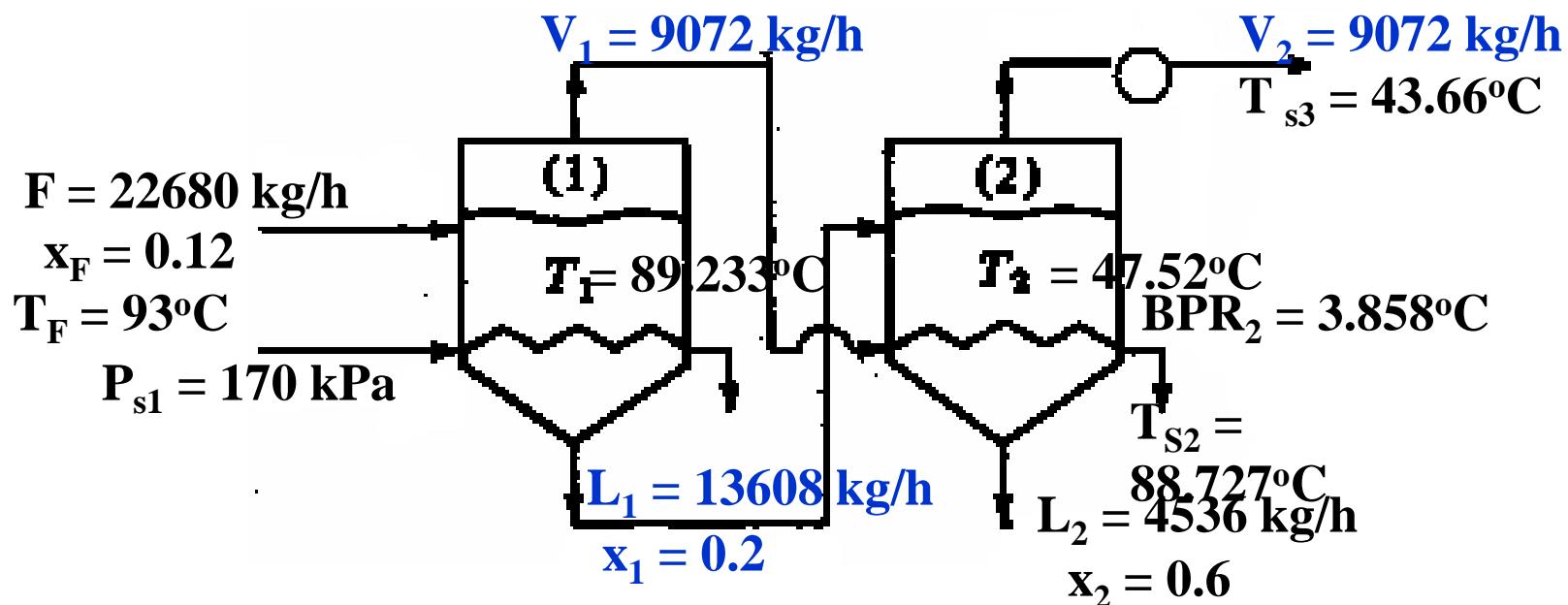
$$T_{s2} = 88.727^\circ\text{C}, H_{s2} = 2658.01 \text{ kJ/kg} \text{ and } h_{s2} = 371.57 \text{ kJ/kg}$$

$$H_1 = 2658.01 + 1.884(0.506) = 2658.97 \text{ kJ/kg}$$

$$22680(356.97) + S(2216.52) = L_1(321.6) + (22680 - L_1)(2658.97) \quad (1)$$

$$\text{Effect 2: } L_1 h_1 + V_1(H_1 - h_{s2}) = L_2 h_2 + V_2 H_2$$

$$L_1(321.6) + (22680 - L_1)(2658.97 - 371.57) = 4536(115.56) + (L_1 - L_2)H_2$$



$$H_2 = H_{s3} + 1.884(BPR_2)$$

At 9 kPa, $T_{s3} = 43.66^\circ\text{C}$, $H_{s3} = 2580.81 \text{ kJ/kg}$

$$H_2 = 2580.81 + 1.884(3.858) = 2588.08 \text{ kJ/kg}$$

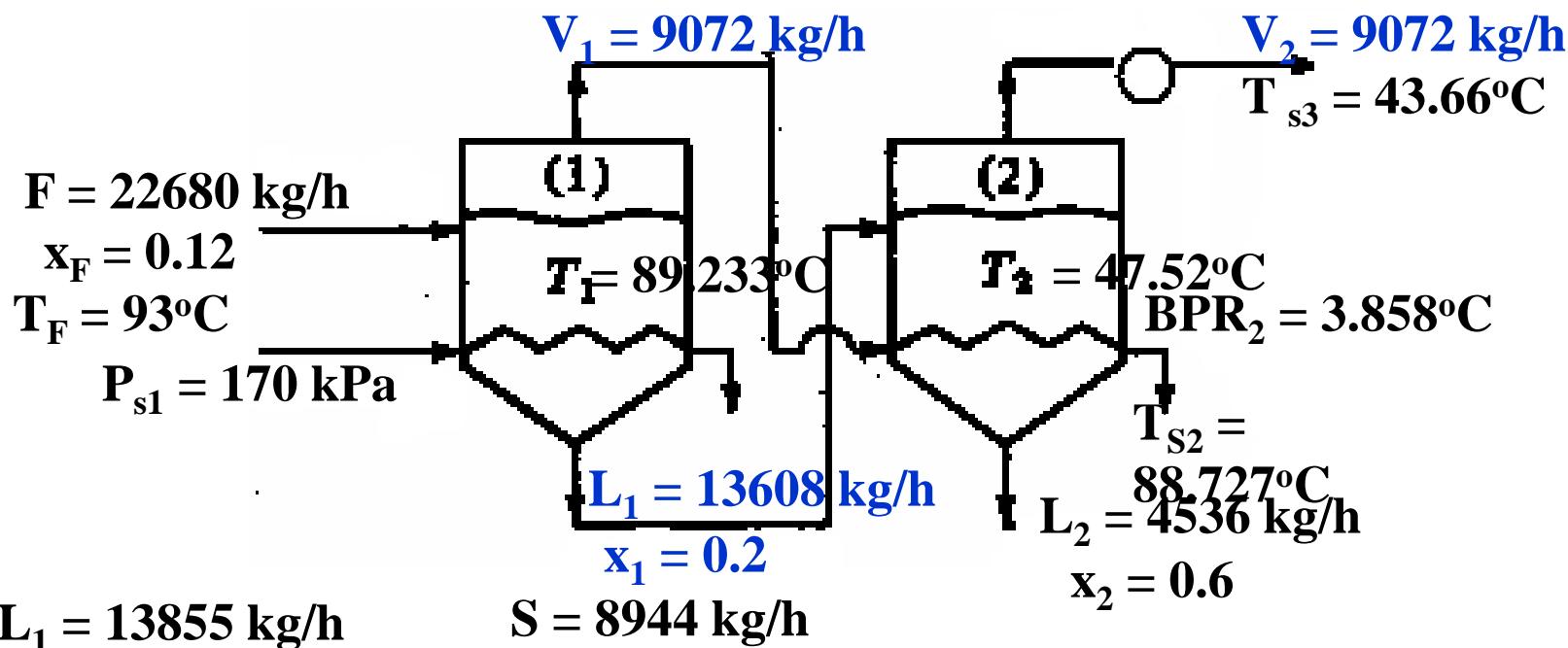
$$L_1(321.6) + (22680 - L_1)(2658.97 - 371.57) = 4536(115.56) + (L_1 - L_2)H_2$$

$$L_1(321.6) + (22680 - L_1)(2658.97 - 371.57) = 4536(115.56) + (L_1 - 4536)(2588.08)$$

$$\therefore L_1 = 13855 \text{ kg/h}$$

$$22680(356.97) + S(2216.52) = L_1(321.6) + (22680 - L_1)(2658.97) \quad (1)$$

$$S = 8944 \text{ kg/h}$$



Substituting L_1 into the material balances of each effect:

$$V_1 = 22680 - L_1 = 22680 - 13855 = 8825 \text{ kg/h}$$

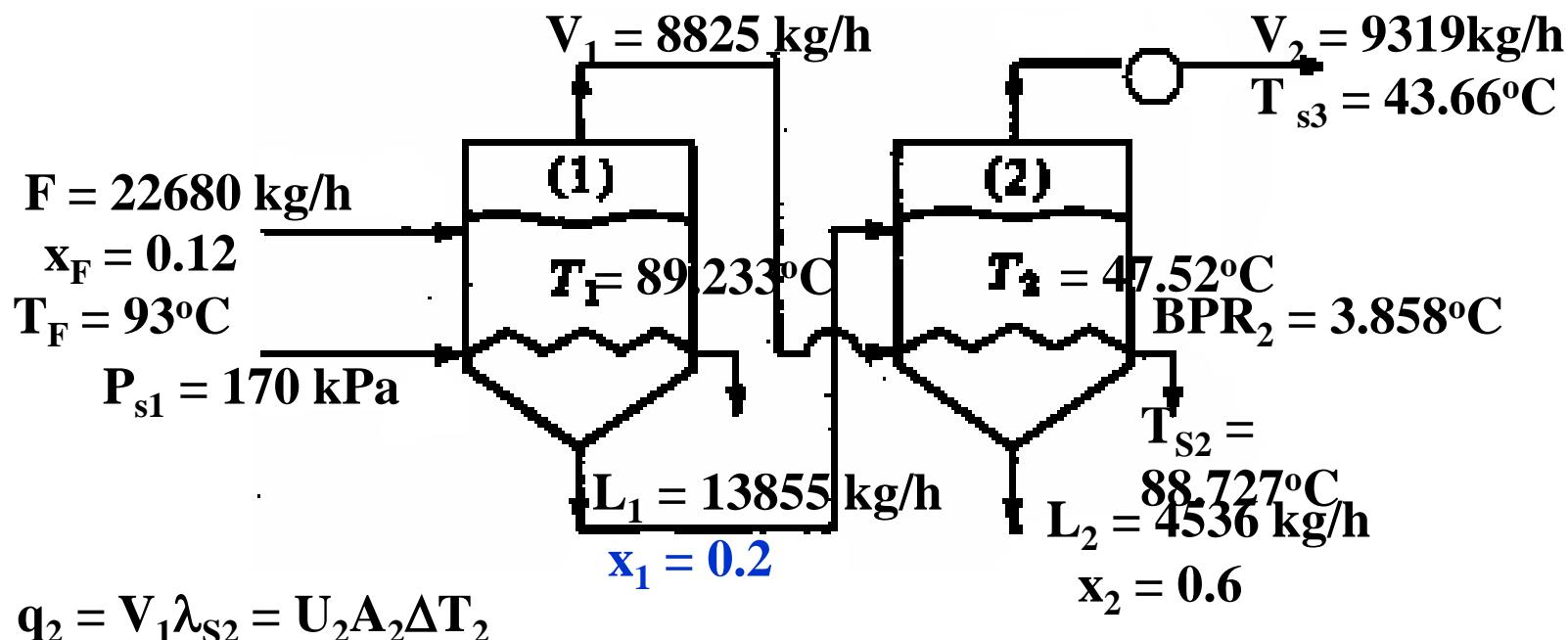
$$V_2 = L_1 - L_2 = 13855 - 4536 = 9319 \text{ kg/h}$$

Previous calculations: $L_1 = 13608 \text{ kg/h}$ (% error = 1.78), $V_1 = 9072 \text{ kg/h}$ (% error = 2.8), $V_2 = 9072 \text{ kg/h}$ (% error = 2.65)

$$q_1 = S\lambda_{s1} = U_1 A_1 \Delta T_1$$

$$q_1 = S\lambda_{s1} = \frac{8944 \text{ kg}}{\cancel{h}} \left| \frac{2216.52 \text{ kJ}}{\cancel{kg}} \right| \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 5507 \text{ kJ/s}$$

$$A_1 = \frac{5507 \times 10^3 \text{ W}}{\cancel{25.767 \text{ K}}} \left| \frac{\text{m}^2 \cancel{K}}{2271 \text{ W}} \right| = 94.11 \text{ m}^2$$



$$q_2 = V_1 \lambda_{S2} = U_2 A_2 \Delta T_2$$

$$q_2 = V_1 \lambda_{S2} = \frac{8825 \text{ kg}}{\text{h}} \times \frac{2287.4 \text{ kJ}}{\text{kg}} \times \frac{1\text{h}}{3600\text{s}} = 5607 \text{ kJ/s}$$

$$A_2 = \frac{5607 \times 10^3 \text{ W}}{\frac{41.209 \text{ K}}{1420 \text{ W}}} = 95.82 \text{ m}^2$$

$$A_{\text{ave.}} = A_m = 94.97 \text{ m}^2$$

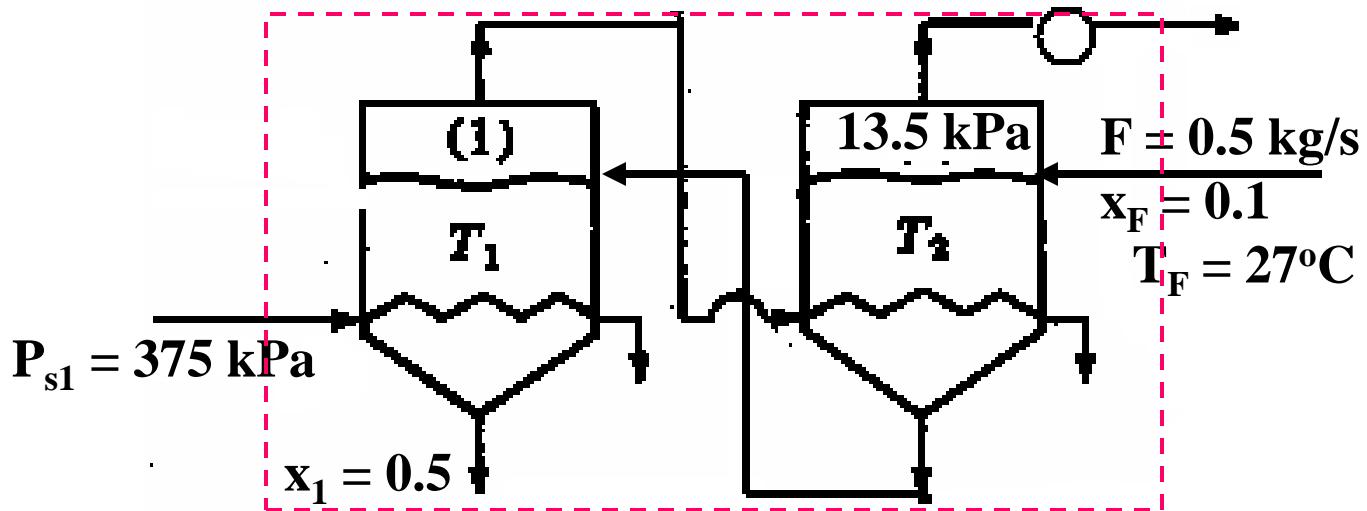
$$A_1 = 94.11 \text{ m}^2 \quad (\% \text{ error} = 0.9)$$

$$A_2 = 95.82 \text{ m}^2 \quad (\% \text{ error} = 0.9)$$

$$\therefore A = 94.97 \text{ m}^2$$

$$\text{Steam economy} = (8825 + 9319)/8944 = 2.03$$

Example



Given: No BPR

$$c_p \text{ kJ/kg.K} = 4.18$$

$$U_1 = 2.0 \text{ kW/m}^2.\text{K}, U_2 = 1.7 \text{ kW/m}^2.\text{K}$$

At 13.5 kPa , $T_{\text{sat3.}} = 51.69^\circ\text{C}$ (from A.2-9 after interpolation) = T_2

