2^{nd} Law Analysis Law Analysis for a Control Volume

Chapter 9

Entropy Balance

From Eq(8.11) for a closed system,

Change in the system = Inlet flow – outlet flow + Generation

$$dS = \frac{\partial Q}{T} + \partial S_{gen} \qquad (8.11)$$

For a closed system, entropy inlet and outlet flow is caused by heat transfer.

Eq.(8.11) can be written in a form of,

$$\frac{dS_{c.m}}{dt} = \frac{\partial Q}{T \partial t} + \frac{\partial S_{gen}}{\partial t} \qquad (8.45)$$

$$\frac{dS_{c.m}}{dt} = \sum \frac{\dot{Q}}{T} + \dot{S}_{gen} \qquad (8.46 \,\&\, 9.1)$$

For open system (Refer to Figure 9.1)

Change in the system = Inlet flow – Outlet flow + Generation

For open system, entropy inlet and outlet flow is from heat transfer and mass transfer.

$$\frac{dS_{c.v}}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{c.v}}{T} + \dot{S}_{gen}$$

OR

$$\frac{dS_{c.v}}{dt} \geq \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{c.v}}{T} \qquad (9.5)$$

Steady State, Steady Flow Process (SSSF)

- **From 2nd Law control volume equation,**

$$\frac{dS_{c.v}}{dt} = 0$$

$$0 = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{c.v}}{T} + \dot{S}_{gen}$$

$$\sum \dot{m}_e s_e - \sum \dot{m}_i s_i = +\sum \frac{\dot{Q}_{c.v}}{T} + \dot{S}_{gen} \qquad (9.7)$$

If steady inlet, steady outlet (SISO)

$$\dot{m}(s_e - s_i) = \sum \frac{\dot{Q}_{c.v}}{T} + \dot{S}_{gen} \qquad (9.8)$$

Uniform State, Uniform Flow Process (USUF)

- **From Equation (9.5),**

$$\frac{d(ms)_{c.v}}{dt} \geq \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{c.v}}{T}$$

Integrate over the time interval, t

$$(m_2 s_2 - m_1 s_1)_{c.v} + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i \geq \sum \frac{\dot{Q}_{c.v}}{T} dt \qquad (9.12)$$

$$(m_2 s_2 - m_1 s_1)_{c.v} + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i \geq \int_0^t \frac{\dot{Q}_{c.v}}{T} dt + {}_1S_{2\,gen} \qquad (9.13)$$

The Reversible SSSF Process

- **1st Law for SSSF and SISO,**

$$q - w = h_e - h_i + \frac{1}{2}(v_e^2 - v_i^2) + g(z_e - z_i) \qquad (6.13)$$

Second Law for SSSF and SISO

$$\dot{m}(s_e - s_i) = \sum \frac{\dot{Q}_{c.v}}{T} + \dot{S}_{gen} \qquad (9.8)$$

Applications on The Reversible, Adiabatic Process for SSSF and SISO

- **For adiabatic , q = 0**
- **Reversible process, $S_{gen} = 0$**
- **Entropy balance from Eq(9.8) becomes:**

$$s_e - s_i = 0 = ds$$

- **From the fundamental property relation, Tds = dh –vdP**

$$dh = vdP$$

$$h_e - h_i = \int_i^e vdP$$

- **Replaced into the Energy balance (6.13)**

$$-w = \int_i^e v\,dP + \frac{1}{2}(v_e^2 - v_i^2) + g(z_e - z_i)$$

$$w = -\int_i^e v\,dP + \frac{1}{2}(v_e^2 - v_i^2) + g(z_e - z_i) \qquad (9.15)$$

Applications on The Reversible, Isothermal Process for SSSF and SISO

- **Reversible, S_{gen} =0**
- **Entropy balance from Eq(9.8) becomes;**

$$\dot{m}(s_e - s_i) = \frac{\dot{Q}_{c.v}}{T}$$

$$T(s_e - s_i) = \frac{\dot{Q}_{c.v}}{\dot{m}} = q$$

- **From fundamental relation property, Tds = dh –vdP**

$$T(s_e - s_i) = (h_e - h_i) - \int_i^e v dP$$

$$(h_e - h_i) = q + \int_i^e v dP$$

Replaced into Energy balance (6.13)

$$w = -\int_i^e v dP + \frac{1}{2}(v_i^2 - v_e^2) + g(z_i - z_e) \qquad (9.15)$$

- **Equation (9.15) is valid for SSSF, reversible process either adiabatic or isothermal.**

Efficiency

- **Thermal efficiency for heat engine,**

$$\eta_{thermal} = \frac{W_{net}}{Q_H}$$

- **Turbine efficiency,**

$$\eta_{turbine} = \frac{\mathrm{W}}{\mathrm{W_s}} = \frac{h_i - h_e}{h_i - h_{es}}$$

- **Turbine efficiency is usually 0.7-0.88**
- **Ideal process is a reversible, isentropic process**

- **Compressor efficiency**

$$\eta_{compressor} = \frac{\text{Work needed if ideal process}}{\text{Work done in real process}}$$

$$\eta_{compressor} = \frac{W_s}{W} = \frac{(h_i - h_{es})}{h_i - h_e}$$

- **The typical compressor efficiency is about 0.7 -0.88.**
- **Compressor will give a better efficiency if the ideal process is considered a reversible isothermal process. It can be done by using ‘inter coolers’.**