Chapter 8

Entropy

A Measure of Disorder or Randomness

Entropy Definition and Change

DEFINE A PROPERTY $\boxed{S}$ ENTROPY

$$\int_{1\,\text{rev}}^{2} \frac{\delta Q}{T} = S_2 - S_1$$

$$\int_{2\,\text{irrev}}^{1} \frac{\delta Q}{T} + S_2 - S_1 \leq 0$$

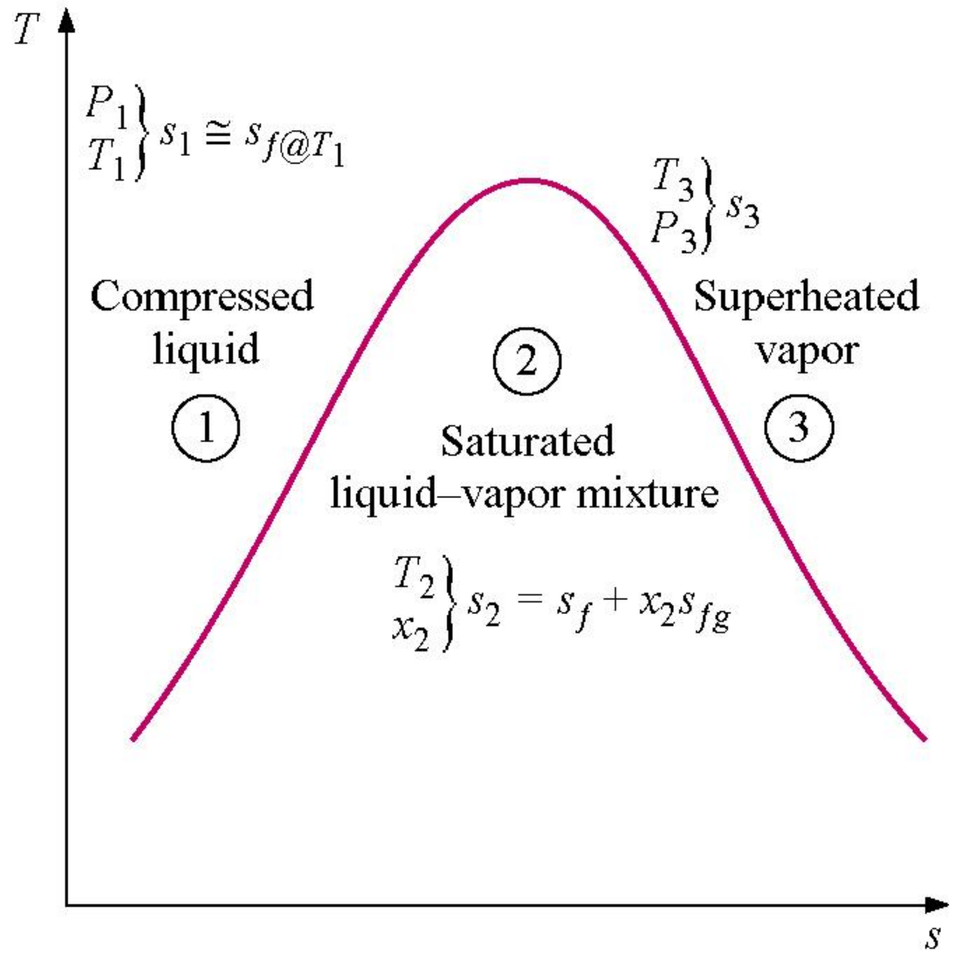
$$\int_{2\,\text{irrev}}^{1} \frac{\delta Q}{T} + ds \leq 0$$

$$ds \geq \int_{2\,\text{irrev}}^{1} \frac{\delta Q}{T}$$

$$ds \geq \int_{\text{irrev}} \frac{\delta Q}{T} \qquad ds = \int_{\text{rev}} \frac{\delta Q}{T}$$

$$\boxed{\delta Q = \int T ds}$$

The temperature-entropy diagrams for water



Entropy for a Pure Substance

Values of specific entropy are given in tables of thermodynamic properties.

The tables are the same as for u, h and v

$$s = (1 - x)s_f + s_g$$

$$OR$$

$$s = s_f + xs_{fg}$$

Example 8-1

Find the entropy and/or temperature of steam at the following states:

P	*T*	Region	*s* kJ/(kg K)
5 MPa	120°C		
1 MPa	50°C		
1.8 MPa	400°C		
40 kPa		Quality, *x* = 0.9	
40 kPa			7.1794