Total balance: $F = (V_1 + V_2) + L_1 = 0.5$

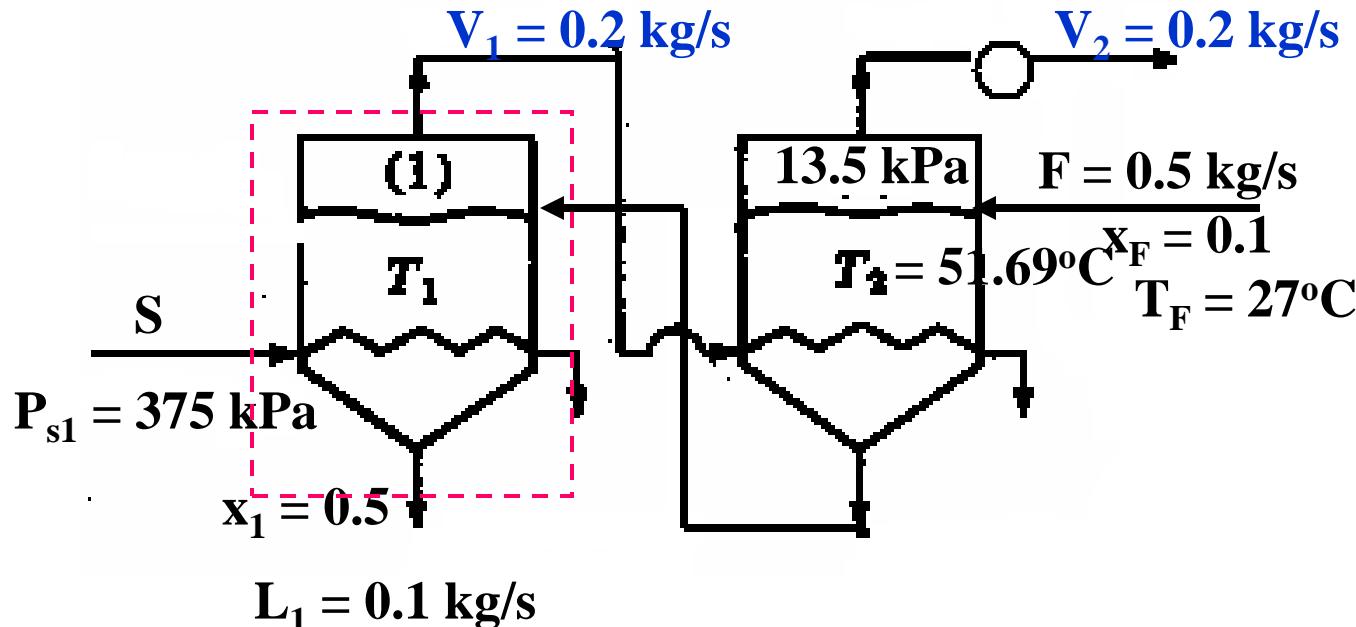
Solid balance: $Fx_F = L_1 x_1$

$$0.5(0.1) = L_1(0.5) \quad \therefore L_1 = 0.1 \text{ kg/s}$$

Total balance: $0.5 = (V_1 + V_2) + 0.1$

$$(V_1 + V_2) = 0.4 \text{ kg/s}$$

Assume $V_1 = V_2$ $V_1 = V_2 = 0.4 / 2 = 0.2 \text{ kg/s}$



Effect 1: Total balance: $L_2 = V_1 + L_1$

$$= 0.2 + 0.1 = 0.3 \text{ kg/s}$$

Solid balance: $L_2 x_2 = L_1 x_1$

$$0.3x_2 = 0.1(0.5) \quad x_2 = 0.1667$$

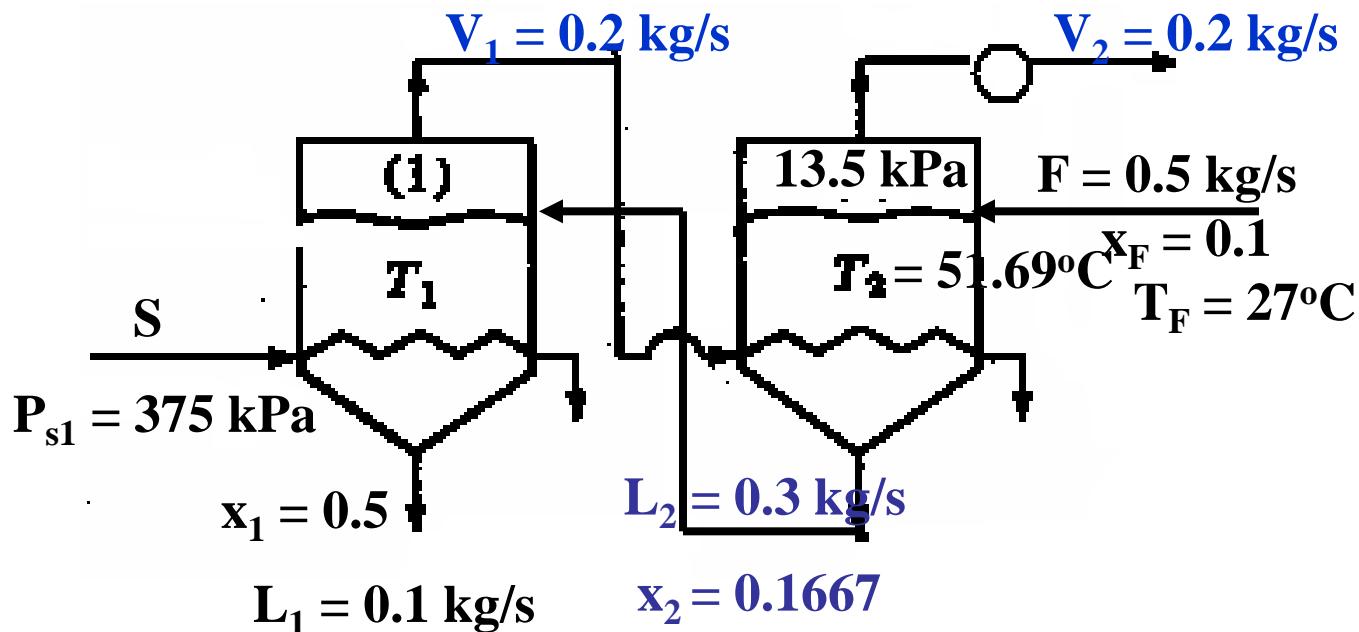
At 375 kPa, $T_{s1} = 142.96^\circ\text{C}$

$$\sum \Delta T = T_{s1} - T_{s3} = 142.96 - 51.69 = 91.27^\circ\text{C}$$

$$\Delta T_1 = \sum \Delta T \frac{1/U_1}{1/U_1 + 1/U_2} = 91.27 \frac{1/2}{1/2 + 1/1.7} = 41.93^\circ\text{C}$$

$$\Delta T_2 = \sum \Delta T - \Delta T_1 = 91.27 - 41.93 = 49.34^\circ\text{C}$$

Since cold feed, ΔT_2 will be readjusted to 51°C and $\Delta T_1 = 40.27^\circ\text{C}$



Since cold feed, ΔT_2 will be readjusted to 51°C and $\Delta T_1 = 40.27^\circ\text{C}$

$$\Delta T_2 = T_{s2} - T_2$$

$$51 = T_{s2} - 51.69$$

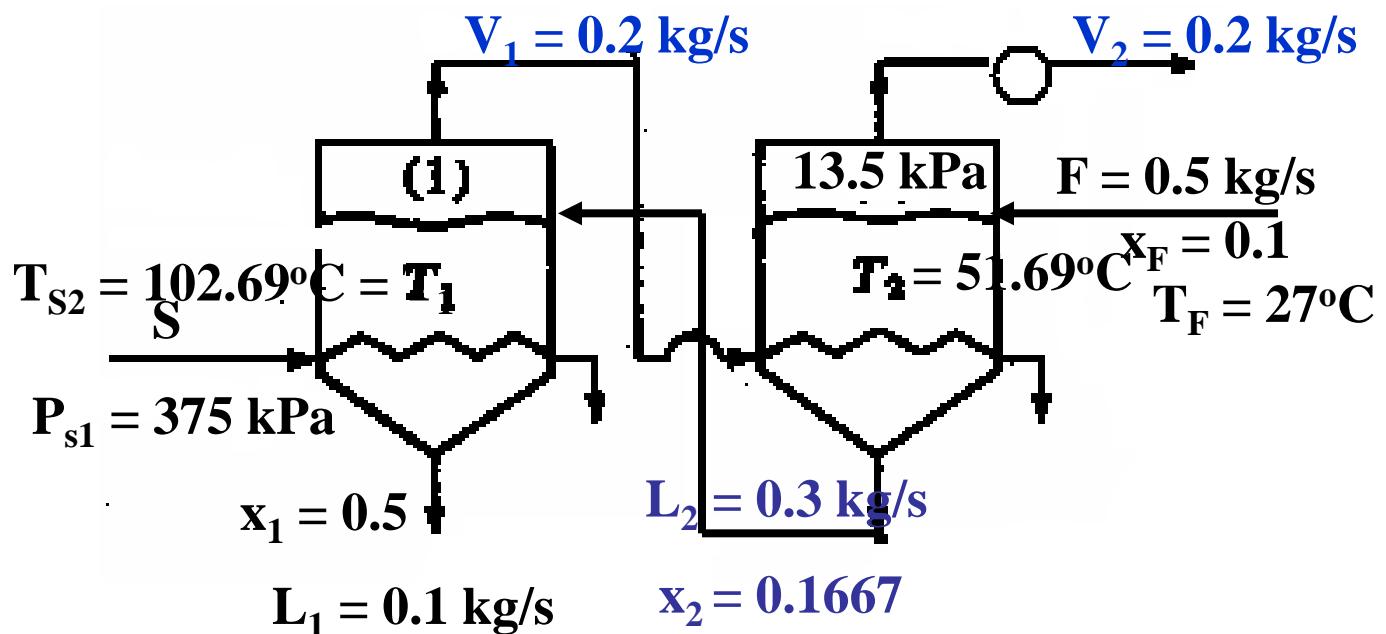
$$\therefore T_{s2} = 102.69^\circ\text{C} = T_1$$

$T_{\text{ref.}} = 0^\circ\text{C}$ (liquid solution),

$$h_F = c_{PF}(T_F - T_{\text{ref.}}) = 4.18(27 - 0) = 112.86 \text{ kJ/kg}$$

$$h_1 = c_{P1}(T_1 - T_{\text{ref.}}) = 4.18(102.69 - 0) = 429.244 \text{ kJ/kg}$$

$$h_2 = c_{P2}(T_2 - T_{\text{ref.}}) = 4.18(51.69 - 0) = 216.06 \text{ kJ/kg}$$



$T_{ref} = 0^\circ\text{C}$ (liquid solution),

$$h_F = 112.86 \text{ kJ/kg} \quad h_1 = 429.244 \text{ kJ/kg} \quad h_2 = 216.06 \text{ kJ/kg}$$

$$T_{S1} = 142.96^\circ\text{C}, H_{S1} = 2737.69 \text{ kJ/kg} \text{ & } h_{S1} = 601.87 \text{ kJ/kg.}$$

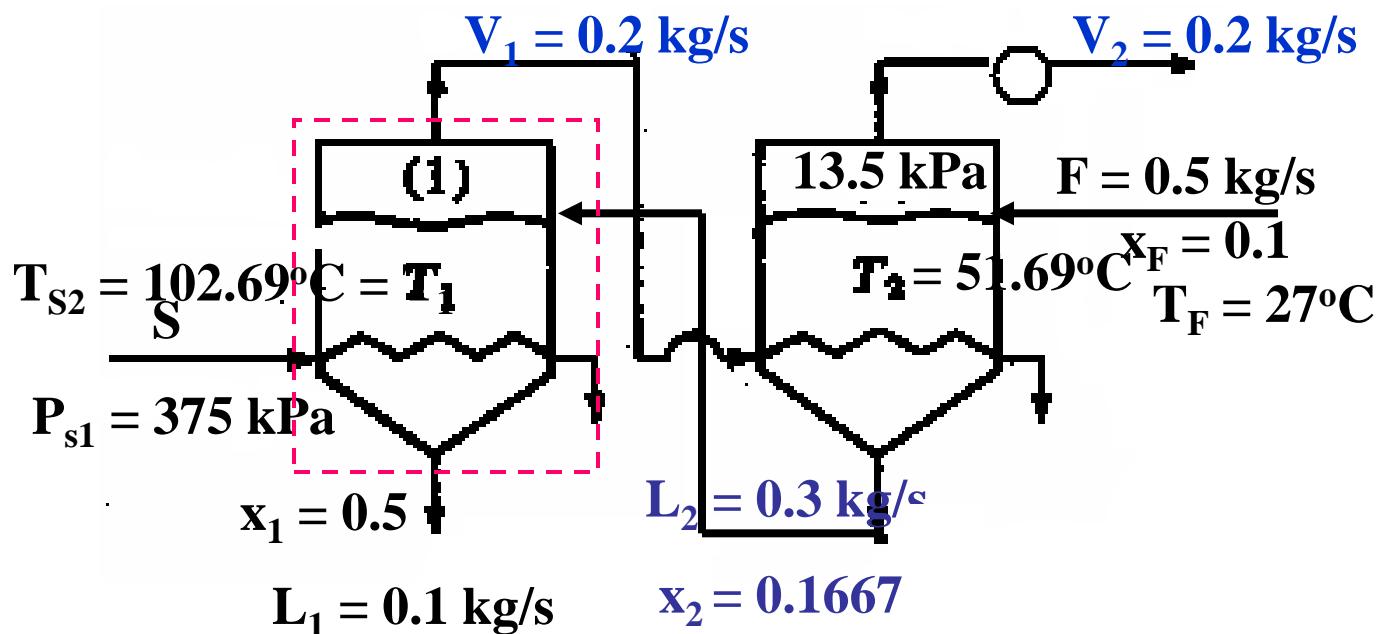
$$\therefore \lambda_{S1} = (2737.69 - 601.87) = 2135.82 \text{ kJ/kg}$$

$$T_{S2} = 102.69^\circ\text{C}, H_{S2} = 2680.24 \text{ kJ/kg} \text{ & } h_{S2} = 430.4 \text{ kJ/kg.}$$

$$\therefore \lambda_{S2} = (2680.24 - 430.4) = 2249.84 \text{ kJ/kg}$$

$$H_1 = H_{S2} = 2680.24 \text{ kJ/kg}$$

$$H_2 = H_{S3} = 2595.07 \text{ kJ/kg}$$



Tref. = 0°C , liquid solution (same as steam table),

$$\begin{aligned}
 h_F &= 112.86 \text{ kJ/kg} & h_1 &= 429.244 \text{ kJ/kg} & h_2 &= 216.06 \text{ kJ/kg} \\
 \lambda_{S1} &= 2135.82 \text{ kJ/kg} & \lambda_{S2} &= 2249.84 \text{ kJ/kg} & H_1 &= 2680.24 \text{ kJ/kg} & H_2 &= 2595.07 \text{ kJ/kg}
 \end{aligned}$$

Effect 1:

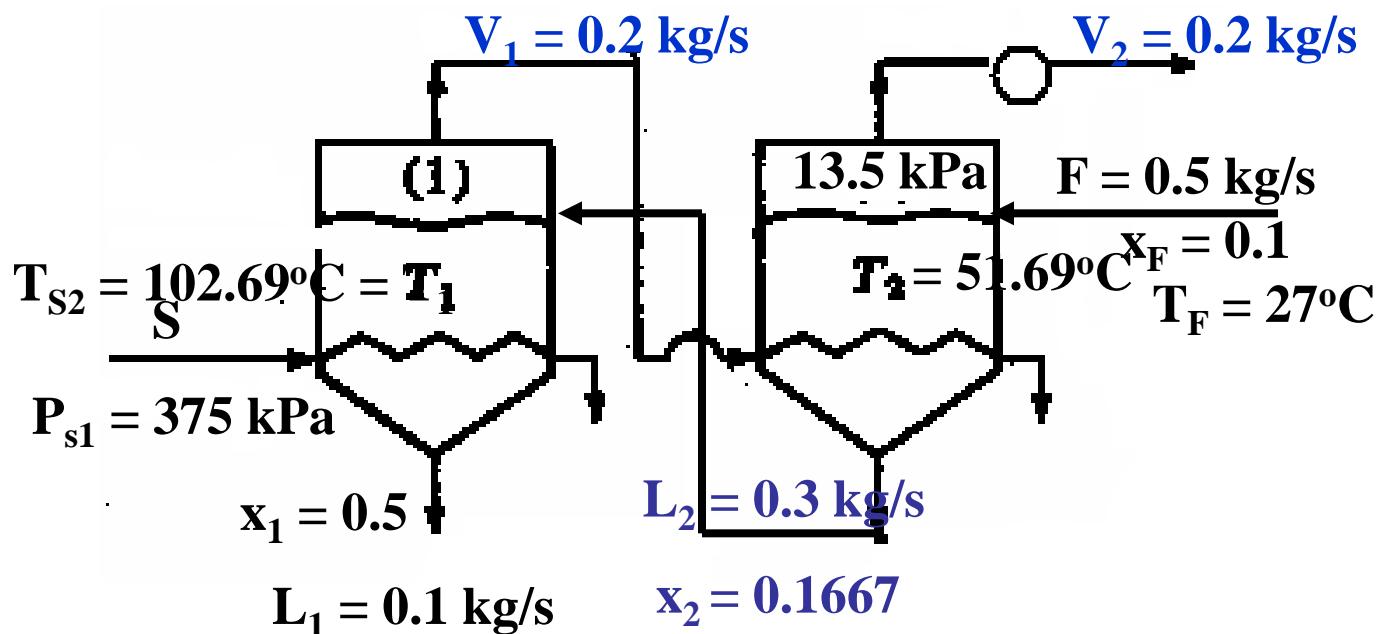
$$\text{Energy balance: } L_2 h_2 + S \lambda_{S1} = L_1 h_1 + V_1 H_1$$

$$\text{Material balance: } L_2 = V_1 + L_1$$

$$V_1 = L_2 - 0.1$$

$$L_2(216.06) + S(2135.82) = 0.1(429.244) + (L_2 - 0.1)2680.24$$

$$2135.82S + 225.1 = 2689.28L_2 \quad (1)$$



Tref. = 0°C (same as steam table),

$$h_F = 112.86 \text{ kJ/kg} \quad h_1 = 429.244 \text{ kJ/kg} \quad h_2 = 216.06 \text{ kJ/kg}$$

$$\lambda_{S1} = 2135.82 \text{ kJ/kg} \quad \lambda_{S2} = 2249.84 \text{ kJ/kg} \quad H_1 = 2680.24 \text{ kJ/kg} \quad H_2 = 2595.07 \text{ kJ/kg}$$

$$\text{Effect 1: } 2135.82S + 225.1 = 2689.28L_2 \quad (1)$$

$$\text{Effect 2: Energy balance: } Fh_F + V_1 \lambda_{S2} = L_2 h_2 + V_2 H_2$$

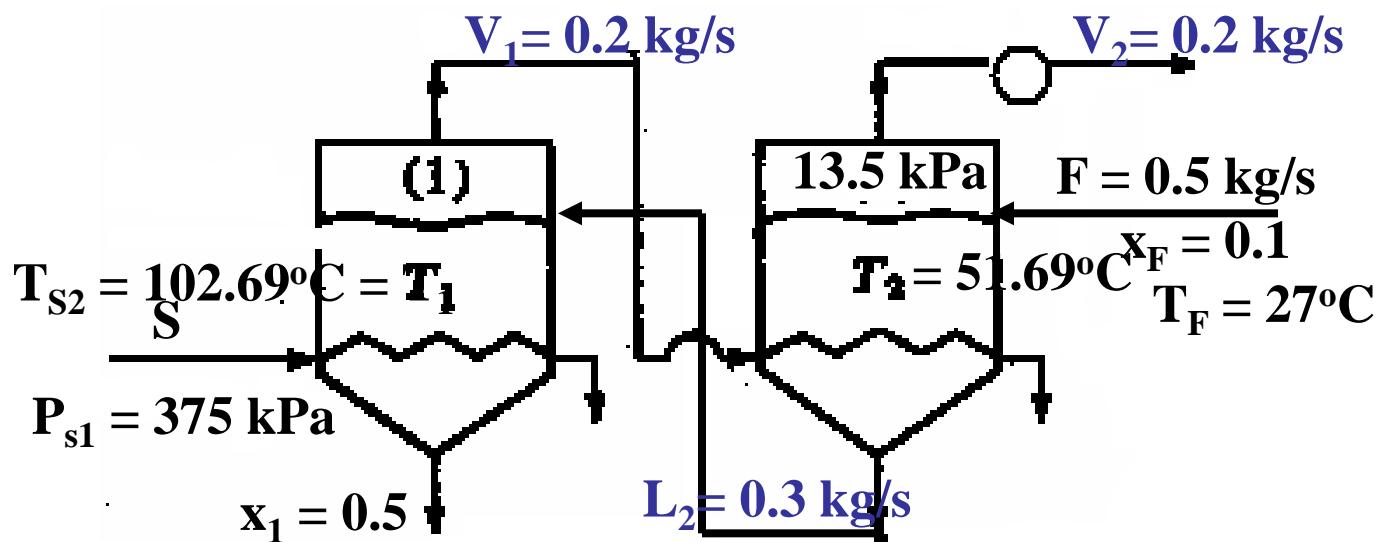
$$\text{Material balance: } F = V_2 + L_2$$

$$V_2 = F - L_2 = 0.5 - L_2$$

$$0.5(112.86) + (L_2 - 0.1)(2249.84) = L_2(216.06) + (0.5 - L_2)2595.07$$

$$L_2(2595.07 - 216.06 + 2249.84) = 1297.535 - 56.43 + 224.984$$

$$\therefore L_2 = 0.3167 \text{ kg/s} \quad S = 0.2933 \text{ kg/s} \quad V_2 = 0.1833 \text{ kg/s} \quad V_1 = 0.2167 \text{ kg/s}$$



$$L_1 = 0.1 \text{ kg/s}$$

$$\therefore L_2 = 0.3167 \text{ kg/s} (\% \text{ error} = 5.3) \quad S = 0.2933 \text{ kg/s}$$

$$V_1 = 0.2167 \text{ kg/s} (\% \text{ error} = 7.7) \quad V_2 = 0.1833 \text{ kg/s} (\% \text{ error} = 9.1)$$

Given: $U_1 = 2.0 \text{ kW/m}^2 \cdot \text{K}$, $U_2 = 1.7 \text{ kW/m}^2 \cdot \text{K}$

$$q_1 = S\lambda_{s1} = U_1 A_1 \Delta T_1 = U_1 A_1 (T_{s1} - T_1)$$

$$q_1 = S\lambda_{s1} = (0.2933)(2135.82) = 626.436 = 2(A_1)(142.96 - 102.69)$$

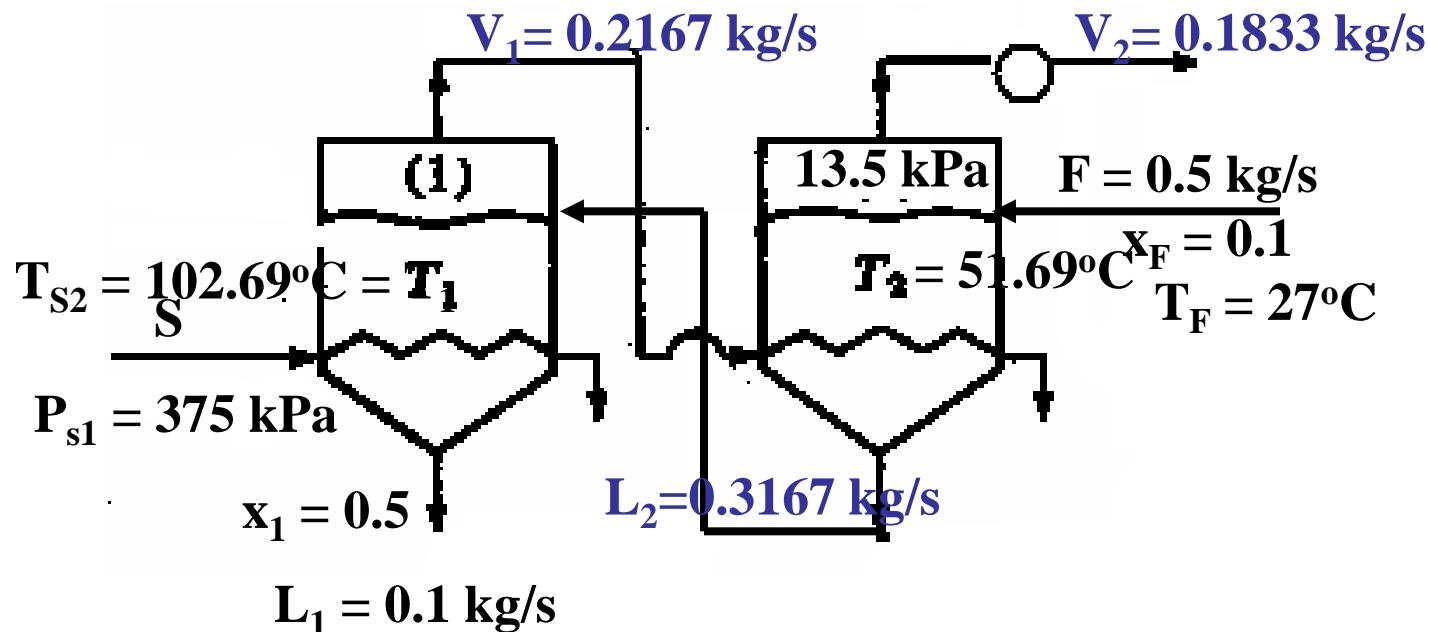
$$A_1 = 7.778 \text{ m}^2$$

$$q_2 = V_1 \lambda_{s2} = U_2 A_2 \Delta T_2 = U_2 A_2 (T_{s2} - T_2)$$

$$q_2 = V_1 \lambda_{s2} = (0.2167)(2249.84) = 1.7(A_2)(51)$$

$$A_2 = 5.623 \text{ m}^2$$

$$A_{\text{ave.}} = A_m = 6.7005 \text{ m}^2$$



$$A_{ave.} = A_m = 6.7005 \text{ m}^2$$

$$(\% \text{ error} = 16.1)$$

$$(\% \text{ error} = 16.1)$$

2nd Trial:

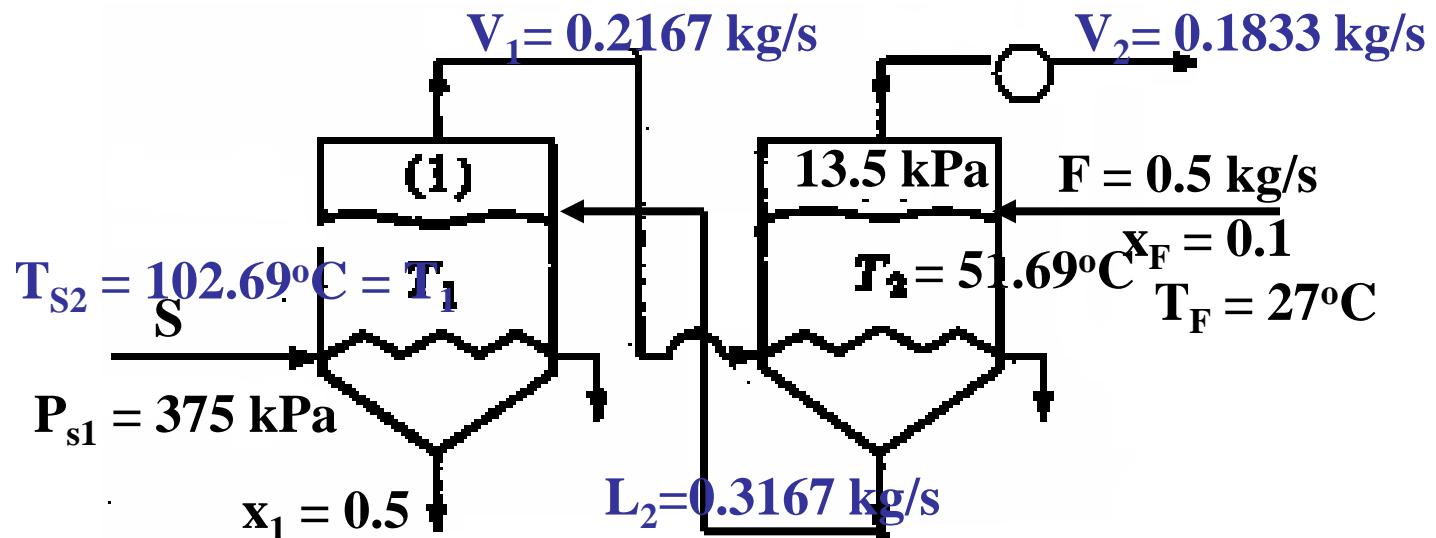
Effect 1: **Total balance:** $L_2 = V_1 + L_1$
 $= 0.2167 + 0.1 = 0.3167 \text{ kg/s}$

Solid balance: $L_2 x_2 = L_1 x_1$

$$0.3167 x_2 = 0.1(0.5)$$

$$x_2 = 0.1579 \text{ kg/kg}$$

$$\sum \Delta T = T_{s1} - T_{s3} = 142.96 - 51.69 = 91.27^\circ\text{C}$$



$$A_{\text{ave.}} = A_m = 6.7005 \text{ m}^2 \quad A_1 = 7.778 \text{ m}^2 \quad A_2 = 5.623 \text{ m}^2$$

$$\sum \Delta T = T_{S1} - T_{S3} = 142.96 - 51.69 = 91.27^\circ\text{C}$$

$$\Delta T_1' = \frac{\Delta T_1 A_1}{A_m} = \frac{(40.27)7.778}{6.7005} = 46.75^\circ\text{C}$$

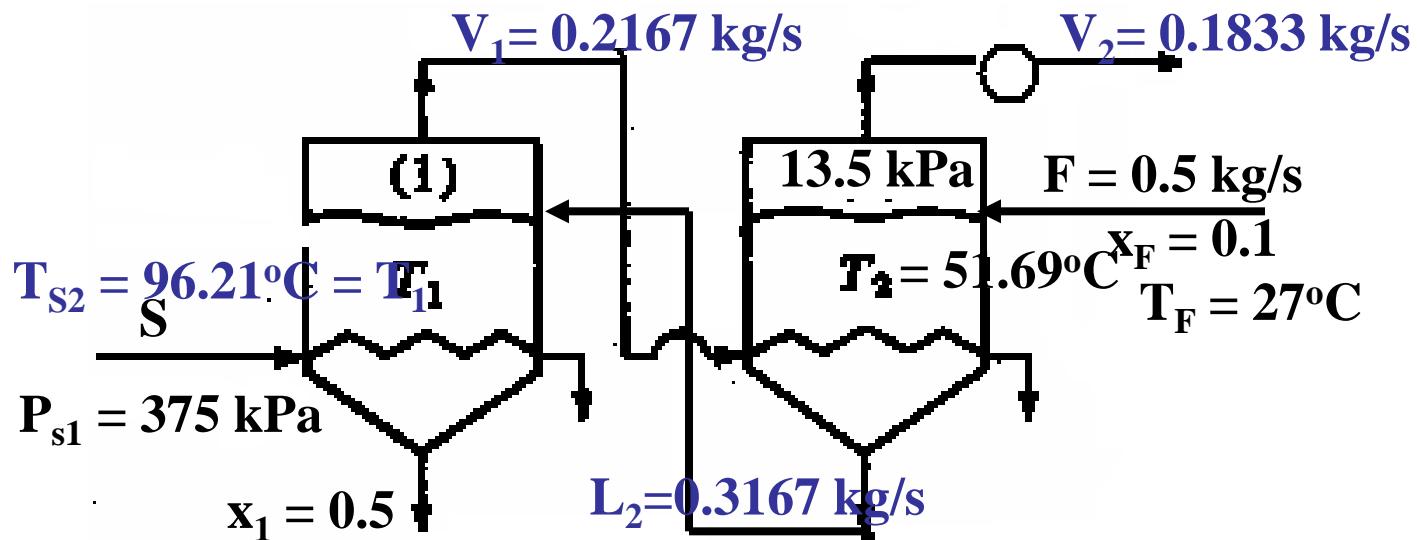
$$\Delta T_2' = \sum \Delta T - \Delta T_1 = 91.27 - 46.75 = 44.52^\circ\text{C}$$

$$\Delta T_2' = T_{S2} - T_2 = T_{S2} - 51.69 = 44.52$$

$$\therefore T_{S2} = 96.21^\circ\text{C} = T_1$$

$$T_{S2} = 96.21^\circ\text{C}, H_{S2} = 2670.04 \text{ kJ/kg} \text{ & } h_{S2} = 403.06 \text{ kJ/kg.}$$

$$\therefore \lambda_{S2} = (2670.04 - 403.06) = 2266.98 \text{ kJ/kg}$$



$T_{ref.} = 0^\circ\text{C}$ (liquid solution),

$$\lambda_{S2} = 2266.98 \text{ kJ/kg} \quad h_F = 112.86 \text{ kJ/kg} \quad h_2 = 216.06 \text{ kJ/kg}$$

$$h_1 = c_{P1}(T_1 - T_{ref.}) = 4.18(96.21 - 0) = 402.16 \text{ kJ/kg}$$

$$H_1 = H_{S2} = 2670.04 \text{ kJ/kg}$$

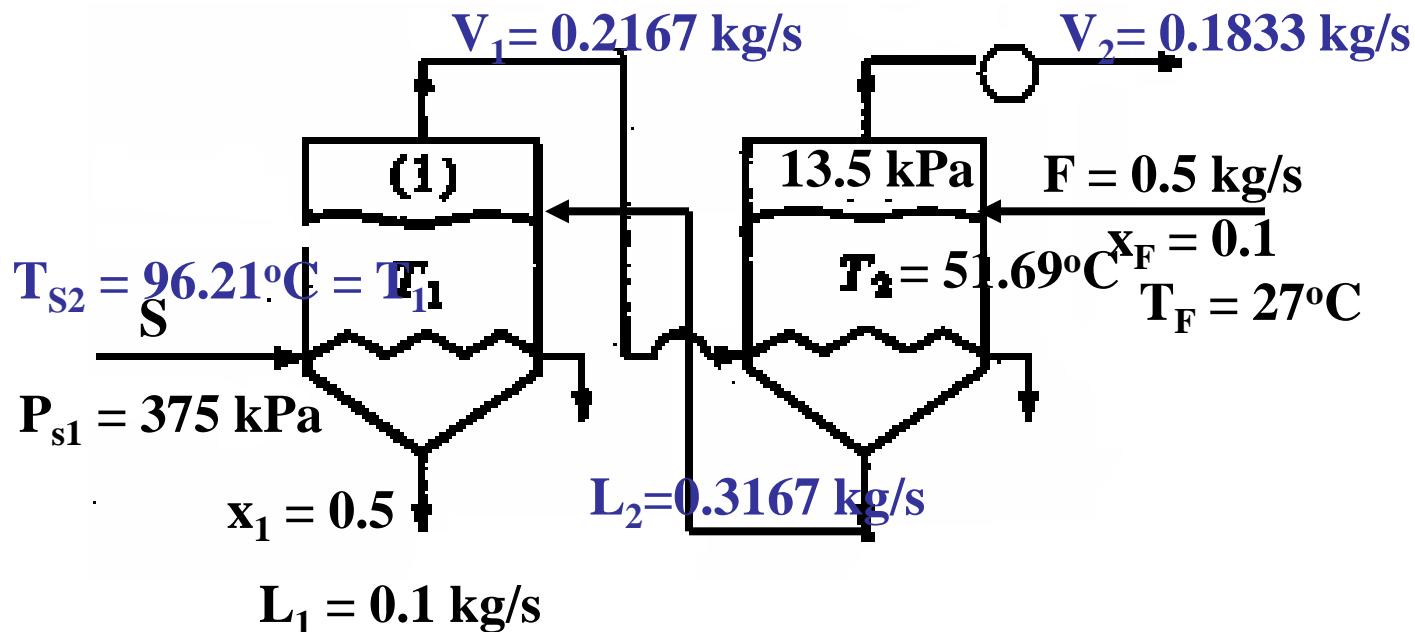
$$H_2 = H_{S3} = 2595.07 \text{ kJ/kg}$$

Effect 2: Energy balance: $Fh_F + V_1 \lambda_{S2} = L_2 h_2 + V_2 H_2$

Material balance: $V_2 = F - L_2 = 0.5 - L_2$

$$0.5(112.86) + (L_2 - 0.1)(2266.98) = L_2(216.06) + (0.5 - L_2)2595.07$$

$$\therefore L_2 = 0.316 \text{ kg/s}$$



Tref. = 0°C , liquid solution (same as steam table),

$$\begin{aligned}
 h_F &= 112.86 \text{ kJ/kg} & h_1 &= 402.16 \text{ kJ/kg} & h_2 &= 216.06 \text{ kJ/kg} \\
 \lambda_{s1} &= 2135.82 \text{ kJ/kg} & \lambda_{s2} &= 2266.98 \text{ kJ/kg} & H_1 &= 2670.04 \text{ kJ/kg} & H_2 &= 2595.07 \text{ kJ/kg}
 \end{aligned}$$

Effect 1: Energy balance: $L_2 h_2 + S \lambda_{s1} = L_1 h_1 + V_1 H_1$

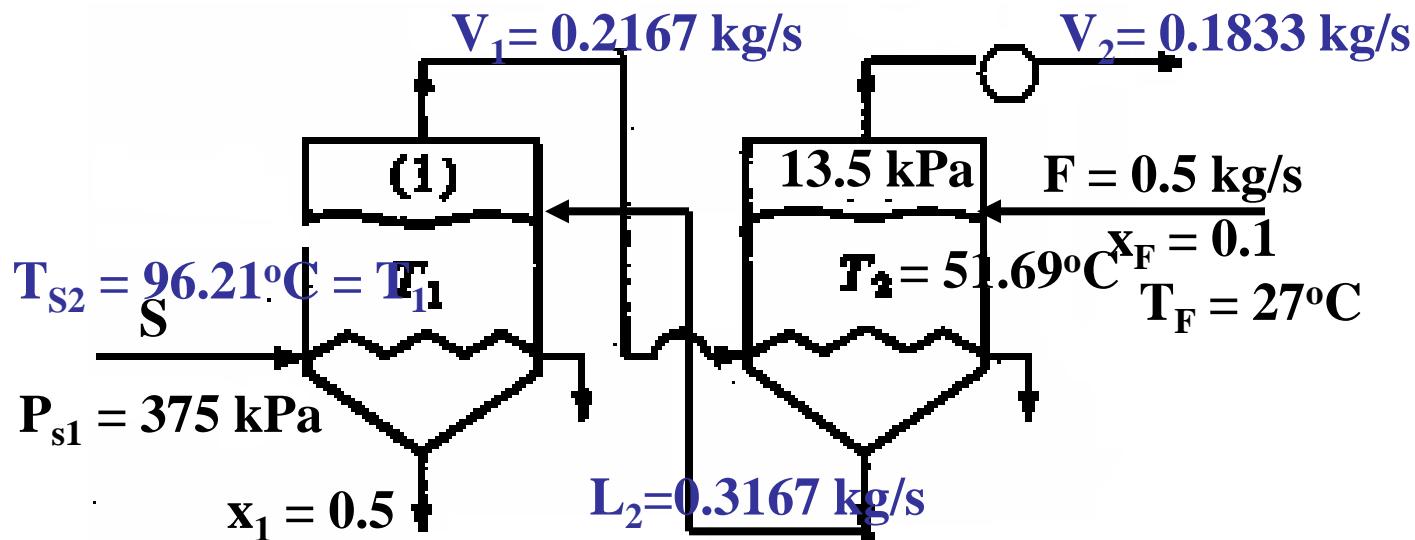
Material balance: $V_1 = L_2 - 0.1$

$$L_2(216.06) + S(2135.82) = 0.1(402.16) + (L_2 - 0.1)2670.04$$

Substituting $L_2 = 0.316 \text{ kg/s}$, $S = 0.257 \text{ kg/s}$

$$V_2 = F - L_2 = 0.5 - L_2 = 0.5 - 0.316 = 0.184 \text{ kg/s}$$

$$V_1 = L_2 - 0.1 = 0.316 - 0.1 = 0.216 \text{ kg/s}$$



$$L_1 = 0.1 \text{ kg/s}$$

Given: $U_1 = 2.0 \text{ kW/m}^2 \cdot \text{K}$, $U_2 = 1.7 \text{ kW/m}^2 \cdot \text{K}$