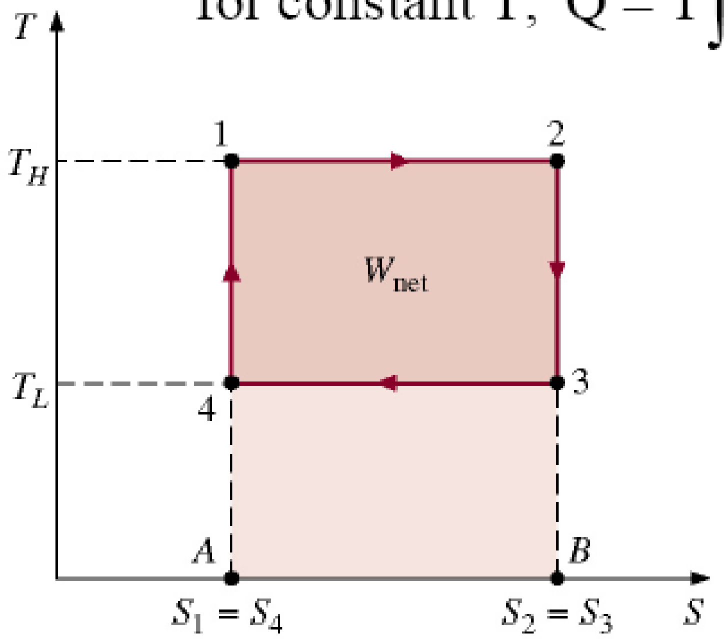
Answer to Example 8-1

P	T	Region	s kJ/kg·K
5 MPa	120°C	Compressed Liquid and in the table	1.5233
1 MPa	50°C	Compressed liquid but not in the table	$s = s_f$ at 50°C = 0.7038
1.8 MPa	400°C	Superheated	7.1794
40 kPa	$T=T_{sat}$ =75.87°C	Quality, $x = 0.9$ Saturated mixture	$s = s_f + x\, s_{fg}$ = 7.0056
40 kPa	$T=T_{sat}$ =75.87°C	$s_f<s<s_g$ at P Saturated mixture $X = (s-s_f)/s_{fg}$ = 0.9262	7.1794

Temperature Entropy Property Diagrams

$$dS = \int \frac{\delta Q}{T}$$

$$\text{for constant T,} \quad Q = T\int dS = T_{constant}\Delta S$$



$$Q_{in} = T_H \Delta S$$
$$Q_{out} = T_L \Delta S$$
$$W_{net} = Q_{in} - Q_{out} = (T_H - T_L)\Delta S$$

The Thermodynamic Property Relation

From First Law for a closed system,

$$\partial Q - \partial W = dU + dKE + dPE$$

$$\partial Q - \partial W = dU$$

For reversible process,

$$\partial Q_{rev} = dU + \partial W_{rev}$$

$$TdS = dU + PdV$$

$$dU = TdS - PdV \qquad (8.5)$$

$$du = Tds - PdV \quad (8.5)$$

From definition of enthalpy,

$$H = U + PV$$

Differential,

$$dH = dU + PdV + VdP$$

Replaced into Eq(8.5),

$$TdS = dH - VdP$$

OR

$$\boxed{dH = TdS + VdP}$$

Entropy Change for Solid and Liquid

$$du = Tds - Pdv$$

$$dh = Tds + vdP$$

For liquid and solid, v is almost negligible; dv≈0 for all process. If dP is small,

$$du = Tds$$

$$dh = Tds$$

From Eq.(5.17) for solid and liquid,

$$dh \approx du \approx CdT \quad \text{(no phase change)}$$

And $\quad Cp \approx Cv \approx C$

- So,

$$ds \approx C\,\frac{dT}{T}$$

- If C is constant,

$$\Delta s = s_2 - s_1 = C\ln\frac{T_2}{T_1} \qquad 8.20$$

Entropy Change for Ideal Gas

- From du = Tds – Pdv
- For ideal gas,

$$C_{vo}dT = Tds - \frac{RT}{v}dv$$

$$ds = C_{vo}\frac{dT}{T} + R\frac{dv}{v}$$

$$\Delta s = \int C_{vo}\frac{dT}{T} + R\ln\frac{v_2}{v_1} \qquad 8.22$$

$$\Delta \mathrm{s} = \mathrm{C_{vo}}\ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

- **From** **dh = Tds +vdP**
- **For ideal gas,**

$$C_{po}dT = Tds + RT\,\frac{dP}{P}$$

$$ds = C_{po}\,\frac{dT}{T} - R\,\frac{dP}{P}$$

$$\boxed{\begin{aligned}\Delta s &= \int C_{po}\,\frac{dT}{T} - R\ln\frac{P_2}{P_1}\\ \Delta s &= C_{po}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\end{aligned}}$$

Reversible, adiabatic process (isentropic)

For ideal gas which is isentropic ($ds = 0$)

$du = Tds - Pdv$

$du = -Pdv$

$k = C_p/C_v$ $\qquad$ $C_p = C_v + R$

$\boxed{C_v = R/k-1}$ and $\boxed{C_p = kR/k-1}$

$$\boxed{T_2/T_1 = (P_2/P_1)^{(k-1)/k} = (v_1/v_2)^{k-1}}$$

Entropy Generation

$$dS = \frac{\partial Q}{T} + \partial S_{gen} \qquad (8.11)$$

where $\partial \mathrm{S}_{\mathrm{gen}} \geq 0$

$\partial \mathbf{S}_{\mathbf{gen}}$ is a entropy generation due to irreversibility of the process. It is a path function.

$\delta \mathrm{S}_{gen} = 0$	For reversible process
$\delta \mathrm{S}_{\mathrm{gen}} \rangle 0$	For irreversible process

From Eq. (8.11), the conclusion can be drawn,

dS_{sys}	Reason
Increased (+)	**1. Heat is transferred into the system** **2. Irreversible process**
Decreased (-)	**Heat is transferred from the system**
Unchanged (0)	**Adiabatic and reversible process**

Entropy equation

$$dS = \frac{\partial Q}{T} + \partial S_{gen} \qquad (8.11)$$

$$\text{where } \partial S_{gen} \geq 0$$

$$s_2 - s_1 = \frac{{}_1Q_2}{T_o} + {}_1S_2$$

Normally we need to calculate ${}_1S_2$ (Sgen)