$L_2 = 0.316 \text{ kg/s}$ (% error = 0.2), $S = 0.257 \text{ kg/s}$, $V_1 = 0.216 \text{ kg/s}$ (% error = 0.32),
 $V_2 = 0.184 \text{ kg/s}$ (% error = 0.38),

$$q_1 = S\lambda_{S1} = U_1 A_1 \Delta T_1$$

$$q_1 = S\lambda_{S1} = (0.257)(2135.82) = 2(A_1)(46.75)$$

$$A_1 = 5.871 \text{ m}^2$$

$$q_2 = V_1 \lambda_{S2} = U_2 A_2 \Delta T_2$$

$$q_2 = V_1 \lambda_{S2} = (0.216)(2266.98) = 1.7(A_2)(44.52)$$

$$A_2 = 6.47 \text{ m}^2$$

$$A_1 = 5.871 \text{ m}^2$$

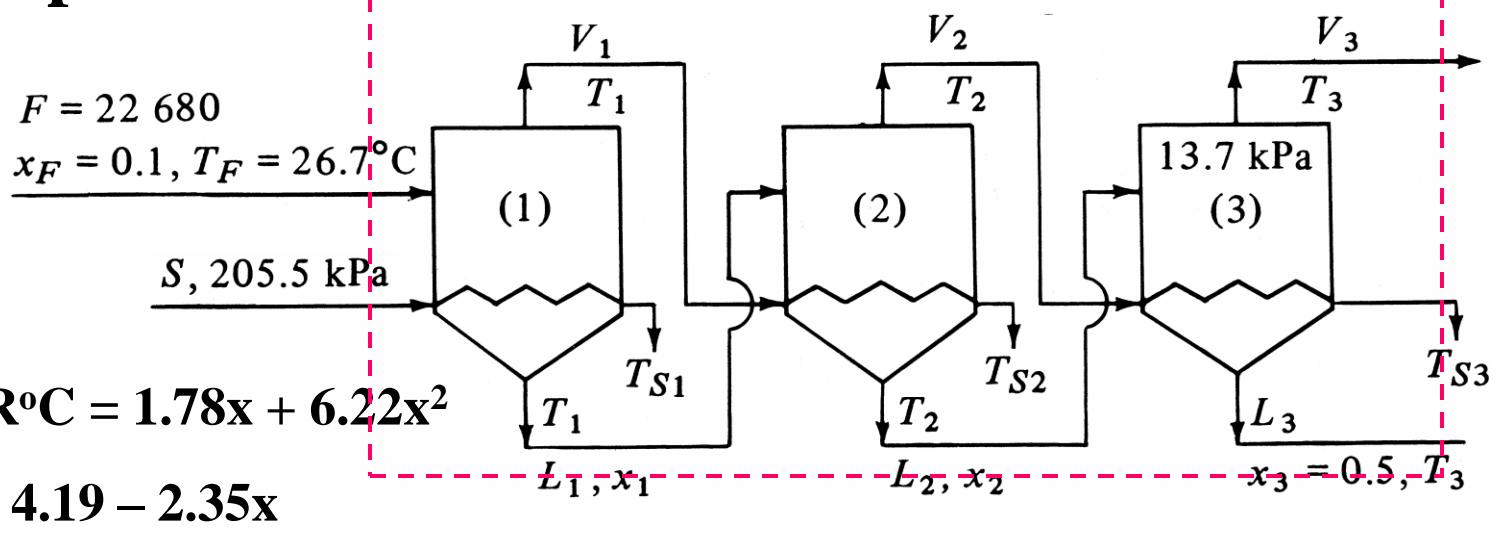
(% error = 5.1)

$$A_{ave.} = A_m = 6.1705 \text{ m}^2$$

$$A_2 = 6.47 \text{ m}^2$$

(% error = 4.86)

Example 8.5-1



Given: $BPR^{\circ}\text{C} = 1.78x + 6.22x^2$

$$c_p = 4.19 - 2.35x$$

$$U_1 = 3123 \text{ W/m}^2\cdot\text{K}, U_2 = 1987 \text{ W/m}^2\cdot\text{K} \text{ & } U_3 = 1135 \text{ W/m}^2\cdot\text{K}$$

1. T_{sat.}, BPR₃ & T₃

At 13.7 kPa, T_{sat.} = 51.67°C (from A.2-9 after interpolation)

$$BPR_3 = 1.78(0.5) + 6.22(0.5)^2 = 2.45^{\circ}\text{C}$$

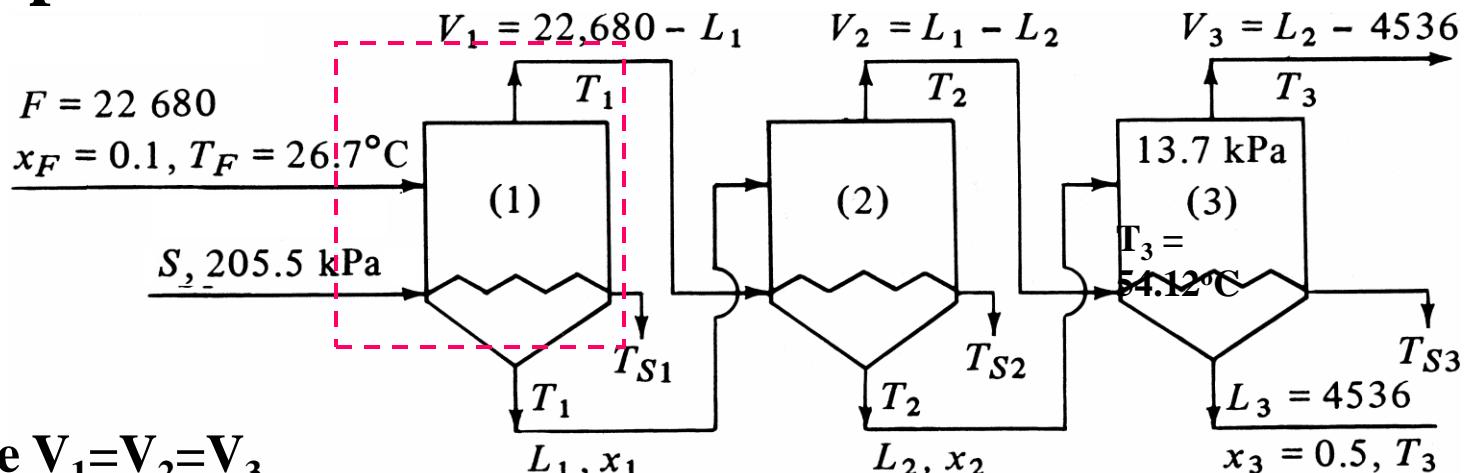
$$\therefore T_3 = T_{sat.} + BPR_3 = 51.67 + 2.45 = 54.12^{\circ}\text{C}$$

2. L₃ & V₁+V₂+V₃. Assume V₁=V₂=V₃

$$\text{Total balance: } F = (V_1 + V_2 + V_3) + L_3 = 22680 \text{ kg/h}$$

$$\text{Sugar balance: } Fx_F = L_3x_3 \quad 22680(0.1) = L_3(0.5) \quad \therefore L_3 = 4536 \text{ kg/h}$$

Example 8.5-1



2. Assume $V_1 = V_2 = V_3$

$$\text{Total balance: } 22680 = (V_1 + V_2 + V_3) + 4536$$

$$(V_1 + V_2 + V_3) = 18\ 144 \text{ kg/h}$$

$$V_1 = V_2 = V_3 = 18\ 144 / 3 = 6048 \text{ kg/h}$$

3. L_1, L_2, x_1 & x_2

Effect 1:

Total balance:

$$F = 22680 = V_1 + L_1$$

$$22680 = 6048 + L_1$$

$$L_1 = 16\ 632 \text{ kg/h}$$

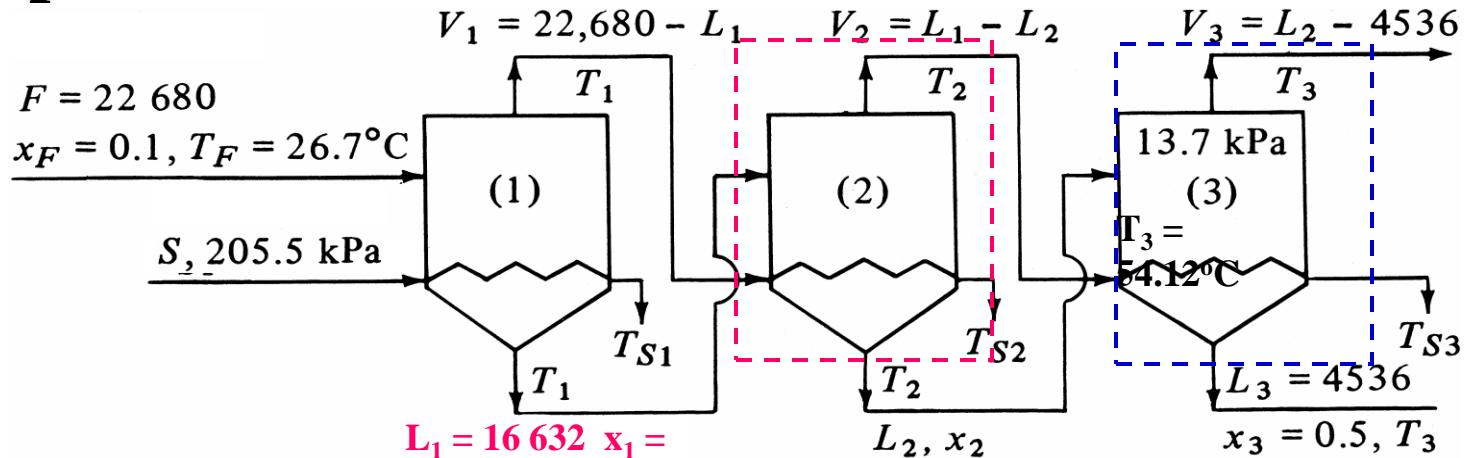
Sugar balance:

$$Fx_F = L_1 x_1$$

$$22680(0.1) = 16\ 632x_1$$

$$x_1 = 0.136$$

Example 8.5-1



Effect 2:

Total balance: $L_1 = 16,632 = V_2 + L_2$

$$16,632 = 6,048 + L_2$$

$$L_2 = 10,584 \text{ kg/h}$$

Sugar balance: $L_1 x_1 = L_2 x_2$

$$16,632(0.136) = 10,584 x_2$$

$$x_2 = 0.214$$

Effect 3 (checking) :

Total balance: $L_2 = 10,584 = V_3 + L_3$

$$10,584 = 6,048 + L_3$$

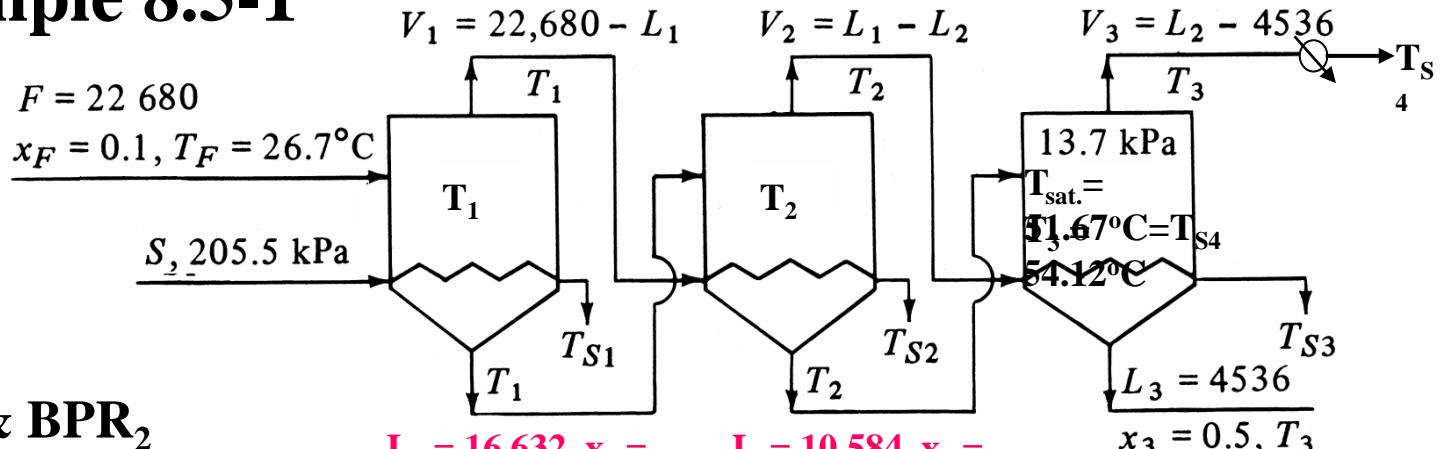
$$L_3 = 4,536 \text{ kg/h}$$

Sugar balance: $L_2 x_2 = L_3 x_3$

$$10,584(0.214) = 4,536 x_3$$

$$x_3 = 0.5$$

Example 8.5-1



4. BPR₁ & BPR₂

$$\text{BPR}^\circ\text{C} = 1.78x + 6.22x^2$$

$$L_1 = 16,632 \quad x_1 =$$

$$L_2 = 10,584 \quad x_2 =$$

$$0.136 \quad 0.214$$

$$\text{BPR}_1 = 1.78x_1 + 6.22x_1^2 = 1.78(0.136) + 6.22(0.136)^2 = 0.36^\circ\text{C}$$

$$\text{BPR}_2 = 1.78x_2 + 6.22x_2^2 = 1.78(0.214) + 6.22(0.214)^2 = 0.65^\circ\text{C}$$

5. $\sum \Delta T$, ΔT_1 , ΔT_2 & ΔT_3 . Readjust if necessary

$$\Delta T_1 = T_{S1} - T_1$$

$$\Delta T_1 = \frac{1/U_1}{1/U_1 + 1/U_2 + 1/U_3}$$

$$\Delta T_2 = T_{S2} - T_2$$

$$\Delta T_2 = \frac{1/U_2}{1/U_1 + 1/U_2 + 1/U_3}$$

$$\Delta T_3 = T_{S3} - T_3$$

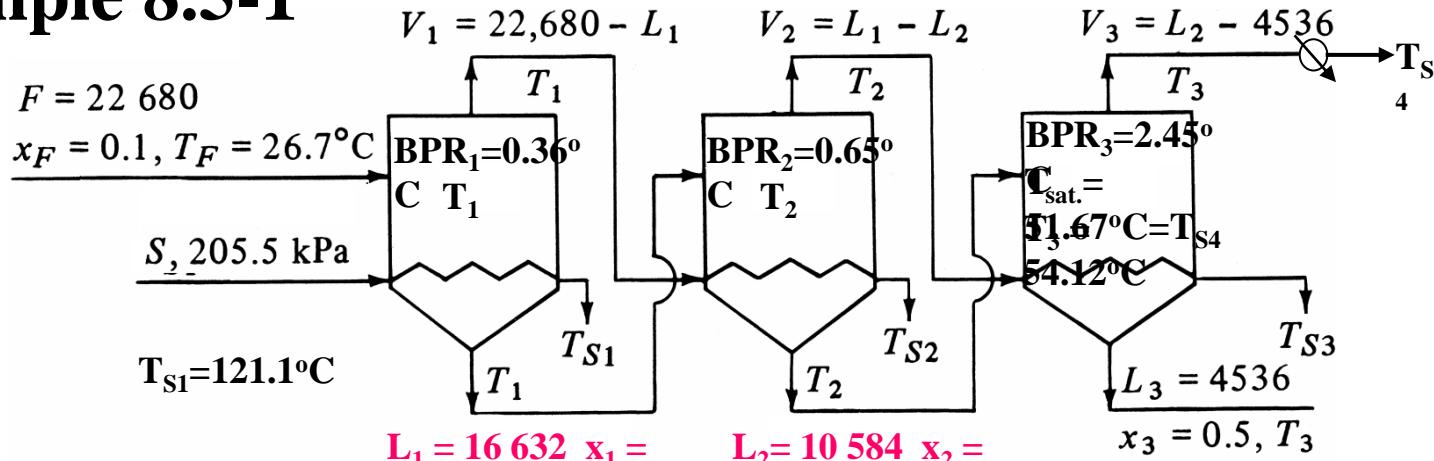
- No BPR: $\Delta T_2 = T_1 - T_2$

- BPR: $\Delta T_2 = T_1 - \text{BPR}_1 - T_2$

- No BPR: $\sum \Delta T = T_{S1} - T_{S4}$

- BPR: $\sum \Delta T = T_{S1} - T_{S4} - (\text{BPR}_1 + \text{BPR}_2 + \text{BPR}_3)$

Example 8.5-1



Given: $U_1 = 3123 \text{ W/m}^2.\text{K}$, $U_2 = 1987 \text{ W/m}^2.\text{K}$ & $U_3 = 1135 \text{ W/m}^2.\text{K}$

5. $\sum \Delta T, \Delta T_1, \Delta T_2 \text{ & } \Delta T_3$. Readjust if necessary

$$\sum \Delta T = T_{S1} - T_{S4} - (\text{BPR}_1 + \text{BPR}_2 + \text{BPR}_3) = 121.1 - 51.46 - (0.36 + 0.65 + 2.45) = 65.97^\circ\text{C}$$

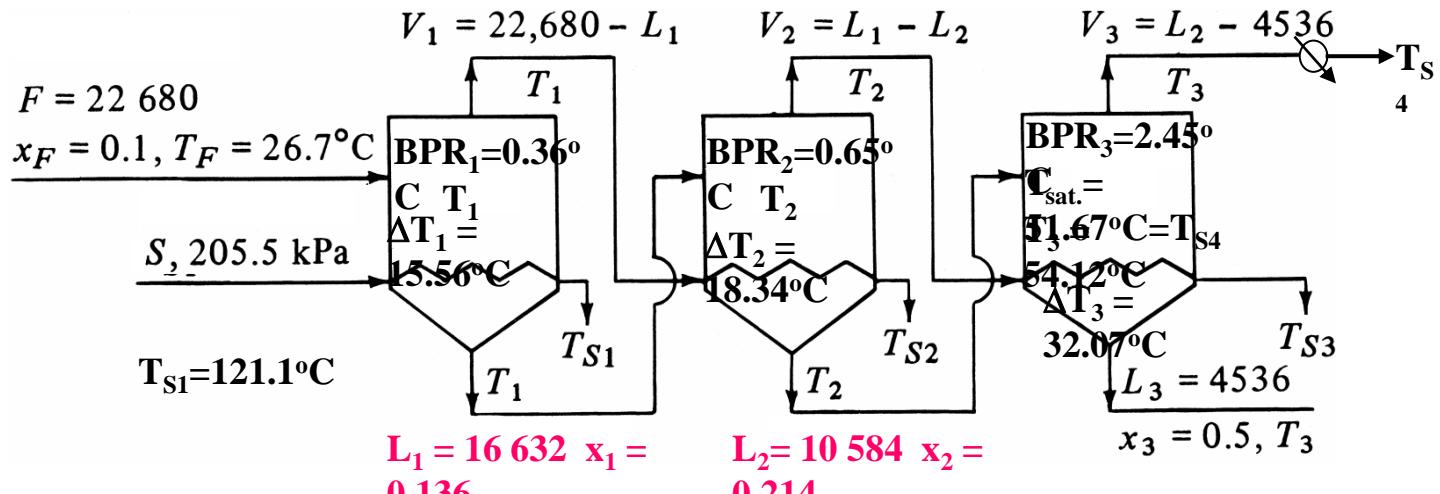
$$\Delta T_1 = \sum \Delta T \frac{1/U_1}{1/U_1 + 1/U_2 + 1/U_3} = 65.97 \frac{1/3123}{1/3123 + 1/1987 + 1/1135} = 12.40^\circ\text{C}$$

$$\Delta T_2 = \sum \Delta T \frac{1/U_2}{1/U_1 + 1/U_2 + 1/U_3} = 65.97 \frac{1/1987}{1/3123 + 1/1987 + 1/1135} = 19.50^\circ\text{C}$$

$$\Delta T_3 = \sum \Delta T \frac{1/U_3}{1/U_1 + 1/U_2 + 1/U_3} = 65.97 \frac{1/1135}{1/3123 + 1/1987 + 1/1135} = 34.07^\circ\text{C}$$

Since $T_F = 26.7^\circ\text{C} < T_3 = 54.12^\circ\text{C}$, the feed is a cold feed. Readjust as the feed need to be heated to T_1 . $\Delta T_1 = 15.56^\circ\text{C}$, $\Delta T_2 = 18.34^\circ\text{C}$, $\Delta T_3 = 32.07^\circ\text{C}$, $\sum \Delta T = 65.97^\circ\text{C}$

Example 8.5-1



6. $T_1, T_{S1}, T_2, T_{S2}, T_3 \& T_{S3}$

$$\Delta T_1 = T_{S1} - T_1$$

$$T_1 = T_{S1} - \Delta T_1 = 121.1 - 15.56 = 105.54^\circ\text{C}$$

$$\Delta T_2 = T_1 - BPR_1 - T_2$$

$$T_2 = T_1 - BPR_1 - \Delta T_2 = 105.54 - 0.36 - 18.34 = 86.84^\circ\text{C}$$

$$\Delta T_2 = T_{S2} - T_2$$

$$T_{S2} = \Delta T_2 + T_2 = 18.34 + 86.84 = 105.18^\circ\text{C}$$

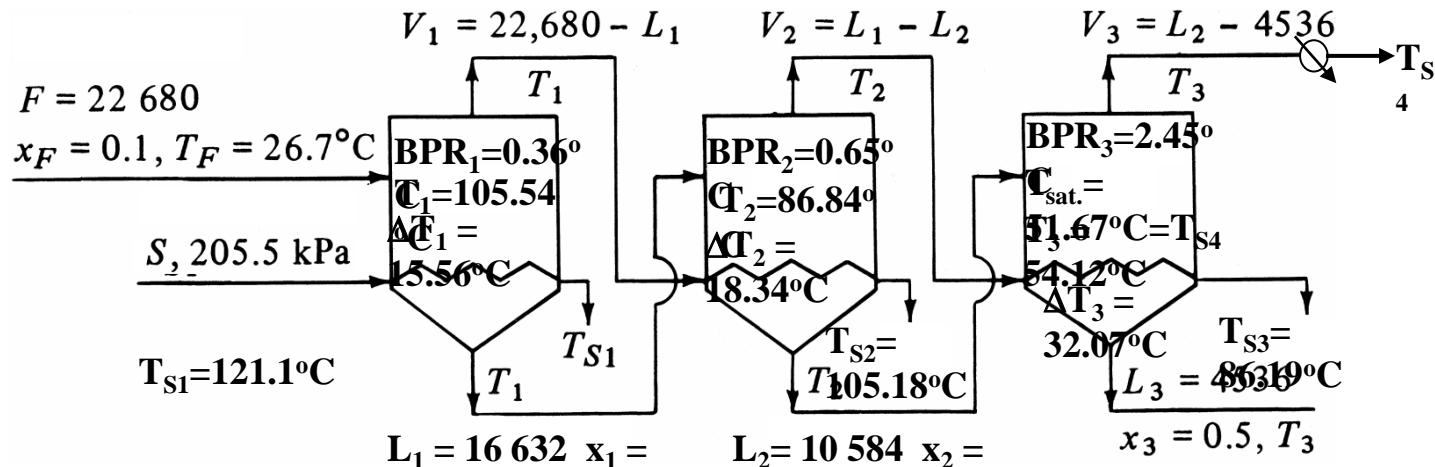
$$\Delta T_3 = T_2 - BPR_2 - T_3$$

$$T_3 = T_2 - BPR_2 - \Delta T_3 = 86.84 - 0.65 - 32.07 = 54.12^\circ\text{C} \text{ (checking)}$$

$$\Delta T_3 = T_{S3} - T_3$$

$$T_{S3} = \Delta T_3 + T_3 = 32.07 + 54.12 = 86.19^\circ\text{C}$$

Example 8.5-1



7. c_p , enthalpies, recalculate L_1, L_2, V_1, V_2, V_3 & S using heat balances

$$\text{Given: } c_p (\text{kJ/kg.K}) = 4.19 - 2.35x$$

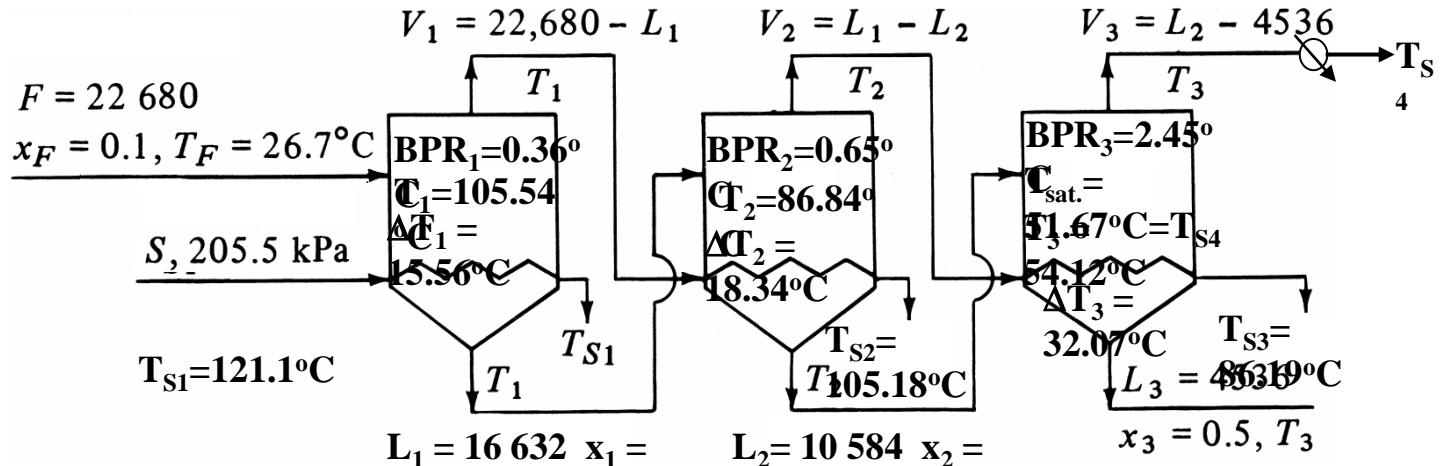
$$c_{pF} = 4.19 - 2.35x_F = 4.19 - 2.35(0.1) = 3.955 \text{ kJ/kg.K}$$

$$c_{pL1} = 4.19 - 2.35x_1 = 4.19 - 2.35(0.136) = 3.869 \text{ kJ/kg.K}$$

$$c_{pL2} = 4.19 - 2.35x_2 = 4.19 - 2.35(0.214) = 3.684 \text{ kJ/kg.K}$$

$$c_{pL3} = 4.19 - 2.35x_3 = 4.19 - 2.35(0.5) = 3.015 \text{ kJ/kg.K}$$

Example 8.5-1



$$c_{pF} = 3.955 \text{ kJ/kg.K} \quad c_{pL1} = 3.869 \text{ kJ/kg.K} \quad c_{pL2} = 3.684 \text{ kJ/kg.K} \quad c_{pL3} = 3.015 \text{ kJ/kg.K}$$

$$h = c_p(T - T_{\text{ref.}})$$

Tref. = 0°C (same as steam table),

$$h_F = c_{pF}(T_F - T_{\text{ref.}}) = 3.955(26.7 - 0) = 105.6 \text{ kJ/kg}$$

$$h_1 = c_{pL1}(T_1 - T_{\text{ref.}}) = 3.869(105.54 - 0) = 408.3 \text{ kJ/kg}$$

$$h_2 = c_{pL2}(T_2 - T_{\text{ref.}}) = 3.684(86.84 - 0) = 319.9 \text{ kJ/kg}$$

$$h_3 = c_{pL3}(T_3 - T_{\text{ref.}}) = 3.015(54.12 - 0) = 163.17 \text{ kJ/kg}$$

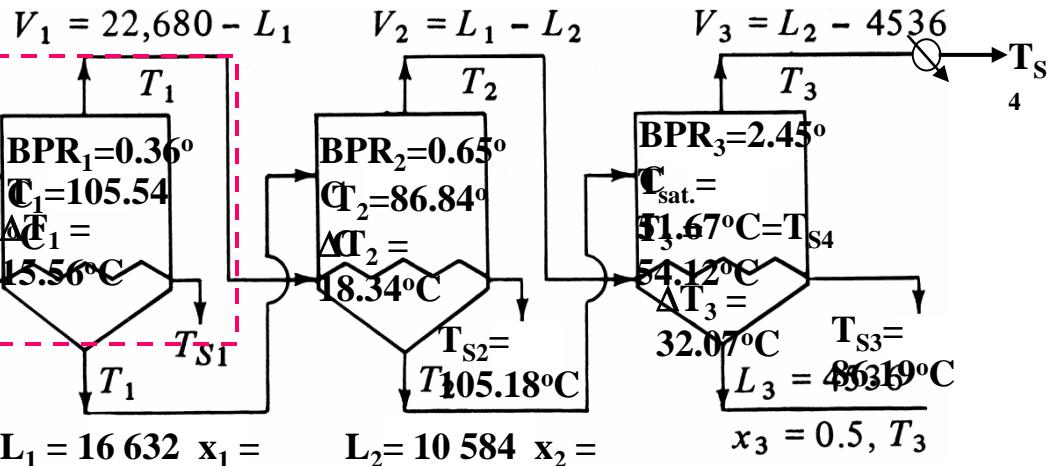
Example 8.5-1

$$F = 22,680$$

$$x_F = 0.1, T_F = 26.7^\circ\text{C}$$

$$S, 205.5 \text{ kPa}$$

$$\underline{T_{S1}=121.1^\circ\text{C}}$$



$$L_1 = 16,632 \quad x_1 =$$

$$L_2 = 10,584 \quad x_2 =$$

$$x_3 = 0.5, T_3$$

$$h_F = 105.6 \text{ kJ/kg}$$

$$h_1 = 408.3 \text{ kJ/kg}$$

$$h_2 = 319.9 \text{ kJ/kg}$$

$$h_3 = 163.17 \text{ kJ/kg}$$

Tref. = 0°C (same as steam table),

Effect 1:

$$\text{Energy balance: } Fh_F + S(H_{S1} - h_{S1}) = L_1h_1 + V_1H_1$$

$$22680h_F + S\lambda_{S1} = L_1h_1 + V_1H_1$$

$$\text{Material balance: } 22680 = V_1 + L_1$$

$$V_1 = 22680 - L_1$$

$$22680h_F + S\lambda_{S1} = L_1h_1 + (22680 - L_1)H_1$$

$$22680(105.6) + S\lambda_{S1} = L_1(408.3) + (22680 - L_1)H_1$$

From the steam table (A.2-9) at $T_{S1}=121.1^\circ\text{C}$ after interpolation, $H_{S1} = 2708 \text{ kJ/kg}$ and $h_{S1} = 508 \text{ kJ/kg}$. $\therefore \lambda_{S1} = (2708 - 508) = 2200 \text{ kJ/kg}$

Example 8.5-1

$$F = 22,680$$

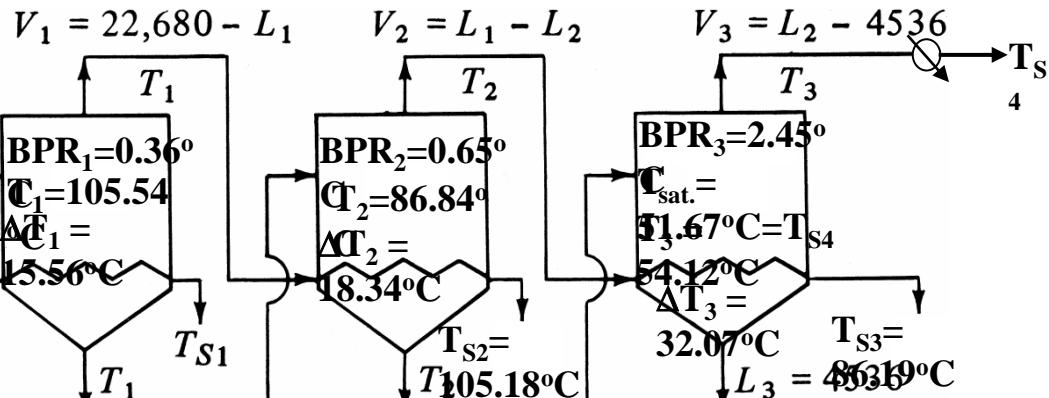
$$x_F = 0.1, T_F = 26.7^\circ\text{C}$$

$S, 205.5 \text{ kPa}$

$$\underline{T_{S1}=121.1^\circ\text{C}}$$

$$\lambda_{S1} = 2200 \text{ kJ/kg}$$

$$L_1 = 16,632 \quad x_1 =$$



$$h_F = 105.6 \text{ kJ/kg}$$

$$h_1 = 408.3 \text{ kJ/kg}$$

$$L_2 = 10,584 \quad x_2 =$$

$$h_2 = 319.9 \text{ kJ/kg}$$

$$h_3 = 163.17 \text{ kJ/kg}$$

Tref. = 0°C (same as steam table),

Effect 1: $22680(105.6) + S\lambda_{S1} = L_1(408.3) + (22680-L_1)H_1$

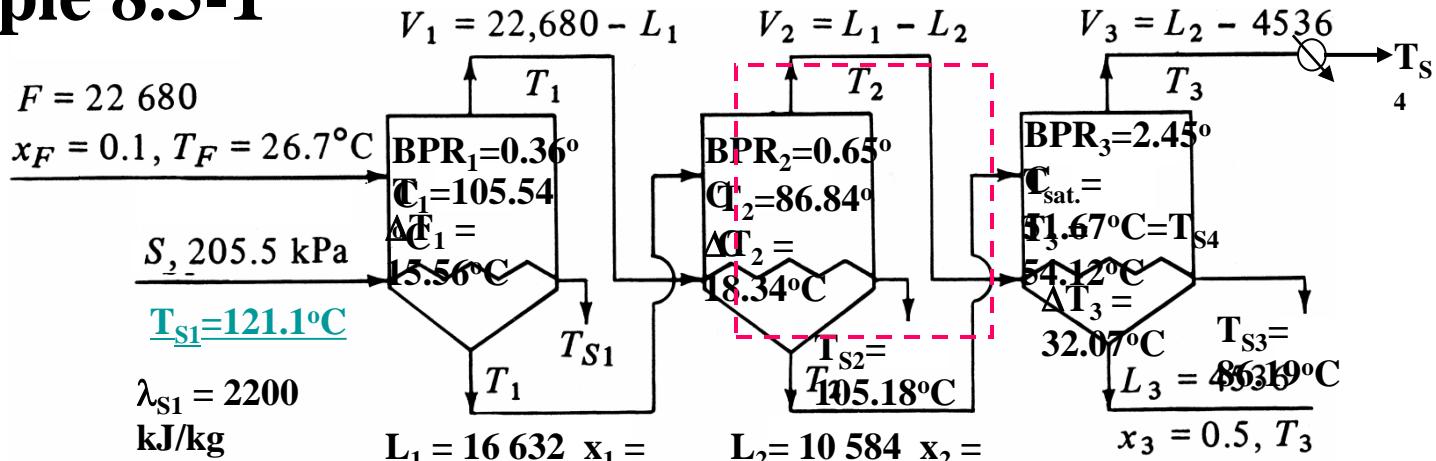
$$H_1 = H_{S2} + 1.884(\text{BPR}_1)$$

From the steam table A.2-9 at $T_{S2}=105.18^\circ\text{C}$, $H_{S2} = 2684$ and $h_{S2} = 441 \text{ kJ/kg}$

$$H_1 = 2684 + 1.884(0.36) = 2685 \text{ kJ/kg}$$

$$22680(105.6) + S(2200) = L_1(408.3) + (22680-L_1)2685 \quad (1)$$

Example 8.5-1



$$\begin{aligned}
 h_F &= 105.6 \text{ kJ/kg} & h_1 &= 408.3 \text{ kJ/kg} & h_2 &= 319.9 \text{ kJ/kg} & h_3 &= 163.17 \text{ kJ/kg} \\
 h_{S2} &= 441 \text{ kJ/kg} & H_1 &= 2685 \text{ kJ/kg}
 \end{aligned}$$

Tref. = 0°C (same as steam table),

$$\text{Effect 2: } L_1 h_1 + V_1 (H_1 - h_{S2}) = L_2 h_2 + V_2 H_2$$

$$L_1 h_1 + (22680 - L_1)(H_1 - h_{S2}) = L_2 h_2 + (L_1 - L_2)H_2$$

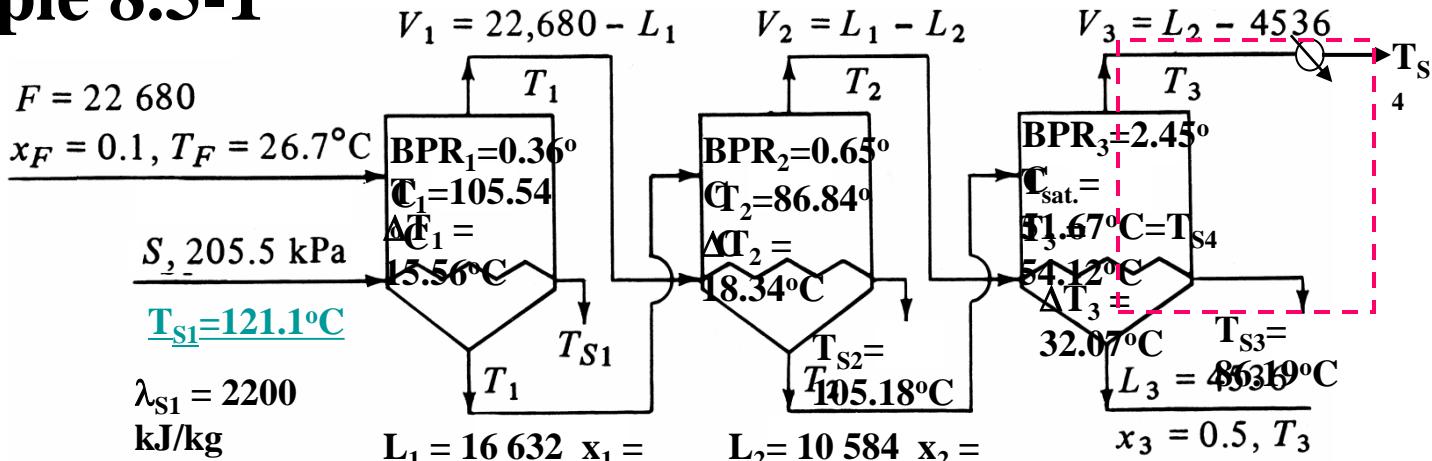
$$H_2 = H_{S3} + 1.884(BPR_2)$$

From the steam table A.2-9 at $T_{S3} = 86.19^\circ\text{C}$, $H_{S3} = 2654$ and $h_{S3} = 361 \text{ kJ/kg}$

$$H_2 = 2654 + 1.884(0.65) = 2655 \text{ kJ/kg}$$

$$\therefore L_1(408.3) + (22680 - L_1)(2685 - 441) = L_2(319.9) + (L_1 - L_2)(2655) \quad (2)$$

Example 8.5-1



$$h_F = 105.6 \text{ kJ/kg} \quad h_1 = 408.3 \text{ kJ/kg} \quad h_2 = 319.9 \text{ kJ/kg} \quad h_3 = 163.17 \text{ kJ/kg}$$

$$h_{S2} = 441 \text{ kJ/kg} \quad H_1 = 2685 \text{ kJ/kg} \quad H_2 = 2655 \text{ kJ/kg} \quad h_{S3} = 361 \text{ kJ/kg}$$

Tref. = 0°C (same as steam table),

$$\text{Effect 3: } L_2 h_2 + V_2 (H_2 - h_{S3}) = L_3 h_3 + V_3 H_3$$

$$L_2 h_2 + (L_1 - L_2)(H_2 - h_{S3}) = L_3 h_3 + (L_2 - 4536)H_3$$

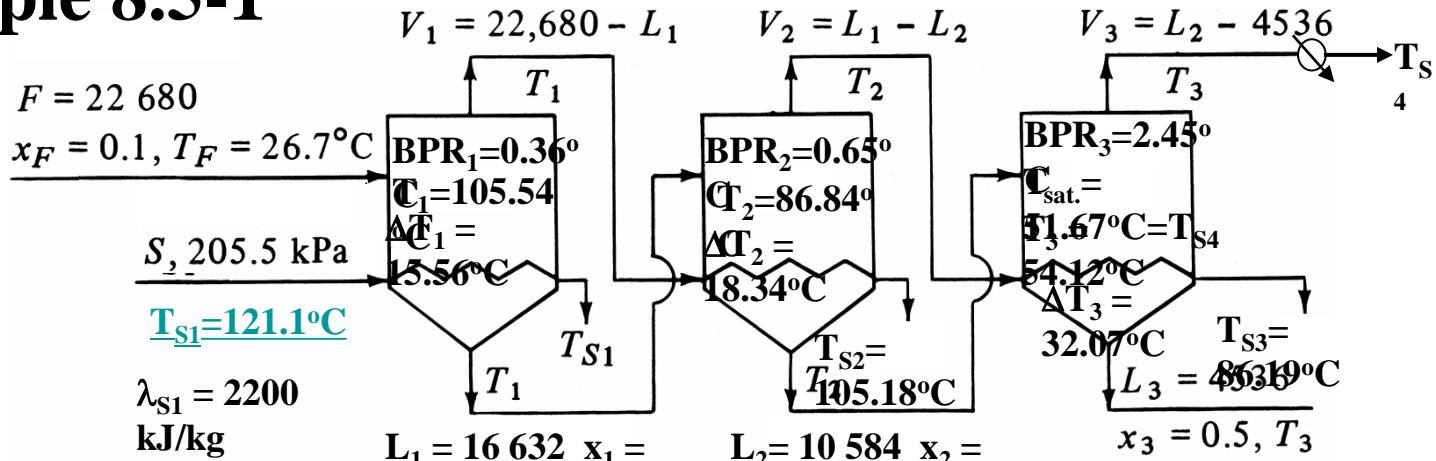
$$H_3 = H_{S4} + 1.884(BPR_3)$$

From the steam table A.2-9 at $T_{S4} = 51.67^\circ\text{C}$, $H_{S4} = 2595$

$$H_3 = 2595 + 1.884(2.45) = 2600 \text{ kJ/kg}$$

$$\therefore L_2(319.9) + (L_1 - L_2)(2655 - 361) = L_3(163.17) + (L_2 - 4536)(2600) \quad (3)$$

Example 8.5-1



$$22680(105.6) + S(2200) = L_1(408.3) + (22680 - L_1)2685 \quad (1)$$

$$L_1(408.3) + (22680 - L_1)(2685 - 441) = L_2(319.9) + (L_1 - L_2)(2655) \quad (2)$$

$$L_2(319.9) + (L_1 - L_2)(2655 - 361) = L_3(163.17) + (L_2 - 4536)(2600) \quad (3)$$

Solving simultaneously eq. (2) and (3):

$$L_1 = 17078 \text{ kg/h and } L_2 = 11068 \text{ kg/h}$$

From eq. (1) after substituting $L_1 = 17078 \text{ kg/h}$ and $L_2 = 11068 \text{ kg/h}$, $S = 8936 \text{ kg/h}$

Substituting L_1 & L_2 into the material balances of each effect:

$$V_1 = 22680 - L_1 = 22680 - 17078 = 5602 \text{ kg/h}$$

$$V_2 = L_1 - L_2 = 17078 - 11068 = 6010 \text{ kg/h}$$

$$V_3 = L_2 - 4536 = 11068 - 4536 = 6532 \text{ kg/h}$$

Example 8.5-1

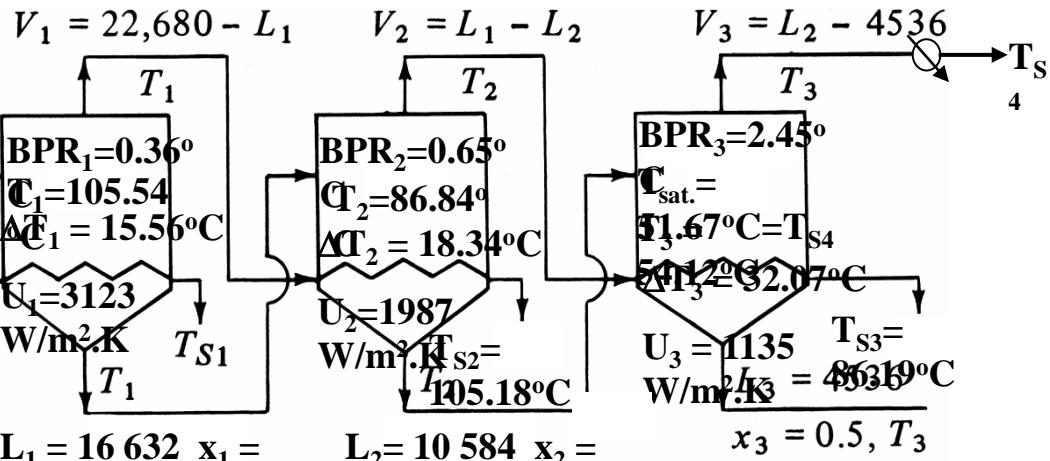
$$F = 22,680$$

$$x_F = 0.1, T_F = 26.7^\circ\text{C}$$

$$S, 205.5 \text{ kPa}$$

$$\underline{T_{S1} = 121.1^\circ\text{C}}$$

$$\lambda_{S1} = 2200 \text{ kJ/kg}$$



Latest calculations: $L_1 = 17078 \text{ kg/h}$, $L_2 = 11068 \text{ kg/h}$, $S = 8936 \text{ kg/h}$, $V_1 = 5602 \text{ kg/h}$, $V_2 = 6010 \text{ kg/h}$ and $V_3 = 6532 \text{ kg/h}$

Previous calculations: $L_1 = 16632 \text{ kg/h}$ (% error = 2.6), $L_2 = 10584 \text{ kg/h}$ (% error = 4.4), $V_1 = 6048 \text{ kg/h}$ (% error = 8.0), $V_2 = 6048 \text{ kg/h}$ (% error = 0.6) and $V_3 = 6048 \text{ kg/h}$ (% error = 7.4)

9. If errors < 10%, calculate $q_1, q_2, q_3, A_1, A_2, A_3$ & A_{ave} . If errors > 10%, revise ΔTs .

$$q_1 = S\lambda_{S1} = U_1 A_1 \Delta T_1$$

$$q_1 = S\lambda_{S1} = \frac{8936 \text{ kg}}{\cancel{1 \text{ h}}} \left| \frac{2200 \text{ kJ}}{\cancel{1 \text{ kg}}} \right| \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 5460 \text{ kJ/s}$$

$$A_1 = \frac{5460 \times 10^3 \text{ W}}{\cancel{15.56 \text{ K}}} \left| \frac{\text{m}^2}{\cancel{3123 \text{ W}}} \right| = 112.4 \text{ m}^2$$

Example 8.5-1

$$F = 22,680$$

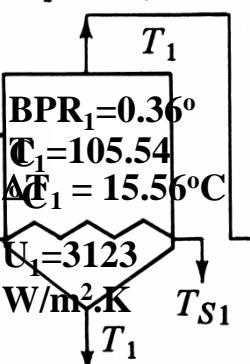
$$x_F = 0.1, T_F = 26.7^\circ\text{C}$$

$$S, 205.5 \text{ kPa}$$

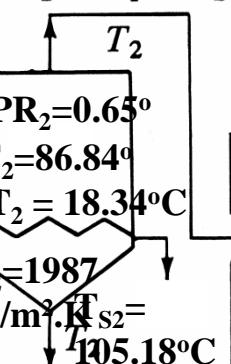
$$\underline{T_{S1} = 121.1^\circ\text{C}}$$

$$\lambda_{S1} = 2200 \text{ kJ/kg}$$

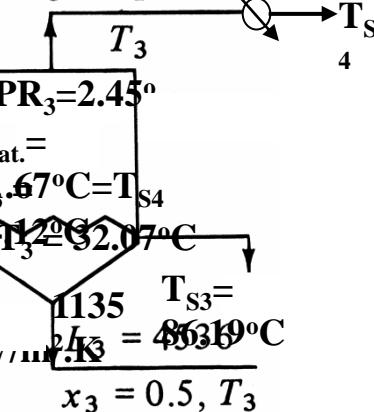
$$V_1 = 22,680 - L_1$$



$$V_2 = L_1 - L_2$$



$$V_3 = L_2 - 4536$$



Latest calculations: $L_1 = 17078 \text{ kg/h}$, $L_2 = 11068 \text{ kg/h}$, $S = 8936 \text{ kg/h}$, $V_1 = 5602 \text{ kg/h}$, $V_2 = 6010 \text{ kg/h}$ and $V_3 = 6532 \text{ kg/h}$

$$q_1 = 5460 \text{ kJ/s}$$

$$A_1 = 112.4 \text{ m}^2$$

$$A_{\text{ave.}} = A_m = 104.4 \text{ m}^2$$

$$q_2 = V_1 \lambda_{S2} = \frac{5602 \text{ kg}}{\cancel{h}} \left| \frac{2244 \text{ kJ}}{\cancel{kg}} \right| \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 3492 \text{ kJ/s}$$

$$A_{\text{ave.}} = A_m = 104.4 \text{ m}^2$$

$$A_2 = \frac{3492 \times 10^3 \text{ W}}{\cancel{18.34 \text{ K}}} \left| \frac{\text{m}^2 \text{ K}}{\cancel{1987 \text{ W}}} \right| = 95.8 \text{ m}^2$$

$$q_3 = V_2 \lambda_{S3} = \frac{6010 \text{ kg}}{\cancel{h}} \left| \frac{2294 \text{ kJ}}{\cancel{kg}} \right| \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 3830 \text{ kJ/s}$$

$$A_3 = \frac{3830 \times 10^3 \text{ W}}{\cancel{32.07 \text{ K}}} \left| \frac{\text{m}^2 \text{ K}}{\cancel{1135 \text{ W}}} \right| = 105.1 \text{ m}^2$$

Example 8.5-1

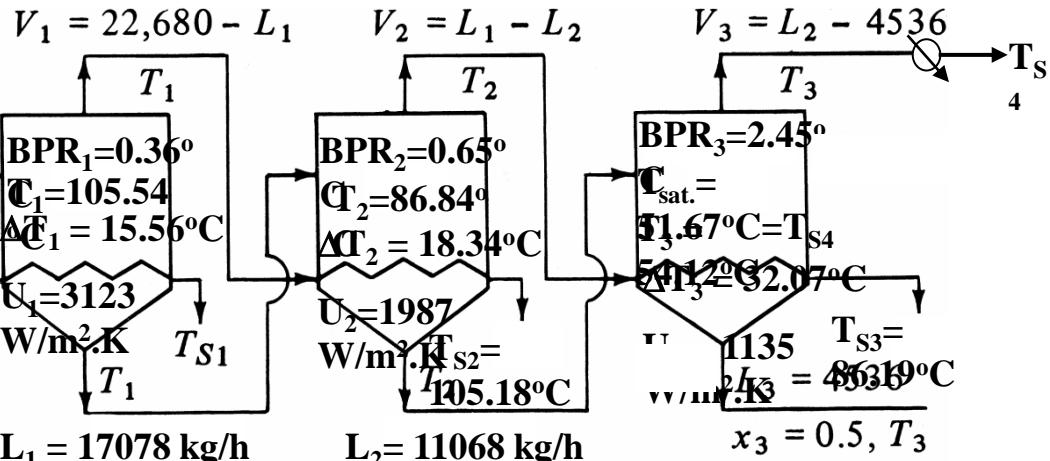
$$F = 22,680$$

$$x_F = 0.1, T_F = 26.7^\circ\text{C}$$

$$S, 205.5 \text{ kPa}$$

$$\underline{T_{S1}=121.1^\circ\text{C}}$$

$$\lambda_{S1} = 2200 \text{ kJ/kg}$$



Latest calculations: $L_1 = 17078 \text{ kg/h}$, $L_2 = 11068 \text{ kg/h}$, $S = 8936 \text{ kg/h}$, $V_1 = 5602 \text{ kg/h}$, $V_2 = 6010 \text{ kg/h}$ and $V_3 = 6532 \text{ kg/h}$

9. If % errors for $A_1, A_2, A_3 > 10\%$, revise ΔTs

$$q_1 = 5460 \text{ kJ/s} \quad A_1 = 112.4 \text{ m}^2 \quad (\% \text{ error} = 7.7)$$

$$q_2 = 3492 \text{ kJ/s} \quad A_2 = 95.8 \text{ m}^2 \quad (\% \text{ error} = 8.2)$$

$$q_3 = 3830 \text{ kJ/s} \quad A_3 = 105.1 \text{ m}^2 \quad (\% \text{ error} = 0.7)$$

$$A_{ave.} = A_m = 104.4 \text{ m}^2$$

$$\therefore A = 104.4 \text{ m}^2$$